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Dimension-six CP-violating operators of the third-family quarks and their effects at colliders

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We list all possible dimension-six CP-violating $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$-invariant operators involving the third-family quarks, which can be generated by new physics at a higher energy scale. The expressions of these operators after electroweak symmetry breaking and the induced effective couplings $Wt\overline{b}$, $Xb\overline{b}$, and $Xt\overline{t}$ $(X=Z,\gamma,g,H)$ are also presented. We evaluate sample contributions of these operators to CP-odd asymmetries of transverse polarization of the top quark in single top quark production at the upgraded Fermilab Tevatron, the same polarization effect in top-quark–top-antiquark pair production at the NLC, and the CP-odd observables of momentum correlations among the top quark decay products at the NLC. The energy and luminosity sensitivity in probing this CP-violating new physics is also studied. [S0556-2821(97)03921-0]

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I. INTRODUCTION

As is well known, for more than 30 years after the discovery of the CP-violating decays of the K_L^0 meson [1], the origin of this phenomenon remains a mystery. The standard model (SM) gives a natural explanation for this phenomenon assuming the existence of a phase in the Kobayashi-Maskawa mixing matrix [2]. In models beyond the SM, additional CP-violating effects can appear rather naturally and such nonstandard CP violations are necessary in order to account for baryogenesis [3]. In Ref. [4], possible effects of non-SM CP-violating interactions have been studied in detail in the form of momentum space representation and involving only weak bosons. In this paper we will focus on CP-violation effects in the model-independent effective Lagrangian approach. So we assume that the new physics terms in Eq. (1) contain both CP-conserving and CP-violating operators.

It has been shown [5] that the Kobayashi-Maskawa (KM) mechanism of CP violation predicts a negligibly small effect for the top quark in the SM, and thus the standard CP-violation effects in top quark production and decays will be unobservable in collider experiments. Therefore, the top quark system will be sensitive to new source of CP violation and may serve as a powerful probe to nonstandard CP violation in association with new physics effects. Nonstandard CP violation in the top quark system as predicted by various new physics models and the strategy for observing these effects have been studied by many authors [6–14]. In this work we initiate a model-independent study of possible dimension-six CP-violating operators in the effective Lagrangian approach to new physics, which involve the third-family quarks and are invariant under the SM transformation. The effects of these operators can be studied at future linear and hadron colliders, and thus their strengths can be constrained. We will evaluate the effects of some of these CP-violating operators possibly measurable at the Fermilab Tevatron and the Next Linear Collider (NLC). Any nonzero value of these CP asymmetries will suggest the existence of new physics as well as new CP-violation effects.

The motivation of new physics and the use of the effective Lagrangian approach have been discussed widely. We refer to, for example, [15] for a brief discussion. Before the electroweak symmetry breaking, we can write the effective Lagrangian as

$$\mathcal{L}_{\mathrm{eff}}=\mathcal{L}_0+\frac{1}{\Lambda^2}\sum_i C_iO_i+O\left(\frac{1}{\Lambda^4}\right), \tag{1}$$

where $\mathcal{L}_0$ is the SM Lagrangian, Λ is the new physics scale, and O_i are $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$-invariant dimension-six operators, and C_i are constants which represent the coupling strengths of O_i. The expansion in Eq. (1) was first discussed in Ref. [16]. Recently, many authors further classified such CP-conserving operators and analyzed their phenomenological implications at current and future colliders [15,17–19].

This paper is organized as follows. In Sec. II we list all possible dimension-six CP-violating $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$-invariant operators. The expressions of these operators after electroweak gauge symmetry breaking are given in the Appendix. In Sec. III we give the induced CP-violating effective couplings $Wt\overline{b}$, $Xb\overline{b}$, and $Xt\overline{t}$ $(X=Z,\gamma,g,H)$. In Sec. IV we evaluate the contributions to some CP-odd quantities at the Tevatron and the NLC. And finally in Sec. V we present the summary.

II. DIMENSION-SIX CP-VIOLATING GAUGE-INVARIANT OPERATORS

The remarkably heavy top quark indicates a significant coupling to the symmetry-breaking sector which may open a new window to physics beyond the standard model. Therefore it is likely that new physics effects can be more readily reveled in processes involving the top quark. Here we assume that the new physics in the quark sector resides in the

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third-quark family. Although new physics can give rise to four-quark operators involving only the third family, such operators are not experimentally relevant here. New physics may also occur in the gauge boson and Higgs sectors, they are not, however, our attention here. Therefore, the operators we are interested in are those containing third-family quarks coupling to gauge and Higgs bosons.

To restrict ourselves to the lowest order, we consider only tree diagrams and to the order of $1/\Lambda^2$. Therefore, only one vertex in a given diagram can contain anomalous couplings. Under these conditions, operators which are allowed to be related by the field equations are not independent. As discussed in [15], to which we refer for the details, the fermion and the Higgs boson equations of motion can be used but the equations of motion of the gauge bosons cannot when writing down the operators in Eq. (1).

We assume all the operators O_i to be Hermitian. Because of our assumption that the available energies are below the unitarity cuts of new-physics particles, no imaginary part can be generated by the new physics effect. Therefore the coefficients C_i in Eq. (1) are real.

The expressions of the CP-violating operators are parallel to their corresponding CP-conserving ones given in [15] [Eqs. (2)–(23), [15]], and a similar classification of the operators can be made. We list all possible dimension-six CP-odd $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$-invariant operators involving third-family quarks but no four-fermion interactions. We follow the standard notation: q_L denotes the third-family left-handed doublet quarks, Φ and $\widetilde{\Phi}$ are the Higgs field and its equivalent complex conjugate representation, $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$ are the SU(3), SU(2), and U(1) gauge boson field tensors in the appropriate matrix forms, and D_μ denotes the appropriate covariant derivatives. For more details of the notation we refer to [15].

(1) Class 1 (contains a t_R field)

$$\overline{O_{t1}}=i\left(\Phi^\dagger\Phi-\frac{v^2}{2}\right)[\overline{q}_L t_R\widetilde{\Phi}-\widetilde{\Phi}^\dagger\,\overline{t}_R q_L], \tag{2}$$

$$\overline{O_{t2}}=[\Phi^\dagger D_\mu\Phi+(D_\mu\Phi)^\dagger\Phi]\,\overline{t}_R\gamma^\mu t_R, \tag{3}$$

$$\overline{O_{t3}}=(\widetilde{\Phi}^\dagger D_\mu\Phi)(\overline{t}_R\gamma^\mu b_R)+(D_\mu\Phi)^\dagger\widetilde{\Phi}(\overline{b}_R\gamma^\mu t_R), \tag{4}$$

$$\overline{O_{Dt}}=i[(\overline{q}_L D_\mu t_R)D^\mu\widetilde{\Phi}-(D^\mu\widetilde{\Phi})^\dagger(\overline{D_\mu t_R}q_L)], \tag{5}$$

$$\overline{O_{tW\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}\tau^I t_R)\widetilde{\Phi}-\widetilde{\Phi}^\dagger(\overline{t}_R\sigma^{\mu\nu}\tau^I q_L)]W^I_{\mu\nu}, \tag{6}$$

$$\overline{O_{tB\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}t_R)\widetilde{\Phi}-\widetilde{\Phi}^\dagger(\overline{t}_R\sigma^{\mu\nu}q_L)]B_{\mu\nu}, \tag{7}$$

$$\overline{O_{tG\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}T^A t_R)\widetilde{\Phi}-\widetilde{\Phi}^\dagger(\overline{t}_R\sigma^{\mu\nu}T^A q_L)]G^A_{\mu\nu}, \tag{8}$$

$$\overline{O_{tG}}=i[\overline{t}_R\gamma^\mu T^A D^\nu t_R-\overline{D^\nu t_R}\gamma^\mu T^A t_R]G^A_{\mu\nu}, \tag{9}$$

$$\overline{O_{tB}}=i[\overline{t}_R\gamma^\mu D^\nu t_R-\overline{D^\nu t_R}\gamma^\mu t_R]B_{\mu\nu}. \tag{10}$$

(2) Class 2 (contains no t_R field)

$$\overline{O_{qG}}=i[\overline{q}_L\gamma^\mu T^A D^\nu q_L-\overline{D^\nu q_L}\gamma^\mu T^A q_L]G^A_{\mu\nu}, \tag{11}$$

$$\overline{O_{qW}}=i[\overline{q}_L\gamma^\mu\tau^I D^\nu q_L-\overline{D^\nu q_L}\gamma^\mu\tau^I q_L]W^I_{\mu\nu}, \tag{12}$$

$$\overline{O_{qB}}=i[\overline{q}_L\gamma^\mu D^\nu q_L-\overline{D^\nu q_L}\gamma^\mu q_L]B_{\mu\nu}, \tag{13}$$

$$\overline{O_{bG}}=i[\overline{b}_R\gamma^\mu T^A D^\nu b_R-\overline{D^\nu b_R}\gamma^\mu T^A b_R]G^A_{\mu\nu}, \tag{14}$$

$$\overline{O_{bB}}=i[\overline{b}_R\gamma^\mu D^\nu b_R-\overline{D^\nu b_R}\gamma^\mu b_R]B_{\mu\nu}, \tag{15}$$

$$\overline{O^{(1)}_{\Phi q}}=[\Phi^\dagger D_\mu\Phi+(D_\mu\Phi)^\dagger\Phi]\overline{q}_L\gamma^\mu q_L, \tag{16}$$

$$\overline{O^{(3)}_{\Phi q}}=[\Phi^\dagger\tau^I D_\mu\Phi+(D_\mu\Phi)^\dagger\tau^I\Phi]\overline{q}_L\gamma^\mu\tau^I q_L, \tag{17}$$

$$\overline{O_{\Phi b}}=[\Phi^\dagger D_\mu\Phi+(D_\mu\Phi)^\dagger\Phi]\overline{b}_R\gamma^\mu b_R, \tag{18}$$

$$\overline{O_{b1}}=i\left(\Phi^\dagger\Phi-\frac{v^2}{2}\right)[\overline{q}_L b_R\Phi-\Phi^\dagger\overline{b}_R q_L], \tag{19}$$

$$\overline{O_{Db}}=i[(\overline{q}_L D_\mu b_R)D^\mu\Phi-(D^\mu\Phi)^\dagger(\overline{D_\mu b_R}q_L)], \tag{20}$$

$$\overline{O_{bW\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}\tau^I b_R)\Phi-\Phi^\dagger(\overline{b}_R\sigma^{\mu\nu}\tau^I q_L)]W^I_{\mu\nu}, \tag{21}$$

$$\overline{O_{bB\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}b_R)\Phi-\Phi^\dagger(\overline{b}_R\sigma^{\mu\nu}q_L)]B_{\mu\nu}, \tag{22}$$

$$\overline{O_{bG\Phi}}=i[(\overline{q}_L\sigma^{\mu\nu}T^A b_R)\Phi-\Phi^\dagger(\overline{b}_R\sigma^{\mu\nu}T^A q_L)]G^A_{\mu\nu}. \tag{23}$$

Note that in $\overline{O_{t1}}$ and $\overline{O_{b1}}$ we subtract the vacuum expectation value, $v^2/2$, from $\Phi^\dagger\Phi$, to avoid the additional mass term for the third-family quarks.

There are nine additional operators which we shall not list here. These operators are not independent of those given in Eqs. (2)–(23) upon the use of the equations of motion of the Higgs boson and fermion fields. The form of these omitted operators can be obtained from the corresponding case of CP-conserving operators treated in [15] by converting them into the corresponding CP-violating ones [see Eqs. (24)–(32), [15]] and the proof of the dependence also parallel to that given in [15].

The expressions of these CP-violating operators Eqs. (2)–(23) after electroweak symmetry breaking are presented in the Appendix. Note that most of the operators clearly show the $\mathrm{U}_{\mathrm{em}}(1)$ gauge invariance. But some of them do not manifest the electroweak gauge invariance straightforwardly, for example, $\overline{O_{Dt}}$ in Eq. (A4). We have checked that the operator gives indeed a $\mathrm{U}_{\mathrm{em}}(1)$ gauge-invariant expression.

III. EFFECTIVE LAGRANGIAN FOR SOME COUPLINGS

We consider the contribution of CP-violating operators to top quark couplings $Wt\overline{b}$, $Zt\overline{t}$, $\gamma t\overline{t}$, $Ht\overline{t}$, $gt\overline{t}$ and the bottom quark coupling $Zb\overline{b}$, $\gamma b\overline{b}$. These couplings can be meaningfully investigated at the CERN e^+e^- collider LEP, Tevatron, NLC, and the CERN Large Hadron Collider (LHC). The status of the contributions of the dimension-six CP-violating operators to these couplings are shown in Table I.

TABLE I. The contribution status of dimension-six CP-violating operators to electroweak and $gt\bar{t}$ couplings. The contribution of a CP-violating operator to a particular vertex is marked by ×.

	$Wt\bar{b}$	$Zt\bar{t}$	$\gamma t\bar{t}$	$Ht\bar{t}$	$gt\bar{t}$	$Zb\bar{b}$	$\gamma b\bar{b}$	$Hb\bar{b}$
$\overline{O}_{t1}$				×				
$\overline{O}_{t2}$				×				
$\overline{O}_{t3}$	×							
$\overline{O}_{Dt}$	×	×		×				
$\overline{O}_{tW\Phi}$	×	×	×					
$\overline{O}_{tB\Phi}$		×	×					
$\overline{O}_{tG\Phi}$					×			
$\overline{O}_{tG}$					×			
$\overline{O}_{tB}$		×	×					
$\overline{O}_{qG}$					×			
$\overline{O}_{qW}$	×	×	×			×	×	
$\overline{O}_{qB}$		×	×			×	×	
$\overline{O}_{bB}$						×	×	
$\overline{O}_{\Phi q}^{(1)}$				×				×
$\overline{O}_{\Phi q}^{(3)}$	×			×				×
$\overline{O}_{\Phi b}$								×
$\overline{O}_{b1}$								×
$\overline{O}_{Db}$	×					×		×
$\overline{O}_{bW\Phi}$	×					×	×	
$\overline{O}_{bB\Phi}$						×	×	

Collecting all the relevant terms we get the CP-violating effective couplings as

$$\begin{aligned}\tilde{\mathcal{L}}_{Wtb}=&-i\frac{C_{\Phi q}^{(3)}}{\Lambda^2}\frac{g_2}{\sqrt{2}}v^2W_\mu^+(\bar{t}\gamma^\mu P_L b)-i\frac{C_{t3}}{\Lambda^2}\frac{v^2}{2}\frac{g_2}{\sqrt{2}}W_\mu^+(\bar{t}\gamma^\mu P_R b)-i\frac{C_{Dt}}{\Lambda^2}\frac{v}{\sqrt{2}}\frac{g_2}{\sqrt{2}}W_\mu^+(i\partial^\mu\bar{t})P_L b\\&-i\frac{C_{Db}}{\Lambda^2}\frac{v}{\sqrt{2}}\frac{g_2}{\sqrt{2}}W_\mu^+\bar{t}P_R(i\partial^\mu b)-i\frac{C_{tW\Phi}}{\Lambda^2}\frac{v}{2}W_{\mu\nu}^+(\bar{t}\sigma^{\mu\nu}P_L b)+i\frac{C_{bW\Phi}}{\Lambda^2}\frac{v}{2}W_{\mu\nu}^+(\bar{t}\sigma^{\mu\nu}P_R b)\\&+i\frac{C_{qW}}{\Lambda^2}\frac{1}{\sqrt{2}}W_{\mu\nu}^+[\bar{t}\gamma^\mu P_L(\partial^\nu b)-(\partial^\nu\bar{t})\gamma^\mu P_L b]+\text{H.c.},\end{aligned}\tag{24}$$

$$\begin{aligned}\tilde{\mathcal{L}}_{Zb\bar{b}}=&i\left(\frac{C_{bW\Phi}}{\Lambda^2}\frac{c_W}{\sqrt{2}}+\frac{C_{bB\Phi}}{\Lambda^2}\frac{v}{\sqrt{2}}s_W\right)Z_{\mu\nu}(\bar{b}\sigma^{\mu\nu}\gamma_5 b)+i\left(\frac{C_{qW}}{\Lambda^2}\frac{c_W}{2}+\frac{C_{qB}}{\Lambda^2}s_W\right)Z_{\mu\nu}(\bar{b}\gamma^\mu P_L\partial^\nu b-\partial^\nu\bar{b}\gamma^\mu P_L b)\\&+i\frac{C_{bB}}{\Lambda^2}s_W Z_{\mu\nu}(\bar{b}\gamma^\mu P_R\partial^\nu b-\partial^\nu\bar{b}\gamma^\mu P_R b)-i\frac{m_Z}{2}Z^\mu\left[i(\bar{b}\gamma_5\partial_\mu b-\partial_\mu\bar{b}\gamma_5 b)\frac{C_{Db}}{\Lambda^2}+i\partial_\mu(\bar{b}b)\frac{C_{Db}}{\Lambda^2}\right],\end{aligned}\tag{25}$$

$$\begin{aligned}\tilde{\mathcal{L}}_{\gamma b\bar{b}}=&i\left(\frac{C_{qB}}{\Lambda^2}c_W-\frac{C_{qW}}{\Lambda^2}\frac{s_W}{2}\right)A_{\mu\nu}(\bar{b}\gamma^\mu P_L\partial^\nu b-\partial^\nu\bar{b}\gamma^\mu P_L b)+i\frac{C_{bB}}{\Lambda^2}c_W A_{\mu\nu}(\bar{b}\gamma^\mu P_R\partial^\nu b-\partial^\nu\bar{b}\gamma^\mu P_R b)\\&+i\left(\frac{C_{bB\Phi}}{\Lambda^2}c_W-\frac{C_{bW\Phi}}{\Lambda^2}\frac{s_W}{2}\right)\frac{v}{\sqrt{2}}A_{\mu\nu}(\bar{b}\sigma^{\mu\nu}\gamma_5 b),\end{aligned}\tag{26}$$

$$\tilde{\mathcal{L}}_{Zt\bar{t}}=i\frac{C_{Dt}}{\Lambda^2}\frac{1}{\sqrt{2}}\frac{m_Z}{2}Z^{\mu}[i\partial_{\mu}(\bar{t}t)]+i\frac{C_{Dt}}{\Lambda^2}\frac{1}{\sqrt{2}}\frac{m_Z}{2}Z^{\mu}(i\bar{t}\gamma_5\partial_{\mu}t-i\partial_{\mu}\bar{t}\gamma_5 t)+i\left(\frac{C_{tB\Phi}}{\Lambda^2}s_W-\frac{C_{tW\Phi}}{\Lambda^2}\frac{c_W}{2}\right)\frac{v}{\sqrt{2}}Z_{\mu\nu}(\bar{t}\sigma^{\mu\nu}\gamma_5 t)$$
$$+i\frac{C_{tB}}{\Lambda^2}s_W Z_{\mu\nu}(\bar{t}\gamma^{\mu}P_R\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_R t)+i\left(\frac{C_{qB}}{\Lambda^2}s_W-\frac{C_{qW}}{\Lambda^2}\frac{c_W}{2}\right)Z_{\mu\nu}(\bar{t}\gamma^{\mu}P_L\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_L t), \tag{27}$$

$$\tilde{\mathcal{L}}_{\gamma t\bar{t}}=i\left(\frac{C_{tW\Phi}}{\Lambda^2}\frac{s_W}{2}+\frac{C_{tB\Phi}}{\Lambda^2}c_W\right)\frac{v}{\sqrt{2}}A_{\mu\nu}(\bar{t}\sigma^{\mu\nu}\gamma_5 t)+i\frac{C_{tB}}{\Lambda^2}c_W A_{\mu\nu}(\bar{t}\gamma^{\mu}P_R\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_R t)$$
$$+i\left(\frac{C_{qB}}{\Lambda^2}c_W+\frac{C_{qW}}{\Lambda^2}\frac{s_W}{2}\right)A_{\mu\nu}(\bar{t}\gamma^{\mu}P_L\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_L t), \tag{28}$$

$$\tilde{\mathcal{L}}_{Ht\bar{t}}=i\frac{C_{t1}}{\Lambda^2}\frac{v^2}{\sqrt{2}}H(\bar{t}\gamma_5 t)+i\frac{C_{Dt}}{\Lambda^2}\frac{1}{2\sqrt{2}}\partial^{\mu}H[\partial_{\mu}(\bar{t}\gamma_5 t)+\bar{t}\partial_{\mu}t-(\partial_{\mu}\bar{t})t]-i\frac{C_{t2}}{\Lambda^2}v(i\partial^{\mu}H)(\bar{t}\gamma_{\mu}P_R t)$$
$$-i\left(\frac{C_{\Phi q}^{(1)}}{\Lambda^2}-\frac{C_{\Phi q}^{(3)}}{\Lambda^2}\right)v(i\partial^{\mu}H)(\bar{t}\gamma_{\mu}P_L t), \tag{29}$$

$$\tilde{\mathcal{L}}_{gt\bar{t}}=i\frac{C_{tG}}{\Lambda^2}[\bar{t}\gamma^{\mu}P_R T^A\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_R T^A t]G^A_{\mu\nu}+i\frac{C_{qG}}{\Lambda^2}[\bar{t}\gamma^{\mu}P_L T^A\partial^{\nu}t-\partial^{\nu}\bar{t}\gamma^{\mu}P_L T^A t]G^A_{\mu\nu}+i\frac{C_{tG\Phi}}{\Lambda^2}\frac{v}{\sqrt{2}}(\bar{t}\sigma^{\mu\nu}\gamma_5 T^A t)G^A_{\mu\nu}, \tag{30}$$

$$\tilde{\mathcal{L}}_{Hb\bar{b}}=-i\frac{1}{\Lambda^2}(C_{\Phi q}^{(1)}+C_{\Phi q}^{(3)})v(i\partial_{\mu}H)\bar{b}\gamma^{\mu}P_L b-i\frac{C_{\Phi b}}{\Lambda^2}v(i\partial_{\mu}H)\bar{b}\gamma^{\mu}P_R b+i\frac{C_{b1}}{\Lambda^2}\frac{v^2}{\sqrt{2}}H(\bar{b}\gamma_5 b)$$
$$+i\frac{C_{Db}}{\Lambda^2}\frac{1}{2\sqrt{2}}\partial^{\mu}H[\bar{b}\partial_{\mu}b-(\partial_{\mu}\bar{b})b+\partial_{\mu}(\bar{b}\gamma_5 b)], \tag{31}$$

where $s_W\equiv\sin\theta_W$, $c_W\equiv\cos\theta_W$ and $P_{L,R}\equiv(1\mp\gamma_5)/2$.

IV. THE CONTRIBUTIONS TO *CP*-ODD QUANTITIES OF TOP QUARK AT COLLIDERS

Various experiments have been suggested to measure CP-violating couplings of the top quark. They include CP-odd quantities such as the polarization asymmetries [7–9] and CP-odd momentum correlations among the decay products [10,11].

In this section we will evaluate the contributions of some of the CP-violating new physics operators to these CP asymmetries. By taking an individual operator as an example, we present numerical results to show at what level of C_i/Λ^2 the CP-violating effect may be visible. We will only consider the CP-odd operators listed in Sec. III and do not include their corresponding CP-even operators whose phenomenologies are different and have been systematically analyzed in [15,17–19]. Furthermore, we restrict ourselves to the electroweak vertices, i.e., Wtb, $Zt\bar{t}$, and $\gamma t\bar{t}$.

A. Transverse polarization[1] asyimmetry of top quark in single top quark production at the Tevatron

The reaction $p\bar{p}\to t\bar{b}X$ at the Tevatron can be used to investigate several different types of CP asymmetries [10]. The complicated coordinate representation of the effective Lagrangian equations (24)–(31) can be simplified in the momentum space when t and b are on shell. The CP-violating contribution to the Wtb vertex, Eqs. (24), can be written in the momentum space as

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[1] In this paper the transverse polarization direction is the one which is perpendicular to the scattering plane.

$$\begin{aligned}\tilde{\mathcal{L}}_{Wtb}=i\frac{g_2}{\sqrt{2}}W_\mu^+\bar{t}\bigg[&F_L\gamma^\mu P_L+F_R\gamma^\mu P_R-i\frac{G_L}{m_t}\sigma^{\mu\nu}k_\nu P_L\\&-i\frac{G_R}{m_t}\sigma^{\mu\nu}k_\nu P_R\bigg]b-i\frac{g_2}{\sqrt{2}}W_\mu^-\bar{b}\bigg[F_L\gamma^\mu P_L+F_R\gamma^\mu P_R\\&-i\frac{G_L}{m_t}\sigma^{\mu\nu}k_\nu P_R-i\frac{G_R}{m_t}\sigma^{\mu\nu}k_\nu P_L\bigg]t,\end{aligned}\tag{32}$$

where $P_{L,R}\equiv(1\mp\gamma_5)/2$, $k=p_t+p_{\bar{b}}$, and

$$F_L=\frac{v^2}{\Lambda^2}\left(-C_{\Phi q}^{(3)}+\frac{C_{Dt}}{2\sqrt{2}}\frac{m_t}{v}\right),\tag{33}$$

$$G_L=\frac{v^2}{\Lambda^2}\left(\frac{C_{Dt}}{2\sqrt{2}}\frac{m_t}{v}+C_{tW\Phi}\frac{\sqrt{2}}{g_2}\frac{m_t}{v}-C_{qW}\frac{1}{g_2}\frac{m_t^2}{v^2}\right),\tag{34}$$

$$F_R=-\frac{v^2}{2\Lambda^2}\left(C_{t3}+\frac{C_{Db}}{\sqrt{2}}\frac{m_t}{v}\right),\tag{35}$$

$$G_R=-\frac{v^2}{\Lambda^2}\left(\frac{C_{Db}}{2\sqrt{2}}\frac{m_t}{v}+C_{tW\Phi}\frac{\sqrt{2}}{g_2}\frac{m_t}{v}\right).\tag{36}$$

We have neglected the scalar and pseudoscalar couplings, k_μ and $k_\mu\gamma_5$, which, in the process $u\bar{d}\rightarrow W\rightarrow t\bar{b}$, give contributions proportional to the initial parton mass. It should be pointed out that in contrast to [10], where the form factors F_L, etc., can be complex, form factors in Eq. (32) are all real because $C_{\Phi q}^{(3)}$, etc., are real as noted in Sec. II above.

The spin of the top quark allows three types of CP-violating polarization asymmetries [10] in the single top quark production via

$$u+\bar{d}\rightarrow t+\bar{b},\quad \bar{u}+d\rightarrow\bar{t}+b.\tag{37}$$

Introducing the coordinate system in the top quark (or top antiquark) rest frame with the unit vectors $\vec{e}_z\propto-\vec{P}_{\bar{b}}$, $\vec{e}_y\propto\vec{P}_u\times\vec{P}_{\bar{b}}$ and $\vec{e}_x=\vec{e}_y\times\vec{e}_z$, the transverse polarization asymmetry is defined as

$$A(\hat{y})=\frac{1}{2}[\Pi(\hat{y})-\overline{\Pi}(\hat{y})],\tag{38}$$

where $\Pi(\hat{y})$ and $\overline{\Pi}(\hat{y})$ are, respectively, the polarizations of the top quark and top antiquark in the direction $\hat{y}$, arising from the interference of the SM and the CP-violating vertices. Only the terms proportional to P_L contribute. The polarizations are given by

$$\Pi(\hat{y})=\frac{N_t(+\hat{y})-N_t(-\hat{y})}{N_t(+\hat{y})+N_t(-\hat{y})},\tag{39}$$

$$\overline{\Pi}(\hat{y})=\frac{N_{\bar{t}}(+\hat{y})-N_{\bar{t}}(-\hat{y})}{N_{\bar{t}}(+\hat{y})+N_{\bar{t}}(-\hat{y})},\tag{40}$$

where $N_t(\pm\hat{y})$ $[N_{\bar{t}}(\pm\hat{y})]$ is the number of $t(\bar{t})$ quarks polarized in the direction $\pm\hat{y}$.

Let us comment briefly on the identification of the momentum directions $\vec{P}_b$, $\vec{P}_{\bar{b}}$, and $\vec{P}_u$. At the Tevatron $\vec{P}_u$ is in the direction of the incoming proton. The identification of $\vec{P}_b$ and $\vec{P}_{\bar{b}}$ are more complicated. Take the single top quark productions as an example. Assuming the W^+ in the top quark decay is identified by a positive charged lepton, the associated hadron jet with which to form the top quark will be the b quark and the accompanying jet will be the $\bar{b}$ quark. With b tagging, a better identification of the b and $\bar{b}$ direction can be made with the price of a reduced number of events to about one-quarter of the number before the b tagging. However, the sign of the b quark jet, i.e., b vs $\bar{b}$, is difficult to obtain even through b tagging.

The asymmetry $A(\hat{y})$ in the parton level is proportional to the real part of the form factor G_L, which is given by [10]

$$A(\hat{y})|_{\mathrm{parton}}=\frac{3\pi}{4}\frac{(1-x)}{(2+x)\sqrt{x}}\mathrm{Re}\quad G_L,\tag{41}$$

where $x=m_t^2/\hat{s}$. This parton level asymmetry can be converted to the hadron level asymmetry by folding in the structure functions. In the absence of an imaginary part F_L makes no contribution to polarization asymmetries.

The measured asymmetry is obtained by folding the parton level asymmetry (41) in the evolution integral. Using the CTEQ3L parton distribution functions [20] with $\mu=\sqrt{\hat{s}}$ and assuming $m_t=175$ GeV, we obtain the asymmetry as

$$A(\hat{y})=\sum_{i,j}\int A(\hat{y})|_{\mathrm{parton}}\ [f_i^A(x_1,\mu)f_j^B(x_2,\mu)+(A\leftrightarrow B)]dx_1dx_2=\begin{cases}-0.41\dfrac{C_{qW}-2C_{tW\Phi}-g_2C_{Dt}/2}{(\Lambda/1\ \mathrm{TeV})^2} & \text{at}\ \sqrt{s}=2\ \ \mathrm{TeV},\\[2ex] -0.84\dfrac{C_{qW}-2C_{tW\Phi}-g_2C_{Dt}/2}{(\Lambda/1\ \mathrm{TeV})^2} & \text{at}\ \sqrt{s}=4\ \ \mathrm{TeV}\end{cases}\tag{42}$$

where A and B denote the incident hadrons and the functions f_i^A and f_j^B are the parton distribution functions. $x_{1,2}$ denoting the longitudinal momentum fractions of the two initial partons.

As analyzed in [10], such an asymmetry of a few percent might be within the reach of experiment at the upgraded Tevatron with $\sqrt{s}=2$ TeV and an integrated luminosity 3–10 fb^{-1}. As the results in Eq. (42) show, the CP asymmetry caused by new physics will be more significant at higher energies, say $\sqrt{s}=4$ TeV. Hence, if the collider can be further upgraded to 4 TeV and/or with increased luminosity [21], it can serve as a more powerful tool for probing CP-violating new physics. It should be noted that the signal for this process is unobservable at the LHC because of the background from $t\bar{t}$ production and single top production via W-gluon fusion, which are much larger than the signal in comparison with the case of Tevatron. At the LHC the signal of single top production due to quark-antiquark annihilation is severely suppressed in comparion with the gluon-W boson fusion process. This is due the fact that the initial antiquarks $\bar{d}$ and $\bar{u}$ are from the sea, while the gluon struction function increases at LHC. More details can be found in [22].

If we assume an observable level of 10%, we see from Eq. (42) that the upgraded Tevatron will probe $(C_{qW}-2C_{tW\Phi}-g_2C_{Dt}/2)/(\Lambda/1\text{ TeV})^2$ to 1/4 and 1/8 for $\sqrt{s}=2$ TeV and $\sqrt{s}=4$ TeV, respectively. This means that with a new physics scale at the order of 1 TeV, the further upgraded Tevatron can probe the coupling strength down to the level of 0.1.

B. Transverse polarization asymmetry of top quark pair production at the NLC

From the polarizations of the top quark and top antiquark in $e^+e^-\to t\bar{t}$, one can construct CP-odd quantities which can be measured through the energy asymmetry of the charged leptons in the t and $\bar{t}$ decays as well as the up-down asymmetry of these leptons with respect to the $t\bar{t}$ production plane [7,8].

Including both the SM couplings and new physics effects, we can write the $Vt\bar{t}$ $(V=Z,\gamma)$ vertices as

$$\Gamma^{\mu}_{Vt\bar{t}}=i\frac{g}{2}\left[\gamma_\mu A_V-\gamma_\mu\gamma_5B_V+\frac{p_t^\mu-p_{\bar{t}}^\mu}{2}(C_V-iD_V\gamma_5)\right], \tag{43}$$

where p_t and $p_{\bar{t}}$ are the momenta of the top quark and top antiquark. We neglect the scalar and pseudoscalar couplings k_μ and $k_\mu\gamma_5$ with $k=p_t+p_{\bar{t}}$, since these terms give contributions proportional to the electron mass. We note that some of these neglected terms are needed to maintain the electromagnetic gauge invariance for the axial vector couplings in Eq. (43). The form factors can be written as

$$X_V=X_V^{\rm SM}+\delta X_V \quad (X=A,B,C,D \text{ and } V=Z,\gamma), \tag{44}$$

where $X_V^{\rm SM}$ and δX_V represent the SM and the new physics contributions, respectively. In the SM, only $A_{\gamma,Z}$ and B_Z exist at the tree level. Beyond the tree level, all of them except the CP-violating form factor D get contributions from loop diagrams. The SM loop contribution to D is completely negligible [5]. Since we are interested in the CP-violation effect, we neglect the SM loop contributions to all form factors. Thus we have

$$A_\gamma^{\rm SM}=\frac{4}{3}s_W, \quad A_Z^{\rm SM}=\frac{1}{2c_W}\left(1-\frac{8}{3}s_W^2\right), \tag{45}$$

$$B_\gamma^{\rm SM}=0, \quad B_Z^{\rm SM}=\frac{1}{2c_W}, \tag{46}$$

$$C_\gamma^{\rm SM}=D_\gamma^{\rm SM}=C_Z^{\rm SM}=D_Z^{\rm SM}=0. \tag{47}$$

For new physics effects, only the form factor D receives CP-violating contributions. Then we obtain

$$\delta A_{\gamma,Z}=\delta B_{\gamma,Z}=\delta C_{\gamma,Z}=0, \tag{48}$$

$$\delta D_\gamma=-\frac{v}{\Lambda^2}\frac{4}{g}\left((C_{qB}-C_{tB})\frac{c_Wm_t}{v}+C_{qW}\frac{s_Wm_t}{2v}-C_{tW\Phi}\frac{s_W}{\sqrt{2}}-C_{tB\Phi}\sqrt{2}c_W\right), \tag{49}$$

$$\delta D_Z=\frac{v}{\Lambda^2}\frac{4}{g}\left((C_{qB}-C_{tB})\frac{s_Wm_t}{v}-C_{qW}\frac{c_Wm_t}{2v}+C_{tW\Phi}\frac{c_W}{\sqrt{2}}-C_{tB\Phi}\sqrt{2}s_W+C_{Dt}\frac{m_Z}{2\sqrt{2}v}\right). \tag{50}$$

The nonvanishing real parts of D can give rise to the following asymmetry [9]:

$$A_T=P_\perp\sin\alpha-\bar{P}_\perp\sin\bar{\alpha}, \tag{51}$$

where $P_\perp\sin\alpha$ $(\bar{P}_\perp\sin\bar{\alpha})$ is the degree of polarization of the t $(\bar{t})$ quark perpendicular to the scattering plane of $e^+e^-\to t\bar{t}$. The angle α depends on the top quark polarization direction as follows: We denote the top quark unit momentum in the center-of-mass frame of the $t\bar{t}$ pair as $\hat{p}=(\sin\theta_t\cos\phi_t,\sin\theta_t\sin\phi_t,\cos\theta_t)$, and the polarization vector of the top quark $s^\mu=P_\perp(0,\hat{s})+(P_\parallel m_t)(|\vec{p}|,E_t\hat{p})$. $P_\perp(P_\parallel)$ is the degree of top quark transverse (longitudinal) polarization, $\hat{s}$ is a unit three-vector perpendicular to $\hat{p}$, and E_t is the energy of the top quark. We choose $\hat{s}$:

$$\hat{s}=(\sin\psi_t\sin\phi_t+\cos\psi_t\cos\theta_t\cos\phi_t,-\sin\psi_t\cos\phi_t+\cos\psi_t\cos\theta_t\sin\phi_t,-\sin\theta_t\cos\phi_t). \tag{52}$$

Then α is given by $\alpha=\phi_t-\psi_t$. For more details we refer to Appendix C of the first article of [9]. The $\bar{\alpha}$ in Eq. (51) is defined similarly for a top antiquark.

$P_\perp\sin\alpha$ and $\bar{P}_\perp\sin\bar{\alpha}$ are given by [9]

$$P_\perp\sin\alpha=\frac{T_\perp}{G}, \quad \bar{P}_\perp\sin\bar{\alpha}=\frac{\bar{T}_\perp}{\bar{G}}, \tag{53}$$

where

$$G= |(+-++)|^2+|(+-+-)|^2+|(+--+)|^2 +|(+---)|^2+|(-+++)|^2+|(-++-)|^2 +|(-+-+)|^2+|(-+--)|^2, \tag{54}$$

$$T_\perp=2\,\mathrm{Im}[(+-++)^*(+--+)+(+-+-)^*(+---) +(-+++)^*(-+-+)+(-++-)^*(-+--)], \tag{55}$$

$$\overline{T_\perp}=2\,\mathrm{Im}[(+-++)^*(+-+-)+(+--+)^*(+---) +(-+++)^*(-++-)+(-+-+)^*(-+--)]. \tag{56}$$

Here the helicity amplitudes $(h_{e^-},h_{e^+},h_t,h_{\bar t})$, where $h_{e^-}=-,+$, etc., indicate, respectively, a left- and right-handed electron, etc., are given by

$$(h_{e^-},h_{e^+},h_t,h_{\bar t})=2g^2E\left[\frac{(h_{e^-},h_{e^+},h_t,h_{\bar t})_Z}{s-M_Z^2} +\frac{(h_{e^-},h_{e^+},h_t,h_{\bar t})_\gamma}{s}\right]. \tag{57}$$

The nonvanishing $(h_{e^-},h_{e^+},h_t,h_{\bar t})_V$ $(V=\gamma,Z)$ can be found in [9] and are listed below:

$$(-+--)_V=e_L^V\sin\theta_t(m_tA_V-K^2C_V+iEKD_V), \tag{58}$$

$$(-+-+)_V=-e_L^V(1+\cos\theta_t)(EA_V+KB_V), \tag{59}$$

$$(-++-)_V=e_L^V(1-\cos\theta_t)(EA_V-KB_V), \tag{60}$$

$$(-+++)_V=e_L^V\sin\theta_t(-m_tA_V+K^2C_V+iEKD_V), \tag{61}$$

$$(+---)_V=e_R^V\sin\theta_t(m_tA_V-K^2C_V+iEKD_V), \tag{62}$$

$$(+--+)_V=e_R^V(1-\cos\theta_t)(EA_V+KB_V), \tag{63}$$

$$(+-+-)_V=-e_R^V(1+\cos\theta_t)(EA_V-KB_V), \tag{64}$$

$$(+-++)_V=e_R^V\sin\theta_t(-m_tA_V+K^2C_V+iEKD_V), \tag{65}$$

where θ_t is the angle between the top quark and the electron, $E=\sqrt{s}/2$, $K=\sqrt{E^2-m_t^2}$ and $e_{L,R}^V$ are the form factors in Ve^-e^+ vertex $ig\gamma^\mu(e_L^VP_L+e_R^VP_R)$, which are given by

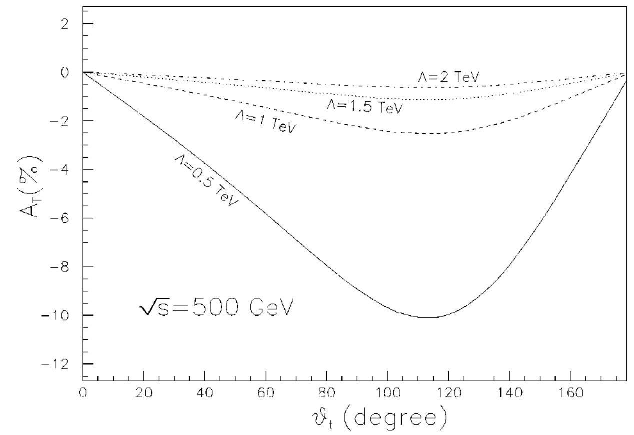

FIG. 1. The asymmetry between the degrees of transverse polarization of the top quark and top antiquark induced by $\overline{O_{qW}}$ as a function of θ_t in top pair production at the NLC for $\sqrt{s}=500$ GeV.

$$e_L^Z=\frac{1}{c_W}\left(-\frac{1}{2}+s_W^2\right),\quad e_R^Z=\frac{1}{c_W}s_W^2, \tag{66}$$

$$e_L^\gamma=e_R^\gamma=-s_W. \tag{67}$$

In the present case the expression of the asymmetry A_T is quite involved and an explicit expression such as Eq. (42) is not available. In order to see the size of the asymmetry we have to resort to numerical demonstration. We do not know, *a priori*, the size of the strength of the various couplings entering in Eqs. (49) and (50). For the purpose of demonstration, we take the couplings to be equivalent to that of the operator $\overline{O_{qW}}$ alone and assume $C_{qW}=0.1$. The numerical value of the asymmetry A_T can then be calculated.

The asymmetry A_T as a function of θ_t in the top pair production at the NLC is plotted in Figs. 1 and 2 for $\sqrt{s}=500$ GeV and $\sqrt{s}=1$ TeV, respectively. Figure 1 shows that if the scale of new physics which generates the operators is below 1.5 TeV, the A_T induced can exceed 1%. Comparing Fig. 1 with Fig. 2, we find that the asymmetry A_T for

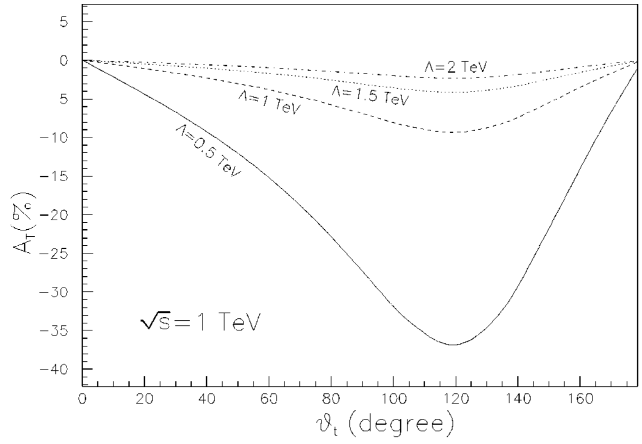

FIG. 2. The asymmetry between the degrees of transverse polarization of the top quark and top antiquark induced by $\overline{O_{qW}}$ as a function of θ_t in top pair production at the NLC for $\sqrt{s}=1$ TeV.

$\sqrt{s}=1$ TeV is larger than that for $\sqrt{s}=500$ GeV. To see more clearly, we compare the values corresponding to $\theta_t=120°$

Λ(TeV)	0.5	1	1.5	2
A_T (%) ($\sqrt{s}=0.5$ TeV)	-9.97	-2.50	-1.11	-0.62
A_T (%) ($\sqrt{s}=1$ TeV)	-36.87	-9.34	-4.15	-2.34

Here we see that the A_T for $\sqrt{s}=1$ TeV is four times larger than that for $\sqrt{s}=500$ GeV. But since the total event rate at a 1 TeV machine is about four times smaller than a 500 GeV machine, the net effect is that a 1 TeV machine cannot provide a better measurement unless it has a higher luminosity.

C. Momentum correlations among the decay products of top quark at the NLC

In the process $e^+e^-\to\gamma^*,Z^*\to t\bar{t}$ with $t\to W^+b$ and $\bar{t}\to W^-\bar{b}$, some CP-odd momentum correlations among the decay products can be constructed [10,11]. One of them, which is CPT even and sensitive to the real part of the dipole moment factor D in Eq. (43), is

$$O_1=(\vec{p}_b\times\vec{p}_{\bar{b}})\cdot\hat{e}_z, \tag{68}$$

where $\hat{e}_z$ is the unit vector along the incoming positron beam direction. However, this observable is not sensitive to possible CP violation of the $t\bar{b}W$ vertex in the top quark decay [10,11]. Thus we consider only the CP-violating new physics effects in the vertices $Vt\bar{t}$ $(V=\gamma,Z)$. In terms of the expression Eq. (43), one gets the average value [12]

$$\begin{aligned}\langle O_1\rangle=-\frac{g}{48}sm_t(1-x)\epsilon^2\beta\Sigma^{-1}\Bigg\{&\frac{1}{s^2}C^{\gamma\gamma}(v_e^\gamma)^2v_t^\gamma\mathrm{Re}D_\gamma+\frac{1}{s(s-m_Z^2)}C^{Z\gamma}v_e^\gamma v_e^Z\left(v_t^Z-\frac{\beta}{3}a_t^Z\right)\mathrm{Re}D_\gamma\\&+\frac{1}{s(s-m_Z^2)}C^{Z\gamma}v_e^\gamma v_e^Z v_t^\gamma\mathrm{Re}D_Z+\frac{1}{(s-m_Z^2)^2}C^{ZZ}[(v_e^Z)^2+(a_e^Z)^2]\left(v_t^Z-\frac{\beta}{3}a_t^Z\right)\mathrm{Re}D_Z\Bigg\},\end{aligned} \tag{69}$$

where

$$x=\frac{4m_t^2}{s},\quad \epsilon=1-\frac{m_W^2}{m_t^2},$$

$$\beta=\frac{m_t^2-2m_W^2}{m_t^2+2m_W^2},$$

$$C^{\gamma\gamma}=-p,\quad C^{Z\gamma}=\frac{a_e^Z}{v_e^Z}-p,$$

$$C^{ZZ}=\frac{2a_e^Zv_e^Z}{(v_e^Z)^2+(a_e^Z)^2}-p,$$

$$v_e^V=\frac{1}{2s_W}(e_L^V+e_R^V),\quad v_t^V=\frac{A_V}{2s_W},$$

$$a_e^V=\frac{B_V}{2s_W},\quad a_t^V=\frac{1}{2s_W}(e_L^V-e_R^V), \tag{70}$$

and

$$\begin{aligned}\Sigma=&\frac{1}{s^2}\left(1+\frac{x}{2}\right)(v_e^\gamma)^2(v_t^\gamma)^2+\frac{2}{s(s-m_Z^2)}\left(1+\frac{x}{2}\right)v_e^\gamma v_t^\gamma\\&\times(v_e^Z-pa_e^Z)v_t^Z+\frac{1}{(s-m_Z^2)^2}[(v_e^Z)^2+(a_e^Z)^2\\&-2pv_e^Za_e^Z]\left[\left(1+\frac{x}{2}\right)(v_t^Z)^2+(1-x)(a_t^Z)^2\right].\end{aligned} \tag{71}$$

In the above equations, s is the center-of-mass energy squared and p is the degree of longitudinal polarization of the initial electron with $p=\pm1$ corresponding to the right- and left-handed helicities, respectively. Note that in our analyses we neglect both the radiative corrections to the couplings Ve^+e^- $(V=\gamma,Z)$ and the electron mass, thus only the left-right and right-left combinations[2] of electron and positron helicities couple to the γ and Z.

Again we take the equivalence of operator $\overline{O}_{qW}$ for $C_{qW}=1$ as an example to show the numerical results. The values of $\langle O_1\rangle$ for different polarizations of the electron beam with new physics scale of 1 TeV are found to be

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[2] Hard collinear emission of a photon from the electron and positron beams can flip helicities. This gives rise to nonzero CP-odd correlations even in the absence of CP-violating interactions and this background should be subtracted. However, as analyzed in [12], there will be no such background at tree level for $\langle O_1\rangle$.

	$e^+e^-_L$	$e^+e^-_R$	e^+e^-
$\langle O_1\rangle$ $[(\mathrm{GeV})^2]$ ($\sqrt{s}=0.5$ TeV)	-3.67	-1.0	-25.5
$\langle O_1\rangle$ $[(\mathrm{GeV})^2]$ ($\sqrt{s}=1$ TeV)	-272.3	-1.7	-183.4

Here we find that the left-polarized electron beam yields the most significant results for $\langle O_1\rangle$ and in this case the result in a 1 TeV accelerator is eight times larger than a 500 GeV accelerator. In the following analyses we will only consider the left-polarized electron beam.

Now we compare the value of $\langle O_1\rangle$ with the expected variance $\langle O_1^2\rangle$ to see what luminosity is needed for the observation to be statistically significant. To observe a deviation from the SM expectation with better than one standard deviation (at the 68% confidence level), we need

$$|\langle O_1\rangle|\geq\sqrt{\frac{\langle O_1^2\rangle}{\mathcal{L}\sigma\kappa}}, \tag{72}$$

where $\mathcal{L}$ is the integrated luminosity, κ is the overall b- and W-tagging efficiency. The variance $\langle O_1^2\rangle$ and the production cross section σ at lowest order are given by [12]

$$\sigma=4\pi\alpha^2 s\sqrt{1-x}\Sigma, \tag{73}$$

$$\begin{aligned}\langle O_1^2\rangle=\frac{s m_t^2\epsilon^4}{2880}\Sigma^{-1}\Bigg(&\frac{1}{s^2}(v_e^\gamma)^2(v_t^\gamma)^2[24+2x-11x^2+4\beta^2(1-x)^2]+\frac{2}{s(s-m_Z^2)}v_e^\gamma v_e^Z\{v_t^\gamma v_t^Z[24+2x-11x^2+4\beta^2(1-x)^2]\\&-2v_t^\gamma a_t^Z(1-x)(6-x)\beta\}+\frac{1}{(s-m_Z^2)^2}[(v_e^Z)^2+(a_e^Z)^2]\{(v_t^Z)^2[24+2x-11x^2+4\beta^2(1-x)^2]\\&+(a_t^Z)^2[24-14x-4\beta^2(1-x)](1-x)-4v_t^Z a_t^Z(1-x)(6-x)\beta\}\Bigg).\end{aligned} \tag{74}$$

For a negative helicity electron beam considered in our analyses, the production rate is

$$\sigma(e^+e^-_L\to t\bar{t})=\begin{cases}775\ \mathrm{fb} & \text{for } \sqrt{s}=500\ \mathrm{GeV},\\ 232\ \mathrm{fb} & \text{for } \sqrt{s}=1\ \mathrm{TeV}.\end{cases} \tag{75}$$

Assuming the coupling strength of the order of unity and an overall b- and W-tagging efficiency of 50%, then the luminosity required to observe the CP-violating effects of $\overline{O}_{qW}$ at 68% confidence level is found to be

$$\mathcal{L}=\begin{cases}25\dfrac{(\Lambda/1\ \mathrm{TeV})^4}{C_{qW}^2}\ \mathrm{fb}^{-1} & \text{at } \sqrt{s}=0.5\ \mathrm{TeV},\\[2ex] 8\dfrac{(\Lambda/1\ \mathrm{TeV})^4}{C_{qW}^2}\ \mathrm{fb}^{-1} & \text{at } \sqrt{s}=1\ \mathrm{TeV}.\end{cases} \tag{76}$$

So, if the new physics scale is 1 TeV, we need a luminosity of 100 fb^{-1} (30 fb^{-1}) to probe the coupling strength C_{qW} down to 0.5 with a confidence level of 68% at $\sqrt{s}=500$ GeV (1 TeV). If a conservative overall b- and W-tagging efficiency of 10% is assumed, the required luminosity will be increased by a factor of 5. If a confidence level of 99.7% is assumed, the required luminosity will be increased by a factor of 9.

From the above results we find that for the same luminosity a 1 TeV collider can do a better measurement than a 500 GeV collider. This is due to the fact that the size of $\langle O_1\rangle$ at $\sqrt{s}=1$ TeV is eight times larger than at $\sqrt{s}=500$ GeV, while the production rate at $\sqrt{s}=1$ TeV is only about four times smaller than at $\sqrt{s}=500$ GeV. Thus the net effect is that a 1 TeV accelerator can do a better measurement than a 500 GeV accelerator.

V. SUMMARY

In this paper we listed all possible dimension-six CP-violating $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ invariant operators involving the third-family quarks, which may be generated by new physics at a higher scale. The expressions of these operators after the electroweak symmetry breaking and the induced effective couplings for $Wt\bar{b}$, $Vb\bar{b}$, and $Vt\bar{t}$ ($V=Z,\gamma,g,H$) were presented.

The contributions of some of these operators to the *CP*-odd asymmetries of the transverse polarization of top quark and top antiquark in single top production at the Tevatron and top pair production at the NLC are evaluated. The numerical results showed that if the new physics scale is around 1 TeV, then both colliders can be used to probe the couplings with a confined strength of 0.1 provided that the asymmetry of the transverse polarization can be measured at a level of a few percent.

We also calculated the effects on a *CP*-odd observable, which involves momentum correlations among the decay products of the top quark, at the NLC and studied the dependence on the energy and luminosity of the NLC. We found that with a luminosity of 100 fb^{-1}, a 500 GeV accelerator can probe the coupling strength to 0.5, assuming that the new physics scale is of the order of 1 TeV. Achieving the same measurement, we need a luminosity of 30 fb^{-1} at a 1 TeV accelerator.

Let us conclude by noting that although the measurements of the *CP*-violating observable are challenging, they are not tempered by SM background as already noted that the KM mechanism of *CP* violation which may enter our consideration in loop order is very small for the top quark process [5].

ACKNOWLEDGMENTS

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APPENDIX *CP*-VIOLATING OPERATORS AFTER ELECTROWEAK SYMMETRY BREAKING

(1) Class 1:

$$\overline{O}_{t1}=\frac{1}{2\sqrt{2}}H(H+2v)(H+v)(\bar{t}\,i\gamma_5 t), \tag{A1}$$

$$\overline{O}_{t2}=(H+v)\partial^\mu H(\bar{t}_R\gamma_\mu t_R), \tag{A2}$$

$$\overline{O}_{t3}=i\frac{1}{2\sqrt{2}}g_2(H+v)^2[-W_\mu^+(\bar{t}_R\gamma^\mu b_R)+W_\mu^-(\bar{b}_R\gamma^\mu t_R)], \tag{A3}$$

$$\begin{aligned}\overline{O}_{Dt}=&\,i\frac{1}{2\sqrt{2}}\partial^\mu H\left[\bar{t}\,\partial_\mu t-(\partial_\mu\bar{t})t+\partial_\mu(\bar{t}\gamma_5 t)-i\frac{4}{3}g_1B_\mu\bar{t}t\right]-\frac{1}{4\sqrt{2}}g_Z(H+v)Z^\mu\left[\partial_\mu(\bar{t}t)+\bar{t}\gamma_5\partial_\mu t-(\partial_\mu\bar{t})\gamma_5 t\right.\\&\left.-i\frac{4}{3}g_1B_\mu\bar{t}\gamma_5 t\right]+\frac{1}{2}g_2(H+v)W_\mu^-\left[\bar{b}_L\partial^\mu t_R-i\frac{2}{3}g_1B^\mu\bar{b}_Lt_R\right]+\frac{1}{2}g_2(H+v)W_\mu^+\left[(\partial^\mu\bar{t}_R)b_L+i\frac{2}{3}g_1B^\mu\bar{t}_Rb_L\right],\end{aligned} \tag{A4}$$

$$\begin{aligned}\overline{O}_{tW\Phi}=&\,i\frac{1}{2\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)[W_{\mu\nu}^3-ig_2(W_\mu^+W_\nu^--W_\mu^-W_\nu^+)]+i\frac{1}{2}(H+v)(\bar{b}_L\sigma^{\mu\nu}t_R)[W_{\mu\nu}^--ig_2(W_\mu^-W_\nu^3-W_\mu^3W_\nu^-)]\\&-i\frac{1}{2}(H+v)(\bar{t}_R\sigma^{\mu\nu}b_L)[W_{\mu\nu}^+-ig_2(W_\mu^3W_\nu^+-W_\mu^+W_\nu^3)],\end{aligned} \tag{A5}$$

$$\overline{O}_{tB\Phi}=i\frac{1}{\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)B_{\mu\nu}, \tag{A6}$$

$$\overline{O}_{tG\Phi}=i\frac{1}{\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5T^At)G_{\mu\nu}^A, \tag{A7}$$

$$\overline{O}_{tG}=i[\bar{t}_R\gamma^\mu T^A\partial^\nu t_R-\partial^\nu\bar{t}_R\gamma^\mu T^At_R]G_{\mu\nu}^A+g_s\bar{t}_R\gamma^\mu\{G^\nu,G_{\mu\nu}\}t_R+\frac{4g_1}{3}\bar{t}_R\gamma^\mu G_{\mu\nu}B^\nu t_R, \tag{A8}$$

$$\overline{O}_{tB}=i[\bar{t}_R\gamma^\mu\partial^\nu t_R-\partial^\nu\bar{t}_R\gamma^\mu t_R]B_{\mu\nu}+2g_s\bar{t}_R\gamma^\mu G^\nu t_RB_{\mu\nu}+\frac{4}{3}g_1\bar{t}_R\gamma^\mu t_RB_{\mu\nu}B^\nu. \tag{A9}$$

(2) Class 2:

$$\overline{O}_{qG}=i[\bar{q}_L\gamma^\mu T^A\partial^\nu q_L-\partial^\nu\bar{q}_L\gamma^\mu T^Aq_L]G_{\mu\nu}^A+g_s\bar{q}_L\gamma^\mu\{G^\nu,G_{\mu\nu}\}q_L+2g_2\bar{q}_L\gamma^\mu W^\nu G_{\mu\nu}q_L+\frac{1}{3}g_1\bar{q}_L\gamma^\mu G_{\mu\nu}B^\nu q_L, \tag{A10}$$

$$\overline{O}_{qW}=\frac{i}{2}W^3_{\mu\nu}[\overline{t}_L\gamma^\mu\partial^\nu t_L-\partial^\nu\overline{t}_L\gamma^\mu t_L-\overline{b}_L\gamma^\mu\partial^\nu b_L+\partial^\nu\overline{b}_L\gamma^\mu b_L]+\frac{i}{\sqrt{2}}[W^+_{\mu\nu}(\overline{t}_L\gamma^\mu\partial^\nu b_L-\partial^\nu\overline{t}_L\gamma^\mu b_L)$$
$$+W^-_{\mu\nu}(\overline{b}_L\gamma^\mu\partial^\nu t_L-\partial^\nu\overline{b}_L\gamma^\mu t_L)]+g_2\overline{q}_L\gamma^\mu[W_\mu,W_\nu]\partial^\nu q_L-g_2\partial^\nu\overline{q}_L\gamma^\mu[W_\mu,W_\nu]q_L+2g_s\overline{q}_L\gamma^\mu G^\nu W_{\mu\nu}q_L$$
$$+\frac{1}{2}g_2(\vec{W}_{\mu\nu}\cdot\vec{W}^\nu)\overline{q}_L\gamma^\mu q_L+\frac{1}{3}g_1B^\nu\overline{q}_L\gamma^\mu W_{\mu\nu}q_L\,, \tag{A11}$$

$$\overline{O}_{qB}=iB_{\mu\nu}\left[\overline{q}_L\gamma^\mu\partial^\nu q_L-\partial^\nu\overline{q}_L\gamma^\mu q_L-2i\overline{q}_L\gamma^\mu\left(g_sG^\nu+g_2W^\nu+\frac{1}{6}g_1B^\nu\right)q_L\right], \tag{A12}$$

$$\overline{O}_{bG}=i[\overline{b}_R\gamma^\mu T^A\partial^\nu b_R-\partial^\nu\overline{b}_R\gamma^\mu T^Ab_R]G^A_{\mu\nu}g_s\overline{b}_R\gamma^\mu\{G^\nu,G_{\mu\nu}\}b_R-\frac{2g_1}{3}\overline{b}_R\gamma^\mu G_{\mu\nu}B^\nu b_R\,, \tag{A13}$$

$$\overline{O}_{bB}=i[\overline{b}_R\gamma^\mu\partial^\nu b_R-\partial^\nu\overline{b}_R\gamma^\mu b_R]B_{\mu\nu}+2g_s\overline{b}_R\gamma^\mu G^\nu b_RB_{\mu\nu}-\frac{2}{3}g_1\overline{b}_R\gamma^\mu b_RB_{\mu\nu}B^\nu, \tag{A14}$$

$$\overline{O}^{(1)}_{\Phi q}=(H+v)\partial_\mu H[\overline{t}_L\gamma^\mu t_L+\overline{b}_L\gamma^\mu b_L], \tag{A15}$$

$$\overline{O}^{(3)}_{\Phi q}=-\overline{O}^{(1)}_{\Phi q}+2(H+v)\partial_\mu H\overline{b}_L\gamma^\mu b_L-\frac{i}{\sqrt{2}}g_2(H+v)^2(W^+_\mu\overline{t}_L\gamma^\mu b_L-W^-_\mu\overline{b}_L\gamma^\mu t_L), \tag{A16}$$

$$\overline{O}_{\Phi b}=(H+v)\partial_\mu H\overline{b}_R\gamma^\mu b_R\,, \tag{A17}$$

$$\overline{O}_{b1}=\frac{1}{2\sqrt{2}}H(H+v)(H+2v)\overline{b}i\gamma_5b, \tag{A18}$$

$$\overline{O}_{Db}=i\frac{1}{2\sqrt{2}}\partial^\mu H\left[\overline{b}\partial_\mu b-(\partial_\mu\overline{b})b+\partial_\mu(\overline{b}\gamma_5b)+i\frac{2}{3}g_1B_\mu(\overline{b}b)\right]+\frac{1}{4}g_Z(H+v)Z^\mu\left[\partial_\mu(\overline{b}b)+\overline{b}\gamma_5\partial_\mu b-(\partial_\mu\overline{b})\gamma_5b\right.$$
$$\left.+\frac{2}{3}g_1B_\mu(\overline{b}i\gamma_5b)\right]+\frac{g_2}{2}(H+v)\left[W^+_\mu\left(\overline{t}_L\partial_\mu b_R+\frac{i}{3}g_1B_\mu\overline{t}_Lb_R\right)+W^-_\mu\left(\partial_\mu\overline{b}_Rt_L-\frac{i}{3}g_1B_\mu\overline{b}_Rt_L\right)\right], \tag{A19}$$

$$\overline{O}_{bW\Phi}=i\frac{1}{2}(H+v)\left[W^+_{\mu\nu}(\overline{t}_L\sigma^{\mu\nu}b_R)-W^-_{\mu\nu}(\overline{b}_R\sigma^{\mu\nu}t_L)-\frac{1}{\sqrt{2}}W^3_{\mu\nu}(\overline{b}\sigma^{\mu\nu}\gamma_5b)+ig_2(W^+_\mu W^3_\nu-W^3_\mu W^+_\nu)(\overline{t}_L\sigma^{\mu\nu}b_R)\right.$$
$$\left.+ig_2(W^-_\mu W^3_\nu-W^3_\mu W^-_\nu)(\overline{b}_R\sigma^{\mu\nu}t_L)+i\frac{g_2}{\sqrt{2}}(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu)(\overline{b}\sigma^{\mu\nu}\gamma_5b)\right], \tag{A20}$$

$$\overline{O}_{bB\Phi}=\frac{i}{\sqrt{2}}(H+v)B_{\mu\nu}(\overline{b}\sigma^{\mu\nu}\gamma_5b), \tag{A21}$$

$$\overline{O}_{bG\Phi}=\frac{i}{\sqrt{2}}(H+v)G^A_{\mu\nu}(\overline{b}\sigma^{\mu\nu}\gamma_5T^Ab). \tag{A22}$$

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[1] J. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).

[2] K. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] For a review see A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Annu. Rev. Nucl. Phys. **43**, 27 (1993).

[4] K. Hagiwara, R. D. Peccei, and D. Zeppanfeld, Nucl. Phys. **B282**, 253 (1987); D. London, Phys. Rev. D **45**, 3186 (1992); X. G. He, J. P. Ma, and B. H. J. Mckellar, Phys. Lett. B **304**, 285 (1993); D. Choudhury and S. S. Rindani, *ibid.* **335**, 198 (1994); G. Belanger and G. Couture, Phys. Rev. D **49**, 5720 (1994); D. Chang, W. Y. Keung, and P. Pal, *ibid.* **51**, 1326 (1995).

[5] C. Jarlskog, Phys. Rev. D **35**, 1685 (1987); C. Jarlskog and R.

Stora, Phys. Lett. B **208**, 268 (1988); G. Eilam, J. Hewett, and A. Soni, Phys. Rev. Lett. **67**, 1979 (1991); B. Grzadkowski, Phys. Lett. B **319**, 526 (1993).

[6] For reviews see, for example, C.-P. Yuan, Mod. Phys. Lett. A **10**, 627 (1995); D. Atwood and A. Soni, in *ICHEP '96*, Proceedings of the 28th International Conference on High Energy Physics, Warsaw, Poland, edited by Z. Ajduk and A. Wroblewski (World Scientific, Singapore, 1997), hep-ph/9609418.

[7] G. L. Kane, J. Pumplin, and W. Repko, Phys. Rev. Lett. **41**, 1689 (1978); A. Devoto, G. L. Kane, J. Pumplin, and W. Repko, *ibid.* **43**, 1062 (1979); **43**, 1540 (1979); Phys. Lett. **90B**, 436 (1980).

[8] J. F. Donoghue and G. Valencia, Phys. Rev. Lett. **58**, 451 (1987); C. R. Schmidt and M. E. Peskin, *ibid.* **69**, 410 (1992); D. Chang, W.-Y. Keung, and I. Phillips, Nucl. Phys. **B408**, 286 (1993).

[9] G. L. Kane, G. A. Ladinsky, and C.-P. Yuan, Phys. Rev. D **45**, 124 (1992); C.-P. Yuan, *ibid.* **45**, 782 (1992); G. A. Ladinsky and C.-P. Yuan, *ibid.* **49**, 4415 (1994).

[10] D. Atwood, S. B. Shalom, G. Eilam, and A. Soni, Phys. Rev. D **54**, 5412 (1996).

[11] W. Bernreuther, T. Schroder, and T. N. Pham, Phys. Lett. B **279**, 389 (1992); W. Bernreuther, O. Nachtmann, P. Overmann, and T. Schroder, Nucl. Phys. **B388**, 53 (1992); W. Bernreuther and A. Brandenburg, Phys. Lett. B **314**, 104 (1993); A. Brandenburg and J. P. Ma, *ibid.* **298**, 211 (1993); W. Bernreuther and A. Brandenburg, Phys. Rev. D **49**, 4481 (1994).

[12] F. Cuypers and S. D. Rindani, Phys. Lett. B **343**, 333 (1995).

[13] D. Atwood and A. Soni, Phys. Rev. D **45**, 2405 (1992); D. Atwood, G. Eilam, A. Soni, R. Mendel, and R. Migneron, Phys. Rev. Lett. **70**, 1364 (1993); D. Atwood, G. Eilam, and A. Soni, *ibid.* **71**, 492 (1993).

[14] B. Grzadkowski and J. F. Gunion, Phys. Lett. B **287**, 237 (1992); R. Cruz, B. Grzadkowski, and J. F. Gunion, *ibid.* **289**, 440 (1992); N. G. Deshpande, B. Margolis, and H. D. Trottier, Phys. Rev. D **45**, 178 (1992); J. L. Diaz-Cruz and G. Lopez Castro, Phys. Lett. B **301**, 405 (1993); C. J. Im, G. L. Kane, and P. J. Malde, *ibid.* **317**, 454 (1993); B. Grzadkowski and W.-Y. Keung, *ibid.* **319**, 526 (1993); D. Chang, W.-Y. Keung, and I. Phillips, Phys. Rev. D **48**, 3225 (1993); M. Nowakowski and A. Pilaftsis, Int. J. Mod. Phys. A **9**, 1097 (1994); A. Ilakovac, B. A. Kniel, and A. Pilaftsis, Phys. Lett. B **320**, 329 (1994).

[15] K. Whisnant, J. M. Yang, B.-L. Young, and X. Zhang, Phys. Rev. D **56**, 467 (1997).

[16] C. J. C. Burgess and H. J. Schnitzer, Nucl. Phys. **B228**, 454 (1983); C. N. Leung, S. T. Love, and S. Rao, Z. Phys. C **31**, 433 (1986); W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).

[17] K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, Phys. Lett. B **283**, 353 (1992); Phys. Rev. D **48**, 2182 (1993).

[18] G. J. Gounaris, F. M. Renard, and C. Verzegnassi, Phys. Rev. D **52**, 451 (1995); R. D. Peccei and X. Zhang, Nucl. Phys. **B337**, 269 (1990); R. D. Peccei, S. Peris, and X. Zhang, *ibid.* **B349**, 305 (1991); C. T. Hill and S. Parke, Phys. Rev. D **49**, 4454 (1994); D. Atwood, A. Kagan, and T. Rizzo, *ibid.* **52**, 6264 (1995); D. O. Carlson, E. Malkawi, and C.-P. Yuan, Phys. Lett. B **337**, 145 (1994); H. Georgi, L. Kaplan, D. Morin, and A. Shenk, Phys. Rev. D **51**, 3888 (1995); T. Han, R. D. Peccei, and X. Zhang, Nucl. Phys. **B454**, 527 (1995); X. Zhang and B.-L. Young, Phys. Rev. D **51**, 6584 (1995); E. Malkawi and T. Tait, *ibid.* **54** 5758 (1996); S. Dawson and G. Valencia, *ibid.* **53**, 1721 (1996); T. G. Rizzo, *ibid.* **53**, 6218 (1996); P. Haberl, O. Nachtman, and A. Wilch, *ibid.* **53**, 4875 (1996); T. Han, K. Whisnant, B.-L. Young, and X. Zhang, Phys. Lett. B **385**, 311 (1996); Phys. Rev. D **55**, 724 (1997); K. Hagiwara, T. Hatsukano, S. Ishihara, and R. Szalapski, Nucl. Phys. **B496**, 66 (1997); G. J. Gounaris, J. Layssac, and F. M. Renard, Phys. Rev. D **55**, 5786 (1997); A. Data and X. Zhang, *ibid.* **55**, 2530 (1997); B.-L. Young, in *Proceedings of the International Symposium on Heavy Flavor and Electroweak Theory*, 1995, edited by C.-H.Chang and C.-S. Huang (World Scientific, Singapore, 1996).

[19] G. J. Gounaris, D. T. Papadamou, and F. M. Renard, hep-ph/9609437.

[20] H. L. Lai *et al.*, Phys. Rev. D **51**, 4763 (1995).

[21] T. Stelzer and S. Willenbrock, Phys. Lett. B **357**, 125 (1995); D. O. Carlson and C.-P. Yuan, in *Particle Theory and Phenomenology*, Proceedings of the International Meeting, Ames, Iowa, 1995, edited by K. Lassila *et al.* (World Scientific, Singapore, 1996), hep-ph/9509208; A. P. Heinson, A. S. Belyaev, and E. E. Boos, hep-ph/9509274; D. Amidei *et al.*, Fermilab Report No. Fermilab-Pub-96/082 (unpublished).

[22] C.P. Yuan, Phys. Rev. D **41**, 42 (1990); D. Carlson and C.P. Yuan, Phys. Lett. B **306**, 386 (1993).