Momentum distribution in the decay $B \to J/\psi + X$

William F. Palmer

Department of Physics, The Ohio State University, Columbus, Ohio 43210

Emmanuel A. Paschos and Peter H. Soldan

Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany

We combine the NRQCD formalism for the inclusive color singlet and octet production of charmonium states with the parton and the ACCMM model, respectively, and calculate the momentum distribution in the decay $B \to J/\psi + X$. Neglecting the kinematics of soft gluon radiation, we find that the motion of the b quark in the bound state can account, to a large extent, for the observed spectrum. The parton model gives a satisfactory presentation of the data, provided that the heavy quark momentum distribution is taken to be soft. To be explicit, we obtain $\varepsilon_p \sim 0.008-0.012$ for the parameter of the distribution function of Peterson *et al.* The ACCMM model can account for the data more accurately. The preferred Fermi momentum $p_F \cong 0.57$ GeV is in good agreement with recent studies of the heavy quark's kinetic energy. [S0556-2821(97)01321-0]

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I. INTRODUCTION

The ARGUS and CLEO Collaborations reported results on inclusive B meson decays to J/ψ, where they identified a sizable component of decays with three or more particles in the final state [1]. In a recent publication [2] CLEO presented an analysis based on their sample of data which is an order of magnitude larger than those of previous studies, corresponding to a reduction of errors by a factor of 2.4. As a result the group finds the direct branching ratio $\mathcal{B}(B \to J/\psi + X) = (0.80 \pm 0.08)\%$.

In the context of the color singlet (wave function) model this process is related to the decay of a b quark in the B meson, at short distances, into a color singlet $c\bar{c}$ pair plus other quarks and gluons. The c and $\bar{c}$ quarks generated by this method have almost equal momenta and reside in the $^3S_1$ angular momentum state. Several authors performed theoretical studies of the branching ratio $\mathcal{B}(b \to J/\psi + X)$, to leading order in α_s, using the color singlet approach [3]. The measurements indicate that this approach underestimates the data by roughly a factor of three. The inconsistency remains when the next-to-leading order perturbative corrections are included [4,5]. Thus it is interesting to consider generalizations of the color singlet model.

The nonrelativistic QCD (NRQCD) factorization formalism developed by Bodwin, Braaten, and Lepage (BBL) [6] makes possible a systematic treatment of soft gluon effects in the inclusive heavy quarkonium production. It allows for the creation, at short distances, of a heavy quark and antiquark pair in a color octet configuration which subsequently evolves into a physical bound state through the emission and absorption of soft gluons.

Color octet contributions to inclusive charmonium production have been recently investigated in hadronic reactions at collider and fixed target experiments [7–10], in e^+e^- annihilations [11], γ-nucleon reactions at fixed target and DESY ep collider HERA energies [12–14], in Z^0 decays at the CERN e^+e^- collider LEP [15], and in lepton-nucleon reactions [16]. A comprehensive review of recent developments in this field can be found in Ref. [17]. In our analysis, we shall refer to previous studies of inclusive B meson decays to J/ψ [14,18,19]. These articles considered the decay of free b quarks, including also the leading color octet contributions to the branching ratio $\mathcal{B}(b \to J/\psi + X)$. However, the branching ratio is very sensitive to the numerical values chosen for the Wilson coefficients and the NRQCD matrix elements. In this situation it is desirable to study additional predictions of the theory. A second comparison deals with the J/ψ momentum distribution. Adopting the NRQCD formalism to leading order in the nonrelativistic expansion, we shall argue that the observed momentum distribution can be attributed, to a large extent, to the B meson bound state corrections.

So far there is no detailed theoretical fit of the momentum spectrum. Palmer and Stech have made a first attempt using a simple wave function formalism [20]. In this paper we investigate two different approaches and compare them with the experimental results. As stated above, a sizable component in the decay consists of nonresonant multiparticle final states. Consequently, for a wide range of the phase space an inclusive description based on quark-hadron duality is appropriate. This approach was applied extensively to the inclusive semileptonic decays of B mesons.

Over the last few years, considerable progress has been made in the calculation of nonperturbative corrections to the lepton energy spectrum of the decay $B \to e\nu + X$ using the operator product expansion (OPE) [21]. The latter approach involves a series in powers of $1/(1-y)m_b$ with y being the normalized lepton energy and incorporates the formalism of the heavy quark effective theory (HQET) [22]. However, an analysis of the decay $B \to J/\psi + X$ using the HQET is of limited validity, since due to the smaller energy release the convergence of the OPE is slower than for the semileptonic decays. Therefore we shall resort to a formalism in which the momentum distribution of the heavy quark in the B meson, the latter representing the dominant bound state effect, is modeled in terms of parameters obtained from experiment. The motion of the b quark will be referred to as the smearing

of its momentum or the Fermi motion within the B meson.

Several effects may contribute to the momentum spectrum of the J/ψ: the motion of the b quark in the bound state; the shifting of the $c\bar{c}$ momentum due to perturbative QCD corrections to the $b \rightarrow c\bar{c}s$ matrix element, carried out beyond the leading logarithm approximation (LLA); and the emission or absorption of soft gluons at the hadronization stage. In this work we focus on the dependence of the spectrum on the smearing of the b quark momentum in the B meson using the LLA for the short-distance matrix element. As far as soft gluon radiation is concerned, we account for the color flow using the NRQCD formalism, but neglect the momentum flow carried by the soft gluons. This approximation corresponds to keeping the leading nonvanishing orders in the v^2 expansion for the color singlet and octet states, respectively (v being the relative velocity of the c and $\bar{c}$). One might note, however, that the v^2 expansion fails to converge near the boundaries of the phase space. In this region, which is sensitive to the mass difference between the partonic $c\bar{c}$ and the hadronic J/ψ final state, the momenta of soft gluons become important and influence the spectrum [23].

By analyzing the dependence of the J/ψ momentum distribution on the initial bound state corrections, we explore one of the two relevant nonperturbative effects numerically, keeping in mind that the soft gluons, which are radiated within the fragmentation process, must be included in the future. To be explicit, we investigate to what extent the b quark motion in the B meson can account for the observed spectrum in $B \rightarrow J/\psi + X$ decays.

The paper is organized as follows. In Sec. II we outline the basic ideas of the NRQCD approach as they apply to the inclusive charmonium production in b decays. We adopt the NRQCD to the decay of a $b \rightarrow J/\psi + X$, where we emphasize that the b quark is considered to be free but the $c\bar{c}$ pairs from both color singlet and octet states convert into the hadronic J/ψ final state. The latter step is treated in the leading nonvanishing orders of the v^2 expansion for the singlet and octet states, respectively. In order to account for the measured J/ψ momentum spectrum, we shall combine this formalism with the B meson bound state corrections, i.e., we consider the smearing of the b quark momentum in the meson. This we do in two models. In Sec. III we present the parton model which gives a satisfactory presentation of the data. Then we calculate in Sec. IV the J/ψ momentum spectrum in the framework of the ACCMM model [24], which is in better agreement with the data. In this context, we take the Fermi motion parameter p_F within the range obtained from theoretical studies of the b quark's kinetic energy. Our analysis confirms that, by including the leading color octet contributions to the charmonium production, one can account for the measured branching ratio. In addition, we find that the b quark motion gives a good representation of the spectrum. The summary and the discussion of our results can be found in Sec. V.

II. $b \rightarrow J/\psi + X$ IN THE NRQCD FORMALISM

The nonrelativistic-QCD (NRQCD) formalism, developed by BBL [6], represents a comprehensive theoretical framework for the study of inclusive charmonium production. De-
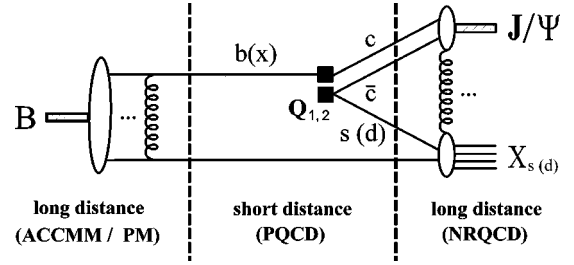

FIG. 1. Separation of the various distances contributing to the $B \rightarrow J/\psi + X$ decays. $Q_{1,2}$ are the four-quark operators which induce the effective $b \rightarrow c\bar{c}q_f$ transitions.

noting by v the relative velocity of the two heavy constituents inside the $(Q\bar{Q})$ bound state, the NRQCD approach introduces the complete structure of the quarkonium Fock space to a given order in v^2 and thus allows for a consistent factorization of long- and short-distance effects. Consequently, the charmonium production rate is represented by a sum of products, each of which consists of a short-distance coefficient, associated with the creation of a heavy quark and antiquark pair in a specific angular and color configuration, and a nonperturbative NRQCD matrix element $\langle 0|O^H(^{2S+1}L_J)_a|0\rangle$ which parametrizes the subsequent evolution of the intermediate $c\bar{c}(^{2S+1}L_J)_a$ state into a physical charmonium bound state H (plus light hadrons). The latter step is generated through the emission and absorption of soft gluons.

The separation of the distance scales (see Fig. 1), and its association to the small value of the constituents' relative velocity ($v_c^2 \sim 0.23$–0.30 for charmonium), makes possible the calculation of the heavy quarkonium production rate in terms of a double series in powers of v^2 and α_s [numerically $\alpha_s(m_c^2) \sim v_c^2$]. This way the production through an intermediate color octet state, although down by powers of v^2 in the Fock state hierarchy, can be numerically relevant in view of possible short-distance enhancements. The authors of Refs. [14,18,19] pointed out that, for inclusive B decays to S-wave charmonia, the color octet production mechanism is indeed important, even though it is of order v^4 with respect to the basic color singlet one, because the Wilson coefficient of the color octet term is strongly enhanced in comparison to that of the color singlet transition. Since we shall use formulas for the decay rate $\Gamma(b \rightarrow J/\psi + X)$ in our analysis of the ACCMM model, we briefly outline the basic ideas of the NRQCD approach.

At the b mass scale, the Hamiltonian which induces the effective $b \rightarrow c\bar{c}q_f$ ($f = s,d$) transitions reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cf}^* [\tfrac{1}{3}(2C_+ - C_-)\bar{c}\gamma_\mu Lc\,\bar{q}_f\gamma^\mu Lb$$
$$+ (C_+ + C_-)\bar{c}\gamma_\mu LT^a c\,\bar{q}_f\gamma^\mu LT^a b], \tag{1}$$

$L = 1 - \gamma_5$, where operators arising from penguin and box diagrams have been neglected. The renormalization (Wilson) coefficients $C_\pm(\mu)$ have been computed up to the next-to-leading order corrections in Ref. [25].

In the leading logarithm approximation (LLA), the decay amplitude refers to the tree level matrix element of the ef-

fective Hamiltonian. Consequently, the renormalization scale dependence contained in the Wilson coefficient functions, $C_\pm(\mu)$, cannot be cancelled by this matrix element. While this work was being completed, a study of higher order perturbative corrections to the matrix element of $b \rightarrow J/\psi + X$ decays appeared [5], in which a double series expansion in α_s and the small ratio of the (LL) color singlet to octet Wilson coefficients was performed. Using the color singlet approximation for charmonium production, the authors obtained a result which carries only weak dependence on the renormalization scale. The predicted branching ratio, $\mathcal{B}(B \rightarrow J/\psi + X) = 0.9^{+1.1}_{-0.3} \times 10^{-3}$ lies well below the experimental value of $(0.80 \pm 0.08)\%$ reported by the CLEO Collaboration. This confirms the supposition that a nonperturbative effect in the $B \rightarrow J/\psi + X$ transition is required, which goes beyond the color singlet approximation, namely the color octet (NRQCD) mechanism for charmonium production. In future studies the leading order result for the color singlet contribution can be replaced by the expression given in Ref. [5]. In this article we keep the leading logarithm approximation, assuming that the modification of the J/ψ momentum distribution will be small.

In order to obtain the nonrelativistic interaction from the effective Hamiltonian of Eq. (1), one expresses the Dirac bilinears of the c and $\bar{c}$ in terms of (two-component) Pauli spinors ξ and η, the constituents' relative three-momentum $\mathbf{q}$, and the Pauli matrices σ_i. This allows one to consider separately states with specific angular momentum and color (for details see Ref. [9]). To linear order in $v = |\mathbf{q}|/m_c$, the short-distance amplitude $\mathcal{A}$ (using a notation in which the spinors do not carry color indices) can be rewritten as [19]

$$\mathcal{A}(\sigma, \tau; c, d, e, f; \mathbf{q})$$

$$= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \Big[\tfrac{1}{3}(2C_+ - C_-) \delta^{cd} \delta^{ef}$$

$$+ (C_+ + C_-) T_{cd}^g T_{ef}^g \Big] \bar{s} \gamma^\mu L b \xi_\sigma^\dagger$$

$$\times [2m_c \Lambda_i^\mu \sigma^i - P^\mu + 2i \Lambda_k^\mu \varepsilon^{mik} q_m \sigma_i] \eta_\tau, \quad (2)$$

c, d, e, f being color indices, σ and τ the individual spins of the charm quarks. P is the total four-momentum of the c and $\bar{c}$ in the laboratory frame, Λ_i^μ is the Lorentz boost matrix that takes a three-vector from the $c\bar{c}$ rest frame to the laboratory frame.

The decay rate to a $c\bar{c}$ pair, calculated in full perturbative QCD, is obtained from

$$\Gamma(b \rightarrow c\bar{c} + q_f)$$

$$= \frac{1}{2E_b} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{P}}{(2\pi)^3 2P_0} \frac{d^3\mathbf{p}_f}{(2\pi)^3 2E_f} (2\pi)^4$$

$$\times \delta^{(4)}(p_b - p_f - P) |\bar{\mathcal{A}}|^2 m_c^{-1} + \cdots, \quad (3)$$

where the dots denote higher order terms in the v^2 expansion. The integrations represent the three-body phase space of the final state. Using perturbative NRQCD, the corresponding decay rate to a $c\bar{c}$ pair of a specific angular and color quantum number state $[n]_a \equiv (^{2S+1}L_J)_a$ reads

$$\Gamma(b \rightarrow c\bar{c}[n]_a + q_f) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} S_a[n] \frac{\langle 0|O_a^{c\bar{c}}[n]|0\rangle}{m_c^2} + \cdots. \quad (4)$$

Replacing the heavy quark field operators, which appear in the $c\bar{c}$ matrix elements, in terms of the Pauli spinors ξ and η permits a matching of the two perturbative results in Eqs. (3) and (4). This determines the short-distance coefficients $S_a[n]$, which equally apply to the inclusive decays to charmonium *bound* states (for more details see Refs. [8,9]).

Consequently, the rate of the transition $b \rightarrow J/\psi + X$ in the NRQCD factorization formalism reads

$$\Gamma(b \rightarrow J/\psi + X) = \sum_{[n]_a} S_a[n] \frac{\langle 0|O_a^{J/\psi}[n]|0\rangle}{m_c^2} \quad (5)$$

$$= \frac{1}{2E_b} \int \frac{d^3\mathbf{P}}{(2\pi)^3 2P_0} \frac{d^3\mathbf{p}_f}{(2\pi)^3 2E_f} (2\pi)^4 \delta^{(4)}(p_b - p_f - P) m_c^{-1} \sum_{[n]_a} \frac{|\bar{\mathcal{A}}|^2 (b \rightarrow c\bar{c}[n]_a + q_f)}{\langle 0|O_a^{c\bar{c}}[n]|0\rangle} \langle 0|O_a^{J/\psi}[n]|0\rangle. \quad (6)$$

Note that the integration over the relative momentum in Eq. (4) properly relates the short-distance $1 \rightarrow 3$ particle transition to the long-distance effective $1 \rightarrow 2$ particle one.

Evaluating the color octet intermediate states in Eq. (6) up to order v^4 relative to the color singlet "baseline," one obtains, in the rest frame of the b meson (with $m_f \simeq 0$ and $|V_{cs}|^2 + |V_{cd}|^2 \simeq 1$) [14,18,19],

$$\Gamma(b \rightarrow J/\psi + X) = \frac{G_F^2}{144\pi} |V_{cb}|^2 m_c m_b^3 \left(1 - \frac{4m_c^2}{m_b^2}\right)^2$$

$$\times \left[a \left(1 + \frac{8m_c^2}{m_b^2}\right) + b \right], \quad (7)$$

where we defined the NRQCD coefficients

$$a = (2C_+ - C_-)^2 \frac{\langle 0|O_1^{J/\psi}(^3S_1)|0\rangle}{3m_c^2} + (C_+ + C_-)^2$$

$$\times \left[\frac{\langle 0|O_8^{J/\psi}(^3S_1)|0\rangle}{2m_c^2} + \frac{\langle 0|O_8^{J/\psi}(^3P_1)|0\rangle}{m_c^4} \right], \qquad (8)$$

$$b = (C_+ + C_-)^2 \frac{3\langle 0|O_8^{J/\psi}(^1S_0)|0\rangle}{2m_c^2}. \qquad (9)$$

We shall apply Eqs. (7)–(9) in Sec. IV in the context of the ACCMM model in order to determine the J/ψ momentum distribution from B meson decays.

In the case of the parton model, which we shall introduce in Sec. III, we have to relate the short- and long-distance matrix elements squared rather than the decay rates. To this end, let us define the matrix element $\mathcal{M}$ of the effective interaction $b \to J/\psi + X$, in which the $b \to q_f$ transition is assumed to be independent of the inclusive charmonium production process, through

$$\Gamma(b \to J/\psi + X) = \frac{1}{2E_b} \int \frac{d^3\mathbf{k}_\psi}{(2\pi)^3 2E_\psi} \frac{d^3\mathbf{p}_f}{(2\pi)^3 2E_f} (2\pi)^4$$

$$\times \delta^{(4)}(p_b - p_f - k_\psi)|\bar{\mathcal{M}}|^2. \qquad (10)$$

If we identify the total four-momentum P of the c and $\bar{c}$ with the momentum of the J/ψ in the final state, $k_\psi \equiv P$, then comparing Eqs. (6) and (10) we obtain

$$|\bar{\mathcal{M}}|^2(b \to J/\psi + X) = m_c^{-1} \sum_{[n]_a} \frac{|\bar{\mathcal{A}}|^2(b \to c\bar{c}[n]_a + q_f)}{\langle 0|O_a^{c\bar{c}}[n]|0\rangle}$$

$$\times \langle 0|O_a^{J/\psi}[n]|0\rangle. \qquad (11)$$

An explicit calculation yields

$$|\bar{\mathcal{M}}|^2 = G_F^2 |V_{cb}|^2 T_{\mu\nu} \langle (\not{p}_f + m_f)\gamma^\mu L(\not{p}_b + m_b)\gamma^\nu L\rangle, \qquad (12)$$

where the J/ψ tensor structure $T_{\mu\nu}$ reads

$$T_{\mu\nu} = |V_{cf}|^2 m_c^3 \frac{1}{9} \left[-ag_{\mu\nu} + (a+b)\frac{1}{M_\psi^2}(k_\psi)_\mu (k_\psi)_\nu \right], \qquad (13)$$

with a and b defined in Eqs. (8) and (9). The momentum identification, described above, is in accordance with the BBL formalism for quarkonium production. To leading order in the v^2 expansion, the soft gluons, radiated in the nonperturbative transition $c\bar{c} \to J/\psi + X$ (and materializing into light hadrons), are assigned no energy or momentum; i.e., the energy dependence of the long-distance interaction is neglected, and thus the NRQCD matrix elements appear as universal constants.

In Sec. III we shall apply Eq. (13) in the context of the parton model. For comparison with data we identify $2m_c$ with the mass M_ψ of the J/ψ, because to leading order in the v^2 expansion $P^2 = 4m_c^2$ (however, one might keep in mind that the expansion fails to converge near the end point of the spectrum).

III. $B \to J/\psi + X$ IN THE PARTON MODEL (PM)

A. Calculation of the differential branching ratio

As we mentioned above, in the context of the NRQCD approach the nonperturbative charmonium production process $c\bar{c} \to J/\psi + X$ is assumed to be independent of the $b \to q_f$ transition. Thus, within the leading logarithm approximation for the short-distance matrix element, the $B \to J/\psi + X$ amplitude can be evaluated using a (generalized) factorization approach; i.e., the amplitude is related to a product of matrix elements of current operators [26],

$$\mathcal{M}(B \to J/\psi + X_f)$$

$$= \frac{G_F}{\sqrt{2}} V_{cb} V_{cf}^* [\tfrac{1}{3}(2C_+ - C_-)\langle X_f^1|\bar{q}_f \gamma_\mu Lb|B\rangle \langle \psi$$

$$+ X^1|\bar{c}\gamma^\mu Lc|0\rangle + (C_+ + C_-)\langle X_f^8|\bar{q}_f \gamma_\mu LT^a b|B\rangle$$

$$\times \langle \psi + X^8|\bar{c}\gamma^\mu LT^a c|0\rangle], \qquad (14)$$

with $X_f^1 + X^1 = X_f$, $X_f^8 + X^8 = X_f'$. Note that, contrary to the color singlet wave function model, Eq. (14) includes the color octet transitions.

The standard NRQCD approach outlined in Sec. II considers charmonium production from a *free b* quark decay. As it becomes obvious from Eq. (6), the corresponding transition $b \to J/\psi + X$ refers to a $1 \to 2$ particle phase space, because we neglect the soft gluon momenta. In this approximation, the J/ψ momentum distribution from B meson decays arises from the initial bound state corrections (and the momentum smearing from the Lorentz boost to the laboratory frame). We incorporate the corrections, attributed to the interaction between heavy and light quark in the initial meson, in the parton model along the lines developed for semileptonic B decays [27].

If we use Eq. (14), which is valid at the level of tree calculations, in the rest frame of the B we can write the decay rate as

$$d\Gamma(B \to J/\psi + X_f) = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^2 M_B} T_{\mu\nu} W^{\mu\nu} \frac{d^3\mathbf{k}_\psi}{2E_\psi}, \qquad (15)$$

with $T_{\mu\nu}$ defined in Eq. (13). Introducing the light-cone dominance, as in Ref. [27], allows us to relate the transition $B \to X_f$ to the distribution function $f(x)$ of the heavy quark momentum,

$$W_{\mu\nu} = 4(S_{\mu\rho\nu\lambda} - i\epsilon_{\mu\rho\nu\lambda}) \int_0^1 dx f(x) P_B^\lambda (xP_B - k_\psi)^\rho$$

$$\times \epsilon[(xP_B - k_\psi)_0] \delta[(xP_B - k_\psi)^2 - m_f^2], \qquad (16)$$

with

$$S_{\mu\rho\nu\lambda} = g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\rho\lambda} + g_{\mu\lambda}g_{\nu\rho} \quad \text{and}$$

$$\epsilon(x) = \begin{cases} +1, & x > 0, \\ -1, & x < 0. \end{cases} \qquad (17)$$

The dependence of the distribution function on the single scaling variable x is a consequence of the light-cone dominance, since in this framework the structure function $f(x)$ is

obtained as the Fourier transform of the reduced bilocal matrix element between the hadronic states [28]:

$$f(x) = \int d(yP_B) e^{ix(yP_B)} \frac{1}{4\pi M_B^2}$$

$$\times \langle B|\bar{b}(0)P_B^\mu \gamma_\mu^L b(y)|B\rangle|_{y^2=0}. \quad (18)$$

The Lorentz invariant width of the decay reads

$$E_B \cdot d\Gamma = \sum_{f=s,d} \frac{C_f}{\pi} \int dx f(x) \epsilon[(xP_B - k_\psi)_0]$$

$$\times \delta^{(1)}[(xP_B - k_\psi)^2 - m_f^2]\left[(a-b)P_B(xP_B - k_\psi)\right.$$

$$\left. + \frac{2}{M_\psi^2}(P_B k_\psi)(xP_B k_\psi - M_\psi^2)(a+b)\right]\frac{d^3 \mathbf{k}_\psi}{2E_\psi}, \quad (19)$$

where

$$C_f = \frac{G_F^2}{72\pi}|V_{cb}|^2|V_{cf}|^2 M_\psi^3. \quad (20)$$

Evaluating Eq. (19) in the rest frame of the B meson, we arrive at the formula for the J/ψ momentum spectrum:

$$\frac{1}{\Gamma_B}\frac{d\Gamma}{d|\mathbf{k}_\psi|}(B \to J/\psi + X)$$

$$= \tau_B \frac{G_F^2}{72\pi M_B}|V_{cb}|^2 M_\psi^3 \frac{|\mathbf{k}_\psi|^2}{E_\psi}\left\{[f(x_+)+f(x_-)]\right.$$

$$\times\left[(a-b)+(a+b)\frac{2E_\psi^2}{M_\psi^2}\right]+[f(x_+)-f(x_-)]$$

$$\left.\times(a+b)\frac{2E_\psi|\mathbf{k}_\psi|}{M_\psi^2}\right\}, \quad (21)$$

within the kinematical range

$$M_\psi \leq E_\psi \leq \frac{M_B^2 + M_\psi^2 - m_f^2}{2M_B}. \quad (22)$$

Here we defined

$$x_\pm = \frac{1}{M_B}(E_\psi \pm \sqrt{|k_\psi|^2 + m_f^2}) \quad (23)$$

and adopted $m_f \simeq 0$ and $|V_{cs}|^2 + |V_{cd}|^2 \simeq 1$. The kinematical range for x_- belongs to a final state quark with negative energy. Therefore the corresponding terms can be associated, formally, with quark pair creation in the B meson whereas the dominant terms proportional to $f(x_+)$ reflect the direct decay.

To compare our result with data from CLEO a Lorentz boost has in addition to be performed from the rest frame of the B meson to a moving B produced at the $\Upsilon(4S)$ resonance ($|\mathbf{p}_B| = 0.34$ GeV). The explicit form of the boost integral is presented in Sec. IV A in the context of the ACCMM model [see Eq. (37)].

We note that the dependence of the decay spectrum on the structure function appears in a factorized form. Thus the shape of the spectrum in Eq. (21) is governed by the functional form of the heavy quark momentum distribution. This is different for the semileptonic decay, in which an integral over the structure function is involved. In the following section we compare the predictions of Eq. (21) with the existing experimental data.

B. Analysis and numerical evaluation

In order to compute the theoretical momentum distribution we must fix the parameters appearing in Eq. (21). In the numerical analysis we use $G_F = 1.1664 \times 10^{-5}$ GeV^{-2}, $M_B = 5.279$ GeV, $M_\psi = 3.097$ GeV, and $C_+(m_b) = 0.868$, $C_-(m_b) = 1.329$. The (leading log) values of the Wilson coefficients correspond to the central value of the pole mass, $m_b = 4.7$ GeV [29]. Moreover we adopt $\tau_B = 1.61$ ps for the B meson lifetime [30] and $|V_{cb}| = 0.040$. The latter value stems from a CLEO II analysis [31] of the inclusive semileptonic B decays[1] and is in accordance with the world average from inclusive and exclusive measurements, $|V_{cb}| = 0.0381 \pm 0.0021$ [33].

As we mentioned above, the numerical values of the non-perturbative NRQCD matrix elements $\langle 0|O_a^{J/\psi}(^{2S+1}L_J)|0\rangle \equiv \langle O_a^{J/\psi}(^{2S+1}L_J)\rangle$ must be determined phenomenologically. For the rest of this article, we use

$$\langle O_1^{J/\psi}(^3S_1)\rangle = 1.1 \text{ GeV}^3,$$

$$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.0066 \text{ GeV}^3,$$

$$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.04 \text{ GeV}^3, \quad (24)$$

$$\langle O_8^{J/\psi}(^3P_0)\rangle/m_c^2 = -0.003 \text{ GeV}^3.$$

In addition we have to impose the (approximate) heavy quark spin symmetry to relate the matrix element $\langle O_8^{J/\psi}(^3P_0)\rangle$ to that arising from the $(^3P_1)$ intermediate state relevant for B decays:

$$\langle O_8^{J/\psi}(^3P_J)\rangle \simeq (2J+1)\langle O_8^{J/\psi}(^3P_0)\rangle. \quad (25)$$

The color singlet matrix element $\langle O_1^{J/\psi}(^3S_1)\rangle$ has been determined from the leptonic width of the J/ψ [34], the corresponding color octet matrix element $\langle O_8^{J/\psi}(^3S_1)\rangle$ stems from the analysis of J/ψ hadroproduction at the Collider Detector at Fermilab (CDF) [8]. The remaining parameters $\langle O_8^{J/\psi}(^1S_0)\rangle$ and $\langle O_8^{J/\psi}(^3P_0)\rangle$ have been extracted from a fit to data on J/ψ leptoproduction [16] (the latter involving only a small dependence on the value chosen for m_c).

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[1] The analysis was based on a modified version of the ISGW model [32] and on ACCMM [24], respectively, using $p_F = 0.3$ GeV for the Fermi momentum parameter. The corresponding uncertainty, among other things due to the ambiguity in the choice of p_F, we postpone to the discussion of the overall uncertainty associated with the normalization of the $B \to J/\psi + X$ decay spectrum.

Note that the renormalized matrix element $\langle O_8^{J/\psi}(^3P_0)\rangle$ is not restricted to positive values, a characteristic due to the subtraction of large power divergences from the (positive definite) bare matrix element (for details see Ref. [16]).

The numerical values of $\langle O_8^{J/\psi}(^1S_0)\rangle$ and $\langle O_8^{J/\psi}(^3P_0)\rangle$ quoted in Eq. (24) are consistent with results from photoproduction [13], pion-nucleon reactions [10] and, roughly, with calculations of J/ψ production in hadronic collisions [8]. Yet one has to note that the various investigations of charmonium production processes within the NRQCD approach involve leading order theoretical expressions. The corresponding uncertainties in the determination of the matrix elements set up a sizable range in the parameter space (for a detailed discussion see Ref. [19]). However, as we discuss next, an important feature of our analysis is that the momentum dependence is largely free of these uncertainties.

In both models, the parton and the ACCMM model, the shapes of the spectra originating from various $c\bar{c}(^{2S+1}L_J)$ intermediate states are essentially identical. This statement is obvious in case of the $(^3L_1)$ vector states, which involve identical Lorentz structures, and thus the relevant NRQCD matrix elements appear in a single constant [see Eq. (8)]; in case of the $(^1S_0)$ scalar state the equivalence follows from the explicit calculation. Consequently, the shape of the entire J/ψ momentum distribution in B meson decays is practically independent of the exact values of the nonperturbative matrix elements. The same statement holds for the numerical values of the parameters occurring in the short-distance coefficients [in particular errors have to be attributed to the Wilson coefficients, due to scale uncertainties, and to the value chosen for the Cabibbo-Kobayashi-Maskawa (CKM) element V_{cb}].

The set of parameters mentioned above solely determines the normalization of the J/ψ momentum spectrum. Within our approach the shape is attributed to the momentum distribution of the b quark in the initial B meson. Further investigation of soft gluon effects is clearly needed. The first step we take here is to ask: can a successful reproduction of the data at hand be achieved without them.

In the parton model, in the absence of direct measurements of the distribution function we use a one-parameter Ansatz and fix the distribution parameter by comparing our results with data. Referring to theoretical studies which pointed out that the distribution and fragmentation function of heavy quarks peak at large values of x [35], we assume, as a working hypothesis, that the functional form of both is similar. The latter is known from experiment and we shall use the Peterson functional form [36]

$$f(x) = N_\varepsilon \frac{x(1-x)^2}{[(1-x)^2 + \varepsilon_p x]^2}, \qquad (26)$$

with ε_p being the free parameter and N_ϵ the corresponding normalization constant. This function has already been applied in the semileptonic decays of the B meson [27].

In Fig. 2(a) we show the distribution function $f(x)$ for various values of ε_p. The kinematical range for the two arguments $x_\pm$ appearing in Eq. (21) reads

$$\frac{M_\psi}{M_B} \leq x_+ \leq 1, \qquad \frac{M_\psi^2}{M_B^2} \leq x_- \leq \frac{M_\psi}{M_B}. \qquad (27)$$
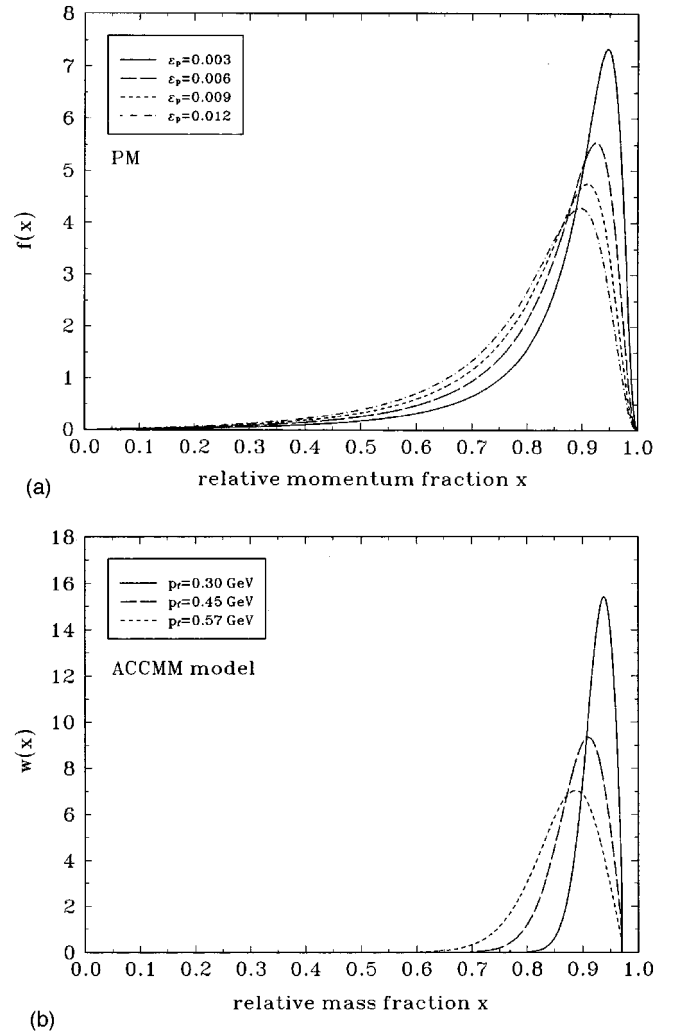

(a)

(b)

FIG. 2. Momentum distribution function $f(x)$ in the PM for various values of ε_p. (b) Mass distribution function $w(x)$ in the ACCMM model for various values of p_F.

Note that the variable x_- only occurs with values at which $f(x_-)$ is small. Therefore the corresponding contribution to the differential branching ratio is small.

We use Eq. (26) to fit the measured momentum spectrum of the J/ψ which was presented by the CLEO group [2]. As pointed out above, the shape of the theoretical spectrum is determined by the distribution function, i.e., by the parameter ε_p.

A general feature of the analysis is the difficulty for reproducing the data over the entire range of phase space. Confronted with this problem, we lay greater emphasis on the appropriate description of the *low* momentum range ($|\mathbf{k}_\psi| \lesssim 1.4$ GeV). Within this region the J/ψ spectrum obtains a sizable contribution from decay channels containing three or more particles in the final state, whereas the high momentum range is mainly determined by the exclusive two-body decays $B \rightarrow J/\psi K^{(*)}$. Therefore incoherence, as a necessary ingredient of the PM, is fulfilled in the former region. In addition, the v^2 expansion fails to converge in the end point domain of the spectrum.

As a result we find that if we apply small values for the parameter $\varepsilon_p \lesssim 0.004 - 0.006$ we cannot account for any part
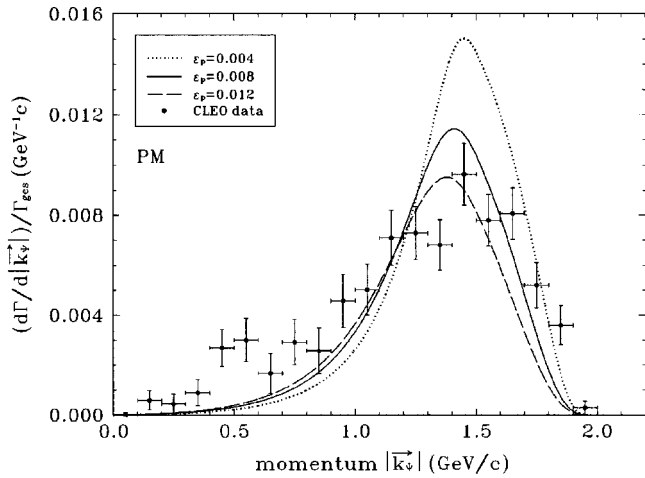
FIG. 3. Theoretical momentum spectrum in the PM for direct inclusive J/ψ production from B decays at the $\Upsilon(4S)$ resonance, shown for various values of ε_p and compared with the CLEO data; the parameters are given in Sec. III B.

of the low momentum region which is underestimated, whereas the high momentum range is overestimated. The situation improves if we apply larger values for ϵ_p ~ 0.008–0.012, corresponding to a soft b quark distribution (see Fig. 3). For $\varepsilon_p = 0.008$ we obtain a first satisfactory fit for the present data. Yet there is still a moderate systematic underestimate. A further increase of the distribution parameter (with the remaining parameters kept unchanged) does not imply significant modifications within the low momentum range. For comparison, one might note that in the studies of semileptonic B decays ε_p was taken between 0.003 and 0.009 [27].

Considering the uncertainties associated with the determination of the NRQCD matrix elements and, consequently, with the normalization of the spectrum, in Fig. 4 we show the J/ψ momentum distribution for $\varepsilon_p = 0.012$ and a modified value $\langle O_8^{J/\psi}(^3P_0) \rangle / m_c^2 = -0.001$ GeV^3. The fit is somewhat better. However, the color octet intermediate states, with reasonable values of the matrix elements, improve the fit of the calculated curves to the observed spectrum. For
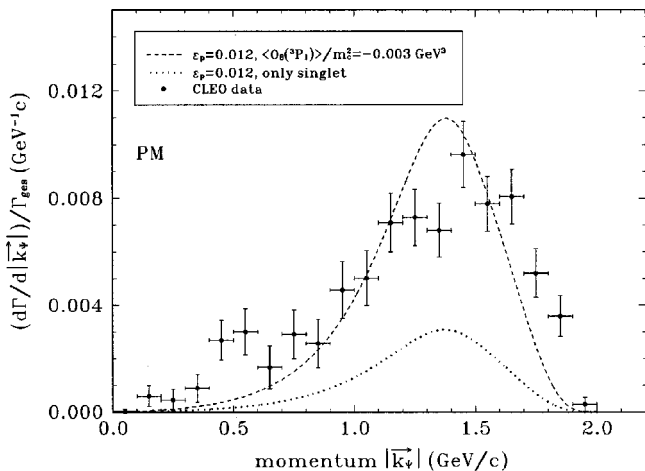

FIG. 4. Same as in Fig. 3, now for $\varepsilon_p = 0.012$ and a modified value of the $(^3P_1)$ matrix element (dashed line). The dotted line shows the contribution from color singlet $(^3S_1)$ intermediate states.

comparison, and in order to point out the importance of the color octet charmonium production mechanism, we also show in Fig. 4 the spectrum arising from the color singlet (wave function) model ($\langle O_8^{J/\psi}(^{2S+1}L_J) \rangle \equiv 0$). The latter underestimates the data by roughly a factor of three.

As a general result we conclude that a soft b quark momentum distribution can, to a good extent, account for the observed spectrum in $B \to J/\psi + X$ decays. We shall see in Section IV B than an analogous statement holds for the Fermi momentum distribution in the ACCMM model.

Finally, the errors in the data, although substantially improved, are still significant and a better test will be possible, when the error bars will be further reduced. With better data one should also include higher order nonperturbative corrections to the quarkonium production, and study the influence of soft gluon radiation within the fragmentation process.

IV. $B \to J/\psi + X$ IN THE ACCMM MODEL

A. Calculation of the differential branching ratio

A second approach which allows us to analyze the momentum distribution of J/ψ in the inclusive decay of the B meson is given by the ACCMM model [24]. In this model the bound state corrections to the free b quark decay are incorporated by attributing to the spectator quark a Fermi motion within the meson. The momentum spectrum of the J/ψ is then obtained by folding the Fermi motion with the spectrum from the b quark decay. In Ref. [37] the shape of the J/ψ momentum distribution resulting from Fermi momentum smearing has been given in the context of the color singlet wave function model (and without consideration of the light spectator quark mass). We shall extend the analysis by including the leading color octet contribution to the charmonium production in the framework of the NRQCD factorization formalism and by comparing our results with experimental data.

The spectator quark is handled as an on-shell particle with definite mass m_{sp} and momentum $|\mathbf{p}| = p$. Consequently, the b quark is considered to be off-shell with a virtual mass W given in the rest frame of the B meson by energy-momentum conservation as

$$W^2(p) = M_B^2 + m_{sp}^2 - 2M_B \sqrt{m_{sp}^2 + p^2}. \tag{28}$$

Altarelli *et al.* introduced in the model a Gaussian probability distribution $\phi(p)$ for the spectator (and thus for the heavy quark) momentum,

$$\phi(p) = \frac{4}{\sqrt{\pi} p_F^3} \exp(-p^2/p_F^2), \tag{29}$$

normalized according to

$$\int_0^\infty dp\, p^2 \phi(p) = 1. \tag{30}$$

The Gaussian width p_F is treated as a free parameter which has to be determined by experiment.

One main difference between the parton model and ACCMM is that in the latter one must consider a b quark in flight. We therefore start from the momentum spectrum of

the J/ψ resulting from the decay $b \rightarrow J/\psi + X_f$ ($f = s, d$) of a b quark of mass W and momentum p which is given by

$$\frac{d\Gamma_b}{d|\mathbf{k}_\psi|}(|\mathbf{k}_\psi|, p) = \gamma_b^{-1} \frac{\Gamma_0}{k_+^{(b)}(p) - |k_-^{(b)}(p)|} [\theta(|\mathbf{k}_\psi| - |k_-^{(b)}(p)|)$$

$$- \theta(|\mathbf{k}_\psi| - k_+^{(b)}(p))]. \tag{31}$$

Here we have defined

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{32}$$

Γ_0 is the width of the analogous decay in the rest frame of the heavy quark as obtained from the NRQCD factorization formalism. According to Eqs. (7)–(9) it reads (again adopting $m_f \simeq 0$ and $|V_{cs}|^2 + |V_{cd}|^2 \simeq 1$)

$$\Gamma_0 = \frac{G_F^2}{144\pi}|V_{cb}|^2 m_c W^3 \left(1 - \frac{4m_c^2}{W^2}\right)^2 \left[a\left(1 + \frac{8m_c^2}{W^2}\right) + b\right], \tag{33}$$

where a and b contain the nonperturbative effects in the charmonium production process.

In Eq. (31) $k_\pm^{(b)}$ give the limits of the momentum range which results from the Lorentz boost from the rest frame of the b quark to a frame where the b has a nonvanishing momentum p,

$$k_\pm^{(b)}(p) = \frac{1}{W}(E_b k_0 \pm p E_0), \tag{34}$$

with

$$k_0 = \frac{1}{2W}(W^2 - M_\psi^2), \quad E_0 = \sqrt{k_0^2 + M_\psi^2}, \tag{35}$$

and γ_b^{-1} being the corresponding Lorentz factor,

$$\gamma_b^{-1} = \frac{W}{E_b}, \quad E_b = \sqrt{W^2 + p^2}. \tag{36}$$

To calculate the momentum spectrum of the J/ψ from the inclusive decay of the B meson one has to fold the heavy quark momentum probability distribution with the spectrum of Eq. (31) resulting from the b quark subprocess. Performing this, we finally arrive at the expression for the differential branching ratio for a B meson in flight,

$$\frac{1}{\Gamma_B}\frac{d\Gamma}{d|\mathbf{k}_\psi|}(B \rightarrow J/\psi + X) = \tau_B \int_{|k_-(|\mathbf{k}_\psi|)|}^{\hat{k}_+(|\mathbf{k}_\psi|)} \frac{d|\mathbf{k}'_\psi|}{k_+(|\mathbf{k}'_\psi|) - |k_-(|\mathbf{k}'_\psi|)|} \int_0^{p_{\max}} dp\, p^2 \phi(p) \frac{d\Gamma_b}{d|\mathbf{k}_\psi|}(|\mathbf{k}'_\psi|, p). \tag{37}$$

Here $p_{\max}$ is the maximum kinematically allowed value of the quark momentum p, i.e., that which makes W in Eq. (28) equal to $W = M_\psi$ (for comparison with data we identify $2m_c$ with the mass M_ψ of the J/ψ),

$$p_{\max} = \frac{1}{2M_B}[(M_B^2 + m_{sp}^2 - M_\psi^2)^2 - 4m_{sp}^2 M_B^2]^{1/2}. \tag{38}$$

The first integration in Eq. (37) results from the transformation from the spectrum for a B meson at rest to the spectrum for a B meson in flight, where

$$k_\pm(|\mathbf{k}_\psi|) = \frac{1}{M_B}(E_B|\mathbf{k}_\psi| \pm |\mathbf{p}_B|E_\psi),$$

$$\hat{k}_+(|\mathbf{k}_\psi|) = \min\{k_+(|\mathbf{k}_\psi|), k_{\max}\}, \tag{39}$$

$k_{\max}$ being the maximum value of the J/ψ momentum from the decay $B \rightarrow J/\psi + X$ in the rest frame of the B,

$$k_{\max} = \frac{1}{2M_B}[(M_B^2 + M_\psi^2 - S_{\min})^2 - 4M_B^2 M_\psi^2]^{1/2}, \quad S_{\min} = m_{sp}^2. \tag{40}$$

(Note that in the PM $S_{\min} = 0$ for vanishing masses of the final state quark q_f.)

In the following section we make use of Eq. (37) to compare the model predictions with experimental data.

B. Analysis and numerical evaluation

Both models, the parton model as well as ACCMM incorporate the bound state structure of the B meson by postulating a momentum and, consequently, a mass distribution for the heavy quark. In Sec. III B we pointed out that in the PM the dependence of the J/ψ momentum spectrum on the exact value of the Peterson parameter ε_p (within the low momentum range $|\mathbf{k}_\psi| \lesssim 1.4$ GeV) is only moderate. We shall see that, contrary to this, in the ACCMM model the dependence of the spectrum on the Fermi parameter p_F is strong, which allows us to perform a detailed analysis.

Introducing in the latter model another x variable as the ratio $x = W/M_B$, we may calculate the appropriate distribution function $w(x)$ of the b quark in the rest frame of the B meson as a function of the relative mass x. In Fig. 2(b) we plot the function $w(x)$ for $m_{sp} = 0.15$ GeV and various values of p_F, this value for the spectator mass being frequently used in studies of the semileptonic decays. Note that the mass distribution shows sizable modifications due to the variation of p_F. Both models have the advantage of avoiding the mass of the heavy quark as an independent parameter. As a consequence, the phase space is treated correctly because they use the mesonic degrees of freedom.

Within the approximations discussed in Sec. I, the shape of the momentum spectrum in the decay $B \rightarrow J/\psi + X$ is determined by the value of p_F. As argued in Ref. [38], the Fermi momentum parameter is not a truly free parameter, but it is directly related to the average kinetic energy of the
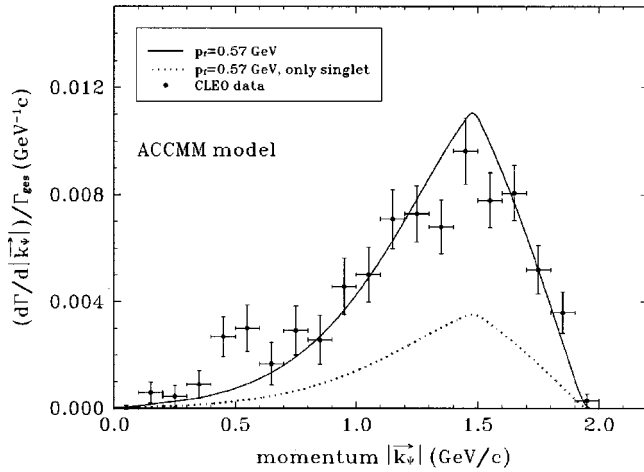
FIG. 5. Theoretical momentum spectrum in the ACCMM model for inclusive B decays to J/ψ with $p_F = 0.57$ GeV, compared with the CLEO data; the parameters are given in Sec. III B. The dashed line shows the contribution from color singlet ($^3S_1$) intermediate states.

FIG. 6. Same as in Fig. 5, now the theoretical momentum spectrum in the ACCMM model for various values of p_F as shown.

heavy quark, $\langle \mathbf{p}^2 \rangle = \frac{3}{2} p_F^2$ through the Gaussian form of the distribution function of Eq. (29). Thus the value of p_F can also be deduced theoretically from a study of the b quark's average kinetic energy. Hwang *et al.* [38] calculated the latter in the relativistic quark model, from which they obtained $p_F \sim 0.5$–0.6 GeV, this value being in good agreement with the one deduced from the QCD sum rule analysis of Ref. [39], $p_F = 0.58 \pm 0.06$ GeV. Both results respect the limit $p_F \gtrsim 0.49$ GeV which follows from an inequality between the expectation value of the b quark's kinetic energy and that of the chromomagnetic operator derived by Bigi *et al.* using the methods of the Heavy Quark Effective Theory [40]. The lower bound from the latter inequality, however, could be considerably weakened by higher order perturbative corrections [41].

The numerical range of the Fermi motion parameter deduced from the heavy quark's kinetic energy is in good agreement with a recent study of the semileptonic B decays: Using dilepton data, the CLEO Collaboration has performed a model independent analysis of the lepton spectrum over essentially the full momentum range [31]. By comparing the results with the theoretical prediction of the ACCMM model, Hwang *et al.* [38] obtained $p_F = 0.54^{+0.16}_{-0.15}$ GeV.

Allowing the Fermi momentum to float within the range $p_F \sim 0.5$–0.6 GeV, we investigate to what extent the b quark motion in the B meson can account for the observed spectrum in $B \to J/\psi + X$ decays. One might note that, in contrast to the energy spectrum in the semileptonic decays, the shape of the J/ψ momentum spectrum is sensitive to the Fermi motion parameter over a wide range of the phase space.

Employing the spectator distribution function of Eq. (29), we calculated the momentum spectrum for the decay of B mesons produced at the $\Upsilon(4S)$ resonance. Figures 5 and 6 show the comparison with the CLEO data for the set of parameters given in Sec. III B and for $m_{sp} = 0.15$ GeV. In Fig. 5 one can see that (considering the fact that the ACCMM model does not include hadronization effects and, consequently, yields an averaged spectrum) the agreement of the
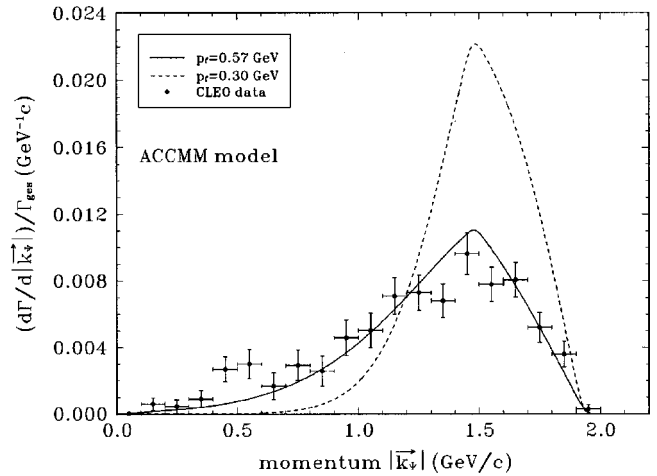
theoretical spectrum with the data is good provided we choose the value $p_F \sim 0.57$ GeV. Again we point out the dominance of the color octet charmonium production mechanism by plotting also the spectrum which arises from the color singlet model, the latter underestimating the data by roughly a factor of three.

To demonstrate the high sensitivity of the spectrum to the Fermi motion parameter, we present in Fig. 6 the spectrum for $p_F = 0.3$ GeV (with the remaining parameters kept unchanged), the latter value being commonly used by experimentalists in the analysis of the semileptonic decays. Note the large discrepancy between theoretical prediction and data. Thus it is obvious that the shape of the measured J/ψ momentum distribution cannot be reproduced in the model, within the approximations we made, when using values significantly smaller than $p_F = 0.5$ GeV. Especially the sizable contribution in the low momentum range requires a soft probability distribution of the heavy quark momentum. Further we want to emphasize that (using $p_F = 0.57$ GeV) not only the shape but, due to the inclusion of color octet intermediate states in the charmonium production, also the normalization of the theoretical spectrum is in good agreement with the data. This way our analysis supports the numerical values of the NRQCD matrix elements given in Eq. (24).

Note that the authors of Ref. [14] could not account for the measured value of the branching ratio $\mathcal{B}(B \to J/\psi + X)$, because they restricted the NRQCD analysis to positive values of the matrix element $\langle O_8^{J/\psi}(^3P_0) \rangle$, and because (adopting $m_b = 5.3$ GeV) they did not take into account corrections due to the bound state structure of the B meson.

We conclude that, using the ACCMM model, the Fermi motion of the b quark can account for the observed momentum distribution in the $B \to J/\psi + X$ decay accurately. The required value of the Fermi momentum $p_F \sim 0.57$ GeV comes out to agree with the range deduced from the studies of the heavy quark's kinetic energy, as well as, with the range recently obtained from the semileptonic B decays.

We close this section with a comment on the distinct secondary bump which appears in the low momentum region of the measured J/ψ spectrum. While this article was being written, Brodsky and Navarra [42] suggested an interpretation of the latter in terms of the three-body decay

$B \to J/\psi \Lambda \bar{p}$, in which they assumed that the underlying dynamics of the *exclusive* mode is reflected in the measured shape of the inclusive momentum distribution. The authors argued that the three-body final state accounts for a distinct enhancement in the inclusive spectrum, in which the position of the maximum ($|\mathbf{k}_\psi| \simeq 0.5$ GeV) coincides with the measured bump. Thus our inclusive approach to the J/ψ momentum distribution, when combined with the analysis of the exclusive three-body mode $J/\psi \Lambda \bar{p}$ [42], yields a spectrum which is in good agreement with the measurement over the whole range of phase space.

V. SUMMARY

The analysis of the decay $B \to J/\psi + X$ requires the color singlet and octet contributions to the intermediate $c\bar{c}$ states. The octet configurations were found to be necessary for improving the predictions for the branching ratio [18]. In this article we include the octet configurations in the analysis of the J/ψ momentum spectrum. We adopt the leading order expansion of NRQCD, where the kinematics of soft gluon radiation associated with the nonperturbative transitions $c\bar{c} \to J/\psi + X$ is neglected. Thus we address the question whether and to what extent the motion of the b quark in the bound state can account for the observed momentum spectrum. To this end, we review in Eqs. (7)–(9) and (13) the changes introduced to the J/ψ tensor through the leading color octet configurations. The structure constants of the tensor depend on the NRQCD matrix elements $\langle O_a^{J/\psi}[n] \rangle$, which parameterize the conversion of a $c\bar{c}$ pair in angular momentum $[n]$ and color a to the J/ψ. Values for the matrix elements are taken from other experiments. With these results we develop a formalism for the inclusive decay within the parton and the ACCMM model, respectively. The shapes of the spectra originating from various $c\bar{c}(^{2S+1}L_J)_a$ configurations are very similar to each other. They are sensitive to the motion of the b quark in the B meson. Explicit formulas

for the decay spectra are given in Eqs. (21) and (37). They depend on the momentum distribution function of the b quark $f(x)$ and the Fermi motion $\phi(p)$ of the spectator quark, respectively.

Numerical calculations in the parton model give a satisfactory presentation of the data, provided that the heavy quark momentum distribution is taken to be soft. To be explicit, we find $\varepsilon_p \sim 0.008 - 0.012$ for the parameter of the Peterson *et al.* distribution function. The ACCMM model can account for the observed momentum spectrum more accurately. The preferred Fermi momentum $p_F \sim 0.57$ GeV is in good agreement with the range $p_F \sim 0.5 - 0.6$ GeV deduced from theoretical studies of the heavy quark's kinetic energy.

Our results are obtained in the approximation of neglecting the gluon momenta at the hadronization stage, i.e., we include only the leading terms in the NRQCD expansion. Very recently, it was shown how the resummation of higher order nonperturbative corrections to the quarkonium production leads to the introduction of universal distribution functions [23]. It will be a challenge for future studies to investigate the importance of these shape functions for the momentum distribution in the decay $B \to J/\psi + X$ and to compare the theory with more accurate data.

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