Dijet production at hadron-hadron colliders in the Balitskii-Fadin-Kuraev-Lipatov approach

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The production in high-energy hadron collisions of a pair of jets with large rapidity separation is studied in an improved Balitskii-Fadin-Kuraev-Lipatov (BFKL) formalism. By recasting the analytic solution of the BFKL equation as an explicit order-by-order sum over emitted gluons, the effects of phase space constraints and the running coupling are studied. Particular attention is paid to the azimuthal angle decorrelation of the jet pair. The inclusion of subleading effects significantly improves the agreement between the theoretical predictions and recent preliminary measurements from the D0 Collaboration. [S0556-2821(97)05021-2]

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I. INTRODUCTION

Fixed-order, renormalization-group-improved QCD perturbation theory has been remarkably successful in describing jet physics at high-energy colliders. For example, the bulk of the inclusive jet distribution at the Fermilab Tevatron $p\bar{p}$ collider is well described by folding subprocess cross sections calculated at next-to-leading order (NLO) with parton distributions extracted from deep inelastic scattering. However there are certain situations where such an approach may be expected to fail. If in the production of a pair of large E_T jets the separation in rapidity Δy of the jets becomes large, then higher-order perturbative corrections become more and more important. Essentially, for $\Delta y \gg 1$ the subprocess cross section has an expansion in powers of $\alpha_s \Delta y$ rather than α_s.

Dijet production with a large rapidity separation is an example of a ''two-large-scale'' process in perturbative QCD, where large logarithms, in this case $\ln(\hat{s}/E_T^2) \sim \Delta y \gg 1$, arising from real and virtual soft gluon emission compensate the strong coupling in the perturbation series. These logarithms can be resummed using the techniques of Balitskii, Fadin, Kuraev, and Lipatov (BFKL) [1]. Indeed it was first pointed out by Mueller and Navelet [2] that dijet production in high-energy hadron-hadron collisions would be a particularly clean environment in which to look for evidence of such resummation. They showed that the subprocess cross section was expected to increase at asymptotic separations according to $\hat{\sigma} \sim \exp(\lambda \Delta y)$ with $\lambda = \alpha_s 12 \ln 2/\pi$. These ideas were taken further in Refs. [3–6]. In particular, it was shown that the *azimuthal decorrelation* of the jet pair, resulting from multiple soft gluon emission in the rapidity interval between them, provides a particularly distinctive signature of BFKL dynamics, free of potentially confounding effects from dependence on parton distributions. The leading BFKL resummation provides a quantitative prediction for the rate of decorrelation with increasing separation Δy.

The azimuthal decorrelation has recently been measured by the D0 Collaboration [7] at the Tevatron $p\bar{p}$ collider. The results are intriguing. The observed rate of decorrelation is larger than that predicted by an ''exact'' next-to-leading order calculation based on the $2 \to 2$ and $2 \to 3$ matrix elements (as implemented in the JETRAD program [8]), but smaller than that predicted by both the leading BFKL resummation [3,4] and an ''improved'' BFKL calculation incorporating certain subleading kinematical effects [5]. In fact the data agree best with the predictions of the HERWIG parton-shower Monte Carlo program [9], based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) multigluon emission with angular ordering.

The bulk of the theoretical attention concerning BFKL dynamics has focused on its application to the physics at the DESY ep collider HERA—originally, to the behavior of the structure function F_2 at small x, and latterly to more exclusive quantities (for a review see Refs. [10,11], and references therein). In particular, ''forward jet'' production has been studied in Refs. [12–16], and forward single particle and single photon production in Ref. [17]. The possibility of observing ''BFKL gluons'' in small-x deep inelastic scattering has been discussed in Ref. [18]. One result of this activity is the realization that subleading corrections to the leading BFKL resummation, from effects such as the running coupling and phase space, are likely to be numerically important in practice; see, for example, Ref. [19]. Unfortunately the complete perturbative next-to-leading logarithmic corrections are not yet available in a form which allows them to be incorporated in a phenomenological analysis, although there has been considerable theoretical progress towards this goal [20–22].

The aim of the present work is to study such subleading effects and to see whether they could account for the discrepancy between the BFKL predictions and the Tevatron data. We do this by recasting the original analytic resummed expression for the parton cross section [2] in the form of an event generator. This not only allows kinematic subasymptotic constraints (energy conservation, for example) and other corrections to be readily implemented, but also gives a better overall picture of the events with large dijet rapidity separation (for example, the distribution of the accompanying BFKL ''minijets''). A further advantage is that experi-

mental acceptance cuts on the jets are straightforward to impose. A similar Monte Carlo approach was used recently in the fixed coupling limit [23] to look at transverse energy flow in dijet production.

In this paper we concentrate on several issues. First, we set up the calculational framework and demonstrate that we are able to reproduce the results based on the analytic resummed expressions of Refs. [2–4]. Second, we examine the effect on the predicted azimuthal decorrelation of including various subasymptotic effects, including kinematic constraints and the running coupling constant. We will show that these effects are indeed important in the kinematic domain accessible to the Tevatron experiments. More general applications of our formalism, together with a more detailed comparison with data, will be presented elsewhere.

II. BFKL FORMALISM FOR DIJET PRODUCTION

A. Summary of analytic results

We wish to describe events in hadron collisions containing two jets with relatively small transverse momenta p_{T1}, p_{T2} and large rapidity separation $\Delta y \equiv y_1 - y_2$. In the limit where the minimum jet transverse momentum P_T is small compared to the parton-parton center-of-mass energy and Δy is large, the inclusive dijet cross section can be written analytically to leading logarithmic order in the BFKL approach [2].

For jets produced in gluon-gluon collisions (the $q\overline{q}$ initial state can be included via the effective subprocess approximation; see below) the differential cross section is given by

$$\frac{d\hat{\sigma}_{gg}}{dp_{T1}^2 dp_{T2}^2 d\Delta\phi} = \frac{\alpha_s^2 C_A^2 \pi}{2 p_{T1}^3 p_{T2}^3} \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} e^{in\Delta\phi} \frac{1}{2\pi} \mathrm{Re} \int_{-\infty}^{+\infty} dz \, \exp[2t\chi_n(z) + iz \ln(p_{T1}^2/p_{T2}^2)] \tag{1}$$

with $t = \alpha_s C_A \Delta y/\pi$ and

$$\chi_n(z) = \mathrm{Re}\{\psi(1) - \psi[\tfrac{1}{2}(1+|n|) + iz]\}. \tag{2}$$

Here ψ is the logarithmic derivative of the Γ function. We have defined

$$\Delta\phi \equiv |\phi_1 - \phi_2| - \pi \tag{3}$$

so that $\Delta\phi = 0$ when the two jets are back to back in the transverse plane.

Integrating over the dijet transverse momenta above a fixed threshold P_T then gives for the azimuthal distribution

$$\left.\frac{d\hat{\sigma}_{gg}}{d\Delta\phi}\right|_{p_{T1}^2, p_{T2}^2 > P_T^2} = \frac{\alpha_s^2 C_A^2 \pi}{2 P_T^2} \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} e^{in\Delta\phi} C_n(t), \tag{4}$$

with

$$C_n(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{dz}{z^2 + \frac{1}{4}} \exp[2t\chi_n(z)]. \tag{5}$$

The total subprocess cross section is simply proportional to $C_0(t)$:

$$\hat{\sigma}_{gg} = \frac{\alpha_s^2 C_A^2 \pi}{2 P_T^2} C_0(t). \tag{6}$$

Its asymptotic behavior is determined by

$$C_0(t) \begin{cases} = 1 & \text{for } t = 0, \\ \sim [\tfrac{1}{2}\pi 7\zeta(3) t]^{-1/2} e^{4 \ln 2 t} & \text{for } t \to \infty, \end{cases} \tag{7}$$

from which we see the characteristic BFKL prediction of an exponential increase in the cross section with large Δy (equivalently, large t). It can also be seen from Eq. (4) that the average cosine of the azimuthal angle difference $\Delta\phi$ defined above is proportional to $C_1(t)$. In fact we have

$$\langle \cos\Delta\phi \rangle = \frac{C_1(t)}{C_0(t)} \tag{8}$$

and as we shall see below, this falls off with increasing t, exhibiting the decorrelation expected with the emission of gluons in the rapidity interval between the jets.

In what follows we will compare the analytic results just described with results from our formalism described in the next subsection. We will pay particular attention to the total subprocess cross section (6) and the azimuthal angle correlation (8).

B. Iterated solution for use in event generators

The assumptions built into the leading order BFKL formalism that allow us to obtain an analytic solution for the cross section are not all easily satisfied in an experimental situation. In particular, implicit in the above solution are integrations over the transverse momenta of intermediate radiated gluons that extend to infinity. Furthermore nonleading effects (which lead for example to the running of α_s) are also neglected. And finally, the analytic solution (4), (5) is symmetric with respect to the two observed jets, while experimental cuts may not be.

In this section we solve the BFKL equation by iteration, which allows us to avoid these assumptions and obtain a solution that is more directly amenable to comparison with experiment. This solution amounts to ''unfolding'' the summation over the intermediate radiated gluons and making their contributions explicit. It is then straightforward to implement this iterated solution in an event generator, as we describe below.

To obtain the iterated solution we begin with the differential cross section

$$\frac{d\hat{\sigma}_{gg}}{d^2p_{T1}d^2p_{T2}d\Delta y}=\frac{\alpha_s^2C_A^2}{p_{T1}^2p_{T2}^2}\,f(\vec{p}_{T1},\vec{p}_{T2},\Delta y). \tag{9}$$

The Laplace transform $\tilde{f}$ of the function f with respect to Δy satisfies the BFKL equation. Defining

$$\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)=\int_0^\infty \mathrm{d}\Delta e^{-\omega\Delta}f(\vec{p}_{T1},\vec{p}_{T2},\Delta), \tag{10}$$

where for convenience we use $\Delta\equiv\Delta y$, we have

$$\omega\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)=\delta(p_{T1}^2-p_{T2}^2)\,\delta(\phi_1-\phi_2)+\left(\frac{\alpha_sC_A}{\pi^2}\right)\int\frac{d^2q_T}{q_T^2}\left[\tilde{f}(\vec{p}_{T1}+\vec{q}_T,\vec{p}_{T2},\omega)-\frac{p_{T1}^2\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)}{q_T^2+(\vec{p}_{T1}+\vec{q}_T)^2}\right] \tag{11}$$

which is the BFKL equation for dijet production in hadron collisions. Infrared divergences from real gluon emission—the first term in the integral on the right-hand side—are cancelled by the virtual gluon contribution in the second term. Note that in writing Eq. (11) we have assumed that α_s is fixed.

Instead of solving Eq. (11) analytically, which would lead to the results shown above, we solve iteratively, using a slightly modified form of the equation. Following Ref. [18], we note that very low-energy gluons are not resolvable and we therefore separate the real gluon integral into ''resolved'' and ''unresolved'' contributions, according to whether they lie above or below a small transverse energy scale μ. The scale μ is assumed to be small compared to the other relevant scales in the problem (the minimum transverse momentum P_T of the ''external'' jets, for example). We then combine the virtual and unresolved contributions into a single, finite integral. The BFKL equation then becomes

$$\omega\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)=\delta(p_{T1}^2-p_{T2}^2)\,\delta(\phi_1-\phi_2)+\left(\frac{\alpha_sC_A}{\pi^2}\right)\int_{q_T^2>\mu^2}\frac{d^2q_T}{q_T^2}\,\tilde{f}(\vec{p}_{T1}+\vec{q}_T,\vec{p}_{T2},\omega)-\left(\frac{\alpha_sC_A}{\pi^2}\right)\int\frac{d^2q_T}{q_T^2}\left[\tilde{f}(\vec{p}_{T1}+\vec{q}_T,\vec{p}_{T2},\omega)\times\theta(\mu^2-q_T^2)-\frac{p_{T1}^2\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)}{q_T^2+(\vec{p}_{T1}+\vec{q}_T)^2}\right]. \tag{12}$$

The combined unresolved and virtual integral can be simplified by noting that $q_T\ll p_{T1}$ by construction because $\mu^2\ll P_T^2$ and $p_{T1}>P_T$. Therefore

$$\tilde{f}(\vec{p}_{T1}+\vec{q}_T,\vec{p}_{T2},\omega)\approx\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega), \tag{13}$$

which allows us to write

$$(\omega-\omega_0)\tilde{f}(\vec{p}_{T1},\vec{p}_{T2},\omega)=\delta(p_{T1}^2-p_{T2}^2)\,\delta(\phi_1-\phi_2)+\left(\frac{\alpha_sC_A}{\pi^2}\right)\int_{q_T^2>\mu^2}\frac{d^2q_T}{q_T^2}\times\tilde{f}(\vec{p}_{T1}+\vec{q}_T,\vec{p}_{T2},\omega), \tag{14}$$

where we have defined

$$\omega_0\equiv\left(\frac{\alpha_sC_A}{\pi^2}\right)\int\frac{d^2q_T}{q_T^2}\left[\theta(\mu^2-q_T^2)-\frac{p_{T1}^2}{q_T^2+(\vec{p}_{T1}+\vec{q}_T)^2}\right]. \tag{15}$$

The virtual and unresolved contributions are now contained in ω_0 and we are left with an integral over resolved real gluons.

We now solve Eq. (21) iteratively, and performing the inverse transform we have

$$f(\vec{p}_{T1},\vec{p}_{T2},\Delta)=\sum_{n=0}^{\infty}f^{(n)}(\vec{p}_{T1},\vec{p}_{T2},\Delta), \tag{16}$$

where the exact form of $f^{(n)}$ depends on whether the coupling α_s is taken to be fixed or running. Strictly speaking, the derivation above only applies for fixed coupling because we have left α_s outside the integrals. The modifications necessary to account for running coupling are discussed below.

1. Solution for fixed α_s

For the case where α_s is fixed (evaluated at the scale P_T^2, for example), the integral in Eq. (15) is straightforward to evaluate and we obtain

$$\omega_0=\left(\frac{\alpha_sC_A}{\pi^2}\right)\ln(\mu^2/p_{T1}^2). \tag{17}$$

Substituting into Eq. (14), solving by iteration and then performing the inverse Laplace transform, we find

$$f^{(0)}(\vec{p}_{T1},\vec{p}_{T2},\Delta)=\left[\frac{\mu^2}{p_{T1}^2}\right]^{\alpha_sC_A\Delta/\pi}\delta^{(2)}(\vec{p}_{T1}+\vec{p}_{T2}) \tag{18}$$

and, for $n\neq 0$,

$$f^{(n)}(\vec{p}_{T1},\vec{p}_{T2},\Delta)=\left(\frac{\alpha_s C_A}{\pi^2}\right)^n\left[\frac{\mu^2}{p_{T1}^2}\right]^{\alpha_s C_A\Delta/\pi}\prod_{i=1}^{n}\int\frac{d^2q_{Ti}}{q_{Ti}^2}\,\theta(q_{Ti}^2-\mu^2)\int_0^{\Delta}dY_1\int_0^{Y_1}dY_2\cdots\int_0^{Y_{n-1}}dY_n\,\delta^{(2)}\left(\vec{p}_{T1}+\vec{p}_{T2}+\sum_{i=1}^{n}\vec{q}_{Ti}\right)$$
$$\times\left[\frac{p_{T1}^2}{(\vec{p}_{T1}+\vec{q}_{T1})^2}\right]^{\alpha_s C_A Y_1/\pi}\left[\frac{(\vec{p}_{T1}+\vec{q}_{T1})^2)}{(\vec{p}_{T1}+\vec{q}_{T1}+\vec{q}_{T2})^2}\right]^{\alpha_s C_A Y_2/\pi}\cdots\left[\frac{(\vec{p}_{T1}+\Sigma_{i=1}^{n-1}\vec{q}_{Ti})^2}{(\vec{p}_{T1}+\Sigma_{i=1}^{n}\vec{q}_{Ti})^2}\right]^{\alpha_s C_A Y_n/\pi}. \tag{19}$$

The inverse transform has given rise to a set of nested integrals over the variables Y_i, which can be interpreted as the rapidities of the emitted gluons. The differential subprocess cross section is then given by

$$\frac{d\hat{\sigma}_{gg}}{d^2p_{T1}d^2p_{T2}d\Delta y}=\frac{\alpha_s^2C_A^2}{p_{T1}^2p_{T2}^2}\sum_{n=0}^{\infty}f^{(n)}(\vec{p}_{T1},\vec{p}_{T2},\Delta). \tag{20}$$

The subprocess cross section is now expressed as an explicit sum over radiated gluons, with corresponding $\vec{q}_{Ti}$ and Y_i integrals over their phase space. It is straightforward to implement this in a Monte Carlo event generator,[1] and to impose energy conservation and experimental cuts via limits on the integration; see below for numerical results.

An interesting feature of the results (18) and (19) is the presence of the form factors

$$\left[\frac{\mu^2}{p_{T1}^2}\right]^{\alpha_s C_A\Delta/\pi}$$

and

$$\left[\frac{(\vec{p}_{T1}+\Sigma_{i=1}^{n-1}\vec{q}_{Ti})^2}{(\vec{p}_{T1}+\Sigma_{i=1}^{n}\vec{q}_{Ti})^2}\right]^{\alpha_s C_A Y_n/\pi},$$

respectively. These form factors arise from the resummation of the unresolved $q_{Ti}^2<\mu^2$ soft gluon emission in the rapidity interval Δ. In particular we see the modification of the naive zeroth-order perturbative result $\delta(\vec{p}_{T1}+\vec{p}_{T2})$ by

$$\left[\frac{\mu^2}{p_{T1}^2}\right]^{\alpha_s C_A\Delta/\pi}<1 \quad \text{for } p_{T1}^2>\mu^2,\ \Delta>0. \tag{21}$$

This is a consequence of the fact that the emission of soft gluons reduces the probability of the dijets having equal transverse momenta and being back to back in azimuth. For $\Delta=0$ all radiation is suppressed (in this approximation) and the form factor is equal to unity.

We can make contact with the analytic results of the previous section by noting that the only additional approximation we have made is Eq. (13), which we used in the computation of ω_0, thereby neglecting momenta smaller than μ compared to p_{T1}. We therefore expect our result to agree with the analytic one up to corrections of $O(\mu^2/p_{Ti}^2)$. In fact one can show that the dijet cross section (20) integrated over the external transverse momenta

$$\frac{d\hat{\sigma}_{gg}}{d\Delta\phi}=\int d^2p_{T1}d^2p_{T2}\,\delta(|\phi_1-\phi_2|-\pi-\Delta\phi)\times\theta(p_{T1}^2-P_T^2)\,\theta(p_{T2}^2-P_T^2)\,\frac{d\hat{\sigma}_{gg}}{d^2p_{T1}d^2p_{T2}d\Delta y} \tag{22}$$

corresponds *exactly* to the analytic result (6) in the limit $\mu^2\to 0$. This will be illustrated numerically below. In the simulations based on Tevatron kinematics to be described in the following section, where $p_{Ti}\gtrsim 10$ GeV, we will use values $\mu\sim 1$ GeV such that the finite $O(\mu^2/p_{Ti}^2)$ corrections are negligible.

2. Solution for running α_s

Higher order corrections are known to lead to the running of the coupling constant α_s [22]. They can therefore be taken into account by including momentum dependence in the coupling associated with the emission of each gluon. We shall see that this leads to two types of modification: (i) the factors of α_s associated with resolved real gluon emission are simply evaluated at the scale of the emitted gluon and (ii) the form factors associated with the unresolved real and virtual gluon emission get modified slightly.

We will include the running to lowest order, taking

$$\alpha_s(q^2)=\frac{1}{b\,\ln(q^2/\Lambda^2)}, \tag{23}$$

where

$$b=\frac{33-2N_f}{12\pi} \tag{24}$$

with $N_f=4$. We then pull the factors of α_s in Eqs. (14) and (15) inside the integrals and make the substitution $\alpha_s\to\alpha_s(q_T^2)$. In addition, we must regulate the behavior of $\alpha_s(q^2)$ to prevent its becoming unphysically large as q^2 becomes small. This can happen, for example, in the momentum integration in the expression for ω_0. Here we simply assume that the value of α_s freezes out below some scale $Q_0>\Lambda$; i.e., we take

$$\alpha_s(q^2)\begin{cases}=\alpha_0\equiv\alpha_s(Q_0^2) & \text{for } q^2\leqslant Q_0^2,\\ =\alpha_s(q^2) & \text{for } q^2\geqslant Q_0^2.\end{cases} \tag{25}$$

In practice choosing values $\Lambda<Q_0<\mu$ so that α_0 is of order 1, as we do below, gives results that are insensitive to the exact choice.

The iterated solution to the BFKL equation in the running coupling case is then given by

[1] See also Ref. [23] for a similar approach.

$$f^{(n)}(\vec{p}_{T1},\vec{p}_{T2},\Delta)=\left[\frac{\alpha_s(p_{T1}^2)}{\alpha_s(\mu^2)}\right]^{C_A\Delta/\pi b}\prod_{i=1}^{n}\int d^2q_{Ti}\,\theta(q_{Ti}^2-\mu^2)\,\frac{\alpha_s(q_{Ti}^2)C_A}{\pi^2 q_{Ti}^2}$$
$$\times\int_0^{\Delta}dY_1\int_0^{Y_1}dY_2\cdots\int_0^{Y_{n-1}}dY_n\,\delta^{(2)}\left(\vec{p}_{T1}+\vec{p}_{T2}+\sum_{i=1}^{n}\vec{q}_{Ti}\right)$$
$$\times\left[\frac{\overline{\alpha_s}((\vec{p}_{T1}+\vec{q}_{T1})^2)}{\overline{\alpha_s}(p_{T1}^2)}\right]^{C_AY_1/\pi b}\left[\frac{\overline{\alpha_s}((\vec{p}_{T1}+\vec{q}_{T1}+\vec{q}_{T2})^2)}{\overline{\alpha_s}((\vec{p}_{T1}+\vec{q}_{T1})^2)}\right]^{C_AY_2/\pi b}\cdots\left[\frac{\overline{\alpha_s}((\vec{p}_{T1}+\Sigma_{i=1}^{n}\vec{q}_{Ti})^2)}{\overline{\alpha_s}((\vec{p}_{T1}+\Sigma_{i=1}^{n-1}\vec{q}_{Ti})^2)}\right]^{C_AY_n/\pi b},\tag{26}$$

$$f^{(0)}(\vec{p}_{T1},\vec{p}_{T2},\Delta)=\left[\frac{\alpha_s(p_{T1}^2)}{\alpha_s(\mu^2)}\right]^{C_A\Delta/\pi b}\delta^{(2)}(\vec{p}_{T1}+\vec{p}_{T2}),\tag{27}$$

where

$$\overline{\alpha_s}(q^2)\begin{cases}=(Q_0^2/q^2)^{\alpha_0 b}\alpha_0 & \text{for } q^2\leqslant Q_0^2,\\ =\alpha_s(q^2) & \text{for } q^2\geqslant Q_0^2.\end{cases}\tag{28}$$

Thus the result for running α_s is obtained by the replacements

$$\alpha_s^n\to\prod_{i=1}^{n}\overline{\alpha_s}(q_{Ti}^2),$$

$$\left[\frac{\mu^2}{q^2}\right]^{\alpha_sC_AY/\pi}\to\left[\frac{\overline{\alpha_s}(q^2)}{\overline{\alpha_s}(\mu^2)}\right]^{C_AY/\pi b}\tag{29}$$

in the fixed coupling results (18), (19) for $f^{(0)}$ and $f^{(n)}$ and

$$\alpha_s^2\to\alpha_s(p_{T1}^2)\alpha_s(p_{T2}^2)\tag{30}$$

in the differential cross section (20). Note that $\overline{\alpha_s}$ is equal to α_s unless it is evaluated at a momentum smaller than the ''freeze-out'' scale Q_0. We maintain the hierarchy of scales $\Lambda<Q_0<\mu\ll P_T$.

C. Cross section

The calculation is completed by weighting the integrand in the definition of $f^{(n)}$ [Eqs. (19), (26)] in the subprocess cross section with parton distributions $G(x_1,Q^2)G(x_2,Q^2)$ where, using the ''effective subprocess approximation,''

$$G(x,Q^2)=g(x,Q^2)+\frac{4}{9}\sum_{q=u,d,s,c}[q(x,Q^2)+\overline{q}(x,Q^2)].\tag{31}$$

The parton momentum fractions x_1 and x_2 are determined by the invariant mass $\sqrt{\hat{s}}$ and rapidity Y of the multijet final state:

$$x_1=\sqrt{\frac{\hat{s}}{s}}\,e^{Y}=\frac{1}{\sqrt{s}}\,e^{-\Delta/2+Y}\left(p_{T1}e^{\Delta}+p_{T2}+\sum_i q_{Ti}e^{Y_i}\right),$$

$$x_2=\sqrt{\frac{\hat{s}}{s}}\,e^{-Y}=\frac{1}{\sqrt{s}}\,e^{\Delta/2-Y}\left(p_{T1}e^{-\Delta}+p_{T2}+\sum_i q_{Ti}e^{-Y_i}\right),\tag{32}$$

where $\sqrt{s}$ is the total collision energy. In the numerical studies to be described below we choose the factorization scale $Q^2=P_T^2$.

Note that the requirement $x_1,x_2\leqslant 1$ effectively imposes an upper limit on the transverse momentum (q_{Ti}) integrals. This in turn means that the analytic results (2), (4) are *not* reproduced in the presence of such a constraint, since they require the internal transverse momenta integrals to extend to infinity. In the original work of Ref. [2] (see also [4]) the strong ordering of the rapidities was used to approximate the right-hand side of Eq. (32) by

$$x_1=\frac{p_{T1}}{\sqrt{s}}\,e^{\Delta/2+Y},$$

$$x_2=\frac{p_{T2}}{\sqrt{s}}\,e^{\Delta/2-Y}\tag{33}$$

so that there was no longer any constraint on the q_{Ti}. An improved approximation where account was taken of the additional energy required for the multigluon emission was studied in Ref. [5]. In our approach all kinematic constraints are applied directly to the multijet final state.

III. NUMERICAL RESULTS

In this section we present numerical results from our event generator for dijet production in the BFKL approach. We will take a brief look at the properties of the subprocess cross section $\hat{\sigma}$ and then present predictions for the Tevatron $p\overline{p}$ collider with center-of-mass energy 1.8 TeV.

In our numerical computations we make the following choices for the relevant parameters. The minimum transverse momentum for each jet of the pair is $P_T=20$ GeV, and for simplicity the two jets are assumed to have equal and opposite rapidities: $y_1=-y_2=\Delta/2$. Unless otherwise noted the scale μ that defines the boundary between resolved and unresolved gluons we take to be $\mu=1$ GeV. In the fixed coupling case we evaluate α_s at the scale P_T^2; for $\Lambda=200$ MeV (as dictated by our choice of parton distributions [24]; see below) and our choice of P_T, this gives $\alpha_s=0.164$. In the running coupling case we again have $\Lambda=200$ MeV and we

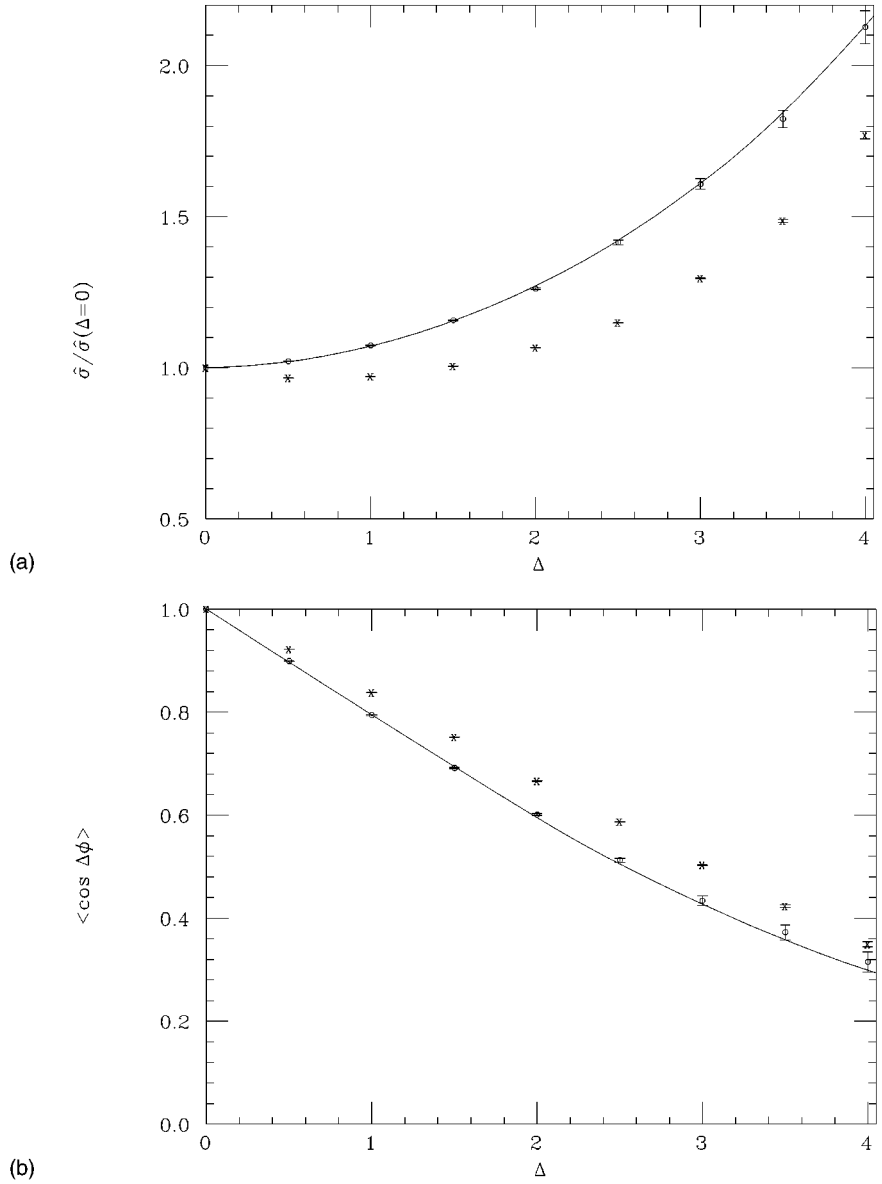

FIG. 1. (a) The subprocess cross section $\hat{\sigma}$ for dijet production as a function of the dijet rapidity difference Δ for α_s fixed (circles) and running (crosses), for minimum jet transverse momentum $P_T = 20$ GeV and $\mu = 1$ GeV. The analytic solution is shown as a solid curve. In each case the cross section is normalized to its value at $\Delta = 0$. Errors are from Monte Carlo statistics. (b) The mean of cos $\Delta\phi$ computed from the subprocess cross section, for the same cases as in (a).

choose $Q_0 = 0.425$ GeV so that $\alpha_0 = 1$. Finally, in our Tevatron calculations we use the leading-order parton distribution functions of Ref. [24].

We begin with the subprocess cross section, shown in Fig. 1(a) as a function of the dijet rapidity difference Δ, normalized to its value at $\Delta = 0$. The results for fixed α_s are shown as open circles (the error bars shown are from Monte Carlo statistics), and we see the characteristic BFKL exponential rise with increasing Δ. We also see that the analytic result, shown as a solid curve, is well-reproduced by the iterated solution. The cross section for running α_s is shown as the points marked by crosses, and we see that the running of the coupling has the effect of slightly suppressing the increase in $\hat{\sigma}$ with Δ compared to the fixed case. In Fig. 1(b) we show $\langle \cos \Delta\phi \rangle$ computed from the subprocess cross section $\hat{\sigma}$ for the same cases as in Fig. 1(a). The fixed-α_s results (circles) agree very well with the analytic prediction (solid curve), while the running α_s results show a slightly slower decorrelation. We will see a further slowing of the decorrelation when conservation of energy and parton distributions are included below.

In Fig. 2 we investigate the sensitivity of our results to the choice of gluon resolution cutoff μ. As noted above, we expect corrections to be of order μ^2/P_T^2 so that the μ dependence becomes stronger as μ approaches P_T, but for μ small enough there should be relatively little sensitivity. This is illustrated in Fig. 2 where we show the cross section $\hat{\sigma}$ as a function of μ for two values of the dijet rapidity difference Δ. In this and the following figures we show results for running α_s. We see the expected behavior and also that larger values of Δ exhibit more sensitivity to μ. This is related to the fact, as we will see explicitly below, that larger Δ means more emitted gluons. This in turn leads to more sensitivity to the resolution parameter. We also note that the presence of

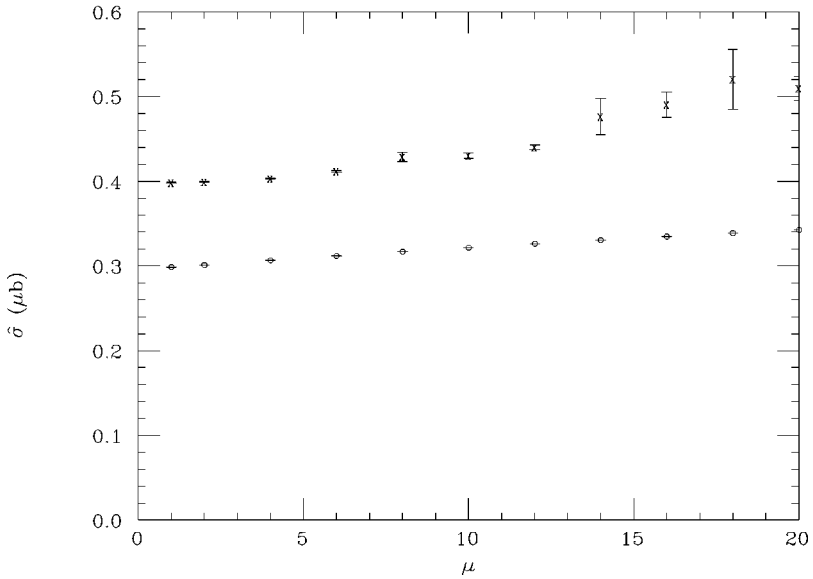

FIG. 2. The subprocess cross section $\hat{\sigma}$ for dijet production as a function of the gluon resolution cutoff μ for $\Delta=1$ (circles) and $\Delta=3$ (crosses) and $P_T=20$ GeV, with running α_s.

other scales in the problem besides P_T (Λ, Q_0) mean that arguments about the size of corrections are not rigorous. In any case, it is clear from the figure that our choice of $\mu=1$ GeV is sufficient to guarantee that we are not sensitive to its exact value.

We examine in Fig. 3 how the cross section is distributed among the contributions from different numbers of emitted gluons for various values of Δ as in the previous figure. As expected, larger rapidity differences between the jets allow for larger numbers of emitted gluons, so that the cross section peaks at higher n_g with increasing Δ. Note that the increasing area of the histograms simply reflects the rise of the cross section with increasing Δ. We should point out that there is *not* a direct correspondence between emitted gluons and physical jets, so that these results for n_g should not be interpreted directly as a prediction for numbers of jets. By the same token, we note that the distribution in n_g depends to some extent on the resolution cutoff parameter μ—larger values of μ lead to fewer resolvable gluons.

We now turn from the subprocess cross section to the total cross section integrated over parton distributions. Figure 4 shows the cross section as a function of dijet rapidity difference Δ for fixed and running α_s. In both cases the falling parton densities more than compensate for the rise in $\hat{\sigma}$ with the net result that the cross section falls off with increasing Δ. Comparing the BFKL results with lowest-order QCD, shown as a solid curve in the figure, shows that the BFKL cross sections fall off more slowly than lowest-order QCD. The latter curve simply reflects the reduction in available phase space as Δ increases. The slower rise in $\hat{\sigma}$ for the

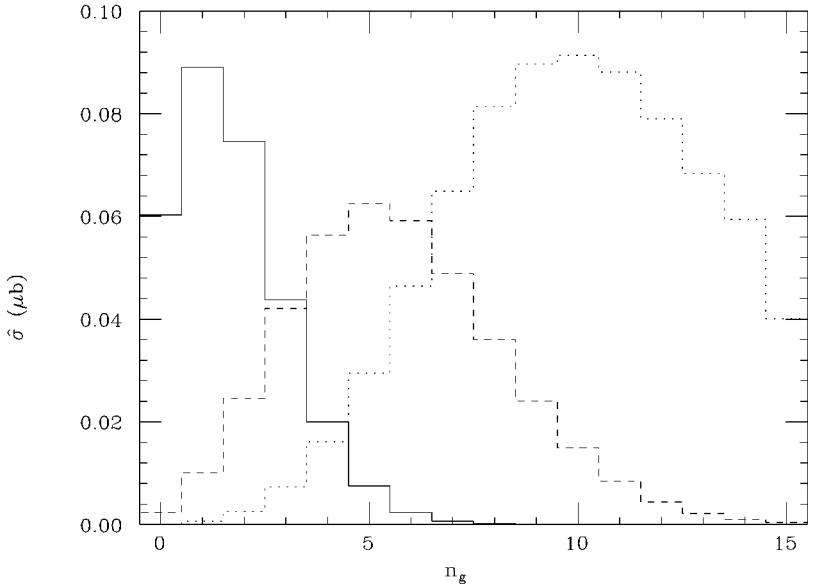

FIG. 3. The contributions to the subprocess cross section $\hat{\sigma}$ for dijet production from different numbers of emitted (i.e., resolved) gluons n_g, for $\Delta=1$ (solid histogram), 3 (dashed), 5 (dotted), running α_s, $P_T=20$ GeV and $\mu=1$ GeV.

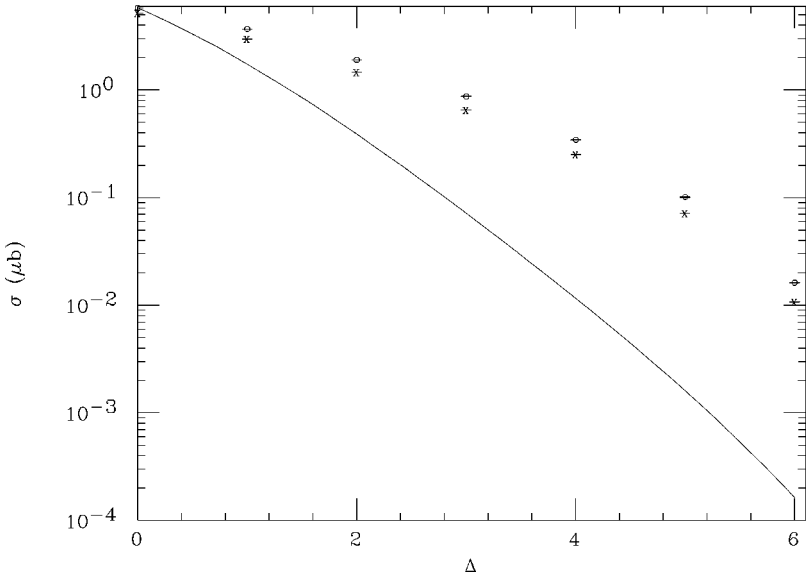

FIG. 4. The total cross section for dijet production at the Tevatron as a function of the dijet rapidity difference Δ for α_s fixed (circles) and running (crosses), for minimum jet transverse momentum $P_T = 20$ GeV and $\mu = 1$ GeV. The lowest order QCD result is shown as a solid curve.

running case that we saw in Fig. 1 translates here into a faster falloff than one expects in the fixed coupling case.

Because of the subtleties involved in untangling the BFKL prediction for $\hat{\sigma}$ from the effects of the falling parton densities in the measured dijet cross section, it was proposed in Refs. [3,4] to measure the azimuthal angle decorrelation between the two jets. This quantity is relatively insensitive to the details of the parton densities and provides a clear distinction between the predictions of next-to-leading-order QCD and BFKL. As noted above, previous comparisons were to BFKL predictions which did not account for subleading effects such as the running of α_s and truncation of transverse momentum integrals for the emitted gluons. In Fig. 5 we present our prediction for $\langle \cos \Delta\phi \rangle$ with these effects included. For comparison, the analytic BFKL prediction is shown as a solid curve. Clearly, subleading effects are quite substantial. In particular we see that the azimuthal decorrelation occurs more slowly with increasing dijet rapidity difference than predicted by the analytic BFKL result. This can be understood partly in terms of phase space availability—for a given transverse momentum threshold, dijets produced at larger rapidities require more energy, leaving less phase space available for emission of gluons. This effect—not present in the analytic solution—partly mitigates

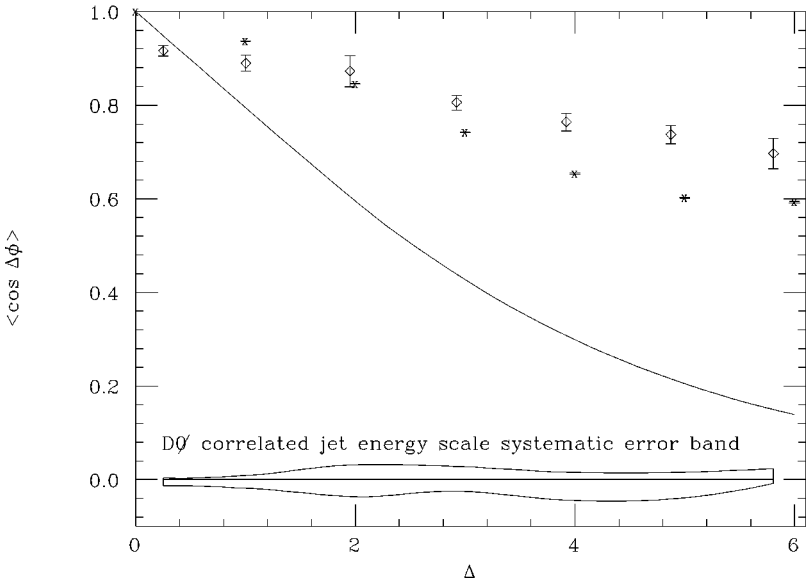

FIG. 5. The azimuthal angle decorrelation in dijet production at the Tevatron as a function of dijet rapidity difference Δ for running α_s (crosses), for minimum jet transverse momentum $P_T = 20$ GeV and $\mu = 1$ GeV. The analytic BFKL solution is shown as a solid curve and a preliminary measurement from D0 [7] is shown as diamonds. Error bars on D0 points represent statistical and uncorrelated systematic errors; correlated jet energy scale systematic errors are shown as an error band. See text for further explanation.

the increasing probability for emitting more gluons in the center-of-mass system. Because the additional emitted gluons are responsible for the decorrelation, the result of including subleading effects is a reduction in azimuthal decorrelation compared to the analytic BFKL solution, as seen in the figure.

Finally, we return to the question that originally motivated this work: can the inclusion of subleading effects improve the agreement between the measured dijet decorrelation and that predicted by BFKL? We show for reference in Fig. 5 some recent preliminary measurements from the D0 Collaboration [7] with the same minimum jet transverse momentum of 20 GeV. The comparison should be taken as a rough guide only, because our predictions (and our kinematic cuts) are at the parton level, and we assume equal and opposite rapidities of the dijets. The D0 measurements (and cuts) are at the jet level, and the net dijet rapidity is allowed to range between ± 0.5. The finite width of the jets will lead, for example, to $\langle \cos \Delta\phi \rangle \neq 1$ even at $\Delta = 0$. Furthermore, the results are not final, and the error bars in the figure represent the statistical and uncorrelated systematic uncertainties only; an error band showing correlated jet energy scale systematic uncertainties appears at the bottom of the figure. See Ref. [7] for a full explanation of the data. Having made those qualifications, we note a marked improvement in agreement between data and the BFKL prediction when subleading effects are included.

IV. CONCLUSIONS

We have presented the formalism and numerical results for dijet production at hadron colliders in the BFKL approach, using an improved formalism incorporating an iterated solution (as described for deep inelastic scattering in [18]) that unfolds the sum over emitted gluons that is implicit in the analytic solution to the BFKL equation. We have cast the iterated solution in the form of an event generator. This allows us to incorporate subleading effects such as energy conservation and other kinematic constraints as well as the running of the strong coupling constant, which are necessarily absent in the analytic approach. It also allows us to examine the properties of dijet events. We find that the subleading effects included in the improved formalism can be substantial, and in particular they lead to improved agreement with measurements of the azimuthal decorrelation in dijet production at the Fermilab Tevatron. Further elaboration of our results for hadron colliders, as well as applications of the formalism to forward jet production in ep collisions, will appear in future work.

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