Nucleon contribution to the neutrino electromagnetic vertex in matter

Juan Carlos D'Olivo
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, 04510 México, D.F., Mexico

José F. Nieves
Laboratory of Theoretical Physics, Department of Physics, P.O. Box 23343, University of Puerto Rico, Río Piedras, Puerto Rico 00931-3343
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We calculate the nucleon contribution to the electromagnetic vertex of a neutrino in a background of particles, including the effect of the anomalous magnetic moment of the nucleons. Explicit formulas for the form factors are given in various physical limits of practical interest. Several applications of the results are mentioned, including the effect of an external magnetic field on the dispersion relation of a neutrino in matter. [S0556-2821(97)04721-8]

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I. INTRODUCTION

It is commonly accepted that the properties of neutrinos that propagate through a thermal background of particles can be very different compared to their properties in the vacuum. This is the case, in particular, for their electromagnetic properties [1]. Some time ago, this notion motivated the study of the electromagnetic properties of neutrinos in a background of electrons [2]. In Ref. [2], the effective electromagnetic interactions of a neutrino that propagates through matter were determined by a one-loop calculation of the electromagnetic vertex function induced by the neutrino interactions with the electrons in the background. The results of those calculations have been applied to determine the rates of various physical processes such as, for example, the radiative neutrino decay [3] and the Cherenkov radiation by massless (chiral) neutrinos [4], the latter of which had been studied previously by different techniques [5]. We mention that the Cherenkov process considered in Refs. [4,5] differs from the one studied in the recent works by Grimus and Neufeld [6] and by Mohanty and Samal [7], which depends on a hypothetical magnetic moment coupling of the neutrino.

It was observed in Ref. [2] that, in the presence of a static magnetic field, the induced electromagnetic interactions produce an additional contribution to the effective potential of the neutrinos, or equivalently to their index of refraction, that modify the Wolfenstein resonance condition [8] for neutrino oscillations in matter. This observation, and the calculations on which it is based, have been generalized and refined subsequently by various authors [9–12].[1]

In the present work we extend the calculations carried out in Ref. [2] by including the contribution to the neutrino electromagnetic vertex coming from the presence of the nucleons in the background. In particular, we take into account the anomalous magnetic moment coupling of the nucleons to the photon and we calculate explicitly the additional terms they induce in the neutrino electromagnetic vertex.

The present calculation is motivated, in part, by the recent interesting work of Kusenko and Segrè suggesting that the observed large birth velocities of pulsars are due to the asymmetric emission of neutrinos from the cooling protoneutron star, which is produced by the resonant neutrino oscillations in the supernova's magnetic field [14]. In a subsequent paper [15], the same authors find that a similar explanation is possible if the oscillations occur between an active (weak interacting) neutrino and a sterile one. While in the oscillation between active neutrinos the neutral-current interaction contribution to the neutrino energy in not relevant, it becomes important for oscillations between an active and a sterile neutrino because the latter has no weak interactions at all. In these contexts, the effects on the neutrino potentials due to the magnetic couplings of the nucleons have been estimated for various limiting cases in Refs. [12,16].

However, the calculations presented here go farther. They are based on the one-loop formula for the neutrino electromagnetic vertex using thermal field theory methods. Apart from the limitations that the one-loop approximation (linear in the electromagnetic field) imply, the formulas obtained for the contribution due to the anomalous nucleon moments are valid for general conditions of the nucleon gas, degenerate or nondegenerate, whether it is relativistic or not. The formulas can be applied, in the context of neutrino oscillations in the presence of a magnetic field, to determine the additional corrections to the neutrino index of refraction in situations in which the nucleons are not necessarily described by one of the idealized limiting cases, and instead a more detailed evaluation of the effects is sought. Besides the application in this context, our calculations can be relevant for other physical processes that have been considered in the literature, such as the induced radiative neutrino decay and the Cherenkov radiation emission by neutrinos mentioned above.

II. CALCULATION OF THE NEUTRINO ELECTROMAGNETIC VERTEX

We follow the method and conventions of Ref. [9]. The background-dependent part of the neutrino electromagnetic

[1] A particularly clear exposition, which also corrects some inaccurate statements contained in Ref. [9], has been given by Smirnov in Ref. [13].

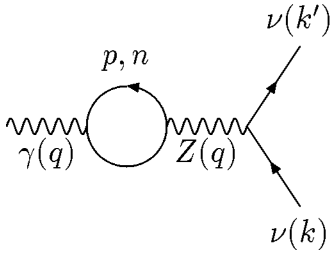

FIG. 1. One-loop diagram for the nucleon contributions $\Gamma_\mu^{(p,n)} \times(k,k')$ to the neutrino electromagnetic vertex.

vertex function is denoted by $\Gamma'(k,k',v)_\mu$ where k and k' denote the momentum of the incoming and outgoing neutrino, respectively, and v^μ is the velocity four-vector of the medium, which from now on we set equal to $(1,\vec{0})$ in our calculations. The electron background contribution to Γ'_μ, which we denote here by $\Gamma_\mu^{(e)}$, was calculated in Ref. [9] while the nucleon background contribution $\Gamma_\mu^{(p)}+\Gamma_\mu^{(n)}$ is the subject of the present work. To lowest order, the diagrams relevant to the calculation are shown in Fig. 1.

For each nucleon $f=n,p$ in the loop, the propagator is given by

$$S_F(p)=(\not p+m_f)\left[\frac{1}{p^2-m_f^2}+2\pi i\,\delta(p^2-m_f^2)\,\eta_f(p)\right], \tag{2.1}$$

where

$$\eta_f(p)=\frac{\theta(p\cdot v)}{e^{\beta(p\cdot v-\mu_f)}+1}+\frac{\theta(-p\cdot v)}{e^{-\beta(p\cdot v-\mu_f)}+1}, \tag{2.2}$$

with θ representing the unit step function, $1/\beta$ the temperature, and μ_f the chemical potential of each nucleon specie.

The electromagnetic couplings of the nucleons are given by

$$L_\gamma=-|e|A^\mu\overline{p}\gamma_\mu p-\frac{\kappa_p}{2}\,\overline{p}\sigma^{\mu\nu}pF_{\mu\nu}-\frac{\kappa_n}{2}\,\overline{n}\sigma^{\mu\nu}nF_{\mu\nu}, \tag{2.3}$$

where $\kappa_{n,p}$ are the anomalous part of the nucleon magnetic moments, given by

$$\kappa_p=1.79\left(\frac{|e|}{2m_p}\right),$$

$$\kappa_n=-1.91\left(\frac{|e|}{2m_n}\right); \tag{2.4}$$

e stands for the electron charge and, as usual, $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$. For the neutral-current couplings we write

$$L_Z=-g_Z Z^\mu\left[\overline{\nu_L}\gamma_\mu\nu_L+\sum_{f=e,p,n}\overline{f}\gamma_\mu(a_f+b_f\gamma_5)f\right], \tag{2.5}$$

where, in the standard model,

$$g_Z=g/(2\cos\theta_W) \tag{2.6}$$

and

$$-a_e=a_p=\frac{1}{2}-2\sin^2\theta_W,$$

$$a_n=-\frac{1}{2},$$

$$b_e=\frac{1}{2},$$

$$b_n=-b_p=\frac{1}{2}g_A, \tag{2.7}$$

with $g_A=1.26$ being the renormalization constant of the axial-vector current of the nucleon. There are several implicit assumptions and simplifications that we have made by adopting the electromagnetic and neutral-current couplings defined by Eqs. (2.3) and (2.5). Their justification is discussed in more detail in Appendix B.

With these couplings, the nucleon contribution to the neutrino electromagnetic vertex is given by

$$\Gamma_\mu^{(\mathrm{nucl})}=(T_{\mu\nu}^{(p)}+T_{\mu\nu}^{(n)})\gamma^\nu L, \tag{2.8}$$

where, as usual $L=(1-\gamma_5)/2$, and

$$\begin{aligned}T_{\mu\nu}^{(f)}=\sqrt{2}G_F\int\frac{d^4p}{(2\pi)^3}\mathrm{Tr}[j_{f\mu}^{(\mathrm{em})}(q)(\not p+m_f)\gamma_\nu(a_f+b_f\gamma_5)\\ \times(\not p-\not q+m_f)]\left\{\frac{\delta(p^2-m_f^2)\,\eta_f(p)}{(p-q)^2-m_f^2}\right.\\ \left.+\frac{\delta[(p-q)^2-m_f^2]\,\eta_f(p-q)}{p^2-m_f^2}\right\},\end{aligned} \tag{2.9}$$

for $f=n,p$. To arrive at this formula we have dropped the term that is independent of the particle density distributions, as well as the term that contains the product of the two δ functions.[2] In Eq. (2.9) $q=k-k'$ denotes the momentum of the outgoing photon, $j_{f\mu}^{(\mathrm{em})}(q)$ is the total electromagnetic current of each nucleon,

$$j_{p\mu}^{(\mathrm{em})}(q)=|e|\gamma_\mu-i\kappa_p\sigma_{\mu\alpha}q^\alpha,$$

$$j_{n\mu}^{(\mathrm{em})}(q)=-i\kappa_n\sigma_{\mu\alpha}q^\alpha, \tag{2.10}$$

and we have also used the relation

$$\frac{g_Z^2}{m_Z^2}=\sqrt{2}G_F. \tag{2.11}$$

[2]The latter contributes only to the absorptive part (provided the appropriate kinematical conditions are satisfied), which we are not considering in this paper.

Let us consider the neutron case first. Making the change of variable $p \rightarrow p+q$ in the integrand corresponding to the second term in curly brackets in Eq. (2.9), and carrying out the traces, we obtain[3]

$$T_{\mu\nu}^{(n)} = 4 m_n \kappa_n \sqrt{2} G_F \int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}} \{ a_n (q^2 g_{\mu\nu} - q_\mu q_\nu)(f_n + f_{\overline{n}}) - 2 i b_n \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta (f_n - f_{\overline{n}}) \} \left[\frac{1}{q^2 + 2p\cdot q} + (q \rightarrow -q) \right], \tag{2.12}$$

where

$$p^\mu = (\mathcal{E}, \vec{\mathcal{P}}), \quad \mathcal{E} = \sqrt{\vec{\mathcal{P}}^2 + m_n^2}. \tag{2.13}$$

We have introduced the number densities of the nucleons

$$f_{n,p}(p) = \frac{1}{e^{\beta(\mathcal{E} - \mu_{n,p})} + 1}, \tag{2.14}$$

and the corresponding quantities $f_{\overline{n,p}}$ for the antiparticles, which are given by a similar formula but with the opposite sign of the chemical potential. Following Ref. [2], $T_{\mu\nu}^{(n)}$ can be decomposed in the form

$$T_{\mu\nu}^{(n)} = T_T^{(n)} R_{\mu\nu} + T_L^{(n)} Q_{\mu\nu} + T_P^{(n)} P_{\mu\nu}, \tag{2.15}$$

where

$$R_{\mu\nu} = \widetilde{g}_{\mu\nu} - Q_{\mu\nu},$$

$$Q_{\mu\nu} = \frac{\widetilde{v}_\mu \widetilde{v}_\nu}{\widetilde{v}^{\,2}},$$

$$P_{\mu\nu} = \frac{i}{\mathcal{Q}} \epsilon_{\mu\nu\alpha\beta} q^\alpha v^\beta, \tag{2.16}$$

with

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \tag{2.17}$$

and

$$\widetilde{v}_\mu = \widetilde{g}_{\mu\nu} v^\nu. \tag{2.18}$$

The functions $T_{T,L,P}^{(n)}$ depend on the variables

$$\Omega = q \cdot v,$$

$$\mathcal{Q} = \sqrt{\Omega^2 - q^2}, \tag{2.19}$$

which are the energy and momentum of the photon, and are given explicitly by

$$T_T^{(n)} = T_L^{(n)} = 4\sqrt{2} G_F m_n \kappa_n a_n q^2 D_n,$$

$$T_P^{(n)} = -8\sqrt{2} G_F m_n \kappa_n b_n \mathcal{Q} C_n, \tag{2.20}$$

where

$$D_{n,p} = \int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}} (f_{n,p} + f_{\overline{n,p}}) \left[\frac{1}{q^2 + 2p\cdot q} + (q \rightarrow -q) \right], \tag{2.21}$$

$$C_{n,p} = \int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}} \left(\frac{\widetilde{v}\cdot p}{\widetilde{v}^{\,2}} \right) (f_{n,p} - f_{\overline{n,p}}) \left[\frac{1}{q^2 + 2p\cdot q} + (q \rightarrow -q) \right]. \tag{2.22}$$

It is useful to recall that

$$\widetilde{v}^{\,2} = -\frac{\mathcal{Q}^2}{q^2},$$

$$\widetilde{v}\cdot p = \mathcal{E} - \left(\frac{q\cdot p}{q^2} \right) \Omega. \tag{2.23}$$

According to Eq. (2.10), the proton contribution to the $\nu\nu\gamma$ vertex contains a term that is similar to the one determined above in the neutron case, plus another one that arises from the ordinary γ_μ coupling. The latter is of the same form as the one calculated in Ref. [2] for the neutral-current contribution in the electron background. Thus, repeating the steps that lead to Eq. (2.24) of Ref. [2] and imitating Eq. (2.20) above, we obtain the total contribution from the proton background

$$T_T^{(p)} = 2\sqrt{2} G_F a_p \left\{ |e| \left(A_p - \frac{B_p}{\widetilde{v}^{\,2}} \right) + 2 m_p \kappa_p q^2 D_p \right\},$$

$$T_L^{(p)} = 4\sqrt{2} G_F a_p \left\{ |e| \frac{B_p}{\widetilde{v}^{\,2}} + m_p \kappa_p q^2 D_p \right\},$$

$$T_P^{(p)} = -4\sqrt{2} G_F b_p \mathcal{Q} C_p (|e| + 2 m_p \kappa_p), \tag{2.24}$$

where

$$A_p = \int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}} (f_p + f_{\overline{p}}) \left[\frac{2m_p^2 - 2p\cdot q}{q^2 + 2p\cdot q} + (q \rightarrow -q) \right],$$

$$B_p = \int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}} (f_p + f_{\overline{p}}) \left[\frac{2(p\cdot v)^2 + 2(p\cdot v)(q\cdot v) - p\cdot q}{q^2 + 2p\cdot q} + (q \rightarrow -q) \right]. \tag{2.25}$$

The expressions in Eqs. (2.22), (2.21), and (2.25) allow us to obtain simple formulas for the coefficients $T_{T,L,P}^{(n,p)}$ in various limiting cases. For Eqs. (2.22) and (2.25) we can borrow

[3]Our conventions are such that $\epsilon^{0123} = +1$.

the results from Ref. [2], where the corresponding quantities for the electron background were denoted by A,B,C. Thus, for example, for the proton background,[4]

$$A_p(\Omega,\mathcal{Q}\to 0)=-3\omega_{0p}^2+\frac{\mathcal{Q}^2\omega_{0p}^2}{\Omega^2}+O(\Omega^2),$$

$$A_p(\Omega\to 0,\mathcal{Q})=\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{d}{d\mathcal{E}}(f_p+f_{\bar{p}})+O(\mathcal{Q}^2), \tag{2.26}$$

$$B_p(\Omega,\mathcal{Q}\to 0)=\frac{\mathcal{Q}^2\omega_{0p}^2}{\Omega^2},$$

$$B_p(\Omega\to 0,\mathcal{Q})=A_p(\Omega\to 0,\mathcal{Q})+O(\mathcal{Q}^2), \tag{2.27}$$

$$C_p(\Omega,\mathcal{Q}\to 0)=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{f_p-f_{\bar{p}}}{\mathcal{E}}\left[1-\frac{2\mathcal{P}^2}{3\mathcal{E}^2}\right]+O(\Omega^2),$$

$$C_p(\Omega\to 0,\mathcal{Q})=\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{d}{d\mathcal{E}}(f_p-f_{\bar{p}})+O(\mathcal{Q}^2), \tag{2.28}$$

where

$$\omega_{0p}^2=\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}(f_p+f_{\bar{p}})\left[1-\frac{\mathcal{P}^2}{3\mathcal{E}^2}\right]. \tag{2.29}$$

In similar fashion, we obtain here

$$D_p(\Omega,\mathcal{Q}\to 0)=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{f_p+f_{\bar{p}}}{\mathcal{E}^2}+O(\Omega^2),$$

$$D_p(\Omega\to 0,\mathcal{Q})=\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{d}{d\mathcal{E}}\left(\frac{f_p+f_{\bar{p}}}{\mathcal{E}}\right)+O(\mathcal{Q}^2). \tag{2.30}$$

We would like to stress that while the previous formulas hold for the limiting values indicated of the photon energy and momentum, no assumption has been made with respect to the background gas. The formulas can be simplified further by restricting the attention to some simple idealized situations, such as the degenerate and nondegenerate cases, in both the relativistic and nonrelativistic limits. Since the nucleons are non-relativistic in the situations of practical interest, we consider this particular case in some detail.

In the nonrelativistic limit, the above expressions reduce to

$$\omega_{0p}^2=\frac{n_p}{4m_p}, \tag{2.31}$$

$$A_p(\Omega\to 0,\mathcal{Q})=\frac{-m_p}{4\pi^2}I_p+O(\mathcal{Q}^2), \tag{2.32}$$

$$C_p(\Omega,\mathcal{Q}\to 0)=\frac{-\omega_{0p}^2}{2m_p}+O(\Omega^2),$$

$$C_p(\Omega\to 0,\mathcal{Q})=\frac{-1}{8\pi^2}I_p+O(\mathcal{Q}^2), \tag{2.33}$$

$$D_p(\Omega,\mathcal{Q}\to 0)=\frac{-\omega_{0p}^2}{2m_p^2}+O(\Omega^2),$$

$$D_p(\Omega\to 0,\mathcal{Q})=\frac{-1}{8\pi^2 m_p}I_p+O(\mathcal{Q}^2), \tag{2.34}$$

where n_p is the total proton number density

$$n_p=2\int\frac{d^3\mathcal{P}}{(2\pi)^3}f_p, \tag{2.35}$$

and

$$I_p=\int_0^\infty d\mathcal{P}f_p. \tag{2.36}$$

The integral in Eq. (2.36) cannot be evaluated without knowing the distribution function and, therefore, it depends on whether the background is degenerate or nondegenerate. For these two limiting cases we obtain

$$I_p=\begin{cases}\pi^2\dfrac{\beta n_p}{m_p} & \text{(nondegenerate)},\\ (3\pi^2 n_p)^{1/3} & \text{(degenerate)}.\end{cases} \tag{2.37}$$

The corresponding formulas for the neutron background are easily obtained from the above by making obvious substitutions. In this manner, via Eqs. (2.20) and (2.24), we determine the nucleon contribution to the neutrino electromagnetic vertex either in the static $(\Omega\to 0)$ or in the long wavelength $(\mathcal{Q}\to 0)$ limit. It is useful for some applications to consider the case in which neither Ω nor $\mathcal{Q}$ is zero, while still satisfying $\Omega,\mathcal{Q}\ll m_{n,p}$, which is a good approximation for most situations of interest. The results for this case are derived in Appendix A.

III. DISCUSSION AND APPLICATIONS

The total matter background contribution to the $\nu\nu\gamma$ vertex is given by

$$\Gamma_\mu=(T_T R_{\mu\nu}+T_L Q_{\mu\nu}+T_P P_{\mu\nu})\gamma^\nu L, \tag{3.1}$$

where

$$T_X=T_X^{(p)}+T_X^{(n)}+T_X^{(e)}\quad (X=T,L,P). \tag{3.2}$$

The electron term $T_{\mu\nu}^{(e)}$ can be decomposed as in Eq. (2.15), with

[4] We take this opportunity to mention the following typographical errors in Ref. [2]: the second formula for C in Eq. (2.24) of Ref. [2] contains the factor f_-+f_+ when it should be f_--f_+, and the left-hand side of Eq. (2.28) of the same reference should be B instead of $B/\tilde{v}^2$. The electron and positron distribution functions are denoted by $f_{e,\bar{e}}$ in the present paper.

$$T_T^{(e)} = 2\sqrt{2} e G_F \left(A - \frac{B}{\tilde{v}^2} \right) \begin{cases} a_e + 1 & (\nu_e), \\ a_e & (\nu_{\mu,\tau}), \end{cases}$$

$$T_L^{(e)} = 4\sqrt{2} e G_F \frac{B}{\tilde{v}^2} \begin{cases} a_e + 1 & (\nu_e), \\ a_e & (\nu_{\mu,\tau}), \end{cases}$$

$$T_P^{(e)} = -4\sqrt{2} e G_F \mathcal{Q} C \begin{cases} b_e - 1 & (\nu_e), \\ b_e & (\nu_{\mu,\tau}), \end{cases} \tag{3.3}$$

The additional contribution for the electron neutrino is due to the charged-current diagram, which is absent for $\nu_{\mu,\tau}$. The functions A, B, C are given by expressions analogous to those in Eqs. (2.22) and (2.25), with $f_{n,p}$ being replaced by the electron distribution f_e. In the nonrelativistic limit they reduce to formulas analogous to those given in Eqs. (2.32) and (2.33).

Since the electrons are relativistic in many situations of interest, it is also useful to summarize the corresponding results in that limit. Thus, for a relativistic electron gas,

$$\omega_0^2 = \frac{1}{6\pi^2} \int_0^\infty d\mathcal{P} \mathcal{P} (f_e + f_{\bar{e}}), \tag{3.4}$$

and

$$A(0,\mathcal{Q}) = -3\omega_0^2 + O(\mathcal{Q}^2),$$

$$C(0,\mathcal{Q}) = -\frac{1}{8\pi^2}(I_e - I_{\bar{e}}) + O(\mathcal{Q}^2),$$

$$C(\Omega,0) = -\frac{1}{24\pi^2}(I_e - I_{\bar{e}}) + O(\Omega^2), \tag{3.5}$$

where

$$I_{e,\bar{e}} = \int_0^\infty d\mathcal{P} f_{e,\bar{e}}. \tag{3.6}$$

The remaining integrals in Eqs. (3.4) and (3.6) cannot be performed without specifying the distribution function. In the limiting cases of a degenerate or nondegenerate gas, they are given by

$$\omega_0^2 = \begin{cases} \dfrac{\beta}{12}(n_e + n_{\bar{e}}) & \text{(nondegenerate)}, \\ \dfrac{1}{3}\left(\dfrac{3}{8\pi}\right)^{2/3} (n_e^{2/3} + n_{\bar{e}}^{2/3}) & \text{(degenerate)}, \end{cases} \tag{3.7}$$

$$I_{e,\bar{e}} = \begin{cases} \dfrac{\pi^2 \beta^2}{2} n_{e,\bar{e}} & \text{(nondegenerate)}, \\ (3\pi^2 n_{e,\bar{e}})^{1/3} & \text{(degenerate)}. \end{cases} \tag{3.8}$$

It should also be remembered that, for either case,

$$A(\Omega, \mathcal{Q} \to 0) = -3\omega_0^2 + \frac{\mathcal{Q}^2 \omega_0^2}{\Omega^2} + O(\Omega^2),$$

$$B(\Omega, \mathcal{Q} \to 0) = \frac{\mathcal{Q}^2 \omega_0^2}{\Omega^2},$$

$$B(\Omega \to 0, \mathcal{Q}) = A(\Omega \to 0, \mathcal{Q}) + O(\mathcal{Q}^2). \tag{3.9}$$

The application of these and the formulas obtained in Sec. II depend on the specific environment under consideration as well as the kinematic regime involved. Let us then consider some particular situations of interest.

From the explicit formulas given in Eq. (2.34), or more generally in Eq. (A13), it is immediately seen that for values of q such that $\Omega, \mathcal{Q} \ll m_{n,p}$, the function $D_{n,p}$ is smaller than A_p and B_p by a factor of order $1/m_{n,p}^2$. Therefore, neglecting such terms in Eqs. (2.20) and (2.24) it follows that Eq. (3.2) reduces to

$$T_T = 2\sqrt{2} |e| G_F a_p \left(A_p - \frac{B_p}{\tilde{v}^2} \right) + T_T^{(e)},$$

$$T_L = 4\sqrt{2} |e| G_F a_p \frac{B_p}{\tilde{v}^2} + T_L^{(e)}, \tag{3.10}$$

with the $T_{T,L}^{(e)}$ given in Eq. (3.3). The relative importance of the electron and the proton contributions in these formulas depend on the kinematic regime involved as well as the conditions of the proton and electron gases.

For illustrative purposes, suppose that the physical situation is such that q satisfies

$$\Omega, \mathcal{Q} \ll m_e, \tag{3.11}$$

in which case Eq. (A1) is satisfied also. If both the electron and proton gases are nondegenerate and nonrelativistic, then from Eqs. (2.26) and (2.31) and the corresponding formulas for the electron we have

$$A_p(\Omega,0) = -\frac{3n_p}{4m_p},$$

$$A(\Omega,0) = -\frac{3n_e}{4m_e}, \tag{3.12}$$

with analogous results for B_p and B. Thus in this case the proton contribution to $T_{T,L}(\Omega,0)$ is negligible. On the other hand, Eqs. (2.32) and (2.37), and the analogous formulas for the electron, imply

$$B_p(0,\mathcal{Q}) = A_p(0,\mathcal{Q}) = -\frac{1}{4}\beta n_p,$$

$$B(0,\mathcal{Q}) = A(0,\mathcal{Q}) = -\frac{1}{4}\beta n_e, \tag{3.13}$$

so that the proton and the electron contributions to $T_{T,L}(0,\mathcal{Q})$ are comparable. This last conclusion remains valid even if the electrons are relativistic. In fact, in that case, their contribution is also given by the result given in Eq. (3.13), as can be easily checked using Eq. (3.7) in Eqs. (3.9) and (3.5). More possibilities can obviously arise if we consider other realistic situations, such as a nondegenerate proton gas but a degenerate electron gas, or a kinematic regime in which $\Omega, \mathcal{Q} \ll m_p$ is still satisfied but Eq. (3.11) is not.

IV. NEUTRINO DISPERSION RELATION IN A MAGNETIC FIELD

In the presence of a static, uniform magnetic field $\vec{B}$, the $\nu\nu\gamma$ modifies the neutrino dispersion relation in the medium according to

$$\omega_k = |\vec{k}| + b - c\hat{k}\cdot\vec{B}, \tag{4.1}$$

where $\vec{k}$ is the momentum vector of the neutrino and b gives the standard Wolfenstein term in the dispersion relation [8,17]. As shown in Ref. [2],

$$c = \left[\frac{T_P(0,\mathcal{Q})}{\mathcal{Q}}\right]_{\mathcal{Q}\to 0}. \tag{4.2}$$

Substituting the formulas for $T_P^{(n,p,e)}$ given in Eqs. (2.20), (2.24), and (3.3), this yields

$$c = -4\sqrt{2}G_F\left[b_p(|e|+2m_p\kappa_p)C_p(0,\mathcal{Q}\to 0) + b_n 2m_p\kappa_n C_n(0,\mathcal{Q}\to 0) \mp \frac{1}{2}eC(0,\mathcal{Q}\to 0)\right], \tag{4.3}$$

where the upper (lower) sign holds for ν_e ($\nu_{\mu\tau}$) and we have put $b_e = \frac{1}{2}$. If the electron gas is degenerate, then

$$C(0,\mathcal{Q}\to 0) = -\frac{1}{8}\left(\frac{3n_e}{\pi^4}\right)^{1/3}, \tag{4.4}$$

for both the relativistic and nonrelativistic cases. If the physical situation is such that the proton gas also is degenerate, then a similar formula holds for $C_p(0,\mathcal{Q}\to 0)$ (with $n_e \to n_p$) and, in a neutral system, the electron and the normal proton contributions tend to cancel for ν_e in Eq. (4.3). In fact, if the effect of the anomalous nucleon magnetic moment as well as the renormalization of the nucleon axial-vector coupling are neglected, then c in Eq. (4.3) would be zero for ν_e. However, the cancellation is not complete once those two effects are taken into account, independently of whether the proton gas is degenerate or nondegenerate. This can have important consequences in the context of the possible explanation of the pulsar birth velocities in terms of resonant oscillations between active and sterile neutrinos [15], as recently pointed out in Refs. [12,16]. Equation (4.3), together with the formulas for $C_{p,n}(0,\mathcal{Q})$ and $C(0,\mathcal{Q})$ given in Sec. II [e.g., Eq. (2.28) and the corresponding formulas for C_n and C] give the magnetic contribution to the neutrino dispersion relation for fairly general conditions of the matter background. Under some circumstances, it may be more appropriate to use other methods to determine this contribution, such as those employed in Refs. [11–13].

V. CONCLUSIONS

In this work we have extended the previous calculations of the electromagnetic properties of neutrinos in a background of electrons, by including the contribution from the nucleon background. In particular, we have taken into account the anomalous electromagnetic and neutral-current couplings of the nucleons. The calculations are based on the one-loop formula for the neutrino electromagnetic vertex using thermal field theory methods and, apart from the limitations of the one-loop approximation, the formulas obtained for the electromagnetic vertex are valid for general conditions of the nucleon gas, degenerate or nondegenerate, whether it is relativistic or not. In the context of neutrino oscillations in the presence of a magnetic field, we applied the formulas to determine the additional corrections to the neutrino index of refraction in those situations in which the nucleons are not necessarily described by one of the idealized limiting cases, and instead a more detailed evaluation of the effects is sought. We have already mentioned that the formulas for the electromagnetic vertex have been the basis for studying other physical processes of neutrinos, such as the induced radiative decay and Cherenkov radiation. Here we have shown that the importance of the nucleon contribution to the functions $T_{T,L,P}$, relative to the electron contribution, depends on the particular physical conditions of the situation under consideration. For example, we indicated how different the results can be depending on whether the nucleon and electron gases are degenerate or not, or whether the kinematic regime is such that $\Omega,\mathcal{Q} \gg m_e$ or $\Omega,\mathcal{Q} \ll m_e$ while still maintaining $\Omega,\mathcal{Q} \ll m_p$, among other possibilities. The formulas that we have given in this paper form a useful starting point to study in more detail the radiative neutrino process in such astrophysical settings as the supernova and the early universe, or perhaps in the context of laboratory experiments involving the coherent neutrino electromagnetic conversion in crystals that has been discussed recently [18].

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APPENDIX A: THE LIMIT OF SMALL PHOTON MOMENTUM

Here we consider the functions A_f, B_f, C_f, D_f for values of the photon momentum satisfying

$$\Omega,\mathcal{Q} \ll m_{p,n}, \tag{A1}$$

but for the case that neither Ω nor $\mathcal{Q}$ is necessarily zero. Consider first the formula for B_p given in Eq. (2.25). It can be written as

$$B_p = \frac{1}{2}\int \frac{d^3\mathcal{P}}{(2\pi)^3}(f_p + f_{\bar{p}})\left[\frac{2\mathcal{E} + 2\Omega + \vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}{q^2 + 2p\cdot q} + (q \to -q)\right], \tag{A2}$$

where $\vec{v}_{\mathcal{P}} \equiv \vec{\mathcal{P}}/\mathcal{E}$ is the velocity of the background particles. We now make the change of variable

$$\vec{\mathcal{P}} \to \vec{\mathcal{P}} - \frac{1}{2}\vec{\mathcal{Q}}. \tag{A3}$$

After expanding the numerator and denominator in the terms inside the brackets and neglecting terms of order $\mathcal{Q}^2/\mathcal{E}^2$,

$$B_p=\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\left[\frac{f_p(\vec{\mathcal{P}}-(1/2)\vec{\mathcal{Q}})+f_{\overline{p}}(\vec{\mathcal{P}}-(1/2)\vec{\mathcal{Q}})}{\Omega-\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}+(q\rightarrow -q)\right]. \tag{A4}$$

Finally, expanding the distribution functions in powers of $\vec{\mathcal{Q}}$, we obtain

$$B_p=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{\vec{\mathcal{Q}}\cdot\nabla_{\mathcal{P}}(f_p+f_{\overline{p}})}{\Omega-\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}, \tag{A5}$$

where $\nabla_{\mathcal{P}}$ is the gradient operator with respect to the momentum variable $\vec{\mathcal{P}}$. Since the distribution functions depend on $\mathcal{P}$ only through $\mathcal{E}$, Eq. (A5) is equivalent to

$$B_p=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\left(\frac{\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}{\Omega-\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}\right)\frac{d}{d\mathcal{E}}(f_p+f_{\overline{p}}). \tag{A6}$$

In the limit $\Omega\rightarrow 0$, this formula reduces to the result quoted in Eq. (2.27). It is also straightforward to show, after performing an integration by parts, that it reproduces the result quoted in Eq. (2.27) for the limit $\mathcal{Q}\rightarrow 0$. However, it is important to remark that Eq. (A6) holds for any arbitrary values of $\Omega,\mathcal{Q}$, subject only to the condition in Eq. (A1).

We can proceed in similar fashion with the function A_p, although the algebra is somewhat more involved in this case. Thus, making the change of variable $\vec{\mathcal{P}}\rightarrow\vec{\mathcal{P}}-\frac{1}{2}\vec{\mathcal{Q}}$ in the formula for A_p given in Eq. (2.25), then expanding the integrand in powers of $\mathcal{Q}/E$ and neglecting terms of order $\mathcal{Q}^2/E^2$ we obtain

$$A_p=B_p+\int\frac{d^3\mathcal{P}}{(2\pi)^3}\left[\frac{\vec{\mathcal{Q}}\cdot\nabla_{\mathcal{P}}(f_p+f_{\overline{p}})(\vec{\mathcal{P}}\cdot\vec{v}_{\mathcal{P}})}{2(\mathcal{E}\Omega-\vec{\mathcal{P}}\cdot\vec{\mathcal{Q}})}-\frac{(3-v_{\mathcal{P}}^2)}{2\mathcal{E}}\times(f_p+f_{\overline{p}})\right]. \tag{A7}$$

With the help of the identity

$$\begin{aligned}(\vec{\mathcal{P}}\cdot\vec{v}_{\mathcal{P}})\vec{\mathcal{Q}}\cdot\nabla_{\mathcal{P}}&=(\vec{\mathcal{P}}\cdot\vec{\mathcal{Q}})\vec{v}_{\mathcal{P}}\cdot\nabla_{\mathcal{P}}\\&=[\mathcal{E}\Omega-(\mathcal{E}\Omega-\vec{\mathcal{P}}\cdot\vec{\mathcal{Q}})]\vec{v}_{\mathcal{P}}\cdot\nabla_{\mathcal{P}},\end{aligned} \tag{A8}$$

Eq. (A7) reduces to

$$A_p=B_p+\frac{\Omega}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{\vec{v}_{\mathcal{P}}\cdot\nabla_{\mathcal{P}}(f_p+f_{\overline{p}})}{\Omega-\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}. \tag{A9}$$

In Eq. (A9) we have omitted the terms

$$\int\frac{d^3\mathcal{P}}{(2\pi)^3}\left[-\frac{1}{2}\vec{v}_{\mathcal{P}}\cdot\nabla_{\mathcal{P}}(f_p+f_{\overline{p}})-\frac{(3-v_{\mathcal{P}}^2)}{2\mathcal{E}}(f_p+f_{\overline{p}})\right] \tag{A10}$$

which, using the fact that

$$\nabla_{\mathcal{P}}\cdot\vec{v}_{\mathcal{P}}=\frac{3-v_{\mathcal{P}}^2}{\mathcal{E}},$$

reduce (apart from a factor of $-1/2$) to the integral of $\nabla_{\mathcal{P}}\cdot[\vec{v}_{\mathcal{P}}(f_p+f_{\overline{p}})]$ and therefore integrate to zero. Equation (A9) can be written in the equivalent form

$$A_p=B_p+\frac{\Omega}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\left(\frac{v_{\mathcal{P}}^2}{\Omega-\vec{v}_{\mathcal{P}}\cdot\vec{\mathcal{Q}}}\right)\frac{d}{d\mathcal{E}}(f_p+f_{\overline{p}}). \tag{A11}$$

It is immediately seen that Eq. (A9), or equivalently (A11), reduces to the results quoted in Eq. (2.26) when $\mathcal{Q}=0$ or $\Omega=0$. However, Eqs. (A9) and (A11) hold also when neither $\mathcal{Q}$ nor Ω is zero. When q is such that $\Omega,\mathcal{Q}\ll m_e$, the functions A,B for the electron are given by similar formulas.

Proceeding in similar form for the functions $C_{n,p}$ and $D_{n,p}$ we obtain

$$C_{n,p}=\frac{q^2}{2\mathcal{Q}^2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{\vec{\mathcal{Q}}\cdot\vec{v}_{\mathcal{P}}}{\Omega-\vec{\mathcal{Q}}\cdot\vec{v}_{\mathcal{P}}}\frac{d}{d\mathcal{E}}(f_{n,p}-f_{\overline{n,p}}), \tag{A12}$$

$$D_{n,p}=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{1}{2\mathcal{E}^2}\left[\frac{\vec{\mathcal{Q}}\cdot\vec{v}_{\mathcal{P}}}{\Omega-\vec{\mathcal{Q}}\cdot\vec{v}_{\mathcal{P}}}\frac{d}{d\mathcal{E}}(f_{n,p}+f_{\overline{n,p}})+\frac{(f_{n,p}+f_{\overline{n,p}})}{\mathcal{E}}\right]. \tag{A13}$$

Once more, Eqs. (A12) and (A13) reduce to the formulas in Eqs. (2.28) and (2.30) in the indicated limits. We stress also that, in deriving Eqs. (A6), (A11), (A12), and (A13), we have made no assumption regarding the conditions of the background gas. Thus, they are valid for degenerate and nondegenerate gases, in the relativistic as well the nonrelativistic limits.

APPENDIX B: ELECTROMAGNETIC AND NEUTRAL-CURRENT COUPLINGS OF THE NUCLEONS

The couplings of the interaction Lagrangian that are relevant to our calculation are given by

$$\begin{aligned}L_{\mathrm{int}}=&-|e|A^\mu(-\overline{e}\gamma_\mu e+J_\mu^{(\mathrm{em})})-g_Z Z^\mu[\overline{\nu_L}\gamma\nu_L\\&+\overline{e}\gamma_\mu(a_e+b_e\gamma_5)e+J_\mu^{(Z)}],\end{aligned} \tag{B1}$$

where, in the standard model a_e and b_e are given in Eq. (2.7) while, in terms of the quark fields,

$$J_\mu^{(\mathrm{em})}=\overline{q}\gamma_\mu\frac{\tau_3}{2}q+\frac{1}{6}\overline{q}\gamma_\mu q, \tag{B2}$$

$$J_\mu^{(Z)}=\overline{q}\gamma_\mu\frac{\tau_3}{2}q-\overline{q}\gamma_\mu\gamma_5\frac{\tau_3}{2}q-2\sin^2\theta_W J_\mu^{(\mathrm{em})}. \tag{B3}$$

We have introduced the notation

$$q=\begin{pmatrix} u \\ d \end{pmatrix}, \tag{B4}$$

and $\tau_{1,2,3}$ stand for the Pauli matrices.

For each nucleon $f=p,n$, the electromagnetic couplings are defined by writing the matrix element

$$\langle f(p')|J_\mu^{(\mathrm{em})}(0)|f(p)\rangle=\overline{u(p')}\left[F_{1f}^{(\mathrm{em})}\gamma_\mu - i\frac{F_{2f}^{(\mathrm{em})}}{2m}\sigma_{\mu\nu}q^\nu\right]u(p), \tag{B5}$$

where $q=p-p'$, m is the nucleon mass and $u(p)$ is a Dirac spinor. The form factors are functions of q^2 and are such that

$$F_{1p}^{(\mathrm{em})}(0)=1, \tag{B6}$$
$$F_{1n}^{(\mathrm{em})}(0)=0,$$
$$F_{2p}^{(\mathrm{em})}(0)=1.79,$$
$$F_{2n}^{(\mathrm{em})}(0)=-1.71.$$

In similar fashion, and using the SU(2) symmetry property of the matrix elements, we can write

$$\left\langle f(p')\left|\overline{q}\gamma_\mu\frac{\tau_3}{2}q\right|f(p)\right\rangle=I_{3f}\overline{u(p')}\left[F_1^{(3)}\gamma_\mu - i\frac{F_2^{(3)}}{2m}\sigma_{\mu\nu}q^\nu\right]u(p),$$

$$\left\langle f(p')\left|\overline{q}\gamma_\mu\gamma_5\frac{\tau_3}{2}q\right|f(p)\right\rangle=I_{3f}\overline{u(p')}F_A^{(3)}\gamma_\mu\gamma_5u(p), \tag{B7}$$

where $I_{3p}=-I_{3n}=1/2$. Since $F_A^{(3)}$ is the same form factor that appears in the charged current matrix element (which is responsible, for example, for β decay)

$$g_A\equiv F_A^{(3)}(0)=1.26. \tag{B8}$$

Further, from the SU(2) decomposition of $J_\mu^{(\mathrm{em})}$ given in Eq. (B2), it follows that

$$F_1^{(3)}=F_{1p}^{(\mathrm{em})}-F_{1n}^{(\mathrm{em})},$$
$$F_2^{(3)}=F_{2p}^{(\mathrm{em})}-F_{2n}^{(\mathrm{em})}, \tag{B9}$$

so that, in particular,

$$F_1^{(3)}(0)=1,$$

$$F_2^{(3)}(0)=3.7. \tag{B10}$$

In principle, the form factors that enter the calculation of the diagram shown in Fig. 1 are not the on-shell form factors we have introduced above, but their off-shell counterpart. However, since we are considering situations in which the photon momentum q is small, we will use their value at $q\to 0$, for which the formulas given above are valid. The matrix element of the neutral-current between nucleon states can then be written in the form

$$\langle f(p')|J_\mu^{(Z)}(0)|f(p)\rangle=\overline{u(p')}j_{f\mu}^{(Z)}(q)u(p), \tag{B11}$$

where

$$j_{f\mu}^{(Z)}(q)=a_f\gamma_\mu+b_f\gamma_\mu\gamma_5-i\frac{c_f}{2m}\sigma_{\mu\nu}q^\nu. \tag{B12}$$

From the decomposition of $J_\mu^{(Z)}$ given in Eq. (B3) together with Eqs. (B5) and (B7), it follows that

$$a_f=I_{3f}-2\sin^2\theta_WQ_f,$$

$$b_f=-I_{3f}g_A,$$

$$c_f=I_{3f}[F_{2p}^{(\mathrm{em})}(0)-F_{2n}^{(\mathrm{em})}(0)]-2\sin^2\theta_WF_{2f}^{(\mathrm{em})}(0), \tag{B13}$$

where $Q_p=1, Q_n=0$ and, as remarked above, the limit $q\to 0$ is implied. Similar considerations apply to the electromagnetic vertices adopted in Eq. (2.10).

In Eq. (2.9) we have neglected the c_f term in the nucleon neutral-current couplings because it appears to be unimportant in the situations of interest. However, for completeness, we summarize below the results of including such term in the calculation.

The effect of including the c_f term in the definition of the nucleon neutral-current vertex is taken into account by making the substitution

$$\gamma_\mu(a_f+b_f\gamma_5)\to j_{f\mu}^{(Z)}(-q) \tag{B14}$$

in Eq. (2.9). The end result of making that substitution is that $T_{\mu\nu}^{(p,n)}$ can still be decomposed as in Eq. (2.15) but, instead of the formulas in Eqs. (2.20) and (2.24) for $T_{T,L}^{(p,n)}$, we have instead

$$T_T^{(p)}=2\sqrt{2}G_Fa_p\left\{|e|\left(A_p-\frac{B_p}{\tilde{v}^2}\right)+2m_p\kappa_pa_pq^2D_p\right\} + \sqrt{2}G_Fc_p\left[2|e|q^2D_p+\frac{\kappa_p}{m_p}\left(A_p'-\frac{B_p'}{\tilde{v}^2}\right)\right],$$

$$T_L^{(p)}=4\sqrt{2}G_Fa_p\left\{|e|\frac{B_p}{\tilde{v}^2}+m_p\kappa_pa_pq^2D_p\right\} + 2\sqrt{2}G_Fc_p\left[|e|q^2D_p+\frac{\kappa_p}{m_p}\frac{B_p'}{\tilde{v}^2}\right], \tag{B15}$$

and

$$T_T^{(n)} = \sqrt{2} G_F m_n \kappa_n \left[4 a_n q^2 D_n + \frac{c_n}{m_n^2} \left(A_n' - \frac{B_n'}{\tilde{v}^2} \right) \right],$$

$$T_L^{(n)} = 2\sqrt{2} G_F m_n \kappa_n \left[2 a_n q^2 D_n + \frac{c_n}{m_n^2} \frac{B_n'}{\tilde{v}^2} \right]. \tag{B16}$$

The functions A_f', B_f' are given by

$$A_f' = q^2 \left(\frac{1}{2} A_f + 3 m_f^2 D_f \right),$$

$$B_f' = -q^2 B_f - \mathcal{Q}^2 \left(\frac{1}{2} A_f + m_f^2 D_f \right). \tag{B17}$$

As expected, the additional terms associated with the c_f neutral-current couplings are accompanied by additional factors of $\mathcal{Q}$ or Ω and therefore are not important in the limit of small photon momentum that we have considered.

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