Study of off-forward parton distributions

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An extensive theoretical analysis of off-forward parton distributions (OFPDs) is presented. The OFPDs and the form factors of the quark energy-momentum tensor are estimated at a low energy scale using a bag model. Relations among the second moments of OFPDs, the form factors, and the fraction of the nucleon spin carried by quarks are discussed. [S0556-2821(97)01121-1]

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I. INTRODUCTION

One of the most important frontiers in strong interaction physics is the study of the structure of the nucleon. Despite considerable experimental and theoretical progress made over the last 40 years, there are still many unanswered questions. An example is the intensive debate which has continued over the spin structure of the nucleon, ever since the European Muon Collaboration (EMC) published their initial data on the spin structure function g_1 [1]. Traditionally, two types of observables related to the nucleon structure have been studied mostly extensively: elastic form factors and parton (quark and gluon) distributions. Electromagnetic form factors of the proton were first measured in the mid 1950s [2], and in recent years measurements of those of the neutron have been attempted and more are planned [3]. Weak form factors are also being measured through parity-violating electron and neutrino scattering [4]. On the other hand, the unpolarized quark and gluon distributions have been systematically probed in deep-inelastic scattering and Drell-Yan processes since the discovery of quarks at SLAC in the late 1960s. The polarized quark distributions have also been studied in a number of experiments in recent years [5], and more data on these are anticipated in the future from CERN, SLAC, the DESY ep collider HERA, and the BNL Relativistic Heavy Ion Collider (RHIC).

In this paper, we present a first detailed study of a new type of nucleon observable: the off-forward parton distribution (OFPD). The OFPDs generalize and interpolate between the ordinary parton distributions, measured for instance in deep-inelastic scattering, and the elastic form factors, and therefore contain rich structural information. There are no data so far on these distributions, and a relatively short theoretical history. Discussion of the OFPDs has arisen independently in several different theoretical studies. Using the operator product expansion, Watanabe [6] analyzed the short-distance behavior of virtual Compton scattering in terms of functions related to off-diagonal matrix elements of twist-2 operators. In the late 1980s, Geyer and collaborators [7] studied the relation between the Altarelli-Parisi evolution for parton distributions and the Brodsky-Lepage evolution for leading-twist meson wave functions. The ''interpolating functions'' introduced in Ref. [7] are essentially the OFPDs which we study in this paper. In the early 1990s, Jain and Ralston [8] studied hard processes involving hadron helicity flip, in terms of an ''off-diagonal transition amplitude'' involving off-forward matrix elements of biquark fields in the nucleon. It was shown that the integral of this amplitude over the quark four-momentum yielded elastic form factors. Recently, one of us [9,10] introduced OFPDs in the study of the spin structure of the nucleon. The main observations in Refs. [9, 10] were that the fractions of the spin carried by quarks and gluons can be determined from form factors of the QCD energy-momentum tensor, and that the latter can be extracted from the OFPDs. Furthermore, the deeply virtual Compton scattering (DVCS) process was proposed [10] as a practical way to measure the new distributions.

From the point of view of parton physics in the infinite momentum frame, the OFPDs have the following meaning: if a nucleon is moving with an infinite momentum in a particular direction, take out a parton with a certain fraction of the momentum, give it a four-momentum transfer Δ^μ, and insert it back into the nucleon. This is illustrated in Fig. 1. The OFPD is then the amplitude characterizing this process. On the other hand, from the point of view of elastic form factors, the moments of OFPDs are form factors of twist-2 quark and gluon operators. For the spin-1 operators one has the ordinary electromagnetic and axial form factors, while

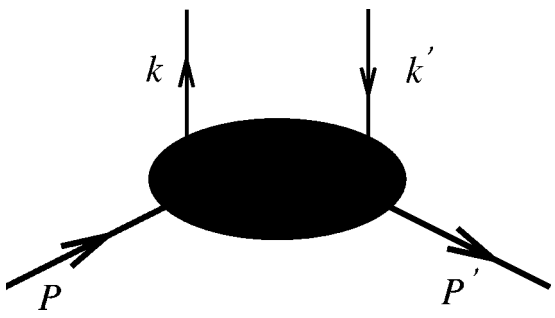

FIG. 1. Off-forward parton distribution. P and P' are the initial and final state nucleon momenta, and k and k' are the active quark momenta.

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for the spin-2 operators one has the form factors of the energy-momentum tensor. Because the form factors of the energy-momentum tensor contain information about the quark and gluon contributions to the nucleon angular momentum, the OFPDs can provide information on the fraction of the nucleon spin carried by quark orbital angular momentum—a subject of considerable current interest [11].

The OFPDs can be measured in diffractive processes in which the nucleon recoils elastically after receiving some momentum transfer. Moreover, one must have in the processes a hard, lightlike momentum so that the parton light-cone correlations are selected. The simplest such process is DVCS [10], in which a deeply virtual photon, supplied by inelastic electron scattering, hits the nucleon and turns into a real, high-energy photon. Such a process is easy to analyze theoretically and is similar to ordinary deep-inelastic scattering. More complicated processes include diffractive meson production, in which one must deal in addition with meson light-cone wave functions [12–18] (these have recently been shown by Collins *et al.* [18] to be factorizable). The best experimental facility to carry out DVCS experiments is the proposed ELFE [19]. However, some studies can already be made at HERA (the HERMES Collaboration) and at Jefferson Lab with a 6 GeV electron beam.

In planning future DVCS experiments, it is important to have a theoretical estimate of the OFPDs. The purpose of the present study is to perform a first analysis of the OFPDs in the MIT bag model [20]. To be sure, the bag model has a number of well-known problems, including breaking of chiral symmetry and translational invariance, absence of explicit gluon degrees of freedom, etc. Nonetheless, it contains quarks, it predicts well the hadron spectrum, it gives reasonable initial input for quark distributions [21–23], and it can describe the electromagnetic form factors [22,24] of the nucleon. Other models of which we are aware have an equally long list of problems and do not seem to provide any obvious advantage for estimating the quark distributions.

After first reviewing in Sec. II the definitions of the off-forward parton distributions and some of their general properties, we present in Sec. III the results of the bag model for their dependence on the various kinematic variables. The calculation accounts for Lorentz boost and spectator quark effects. In Sec. IV we analyze the form factors of the energy-momentum tensor, and evaluate their t dependence. Finally, conclusions are noted in Sec. V, and possible extensions of this work outlined.

II. BASICS OF OFF-FORWARD PARTON DISTRIBUTIONS

In this section we review the definitions and model independent results for the OFPDs and their moments discussed in Ref. [9]. Other definitions of OFPDs exist in the literature [7,12]; however, the definition introduced in Ref. [9] has a number of advantages, such as explicit hermiticity, and a simple connection with local operators and their form factors. We shall henceforth use the definition from Ref. [9]. The OFPDs can be defined for both quarks and gluons; however, in this paper we focus primarily on quark distributions, since the description of the wave function of gluons in the nucleon is a much more difficult problem.

To start, consider the bilocal operator $\overline{\psi}(-\lambda n/2)\mathcal{L}\Gamma^\mu\psi(\lambda n/2)$, where λ is a scalar parameter, ψ is a quark field of a certain flavor, and $\Gamma^\mu=\gamma^\mu$ or $\gamma^\mu\gamma_5$. The lightlike vector n^μ is proportional to $(1;0,0,-1)$, with a coefficient depending on the choice of coordinates. The gauge link $\mathcal{L}$ is along a straight line segment extending from one quark field to the other, which makes the bilocal operator gauge invariant. In the following, we work in the lightlike gauge $A\cdot n=0$, so that the gauge link can be ignored. We have also ignored the time-ordering of the quark fields, since the difference between the time-ordered and non-time-ordered operators is an anticommutator of the fields along the light cone, which is simply a constant [25].

One can now proceed to take the matrix element of the bilocal operator between the nucleon states of momenta P^μ and $P'^\mu=P^\mu+\Delta^\mu$, where Δ^μ is the four-momentum transfer. The matrix element must be expressible in terms of nucleon spinors, Dirac matrices, and the four-vectors P^μ, Δ^μ, and n^μ. Since we are only interested in the leading-twist contributions which are proportional to P^μ or P'^μ in the infinite momentum frame, we keep terms that are nonvanishing after multiplication by n^μ [9,10]:

$$\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\overline{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\xi,t)\overline{u}(P')\gamma^\mu u(P) + E(x,\xi,t)\overline{u}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}u(P)+\cdots, \tag{1a}$$

$$\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\overline{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \widetilde{H}(x,\xi,t)\overline{u}(P')\gamma^\mu\gamma_5 u(P) + \widetilde{E}(x,\xi,t)\overline{u}(P')\frac{\Delta^\mu\gamma_5}{2M}u(P)+\cdots, \tag{1b}$$

where $t\equiv\Delta^2$ and $\xi\equiv -n\cdot\Delta$, with $u(P)$ the nucleon spinor, and the ellipsis $(\cdots)$ denotes higher-twist contributions. It is possible to construct other Dirac structures that appear to be leading twist; however, using Gordon identities and throwing away subleading terms one can always reduce these to the form in Eqs. (1). The structures in Eqs. (1) are the same as those in the definition of the nucleon's elastic form factors. Examination of the helicity structure of quark-nucleon scattering shows that there are exactly four independent amplitudes. The chiral-even distributions H and $\widetilde{H}$ survive in the forward limit in which the nucleon helicity is conserved, while the chiral-odd distributions E and $\widetilde{E}$ arise from the nucleon helicity flip associated with a finite momentum transfer.

The OFPDs are depicted graphically in Fig. 1, where k^μ and k'^μ are the four-momenta of the active partons. The physical meaning of the distributions becomes clearer if one introduces a conjugate lightlike vector p^μ of n^μ, with $p\cdot n=1$. Expanding $P^\mu=(P+P')^\mu/2$ and Δ^μ in terms of the vectors p^μ and n^μ then gives

$$\overline{P^\mu}=p^\mu+(\overline{M^2}/2)n^\mu, \tag{2a}$$

$$\Delta^\mu = -\xi(p^\mu - (\overline{M^2}/2)n^\mu) + \Delta_\perp^\mu, \tag{2b}$$

where $\overline{M^2} = \overline{P^2} = M^2 - t/4$, and the spatial components of $\overline{P^\mu}$ have been chosen along the z direction. If we focus on the p^μ components of the momenta, the initial and final nucleons have longitudinal momenta $(1+\xi/2)p^\mu$ and $(1-\xi/2)p^\mu$, and the outgoing and incoming quarks carry $(x+\xi/2)p^\mu$ and $(x-\xi/2)p^\mu$, respectively. Since the nucleon cannot have negative longitudinal momentum, the limit on ξ is obviously

$$0 < \xi < 2. \tag{3}$$

A more careful analysis using $\underline{\Delta_\perp^2} > 0$ leads to the more stringent constraint $0 < \xi < \sqrt{-t}/M$. On the other hand, since quarks cannot carry more longitudinal momentum than the parent nucleon, one has the constraint on x:

$$-1 < x < 1. \tag{4}$$

When $x > \xi/2$ both quark propagators in Fig. 1 represent quarks. When $x < -\xi/2$ they represent antiquarks. In these regions, the OFPDs are analogous to the usual parton distributions. In the intermediate region $-\xi/2 < x < \xi/2$, the quark propagators contain one quark and one antiquark, and here the distributions resemble a meson's wave function amplitude.

The off-forward parton distributions display characteristics of both the forward parton distributions and nucleon form factors. In fact, in the limit of $\Delta^\mu \to 0$, one finds [9]

$$H(x,0,0) = q(x), \quad \widetilde{H}(x,0,0) = \Delta q(x), \tag{5}$$

where $q(x)$ and $\Delta q(x)$ are the forward quark and quark helicity distributions, defined through similar light-cone correlations [23]. It must be pointed out that while the $\Delta^\mu \to 0$ limit is quite simple and natural for the OFPDs, it cannot be taken literally for the kinematics of the DVCS process, where a finite t-channel momentum transfer is essential to simultaneously maintain the initial photon deeply virtual and the final state photon real.

On the other hand, the first moment of the off-forward distributions are related to the nucleon form factors by the following sum rules [8,9]:

$$\int_{-1}^{1} dx H(x,\xi,t) = F_1(t), \tag{6a}$$

$$\int_{-1}^{1} dx E(x,\xi,t) = F_2(t), \tag{6b}$$

$$\int_{-1}^{1} dx \widetilde{H}(x,\xi,t) = G_A(t), \tag{6c}$$

$$\int_{-1}^{1} dx \widetilde{E}(x,\xi,t) = G_P(t). \tag{6d}$$

Here $F_1(t)$ and $F_2(t)$ are the Dirac and Pauli form factors, and $G_A(t)$ and $G_P(t)$ are the axial-vector and pseudoscalar form factors of the nucleon. The x-integrated distributions on the left-hand side of Eqs. (6) are in fact independent of ξ. The sum rules (6) provide important constraints on any model calculation of the OFPDs.

Generalizing the sum rules (6), let us multiply Eq. (1a) by x^{n-1} and integrate over x from -1 to $+1$, which gives

$$n^{\mu_1} n^{\mu_2} \cdots n^{\mu_n} \langle P' | \overline{\psi} i\overleftrightarrow{D}^{\mu_1} \cdots i\overleftrightarrow{D}^{\mu_{n-1}} \gamma^{\mu_n} \psi | P \rangle$$
$$= H_n(\xi,t)\overline{u}(P')\not{n}u(P) + E_n(\xi,t)\overline{u}(P')\frac{i\sigma^{\mu\nu}n_\mu \Delta_\nu}{2M}u(P), \tag{7}$$

where

$$H_n(\xi,t) = \int_{-1}^{1} dx x^{n-1} H(x,\xi,t), \tag{8}$$

and likewise for $E_n(\xi,t)$. The derivative $\overleftrightarrow{D}^\mu$ is defined as

$$\overleftrightarrow{D}^\mu = \frac{1}{2}(\vec{D}^\mu - \overleftarrow{D}^\mu), \tag{9}$$

where

$$\vec{D}^\mu = \vec{\partial}^\mu + igA^\mu, \tag{10a}$$

$$\overleftarrow{D}^\mu = \overleftarrow{\partial}^\mu - igA^\mu. \tag{10b}$$

The left-hand side of Eq. (7) is an off-forward matrix element of the twist-2 operator

$$O_2^{\mu_1 \cdots \mu_n} = \overline{\psi} i\overleftrightarrow{D}^{\{\mu_1} \cdots i\overleftrightarrow{D}^{\mu_{n-1}} \gamma^{\mu_n\}} \psi - \text{traces}, \tag{11}$$

where the braces $\{\cdots\}$ represent symmetrization of indices. On general grounds, a matrix element of $O_2^{\mu_1 \cdots \mu_n}$ is a sum of terms composed of form factors (which are functions of t only), appropriate Lorentz structures constructed from $\overline{P^\mu}$, Δ^μ and the Dirac matrices, and nucleon spinors. The ξ dependence in Eq. (7) arises only from contracting a vector Δ^μ in any of the Lorentz structures with the null vector n^μ. Therefore the ξ dependence of the moments of the OFPDs is in the form of polynomials. To find the degree of the polynomials, notice that there are at most n contractions of n^μ with Δ^μ in the nth moment. Thus $H_n(\xi,t)$ and $E_n(\xi,t)$ are polynomials of degree n in ξ, a result which is not obvious from the definition itself. The same considerations apply to the bilocal operator with γ_5 dependence.

III. BAG MODEL ESTIMATE OF OFF-FORWARD PARTON DISTRIBUTIONS

Having outlined the basic definitions and results for the OFPDs, we now present a calculation of the OFPDs in a simple version of the MIT bag model [20]. As mentioned in the Introduction, our choice of the MIT bag is based on the fact that it has quark degrees of freedom, and gives reasonable results for the electromagnetic form factors [22,24], as well as for polarized and unpolarized parton distributions [21–23]. However, being a model, it has a number of unwanted artifacts, such as a sharp boundary, absence of gluons, and the breaking of translational invariance and chiral symmetry. Nonetheless, we believe that our results should

provide a reasonable first guess of the unknown distributions at low-energy scales. There are, of course, many other nucleon models in existence in which the OFPDs could be calculated; however, we see no clear reason why these models would be more reliable than that considered here. The issue of evolution of the distributions to higher-energy scales will be addressed in a separate publication.

The starting point of our calculation is Eq. (1), which is written in terms of a coordinate system for which $\vec{P}+\vec{P}'$ is chosen to be in the z direction. The lightlike vectors p^μ and n^μ are given by

$$p^\mu=\Lambda(1;0,0,1), \quad n^\mu=\frac{1}{2\Lambda}(1;0,0,-1), \tag{12}$$

where Λ specifies the frame of reference. According to Eq. (12) the ''+'' and ''−'' components of a vector k^μ are proportional to $k\cdot n$ and $k\cdot p$, respectively. We choose $\Lambda=\overline{M}/2$ so that the initial and final nucleon momenta are

$$P^\mu=(\overline{M};-\vec{\Delta}/2), \quad P'^\mu=(\overline{M};\vec{\Delta}/2), \tag{13}$$

which corresponds to the Breit frame. Obviously in such a frame the three-momentum of the nucleon has transverse components. The t-channel momentum transfer squared becomes

$$t=-\vec{\Delta}^2=4(M^2-\overline{M^2}). \tag{14}$$

The variable ξ in this frame is therefore related to the projection of $\vec{\Delta}$ in the z direction,

$$\xi=-\Delta_z/\overline{M}, \tag{15}$$

i.e., $\vec{\Delta}=(\vec{\Delta}_\perp,-\xi\overline{M})$. For calculations in a model without exact translational invariance, the choice of frame is part of the model assumptions. In principle, a different result could be obtained if, for instance, one chooses instead a frame where the initial nucleon was at rest. Nevertheless, we believe that the main features of our result will be weakly frame dependent, as in many other similar types of calculations.

Recall that the coordinate space wave function of a quark in the rest frame of the MIT bag is given by

$$\psi(\vec{r})=\sqrt{4\pi}NR^3\begin{pmatrix} j_0(\epsilon_0 r) \\ i\vec{\sigma}\cdot\hat{r}j_1(\epsilon_0 r)\end{pmatrix}\chi, \tag{16}$$

where R is the bag radius, $\epsilon_0=\omega_0/R$ is the quark energy, and $\omega_0=2.04$ is lowest-frequency solution of the bag eigenequation, $\tan\omega_0=\omega_0/(1-\omega_0)$. The functions $j_{0,1}$ are the spherical Bessel functions ($r\equiv|\vec{r}|$), and χ is the quark spinor, equal to $\binom{1}{0}$ for a nucleon with spin in the $+z$ direction, and $\binom{0}{1}$ for spin in the $-z$ direction. The normalization N is given by

$$N^2=\frac{\omega_0}{2R^3(\omega_0-1)j_0^2(\omega_0)}. \tag{17}$$

The radius in the basic version of the bag model is given by the relation: $RM=4\omega_0$ [20,23].

Calculation of the OFPDs requires the wave function of a moving nucleon. One must therefore boost the rest frame wave function (16) to a frame moving with velocity $\vec{v}$. Including the time dependence of the quark wave function explicitly, $\psi(t,\vec{r})=\exp(-i\epsilon_0 t)\psi(\vec{r})$, the effect of a Lorentz boost on the wave function can be represented by [24]

$$\begin{aligned}\psi_{\vec{v}}(t,\vec{r})&=S(\Lambda_{\vec{v}})\psi((t-\vec{v}\cdot\vec{r})\cosh\omega;\vec{r}+\hat{v}\cdot\vec{r}(\cosh\omega-1)\\&\quad-\vec{v}t\,\cosh\omega)\\&=\exp[-i\epsilon_0(t-\vec{v}\cdot\vec{r})\cosh\omega]S(\Lambda_{\vec{v}})\\&\quad\times\psi(\vec{r}+\hat{v}\cdot\vec{r}(\cosh\omega-1)-\vec{v}t\,\cosh\omega),\end{aligned} \tag{18}$$

where

$$S(\Lambda_{\vec{v}})=\exp\left(\frac{\omega\hat{v}\cdot\vec{\alpha}}{2}\right)=\cosh\frac{\omega}{2}+\hat{v}\cdot\vec{\alpha}\,\sinh\frac{\omega}{2}. \tag{19}$$

Here the rapidity ω is related to the velocity by $\omega=\tanh^{-1}v$, where $v\equiv|\vec{v}|$. In the Breit frame the velocity of the initial nucleon is $\vec{v}=-\vec{\Delta}/2\overline{M}$, so that

$$\cosh\omega=\frac{\overline{M}}{M}, \quad \sinh\omega=\frac{|\vec{\Delta}|}{2M}. \tag{20}$$

Here we basically treat the independent quarks in the bag as free particles, ignoring the fact that they are confined by the bag boundary, which again is a part of the model assumptions.

In practical calculations, it will be more convenient to use a momentum space wave function $\varphi(k)$, which is simply related to the coordinate space wave function $\psi_{\vec{v}}(t,\vec{r})$ by a Fourier transformation:

$$\psi_{\vec{v}}(t,\vec{r})=S(\Lambda_{\vec{v}})\int\frac{d^3\vec{k}}{(2\pi)^3}\exp[-i(\tilde{\epsilon}_0 t-\tilde{\vec{k}}\cdot\vec{r})]\varphi(\vec{k}), \tag{21}$$

where $\tilde{\epsilon}_0=(\epsilon_0+\vec{k}\cdot\vec{v})\cosh\omega$, $\tilde{\vec{k}}_\perp=\vec{k}_\perp$, and $\tilde{\vec{k}}_\parallel=(\epsilon_0\vec{v}+\vec{k}_\parallel)\cosh\omega$, with $\vec{k}_\parallel=(\vec{k}\cdot\hat{v})\hat{v}$. The momentum space wave function is given by

$$\varphi(\vec{k})=\sqrt{4\pi}NR^3\begin{pmatrix} t_0(k) \\ \vec{\sigma}\cdot\hat{k}t_1(k)\end{pmatrix}\chi, \tag{22}$$

where $k\equiv|\vec{k}|$, and the functions $t_{0,1}$ are given by:

$$t_0(k)=\frac{j_0(\omega_0)\cos(kR)-j_0(kR)\cos\omega_0}{\omega_0^2-\vec{k}^2R^2}, \tag{23a}$$

$$t_1(k)=\frac{j_0(kR)j_1(\omega_0)kR-j_0(\omega_0)j_1(kR)\omega_0}{\omega_0^2-\vec{k}^2R^2}. \tag{23b}$$

One of the most important issues in calculating the off-forward matrix elements of single-particle operators in independent particle models like the bag is momentum conserva-

tion. In the following, we discuss this issue in some detail, which in the end will motivate the approach we take in performing the calculation. In independent particle models, strictly speaking, the form factors of any one-body operator must be zero. Since the momentum transfer through the one-body operator affects the active quark only, the remaining spectator quarks maintain their original states. On the other hand, if a momentum transfer $\vec{\Delta}$ is given to the whole nucleon, each of the quarks must receive a momentum transfer $\vec{\Delta}/3$, since before and after the interaction, the model nucleon must move as a whole. Thus, as a result of the momentum mismatch, the form factors must vanish. In a realistic situation, however, the nucleon wave function contains correlations. The momentum transfer injected to a single quark is in turn transferred through correlations to the other constituents, and asymptotically is equally shared among them. In independent particle models, however, these vital correlations needed for form factor calculations are missing.

In the literature, several common approaches have been adopted to deal with this issue. In one approach, model wave functions with no definite center-of-mass momentum are used [22,24]. Form factors are calculated from a Fourier component of the single-particle operator, $\hat{O}(\vec{\Delta})=\int d^3\vec{r}e^{i\vec{r}\cdot\vec{\Delta}}\hat{O}(\vec{r})$. In this type of calculation, the momentum transfer to the model nucleon and to the individual quarks is not well defined. If a boosted single-particle wave function is used, roughly speaking, the momentum transfer to each spectator quark is $\vec{\Delta}/3$. On the other hand, the momentum transfer through the active quark is approximately $(1-\epsilon_0/M)\vec{\Delta}$, which arises from the combined effects of boost and the action of the single-particle operator. (The relative sign of the two effects appears somewhat counterintuitive, but is nonetheless correct.)

In principle, a better approach would be to use initial and final nucleon wave functions with definite momentum, which can be approximately obtained, for instance, through the Peierls-Yoccoz projection [26] or the center-of-mass freedom separation method [27]. In such calculations, all quarks share equally the momentum transfer to the nucleon. In particular, the active quark in which the single-particle operator acts is injected with a momentum $\vec{\Delta}/3$ only. Since the state of the two spectator quarks also changes from the initial to the final nucleon, the effective operator which induces such a transition is actually a three-body operator.

A calculation of the OFPDs incorporating the effects of Lorentz boosts and projections is rather involved, and in practice not particularly illuminating. Instead, we consider a simpler alternative, by modifying the momentum transfer through the active quark in the approach of Ref. [24]. Namely, we let the effective momentum transfer through the active quark be $\eta\vec{\Delta}$, where η is taken to be a parameter. As mentioned above, $\eta=1-\epsilon_0/M$ in an unprojected calculation. Ultimately, η in our calculation will be determined by fitting the electromagnetic form factors of the nucleon, but one can expect η to be around 1/3, in the spirit of the momentum-projected calculation discussed above.

The matrix element of the bilocal operator is calculated using the boosted wave function of the active quark,

$$2\overline{M}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\int d^3\vec{r}e^{i\vec{\Delta}\cdot\vec{r}}\overline{\psi_{-\vec{v}}}(-\lambda n/2+\vec{r})\Gamma\psi_{\vec{v}}(\lambda n/2+\vec{r})=\frac{2\overline{M}}{\cosh\omega}\int\frac{d^3k}{(2\pi)^3}\,\delta\left(x-n_-\left(\tilde{k}_++\frac{\Delta_+}{2}\right)\right)\times\{\varphi^\dagger(k')S(\Lambda_{-\vec{v}})\gamma_0\Gamma S(\Lambda_{\vec{v}})\varphi(k)\},\tag{24}$$

where $\Gamma=\not{n}$ or $\not{n}\gamma_5$ and $k'\equiv|\vec{k}'|$, $\vec{k}'=\vec{k}+\bar{\bar{\vec{\Delta}}}$. The effective momentum transfer $\bar{\bar{\vec{\Delta}}}$ is given by

$$\bar{\bar{\vec{\Delta}}}=\eta\frac{\vec{\Delta}}{\cosh\omega}.\tag{25}$$

Choosing for simplicity $\Delta_y=0$, and using cylindrical coordinates to perform the k integration, the δ function in Eq. (24) reduces to a constraint on the z component of $\vec{k}$:

$$k_z=\frac{\overline{M}}{1-(\cosh\omega-1)\Delta_z^2/t}\left[x-\frac{1}{2\overline{M}}\left((2\epsilon_0+\tilde{\Delta}_z)\cosh\omega+|\bar{\bar{\vec{\Delta}}}|\sinh\omega-\frac{2\Delta_x\Delta_z}{t}k_x(\cosh\omega-1)\right)\right].\tag{26}$$

Evaluating the expression in the braces in Eq. (24) explicitly for $\Gamma=\not{n}$, and equating the spin-independent components on both sides, leads to

$$H(x,\xi,t)+\frac{t}{4M^2}E(x,\xi,t)=Z^2(t)(4\pi N^2R^6)\,\frac{\overline{M}}{1-(\cosh\omega-1)\Delta_z^2/t}\int\frac{dk_\perp d\varphi}{(2\pi)^3}k_\perp\left\{t_0(k)t_0(k')+\left[k_z'\cosh\omega+\frac{2\Delta_z}{t}\vec{k}'\cdot\vec{\Delta}\,\sinh^2\frac{\omega}{2}\right]\frac{t_0(k)t_1(k')}{k'}+\left[k_z\cosh\omega+\frac{2\Delta_z}{t}\vec{k}\cdot\vec{\Delta}\,\sinh^2\frac{\omega}{2}\right]\frac{t_1(k)t_0(k')}{k}+[\vec{k}\cdot\vec{k}'-\hat{\Delta}\cdot(\vec{k}\tilde{\Delta}_z-\bar{\bar{\vec{\Delta}}}k_z)\sinh\,\omega]\,\frac{t_1(k)t_1(k')}{kk'}\right\},\tag{27}$$

where the effects of the spectator quarks are included in the factor $Z(t)$ [24]:

$$Z(t)=\frac{N^2}{\cosh\omega}\int_0^R dr r^2 j_0(\epsilon_0|\vec{\Delta}|r/\overline{M})[j_0^2(\epsilon_0 r)+j_1^2(\epsilon_0 r)]. \tag{28}$$

If one compares the σ_y components in Eq. (24), on the other hand, a different combination of H and E arises:

$$\begin{aligned}H(x,\xi,t)+E(x,\xi,t)=Z^2(t)(4\pi N^2R^6)\,\frac{2M\overline{M}}{1-(\cosh\omega-1)\Delta_z^2/t}\int\frac{dk_\perp d\varphi}{(2\pi)^3}k_\perp\Bigg\{\frac{\sinh\omega}{|\vec{\Delta}|}t_0(k)t_0(k')+\Bigg[\frac{k_x'}{\Delta_x}\cosh\omega\\ -\frac{2\Delta_z}{t}\frac{(\vec{k}\times\vec{\Delta})_y}{\Delta_x}\sinh^2\frac{\omega}{2}\Bigg]\frac{t_0(k)t_1(k')}{k'}-\Bigg[\frac{k_x}{\Delta_x}\cosh\omega-\frac{2\Delta_z}{t}\frac{(\vec{k}\times\vec{\Delta})_y}{\Delta_x}\sinh^2\frac{\omega}{2}\Bigg]\frac{t_1(k)t_0(k')}{k}\\ +\Bigg[\frac{(\vec{k}\times\vec{\Delta})_y}{\Delta_x}-\frac{\vec{k}\cdot\vec{k}'-2k_y^2}{|\vec{\Delta}|}\sinh\omega\Bigg]\frac{t_1(k)t_1(k')}{kk'}\Bigg\}.\end{aligned} \tag{29}$$

Expressions for the individual functions H and E are then obtained by solving Eqs. (27) and (29) directly.

For the helicity-dependent case $\Gamma=\not{n}\gamma_5$ again one can obtain two independent combinations of $\widetilde{H}$ and $\widetilde{E}$ by comparing different spin components in Eq. (24). Equating coefficients of σ_z leads to

$$\begin{aligned}2\Bigg(1-\frac{\Delta_z^2}{4\overline{M}(M+\overline{M})}\Bigg)\widetilde{H}(x,\xi,t)-\frac{\Delta_z^2}{2M\overline{M}}\widetilde{E}(x,\xi,t)=Z^2(t)(4\pi N^2R^6)\,\frac{2M}{1-(\cosh\omega-1)\Delta_z^2/t}\int\frac{dk_\perp d\varphi}{(2\pi)^3}k_\perp\Bigg\{\Bigg[\cosh\omega\\ +\frac{2\Delta_z^2}{t}\sinh^2\frac{\omega}{2}\Bigg]t_0(k)t_0(k')+\Bigg[k_z'+\frac{\Delta_x k_x'}{|\vec{\Delta}|}\sinh\omega\Bigg]\frac{t_0(k)t_1(k')}{k'}\\ +\Bigg[k_z-\frac{\Delta_x k_x}{|\vec{\Delta}|}\sinh\omega\Bigg]\frac{t_1(k)t_0(k')}{k}+\Bigg[(2k_zk_z'-\vec{k}\cdot\vec{k}')\cosh\omega\\ +\frac{2\Delta_z}{t}(k_z(\vec{k}+\vec{k}')\cdot\vec{\Delta}-\Delta_z\vec{k}^2)\sinh^2\frac{\omega}{2}\Bigg]\frac{t_1(k)t_1(k')}{kk'}\Bigg\},\end{aligned} \tag{30}$$

while the σ_x components give

$$\begin{aligned}-\frac{\Delta_x\Delta_z}{2\overline{M}(M+\overline{M})}\widetilde{H}(x,\xi,t)-\frac{\Delta_x\Delta_z}{2M\overline{M}}\widetilde{E}(x,\xi,t)=Z^2(t)(4\pi N^2R^6)\,\frac{2M}{1-(\cosh\omega-1)\Delta_z^2/t}\int\frac{dk_\perp d\varphi}{(2\pi)^3}k_\perp\\ \times\Bigg\{\Bigg[\frac{2\Delta_x\Delta_z}{t}\sinh^2\frac{\omega}{2}\Bigg]t_0(k)t_0(k')+\Bigg[k_x'-\frac{\Delta_x k_z'}{|\vec{\Delta}|}\sinh\omega\Bigg]\frac{t_0(k)t_1(k')}{k'}\\ +\Bigg[k_x+\frac{\Delta_x k_z}{|\vec{\Delta}|}\sinh\omega\Bigg]\frac{t_1(k)t_0(k')}{k}\\ +\Bigg[(k_xk_z'+k_zk_x')\cosh\omega\\ +\frac{2\Delta_z}{t}(k_x(\vec{k}+\vec{k}')\cdot\vec{\Delta}-\Delta_x\vec{k}^2)\sinh^2\frac{\omega}{2}\Bigg]\frac{t_1(k)t_1(k')}{kk'}\Bigg\}.\end{aligned} \tag{31}$$

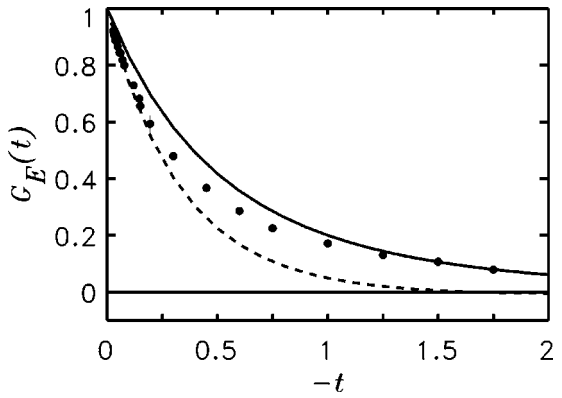

FIG. 2. t dependence of the proton electric form factor $G_E(t)$ for $\eta=0.35$ (solid) and $\eta=0.55$ (dashed). The bag radius in this and subsequent figures is fixed by $RM=4\omega_0$. The data are from Refs. [28,29].

By fixing the bag radius to be $R=4\omega_0/M$ [20,23], the model then has essentially one parameter η. This can be constrained by comparing the model predictions for the Sachs electric and magnetic form factors of the proton,

$$G_E(t)=F_1(t)+\frac{t}{4M^2}F_2(t), \tag{32a}$$

$$G_M(t)=F_1(t)+F_2(t), \tag{32b}$$

with the available data [28,29]. The form factors can be obtained from Eqs. (6a) and (6b) by integrating the H and E distributions in Eqs. (27) and (29) directly. Note that the results are in fact equivalent to those given for G_E and G_M in coordinate space in Ref. [24]. Furthermore, in a calculation for the proton with an SU(6)-symmetric wave function, the combinations of the electric, baryonic, and axial charge factors appear in such a way as to reduce the final result to that for a single quark. In Fig. 2 we show the predicted t dependence of G_E for two values of the parameter η ($\eta=0.35$ and 0.55). The t dependence of the magnetic form factor G_M is shown in Fig. 3. In both cases the bag model results are in quite good agreement with the data. Because the effects of boosting the bag are enhanced for larger $|t|$, one would expect in general the bag model to be more reliable at smaller

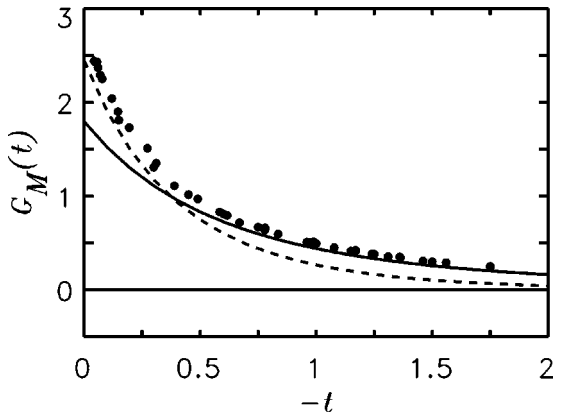

FIG. 3. t dependence of the proton magnetic form factor $G_M(t)$ for $\eta=0.35$ (solid line) and $\eta=0.55$ (dashed line). The data are from Refs. [28,29].

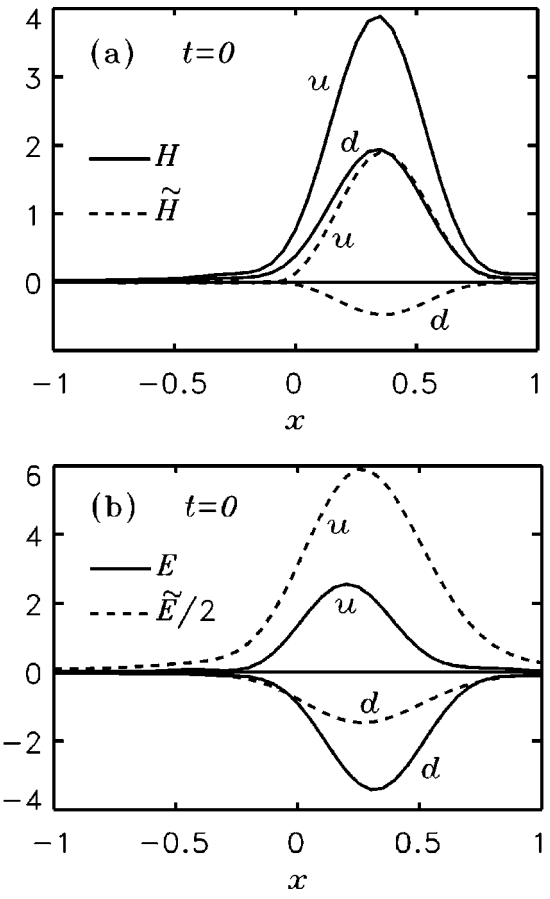

FIG. 4. Off-forward parton distributions at $t=0$ for the u and d flavors: (a) H and $\widetilde{H}$, which correspond to the usual spin-averaged and spin-dependent distributions $q(x)$ and $\Delta q(x)$, respectively; (b) E and $\widetilde{E}$ for $\eta=0.55$. Note that $\widetilde{E}/2$ is plotted.

values of $|t|$. The small-$|t|$ data (including the charge radius and magnetic moment) do favor the larger value $\eta=0.55$, while a better fit can be achieved at large $|t|$ with $\eta=0.35$. Note that while $G_E(0)$ is independent of R and η, the value of the magnetic moment $G_M(0)$ depends on the bag radius and η. This shows that different prescriptions of calculating form factors within the model can give different answers even at small momentum transfers. This is just one of the artifacts of the explicit breaking of translational invariance in the bag model.

Having fixed the model parameters, one can now calculate the individual OFPDs as a function of x and ξ, for different values of t. Again, assuming the SU(6) wave function for the proton, one multiplies the right-hand side of Eq. (27) by a factor 2 (1) and that of Eqs. (29), (30), and (31) by a factor 4/3 ($-1/3$) to solve for the up (down) quark distributions. In Figs. 4(a) and 4(b) we first show the distributions at $t=0$ (and $\xi=0$) for $-1<x<1$, for both the u and d quark flavors. Because the small-$|t|$ form factors are better described with a larger η value, for consistency we use here $\eta=0.55$. Note that the distributions H and $\widetilde{H}$ in Fig. 4(a) for u and d quarks are just the forward unpolarized $u(x)$ and $d(x)$, and polarized $\Delta u(x)$ and $\Delta d(x)$ parton distributions, respectively. Both H and $\widetilde{H}$ are in fact independent of η. Furthermore, the first moments of H and $\widetilde{H}$ at $t=0$ explicitly satisfy the normalization conditions [equal to 2 and 1 for

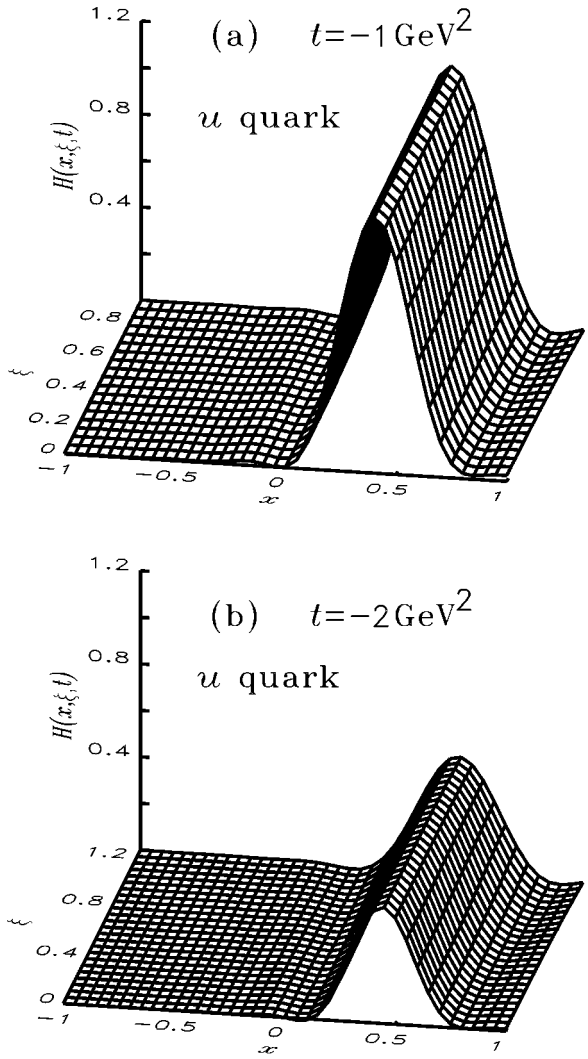

FIG. 5. Off-forward parton distribution $H(x,\xi,t)$ for the u quark, as a function of x and ξ, for (a) $t=-1\,\mathrm{GeV}^2$ and (b) $t=-2\,\mathrm{GeV}^2$.

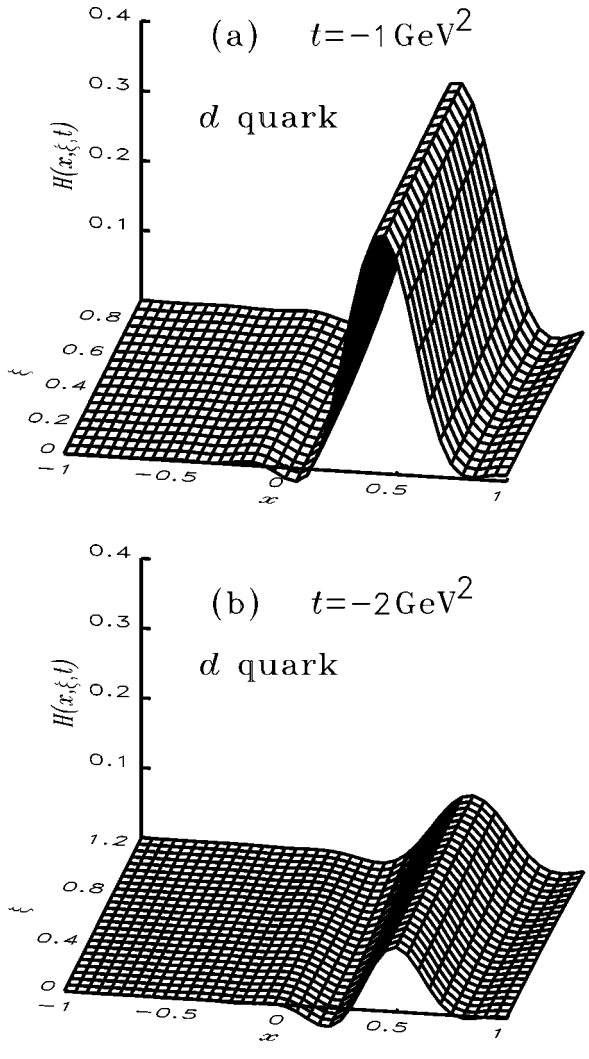

FIG. 6. As for Fig. 5, but for the d quark.

unpolarized u and d quarks, and $(4/3)g_A$ and $(-1/3)g_A$, where $g_A=0.65$ for a quark, in the polarized case, respectively]. The tensor and pseudoscalar distributions E and $\widetilde{E}$, however, shown in Fig. 4(b), do depend rather strongly on η. The peak value of $\widetilde{E}$ for the u quark, for instance, would be ~ 6 for $\eta=1-\epsilon_0/M=0.75$ (with $R=4\omega_0/M$). The first moment of E at $t=0$, summed over both the u and d flavors, is equal to $G_M(0)-1$ (as in Fig. 3), while the first moment of the flavor singlet $\widetilde{E}$ is $G_P(0)\approx 6$ for $\eta=0.55$.

Note also that the calculated x distributions do not vanish entirely at $x=1$, which simply reflects the fact that the initial and final nucleons are not good momentum eigenstates. However, the effect is only slightly noticeable for $\widetilde{E}$, and is negligible for the other distributions. In addition, because the bag model does not have the correct Regge physics built in, we cannot expect the calculated distributions to reproduce the small-x behavior of the structure functions. As for the forward parton distributions, the results are more reliable at moderate values of $x\sim 1/3$, where both the calculated and empirical distributions have a strong peak.

In Figs. 5–10 the distributions for both the u and d quarks are shown as a function of x and ξ, for $t=-1$ and $t=-2\,\mathrm{GeV}^2$. Throughout we use the value $\eta=0.35$ on account of the better agreement with the form factor data at large $|t|$ (see Figs. 2 and 3). One can see that the dependence on ξ is quite weak. According to the discussion in the last section, this means that the form factors of the twist-2 operators associated with the structure $\overline{P^{\mu_1}}\cdots\overline{P^{\mu_{n-1}}}\,\overline{u(P')}\,\gamma^{\mu_n}u(P)$ dominate over other form factors. We do not have a simple explanation for this. The t dependence of OFPDs, however, is rather strong, as expected from the above form factor behavior. Experimentally, the most interesting region for DVCS is $1<-t<2\,\mathrm{GeV}^2$. For too small $|t|$, QED radiative effects mask the processes sensitive to the OFPDs. On the other hand, for too large $|t|$, the distributions become too small to be measurable.

Since the moments of $H(x,\xi,t)$ and $\widetilde{H}(x,\xi,t)$ are related via Eq. (7) to form factors of twist-2 operators, the same QCD counting rule arguments [30] can be applied to study the large-$|t|$ behavior of the OFPDs as for the $n=0$ case. Simple analysis shows that the helicity-conserving form factors, and therefore also the distributions H and $\widetilde{H}$, should fall off like $1/t$ as $-t\to\infty$.

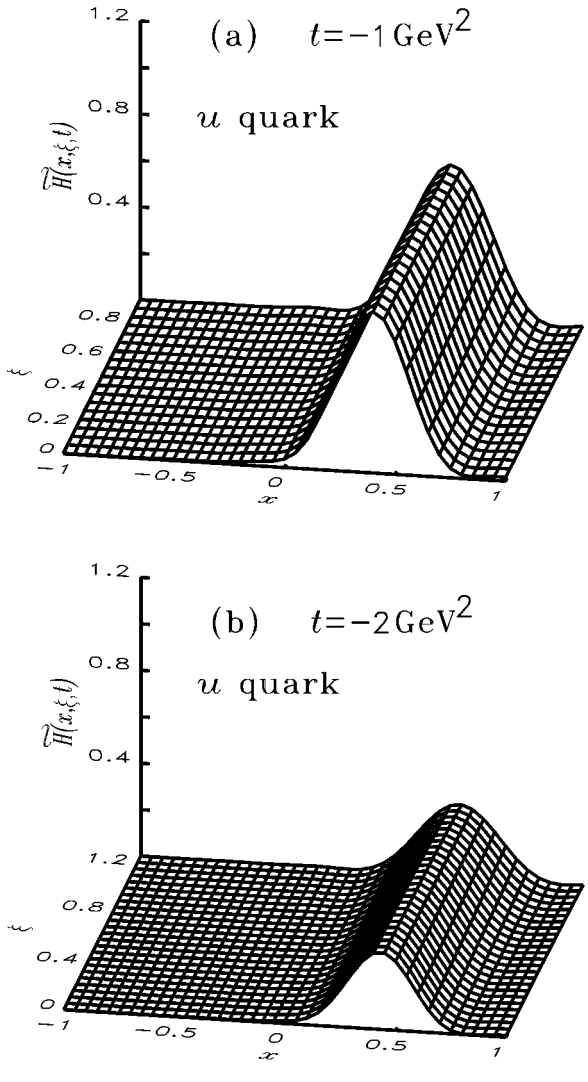

FIG. 7. Off-forward parton distribution $\widetilde{H}(x,\xi,t)$ for the u quark, for (a) $t=-1\ \mathrm{GeV}^2$ and (b) $t=-2\ \mathrm{GeV}^2$. The d quark distribution is obtained by multiplying by $-1/4$.

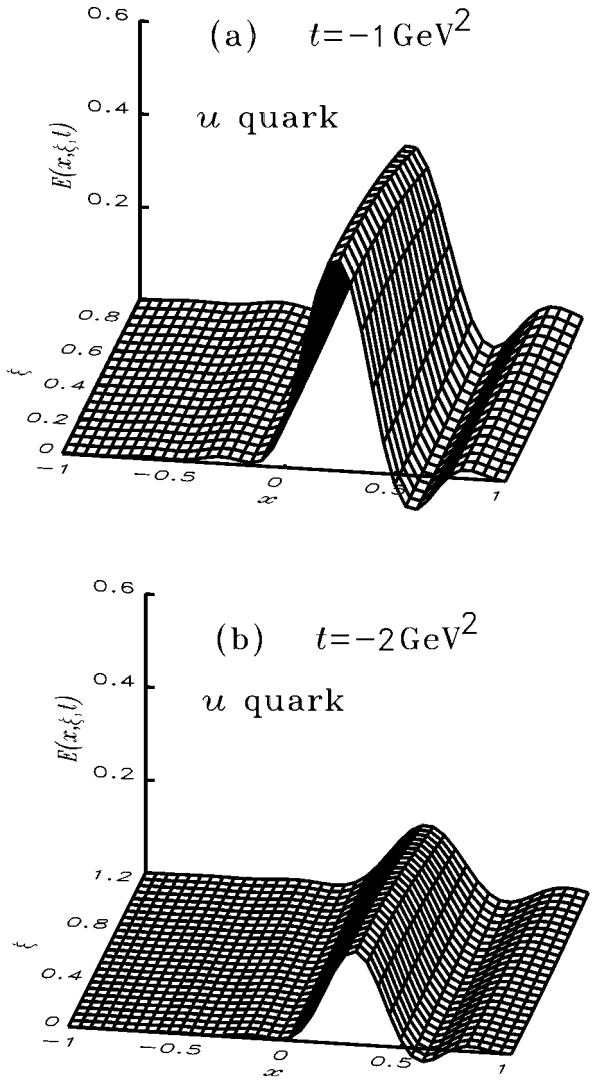

FIG. 8. Off-forward parton distribution $E(x,\xi,t)$ for the u quark, for (a) $t=-1\ \mathrm{GeV}^2$ and (b) $t=-2\ \mathrm{GeV}^2$.

IV. FORM FACTORS OF THE ENERGY-MOMENTUM TENSOR

In this section we review the role of the form factors of the QCD energy-momentum tensor played in the spin structure of the nucleon and their relation with the OFPDs. In particular, we present the first model calculation of the form factors of the quark part of the energy-momentum tensor.

Since the publication of the European Muon Collaboration (EMC) measurement [1] of the fraction of the proton's spin carried by quarks, the spin structure of the nucleon has been studied extensively in the literature. A deeper understanding of the problem, however, requires one to examine more closely the angular momentum operator in QCD. This can be written as a sum of quark and gluon contributions [9]:

$$\vec{J}_{\mathrm{QCD}}=\vec{J}_q+\vec{J}_g, \tag{33}$$

where

$$\vec{J}_q=\int d^3r\vec{r}\times\vec{T}_q=\int d^3r\left[\frac{1}{2}\psi^\dagger\vec{\Sigma}\psi+\psi^\dagger\vec{r}\times(-i\vec{D})\psi\right], \tag{34a}$$

$$\vec{J}_g=\int d^3r\vec{r}\times(\vec{E}\times\vec{B}), \tag{34b}$$

where summations over flavor and color indices are implicit, and $\vec{T}_q$ and $\vec{E}\times\vec{B}$ are the quark and gluon momentum densities, respectively. The Dirac spin matrix is denoted by $\vec{\Sigma}$, and $\vec{D}$ is the covariant derivative. By analogy with the magnetic moment, the separate quark and gluon contributions to the nucleon spin can be obtained from the form factors of the momentum density or, equivalently, the QCD energy-momentum tensor at zero momentum transfer.

The symmetric, conserved, gauge-invariant energy-momentum tensor $T^{\mu\nu}$ of QCD can be separated into quark and gluon components:

$$T^{\mu\nu}=T_q^{\mu\nu}+T_g^{\mu\nu}, \tag{35}$$

where the quark part is

$$T_q^{\mu\nu}=\frac{1}{2}\,(\overline{\psi}\gamma^{\{\mu}i\vec{D}^{\nu\}}\psi+\overline{\psi}\gamma^{\{\mu}i\overleftarrow{D}^{\nu\}}\psi), \tag{36}$$

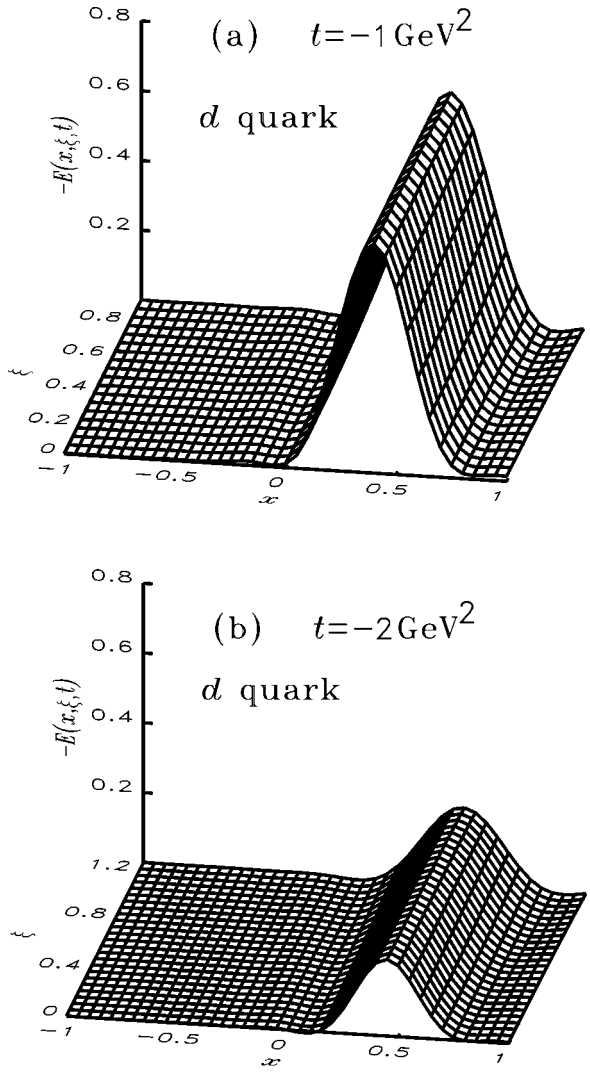

FIG. 9. As for Fig. 8, but for the d quark. Note that it is $-E(x,\xi,t)$ which is plotted.

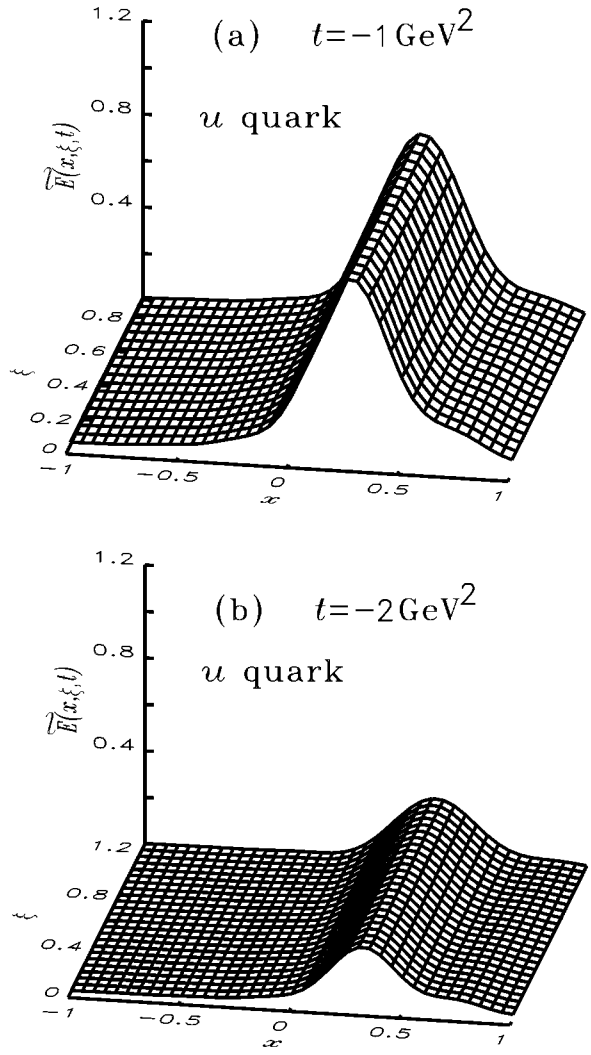

FIG. 10. Off-forward parton distribution $\widetilde{E}(x,\xi,t)$ for the u quark, for (a) $t=-1\ \mathrm{GeV}^2$ and (b) $t=-2\ \mathrm{GeV}^2$. The d quark distribution is obtained by multiplying by $-1/4$.

and the gluon part is

$$T_g^{\mu\nu}=\frac{1}{4}g^{\mu\nu}F^2-F^{\mu\alpha}F^{\nu}_{\alpha}. \tag{37}$$

Using Lorentz covariance and invariance under the discrete symmetries, one can expand the matrix elements of $T_{q,g}^{\mu\nu}$ in terms of four form factors:

$$\begin{aligned}\langle P'|T_{q,g}^{\mu\nu}|P\rangle=&\overline{u(}P')[A_{q,g}(t)\gamma^{\{\mu}\overline{P^{\nu\}}}\\&+B_{q,g}(t)\overline{P^{\{\mu}i\sigma^{\nu\}\alpha}}\Delta_{\alpha}/2M+C_{q,g}(t)(\Delta^{\mu}\Delta^{\nu}\\&-g^{\mu\nu}t)/M+\overline{C_{q,g}}(t)g^{\mu\nu}M]u(P),\end{aligned} \tag{38}$$

where the braces $\{\cdots\}$ on the superscripts denote symmetrization. Substituting the above into the nucleon matrix element of $\vec{J}_{q,g}$, one finds fractions of the nucleon spin carried by quarks, J_q, and gluons, J_g [9]:

$$J_{q,g}=\frac{1}{2}[A_{q,g}(0)+B_{q,g}(0)], \tag{39a}$$

$$J_q+J_g=\frac{1}{2}, \tag{39b}$$

where in the rest frame of the nucleon,

$$J_{q,g}\vec{S}=\langle P|\vec{J}_{q,g}|P\rangle, \tag{40}$$

with $\vec{S}$ the polarization vector of the nucleon. This role of the form factors of the total energy-momentum tensor has been first noted by Jaffe and Manohar [31].

Measuring the form factors of the energy-momentum tensor in practice is very difficult, partly because there is no fundamental probe which couples to them. (The graviton does, but only to the total tensor.) Because of asymptotic freedom, the form factors can be measured through deep-inelastic sum rules, as explained in Ref. [9]. According to our definition of OFPDs, it is simple to show that

$$\int_{-1}^{1}dx x H(x,\xi,t)=A(t)+\xi^2C(t), \tag{41}$$

$$\int_{-1}^{1} dx x E(x,\xi,t) = B(t) - \xi^2 C(t). \tag{42}$$

Combining these one obtains

$$\int_{-1}^{1} dx x [H(x,\xi,t) + E(x,\xi,t)] = A(t) + B(t), \tag{43}$$

where the ξ dependence drops out.

For the spin structure of the nucleon only small values of t are relevant. However, reaching small t in real experiments will be difficult; so some knowledge of the t dependence of the form factors is essential. In the remainder of this section, we will present a detailed calculation of the form factors of the (quark flavor-singlet part of the) energy-momentum tensor in the bag model. In principle, one can already obtain the form factors from the sum rules (41)–(43). However, here we will present an independent derivation, from which not only can one check the calculations on OFPDs, but also obtain the form factors not accessible from the above sum rules.

The bag energy-momentum tensor is a sum of quark and empty-bag contributions. The quark part can be written

$$T^{\mu\nu}_{q,\mathrm{bag}} = \frac{1}{2}\,(\overline{\psi}\gamma^{\{\mu} i\overrightarrow{\partial}^{\nu\}}\psi + \overline{\psi}\gamma^{\{\mu} i\overrightarrow{\partial}^{\nu\}}\psi), \tag{44}$$

where ψ is the quark field in the bag. Using the bag's equation of motion, $i\partial\!\!\!/\psi = \delta(r-R)$, it is easy to see that $T^{\mu\nu}_{q,\mathrm{bag}}$ is traceless. From the general theorem proved in Ref. [32], the bag quarks contribute 3/4 of the nucleon's mass. However, as we shall see below, the quarks in the bag carry all of the momentum of the nucleon.

The form factors A, B, C, and $\overline{C}$ are calculated from the matrix elements of $T^{\mu\nu}_{q,\mathrm{bag}}$ by taking various components of $T^{\mu\nu}_{q,\mathrm{bag}}$ and comparing with the form in Eq. (38). Taking $\vec{\Delta}$ to be in the z direction, one finds, from the $T^{0i}_{q,\mathrm{bag}}$ components,

$$\begin{aligned} A(t)+B(t) = 2Z^2(t)(4\pi N^2 R^6) \int \frac{d^3k}{(2\pi)^3} \\ \times \left\{ \frac{k_x^2}{\cosh\omega} \frac{t_1(k)t_1(k')}{kk'} + \frac{M}{|\vec{\Delta}|}\left(\frac{\epsilon_0}{M} - \frac{\eta t}{4\overline{M^2}}\right) \right. \\ \times \left[\left(t_0(k)t_0(k') - \frac{k_z k_z'}{kk'} t_1(k)t_1(k') \right) \sinh\omega \right. \\ + \left(\frac{k_z'}{k'} t_0(k)t_1(k') - \frac{k_z}{k} t_1(k)t_0(k') \right) \\ \left. \left. \times \cosh\omega \right] \right\}. \end{aligned} \tag{45}$$

In the case of no Lorentz boost, $\omega = 0$, and the combination $A+B$ is unity at $t=0$. According to Eq. (39a), this represents the fact that the total angular momentum (spin plus orbital) of quarks in the bag is 1/2. In Fig. 11 this is illustrated by the dotted curve. With a Lorentz boost and a momentum transfer

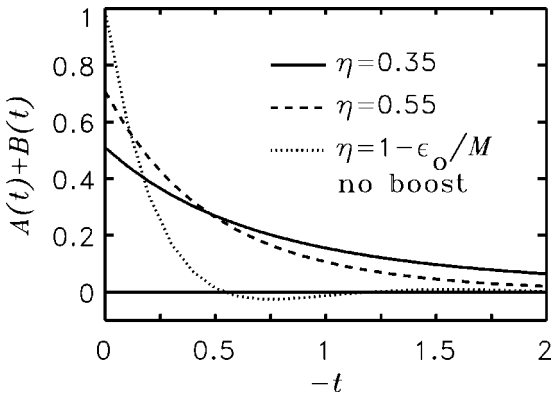

FIG. 11. t dependence of the form factor $A+B$ of the nucleon energy-momentum tensor, for the unboosted calculation with $\eta = 1-\epsilon_0/M$ (dotted line), and for the boosted result with $\eta = 0.55$ (dashed line), and $\eta = 0.35$ (solid line).

fraction to the quark $\eta < 1-\epsilon_0/M$, the value of $A(0)+B(0)$ is no longer unity. This can be understood from the fact that the boosted bag wave function does not have the correct Lorentz symmetry. For a smaller value $\eta = 0.35$ (solid line) one finds that $A+B$ at $t=0$ is now ≈ 0.5. However, as discussed in the previous section, the small-t form factors are more accurately described with a larger η, $\eta = 0.55$ (dashed line) which gives $A(0)+B(0) \approx 0.7$.

In dispersion theory, the t dependence of form factors is controlled by the nearest singularities in the t channel. For $A(t)+B(t)$, the quantum number of the channel is the exotic state with $J^{PC} = 1^{-+}$. There is no conclusive evidence at the present time for the existence of resonances in this channel, although theoretical investigations indicate that hybrid $q\overline{q}g$ mesons could exist in the mass range of 1.3–1.9 GeV [33]. If the dispersive behavior of $A(t)+B(t)$ is dominated by large-mass resonances, then $A+B$ will vary slowly with t, at least noticeably slower than the electromagnetic form factor. Although the bag calculation does indicate such a trend, the evidence is not strong. We suspect therefore that either the t dependence of the bag prediction is not reliable or multipion cuts in the form factor are important. Further study in this direction is called for.

To obtain the individual form factors A, B, C, and $\overline{C}$, one can take other components of the tensor $T^{\mu\nu}_{q,\mathrm{bag}}$. The $T^{00}_{q,\mathrm{bag}}$ component gives

$$\begin{aligned} A(t) + \frac{t}{4M^2}B(t) - \frac{t}{M^2}C(t) + \overline{C}(t) \\ = 3Z^2(t)(4\pi N^2 R^6)\left(\frac{\epsilon_0}{M} - \frac{\eta t}{4\overline{M^2}}\right) \\ \times \int \frac{d^3k}{(2\pi)^3} \{t_0(k)t_0(k') + \hat{k}\cdot\hat{k}' t_1(k)t_1(k')\}, \end{aligned} \tag{46}$$

where the factor of 3 is the number of valence quarks, and from the $T^{xx}_{q,\mathrm{bag}}$ (or $T^{yy}_{q,\mathrm{bag}}$) components one gets

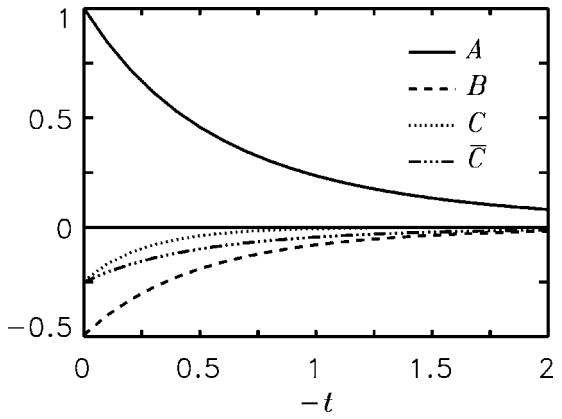

FIG. 12. t dependence of the form factors A, B, C, and $\overline{C}$ of the energy-momentum tensor, for $\eta=0.35$.

$$\frac{t}{M^2}C(t)-\overline{C}(t)=3Z^2(t)(4\pi N^2R^6)\,\frac{1}{M\,\cosh\omega}\times\int\frac{d^3k}{(2\pi)^3}k_x^2\left\{\left[\frac{t_0(k)t_1(k')}{k'}+\frac{t_1(k)t_0(k')}{k}\right]\cosh\omega+\frac{t_1(k)t_1(k')}{kk'}\left|\tilde{\Delta}\right|\sinh\omega\right\}. \tag{47}$$

Finally, the $T^{zz}_{q,\mathrm{bag}}$ components give a fourth equation:

$$\overline{C}(t)=-3Z^2(t)(4\pi N^2R^6)\,\frac{1}{2M}\int\frac{d^3k}{(2\pi)^3}(k_z+k_z')\times\left\{\frac{k_z'}{k'}t_0(k)t_1(k')+\frac{k_z}{k}t_1(k)t_0(k')\right\}. \tag{48}$$

Solving Eqs. (45)–(48), we plot in Fig. 12 the resulting form factors as a function of t for $\eta=0.35$. All form factors fall off monotonically as $-t$ increases, with $A(t)$ being the largest (and positive), and others relatively smaller (and negative). Note that for a radius $R=4\omega_0/M$, the form factor $A(0)=1$, which according to the definition of the form factors simply reflects the fact that all the momentum of the nucleon is carried by quarks.

The trace condition on the quark part of the bag energy-momentum tensor implies that not all of the form factors are independent. In fact, it is easy to show that

$$A(t)+\frac{t}{4M^2}B(t)-\frac{3t}{M^2}C(t)+4\overline{C}(t)=0. \tag{49}$$

We have checked that this relation is in fact satisfied to a very good approximation in the bag model, despite the violation of Lorentz symmetry after boosts. The above equation indicates that the normalization of the nonconserving form factor is fixed to $\overline{C}(0)=1/4$. It must be pointed out, however, that the above relation is not true in QCD because the renormalized quark part of the energy-momentum tensor has a trace anomaly [34,35].

V. CONCLUSION

In this paper we have presented a detailed study of a new class of nucleon observables, the off-forward parton distributions. The physical significance of the distributions has been explained in a partonic language, and their relations to form factors of twist-2 operators made explicit. The ξ dependence of the moments of the OFPDs is found to have a rather simple polynomial form. We have made the first model calculation of the distributions using the MIT bag model, with the specific details of the model fixed by requiring one to reproduce the electromagnetic form factors of the proton and the gross features of the parton distributions at a low-energy scale. In relation to the spin structure of the nucleon, we have also studied the form factors of the energy-momentum tensor in the bag model, focusing on the t dependence in the range of 0 to $-2\ \mathrm{GeV}^2$.

The model calculation includes the effects of Lorentz boosts of the quark wave function and spectator quarks. The ξ dependence of the distributions turns out to be extremely weak, indicating that the form factors of the twist-2 operators associated with the Lorentz structure $P^{\mu_1}\cdots P^{\mu_{n-1}}\overline{u(P')}\gamma^{\mu_n}u(P)$ are dominant. The t dependence of the OFPDs, on the other hand, is much stronger, and roughly follows that of the elastic form factors. Our results for the combination $A+B$ of the tensor form factors provide the first concrete indication of their possible t dependence, which will be important for the extraction of the value at $t=0$, since this gives the total angular momentum carried by quarks in the nucleon.

An important issue to be addressed in the future is the Q^2 evolution of the OFPDs. There have been many attempts in the literature to connect bag model calculations with low-energy observables and develop a phenomenology at higher energies though Q^2 evolution, typically starting from scales of $\mathcal{O}\sim 0.2\ \mathrm{GeV}^2$ [21]. In this approach the OFPDs would firstly need to be evolved before one could make direct estimates of the experimental cross sections. The evolution equations for the off-forward distributions, which interpolate the Brodsky-Lepage [36] and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [37] evolution equations, have been derived in Refs. [10,12,13]. The technology for solving these new evolution equations is currently being developed. In addition, Balitsky and Braun [38] have shown the Brodsky-Lepage equations to be equivalent to evolution equations for so-called string operators [39] on the light-cone. Apart from the evolution, the model calculations of the OFPDs could also be refined in future by including Lorentz boosts together with momentum projections.

The most relevant experiment from which the off-forward distributions and tensor form factors can be extracted is deeply virtual Compton scattering [9,10]. In practice, the DVCS process will be overwhelmed at small t by the Bethe-Heitler process, whose cross section has a QED infrared divergence at $t=0$, so that one cannot go to too small t. One may get around this to some extent by subtracting the (calculable) Bethe-Heitler contribution or by isolating the virtual Compton and Bethe-Heitler interference term [40] through

single-spin asymmetry or combined lepton-antilepton scattering [10]. Other processes which can also provide information on the OFPDs include diffractive ρ and J/ψ production [13–18], which are sensitive to the off-forward gluon distribution. A series of dedicated experiments at suitable machines, such as HERMES, ELFE, or at Jefferson Lab, can in future explore these new distributions, and thus provide valuable information on the distribution of spin in the nucleon.

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