

Flavor alignment solutions to the strong CP problem in supersymmetry

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An approach to solving the strong CP problem in supersymmetric theories is discussed which uses Abelian family symmetries to align the mass matrices of the quarks and squarks. In this way both the strong CP problem and the characteristic flavor and CP problems of supersymmetry can be solved in a single way. [S0556-2821(97)05619-1]

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It is well known that low-energy supersymmetry exacerbates the ‘‘flavor problem’’ [1]. First, there are new contributions to various flavor-changing processes [2]. In particular, since there is in general no Glashow-Iliopoulos-Maiani (GIM) mechanism in the squark sector, $\Delta S=2$ box diagrams involving squarks and gluinos can lead to excessive $K^0-\bar{K}^0$ mixing. Second, there are one-loop contributions to the u and d quark electric dipole moments, coming from new CP-violating phases that appear in the soft terms that break supersymmetry [3]. These contributions are naturally two orders of magnitude larger than the experimental bounds. And, third, there are new contributions to $\bar{\theta}$ from diagrams involving gluinos and squarks. These create difficulties for nonaxion solutions to the strong CP problem [4].

These various problems have led to a renewed interest in flavor symmetry [5] and in spontaneously broken CP symmetry [6–9] as a way to control excessive violations of flavor and CP in the supersymmetrized standard model. Significantly, even before the advent of supersymmetry, it was suggested that a combination of flavor symmetry and spontaneously broken CP could solve the strong CP problem. Various models were proposed [10] that implemented this idea. It therefore seems reasonable in the context of supersymmetry to attempt to find a unified approach to all the problems of flavor and CP violation, or in other words, to treat the strong CP problem not as a separate problem requiring a separate solution (the axion), but as a particularly severe aspect of a more general problem [6,9]. An advantage of such a combined approach is that it may lead to a more constrained set of possible solutions, and perhaps even to a unique one.

In a recent paper [9] we showed that an Abelian flavor symmetry can cause a ‘‘flavor alignment’’ [11] of the quarks and squarks that makes the supersymmetric contributions to $\bar{\theta}$ sufficiently small. However, the example presented there did not deal with the other aspects of the flavor problem. Here we propose a model, or rather a class of models, which does, and which also has the virtue of more comfortably satisfying the bound on $\bar{\theta}$.

The class of models that we propose is characterized by the mass matrices

$$M_u = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{pmatrix} \langle H_u \rangle \sim \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} v \quad (1)$$

and

$$M_d = \begin{pmatrix} s'_{11} & 0 & 0 \\ 0 & s'_{22} & s'_{23} \\ 0 & 0 & s'_{33} \end{pmatrix} \langle H_d \rangle \sim \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v'. \quad (2)$$

Here $s_{ij} = \lambda_{ij} \langle S_{ij} \rangle / M$, and $s'_{ij} = \lambda'_{ij} \langle S'_{ij} \rangle / M$, where λ_{ij} and λ'_{ij} are dimensionless effective coupling constants, and S_{ij} and S'_{ij} are chiral superfields which are singlets under the standard model gauge group, and which get vacuum expectation values that break the flavor group. For now we will treat these singlets as distinct fields, in which case there are at least nine such fields [cf. Eqs. (1) and (2)]. The scale of flavor breaking is M , which is assumed to be somewhat below the Planck scale, but far above the Fermi scale. λ is the Wolfenstein parameter (≈ 0.2). These nonrenormalizable Yukawa terms come from integrating out states of mass M in a renormalizable theory [9]. It is also assumed that CP is a spontaneously broken symmetry, so that all the Yukawa couplings are real. Both the flavor symmetry and CP invariance are broken by the vacuum expectation values (VEV’s) of the singlets at the scale M .

If one assumes that H_u and H_d do not transform under the flavor symmetry and that the S_{ij} and S'_{ij} are all distinct fields, then the flavor symmetry enjoyed by these Yukawa terms is $\tilde{G}_F = U(1)^9$, corresponding to rotating the phases of the nine quark fields Q_i , U_i^c , and D_i^c ($i=1,2,3$), independently. Let $U(1)_{(q_i, u_i, d_i)}$ be the particular U(1) subgroup of \tilde{G}_F under which these quark fields have charges q_i , u_i , and d_i , respectively. Then the eight U(1)’s that satisfy $\sum_i (2q_i + u_i + d_i) = 0$ will have no $SU(3)_c^2 \times U(1)$ anomaly. Let us call this color-anomaly-free U(1)⁸ flavor group G_F .

It is clear that there are two nontrivial combinations of the singlet fields S_{ij} that are G_F invariant. (By nontrivial we mean to exclude such combinations as $S_{ij} S_{ij}^*$.) These are

$$c \equiv s'_{33} s'_{23} s'_{22} s_{12} s_{13} s_{33}^* \sim \lambda^{16} \quad (3)$$

and

$$d \equiv s_{11} s_{22} s_{33} s'_{11} s'_{22} s'_{33}, \quad (4)$$

where the fields have been divided by M to make the quantities c and d dimensionless. If the full group G_F were gauged, then the only two physically meaningful phases in

the theory would be those of c and d . The latter of these appears in the expression for the determinant of the tree-level quark mass matrices. We assume that the minimization of the Higgs potential leads d to be real, at least at tree level. Then $\bar{\theta} = 0$ at tree level.

The CP violation in such a model comes exclusively, therefore, from the phase of c . It is because of this, and because c is so high order in λ , that it will turn out that $\bar{\theta}$ is sufficiently small. On the other hand, the Kobayashi-Maskawa (KM) phase δ is of the same order as $\arg(c)$, which will be assumed to be of order unity. This is easy to see from the fact that the invariant combination of KM elements $V_{td}V_{ts}^*V_{cs}V_{cd}^*$ is given to leading order in λ by $(s_{13}/s_{33})(s'_{23}/s'_{33})^*(1)(s_{12}/s_{22})^*$, which is in turn equal to $c/(|s_{33}s_{22}s'_{33}|^2)$. Thus its phase is simply the phase of c .

To estimate the radiatively induced value of $\bar{\theta}$ it is necessary to examine the squark mass matrices. Assuming for the time being the flavor group to be $G_F = U(1)^8$, the left-right squark mass [2] matrices have the same forms as the quark mass matrices. That is, $(M_{LR}^{d2})_{ij} \sim A(M_d)_{ij} \sim Av's'_{ij}$, and similarly for the up quark sector. Of course there can be other, nonholonomic contributions to the left-right squark masses coming from a variety of sources [8]. But these will either suppressed by powers of $\langle S_{ij}^{(\prime)} \rangle / M_{Pl}$, or by powers of the $s_{ij}^{(\prime)}$ and hence high powers of λ .

The left-left mass [2] matrix M_{LL}^{d2} has the form

$$(M_{LL}^{d2})_{ij} = a_{Li} \delta_{ij} m_0^2 + (M_d M_d^\dagger)_{ij} + O(\ln(M/M_W)/16\pi^2)(M_u M_u^\dagger)_{ij}. \quad (5)$$

The first term represents the diagonal terms, which do not break the flavor group G_F , and hence are unsuppressed. The a_{Li} are dimensionless numbers of order unity, which have no reason to show any degeneracy. The second term is just the supersymmetric contribution. The third term results from loops involving charged Higgs boson. Thus, while the diagonal entries are of order unity times the square of the supersymmetry- (SUSY-) breaking scale, the (ij) element, where $i \neq j$, is proportional either to factors of $s'_{ik}s'_{jk}^*$ or to loop factors times $s_{ik}s_{jk}^*$, and therefore to powers of the Wolfenstein parameter λ . The same discussion applies to the matrix M_{LL}^{u2} , with the roles of s and s' interchanged.

The right-right mass [2] matrices of the squarks have analogous forms:

$$(M_{RR}^{d2})_{ij} = a_{Ri} \delta_{ij} m_0^2 + (M_d^T M_d^*)_{ij}, \quad (6)$$

with a similar expression for the up quark sector. Here there are no one-loop corrections analogous to the third term in Eq. (5).

There may be contributions to the squark mass [2] matrices which have a different form, especially if only a subgroup of G_F is gauged, so that other invariants than c and d are allowed by local symmetry. However, if induced by Planck-scale physics, these contributions will be suppressed by powers of M/M_{Pl} , which we are taking to be small. If they are induced by loops at the scale M , they will derive from the forms given in Eqs. (1) and (2), and thus will in-

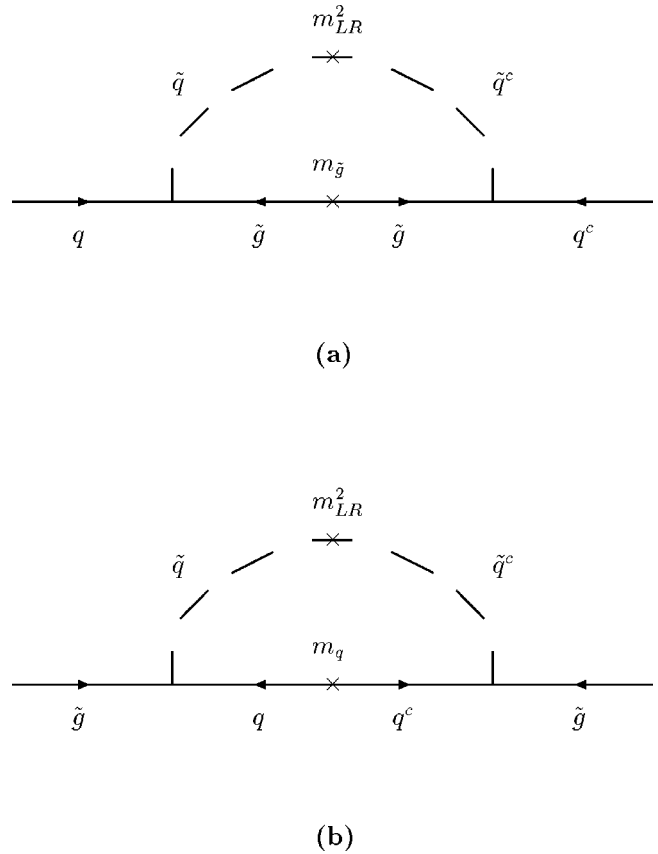


FIG. 1. In supersymmetric models these diagrams give contributions to $\bar{\theta}$ through (a) the phases of quark masses, and (b) through the phase of the gluino mass.

volve no CP -violating flavor invariant except c . Therefore, in looking for the leading contribution to $\bar{\theta}$, the forms given in Eqs. (5) and (6) are sufficient.

The leading contribution to $\bar{\theta}$ comes from the diagram in Fig. 1(a). If one ignored flavor violation in the left-left and right-right squark mass [2] matrices, the contribution of this graph to M_d would be of the form $\delta M_d = O(\alpha_s/4\pi)(M_{LR}^{d2}/m_0)f(m_g^-/m_0)$. But since this has the same form as M_d itself, this gives no contribution to $\bar{\theta}$. Indeed, it is clear from the fact that the only CP -violating invariant c involves elements of both S_{ij} and S'_{ij} , that one must take into account the piece of the M_{LL}^{d2} matrix that involves M_u , namely, the third term in Eq. (5). Effectively, then, $\bar{\theta}$ is a two-loop effect. This is a central idea behind these models and of the forms given in Eqs. (1) and (2). Because the invariant that violates CP involves both M_d and M_u , the exchange of charged states, either W^\pm or H^\pm , is required to bring it into play, thus necessitating higher loops.

When one includes the effect of the third term of Eq. (5) in the diagram of Fig. 1(a), one finds straightforwardly that

$$\delta \bar{\theta} \sim \left(\frac{\alpha_s}{4\pi}\right) O\left(\frac{\ln(M^2/M_W^2)}{16\pi^2}\right) (c/|s'_{33}|^2) \leq 3 \times 10^{-3} \lambda^{12} \lesssim 10^{-10}. \quad (7)$$

The analogous contribution to M_u gives a smaller contribution to $\bar{\theta}$.

There is also a contribution to $\bar{\theta}$ from the diagram in Fig. 1(b). It is straightforward to see that this gives $\delta\bar{\theta} \lesssim (\alpha_s/4\pi)(A/m_{\tilde{g}})(v'/m_0)^2 \arg(c) \sim 10^{-2} \lambda^{16}/\tan^2\beta \sim 10^{-12}/\tan^2\beta$. This is evidently much smaller than the contribution from Fig. 1(a).

The problem of excessive flavor changing in supersymmetric models is here solved in the same way as in the models of ‘‘flavor alignment’’ proposed by Nir and Seiberg [11]. In particular, the danger of excessive K^0 - \bar{K}^0 mixing coming from gluino box diagrams is obviated by the absence of a 12 element in M_d . This means that, as in the Nir-Seiberg models, the Cabibbo mixing must come from the up-quark sector, which in turn implies that the mixing in the D^0 - \bar{D}^0 sector is near the experimental bound [11].

Finally, there is the question of excessive electric dipole moments (or chromoelectric dipole moments) for the u , d , or s quarks. It is easy to see that these, since they also must involve the invariant c , are suppressed by large powers of λ . In fact they are less than or of order $e(\alpha_s/4\pi)(A m_{\tilde{g}} v'^2 v/m_0^6)\lambda^{16}$, which is less than $10^{-28}e$ cm, or about three orders of magnitude below the experimental bound.

The pattern or ‘‘texture’’ given in Eqs. (1) and (2) is unique in the following sense. There are several other textures that give the right amount of KM mixing, the right hierarchy of quark masses, have vanishing 12 element for M_d in order to avoid excessive K^0 - \bar{K}^0 mixing, and have $\bar{\theta}$ vanish at tree level and suppressed by several powers of λ at one-loop level. However, none of them suppress $\bar{\theta}$ by as many powers of λ as the forms given in Eqs. (1) and (2). There are two forms that give $\bar{\theta}$ to be of order $(\alpha_s/4\pi)\lambda^{10}$ [cf. Eq. (7)]. One of these is the same as the forms in Eqs. (1) and (2) except that the 13 element of M_d rather than of M_u is nonvanishing. The other is the same as Eqs. (1) and (2) except that M_d has vanishing 23 element and nonvanishing 13 element, while M_u has vanishing 13 element, and nonvanishing 23 element. Other forms have $\bar{\theta}$ arising at even lower order in λ . For example, if M_d has a diagonal form, and M_u has a triangular form, then $\bar{\theta}$ arises at order λ^6 as in Ref. [9].

There are many ways to construct a Higgs boson superpotential that ensures that at tree level d is real and c complex. An example which is easy to analyze is the following. Let $W_{\text{Higgs}} = W_0 + W_d + W_c$. W_0 has the form $\sum_{ij}(S_{ij}\bar{S}_{ij} - M_{ij}^2)Y_{ij} + \sum_{ij}(S'_{ij}\bar{S}'_{ij} - M'^2_{ij})Y'_{ij}$. Here all the M_{ij}^2 are taken to be real and positive, except M_{13}^2 which is real and negative. This ensures that $\langle \bar{S}_{ij} \rangle = \langle S_{ij} \rangle^*$, and similarly for the S'_{ij} , except that $\langle \bar{S}_{13} \rangle = -\langle S_{13} \rangle^*$.

W_d fixes the phase of d and can be taken to have the form $\sum_k S_{kk} S'_{kk} A_k + \sum_k \bar{S}_{kk} \bar{S}'_{kk} \bar{A}_k + A_1 A_2 A_3 + \bar{A}_1 \bar{A}_2 \bar{A}_3 + \sum_k m_k^2 A_k \bar{A}_k$. Integrating out the A_k and \bar{A}_k gives an effective term of the form $S_{11} S_{22} S_{33} S'_{11} S'_{22} S'_{33} \sim d$ and $\bar{S}_{11} \bar{S}_{22} \bar{S}_{33} \bar{S}'_{11} \bar{S}'_{22} \bar{S}'_{33} \sim \bar{d}$. Together, the conditions $F_{S_{kk}} = 0$ and $F_{\bar{S}_{kk}} = 0$, imply that $\langle d \rangle = \langle \bar{d} \rangle = \langle d \rangle^*$ and therefore that $\langle d \rangle$ is real.

W_c fixes the phase of c and may be taken to be of the form $W_c = S'_{33} \bar{S}'_{23} B_{23} + S_{22} \bar{S}_{12} B_{21} + S_{13} \bar{S}_{33} B_{13} + (S \leftrightarrow \bar{S}$,

$B \leftrightarrow \bar{B})$. Integrating out the B_{ij} , and using the equations $F_{S_{ij}} = 0$, one finds $\langle c \rangle = \langle \bar{c} \rangle$, in an obvious notation. From the relation $\langle \bar{S}_{13} \rangle = -\langle S_{13} \rangle^*$, it follows that $\langle \bar{c} \rangle = -\langle c \rangle^*$ and therefore that $\langle c \rangle$ is pure imaginary. This is not realistic, since $\arg(c) = \arg(V_{td} V_{ts}^* V_{cs} V_{cd}^*) = \arg(1 - \rho - i\eta)$ in Wolfenstein parametrization, and therefore $\arg(c) \neq \pi$. But it is easy to construct superpotentials that give other phases to c .

It is possible to gauge some subset of $G_F = \text{U}(1)^8$, the full flavor symmetry of the quark Yukawa terms. Since there is no $\text{SU}(3)_c^2 \times G_F$ anomaly, by construction, the gauge anomalies can be cancelled by auxiliary leptons, whose presence has no effect on $\bar{\theta}$. There are nine fields, S_{ij} and S'_{ij} , whose VEV's break G_F , but two combinations of these fields, c and d , are G_F invariant. Thus the VEV's of the singlet fields break $\text{U}(1)^8$ down to a single $\text{U}(1)$ factor, which is obviously the $\text{U}(1)$ of baryon number as far as its action on the quarks is concerned. If the broken $\text{U}(1)^7$ is gauged, and the unbroken $\text{U}(1)$ is global, there are no goldstone bosons or pseudogoldstone bosons associated with flavor breaking, and all the flavor gauge bosons will have mass of order M , which is safely heavy.

While the group G_F is convenient for analysis, it is not necessary that the local flavor group actually be this large. Nor is it necessary that there be as many singlet fields S as has been assumed to this point. This is shown by the following example which has a single gauged $\text{U}(1)$ flavor group and six flavor-breaking singlet fields, but essentially the same flavor structure as in Eqs. (1) and (2). Let there be the following singlet fields: $S_2, S_3, S'_3, S_4, S'_4$, and S_6 . The subscripts correspond to the order in λ of each field's vacuum expectation value. For example, $\langle S'_3 \rangle / M \sim \lambda^3$. The quark mass matrices have the form (where the Yukawa couplings, assumed to be of order unity, are not indicated)

$$M_u = \begin{pmatrix} s_6 & s'_4 & s'_3 \\ 0 & s_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \langle H_u \rangle \quad (8)$$

and

$$M_d = \begin{pmatrix} s_6 & 0 & 0 \\ 0 & s_4 & s'_4 \\ 0 & 0 & s_2 \end{pmatrix} \langle H_d \rangle. \quad (9)$$

The Higgs boson superpotential can be arranged so that (in some phase convention) the vacuum expectation value of S'_3 is pure imaginary while those of the other singlets are real (as was the case in the previous model). These forms of the quark Yukawa matrices can be enforced by a family $\text{U}(1)$ under which the doublet Higgs fields H_u and H_d are neutral and the singlet fields have the charges $Q(S_2, S_3, S'_3, S_4, S'_4, S_6) = [x, y, z, t, \frac{1}{2}(x+y+z), -\frac{1}{2}(x+y+t)]$, and the quark fields have the charges

$$\begin{aligned}
 & Q(Q_1, Q_2, Q_3, U_1^c, U_2^c, U_3^c, D_1^c, D_2^c, D_3^c) \\
 &= [-z, \frac{1}{2}(x-y-z), 0, \frac{1}{2}(x+y+t) + z, \\
 & \frac{1}{2}(-x-y+z), 0, \frac{1}{2}(x+y+t) \\
 & + z, \frac{1}{2}(-x+y+z) - t, -x].
 \end{aligned}$$

This U(1) has no $SU(3)_c^2 \times U(1)$ anomaly.

The values of x , y , z , and t must satisfy several conditions. In particular, the resulting charges of the fields must be such that there are no additional terms allowed in the matrices in Eqs. (8) and (9). The zeros must stay zeros, and the nonzero entries must arise from a single field. Moreover, the Higgs boson superpotential must contain enough distinct kinds of terms to prevent unwanted accidental global flavor symmetries, but no terms which make the invariant d have a complex vacuum expectation value. There are many solutions to these conditions. One example is $(x, y, z, t) = (+1, -1, -4, -6)$. This allows the terms $S_2 S_3$, $S_4 S_6 S_6$, $S_2 S_2 S_4'$, and $S_3' S_2 S_6$ to appear in the superpotential, which thus prevents accidental flavor U(1) symmetries from arising. It is possible to construct a superpotential so that the vacuum expectation value of each of these four invariants is real (in which case d is also, since it is the product of the first two of them), while c has a complex VEV.

It must be admitted that in both the examples presented above the cancellation of anomalies would be a nontrivial problem, and would involve a set of additional fields which would doubtless appear complicated and ugly. Constructing a complete superpotential for such a theory would be a daunting task. This is a general problem with models with Abelian flavor symmetries, and if nature makes use of such symmetries one assumes that among the large number of possibilities there must exist some cases where anomaly cancellation is simple.

There are presumably a variety of other sets of singlets and Abelian family symmetries which implement the general Yukawa pattern of Eqs. (1) and (2). An unsatisfactory feature of the examples presented above is that they do not explain the hierarchy in quark masses, as is done, for example, in models of the Froggatt-Nielsen type [12], where terms of higher order in λ arise from higher powers of a flavor-breaking field. There is clearly something quite *ad hoc* about the second example presented. Of more significance are the general features of these models, which it is useful to contrast with other types of models invented to solve the strong CP problem.

One class of models, proposed almost twenty years ago [10] in a non-SUSY context, was similar to the kind of model proposed here in that they used symmetries to restrict the form of the quark mass matrices in such a way that they had (at tree level at least) real determinants in spite of having some elements with phases of order unity. However, most of those models had nonminimal Higgs bosons, and in particular several Higgs boson doublets that contributed to the masses of quarks of a given charge. This, as is well known, leads to problems with Higgs boson mediated flavor viola-

tion [13]. The same feature also typically gave rise to one-loop contributions to $\bar{\theta}$ that tended to be somewhat too large. With minimal Higgs boson structure, there are only the two Yukawa matrices, proportional to M_u and M_d . Thus a one-Higgs-boson-loop contribution to the down quark mass matrix would have the form $M_i M_i^\dagger M_d$, where $i = u$ or d . But then

$$\begin{aligned}
 \bar{\theta} &= \text{argdet}(M_d + \text{const} M_i M_i^\dagger M_d) \\
 &= \text{arg}[\det(\text{Hermitian}) \det M_d] \\
 &= 0.
 \end{aligned}$$

The same is true for one-loop corrections to M_u . But with several Higgs boson doublets contributing to M_d , as in the models of Ref. [10], there are several Yukawa coupling matrices, Y_d^k , for the down quarks. Thus the tree plus one-loop contributions to M_d have the form $(M_d + \text{const} Y_d^k Y_d^{l\dagger} Y_d^m)$, which has no reason to have a real determinant.

An advantage of the present models consists in the fact that there is a minimal *doublet* Higgs boson structure. Instead of there being several Higgs boson doublets which couple differently in flavor and which violate CP spontaneously, there are in the present models several singlet scalars, S_{ij} and S'_{ij} , which perform the same tasks. In this respect the models proposed here are similar to the models proposed by Nelson in Ref. [14]. Of course, as in the models of Ref. [14], there can be one-loop contributions to $\bar{\theta}$ coming from the emission and reabsorption of the heavy singlet fields. In non-supersymmetric Nelson models for such loops to be made sufficiently small requires certain Yukawa couplings to be less than about 10^{-2} (which is not unreasonable). Here, such loops are suppressed by m_{SUSY}/M .

The Nelson-type models have problems, however, in the context of supersymmetry (unless supersymmetry breaking happens at low scales and is mediated by gauge interactions [4,9]). The problem is that even with minimal Higgs structure, other matrices in flavor space exist besides the Yukawa matrices, namely the squark mass [2] matrices. These allow one-loop contributions to $\bar{\theta}$ from diagrams involving squarks and gluinos (cf. Fig. 1). In the present models these are suppressed by ‘‘flavor alignment,’’ somewhat in the spirit of the old nonsupersymmetric models rather than the Nelson models. The models proposed here can therefore be regarded as somewhat of a hybrid between the two approaches, using features of each to suppress all one-loop contributions to the QCD angle.

An important feature of the ‘‘flavor alignment’’ here is that the nonzero elements in the quark mass ‘‘textures’’ have a pure form. That is, each element is generated by the VEV of a single S field. This is in contrast to both the supersymmetric Nelson models discussed in Ref. [6] and to the models of Nir and Rattazzi [8].

If the flavor alignment idea in the form presented here, where all CP -violating effects come from a single flavor invariant c of high order in the Wolfenstein parameter, is the true solution to the strong CP problem, one would expect the

following signatures: $\bar{\theta}$ should be observed not far below the 10^{-10} level (compared to the value 10^{-15} typical of most invisible axion models), $D^0 - \bar{D}^0$ mixing should be seen not far below the present limits, the electric dipole moment of the electron should be less than about 10^{-28} e cm, and that

of the neutron should come predominantly from $\bar{\theta}$ and therefore be not much below 10^{-26} e cm.

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