Predictions for s-wave and p-wave heavy baryons from sum rules and the constituent quark model: Strong interactions

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We study the strong interactions of the $L=1$ orbitally excited baryons with one heavy quark in the framework of the heavy hadron chiral perturbation theory. To leading order in the heavy mass expansion, the interaction Lagrangian describing the couplings of these states among themselves and with the ground state heavy baryons contains 45 unknown couplings. We derive sum rules analogous to the Adler-Weisberger sum rule which constrain these couplings and relate them to the couplings of the s-wave heavy baryons. Using a spin-3/2 baryon as a target, we find a sum rule expressing the deviation from the quark model prediction for pion couplings to s-wave states in terms of couplings of the p-wave states. In the constituent quark model these couplings are related and can be expressed in terms of only six reduced matrix elements. Using recent CLEO data on Σ_c^* and Λ_{c1}^+ strong decays, we determine some of the unknown couplings in the chiral Lagrangian and two of the six quark model reduced matrix elements. Specific predictions are made for the decay properties of a few other $L=1$ charmed baryons. [S0556-2821(97)01521-X]

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I. INTRODUCTION

Baryons containing one heavy quark offer an important testing ground for the ideas and predictions of heavy quark spin-flavor SU(4) and light flavor SU(3) symmetries. These symmetries become manifest in QCD in the limits of infinite heavy quark masses $m_b, m_c \to \infty$ and identical light quark masses $m_u = m_d = m_s$. Although implications of these symmetries for the spectroscopy and decay properties of the heavy baryons are well known (for a review see, e.g., [1]), so far very few predictions, if any, can be compared with experiments due to lack of both data and theoretical knowledge of the unknown parameters.

This is the first of a series of papers in which we will study the properties of the excited heavy baryons, focusing on the first orbital excitations, the p-wave baryons with one heavy quark. In a sequel we will consider the radiative decays of these states.

The spectroscopy of these states is reviewed in Sec. II in the language of the constituent quark model. The constraints imposed by heavy quark symmetry on the possible structure of these couplings can be automatically incorporated by describing them in the framework of heavy hadron chiral perturbation theory [2,3]. The resulting chiral Lagrangian is presented in Sec. III. We include all possible strong interaction couplings among and between s-wave and p-wave baryons to leading order in $1/m_Q$ and chiral expansion. There are a total of 45 independent coupling constants up to and including D-wave interactions, which, in principle, have to be extracted from experiment. Recent data from Fermilab [5,7] and CLEO [6] make it possible to test and constrain the parameters of the theory.

We derive in Sec. IV model-independent sum rules which constrain these couplings and relate them to properties of the lowest-lying baryons. For the strong decay amplitudes these sum rules can be derived in analogy with the Adler-Weisberger (AW) sum rule familiar from current algebra. With a spin-3/2 baryon as a target, the two spin projections 3/2 and 1/2 along the incident pion's momentum give rise to two sum rules. One of these can be used to parametrize the deviation from the quark model relation among the two pion couplings g_1, g_2 to the s-wave heavy baryons, expressing it in terms of the pion couplings of the p-wave baryons.

In Sec. V we derive predictions for the strong couplings of the p-wave baryons in the constituent quark model. Many of these coupling constants can be computed in the quark model whereas others can be related in a simple way. In fact all but six of the 45 coupling constants are determined in the quark model. Furthermore, four of these six couplings are constrained to satisfy an AW sum rule.

In Sec. VI we discuss a few phenomenological applications of our results. We extract one of the pion couplings to the s-wave baryons g_2 from recent CLEO measurements on the Σ_c^* width. The extracted value for g_2 is consistent with the quark model prediction. This is used in turn to determine the S-wave and D-wave couplings of two p-wave charmed baryons Λ_{c1}^+ from their two-pion widths. Taken together with the quark model relations in Sec. V, these couplings can be used to estimate the strong couplings of some of the other p-wave charmed baryons. A few specific predictions are presented for some decay modes of these states. We conclude with some comments in Sec. VII.

II. SPECTROSCOPY OF HEAVY BARYONS

The heavy baryons fall into the $\overline{\mathbf{3}}$ and $\mathbf{6}$ representations of flavor SU(3), into which the product $\mathbf{3}\otimes\mathbf{3} = \overline{\mathbf{3}}\oplus\mathbf{6}$ is decomposed, corresponding to the two light quarks in the baryon.

TABLE I. The p-wave charmed baryons and their quantum numbers. S (the total spin of the two light quarks) is a good quantum number only in the constituent quark model. In the quark model, the first (last) four multiplets have even (odd) orbital wave functions under a permutation of the two light quarks.

	SU(3)	S	$s_\ell^{\pi_\ell}$
$\Lambda_{c1}(\frac{1}{2},\frac{3}{2})$	$\overline{3}$	0	1^-
$\Sigma_{c0}(\frac{1}{2})$	6	1	0^-
$\Sigma_{c1}(\frac{1}{2},\frac{3}{2})$	6	1	1^-
$\Sigma_{c2}(\frac{3}{2},\frac{5}{2})$	6	1	2^-
$\Sigma'_{c1}(\frac{1}{2},\frac{3}{2})$	6	0	1^-
$\Lambda'_{c0}(\frac{1}{2})$	$\overline{3}$	1	0^-
$\Lambda'_{c1}(\frac{1}{2},\frac{3}{2})$	$\overline{3}$	1	1^-
$\Lambda'_{c2}(\frac{3}{2},\frac{5}{2})$	$\overline{3}$	1	2^-

The lowest-lying states transform as an $\overline{\mathbf{3}}$ and can be represented either as an antisymmetric matrix $B_{\overline{3}}[3]$ or as a vector T [8]:

$$T^i=\frac{1+\not{v}}{2}(\Xi_c^0-\Xi_c^+\Lambda_c^+)_i=\frac{1}{2}\epsilon_{ijk}(B_{\overline{3}})_{jk}. \qquad (2.1)$$

We have taken as a heavy quark a charm quark. This multiplet contains an isospin doublet $(\Xi_c^0-\Xi_c^+)$ and a singlet Λ_c^+. In the heavy quark limit, the angular momentum and parity of the light constituents in a heavy baryon become good quantum numbers and the multiplet (2.1) has $s_\ell^{\pi_\ell}=0^+$.

Above this multiplet lie other s-wave states with $s_\ell^{\pi_\ell}=1^+$ which transform as a **6** under light SU(3). When combining the spin 1 of the light degrees of freedom with the heavy quark spin 1/2, one almost degenerate doublet is obtained, with total spins $J=1/2,3/2$. Both these states can be grouped together into one superfield as [8]

$$S_\mu^{ij}=\frac{1}{\sqrt{3}}(\gamma_\mu+v_\mu)\gamma_5\frac{1+\not{v}}{2}B_6^{ij}+\frac{1+\not{v}}{2}B_{6\mu}^{*ij}. \qquad (2.2)$$

The matrices B_6 and $B_{6\mu}^*$ are defined in [3]

$$(B_6)_{ij}=\begin{pmatrix}\Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{+\prime} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{0\prime} \\ \frac{1}{\sqrt{2}}\Xi_c^{+\prime} & \frac{1}{\sqrt{2}}\Xi_c^{0\prime} & \Omega_c^0\end{pmatrix}_{ij} \qquad (2.3)$$

and analogously for the sextet of spin-3/2 Rarita-Schwinger baryon fields $B_{6\mu}^*$.

The spectroscopy of the p-wave heavy baryons is more complex. There are altogether eight heavy quark symmetry multiplets of p-wave baryons, represented in Table I together with their quantum numbers.[1] They can be classified into two distinct groups, corresponding in the constituent quark model to symmetric and antisymmetric orbital wave functions, respectively, under a permutation of the two light quarks [9,10]. We will refer to them as symmetric and antisymmetric states. Potential models [9,11] indicate that the former lie about 150 MeV below the latter.

The lowest-lying p-wave states arise from combining the heavy quark spin with light constituents in an $s_\ell^{\pi_\ell}=1^-$ symmetric state. The corresponding heavy baryon states have spin and parity $J^P=1/2^-,3/2^-$. The $I=0$ members of these multiplets have been observed experimentally [4–7] and are known as $\Lambda_{c_1}(1/2,3/2)$. Their fields can be combined again into a superfield as [12]

$$R_\mu^i=\frac{1}{\sqrt{3}}(\gamma_\mu+v_\mu)\gamma_5R^i+R_\mu^{*i} \qquad (2.4)$$

with

$$R_i=\frac{1+\not{v}}{2}(\Xi_{c_1}^0-\Xi_{c_1}^+\Lambda_{c_1}^+)_i,$$

$$R_\mu^{*i}=\frac{1+\not{v}}{2}(\Xi_{c_1\mu}^{*0}-\Xi_{c_1\mu}^{*+}\Lambda_{c_1\mu}^{*+})_i. \qquad (2.5)$$

Above these states lie three other p-wave **6** symmetric multiplets with quantum numbers of the light degrees of freedom $s_\ell^{\pi_\ell}=0^-,1^-,2^-$. Their $I=1$ members will be denoted as $\Sigma_{c0}(\frac{1}{2}),\Sigma_{c1}(\frac{1}{2},\frac{3}{2})$ and $\Sigma_{c2}(\frac{3}{2},\frac{5}{2})$. The $s_\ell^{\pi_\ell}=0^-$ multiplet will be represented as a symmetric matrix $(U)_{ij}$ defined as in Eq. (2.3) and the $s_\ell^{\pi_\ell}=1^-$ multiplet will be represented as a superfield similar to Eq. (2.4) but with a symmetric matrix V_μ^{ij}:

$$V_\mu^{ij}=\frac{1}{\sqrt{3}}(\gamma_\mu+v_\mu)\gamma_5V^{ij}+V_\mu^{*ij}. \qquad (2.6)$$

The superfield corresponding to the $s_\ell^{\pi_\ell}=2^-$ baryons is constructed as [13]

$$X_{\mu\nu}^{ij}=X_{\mu\nu}^{*ij}+\frac{1}{\sqrt{10}}\{(\gamma_\mu+v_\mu)\gamma_5g_{\nu\alpha}+(\gamma_\nu+v_\nu)\gamma_5g_{\mu\alpha}\}X_\alpha^{ij} \qquad (2.7)$$

with $X_{\mu\nu}^{*ij}$ a spin-5/2 Rarita-Schwinger field and X_α^{ij} its spin-3/2 heavy quark symmetry partner.

The antisymmetric p-wave states are constructed in complete analogy to the symmetric ones. There is a sextet $\Sigma'_{c1}(\frac{1}{2},\frac{3}{2})$ with quantum numbers $s_\ell^{\pi_\ell}=1^-$, which will be represented again by a superfield $R_\mu'^{ij}$ constructed in analogy to Eq. (2.2). In addition to this, there are three antitriplets, whose $I=0$ members are denoted by $\Lambda_{c0}'^+,\Lambda_{c1}'^+,\Lambda_{c2}'^+$. Their superfields will be denoted as $U'_i,V_\mu'^i,X_{\mu\nu}'^i$.

[1]The terminology adopted here is particularly suggestive, with the subscript labeling the angular momentum of the light degrees of freedom.

III. STRONG COUPLINGS OF THE HEAVY BARYONS

The couplings of the heavy baryons to the Goldstone bosons are described most compactly when expressed in terms of their superfields (2.2), (2.4), (2.6). The leading terms describe P-wave couplings among the s-wave baryons and S-wave couplings between the s-wave and p-wave baryons:

$$\mathcal{L}_{\rm int}=\frac{3}{2}ig_1\epsilon_{\mu\nu\sigma\lambda}{\rm tr}(\overline{S^{\mu}}v^{\nu}A^{\sigma}S^{\lambda})-\sqrt{3}g_2{\rm tr}(\overline{B_{\bar 3}}A^{\mu}S_{\mu}+\overline{S^{\mu}}A_{\mu}B_{\bar 3})+h_2\{\epsilon_{ijk}\overline{R^i_{\mu}}v_{\nu}A^{\nu}_{jl}S^{kl}_{\mu}+\epsilon_{ijk}\overline{S^{kl}_{\mu}}v_{\nu}A^{\nu}_{lj}R^i_{\mu}\}+h_3{\rm tr}(\overline{B_{\bar 3}}v_{\mu}A^{\mu}U$$
$$+\overline{U}v^{\mu}A_{\mu}B_{\bar 3})+h_4{\rm tr}\{\overline{V_{\mu}}v_{\nu}A^{\nu}S_{\mu}+\overline{S_{\mu}}v_{\nu}A^{\nu}V_{\mu}\}+h_5{\rm tr}(\overline{R'_{\mu}}v_{\nu}A^{\nu}S^{\mu}+\overline{S^{\mu}}v_{\nu}A^{\nu}R'_{\mu})+h_6(\overline{T_i}v_{\nu}A^{\nu}_{ji}U'_j+\overline{U'_i}v_{\nu}A^{\nu}_{ji}T_j)$$
$$+h_7\{\epsilon_{ijk}\overline{V'^{i}_{\mu}}v_{\nu}A^{\nu}_{jl}S^{\mu}_{kl}+\epsilon_{ijk}\overline{S^{\mu}_{kl}}v_{\nu}A^{\nu}_{lj}V'^{i}_{\mu}\}. \tag{3.1}$$

The Goldstone bosons couple to the matter fields through the nonlinear axial field A_μ defined as

$$A_{\mu}=\frac{i}{2}(\xi^{\dagger}\partial_{\mu}\xi-\xi\partial_{\mu}\xi^{\dagger}), \tag{3.2}$$

with $\xi=\exp(iM/f_\pi)$, $M=(1/\sqrt{2})\pi^a\lambda^a$, and $f_\pi=132$ MeV [2,3].

The couplings g_1, g_2 are defined[2] as in [3] and the coupling of the $\overline{\mathbf{3}}$ p-wave baryons h_2 is chosen as in [12]. We introduced new constants h_3-h_7 describing all the S-wave pion couplings of the p-wave to the s-wave baryons which are allowed by heavy quark symmetry.

The D-wave couplings of the p-wave baryons to s-wave baryons are described by dimension-five terms in the effective Lagrangian

$$\mathcal{L}_D=ih_8\epsilon_{ijk}\overline{S^{kl}_{\mu}}\left(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu}+\frac{2}{3}g_{\mu\nu}(v\cdot\mathcal{D})(v\cdot A)\right)_{lj}R^i_{\nu}+ih_9{\rm tr}\left\{\overline{S_{\mu}}\left(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu}+\frac{2}{3}g_{\mu\nu}(v\cdot\mathcal{D})(v\cdot A)\right)V_{\nu}\right\}$$
$$+ih_{10}\epsilon_{ijk}\overline{T_i}(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu})_{jl}X^{\mu\nu}_{kl}+h_{11}\epsilon_{\mu\nu\sigma\lambda}{\rm tr}\{\overline{S_{\mu}}(\mathcal{D}_{\nu}A_{\alpha}+\mathcal{D}_{\alpha}A_{\nu})X_{\alpha\sigma}\}v_{\lambda}$$
$$+ih_{12}{\rm tr}\left\{\overline{S_{\mu}}\left(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu}+\frac{2}{3}g_{\mu\nu}(v\cdot\mathcal{D})(v\cdot A)\right)R'_{\nu}\right\}+ih_{13}\epsilon_{ijk}\overline{S^{kl}_{\mu}}\left(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu}+\frac{2}{3}g_{\mu\nu}(v\cdot\mathcal{D})(v\cdot A)\right)_{lj}V'^{i}_{\nu}$$
$$+ih_{14}\overline{T_i}(\mathcal{D}_{\mu}A_{\nu}+\mathcal{D}_{\nu}A_{\mu})_{ji}X'^{j}_{\mu\nu}+h_{15}\epsilon_{\mu\nu\sigma\lambda}\epsilon_{ijk}\overline{S^{kl}_{\mu}}(\mathcal{D}_{\nu}A_{\alpha}+\mathcal{D}_{\alpha}A_{\nu})_{lj}X'^{i}_{\alpha\sigma}v_{\lambda}. \tag{3.3}$$

The covariant derivative of the axial field A_μ is defined as $\mathcal{D}_\mu A_\nu=\partial_\mu A_\nu+[V_\mu, A_\nu]$ with

$$V_{\mu}=\frac{1}{2}(\xi^{\dagger}\partial_{\mu}\xi+\xi\partial_{\mu}\xi^{\dagger}), \tag{3.4}$$

and satisfies the relation $\mathcal{D}_\mu A_\nu-\mathcal{D}_\nu A_\mu=0$. The structure $\mathcal{D}_\mu A_\nu+\mathcal{D}_\nu A_\mu+\frac{2}{3}g_{\mu\nu}(v\cdot\mathcal{D})(v\cdot A)$ appearing in the $\overline{S}R$ and $\overline{S}V$ couplings projects out a pure D wave.

In addition to the couplings described by the Lagrangians (3.1), and (3.3), the p-wave baryons can couple also among themselves and to the Goldstone bosons. The most general Lagrangian allowed by heavy quark symmetry describing the P-wave couplings of the symmetric p-wave states has the form

$$\mathcal{L}_{pp\pi}=if_1\epsilon^{\mu\nu\sigma\lambda}\overline{R^i_{\mu}}v_{\nu}A^{ji}_{\sigma}R^j_{\lambda}+if_2\epsilon^{\mu\nu\sigma\lambda}{\rm tr}(\overline{V_{\mu}}v_{\nu}A_{\sigma}V_{\lambda})+if_3\epsilon^{\mu\nu\sigma\lambda}{\rm tr}(\overline{X_{\mu\alpha}}v_{\nu}A_{\sigma}X_{\alpha\lambda})+f_4\epsilon_{ijk}\overline{R^{i\mu}}A^{jl}_{\mu}U^{kl}$$
$$+if_5\epsilon^{\mu\nu\sigma\lambda}\epsilon_{ijk}\overline{R^{i\mu}}v_{\nu}A^{jl}_{\sigma}V^{kl}_{\lambda}+f_6\epsilon_{ijk}(\overline{R^{i\mu}}A^{\nu jl}+\overline{R^{i\nu}}A^{\mu jl})X^{kl}_{\mu\nu}+f_7{\rm tr}(\overline{U}A^{\mu}V_{\mu})+f_8{\rm tr}[(\overline{V_{\mu}}A_{\nu}+\overline{V_{\nu}}A_{\mu})X_{\mu\nu}]+{\rm H.c.} \tag{3.5}$$

We note that the Goldstone bosons do not couple to the field U alone, as the only possible coupling ${\rm tr}(\overline{U}v\cdot AU)$ does not conserve parity.

The couplings of the antisymmetric p-wave states to the Goldstone bosons are described by a Lagrangian similar to Eq. (3.5):

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[2]The couplings in [8] are related to the ones in Eq. (3.1) by $(g_2)_{\rm Cho}=3/2g_1$ and $(g_3)_{\rm Cho}=-\sqrt{3}g_2$.

$$\mathcal{L}_{p'p'\pi}=if_1'\epsilon^{\mu\nu\sigma\lambda}\mathrm{tr}(\overline{R'_\mu}v_\nu A_\sigma R'_\lambda)+if_2'\epsilon^{\mu\nu\sigma\lambda}\overline{V'^i_\mu}v_\nu A^{ji}_\sigma V'^j_\lambda+if_3'\epsilon^{\mu\nu\sigma\lambda}\overline{X'^i_{\mu\alpha}}v_\nu A^{ji}_\sigma X'^j_{\alpha\lambda}+f_4'\epsilon_{ijk}\overline{R'^{kl\mu}}A^{lj}_\mu U'^i$$
$$+if_5'\epsilon^{\mu\nu\sigma\lambda}\epsilon_{ijk}\overline{R'^{kl\mu}}v_\nu A^{lj}_\sigma V'^i_\lambda+f_6'\epsilon_{ijk}(\overline{R'^{kl\mu}}A^{\nu lj}+\overline{R'^{kl\nu}}A^{\mu lj})X'^i_{\mu\nu}+f_7'\overline{U'^i}A^{ji\mu}V'^j_\mu+f_8'(\overline{V'^i_\mu}A^{ji}_\nu+\overline{V'^i_\nu}A^{ji}_\mu)X'^j_{\mu\nu}+\mathrm{H.c.} \tag{3.6}$$

Finally, the couplings of the symmetric to antisymmetric p-wave states are given by a Lagrangian containing 13 additional couplings:

$$\mathcal{L}_{pp'\pi}=if_1''\epsilon_{\mu\nu\sigma\lambda}\epsilon_{ijk}\overline{R^i_\mu}v_\nu A^{lj}_\sigma R'^{kl}_\lambda+f_2''\mathrm{tr}(\overline{U}A_\mu R'_\mu)+if_3''\epsilon_{\mu\nu\sigma\lambda}\mathrm{tr}(\overline{V}_\mu v_\nu A_\sigma R'_\lambda)+f_4''\mathrm{tr}[\overline{X}_{\mu\nu}(A_\mu R'_\nu+A_\nu R'_\mu)]+f_5''\overline{R^i_\mu}A^{ji}_\mu U'^j$$
$$+f_6''\epsilon_{ijk}\overline{V^{kl}_\mu}A^{lj}_\mu U'^i+if_7''\epsilon_{\mu\nu\sigma\lambda}\overline{R^i_\mu}v_\nu A^{ji}_\sigma V'^j_\lambda+f_8''\epsilon_{ijk}\overline{U^{kl}}A^{lj}_\mu V'^i_\mu+if_9''\epsilon_{\mu\nu\sigma\lambda}\epsilon_{ijk}\overline{V^{kl}_\mu}v_\nu A^{lj}_\sigma V'^i_\lambda+f_{10}''\epsilon_{ijk}\overline{X^{kl}_{\mu\nu}}(A^{lj}_\nu V'^i_\mu+A^{lj}_\mu V'^i_\nu)$$
$$+f_{11}''(\overline{R^i_\mu}A^{ji}_\nu+\overline{R^i_\nu}A^{ji}_\mu)X'^j_{\mu\nu}+f_{12}''\epsilon_{ijk}(\overline{V^{kl}_\mu}A^{lj}_\nu+\overline{V^{kl}_\nu}A^{lj}_\mu)X'^i_{\mu\nu}+if_{13}''\epsilon_{ijk}\epsilon_{\mu\alpha\beta\lambda}\overline{X^{kl}_{\mu\nu}}v_\alpha A^{lj}_\beta X'^i_{\nu\lambda}\,. \tag{3.7}$$

We neglected in Eqs. (3.5)–(3.7) interaction terms describing F-wave couplings, as they are expected to be highly suppressed on dimensional grounds.

The Lagrangian (3.1) gives the following typical decay widths:

$$\Gamma(\Sigma_c^{++}\to\pi^+\Lambda_c^+)=\frac{g_2^2}{2\pi f_\pi^2}\frac{M_{\Lambda_c^+}}{M_{\Sigma_c^{++}}}|\vec{p}_\pi|^3,\quad \Gamma(\Sigma_c^{++*}\to\pi^+\Sigma_c^+)=\frac{g_1^2}{16\pi f_\pi^2}\frac{M_{\Sigma_c^+}}{M_{\Sigma_c^{++*}}}|\vec{p}_\pi|^3,$$

$$\Gamma\left[\Lambda_{c_1}^+\left(\frac{1}{2}\right)\to\pi^+\Sigma_c^0\right]=\frac{h_2^2}{2\pi f_\pi^2}\frac{M_{\Sigma_c^0}}{M_{\Lambda_{c1}^+}}E_\pi^2|\vec{p}_\pi|,\quad \Gamma\left[\Sigma_{c0}^{++}\left(\frac{1}{2}\right)\to\pi^+\Lambda_c^+\right]=\frac{h_3^2}{2\pi f_\pi^2}\frac{M_{\Lambda_c^+}}{M_{\Sigma_{c0}^{++}}}E_\pi^2|\vec{p}_\pi|,$$

$$\Gamma\left[\Sigma_{c1}^{++}\left(\frac{1}{2}\right)\to\pi^+\Sigma_c^+\right]=\frac{h_4^2}{4\pi f_\pi^2}\frac{M_{\Sigma_c^+}}{M_{\Sigma_{c1}^{++}}}E_\pi^2|\vec{p}_\pi|,\quad \Gamma\left[\Sigma_{c1}'^{++}\left(\frac{1}{2}\right)\to\pi^+\Sigma_c^+\right]=\frac{h_5^2}{4\pi f_\pi^2}\frac{M_{\Sigma_c^+}}{M_{\Sigma_{c1}'^{++}}}E_\pi^2|\vec{p}_\pi|, \tag{3.8}$$

$$\Gamma\left[\Xi_{c0}'^0\left(\frac{1}{2}\right)\to\pi^-\Xi_c^+\right]=\frac{h_6^2}{2\pi f_\pi^2}\frac{M_{\Xi_c^+}}{M_{\Xi_{c0}'^0}}E_\pi^2|\vec{p}_\pi|,\quad \Gamma\left[\Lambda_{c1}'^+\left(\frac{1}{2}\right)\to\pi^+\Sigma_c^0\right]=\frac{h_7^2}{2\pi f_\pi^2}\frac{M_{\Sigma_c^0}}{M_{\Lambda_{c1}'^+}}E_\pi^2|\vec{p}_\pi|.$$

IV. ADLER-WEISBERGER SUM RULES FOR HEAVY BARYONS

A. Narrow width sum rules

One can derive an analog of the Adler-Weisberger sum rule involving the coupling g_2 by considering a dispersion relation for pion scattering on an s-wave $\overline{\mathbf{3}}$ baryon (similar sum rules have been discussed in [14–18] for the heavy meson case). One possible derivation, which will prove most convenient in the following, is based on the use of the forward (spin-averaged) matrix element of the retarded commutator:[3]

$$F^{ab}_{ji}(\nu)=i\int d^4x e^{iq\cdot x}\theta(x_0)\langle P_j|[D^a(x),D^b(0)]|P_i\rangle$$
$$=F^{(+)}(\nu)\frac{1}{2}\{\tau^a,\tau^b\}_{ji}+F^{(-)}(\nu)\frac{1}{2}[\tau^a,\tau^b]_{ji}\,, \tag{4.1}$$

where $D^a=\partial^\mu[\overline{q}\gamma_\mu(\tau^a/2)q]$ and $\nu=q^0$ is the pion energy in the baryon rest frame. Usual manipulations with current density commutators give the relation [19]

$$\frac{\partial}{\partial\nu}F^{(-)}(\nu)|_{q=0}=I_3 \tag{4.2}$$

with I_3 the isospin of the target. The states in Eq. (4.1) are normalized according to $\langle P_i|P_j\rangle=(E_i/m)(2\pi)^3\delta(\vec{P}_i-\vec{P}_j)$.

Inserting a complete set of states in Eq. (4.1) gives

[3]The derivation presented here is a slightly modified version of the one given in [19].

$$F_{ji}^{ab}(\nu)=(2\pi)^3\sum_{\Gamma}\int d\mu(\Gamma)\frac{\langle P_j|D^a(0)|\Gamma\rangle\langle\Gamma|D^b(0)|P_i\rangle}{-\nu-E+E_\Gamma-i\epsilon}\delta(\vec{q}+\vec{P}-\vec{P}_\Gamma)$$
$$-(2\pi)^3\sum_{\Gamma'}\int d\mu(\Gamma')\frac{\langle P_j|D^b(0)|\Gamma'\rangle\langle\Gamma'|D^a(0)|P_i\rangle}{-\nu+E-E_{\Gamma'}-i\epsilon}\delta(\vec{q}-\vec{P}+\vec{P}_{\Gamma'}). \tag{4.3}$$

Assuming that the target has isospin $I_3=+1/2$, the isospin-odd component $F^{(-)}(\nu)$ can be extracted as $F^{(-)}(\nu)=1/4[F^{1+i2,1-i2}(\nu)-F^{1-i2,1+i2}(\nu)]$. Furthermore, noting from Eq. (4.3) that $F^{ab}(\nu)$ has no singularities in the upper half-plane ν, the Cauchy theorem can be applied on a closed contour extending along the real axis and closed in the upper half-plane. This gives the dispersion relation,[4] assumed to require no subtraction

$$\mathrm{Re}F^{(-)}(\nu)=\frac{2\nu}{\pi}\mathcal{P}\int_0^\infty d\zeta\frac{\mathrm{Im}F^{(-)}(\zeta)}{\zeta^2-\nu^2}. \tag{4.4}$$

By adding the imaginary part of $F^{(-)}(\nu)$ to both sides, Eq. (4.4) can be rewritten as

$$F^{(-)}(\nu)=\frac{2\nu}{\pi}\int_0^\infty d\zeta\frac{\mathrm{Im}F^{(-)}(\zeta)}{\zeta^2-\nu^2}, \tag{4.5}$$

where ν is understood to have a small positive imaginary part.

The imaginary part of $F^{(-)}(\nu)$ can be obtained from Eq. (4.3). For $\nu>0$ only the first term will contribute (because the target is the lowest-lying state) and the result can be expressed in terms of inclusive cross sections for π scattering on an antitriplet baryon as

$$\mathrm{Im}F^{(-)}(\nu)=\frac{1}{8}(2\pi)^4\sum_{\Gamma}\int d\mu(\Gamma)|\langle P|D^{1+i2}|\Gamma\rangle|^2\delta(q+P-P_\Gamma)-\frac{1}{8}(2\pi)^4\sum_{\Gamma'}\int d\mu(\Gamma')|\langle P|D^{1-i2}|\Gamma'\rangle|^2\delta(q+P-P_{\Gamma'})$$
$$=\frac{f_\pi^2\nu}{4}[\sigma_0(\pi^-\Xi_c^+\to X)-\sigma_0(\pi^+\Xi_c^+\to X)]. \tag{4.6}$$

The cross sections σ_0 correspond to off-shell incident pions of momentum q with $q^2=0$. This value of q^2 is needed in order to be able to make use of relation (4.2) on the left-hand side (LHS) of the fixed-q^2 dispersion relation (4.5).

Inserting Eqs. (4.5) and (4.6) into (4.2) one obtains the well-known result for the Adler-Weisberger sum rule on an antitriplet baryon target

$$1=\frac{f_\pi^2}{\pi}\int_{m_\pi}^\infty\frac{d\nu}{\nu}[\sigma_0(\pi^-\Xi_c^+\to X)-\sigma_0(\pi^+\Xi_c^+\to X)]. \tag{4.7}$$

Let us assume in the following that the resonances dominate the integral in Eq. (4.7), that is, the contribution of the continuum states can be neglected. We will estimate the error induced by this approximation later in this section. Then $\sigma_0(\pi^+\Xi_c^+\to X)$ vanishes as this state has isospin 3/2 and there are no heavy baryons with this quantum number. Furthermore, the remaining cross section in Eq. (4.7) can be expressed in terms of the pionic width (into off-shell pions with $q^2=0$) of the respective excited state as

$$\sigma_0(\pi^-\Xi_c^+\to X)=2\pi^2\sum_{\mathrm{res}}(2J+1)\frac{\Gamma_0(X_{\mathrm{res}}\to\pi^-\Xi_c^+)}{\nu^2}\delta(\nu-\delta M_{\mathrm{exc}}). \tag{4.8}$$

Thus, the Adler-Weisberger sum rule on a $\bar{\mathbf{3}}$ baryon target reads, when only resonances are retained,

$$1=\pi f_\pi^2\sum_{res}(2J+1)\frac{\Gamma(X_{res}\to\pi^-\Xi_c^+)}{\nu^3}$$
$$=\overbrace{\frac{3}{2}g_2^2}^{P\ wave}+\overbrace{\frac{1}{2}h_3^2+h_6^2+\cdots}^{S\ wave}+\overbrace{\frac{4}{3}h_{10}^2|\vec{p}_\pi|^2+\frac{8}{3}h_{14}^2|\vec{p}_\pi|^2+\cdots}^{D\ wave}, \tag{4.9}$$

[4]We used here the relation $F^{(-)}(-\nu)=-F^{(-)*}(\nu)$ which can be obtained from a simple examination of Eq. (4.3).

where we have accounted explicitly for the contribution of the sextet s-wave baryons and of the p-wave baryons. The ellipsis stands for contributions from higher states which can decay to the ground state $\bar{\mathbf{3}}$ baryons with emission of one pion. The pion momenta in Eq. (4.9) are independent of the heavy quark mass in the infinite mass limit, so the sum rule holds for any species of heavy quark.

In a completely analogous way one can derive a sum rule involving the coupling g_1 from the scattering amplitude of pions off a sextet baryon. We define the corresponding retarded commutator averaged over the baryon spin as in Eq. (4.1) and take the baryon to be a member of an isospin doublet with $I_3=1/2$ and spin 1/2. There is, however, a difference when compared with the previous case, due to the fact that now there exist states lighter than the target: the s-wave $\bar{\mathbf{3}}$ baryons. As a result the two cuts of the function $F^{(-)}(\nu)$ along the real axis touch each other and partly overlap. However, the dispersion relation (4.5) remains valid, due to our deliberate choice of working with the retarded commutator instead of the more usual time-ordered product (see, e.g. [20]). In the case of the time-ordered product, the left-hand cut [due to the second term in Eq. (4.3)] sits above the real axis. This is no problem as long as the two cuts do not touch, as the contour can be taken to run above and below the cuts and close on a circle in the upper and lower half-planes. When the cuts touch and overlap, such a choice of the contour is not possible anymore. The method adopted here avoids these complications, as the cuts are always under the real axis and they never get to pinch the contour.

Because of the presence of states lighter than the target, the second term in Eq. (4.3) starts to contribute to Im $F^{(-)}(\nu)$ for positive ν. For this case, relation (4.6) is modified and reads (for $\nu>0$)

$$\mathrm{Im}F^{(-)}(\nu)=\frac{1}{8}(2\pi)^4\sum_{\Gamma}\int d\mu(\Gamma)\{|\langle P|D^{1+i2}|\Gamma_>\rangle|^2\delta(q+P-P_{\Gamma_>})+|\langle P|D^{1+i2}|\Gamma_<\rangle|^2\delta(q-P+P_{\Gamma_<})\}$$
$$-\frac{1}{8}(2\pi)^4\sum_{\Gamma'}\int d\mu(\Gamma')\{|\langle P|D^{1-i2}|\Gamma'_>\rangle|^2\delta(q+P-P_{\Gamma'>})+|\langle P|D^{1-i2}|\Gamma'_<\rangle|^2\delta(q-P+P_{\Gamma_<})\}. \quad (4.10)$$

We denoted here by $\Gamma_>$ ($\Gamma_<$) the states lying above (below) the target mass. The sum over $\Gamma_>$ can be expressed as before in terms of inclusive cross sections for π scattering, and the one over $\Gamma_<$ can be computed in terms of the decay width for the process "target $\to\Gamma_<\pi$" (summed over the spin of Γ, s_r)

$$\Gamma_0(T\to\Gamma_<\pi^+)=\frac{1}{2\pi}\sum_{s_\Gamma}\nu|\langle\pi^+\Gamma_<|T\rangle|^2. \quad (4.11)$$

The contribution of the states Γ which are degenerate with the target will be extracted explicitly. There are two such states, the **6** baryons with spins 1/2 and 3/2 (for the sum rule on a $\bar{\mathbf{3}}$ baryon this contribution vanished as pions do not couple to the $\bar{\mathbf{3}}$ states). Their contributions on the left-hand side of Eq. (4.2) can be obtained from Eq. (4.3) and are

$$\frac{\partial}{\partial\nu}F^{(-)}_{\mathrm{pole}}(\nu)|_{\nu=0}=\frac{g_1^2}{8}+\frac{g_1^2}{16}=\frac{3g_1^2}{16}. \quad (4.12)$$

The total contribution of all states which are not degenerate with the target to Eq. (4.10) can be written as

$$\mathrm{Im}F^{(-)}(\nu)=\frac{1}{2}f_\pi^2\pi^2\sum_{\Gamma_<}\{\Gamma_0(T\to\pi^+\Gamma)-\Gamma_0(T\to\pi^-\Gamma)\}\frac{1}{\nu}\delta(m_T-\nu-m_\Gamma)+\frac{f_\pi^2\nu}{4}[\sigma_0(\pi^-T\to\Gamma)-\sigma_0(\pi^+T\to\Gamma)]. \quad (4.13)$$

Keeping, as before, just the one-body states as intermediate states, one has (taking into account the fact that we have chosen the target T to have $I_3=+1/2$) that $\Gamma(T\to\pi^-\Gamma)=0$ and $\sigma(\pi^+T\to\Gamma)=0$. Inserting Eqs. (4.13) and (4.5) into Eq. (4.2) and keeping explicitly the contributions of the s-wave $\bar{\mathbf{3}}$ and p-wave baryons, one obtains the following form for the Adler-Weisberger sum rule on a target **6** baryon:

$$1=\overbrace{\frac{3g_1^2}{8}+\frac{g_2^2}{2}}^{P\ wave}+\overbrace{\frac{h_2^2}{2}+\frac{h_4^2}{4}+\frac{h_5^2}{4}+\frac{h_7^2}{2}+\cdots}^{S\ wave}$$
$$+\overbrace{\frac{4}{9}h_8^2|\vec{p}_\pi|^2+\frac{2}{9}h_9^2|\vec{p}_\pi|^2+\frac{1}{3}h_{11}^2|\vec{p}_\pi|^2+\frac{2}{9}h_{12}^2|\vec{p}_\pi|^2+\frac{4}{9}h_{13}^2|\vec{p}_\pi|^2+\frac{2}{3}h_{15}^2|\vec{p}_\pi|^2+\cdots}^{D-wave}. \quad (4.14)$$

The ellipses stand again for contributions from higher excited states which can decay to the **6** baryons with emission of a single pion.

Taking as target a polarized spin-3/2 sextet baryon gives new sum rules. For spin projection $m_z=+1/2$ along the incident pion direction we obtain

$$1=\overbrace{\frac{3g_1^2}{16}+g_2^2}^{P\ wave}+\overbrace{\frac{h_2^2}{2}+\frac{h_4^2}{4}+\frac{h_5^2}{4}+\frac{h_7^2}{2}+\cdots}^{S\ wave}$$
$$+\overbrace{\frac{2}{3}h_8^2|\vec{p}_\pi|^2+\frac{1}{3}h_9^2|\vec{p}_\pi|^2+\frac{1}{6}h_{11}^2|\vec{p}_\pi|^2+\frac{1}{3}h_{12}^2|\vec{p}_\pi|^2+\frac{2}{3}h_{13}^2|\vec{p}_\pi|^2+\frac{1}{3}h_{15}^2|\vec{p}_\pi|^2+\cdots}^{D\ wave}, \qquad (4.15)$$

and for $m_z=+3/2$

$$1=\overbrace{\frac{9g_1^2}{16}}^{P\ wave}+\overbrace{\frac{h_2^2}{2}+\frac{h_4^2}{4}+\frac{h_5^2}{4}+\frac{h_7^2}{2}+\cdots}^{S\ wave}$$
$$+\overbrace{\frac{2}{9}h_8^2|\vec{p}_\pi|^2+\frac{1}{9}h_9^2|\vec{p}_\pi|^2+\frac{1}{2}h_{11}^2|\vec{p}_\pi|^2+\frac{1}{9}h_{12}^2|\vec{p}_\pi|^2+\frac{2}{9}h_{13}^2|\vec{p}_\pi|^2+h_{15}^2|\vec{p}_\pi|^2+\cdots}^{D\ wave}, \qquad (4.16)$$

In fact only one of these sum rules is new: by taking their average the unpolarized sum rule (4.14) is recovered. We will take as the new independent sum rule the difference of Eqs. (4.15) and (4.16) written as

$$\frac{3g_1^2}{8}-g_2^2=\frac{4}{9}h_8^2|\vec{p}_\pi|^2+\frac{2}{9}h_9^2|\vec{p}_\pi|^2-\frac{1}{3}h_{11}^2|\vec{p}_\pi|^2+\frac{2}{9}h_{12}^2|\vec{p}_\pi|^2$$
$$+\frac{4}{9}h_{13}^2|\vec{p}_\pi|^2-\frac{2}{3}h_{15}^2|\vec{p}_\pi|^2+\cdots. \qquad (4.17)$$

One can see that the contributions of the S-wave couplings have canceled out in taking the difference. The phenomenological consequences of this sum rule will be discussed in Sec. VI.

B. Continuum contributions

We have neglected in the above considerations the contributions to the sum rule from continuum states. This is likely to be a good approximation in nature, where the heavy baryons are seen as narrow states with widths much smaller than their mass separation. A similar approximation has been justified in the meson case [17] by using large-N_c arguments: the contribution of two-body states to the sum rule is suppressed relative to the one of the resonances by $1/N_c$. The situation in the baryon case is, however, completely different. In the following we will enumerate the contributions of a few intermediate states to the sum rule in the large-N_c limit, following [21,22].

The coupling of an s-wave heavy baryon to the Goldstone bosons scales as $\sqrt{N_c}$, which can be understood by recalling that the pion can couple to each of the N_c-1 light quarks in the baryon (the factor of $1/f_\pi$ gives an additional suppression of $1/\sqrt{N_c}$). This gives that g_1 and g_2 scale as N_c. The similar s-wave to p-wave couplings are however only of order 1, because the Goldstone boson can only couple to the quark in a p-wave state. This implies that the couplings of the p-wave states h_i scale like $\sqrt{N_c}$.

On the other hand, the amplitude for the process pion + s-wave baryon → pion + s-wave baryon is also of order 1 [21,22]. This means that the two-body states (π, s-wave baryon) contribute to the sum rule at the same order in $1/N_c$ as the one-body states with p-wave baryons. This fact could potentially upset the resonance saturation approximation of the sum rules made above. Therefore an estimate of the continuum contribution is necessary.

We will restrict ourselves to the study of the continuum contributions to the sum rule on a $\overline{\mathbf{3}}$ baryon (4.9). Even without an explicit calculation it can be argued, as in the heavy meson case [17], that the continuum contribution must be positive since there are more states containing one heavy quark with isospin 1/2 than 3/2. Our explicit calculations will confirm this conjecture, at least for the low-energy region where we can compute the continuum contribution. This implies that the sum rules (4.9) and (4.14)–(4.17) should in fact be considered as inequalities.

In the absence of experimental data, the only reliable information we have about the continuum contribution comes from chiral perturbation theory (χPT). Unfortunately its validity is restricted to the low-energy region in the vicinity of the threshold for $\pi-\overline{\mathbf{3}}$ scattering. In this subsection we compute the continuum contribution from threshold up to the cutoff $\Lambda=345$ MeV, which will be shown to mark the limit of validity of χPT in this system. In the next section (IV C) we go beyond χPT and use unitarity to bound the total contributions of the S-wave and P-wave channels to the AW sum rule.

The contribution of the (π, s-wave baryon) continuum to the AW sum rule on a $\overline{\mathbf{3}}$ baryon (4.9) is expressed in terms of the cross sections appearing in Eq. (4.7). In accordance with our previous discussion we keep only the contributions of the following channels:

$$\sigma_-(\nu)=\sigma_0(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{1/2})+\sigma_0(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})+\sigma_0(\pi^-\Xi_c^+\to(\pi\mathbf{6})_{1/2})+\sigma_0(\pi^-\Xi_c^+\to(\pi\mathbf{6})_{3/2}), \tag{4.18}$$

$$\sigma_+(\nu)=\sigma_0(\pi^+\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})+\sigma_0(\pi^+\Xi_c^+\to(\pi\mathbf{6})_{3/2}). \tag{4.19}$$

We have separated in $\sigma_-(\nu)$ the contributions of the continuum states with isospins $I=1/2$ and 3/2. The corresponding amplitudes are given by the usual rules for isospin addition as

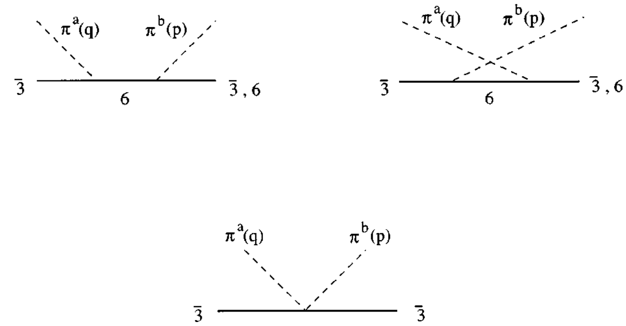

FIG. 1. Feynman diagrams for the continuum contribution to the AW sum rule on a $\overline{\mathbf{3}}$ baryon.

$$\mathcal{M}(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{1/2})=\frac{1}{\sqrt{3}}\mathcal{M}(\pi^-\Xi_c^+\to\pi^0\Xi_c^0)-\sqrt{\frac{2}{3}}\mathcal{M}(\pi^-\Xi_c^+\to\pi^-\Xi_c^+), \tag{4.20}$$

$$\mathcal{M}(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})=\sqrt{\frac{2}{3}}\mathcal{M}(\pi^-\Xi_c^+\to\pi^0\Xi_c^0)+\frac{1}{\sqrt{3}}\mathcal{M}(\pi^-\Xi_c^+\to\pi^-\Xi_c^+). \tag{4.21}$$

The evaluation of the diagrams shown in Fig. 1 gives

$$\mathcal{M}(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{1/2})=-\frac{i}{2}\sqrt{\frac{3}{2}}\frac{1}{f_\pi^2}\overline{u(v,s')}\left\{\frac{8}{3}v\cdot q+g_2^2[(v\cdot q)^2-p\cdot q]\left(\frac{1}{-v\cdot q-\Delta+i\Gamma_{6*}/2}-\frac{3}{v\cdot q-\Delta+i\Gamma_{6*}/2}\right)\right\}u(v,s), \tag{4.22}$$

$$\mathcal{M}(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})=-\frac{i\sqrt{3}}{2f_\pi^2}\overline{u(v,s')}\left\{\frac{2}{3}v\cdot q+g_2^2\frac{(v\cdot q)^2-p\cdot q}{-v\cdot q-\Delta+i\Gamma_{6*}/2}\right\}u(v,s), \tag{4.23}$$

$$\mathcal{M}(\pi^+\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})=-\frac{i}{2f_\pi^2}\overline{u(v,s')}\left\{2v\cdot q+3g_2^2\frac{(v\cdot q)^2-p\cdot q}{-v\cdot q-\Delta+i\Gamma_6/2}\right\}u(v,s). \tag{4.24}$$

Note that, as explained above, the incoming pion has $q^2=0$; however, the final one is on the mass shell $p^2=m_\pi^2$. We have denoted here $\Delta=M_{\mathbf{6}}-M_{\overline{\mathbf{3}}}$, the mass splitting between the $\overline{\mathbf{3}}$ and $\mathbf{6}$ multiplets. The widths in the denominators include both the charged and neutral pion channels and correspond to on-shell final pions. In the heavy mass limit they are equal and are given by

$$\Gamma_6=\Gamma_{6*}=\frac{3g_2^2}{8\pi f_\pi^2}(\Delta^2-m_\pi^2)^{3/2}. \tag{4.25}$$

The amplitudes (4.22)–(4.24) give, after squaring and integrating over the phase space of the final pion, the following cross sections:

$$\sigma_0(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{1/2})=\frac{3}{64\pi f_\pi^4}\left\{\frac{128}{9}E_\pi|\vec{p}_\pi|+\frac{2}{3}g_2^4E_\pi|\vec{p}_\pi|^3\left|\frac{1}{-E_\pi-\Delta+i\Gamma_{6*}/2}-\frac{3}{E_\pi-\Delta+i\Gamma_{6*}/2}\right|^2\right\}, \tag{4.26}$$

TABLE II. Continuum contributions to the AW sum rule on a $\overline{\mathbf{3}}$ baryon corresponding to different partial waves.

	Cutoff energy (MeV)						
	345	370	395	420	445	470	495
$(\overline{\mathbf{3}}+\pi)_S$	0.117	0.141	0.167	0.195	0.225	0.257	0.290
$(\overline{\mathbf{3}}+\pi)_P$	0.020	0.033	0.045	0.057	0.068	0.078	0.089
$(\mathbf{6}+\pi)_P$	0.000	0.000	0.000	0.001	0.001	0.002	0.003
Total	0.137	0.174	0.213	0.253	0.294	0.337	0.382

$$\sigma_0(\pi^-\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})=\frac{3}{32\pi f_\pi^4}\left\{\frac{8}{9}E_\pi|\vec{p}_\pi|+\frac{2}{3}g_2^4E_\pi|\vec{p}_\pi|^3\frac{1}{(E_\pi+\Delta)^2+\Gamma_{6*}^2/4}\right\}, \tag{4.27}$$

$$\sigma_0(\pi^+\Xi_c^+\to(\pi\overline{\mathbf{3}})_{3/2})=\frac{1}{32\pi f_\pi^4}\left\{8E_\pi|\vec{p}_\pi|+6g_2^4E_\pi|\vec{p}_\pi|^3\frac{1}{(E_\pi+\Delta)^2+\Gamma_6^2/4}\right\}. \tag{4.28}$$

An important point which must be taken into account is that expression (4.26) contains a resonant piece. Upon insertion in the AW sum rule (4.7), it reproduces, in the narrow width approximation, the contribution of the one-body states with a sextet baryon shown in Eq. (4.9). Therefore, to avoid double counting of this term, the true continuum contribution to the sum rule is obtained by explicitly subtracting it. To see this explicitly, we insert the resonant term in Eq. (4.26) into the integral on the right-hand side (RHS) of the AW sum rule. We obtain in the narrow width approximation

$$I_{\mathrm{AW}}=\frac{f_\pi^2}{\pi}\int\frac{d\nu}{\nu}\sigma_-^{\mathrm{res}}(\nu)=\frac{9g_2^4}{32\pi^2f_\pi^2}\int d\nu\frac{(\nu^2-m_\pi^2)^{3/2}}{(\nu-\Delta)^2+\Gamma_6^2/4}\to\frac{3}{2}g_2^2 \tag{4.29}$$

as $\Gamma\to0$. In the last step we used the well-known representation of the δ function

$$\frac{\Gamma}{2\pi}\frac{1}{(\nu-\Delta)^2+\Gamma^2/4}\to\delta(\nu-\Delta)\quad(\Gamma\to0). \tag{4.30}$$

The remaining cross sections needed for the continuum corrections to the sum rule (4.9) correspond to final states with a sextet baryon of spin 1/2 and 3/2 plus one pion. They can be computed analogously with the result

$$\sigma_0(\pi^-\Xi_c^+\to\pi\mathbf{6})=\frac{g_1^2g_2^2}{128\pi f_\pi^4}E_\pi(E_\pi'^2-m_\pi^2)^{3/2}\left|\frac{1}{-E_\pi'-\Delta+i\Gamma_6/2}+\frac{3}{E_\pi-\Delta+i\Gamma_6/2}\right|^2+\frac{g_1^2g_2^2}{64\pi f_\pi^4}\frac{E_\pi(E_\pi'^2-m_\pi^2)^{3/2}}{(E_\pi'+\Delta)^2+\Gamma_6^2/4},$$
$$E_\pi'=E_\pi-\Delta, \tag{4.31}$$

$$\sigma_0(\pi^+\Xi_c^+\to\pi\mathbf{6})=\frac{3g_1^2g_2^2}{64\pi f_\pi^4}\frac{E_\pi(E_\pi'^2-m_\pi^2)^{3/2}}{(E_\pi'+\Delta)^2+\Gamma_6^2/4}, \tag{4.32}$$

$$\sigma_0(\pi^-\Xi_c^+\to\pi\mathbf{6}^*)=2\sigma_0(\pi^-\Xi_c^+\to\pi\mathbf{6}), \tag{4.33}$$

$$\sigma_0(\pi^+\Xi_c^+\to\pi\mathbf{6}^*)=2\sigma_0(\pi^+\Xi_c^+\to\pi\mathbf{6}). \tag{4.34}$$

The first (second) term in Eq. (4.31) corresponds to final states with isospin 1/2 (3/2).

These cross sections are inserted in the AW sum rule (4.7) and the integration is done numerically. We included also the first term in the elastic cross section (4.26) although it is formally of higher order in $1/N_c$ than the contributions we are interested in. In fact it will be seen to dominate the continuum contribution. The mass splitting between the $\overline{\mathbf{3}}$ and $\mathbf{6}$ multiplets will be taken $\Delta=225$ MeV, corresponding to the charmed baryons' case. The quark model values (5.1) for the couplings g_1 and g_2 will be used (with $g_A=0.75$). We obtain for the continuum contribution to the AW sum rule (4.9) for a few values of the upper limit of integration the numbers shown in Table II. Especially for larger values of the upper cutoff, these corrections appear as being significant when compared with the one-body sextet contribution to the sum rule $3/2g_2^2=0.562$.

In reality we will see that these large contributions are simply an effect of the limited applicability of chiral perturbation theory for the particular process of pion scattering on a static heavy baryon. It will be shown in the following section that unitarity gives an upper bound on the elastic pion scattering cross sections, which is exceeded in the $L=0$, $I=1/2$ channel already above $E_\pi=345$ MeV. Therefore the only trustworthy values in Table II are those corresponding to the lowest value of the cutoff.

C. Unitarity constraints

The amplitude for elastic pion scattering $\pi^a T_i \to \pi^b T_j$ on a static target T with isospin 1/2 can be written as in Eq. (4.1) in terms of two functions $T^{(\pm)}$

$$T_{ji}^{ba} = T^{(+)}\delta_{ij}\delta_{ab} + T^{(-)}\frac{1}{2}[\tau^a,\tau^b]_{ji}. \tag{4.35}$$

Each of these functions can be expressed in terms of two amplitudes $f^{(\pm)}$ and $g^{(\pm)}$, corresponding to spin-nonflip and, respectively, spin-flip transitions, defined by

$$T^{(\pm)}(E_\pi,\cos\theta) = u^\dagger(v,s')[f^{(\pm)}(E_\pi,\cos\theta) + ig^{(\pm)}(E_\pi,\cos\theta) \times \vec{\sigma}\cdot(\hat{q}'\times\hat{q})]u(v,s). \tag{4.36}$$

We denoted here by E_π and $\cos\theta$ the pion energy and scattering angle in the rest frame of the target. $\hat{q},\hat{q}'$ are unit vectors along the directions of the initial and final pions and $u(v,s)$ are nonrelativistic two-spinors in the rest frame of T.

The functions $f^{(\pm)}$ and $g^{(\pm)}$ have partial wave expansions of the form (see, e.g., [23])

$$f^{(\pm)}(E_\pi,\cos\theta) = \sum_{l=0}^{\infty}[(l+1)f_{l+}^{(\pm)}(E_\pi) + l f_{l-}^{(\pm)}(E_\pi)] \times P_l(\cos\theta), \tag{4.37}$$

$$g^{(\pm)}(E_\pi,\cos\theta) = \sum_{l=1}^{\infty}[f_{l-}^{(\pm)}(E_\pi) - f_{l+}^{(\pm)}(E_\pi)]P_l'(\cos\theta). \tag{4.38}$$

From a physical point of view it is more transparent to work instead of the amplitudes $T^{(\pm)}$ with amplitudes of well-defined isospin, given by

$$T^{(1/2)} = T^{(+)} - 2T^{(-)}, \tag{4.39}$$

$$T^{(3/2)} = T^{(+)} + T^{(-)}. \tag{4.40}$$

These amplitudes have partial-wave expansions similar to those in Eqs. (4.37) and (4.38). The corresponding amplitudes will be called $f_{l\pm}^{(I)}(E_\pi)$, with $I=1/2,3/2$ and have physical interpretation of scattering amplitudes in channels with total angular momentum $j=l\pm\frac{1}{2}$, parity $-(-1)^l$ and isospin I.

Unitarity imposes a well-known constraint on the partial wave amplitudes

$$\mathrm{Im} f_{l\pm}^{(I)} \geq \frac{|\vec{p}_\pi|}{4\pi}|f_{l\pm}^{(I)}|^2. \tag{4.41}$$

For small energies where only elastic scattering is allowed, the inequality turns into an equality. The partial wave amplitudes are usually parametrized in this region in terms of phase shifts $\delta_{l\pm}^{(I)}$ as

$$f_{l\pm}^{(I)} = \frac{4\pi}{|\vec{p}_\pi|}\sin\delta_{l\pm}^{(I)} e^{i\delta_{l\pm}^{(I)}}. \tag{4.42}$$

From Eq. (4.41) one can derive an upper bound on the partial wave amplitudes, valid both in the elastic and inelastic cases:

$$|f_{l\pm}^{(I)}| \leq \frac{4\pi}{|\vec{p}_\pi|}. \tag{4.43}$$

We will use unitarity to derive an upper bound on the inclusive cross sections into all possible final states. The method makes use of the optical theorem which expresses an inclusive cross section in terms of the imaginary part of a forward scattering amplitude. Explicitly this gives the following expressions for the inclusive cross sections for pions incident on the $I_3=+1/2$ member of an isospin doublet:

$$\sigma(\pi^- T\to X) = \frac{1}{|\vec{p}_\pi|}\mathrm{Im}\mathcal{M}(\pi^- T\to\pi^- T) = \frac{1}{3|\vec{p}_\pi|}\mathrm{Im}[T^{(3/2)}(E_\pi,1) + 2T^{(1/2)}(E_\pi,1)], \tag{4.44}$$

$$\sigma(\pi^+ T\to X) = \frac{1}{|\vec{p}_\pi|}\mathrm{Im}\mathcal{M}(\pi^+ T\to\pi^+ T) = \frac{1}{|\vec{p}_\pi|}\mathrm{Im}T^{(3/2)}(E_\pi,1). \tag{4.45}$$

Inserting here the partial wave expansions (4.37) and (4.38) gives

$$\sigma(\pi^- T\to X) = \frac{1}{3|\vec{p}_\pi|}\sum_{l=0}^{\infty}[(l+1)\mathrm{Im}(f_{l+}^{(3/2)} + 2f_{l+}^{(1/2)}) + l\,\mathrm{Im}(f_{l-}^{(3/2)} + 2f_{l-}^{(1/2)})], \tag{4.46}$$

$$\sigma(\pi^+ T\to X) = \frac{1}{|\vec{p}_\pi|}\sum_{l=0}^{\infty}[(l+1)\mathrm{Im}f_{l+}^{(3/2)} + l\,\mathrm{Im}f_{l-}^{(3/2)}]. \tag{4.47}$$

Taking the difference we obtain

$$\sigma(\pi^- T\to X) - \sigma(\pi^+ T\to X) = \frac{2}{3|\vec{p}_\pi|}\sum_{l=0}^{\infty}[(l+1)\mathrm{Im}(f_{l+}^{(1/2)} - f_{l+}^{(3/2)}) + l\,\mathrm{Im}(f_{l-}^{(1/2)} - f_{l-}^{(3/2)})]. \tag{4.48}$$

From Eq. (4.41) one can see that $\mathrm{Im}f_{l\pm}^{(I)}$ is positive and bounded from above by Eq. (4.43). Therefore our strategy in the following will be to set an absolute upper bound on the difference (4.48) by using Eq. (4.43) for $\mathrm{Im}f_{l\pm}^{(1/2)}$ and taking $\mathrm{Im}f_{l\pm}^{(3/2)}=0$ everywhere outside the domain of applicability of chiral perturbation theory.

Strictly speaking the cross sections appearing in the AW sum rule are not the physical on-shell cross sections (4.44) and (4.45) but rather, cross sections with a massless incident pion. Let us explicitly write the dependence of the partial wave amplitudes $f_{l\pm}^{(I)}(E_\pi;m_{\pi f},m_{\pi i})$ on the masses of the initial and final pions. Then the correct amplitudes to be used in Eq. (4.48) would be $f_{l\pm}^{(I)}(E_\pi;0,0)$. Unfortunately, no unitarity bound can be written for the absolute value of this amplitude. To see this, we write the inequality (4.41) keeping explicit the pion mass dependence:

$$\mathrm{Im} f_{l\pm}^{(I)}(E_\pi;0,0) \geqslant \frac{|\vec{p}_\pi|}{4\pi}|f_{l\pm}^{(I)}(E_\pi;m_\pi,0)|^2. \tag{4.49}$$

The quantities on the two sides of this inequality are different and thus no useful information can be extracted about them. We will use nevertheless the physical on-shell partial waves in Eq. (4.48), as the pion mass effects can be expected to be less important in the high-energy region where this relation will be applied.

At low energies we will use the lowest order chiral perturbation theory result obtained from an evaluation of the graphs in Fig. 1. Performing a partial wave decomposition we find that only the $l=0,1$ waves are present at this order. Furthermore, there is no spin-flip, which gives $f_{l+}=f_{l-}=f_l$. We obtain

$$f_0^{(1/2)}(E_\pi)=\frac{2}{f_\pi^2}E_\pi, \tag{4.50}$$

$$f_1^{(1/2)}(E_\pi)=\frac{g_2^2}{4f_\pi^2}|\vec{p}_\pi|^2\left(\frac{1}{-E_\pi-\Delta+i\Gamma_6/2}-\frac{3}{E_\pi-\Delta+i\Gamma_6/2}\right), \tag{4.51}$$

$$f_0^{(3/2)}(E_\pi)=-\frac{1}{f_\pi^2}E_\pi, \tag{4.52}$$

$$f_1^{(3/2)}(E_\pi)=-\frac{g_2^2}{2f_\pi^2}|\vec{p}_\pi|^2\frac{1}{-E_\pi-\Delta+i\Gamma_6/2}. \tag{4.53}$$

From these expressions one can compute the pion energy $(E_\pi)_{\max}=\Lambda_l^{(I)}$ at which the unitarity bound (4.43) is reached in each channel. We obtain $\Lambda_S^{(1/2)}=345$ MeV, $\Lambda_P^{(1/2)}=449$ MeV, $\Lambda_S^{(3/2)}=477.8$ MeV, and $\Lambda_P^{(3/2)}=1189.5$ MeV. We used in Eqs. (4.51) and (4.53) the same values for Δ and g_2 as in our previous estimates. We will consider in the following these values as marking the limits of validity of chiral perturbation theory in each channel.

We can compute now the contributions to the right-hand side of the AW sum rule for each channel, following the prescription outlined above.

(a) $I=1/2$, S wave. Below the unitarity limit $\Lambda_S^{(1/2)}$ we insert the χPT results for the cross sections (4.26) and (4.31) into the AW sum rule. We obtain

$$I_{S_1}^{(1/2)}=\frac{f_\pi^2}{\pi}\int_{m_\pi}^{\Lambda_S^{(1/2)}}\frac{d\nu}{\nu}\sigma_0[\pi^- T\to(\pi T)_{1/2}]_S=0.156. \tag{4.54}$$

Above $\Lambda_S^{(1/2)}$ we use the unitarity limit (4.43)

$$I_{S_2}^{(1/2)}\leqslant\frac{f_\pi^2}{\pi}\int_{\Lambda_S^{(1/2)}}^{\infty}\frac{d\nu}{\nu}\frac{8\pi}{3(\nu^2-m_\pi^2)}=0.212. \tag{4.55}$$

Adding these two contributions we obtain an upper limit on the contribution of this channel to the AW sum rule:

$$I_S^{(1/2)}=I_{S_1}^{(1/2)}+I_{S_2}^{(1/2)}\leqslant 0.368. \tag{4.56}$$

(b) $I=1/2$, P wave. The contribution of this channel to the AW sum rule can be split up as in the previous case into two terms. The low-energy part contains contributions from the both possible final states $(\pi\overline{\mathbf{3}})$ and $(\pi\mathbf{6})$:

$$\begin{aligned}I_{P_1}^{(1/2)}&=\frac{f_\pi^2}{\pi}\int_{m_\pi}^{\Lambda_P^{(1/2)}}\frac{d\nu}{\nu}\{\sigma_0[\pi^- T\to(\pi T)_{1/2}]_P\\&\quad+\sigma_0[\pi^- T\to(\pi S)_{1/2}]_P\}\\&=\frac{3}{2}g_2^2+0.072+0.001\\&=\frac{3}{2}g_2^2+0.073.\end{aligned} \tag{4.57}$$

The contribution of the one-body state with a sextet s-wave baryon has been extracted explicitly, as discussed above [see the paragraph following Eq. (4.28)]. Above $E_\pi=\Lambda_P^{(1/2)}$ we use again the unitarity limit

$$I_{P_2}^{(1/2)}\leqslant\frac{f_\pi^2}{\pi}\int_{\Lambda_P^{(1/2)}}^{\infty}\frac{d\nu}{\nu}3\frac{8\pi}{3(\nu^2-m_\pi^2)}=0.362. \tag{4.58}$$

Taking the sum gives

$$I_P^{(1/2)}=I_{P_1}^{(1/2)}+I_{P_2}^{(1/2)}\leqslant\frac{3}{2}g_2^2+0.435\,. \tag{4.59}$$

(c) $I=3/2$, S wave. For the $I=3/2$ channel we include, as discussed above, only the contribution of the low-energy region, so as to maximize the integral appearing in the AW sum rule. The S-wave contributes

$$\begin{aligned}I_S^{(3/2)}&=-\frac{f_\pi^2}{\pi}\int_{m_\pi}^{\Lambda_S^{(3/2)}}\frac{d\nu}{\nu}\{\sigma_0[\pi^- T\to(\pi T)_{3/2}]_S\\&\quad-\sigma_0[\pi^+ T\to(\pi T)_{3/2}]_S\}\\&=-0.089.\end{aligned} \tag{4.60}$$

(d) $I=3/2$, P wave. This channel receives significant contributions only from the $(\pi\overline{\mathbf{3}})$ final states:

$$\begin{aligned}I_P^{(3/2)}&=-\frac{f_\pi^2}{\pi}\int_{m_\pi}^{\Lambda_P^{(3/2)}}\frac{d\nu}{\nu}\{\sigma_0[\pi^- T\to(\pi T)_{3/2}]_P\\&\quad-\sigma_0[\pi^+ T\to(\pi T)_{3/2}]_P\}\\&=-0.039.\end{aligned} \tag{4.61}$$

The states $(\pi\mathbf{6})$ give a very small contribution of -0.0004.

The quantity on the LHS of the bound (4.56) includes, just as in Eq. (4.57), also contributions from one-body states.

These states are heavy baryons which can decay to the s-wave $\overline{\mathbf{3}}$ baryon by S-wave pion emission. Their contribution to the sum rule has been given in Eq. (4.9). Combining Eq. (4.9) with the limit (4.56) gives the constraint

$$\frac{1}{2}h_3^2+h_6^2+(S\text{ wave},\ I=1/2\text{ continuum})\leq 0.368. \tag{4.62}$$

Taken alone, this inequality tells us absolutely nothing about the couplings h_3 and h_6. To do so, it must be supplemented with additional information about the continuum. For example, assuming that the continuum contribution is positive, Eq. (4.62) gives an upper bound on these couplings.[5] Experience with the lowest order χ PT results shows that this is very likely the case.

A similar prediction can be made about the P-wave $I=1/2$ channel. Relation (4.59) limits the contribution of this channel to the sum rule:

$$I_P^{(1/2)}\leq\frac{3}{2}g_2^2+0.396. \tag{4.63}$$

The only bound states (besides the s-wave **6** whose contribution is made explicit) contributing in this channel are radial excitations of s-wave baryons. Expression (4.63) gives an upper bound on their contribution plus continuum to the sum rule. Note, however, that in contrast to the bound (4.62) which is parameter-free, the numerical bound in Eq. (4.63) has been obtained with the quark model value for g_2.

Unfortunately, not much can be said with the help of these methods about the D-wave contributions to the sum rule. We do not have any control over the low-energy behavior of these partial waves, which would require a next-to-leading calculation in chiral perturbation theory. Also, the use of unitarity for the high-energy region gives an upper bound which is too large to be useful.

V. CONSTITUENT QUARK MODEL PREDICTIONS FOR STRONG DECAYS OF HEAVY BARYONS

So far everything has been completely general and the sum rules (4.9) and (4.14)–(4.17) are model independent. We specialize now to the case of the constituent quark model, where some of the couplings are related. For example, the couplings of the s-wave baryons are given in the quark model by [3]

$$g_1=-\frac{4}{3}g_A,\quad |g_2|=\sqrt{\frac{2}{3}}g_A \tag{5.1}$$

with $g_A\simeq 0.75$ the constituent pion-quark coupling. The strong and electromagnetic decays of the charmed baryons have been also studied in the quark model with SU(6) symmetry in [24].

We will show in the following that the S-wave couplings of the p-wave baryons are also related in the quark model as

$$\frac{|h_3|}{|h_4|}=\frac{\sqrt{3}}{2},\quad \frac{|h_2|}{|h_4|}=\frac{1}{2},\quad \frac{|h_5|}{|h_6|}=\frac{2}{\sqrt{3}},\quad \frac{|h_5|}{|h_7|}=1. \tag{5.2}$$

One possible way of deriving these relations is by comparing the matrix elements of the axial current between s-wave and p-wave baryons computed in two different ways. First, the axial vector current in the effective theory is given by the Noether theorem. To the lowest order in the pion field, it is given by the coefficient of $-(\sqrt{2}/f_\pi)\partial_\mu\pi^a$ in the interaction Lagrangian. For Eq. (3.1) this gives

$$\begin{aligned}J_\mu^a=&h_2\epsilon_{ijk}\overline{S}_\nu^{kl}v_\mu t_{lj}^a R_\nu^i+h_3\text{tr}(\overline{B_{\bar 3}}v_\mu t^a U)+h_4\text{tr}(\overline{S}_\nu v_\mu t^a V_\nu)\\&+h_5\text{tr}(\overline{S}_\nu v_\mu t^a R'_\nu)+h_6\overline{T}_i v_\mu t_{ji}^a U'_j+h_7\epsilon_{ijk}\overline{S}_{kl}^\nu v_\mu t_{lj}^a V'^i_\nu\\&+\text{pion terms.}\end{aligned} \tag{5.3}$$

We denoted here $t^a=\lambda^a/2$, the generators of the diagonal flavor SU(3) group. One can see that only the matrix elements of the $\mu=0$ component are nonvanishing in the hadron rest frame.

On the other hand, the matrix elements of J_0^a can be computed between the corresponding constituent quark model states. In the nonrelativistic limit, the matrix element of J_0^a can be written as

$$\langle f|\overline{q}\gamma_0\gamma_5 t^a q|i\rangle=\left\langle f\left|\sum_{q=1}^{2}\left(\frac{\vec\sigma\cdot\vec p}{2m_u}+\frac{\vec\sigma\cdot\vec p\,'}{2m_d}\right)_q t_q^a\right|i\right\rangle, \tag{5.4}$$

where m_u,m_d are the constituent quark masses, assumed for simplicity equal in the following. The q summation runs over the two light quarks in the baryon. The structure of this matrix element in the quark model is different for symmetric and antisymmetric p-wave states $|i\rangle$. Writing explicitly the dependence on spatial $[\phi(\vec r_1,\vec r_2)]$ and spin-flavor $[\chi(1,2)]$ variables of the wave function, the matrix element (5.4) is given by

$$\left\langle \phi_S(\vec r_1,\vec r_2)\chi_S(1,2)\left|\sum_{q=1}^{2}\left(\frac{\vec\sigma\cdot\vec p}{2m_u}+\frac{\vec\sigma\cdot\vec p\,'}{2m_d}\right)_q t_q^a\right|\phi_P(\vec r_1,\vec r_2)\chi_P(1,2)\right\rangle=\langle\chi_S(1,2)|(\vec\sigma_1\cdot\vec a)t_1^a\pm(\vec\sigma_2\cdot\vec a)t_2^a|\chi_P(1,2)\rangle. \tag{5.5}$$

[5] A similar reasoning applied to pion-heavy meson scattering gives the inequality h^2 + (S wave, $I=1/2$ continuum) ≤ 0.368 (with the notations of [17]). In this case however we can invoke large-N_c arguments to argue that the continuum is suppressed.

The upper (lower) sign on the RHS corresponds to a symmetric (antisymmetric) decaying p-wave state [for which $\phi_P(\vec{r}_2,\vec{r}_1)=+(-)\phi_P(\vec{r}_1,\vec{r}_2)$]. The spatial part of the matrix element is encoded into the constant vector $\vec{a}$, defined by

$$\vec{a}=\left\langle \phi_S(\vec{r}_1,\vec{r}_2)\left|\left(\frac{\vec{p}}{2m_u}+\frac{\vec{p}\,'}{2m_d}\right)_1\right|\phi_P(\vec{r}_1,\vec{r}_2)\right\rangle. \quad (5.6)$$

Although the notation used in Eq. (5.5) does not make this apparent, one must keep in mind that $\vec{a}$ is in general different for symmetric and antisymmetric states.

The first two relations in Eq. (5.2) are obtained by considering the following matrix elements in the limit $q\to 0$:

$$\left\langle \Sigma_c^0\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Lambda_{c1}^+\left(\frac{1}{2}\right)\uparrow\right\rangle=-h_2, \quad (5.7)$$

$$\left\langle \Lambda_c^+\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Sigma_{c0}^{++}\left(\frac{1}{2}\right)\uparrow\right\rangle=h_3, \quad (5.8)$$

$$\left\langle \Sigma_c^+\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Sigma_{c1}^{++}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\frac{1}{\sqrt{2}}h_4. \quad (5.9)$$

One expects the matrix elements (5.7)–(5.9) to be related in the quark model, as the baryon states on the LHS have the same orbital wave function (as do the baryons on the RHS). The flavor-spin wave functions of the baryon states shown are given in the Appendix. The p-wave states are represented as linear combinations of basis vectors $|c(m_Q)L(m_{\ell})q(m_1)q(m_2)\rangle$ with m_Q the z projection of the heavy quark spin, m_{ℓ} the projection of the total orbital angular momentum, and $m_{1,2}$ the projections of the light quarks' spins. The operators in Eq. (5.5) can be written in terms of spin-raising, spin-lowering operators and σ_3 as

$$\vec{\sigma}\cdot\vec{a}=\sqrt{2}\sigma_+a_- - \sqrt{2}\sigma_-a_+ + \sigma_3 a_z, \quad (5.10)$$

where (a_-,a_z,a_+) form the components of a $J=1$ spherical tensor. The matrix element (5.5) can be then simply evaluated with the help of the Wigner-Eckart theorem. Using the wave functions (A1)–(A5) one obtains

$$\left\langle \Sigma_c^0\uparrow\left|\vec{\sigma}\cdot\vec{a}\right|\Lambda_{c1}^+\left(\frac{1}{2}\right)\uparrow\right\rangle=-\sqrt{\frac{2}{3}}I_S, \quad (5.11)$$

$$\left\langle \Lambda_c^+\uparrow\left|\vec{\sigma}\cdot\vec{a}\right|\Sigma_{c0}^{++}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\sqrt{2}I_S, \quad (5.12)$$

$$\left\langle \Sigma_c^+\uparrow\left|\vec{\sigma}\cdot\vec{a}\right|\Sigma_{c1}^{++}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\frac{2}{\sqrt{3}}I_S, \quad (5.13)$$

with I_S an unknown reduced matrix element. Comparing these relations with Eqs. (5.7)–(5.9) gives immediately the first two relations (5.2).

The other relations in Eq. (5.2) can be obtained in an analogous way by starting with the matrix elements

$$\left\langle \Sigma_c^+\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Sigma_{c1}^{\prime ++}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\frac{1}{\sqrt{2}}h_5, \quad (5.14)$$

$$\left\langle \Xi_c^0\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Xi_{c0}^{\prime +}\left(\frac{1}{2}\right)\uparrow\right\rangle=-h_6, \quad (5.15)$$

$$\left\langle \Sigma_c^0\uparrow\left|\overline{d}\gamma_0\gamma_5 u\right|\Lambda_{c1}^{\prime +}\left(\frac{1}{2}\right)\uparrow\right\rangle=-h_7. \quad (5.16)$$

They can be also computed in the quark model with the help of Eq. (5.5). Picking the lower sign in this relation we obtain for the quark model counterparts of these matrix elements

$$\left\langle \Sigma_c^+\uparrow\left|\vec{\sigma}\cdot\vec{a}\,'\right|\Sigma_{c1}^{\prime ++}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\sqrt{\frac{2}{3}}I_S', \quad (5.17)$$

$$\left\langle \Xi_c^0\uparrow\left|\vec{\sigma}\cdot\vec{a}\,'\right|\Xi_{c0}^{\prime +}\left(\frac{1}{2}\right)\uparrow\right\rangle=I_S', \quad (5.18)$$

$$\left\langle \Sigma_c^0\uparrow\left|\vec{\sigma}\cdot\vec{a}\,'\right|\Lambda_{c1}^{\prime +}\left(\frac{1}{2}\right)\uparrow\right\rangle=-\frac{2}{\sqrt{3}}I_S', \quad (5.19)$$

with I_S' another reduced matrix element. Comparison with Eqs. (5.14)–(5.16) gives the last two coupling constant ratios (5.2).

Similar relations can be derived in the quark model among the D-wave amplitudes produced by the interaction Lagrangian (3.3), for which we obtain

$$|h_8|=|h_9|=|h_{10}|,\quad \frac{|h_{11}|}{|h_{10}|}=\sqrt{2},\quad \frac{|h_{12}|}{|h_{13}|}=2\,,$$

$$\frac{|h_{14}|}{|h_{13}|}=1,\quad \frac{|h_{15}|}{|h_{14}|}=\sqrt{2}. \quad (5.20)$$

The derivation proceeds analogously as in the case of Eq. (5.2). The Noether axial current associated with the terms (3.3) in the chiral Lagrangian is given by (up to terms containing pion fields)

$$J^a_\mu = -ih_8\epsilon_{ijk}\left\{\partial_\alpha[\overline{S^{kl}_\mu}t^a_{lj}R^i_\alpha]+\partial_\alpha[\overline{S^{kl}_\alpha}t^a_{lj}R^i_\mu]+\frac{2}{3}v_\mu v\cdot\partial[\overline{S^{kl}_\alpha}t^a_{lj}R^i_\alpha]\right\}-ih_9\mathrm{tr}\left\{\partial_\alpha[\overline{S_\mu}t^aV_\alpha]+\partial_\alpha[\overline{S_\alpha}t^aV_\mu]+\frac{2}{3}v_\mu v\cdot\partial[\overline{S_\alpha}t^aV_\alpha]\right\}$$
$$-2ih_{10}\epsilon_{ijk}\partial_\nu[\overline{T_i}t^a_{jl}X^{kl}_{\mu\nu}]-h_{11}\{\epsilon^{\alpha\mu\sigma\lambda}\partial_\nu\mathrm{tr}[\overline{S_\alpha}t^aX_{\nu\sigma}]v_\lambda+\epsilon^{\alpha\nu\sigma\lambda}\partial_\nu\mathrm{tr}[\overline{S_\alpha}t^aX_{\mu\sigma}]v_\lambda\}$$
$$-ih_{12}\mathrm{tr}\left\{\partial_\alpha[\overline{S_\mu}t^aR'_\alpha]+\partial_\alpha[\overline{S_\alpha}t^aR'_\mu]+\frac{2}{3}v_\mu v\cdot\partial[\overline{S_\alpha}t^aR'_\alpha]\right\}$$
$$-ih_{13}\epsilon_{ijk}\left\{\partial_\alpha[\overline{S^{kl}_\mu}t^a_{lj}V'^i_\alpha]+\partial_\alpha[\overline{S^{kl}_\alpha}t^a_{lj}V'^i_\mu]+\frac{2}{3}v_\mu v\cdot\partial[\overline{S^{kl}_\alpha}t^a_{lj}V'^i_\alpha]\right\}-2ih_{14}\partial_\nu[\overline{T_i}t^a_{ji}X'^j_{\mu\nu}]$$
$$-h_{15}\epsilon_{ijk}\{\epsilon^{\alpha\mu\sigma\lambda}\partial_\nu[\overline{S^{kl}_\alpha}t^a_{lj}X'^i_{\nu\sigma}]v_\lambda+\epsilon^{\alpha\nu\sigma\lambda}\partial_\nu[\overline{S^{kl}_\alpha}t^a_{lj}X'^i_{\mu\sigma}]v_\lambda\}. \tag{5.21}$$

For this case it is the matrix elements of the space components of J^a_μ which are nonvanishing. In the quark model they can be expressed as

$$\langle f|\overline{q}\gamma^i\gamma_5t^aq|i\rangle\to\langle f|\sigma^it^ae^{i\vec{q}\cdot\vec{r}}|i\rangle=\langle f|\sigma^it^a|i\rangle+\frac{i}{2}\left\langle f\left|\left(\sigma^i(\vec{q}\cdot\vec{r})+r^i(\vec{q}\cdot\vec{\sigma})-\frac{2}{3}q^i(\vec{\sigma}\cdot\vec{r})\right)t^a\right|i\right\rangle+\text{higher multipoles}, \tag{5.22}$$

where $\vec{q}$ denotes the momentum of the current. The first term vanishes as i and f have different orbital angular momenta. The second one is nonvanishing and will produce the terms proportional to the D-wave couplings in Eq. (5.21). Separating again the contributions from the spatial and spin-flavor components of the wave function, this matrix element can be written as

$$\left\langle\phi_S(\vec{r}_1,\vec{r}_2)\chi_S(1,2)\left|\sum_{q=1}^{2}\left(\sigma^i(\vec{q}\cdot\vec{r})+r^i(\vec{q}\cdot\vec{\sigma})-\frac{2}{3}q^i(\vec{\sigma}\cdot\vec{r})\right)_q t^a_q\right|\phi_P(\vec{r}_1,\vec{r}_2)\chi_P(1,2)\right\rangle$$
$$=\left\langle\chi_S(1,2)\left|\sum_{q=1}^{2}(\pm)^{1-q}\left(\sigma^i(\vec{q}\cdot\vec{a})+a^i(\vec{q}\cdot\vec{\sigma})-\frac{2}{3}q^i(\vec{\sigma}\cdot\vec{a})\right)_q t^a_q\right|\chi_P(1,2)\right\rangle. \tag{5.23}$$

The upper (lower) sign on the RHS corresponds again to symmetric (antisymmetric) initial p-wave states. We denoted here with $\vec{a}$ the constant vector

$$\vec{a}=\langle\phi_S(\vec{r}_1,\vec{r}_2)|\vec{r}_1|\phi_P(\vec{r}_1,\vec{r}_2)\rangle. \tag{5.24}$$

From Eq. (5.21) we read off the following expressions for a few typical matrix elements:

$$\left\langle\Sigma^0_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Lambda^+_{c1}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{\sqrt{2}}{3}h_8(q_1^2+q_2^2-2q_3^2), \tag{5.25}$$

$$\left\langle\Sigma^+_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Sigma^{++}_{c1}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{3}h_9(q_1^2+q_2^2-2q_3^2), \tag{5.26}$$

$$\left\langle\Lambda^+_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Sigma^{++}_{c2}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=-\frac{2}{\sqrt{15}}h_{10}(q_1^2+q_2^2-2q_3^2), \tag{5.27}$$

$$\left\langle\Sigma^+_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Sigma^{++}_{c2}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{10}}h_{11}(q_1^2+q_2^2-2q_3^2). \tag{5.28}$$

The quark model counterpart of these matrix elements can be computed with the help of Eqs. (5.22) and (5.23) and using the wave functions in the Appendix. We obtain

$$\left\langle\Sigma^0_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Lambda^+_{c1}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{3}I_D(q_1^2+q_2^2-2q_3^2), \tag{5.29}$$

$$\left\langle\Sigma^+_c\left(\frac{1}{2}\right),+\frac{1}{2}\left|q^\mu J^{1-i2}_\mu\right|\Sigma^{++}_{c1}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=-\frac{1}{3\sqrt{2}}I_D(q_1^2+q_2^2-2q_3^2), \tag{5.30}$$

$$\left\langle \Lambda_c^+\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Sigma_{c2}^{++}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = -\frac{2}{\sqrt{30}} I_D(q_1^2+q_2^2-2q_3^2), \tag{5.31}$$

$$\left\langle \Sigma_c^+\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Sigma_{c2}^{++}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{10}} I_D(q_1^2+q_2^2-2q_3^2). \tag{5.32}$$

I_D is the reduced matrix element of the $J=1$ spherical tensor (a_-,a_3,a_+). Comparing Eqs. (5.25)–(5.28) with Eqs. (5.29)–(5.32) gives relations (5.20) among the D-wave couplings h_8-h_{11}.

The ratios among the couplings of the antisymmetric states (5.20) can be computed in a similar way by considering the matrix elements

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Sigma_{c1}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = \frac{1}{3} h_{12}(q_1^2+q_2^2-2q_3^2), \tag{5.33}$$

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Lambda_{c1}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = \frac{\sqrt{2}}{3} h_{13}(q_1^2+q_2^2-2q_3^2), \tag{5.34}$$

$$\left\langle \Xi_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Xi_{c2}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = -\frac{2}{\sqrt{15}} h_{14}(q_1^2+q_2^2-2q_3^2), \tag{5.35}$$

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Lambda_{c2}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{5}} h_{15}(q_1^2+q_2^2-2q_3^2). \tag{5.36}$$

The same matrix elements can also be computed in the quark model in terms of a new common reduced matrix element I_D':

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Sigma_{c1}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = \frac{1}{3\sqrt{3}} I_D'(q_1^2+q_2^2-2q_3^2), \tag{5.37}$$

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Lambda_{c1}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = -\frac{1}{3\sqrt{6}} I_D'(q_1^2+q_2^2-2q_3^2), \tag{5.38}$$

$$\left\langle \Xi_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Xi_{c2}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = \frac{1}{3\sqrt{5}} I_D'(q_1^2+q_2^2-2q_3^2), \tag{5.39}$$

$$\left\langle \Sigma_c^0\left(\frac{1}{2}\right),+\frac{1}{2}\right| q^\mu J_\mu^{1-i2} \left| \Lambda_{c2}^{\prime +}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{30}} I_D'(q_1^2+q_2^2-2q_3^2). \tag{5.40}$$

Comparing these expressions with Eqs. (5.33)–(5.36) gives the remaining relations in Eq. (5.20) among the couplings $h_{12}-h_{15}$.[6]

The coupling constants of the symmetric p-wave baryons among themselves appearing in the Lagrangian (3.5) can be also estimated in the constituent quark model. We obtain (in units of g_A)

$$\begin{gathered} f_1=0, \quad f_2=-1, \quad f_3=2, \quad |f_4|=\sqrt{\frac{2}{3}}, \\ |f_5|=1, \quad |f_6|=\frac{1}{\sqrt{2}}, \quad |f_7|=2\sqrt{\frac{2}{3}}, \quad |f_8|=\frac{1}{\sqrt{2}}. \end{gathered} \tag{5.41}$$

The values taken by the couplings of the antisymmetric p-wave states (3.6) in the constituent quark model are (also in units of g_A)

$$f_1'=0, \quad f_2'=-\frac{1}{2}, \quad f_3'=1, \quad |f_4'|=\sqrt{\frac{2}{3}}, \tag{5.42}$$

[6]Similar relations among pion couplings in the quark model have been obtained in [25].

$$|f_5'|=1,\quad |f_6'|=\frac{1}{\sqrt{2}},\quad |f_7'|=\sqrt{\frac{2}{3}},\quad |f_8'|=\frac{1}{2\sqrt{2}}.$$

The couplings of the symmetric to antisymmetric p-wave states contained in the interaction Lagrangian (3.7) can be computed also by considering the matrix elements of the space component of the axial current $\overline{q}\gamma^i t^a q$. In the quark model, the matrix elements of this operator are given by a nonrelativistic expansion

$$\langle f|\overline{q}\gamma^i t^a q|i\rangle=\left\langle f\left|\sum_q \sigma_q^i t_q^a\right|i\right\rangle+\langle f|\sum_q\left(\frac{\vec{p}^2+\vec{p}\,'^2}{8m^2}\sigma^i+\frac{(\vec{\sigma}\cdot\vec{p}\,')\sigma^i(\vec{\sigma}\cdot\vec{p})}{4m^2}\right)_q t_q^a|i\rangle+O(p^4/m^4)\to\left\langle f\left|\sum_q \sigma_q^i t_q^a\right|i\right\rangle$$
$$+\left\langle f\left|\sum_q \frac{p^i(\vec{\sigma}\cdot\vec{p})}{2m^2}t^a\right|i\right\rangle+O(p^4/m^4),\quad (\vec{p}\,'\to\vec{p}). \tag{5.43}$$

The first term in this expansion vanishes as a consequence of the orthogonality of the spatial wave functions of the two different types of p-wave states. Therefore the leading contribution to these couplings arises from the next term and is formally of order $\vec{p}^2/m^2$.

Writing explicitly the dependence of the wave functions on the orbital and spin-flavor degrees of freedom, the matrix element (5.43) can be expressed as

$$\langle f|\overline{q}\gamma^i t^a q|i\rangle=\left\langle \phi_P(\vec{r}_1,\vec{r}_2)\chi_P(1,2)\left|\sum_q \frac{p^i(\vec{\sigma}\cdot\vec{p})}{2m^2}t^a\right|\phi_{P'}(\vec{r}_1,\vec{r}_2)\chi_{P'}(1,2)\right\rangle$$
$$=\langle\chi_P(1,2)|(T_{ij}\sigma_1^j t_1^a+\kappa_0\sigma_1^i t_1^a)-(T_{ij}\sigma_2^j t_2^a+\kappa_0\sigma_2^i t_2^a)|\chi_{P'}(1,2)\rangle, \tag{5.44}$$

where

$$T_{ij}=\left\langle \phi_P(\vec{r}_1,\vec{r}_2)\left|\frac{1}{2m^2}\left(p_ip_j-\frac{1}{3}\delta_{ij}\vec{p}^2\right)_1\right|\phi_{P'}(\vec{r}_1,\vec{r}_2)\right\rangle, \tag{5.45}$$

$$\kappa_0=\left\langle \phi_P(\vec{r}_1,\vec{r}_2)\left|\frac{\vec{p}_1^{\,2}}{6m^2}\right|\phi_{P'}(\vec{r}_1,\vec{r}_2)\right\rangle. \tag{5.46}$$

The tensor T_{ij} and the scalar κ_0 parametrize the orbital part of the matrix element and correspond to the two irreducible tensors with $J=2,0$ into which the operator p^ip^j can be decomposed. It is a simple matter to compute the remaining matrix elements over the spin-flavor coordinates, using the wave functions in the Appendix. As a result, all 13 couplings $f_i{}'$ can be expressed in terms of two reduced matrix elements κ_0,κ_2:

$$f_1''=0, \tag{5.47a}$$

$$f_2''=-\frac{2}{3}\sqrt{5}\kappa_2+\frac{2}{\sqrt{3}}\kappa_0, \tag{5.47b}$$

$$f_3''=\sqrt{\frac{5}{6}}\kappa_2+\sqrt{2}\kappa_0, \tag{5.47c}$$

$$f_4''=\frac{1}{2\sqrt{15}}\kappa_2-\kappa_0, \tag{5.47d}$$

$$f_5''=\frac{\sqrt{5}}{3}\kappa_2-\frac{1}{\sqrt{3}}\kappa_0, \tag{5.47e}$$

$$f_6''=-\frac{\sqrt{5}}{3}\kappa_2-\frac{2}{\sqrt{3}}\kappa_0, \tag{5.47f}$$

$$f_7''=-\frac{1}{2}\sqrt{\frac{5}{6}}\kappa_2-\frac{1}{\sqrt{2}}\kappa_0, \tag{5.47g}$$

$$f_8''=-\frac{\sqrt{5}}{3}\kappa_2-\frac{2}{\sqrt{3}}\kappa_0, \tag{5.47h}$$

$$f_9''=\sqrt{\frac{5}{6}}\kappa_2-\frac{1}{\sqrt{2}}\kappa_0, \tag{5.47i}$$

$$f_{10}''=\frac{1}{\sqrt{15}}\kappa_2-\frac{1}{2}\kappa_0, \tag{5.47j}$$

$$f_{11}''=-\frac{1}{4\sqrt{15}}\kappa_2+\frac{1}{2}\kappa_0, \tag{5.47k}$$

$$f_{12}''=\frac{1}{\sqrt{15}}\kappa_2-\frac{1}{2}\kappa_0, \tag{5.47l}$$

$$f_{13}''=-\sqrt{\frac{2}{15}}\kappa_2-\sqrt{2}\kappa_0. \tag{5.47m}$$

To summarize, in the constituent quark model, the 45 coupling constants introduced in Sec. III can be expressed in terms of seven reduced matrix elements. Among these, g_A is of order unity, I_S, I'_S, I_D, and I'_D are of order (formally) $O(|\vec{p}|/m)$, and κ_2, κ_0 are of order (formally) $O(\vec{p}^2/m^2)$.

VI. PHENOMENOLOGICAL APPLICATIONS

The sum rules presented in this paper can be used to provide model-independent constraints on the pion couplings of the heavy baryons which will have to be satisfied by model computations or phenomenological determinations. For example, the sum rule (4.9) gives the upper bound

$$g_2^2 \leqslant \frac{2}{3}. \tag{6.1}$$

This inequality is satisfied by the SU(2) Skyrme model calculation of [26] who find

$$|g_2| = \frac{1}{\sqrt{2}} G_A - \frac{1}{3\sqrt{2}} g = 0.717 - 0.813 \tag{6.2}$$

with $G_A = 1.25$ the nucleon axial charge and g the $BB^*\pi$ coupling. The numerical values shown correspond to the experimental bounds for g given in [27], $g^2 = 0.09 - 0.5$ (at 90% confidence limit). The constituent quark model prediction (5.1) [3] for $g_2 = 0.612$ (with $g_A = 0.75$) also satisfies Eq. (6.1), saturating it in the limit $g_A \to 1$.

One can also derive an upper bound on the coupling g_1 from the AW sum rule (4.14). It can be made stronger by assuming that g_2 is known

$$g_1^2 \leqslant \frac{8}{3} - \frac{4}{3} g_2^2. \tag{6.3}$$

The calculation of [26] gives

$$|g_1| = G_A - \frac{1}{3} g = 1.014 - 1.150 \tag{6.4}$$

which satisfies Eq. (6.3). The constituent quark model with $g_A = 0.75$ predicts a somewhat smaller value $|g_1| = 1$ (5.1).

The CLEO Collaboration recently measured the masses and widths of the Σ_c^{*++} and Σ_c^{*0} baryons [28] (for an earlier measurement of the masses of these states see [29]). They find

$$\Delta_{\Sigma_c^{*++}} = M_{\Sigma_c^{*++}} - M_{\Lambda_c^+} = 234.5 \pm 1.36 \text{ MeV},$$

$$\Gamma(\Sigma_c^{*++}) = 17.9^{+5.5}_{-5.1} \text{ MeV}, \tag{6.5}$$

$$\Delta_{\Sigma_c^{*0}} = M_{\Sigma_c^{*0}} - M_{\Lambda_c^+} = 232.6 \pm 1.28 \text{ MeV},$$

$$\Gamma(\Sigma_c^{*0}) = 13.0^{+5.45}_{-5.0} \text{ MeV}. \tag{6.6}$$

The mass of Λ_c^+ is given by the Particle Data Group [31] to be

$$M_{\Lambda_c^+} = 2284.9 \pm 0.6 \text{ MeV}. \tag{6.7}$$

Neglecting the radiative decay width associated with the decays $\Sigma_c^* \to \Sigma_c \gamma$, the total width of these states is given by

$$\Gamma(\Sigma_c^{*++}) = \frac{g_2^2}{2\pi f_\pi^2} \left(\frac{M_{\Lambda_c^+}}{M_{\Sigma_c^{*++}}} \right) |\vec{p}_\pi|^3 = g_2^2 (47.971^{+1.24}_{-1.22}) \text{ MeV}, \tag{6.8}$$

$$\Gamma(\Sigma_c^{*0}) = \frac{g_2^2}{2\pi f_\pi^2} \left(\frac{M_{\Lambda_c^+}}{M_{\Sigma_c^{*0}}} \right) |\vec{p}_\pi|^3 = g_2^2 (46.268^{+1.14}_{-1.13}) \text{ MeV}. \tag{6.9}$$

The CLEO data (6.5) and (6.6) thus offer the possibility of extracting the coupling g_2. From Eq. (6.8) we obtain the value $|g_2| = 0.611^{+0.097}_{-0.101}$ and from Eq. (6.9) $|g_2| = 0.530^{+0.109}_{-0.119}$. Averaging these two values we obtain our final result

$$|g_2| = 0.570^{+0.137}_{-0.159}, \tag{6.10}$$

which [especially the one following from Eq. (6.8)] is in good agreement with the constituent quark model prediction $|g_2| = 0.612$. Similar determinations of g_2 have been presented recently in [32,33]. Our result comes closer to the one in [33].

One can use Eq. (6.10) to predict the widths of the Σ_c baryons. The fit of [31] gives the mass values

$$\Delta_{S^{++}} = M_{\Sigma_c^{++}} - M_{\Lambda_c^+} = 167.95 \pm 0.25 \text{ MeV}, \tag{6.11}$$

$$\Delta_{S^0} = M_{\Sigma_c^0} - M_{\Lambda_c^+} = 167.2 \pm 0.4 \text{ MeV}. \tag{6.12}$$

The Σ_c widths are

$$\Gamma(\Sigma_c^{++}) = \frac{g_2^2}{2\pi f_\pi^2} \left(\frac{M_{\Lambda_c^+}}{M_{\Sigma_c^{++}}} \right) |\vec{p}_\pi|^3 = g_2^2 (6.232^{+0.089}_{-0.088}) \text{ MeV}$$
$$= 2.025^{+1.134}_{-0.987} \text{ MeV}, \tag{6.13}$$

$$\Gamma(\Sigma_c^0) = \frac{g_2^2}{2\pi f_\pi^2} \left(\frac{M_{\Lambda_c^+}}{M_{\Sigma_c^0}} \right) |\vec{p}_\pi|^3 = g_2^2 (5.969^{+0.140}_{-0.139}) \text{ MeV}$$
$$= 1.939^{+1.114}_{-0.954} \text{ MeV}. \tag{6.14}$$

These states are significantly narrower than their spin-3/2 counterparts, which explains why their widths have not been yet measured.

We also present predictions for the $I = 1/2$ spin-3/2 charmed baryons Ξ_c^{*0} and Ξ_c^{*+}. The latter has only been recently discovered [34]. Their measured parameters are

$$M_{\Xi_c^{*0}} = M_{\Xi_c^+} + (178.2 \pm 1.1) \text{ MeV},$$

$$\Gamma(\Xi_c^{*0}) < 5.5 \text{ MeV [35]}, \tag{6.15}$$

$$M_{\Xi_c^{*+}} = M_{\Xi_c^0} + (174.3 \pm 1.1) \text{ MeV},$$

$$\Gamma(\Xi_c^{*+}) < 3.1 \text{ MeV [34]}. \tag{6.16}$$

The masses of the ground state $I=1/2$ baryons are $M_{\Xi_c^+}=2465.6\pm 1.4$ MeV and $M_{\Xi_c^0}=2470.3\pm 1.8$ MeV [31]. We shall neglect the small admixture of SU(3) sextet into the ground states as the corresponding mixing angle is small, of the order of a few degrees [36,37,32]. Including both the charged and neutral pion channels we obtain

$$\Gamma(\Xi_c^{*0})=g_2^2(7.712\pm 0.436)\ \text{MeV}=1.230-4.074\ \text{MeV}, \tag{6.17}$$

$$\Gamma(\Xi_c^{*+})=g_2^2(7.496\pm 0.446)\ \text{MeV}=1.191-3.971\ \text{MeV}, \tag{6.18}$$

which are consistent with the bounds (6.15) and (6.16).

It is interesting to note that the difference of couplings on the LHS of the polarized AW sum rule (4.17) vanishes when the constituent quark model relations (5.1) are used; so does the RHS of Eq. (4.17) when the constituent quark model relations (5.20) are inserted in this relation. If the quark model relations (5.20) are not used, the sum rule (4.17) can be employed to give a model-independent proof of the constituent quark model relations (5.1) in the large-N_c limit. Recalling the scaling relations of the couplings discussed in Sec. IV B one can see that in the large-N_c limit the RHS is suppressed by one power of $1/N_c$ relative to the LHS.

It is, of course, a well-known fact that the predictions of the constituent quark model for low-lying s-wave baryon states become exact in the large-N_c limit [38]. What is new here is that the relation (4.17) also gives the corrections to this result, expressed in terms of couplings of the higher states.

As a matter of fact, the sum rule (4.17) suggests that the quark model relations (5.1) might work better than one would expect from large-N_c alone. First, only D-wave couplings appear in Eq. (4.17), whose contributions are suppressed by factors of Δ^2/Λ_χ^2, with Δ the excitation energies of the p-wave states and $\Lambda_\chi\simeq 1$ GeV the chiral symmetry breaking scale. An explicit calculation using the upper limit on h_8 determined below (6.44) shows that the contribution of the h_8^2 term on the RHS of the sum rule (4.17) is under 0.06. Second, the alternating signs of the terms on the RHS of Eq. (4.17) could enhance further the above-mentioned suppression.

We can expect therefore the quark model relation between g_1 and g_2 to be valid at the order of 10% or better. We obtain in this way the following prediction for g_1:

$$|g_1|=2\sqrt{\frac{2}{3}}|g_2|=0.931^{+0.224+0.093}_{-0.260-0.093}. \tag{6.19}$$

The possibility of determining g_1 in this way is particularly welcome as the decay $\Sigma_c^*\to\Sigma_c\pi$ is kinematically forbidden, making a direct extraction of g_1 impossible (for an alternative method see [33,39]).

In contrast to g_1 and g_2, the couplings of the p-wave baryons are not completely predicted by the simple constituent quark model. To do so, it must be supplemented with additional dynamical assumptions, which in turn will have to be used to determine the wave function of the constituent quarks. The AW sum rules discussed in this paper could be used to set constraints on the parameters of the quark model calculation. Inserting the quark model relations (5.1), (5.2), and (5.20) into the sum rules (4.9) and (4.14) one common relation is obtained

$$1=g_A^2+\frac{3}{8}h_4^2+h_6^2+\frac{4}{3}h_{10}^2|\vec{p}_\pi|^2+\frac{8}{3}h_{14}^2|\vec{p}_\pi|^2+\cdots. \tag{6.20}$$

This result demonstrates the consistency of the constituent quark model with the AW sum rules (4.9) and (4.14).

In the following we present an extraction of the couplings h_2 and h_8 of the lowest-lying p-wave baryons from experimental data. These will be subsequently used, together with the quark model relations (5.2) and (5.20), to predict the couplings of all the other symmetric p-wave baryons.

We will determine the allowed range of values for h_2 and h_8 by using two different measurements. The first is the CLEO measurement of the $\Lambda_{c_1}^+(2593)$ width [6]

$$\Gamma(\Lambda_{c_1}^+(2593))=3.9^{+2.4}_{-1.6}\ \text{MeV}. \tag{6.21}$$

The state $\Lambda_{c_1}^+(2593)$ is the $J^P=1/2^-$ member of the $s_\ell^{\pi_\ell}=1^-$ p-wave heavy quark doublet. Its general properties have been discussed already by Cho [12] (see also [30]). It is known that it decays predominantly in the two-pion mode, which is enhanced by resonant effects due to the proximity of the Σ_c pole. These theoretical expectations have been confirmed by experiment [6,7].

The decay rate for the process $\Lambda_{c_1}^+(2593)\to\Lambda_c^+\pi^+\pi^-$ has been computed in [12]. To lowest order in chiral perturbation theory and in the heavy mass limit it is given by

$$\frac{d\Gamma(\Lambda_{c1}^+(2593)\to\Lambda_c^+\pi^+(E_1)\pi^-(E_2))}{dE_1dE_2}=\frac{g_2^2}{16\pi^3f_\pi^4}M_{\Lambda_c^+}\{\vec{p}_2^{\,2}|A|^2+\vec{p}_1^{\,2}|B|^2+2[E_1E_2-p_1\cdot p_2]\text{Re}(AB^*)\} \tag{6.22}$$

with

$$A(E_1,E_2)=\frac{h_2E_1}{\Delta_R-\Delta_{\Sigma_c^0}-E_1+i\Gamma_{\Sigma_c^0}/2}-\frac{\frac{2}{3}h_8\vec{p}_1^{\,2}}{\Delta_R-\Delta_{\Sigma_c^{*0}}-E_1+i\Gamma_{\Sigma_c^{*0}}/2}+\frac{2h_8[E_1E_2-p_1\cdot p_2]}{\Delta_R-\Delta_{\Sigma_c^{*++}}-E_2+i\Gamma_{\Sigma_c^{*++}}/2}, \tag{6.23}$$

$$B(E_1,E_2;\Delta_{\Sigma_c^{(*)0}},\Delta_{\Sigma_c^{(*)++}})=A(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)0}}). \tag{6.24}$$

The boundaries of the Dalitz plot for these decays are given by

$$(E_1)_{\min}=m_\pi,$$

$$(E_1)_{\max}=\frac{M^2_{\Lambda^+_{c1}}-2M_{\Lambda^+_c}m_\pi-M^2_{\Lambda^+_c}}{2M_{\Lambda^+_{c1}}},$$

$$(E_2)_{\min,\max}=\frac{(E_1-M_{\Lambda^+_{c1}})(M^2_{\Lambda^+_{c1}}+2m^2_\pi-2M_{\Lambda^+_{c1}}E_1-M^2_{\Lambda^+_c})\pm\sqrt{\Delta}}{2(2M_{\Lambda^+_{c1}}E_1-M^2_{\Lambda^+_{c1}}-m^2_\pi)}, \tag{6.25}$$

with

$$\Delta=(E_1^2-m_\pi^2)[(M^2_{\Lambda^+_{c1}}-2M_{\Lambda^+_{c1}}E_1-M^2_{\Lambda^+_c})^2-4M^2_{\Lambda^+_c}m^2_\pi]. \tag{6.26}$$

The boundaries of the Dalitz plots for the two-pion decays of the Λ^+_{c1} states are shown in Fig. 2. In the limit when the energy release Δ_R is much smaller than the mass of the decaying baryon $M_{\Lambda^+_{c1}}$ the phase space degenerates to the line $E_1+E_2=\Delta_R$ and the decay rate reduces to the expression originally derived in [12] (we included here also the contribution of the D-wave couplings which was neglected in [12])

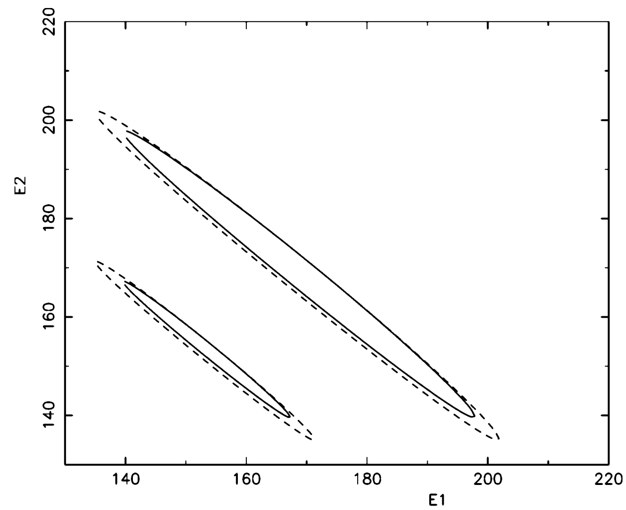

FIG. 2. Boundaries of the Dalitz plots for the decays $\Lambda^+_{c1}(2593)\to\Lambda^+_{c1}\pi\pi$ (lower) and $\Lambda^+_{c1}(2625)\to\Lambda^+_{c1}\pi\pi$ (upper). The continuous lines correspond to charged pions and dashed lines to neutral pions in the final state, respectively.

$$\frac{d\Gamma(\Lambda^+_{c1}(2593)\to\Lambda^+_c\pi^+(E_1)\pi^-(E_2))}{dE_1}=\frac{g_2^2}{8\pi^3f_\pi^4}\left(\frac{M_{\Lambda^+_c}}{M_{\Lambda^+_{c_1}}}\right)\sqrt{(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)}\Bigg\{h_2^2\left(\frac{E_1^2(E_2^2-m_\pi^2)}{(\Delta_R-\Delta_{\Sigma_c^0}-E_1)^2+\frac14\Gamma^2_{\Sigma_c^0}}+\frac{E_2^2(E_1^2-m_\pi^2)}{(\Delta_R-\Delta_{\Sigma_c^{++}}-E_2)^2+\frac14\Gamma^2_{\Sigma_c^{++}}}\right)+\frac89h_8^2(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)\times\left(\frac{E_2^2-m_\pi^2}{(\Delta_R-\Delta_{\Sigma_c^{*++}}-E_2)^2+\frac14\Gamma^2_{\Sigma_c^{*++}}}+\frac{E_1^2-m_\pi^2}{(\Delta_R-\Delta_{\Sigma_c^{*0}}-E_1)^2+\frac14\Gamma^2_{\Sigma_c^{*0}}}\right)\Bigg\}. \tag{6.27}$$

We denoted by Δ_i the excitation energies of the respective states with respect to Λ^+_c. We will use in our analysis below the value [31]

$$\Delta_R=M_{\Lambda^+_{c_1}}-M_{\Lambda^+_c}=308.6\pm0.8\text{ MeV}. \tag{6.28}$$

The widths in the propagator denominators are given by

$$\Gamma_{\Sigma_c^{(*)}}=\frac{g_2^2}{2\pi f_\pi^2}\frac{M_{\Lambda^+_c}}{M_{\Sigma_c^{(*)}}}|\vec{p}_\pi|^3. \tag{6.29}$$

The rate for $\Lambda^+_{c_1}(2593)\to\Lambda^+_c\pi^0\pi^0$ is given by formulas identical to Eqs. (6.22) and (6.27) with an additional factor of 1/2 due to the identity of the pions in the final state. In addition, the substitutions

$$\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)0}}\to\Delta_{\Sigma_c^{(*)+}} \tag{6.30}$$

have to be made. We will use in our calculations the numerical values

$$\Delta_{\Sigma_c^+}=M_{\Sigma_c^+}-M_{\Lambda_c^+}=168.5\pm0.7\text{ MeV [31]}, \tag{6.31}$$

$$\Delta_{\Sigma_c^{*+}}=M_{\Sigma_c^{*+}}-M_{\Lambda_c^+}=233.5^{+2.4}_{-2.2}\text{ MeV}. \tag{6.32}$$

The state Σ_c^{*+} has not yet been observed so we will use for its mass the average of its two isospin partners (6.5) and (6.6).

The second experimental input we will use is an upper bound on the width of $\Lambda_{c1}^+(2625)$ also obtained by CLEO [6]:

$$\Gamma(\Lambda_{c_1}^+(2625))<1.9 \text{ MeV}. \tag{6.33}$$

$\Lambda_{c_1}^+(2625)$ is the spin-3/2 heavy quark symmetry partner of $\Lambda_{c_1}^+(2593)$. To leading order in the heavy mass expansion, the two-pion decay rate of this state is given by

$$\begin{aligned}\frac{d\Gamma(\Lambda_{c1}^+(2625)\to\Lambda_c^+\pi^+(E_1)\pi^-(E_2))}{dE_1dE_2} =& \frac{g_2^2}{16\pi^3 f_\pi^4}M_{\Lambda_c^+}\{\vec{p}_1^{\,2}|C|^2+\vec{p}_2^{\,2}|E|^2+2[E_1E_2-p_1\cdot p_2]\text{Re}(CE^*)\\ &+[\vec{p}_1^{\,2}\vec{p}_2^{\,2}-(E_1E_2-p_1\cdot p_2)^2][\vec{p}_1^{\,2}|D|^2+\vec{p}_2^{\,2}|F|^2-\text{Re}(CF^*)+\text{Re}(DE^*)\\ &+2(E_1E_2-p_1\cdot p_2)\text{Re}(DF^*)]\}\end{aligned} \tag{6.34}$$

with

$$\begin{aligned}C(E_1,E_2)=&\left(h_2E_2-\frac{2}{3}h_8\vec{p}_2^{\,2}\right)\frac{1}{\Delta_{R^*}-\Delta_{\Sigma_c^{*++}}-E_2+i\Gamma_{\Sigma_c^{*++}}/2}+\frac{2}{3}h_8(E_1E_2-p_1\cdot p_2)\left(\frac{1}{\Delta_{R^*}-\Delta_{\Sigma_c^0}-E_1+i\Gamma_{\Sigma_c^0}/2}\right.\\ &\left.+\frac{2}{\Delta_{R^*}-\Delta_{\Sigma_c^{*0}}-E_1+i\Gamma_{\Sigma_c^{*0}}/2}\right),\end{aligned} \tag{6.35}$$

$$D(E_1,E_2)=\frac{2}{3}h_8\left(-\frac{1}{\Delta_{R^*}-\Delta_{\Sigma_c^0}-E_1+i\Gamma_{\Sigma_c^0}/2}+\frac{1}{\Delta_{R^*}-\Delta_{\Sigma_c^{*0}}-E_1+i\Gamma_{\Sigma_c^{*0}}/2}\right), \tag{6.36}$$

$$E(E_1,E_2;\Delta_{\Sigma_c^{(*)0}},\Delta_{\Sigma_c^{(*)++}})=C(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)0}}), \tag{6.37}$$

$$F(E_1,E_2;\Delta_{\Sigma_c^{(*)0}},\Delta_{\Sigma_c^{(*)++}})=-D(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)0}}). \tag{6.38}$$

In analogy to the previous case, this decay rate simplifies in the heavy mass limit and is given by

$$\begin{aligned}\frac{d\Gamma(\Lambda_{c_1}^+(2625)\to\Lambda_c^+\pi^+(E_1)\pi^-(E_2))}{dE_1} =& \frac{g_2^2}{8\pi^3 f_\pi^4}\left(\frac{M_{\Lambda_c^+}}{M_{\Lambda_{c_1}^+}}\right)|\vec{p}_1||\vec{p}_2|\left\{h_2^2\left(\frac{E_1^2\vec{p}_2^{\,2}}{(\Delta_{R^*}-\Delta_{\Sigma_c^{*0}}-E_1)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^{*0}}}\right.\right.\\ &\left.+\frac{E_2^2\vec{p}_1^{\,2}}{(\Delta_{R^*}-\Delta_{\Sigma_c^{*++}}-E_2)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^{*++}}}\right)+\frac{4}{9}h_8^2\vec{p}_1^{\,2}\vec{p}_2^{\,2}\left[\vec{p}_1^{\,2}\left(\frac{1}{(\Delta_{R^*}-\Delta_{\Sigma_c^0}-E_1)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^0}}\right.\right.\\ &\left.+\frac{1}{(\Delta_{R^*}-\Delta_{\Sigma_c^{*0}}-E_1)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^{*0}}}\right)+\vec{p}_2^{\,2}\left(\frac{1}{(\Delta_{R^*}-\Delta_{\Sigma_c^{++}}-E_2)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^{++}}}\right.\\ &\left.\left.\left.+\frac{1}{(\Delta_{R^*}-\Delta_{\Sigma_c^{*++}}-E_2)^2+\frac{1}{4}\Gamma^2_{\Sigma_c^{*++}}}\right)\right]\right\}.\end{aligned} \tag{6.39}$$

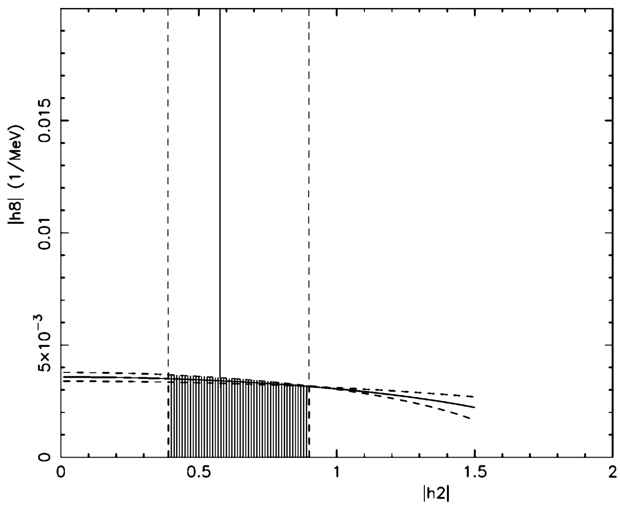

FIG. 3. Constraints on the pion couplings of the Λ_{c1}^{+} p-wave baryons h_2, h_8 from data on their decay widths. The continuous lines give central values and the dashed lines show 1σ deviations.

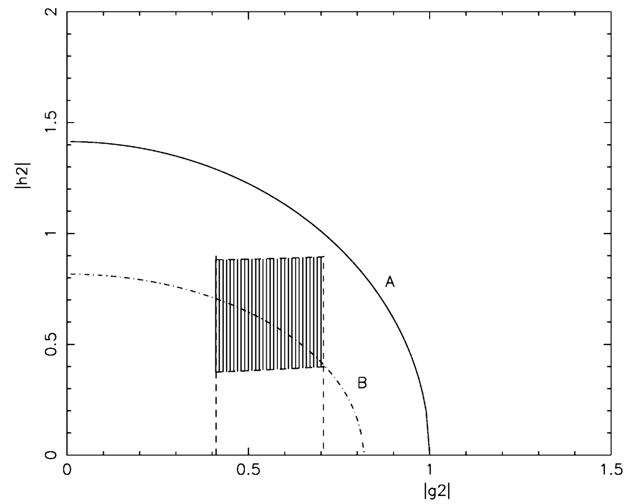

FIG. 4. Allowed region for the couplings (g_2,h_2) from the decay width of $\Lambda_{c1}(2593)$ and the Adler-Weisberger sum rule (6.30) (curve A). The dash-dotted line B shows the constraint imposed by the constituent quark model AW sum rule (6.31).

The rate for the neutral pions channel can again be obtained by adding a symmetry factor 1/2 and making the substitutions (6.30) in this formula. The excitation energy of this state is [31]

$$\Delta_{R*}=M_{\Lambda_{c_1}^{+}(3/2)}-M_{\Lambda_c^{+}}=341.5\pm 0.8\ \text{MeV}. \qquad (6.40)$$

We will assume the widths of the two Λ_{c1}^{+} baryons to be dominated by their two-pion decay modes and neglect the contribution of the multipion and radiative $\Lambda_{c1}^{+}\to\Lambda_c^{+}\gamma$ modes. The latter approximation is supported by model computations of the partial width for this mode [40] which gave the small values $\Gamma(\Lambda_{c_1}^{+}(2593)\to\Lambda_c^{+}\gamma)=0.016$ MeV and $\Gamma(\Lambda_{c_1}^{+}(2625)\to\Lambda_c^{+}\gamma)=0.021$ MeV.

The total widths of the Λ_{c1}^{+} states obtained by integrating Eqs. (6.22) and (6.34) (including also the $\pi^0\pi^0$ channel) can be represented, for given g_2, as two ellipses in the (h_2,h_8) plane. For example, the decay widths of the two Λ_{c1}^{+} states are given, for the central values of g_2 and hadron masses, by

$$\Gamma(\Lambda_{c1}^{+}(2593))=11.902h_2^2+13.817h_8^2-0.042h_2h_8\ (\text{MeV}), \qquad (6.41)$$

$$\Gamma(\Lambda_{c1}^{+}(2625))=0.518h_2^2+(0.148\times 10^6)h_8^2 -5.229h_2h_8\ (\text{MeV}). \qquad (6.42)$$

The constraints on (h_2,h_8) are plotted in Fig. 3 for the interval of values for g_2 (6.10). At the scale of the plot the two ellipses appear very elongated, the vertical lines corresponding to the limits on $\Gamma(\Lambda_{c_1}^{+}(2593))$ in Eq. (6.21) and the horizontal line giving an upper bound on $|h_8|$ arising from Eq. (6.33). From Fig. 3 we read off the following values:

$$|h_2|=0.572^{+0.322}_{-0.197}, \qquad (6.43)$$

$$|h_8|\leqslant(3.50-3.68)\times 10^{-3}\ \text{MeV}^{-1}. \qquad (6.44)$$

The errors shown are mainly due to the uncertainties in the masses (as the resonant decay $\Lambda_{c1}^{+}\to\Sigma_c\pi$ takes place very close to threshold) and in the total widths of $\Lambda_{c1}^{+}(2593)$ and $\Lambda_{c1}^{+}(2625)$. The error due to the unknown relative sign of h_2 and h_8 arising from the last terms in Eqs. (6.41) and (6.42) is negligible. The upper bound on the ratio $h_8/h_2\leqslant 10^{-2}$ MeV^{-1} is one order of magnitude above the naive dimensional analysis estimate $h_8/h_2\simeq 1/\Lambda_\chi\simeq 10^{-3}$ MeV^{-1}. The analogous couplings in the strange hyperons' system have been used in [30] to obtain an estimate for the charm case. Their results $|h_2|=0.54$, $|h_8|=0.55/\Lambda_\chi$ are compatible with the values (6.43) and (6.44) and suggest that the upper bound (6.44) on h_8 overestimates the real value of this coupling by a factor of 10. For illustration we quote also the values of the couplings obtained when the simplified formulas (6.27) and (6.39) are used instead of Eqs. (6.22) and (6.34): $h_2=0.553^{+0.310}_{-0.191}$, $|h_8|<3.63\times 10^{-3}$ MeV^{-1}.

Our results show that the width of the $\Lambda_{c1}^{+}(2625)$ state is possibly dominated by the D-wave term proportional to h_8^2 which can become resonant. The S-wave contribution to the width proportional to h_2^2 accounts for 0.096–0.250 MeV of the total.

In Fig. 4 we show the allowed region for the couplings in the (g_2,h_2) plane, together with the constraints imposed by the model-independent AW sum rule (4.15)

$$1\geqslant g_2^2+\frac{1}{2}h_2^2 \qquad (6.45)$$

and the constituent quark model version (6.20) of the AW sum rule

$$1\geqslant \frac{3}{2}g_2^2+\frac{3}{2}h_2^2. \qquad (6.46)$$

We neglected the contribution of the D-wave couplings on the RHS of these sum rules.

The inequality (6.45) is satisfied for all values of the (g_2,h_2) parameters following from the constraint on the width of $\Lambda_{c1}^{+}(2593)$. On the other hand the constituent quark sum rule (6.46) is more restrictive. It favors smaller values for h_2: $|h_2|=0.375-0.705$.

We can use the extracted values for h_2 and h_8 (6.43) and (6.44) to compute the resonant branching ratios of the $\Lambda_{c1}^{+}(2593)$ baryon. These quantities have been measured by the CLEO and E687 Collaborations [6,7,41]. The CLEO results are

$$f_{\Sigma_c^{++}}=\frac{B(\Lambda_{c_1}^{+}(2593)\to\Sigma_c^{++}\pi^-)}{B(\Lambda_{c_1}^{+}(2593)\to\Lambda_c^{+}\pi^+\pi^-)}=0.36\pm0.13, \tag{6.47}$$

$$f_{\Sigma_c^{0}}=\frac{B(\Lambda_{c_1}^{+}(2593)\to\Sigma_c^{0}\pi^-)}{B(\Lambda_{c_1}^{+}(2593)\to\Lambda_c^{+}\pi^+\pi^-)}=0.42\pm0.13, \tag{6.48}$$

whereas the E687 Collaboration [7,41] only quotes the total resonant branching ratio cummulated over the type of Σ_c

$$\frac{B(\Lambda_{c_1}^{+}(2593)\to\Sigma_c\pi^{\pm})}{B(\Lambda_{c_1}^{+}(2593)\to\Lambda_c^{+}\pi^+\pi^-)}=0.90\pm0.25$$

$$(>51\% \text{ at the 90\% confidence level}). \tag{6.49}$$

The experimental procedure for determining the resonant branching fractions (6.47) and (6.48) is based on measuring the ratio

$$f_{\Sigma_c}=\frac{B(\Lambda_{c_1}^{+}(2593)\to\Sigma_c\pi^{\pm})}{B(\Lambda_{c_1}^{+}(2593)\to\Lambda_c^{+}\pi^+\pi^-)}=\frac{Y_{\Sigma_c\text{region}}-Y_{\text{sideband}}}{Y_{\text{total}}}. \tag{6.50}$$

Here $Y_{\Sigma_c\text{region}}$ is the number of events for which the invariant mass of a pion and the Λ_c^{+} is within ±4 MeV from the mass of the corresponding Σ_c state. Y_{sideband} is the number of events for which E_1 (the energy of the positively charged pion) is contained in the sidebands (150.7, 155.1) MeV for a Σ_c^0 and (153.7, 158.1) MeV for a Σ_c^{++} (for the E687 experiment [7,41]). These sidebands are introduced to eliminate the background and have been chosen such that Eq. (6.50) vanishes for a completely nonresonant process.

We do not know the sideband parameters for the CLEO experiment [6] so we will neglect Y_{sideband} in Eq. (6.50). We obtain, with the help of the theoretical expression (6.27),

$$f_{\Sigma_c^{++}}=0.372^{+0.055}_{-0.074},\quad f_{\Sigma_c^{0}}=0.512^{+0.054}_{-0.061} \tag{6.51}$$

for the individual branching fractions. For the cumulated branching fraction we use the E687 sideband parameters quoted above to obtain

$$f_{\Sigma_c}=0.860^{+0.085}_{-0.086}. \tag{6.52}$$

These values (except $f_{\Sigma_c^0}$ which is off by 1σ) are in good agreement with the experimental data (6.47)–(6.49). The errors in (6.51) and (6.52) depend almost exclusively on g_2 and the hadron masses and are insensitive to the precise value of h_2. This is due to the negligibly small contribution made by the term proportional to h_8^2 to the total rate of $\Lambda_{c1}^{+}(2593)$.

The CLEO Collaboration recently reported evidence for another p-wave charmed baryon which decays into $\Xi_c^{+}\pi^-\pi^+$ via an intermediate Ξ_c^{*0} [42]. The measured excitation energy of this state, which is identified with the strange $c[s,u]$ $J^P=\frac{3}{2}^-$ counterpart of the Λ_{c1}^{+}, is

$$M_{\Xi_{c1}^{+}}-M_{\Xi_c^{+}}=349.4\pm0.7\pm1.0 \text{ MeV}. \tag{6.53}$$

There exists at present only an upper bound on the decay width of this state $\Gamma(\Xi_{c1}^{*+})<2.4$ MeV (90% confidence level) [42]. The dominant decay modes for this state are expected to be, as in the case of the Λ_{c1}^{+}, into Ξ_c plus two pions. In addition, the single pion mode $\Xi_{c1}^{+}\to\Xi_c\pi$ with the pion emitted in a D wave is also allowed at order $1/m_Q$ in the heavy mass expansion. However, this mode can be expected to be very much suppressed and will be neglected in the following.

Assuming SU(3) symmetry we can make some predictions for the decay properties of this state. The decay rates are given by

$$\frac{d\Gamma(\Xi_{c1}^{(*)+}\to\Xi_c^{+}\pi^+(E_1)\pi^-(E_2))}{dE_1dE_2}=\frac{g_2^2h_2^2}{64\pi^3f_\pi^4}M_{\Xi_c^+}\frac{E_1^2\vec{p}_2^{\,2}}{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)0}}-E_1)^2+\frac{1}{4}\Gamma_{S^{(*)0}}^2}, \tag{6.54}$$

$$\begin{aligned}\frac{d\Gamma(\Xi_{c1}^{(*)+}\to\Xi_c^{0}\pi^+(E_1)\pi^0(E_2))}{dE_1dE_2}&=\frac{g_2^2h_2^2}{128\pi^3f_\pi^4}M_{\Xi_c^0}\Bigg\{\frac{E_1^2\vec{p}_2^{\,2}}{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)0}}-E_1)^2+\frac{1}{4}\Gamma_{S^{(*)0}}^2}+\frac{E_2^2\vec{p}_1^{\,2}}{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_2)^2+\frac{1}{4}\Gamma_{S^{(*)+}}^2}\\&+2E_1E_2(E_1E_2-p_1\cdot p_2)\frac{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)0}}-E_1)(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_2)+\frac{1}{4}\Gamma_{S^{(*)0}}\Gamma_{S^{(*)+}}}{[(\Delta_{R^{(*)+}}-\Delta_{S^{(*)0}}-E_1)^2+\frac{1}{4}\Gamma_{S^{(*)0}}^{0}][(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_2)^2+\frac{1}{4}\Gamma_{S^{(*)+}}^2]}\Bigg\},\end{aligned} \tag{6.55}$$

$$\frac{d\Gamma(\Xi_{c1}^{(*)+}\to\Xi_c^+\pi^0(E_1)\pi^0(E_2))}{dE_1dE_2}$$
$$=\frac{g_2^2h_2^2}{256\pi^3f_\pi^4}M_{\Xi_c^+}\Bigg\{\frac{E_1^2\vec{p}_2^{\,2}}{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_1)^2+\frac{1}{4}\Gamma^2_{S^{(*)+}}}+(1\leftrightarrow 2)+2E_1E_2(E_1E_2-p_1\cdot p_2)$$
$$\times\frac{(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_1)(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_2)+\frac{1}{4}\Gamma^2_{S^{(*)+}}}{[(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_1)^2+\frac{1}{4}\Gamma^2_{S^{(*)+}}][(\Delta_{R^{(*)+}}-\Delta_{S^{(*)+}}-E_2)^2+\frac{1}{4}\Gamma^2_{S^{(*)+}}]}\Bigg\}. \tag{6.56}$$

We neglected in these expressions the contributions of the D-wave coupling. This is justified in this case as the S-wave contribution can become resonant for both possible initial states $\Xi_{c1}^{(*)+}$ and is thus dominant.

Numerical integration of the rate formulas (6.54)–(6.56) gives [after adding a factor of 1/2 to the decay rate of $\Gamma(\Xi_{c1}^{(*)+}\to\Xi_c^+\pi^0\pi^0)$ due to the identity of the pions]

$$\Gamma(\Xi_{c1}^{*+}\to\Xi_c^+\pi^+\pi^-)=(7.948^{+0.448}_{-0.454})h_2^2\ \text{MeV}, \tag{6.57}$$

$$\Gamma(\Xi_{c1}^{*+}\to\Xi_c^0\pi^+\pi^0)=(7.301^{+0.408}_{-0.413})h_2^2\ \text{MeV}, \tag{6.58}$$

$$\Gamma(\Xi_{c1}^{*+}\to\Xi_c^+\pi^0\pi^0)=(2.550^{+0.132}_{-0.137})h_2^2\ \text{MeV}, \tag{6.59}$$

so that we obtain for the total decay rate $\Gamma(\Xi_{c1}^{*+})=(16.79-18.77)h_2^2$ MeV=(2.37−15.00) MeV. This result is marginally compatible with the upper limit quoted by CLEO [42] $\Gamma(\Xi_{c1}^{*+})<2.4$ MeV. The finite width of the Ξ_c^* in the intermediate state suppresses the decay rate by about 10% compared to its value in the narrow-width approximation.

As mentioned previously, our results (6.43) and (6.44) can be used in conjunction with the quark model relations (5.2) and (5.20) to predict the couplings of all symmetric p-wave baryons. Unfortunately, except for model calculations [9,11], the masses of these states are not yet known. Eventually they will be measured experimentally. For the time being we will limit ourselves to illustrating this application by using the results of the model calculation [9] for the baryon masses.

The next excitations above Λ_{c1}^+ are expected to be the $\Sigma_{c0}(\frac{1}{2})$ baryons, which have the light degrees of freedom in a $s_\ell^{\pi_\ell}=0^-$ state. Their excitation energy is estimated to be [9]

$$\Delta_U=M_{\Sigma_{c0}^{++}}-M_{\Lambda_c^+}\simeq 500\ \text{MeV}. \tag{6.60}$$

This state can decay directly to $\Lambda_c^+\pi^+$ in an S-wave with a width

$$\Gamma(\Sigma_{c0}^{++})=\frac{h_3^2}{2\pi f_\pi^2}\frac{M_{\Lambda_c^+}}{M_{\Sigma_{c0}^{++}}}E_\pi^2|\vec{p}_\pi|. \tag{6.61}$$

Using the quark model relation $|h_3|=\sqrt{3}|h_2|$ (5.2) together with the value (6.43) for h_2 we obtain[7]

$$\Gamma(\Sigma_{c0}^{++})\simeq h_2^2(2.066)\ \text{GeV}=0.676^{+0.975}_{-0.385}\ \text{GeV}. \tag{6.62}$$

This state is so broad that it might be very difficult to observe it.

The next symmetric baryons are Σ_{c1}, represented by the superfield V in Eq. (3.1). Their dominant decay mode is expected to be, as in the case of Λ_{c1}^+, into two pions. The decay rate of the $\Sigma_{c1}^{++}(\frac{1}{2})$ is

$$\frac{d\Gamma(\Sigma_{c1}^{++}(\frac{1}{2})\to\Lambda_c^+\pi^+(E_1)\pi^0(E_2))}{dE_1dE_2}=\frac{g_2^2}{32\pi^3f_\pi^4}M_{\Lambda_c^+}\{\vec{p}_2^{\,2}|A|^2+\vec{p}_1^{\,2}|B|^2+2[E_1E_2-p_1\cdot p_2]\text{Re}(AB^*)\} \tag{6.63}$$

with

$$A(E_1,E_2)=\frac{h_4E_1}{\Delta_V-\Delta_{\Sigma_c^+}-E_1+i\Gamma_{\Sigma_c^+}/2}-\frac{\frac{2}{3}h_9\vec{p}_1^{\,2}}{\Delta_V-\Delta_{\Sigma_c^{*+}}-E_1+i\Gamma_{\Sigma_c^{*+}}/2}-\frac{2h_9[E_1E_2-p_1\cdot p_2]}{\Delta_V-\Delta_{\Sigma_c^{*++}}-E_2+i\Gamma_{\Sigma_c^{*++}}/2}, \tag{6.64}$$

$$B(E_1,E_2;\Delta_{\Sigma_c^{(*)+}},\Delta_{\Sigma_c^{(*)++}})=-A(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)+}}). \tag{6.65}$$

[7]This estimate for h_3 is compatible, within its error bounds, with the unitarity bound (4.62) $|h_3|\leqslant 0.858$.

The decay rates for the isospin partners of Σ_{c1}^{++} are related as $\Gamma(\Sigma_{c1}^{(*)++,0}\to\Lambda_c^+\pi^0\pi^\pm)=\Gamma(\Sigma_{c1}^{(*)+}\to\Lambda_c^+\pi^+\pi^-)$. The decay $\Sigma_{c1}^{(*)+}\to\Lambda_c^+\pi^0\pi^0$ is forbidden in the limit of exact isospin symmetry, due to the fact that the two neutral pions cannot have isospin 1.

The decay rate of the $\Sigma_{c1}^{++}(\frac{3}{2})$ state is given by a formula analogous to Eq. (6.34):

$$\begin{aligned}\frac{d\Gamma(\Sigma_{c1}^{++}\to\Lambda_c^+\pi^+(E_1)\pi^0(E_2))}{dE_1dE_2}=&\frac{g_2^2}{32\pi^3f_\pi^4}M_{\Lambda_c^+}\{\vec{p}_1^{\,2}|C|^2+\vec{p}_2^{\,2}|E|^2+2[E_1E_2-p_1\cdot p_2]\mathrm{Re}(CE^*)\\&+[\vec{p}_1^{\,2}\vec{p}_2^{\,2}-(E_1E_2-p_1\cdot p_2)^2][\vec{p}_1^{\,2}|D|^2+\vec{p}_2^{\,2}|F|^2-\mathrm{Re}(CF^*)+\mathrm{Re}(DE^*)\\&+2(E_1E_2-p_1\cdot p_2)\mathrm{Re}(DF^*)]\}\end{aligned}\tag{6.66}$$

with

$$\begin{aligned}C(E_1,E_2)=&-\left(h_4E_2-\frac{2}{3}h_9\vec{p}_2^{\,2}\right)\frac{1}{\Delta_{V*}-\Delta_{\Sigma_c^{*++}}-E_2+i\Gamma_{\Sigma_c^{*++}}/2}+\frac{2}{3}h_9(E_1E_2-p_1\cdot p_2)\left(\frac{1}{\Delta_{V*}-\Delta_{\Sigma_c^+}-E_1+i\Gamma_{\Sigma_c^+}/2}\right.\\&\left.+\frac{2}{\Delta_{V*}-\Delta_{\Sigma_c^{*+}}-E_1+i\Gamma_{\Sigma_c^{*+}}/2}\right),\end{aligned}\tag{6.67}$$

$$D(E_1,E_2)=\frac{2}{3}h_9\left(-\frac{1}{\Delta_{V*}-\Delta_{\Sigma_c^+}-E_1+i\Gamma_{\Sigma_c^+}/2}+\frac{1}{\Delta_{V*}-\Delta_{\Sigma_c^{*+}}-E_1+i\Gamma_{\Sigma_c^{*+}}/2}\right),\tag{6.68}$$

$$E(E_1,E_2;\Delta_{\Sigma_c^{(*)+}},\Delta_{\Sigma_c^{(*)++}})=-C(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)+}}),\tag{6.69}$$

$$F(E_1,E_2;\Delta_{\Sigma_c^{(*)+}},\Delta_{\Sigma_c^{(*)++}})=D(E_2,E_1;\Delta_{\Sigma_c^{(*)++}},\Delta_{\Sigma_c^{(*)+}}).\tag{6.70}$$

In addition to s-wave sextet baryons, the Σ_{c0} can appear also in the intermediate state. The contribution of these diagrams can be expected to be suppressed due to the large width of these states. Neglecting them we obtain (for the central values of g_2 and Σ_c masses)

$$\Gamma\left(\Sigma_{c1}\left(\frac{1}{2}\right)\right)=81.3h_4^2+(1.4\times10^6)h_9^2-33.4h_4h_9\ (\mathrm{MeV})=106.4^{+153.5}_{-60.7}\ \mathrm{MeV},\tag{6.71}$$

$$\Gamma\left(\Sigma_{c1}\left(\frac{3}{2}\right)\right)=72.3h_4^2+(2.5\times10^6)h_9^2+617.7h_4h_9\ (\mathrm{MeV})=94.7^{+136.6}_{-54.0}\ \mathrm{MeV}\tag{6.72}$$

for $\Delta_{V(*)}=465.1$ MeV (corresponding to $M_{\Sigma_{c1}}=2750$ MeV) and

$$\Gamma\left(\Sigma_{c1}\left(\frac{1}{2}\right)\right)=65.5h_4^2+(5.1\times10^6)h_9^2-87.1h_4h_9\ (\mathrm{MeV})=85.8^{+123.7}_{-48.9}\ \mathrm{MeV},\tag{6.73}$$

$$\Gamma\left(\Sigma_{c1}\left(\frac{3}{2}\right)\right)=132.5h_4^2+(4.7\times10^6)h_9^2+1765.2h_4h_9\ (\mathrm{MeV})=173.4^{+250.1}_{-98.8}\ \mathrm{MeV}\tag{6.74}$$

for $\Delta_{V(*)}=515.1$ MeV (corresponding to $M_{\Sigma_{c1}}=2800$ MeV), respectively [9]. In the last equality the quark model relations $|h_4|=2|h_2|$ and $|h_9|=|h_8|$ have been used together with Eqs. (6.43) and (6.44). The terms proportional to h_9 have been neglected, as they are of the order of a few MeV. These states appear to be considerably broader than Λ_{c1}^+ due to the larger available phase space.

On the other hand, the Σ_{c2} baryons have only D-wave couplings. Their dominant decay mode is expected to be two-body decay to $\Lambda_c^+\pi^+$

$$\Gamma\left(\Sigma_{c2}^{++}\ \left(\frac{3}{2},\frac{5}{2}\right)\to\Lambda_c^+\pi^+\right)=\frac{4h_{10}^2}{15\pi f_\pi^2}\frac{M_{\Lambda_c^+}}{M_{\Sigma_{c2}^{++}}}|\vec{p}_\pi|^5\simeq12\ \mathrm{MeV},\tag{6.75}$$

where we used $M_{\Sigma_{c2}}=2800$ MeV [9] and the naive dimensional analysis estimate $|h_{10}|=0.4\times10^{-3}$ MeV^{-1}. In addition to this mode, the Σ_{c2} baryons can also decay to $\Sigma_c^{(*)}\pi$. The corresponding partial widths are

$$\Gamma\left(\Sigma_{c2}^{++}\left(\frac{3}{2}\right)\to\Sigma_c^{+}\pi^{+}\right)+\Gamma\left(\Sigma_{c2}^{++}\left(\frac{3}{2}\right)\to\Sigma_c^{*+}\pi^{+}\right)=\frac{h_{11}^2}{10\pi f_\pi^2}\frac{M_{\Sigma_c^+}}{M_{\Sigma_{c2}^{++}}}|\vec{p}_\pi|^5+\frac{h_{11}^2}{10\pi f_\pi^2}\frac{M_{\Sigma_c^{*+}}}{M_{\Sigma_{c2}^{++}}}|\vec{p}_\pi|^5, \tag{6.76}$$

$$\Gamma\left(\Sigma_{c2}^{++}\left(\frac{5}{2}\right)\to\Sigma_c^{+}\pi^{+}\right)+\Gamma\left(\Sigma_{c2}^{++}\left(\frac{5}{2}\right)\to\Sigma_c^{*+}\pi^{+}\right)=\frac{2h_{11}^2}{45\pi f_\pi^2}\frac{M_{\Sigma_c^+}}{M_{\Sigma_{c2}^{++}}}|\vec{p}_\pi|^5+\frac{7h_{11}^2}{45\pi f_\pi^2}\frac{M_{\Sigma_c^{*+}}}{M_{\Sigma_{c2}^{++}}}|\vec{p}_\pi|^5, \tag{6.77}$$

and identical formulas for the $\Sigma_c^{(*)++}\pi^0$ final states. Adding together the contributions of all possible final states we obtain

$$\Gamma\left(\Sigma_{c2}^{++}\left(\frac{3}{2}\right)\to\Sigma_c^{(*)}\pi\right)=(9.86\times10^{-6})h_{11}^2\simeq3.16\text{ MeV}, \tag{6.78}$$

$$\Gamma\left(\Sigma_{c2}^{++}\left(\frac{5}{2}\right)\to\Sigma_c^{(*)}\pi\right)=(6.88\times10^{-6})h_{11}^2\simeq2.20\text{ MeV}, \tag{6.79}$$

where we used the quark model relation $h_{11}^2=2h_{10}^2$ and the above-mentioned dimensional analysis estimate for h_{10}. It must be mentioned that even a small mixing of $\Sigma_{c2}(\frac{3}{2})$ with the broader $\Sigma_{c1}(\frac{3}{2})$ could enhance its decay width.

The strong couplings of the antisymmetric p-wave baryons $\Sigma_{c1}',\Lambda_{c0}',\Lambda_{c1}',\Lambda_{c2}'$ cannot be related in the quark model to h_2,h_8, as the corresponding reduced matrix elements are different. Therefore we are not able, at this stage, to make quantitative predictions about their decay properties. We mention here only one possible effect of these states on our predictions for symmetric p-wave baryons, connected with the possible mixing among states with identical quantum numbers. This could be an important effect with the close pairs of states $(\Sigma_{c1}(\frac{1}{2}),\Sigma_{c1}'(\frac{1}{2}))$ and $(\Sigma_{c1}(\frac{3}{2}),\Sigma_{c1}'(\frac{3}{2}))$, which can mix even in the heavy mass limit. On the other hand, mixing between states belonging to heavy quark doublets with different values for the quantum numbers of the light degrees of freedom $s_\ell^{\pi_\ell}$ is a $1/m_Q$ effect. Still, experimental evidence of mixing in the system of charmed p-wave mesons [31] shows that such $1/m_Q$ effects can be significant in the case of the charm heavy quark.

Another possible decay mode for the antisymmetric p-wave baryons is to the channels $[ND]$ and $[ND^*]$. The threshold for the first one is at 2810 MeV and for the second one at 2950 MeV. Quark model calculations [9,11] suggest that the p-wave baryons must be lighter than 3 GeV with some of the states lying above these thresholds such that these modes may well turn out to be significant. Unfortunately, at the present time it is not possible to treat these processes in a chiral perturbation theory framework, as done in the pion decay case. There are, nevertheless, a few model-independent predictions which can be made about these decays, following [43].

The dominant decays can be expected to be (if kinematically allowed)

$$\Lambda_{c0}'\left(\frac{1}{2}\right)\to[ND]_S,[ND^*]_S, \tag{6.80}$$

$$\Sigma_{c1}'\left(\frac{1}{2}\right),\Lambda_{c1}'\left(\frac{1}{2}\right)\to[ND]_S,[ND^*]_S, \tag{6.81}$$

$$\Sigma_{c1}'\left(\frac{3}{2}\right),\Lambda_{c1}'\left(\frac{3}{2}\right)\to[ND^*]_S, \tag{6.82}$$

which can proceed by S waves. The decay $\Lambda_{c2}'(\frac{3}{2})\to[ND^*]_S$, although allowed by angular momentum and parity conservation, is forbidden in the heavy mass limit.

Heavy quark symmetry predicts the following typical decay rate ratios:

$$\Gamma\left(\Lambda_{c0}'\left(\frac{1}{2}\right)\to[ND]_S\right):\Gamma\left(\Lambda_{c0}'\left(\frac{1}{2}\right)\to[ND^*]_S\right)=1:3, \tag{6.83}$$

$$\Gamma\left(\Lambda_{c1}'\left(\frac{1}{2}\right)\to[ND]_S\right):\Gamma\left(\Lambda_{c1}'\left(\frac{1}{2}\right)\to[ND^*]_S\right):\Gamma\left(\Lambda_{c1}'\left(\frac{3}{2}\right)\to[ND]_S\right):\Gamma\left(\Lambda_{c1}'\left(\frac{3}{2}\right)\to[ND^*]_S\right)=\frac{3}{4}:\frac{1}{4}:0:1, \tag{6.84}$$

$$\Gamma\left(\Lambda_{c1}'\left(\frac{1}{2}\right)\to[ND]_D\right):\Gamma\left(\Lambda_{c1}'\left(\frac{1}{2}\right)\to[ND^*]_D\right):\Gamma\left(\Lambda_{c1}'\left(\frac{3}{2}\right)\to[ND]_D\right):\Gamma\left(\Lambda_{c1}'\left(\frac{3}{2}\right)\to[ND^*]_D\right)=0:1:\frac{3}{8}:\frac{5}{8}, \tag{6.85}$$

$$\Gamma\left(\Lambda_{c2}'\left(\frac{3}{2}\right)\to[ND]_D\right):\Gamma\left(\Lambda_{c2}'\left(\frac{3}{2}\right)\to[ND^*]_D\right):\Gamma\left(\Lambda_{c2}'\left(\frac{5}{2}\right)\to[ND]_D\right):\Gamma\left(\Lambda_{c2}'\left(\frac{5}{2}\right)\to[ND^*]_D\right)=\frac{5}{8}:\frac{11}{8}:\frac{5}{12}:\frac{19}{12}. \tag{6.86}$$

Kinematical effects such as mass splittings within the heavy quark symmetry doublets and mixings among different states will certainly modify these results. After accounting for these corrections, the width ratios (6.83)–(6.86) can be expected to be useful in identifying the heavy quark symmetry assignments of these states.

VII. CONCLUSIONS

In this paper we have made a systematic study of the strong interactions of the s- and p-wave baryons containing a heavy quark. The dynamics of these baryons are very rich. The richness is reflected by the large number of multiplets (2 for s wave and 8 for p wave), and the large number of coupling constants necessary to describe all the interactions. We have found that the constituent quark model in conjunction with the Adler-Weisberger sum rules provides a powerful tool to handle the system. The quark model reduces the number of coupling constants from 45 to 7 which are further constrained by an AW sum rule. One of the seven parameters is the axial vector coupling g_A for the single quark transition $u\to d$. If we assume that g_A is independent of the light quark environment, then its value is known to be $g_A=0.75$ from the nucleon β decay. The recent data on charmed baryons from Fermilab and CLEO are consistent with this value of g_A and give strong constraints on two of the other four unknowns as discussed in Sec. VI.

Through the common value of g_A the single coupling constant which is needed to describe the s-wave heavy mesons is related to many of the coupling constants in the heavy baryons. The choice of $g_A=0.75$ gives a satisfactory rendition of the branching ratios of D^* [44] and the decay widths of the charmed baryons as we have seen in the last Section. However, this value of g_A implies a value for the $DD^*\pi$ coupling constant g to be order of 0.7 [33] which is much larger than the values around 0.3 obtained by other approaches such as QCD sum rules (see the references cited in [33]). It is therefore of great importance to measure the width of D^* to give a direct measurement of g. It will also confirm or reject the hypothesis of environmental independence of g_A.

By heavy quark symmetry the bottom baryons are described by the same interactions and the same coupling constants as those studied in this paper. In fact, heavy quark symmetry should work even better for the heavier bottom baryons. With more forthcoming data on charmed baryons from Fermilab (FOCUS) and CLEO and a wealth of data on bottom baryons expected from the CERN e^+e^- collider LEP and the B factories under construction in a few years, we hope that many of our predictions will be tested experimentally in the near future.

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APPENDIX: QUARK MODEL WAVE FUNCTIONS FOR HEAVY BARYONS

s-wave:

$$|\Lambda_c^+\uparrow\rangle=|c\uparrow\rangle\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle), \tag{A1}$$

$$|\Sigma_c^+\uparrow\rangle=\left(\sqrt{\frac{2}{3}}|c\downarrow\rangle|\uparrow\uparrow\rangle-\frac{1}{\sqrt{6}}|c\uparrow\rangle(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)\right)\frac{1}{\sqrt{2}}(|ud\rangle+|du\rangle). \tag{A2}$$

For the p-wave baryons our phase convention corresponds to combining the total spin $S=s_1+s_2$ with the orbital momentum L in the order $S\otimes L$.

p wave (symmetric):

$$\left|\Lambda_{c1}^+\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=\left(\sqrt{\frac{2}{3}}|L(+1)c\downarrow\rangle-\frac{1}{\sqrt{3}}|L(0)c\uparrow\rangle\right)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle), \tag{A3}$$

$$\left|\Lambda_{c1}^+\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\left(\frac{1}{\sqrt{3}}|c\downarrow L(+1)\rangle+\sqrt{\frac{2}{3}}|c\uparrow L(0)\rangle\right)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle), \tag{A4}$$

$$\left|\Sigma_{c0}^{++}\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}|c\uparrow\rangle\left(|L(+1)u\downarrow u\downarrow\rangle-\frac{1}{\sqrt{2}}|L(0)u\uparrow u\downarrow\rangle-\frac{1}{\sqrt{2}}|L(0)u\downarrow u\uparrow\rangle+|L(-1)u\uparrow u\uparrow\rangle\right), \tag{A5}$$

$$\left|\Sigma_{c1}^{++}\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=-\frac{1}{\sqrt{6}}|c\downarrow L(+1)\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)+\frac{1}{\sqrt{3}}|c\downarrow L(0)u\uparrow u\uparrow\rangle+\frac{1}{\sqrt{6}}|c\uparrow L(+1)u\downarrow u\downarrow\rangle-\frac{1}{\sqrt{6}}|c\uparrow L(-1)u\uparrow u\uparrow\rangle, \tag{A6}$$

$$\left|\Sigma_{c1}^{++}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{6}}|c\downarrow\rangle\left(|L(0)u\uparrow u\uparrow\rangle-\frac{1}{\sqrt{2}}|L(+1)\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)\right)+\frac{1}{\sqrt{3}}|c\uparrow\rangle[|L(-1)u\uparrow u\uparrow\rangle-|L(+1)u\downarrow u\downarrow\rangle], \tag{A7}$$

$$\left|\Sigma_{c2}^{++}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=|c\downarrow\rangle\left(\sqrt{\frac{3}{10}}|L(0)u\uparrow u\uparrow\rangle+\sqrt{\frac{3}{20}}|L(+1)\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)\right)-\frac{1}{\sqrt{15}}|c\uparrow\rangle[|L(-1)u\uparrow u\uparrow\rangle + \sqrt{2}|L(0)\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)+|L(+1)u\downarrow u\downarrow\rangle], \tag{A8}$$

p wave (antisymmetric):

$$\left|\Sigma_{c1}'^{++}\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=\left(\sqrt{\frac{2}{3}}|L(+1)c\downarrow\rangle-\frac{1}{\sqrt{3}}|L(0)c\uparrow\rangle\right)\frac{1}{\sqrt{2}}(|u\uparrow u\downarrow\rangle-|u\downarrow u\uparrow\rangle), \tag{A9}$$

$$\left|\Sigma_{c1}'^{++}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\left(\sqrt{\frac{1}{3}}|L(+1)c\downarrow\rangle+\sqrt{\frac{2}{3}}|L(0)c\uparrow\rangle\right)\frac{1}{\sqrt{2}}(|u\uparrow u\downarrow\rangle-|u\downarrow u\uparrow\rangle), \tag{A10}$$

$$\left|\Xi_{c0}'^{+}\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{6}}|c\uparrow\rangle\left(|L(-1)\rangle(|u\uparrow s\uparrow\rangle-|s\uparrow u\uparrow\rangle)-\frac{1}{\sqrt{2}}|L(0)\rangle(|u\uparrow s\downarrow\rangle-|s\uparrow u\downarrow\rangle+|u\downarrow s\uparrow\rangle-|s\downarrow u\uparrow\rangle) +|L(+1)\rangle(|u\downarrow s\downarrow\rangle-|s\downarrow u\downarrow\rangle)\right), \tag{A11}$$

$$\left|\Lambda_{c1}'^{+}\left(\frac{1}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}|c\downarrow\rangle\left(|L(0)\uparrow\uparrow\rangle-\frac{1}{\sqrt{2}}|L(+1)\rangle(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)\right)\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle)-\frac{1}{\sqrt{6}}|c\uparrow\rangle[|L(-1)\uparrow\uparrow\rangle-|L(+1)\downarrow\downarrow\rangle] \times\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle), \tag{A12}$$

$$\left|\Lambda_{c1}'^{+}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{6}}|c\downarrow\rangle\left(|L(0)\uparrow\uparrow\rangle-\frac{1}{\sqrt{2}}|L(+1)\rangle(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)\right)\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle) +\frac{1}{\sqrt{3}}|c\uparrow\rangle[|L(-1)\uparrow\uparrow\rangle-|L(+1)\downarrow\downarrow\rangle]\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle), \tag{A13}$$

$$\left|\Xi_{c2}'^{+}\left(\frac{3}{2}\right),+\frac{1}{2}\right\rangle=\sqrt{\frac{3}{10}}|c\downarrow\rangle\left(|L(0)\uparrow\uparrow\rangle+\frac{1}{\sqrt{2}}|L(+1)\rangle(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)\right)\frac{1}{\sqrt{2}}(|us\rangle-|su\rangle)-\frac{1}{\sqrt{15}}|c\uparrow\rangle \times[|L(-1)\uparrow\uparrow\rangle+\sqrt{2}|L(0)\rangle(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)+|L(+1)\rangle|\downarrow\downarrow\rangle]\frac{1}{\sqrt{2}}(|us\rangle-|su\rangle). \tag{A14}$$

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