

***CP* violation and baryogenesis due to heavy Majorana neutrinos**

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We analyze the scenario of baryogenesis through leptogenesis induced by the out-of-equilibrium decays of heavy Majorana neutrinos and pay special attention to *CP* violation. Extending a recently proposed resummation formalism for two-fermion mixing to decay amplitudes, we calculate the resonant phenomenon of *CP* violation due to the mixing of two nearly degenerate heavy Majorana neutrinos. Solving numerically the relevant Boltzmann equations, we find that the isosinglet Majorana mass may range from 1 TeV up to the grand unification scale, depending on the mechanism of *CP* violation and/or the flavor structure of the neutrino mass matrix assumed. Finite temperature effects and possible constraints from the electric dipole moment of electron and other low-energy experiments are briefly discussed. [S0556-2821(97)04121-0]

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I. INTRODUCTION

Based on the assumption that the Universe was created initially in a symmetric state with a vanishing baryon number B , Sakharov [1] derived the three known necessary conditions that may explain the small baryon-to-photon ratio of number densities, $n_B/n_\gamma = (4-7) \times 10^{-10}$, which is found by present observations. The first necessary ingredient is the existence of B -violating interactions. With the advent of grand unified theories (GUT's), this requirement can naturally be satisfied at very-high-energy scales [2], through the decay of superheavy bosons with masses near to the grand unification scale $M_X \approx 10^{15}$ GeV. However, such a solution to the baryon asymmetry in the Universe (BAU) faces some difficulties. In fact, Sakharov's second requirement for generating the BAU prescribes that, by the same token, the B -violating interactions should violate the discrete symmetries of charge conjugation (C) and that resulting from the combined action of charge and parity (CP) transformations. One major drawback of the solution suggested is that minimal scenarios of grand unification generally predict very small CP violation, since it occurs at very high orders in perturbation theory. Therefore, one has to rely on non-minimal representations of GUT's in order to obtain appreciable CP violation. Furthermore, experiments on the stability of the proton put tight constraints on the masses of the GUT bosons mediating B violation and their couplings to the matter.

The most severe limits on scenarios for baryogenesis, however, come from Sakharov's last requirement that the B - and CP -violating interactions must be out of thermal equilibrium [3,4]. In the standard model (SM), the sum of B and the lepton number L , $B+L$, is violated anomalously [5] through topologically extended solutions, known as sphalerons. In contrast to $B+L$ nonconservation, sphalerons preserve the quantum number $B-L$. The authors in [6] have found that $B+L$ anomalous violation may be large at high temperatures T above the critical temperature T_c of the elec-

troweak phase transition. For values of T not much larger than the W -boson mass, M_W , i.e., $T > 200$ GeV, up to temperatures of $T \approx 10^{12}$ GeV, the anomalous ($B+L$) rate may exceed the expansion rate of the Universe [7,8]. Therefore, any primordial BAU generated at the GUT scale should not rely on ($B+L$)-violating operators, since sphalerons being in thermal equilibrium will then wash it out. The latter appears to be a generic feature of most GUT's, where ($B-L$)-violating terms may be suppressed against ($B+L$)-violating interactions, thus leading to the net effect of a vanishing BAU. On the other hand, it was suggested [6] that the same anomalous $B+L$ electroweak interactions may also be utilized to produce the observed excess in B during a first-order electroweak phase transition. Given the fact that the experimental lower mass bound of the Higgs boson H is about $M_H > 80$ GeV, this scenario of electroweak baryogenesis must now be considered to be rather improbable to explain the observed BAU [9,10] within the minimal SM.

It is therefore important to note that baryogenesis not only provides the strongest indication against the completeness of the SM but also poses limits on its possible new-physics extensions. An attractive scenario that may lead to a consistent solution to the problem of the BAU is the one proposed by Fukugita and Yanagida [11],¹ in which the baryon number is generated by out-of-equilibrium L -violating decays of heavy Majorana neutrinos N_i with masses $m_{N_i} \gg T_c$. Moreover, it was argued [11] that the excess in L will then be converted into the desired excess in B by means of ($B+L$)-violating sphaleron interactions, which are in thermal equilibrium above the critical temperature T_c . Many studies have been devoted to this mechanism of baryogenesis through leptogenesis [12-16] over the last years.

Evidently, possible mechanisms for enhancing CP violation play a decisive role in understanding the BAU. In [11], the necessary CP violation in heavy Majorana neutrino decays results from the interference between the tree-level

¹The authors in [6] presented an analogous scenario, which was, however, based on anomalous electroweak decays of exotic Dirac leptons into quarks.

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graph and the absorptive part of the one-loop vertex. Since CP violation originates entirely from the decay amplitude in this case, we shall attach the characterization of the ε' type to this kind of CP violation, thereby making contact with the terminology known from the $K^0\bar{K}^0$ system [17]. The ε' -type CP violation has been discussed extensively in the literature [12–14]. Provided all Yukawa couplings of the Higgs fields to N_i and the ordinary lepton isodoublets are of comparable order [12,14], baryogenesis through the ε' -type mechanism requires very heavy Majorana neutrinos with masses not much smaller than 10^7 – 10^8 GeV. If a hierarchical pattern for the Yukawa couplings and the heavy Majorana neutrino masses is assumed [12,13], the above high mass bound may be lifted and the lightest heavy neutrino can have a mass as low as 1 TeV. Taking out-of-equilibrium constraints on scatterings involving heavy Majorana neutrinos into account [12], one finds that ε' may reach values up to 10^{-7} – 10^{-6} in such a scenario. This must be compared with the usual scenario in [14], for which $\varepsilon' < 10^{-15}$ for $m_{N_i} \approx 1$ TeV, and hence very heavy neutrinos are needed to account for the BAU.

In Refs. [18,6], the authors pointed out that CP violation in heavy particle decays responsible for the baryon (lepton) asymmetry may be further enhanced if one considers the absorptive part of the Higgs self-energies, which was neglected in subsequent studies. Since this kind of CP violation may resemble the known mechanism of CP violation through $K^0\bar{K}^0$ mixing in the kaon complex [17], we shall call it hereafter as CP violation of the ε type. Exploiting this idea, Botella and Roldan [19] gave some estimates for ε -type CP violation in Higgs decays. They found that the ratio ε/ε' may indeed be large in an extended SU(5) unified model, since the self-energy and vertex contributions to CP violation may have different Yukawa coupling structures. Applying this mechanism, the authors in [15] concluded that ε/ε' is of order unity in scenarios for leptogenesis, and hence the known results [11] obtained for ε' -type CP violation may not need be modified drastically. Moreover, using an effective Hamiltonian approach based on the Weisskopf-Wigner (WW) approximation [20], the same authors [21] reached the conclusion that ε -type CP violation may be rather suppressed when the two mixed heavy Majorana neutrinos are nearly degenerate and that CP violation vanishes completely in the limit in which the two mass eigenvalues of the effective Hamiltonian are *exactly* equal.

Recently, there has been renewed interest in the ε -type CP violation due to the mixing of heavy Majorana neutrinos and the implications of this mechanism for the BAU [16]. It has been observed in [16] that CP violation can be considerably enhanced through the mixing of two nearly degenerate heavy Majorana neutrinos. Using exact solutions for the wave functions, which were obtained from diagonalizing the effective Hamiltonian, the authors [16] have calculated ε and found that it can be larger than ε' by two or even three orders of magnitude. This result makes the leptogenesis scenario very attractive. The enhancement of the CP -violating phenomenon is in agreement with earlier articles on resonant CP violation in scatterings involving top quarks, supersymmetric quarks, or Higgs particles in the intermediate state [22–25], as well as with a remark [6] concerning CP viola-

tion in the decays of exotic neutral leptons to quarks.

The existing difference between earlier articles, which found values of ε/ε' of order one [15,21], and recent authors [24,16,25], who discovered that ε could be even of order unity [24,25], may be attributed to the problem of the proper treatment of two nearly degenerate states. It is known that conventional perturbation field theory breaks down in the limit of degenerate particles. For example, the wave-function amplitude that describes the CP -asymmetric mixing of two heavy Majorana neutrinos, N_1 and N_2 , say, is inverse proportional to the mass splitting $m_{N_1} - m_{N_2}$, and it becomes singular if the degeneracy is exact [21]. Solutions to this problem have been based on the wave-function formalism in the WW approximation [21,16]. Obviously, a more rigorous field-theoretic approach to the resonant phenomenon of CP violation through nearly degenerate heavy Majorana neutrinos is still necessary. Therefore, it is rather important to provide a field-theoretic solution to the problem of ε -type CP violation and compare the so-derived results with those found with other methods.

Since the dynamics of ε -type CP violation is quite closely related to CP violation induced by particle widths [22], one is therefore compelled to rely on resummation approaches, which treat unstable particles in a consistent way. In the context of gauge field theories, a gauge-independent resummation approach to resonant transition amplitudes has been formulated, which is implemented by the pinch technique [26]. Subsequently, this formalism has been extended to the case of mixing between two intermediate resonant states in scattering processes [24,25]. Here, we develop a related formalism for decays, which can effectively take into account phenomena of mixing of states during the decay of particles.

Consequently, our main interest in this paper will be to study the ε - and ε' -type mechanisms of CP violation in some detail, within the framework of an effective field-theoretic formalism devised for decay amplitudes. This formalism consistently describes the phenomenon of resonantly enhanced CP violation through the mixing of nearly degenerate heavy neutrinos and can therefore be applied to any analogous system responsible for baryogenesis, in which CP violation is of the ε type. The analytic results obtained for ε -type CP violation with our field-theoretic approach do not display singularities and exhibit a physically correct analytic behavior in transition amplitudes. The fact that resonant CP violation through mixing may be of order one [24,25] can lead to scenarios in which the heavy Majorana neutrinos are relatively light with masses as low as 1 TeV. This is not far above the mass scale at which the electroweak phase transition occurs. Most interestingly, this mechanism can produce significant ε -type CP violation, even if all Yukawa couplings are of the same magnitude and the Majorana masses are of few TeV. This novel phenomenological consequence of resonant CP violation has not yet been studied in this leptogenesis scenario. Furthermore, it is worth investigating the influence of other possible phenomena on this resonant CP -violating mechanism, such as low-energy constraints due to the electric dipole moment (EDM) of electron or finite-temperature effects.

The paper is organized as follows. In Sec. II, we describe

minimally extended models that include heavy Majorana neutrinos. At low energies, these models amount to adding isosinglet neutrino states to the field content of an effective one-Higgs-doublet model. Such scenarios may be embedded into certain SO(10) and/or E_6 unified theories, which can naturally predict nearly degenerate heavy Majorana neutrinos as light as 100 GeV. Moreover, we discuss the renormalizability of the effective model. In Sec. III, we present a resummation formalism for resonant transitions between fermions in decay amplitudes. Furthermore, we illustrate some of the advantages of this approach when compared to other existing methods. Making use of our formalism, we calculate the analytic expressions of the relevant transition amplitudes in Sec. IV. In Sec. V, we give estimates of possible constraints coming from low-energy data, such as the EDM of electron, which turn out to be quite weak in order to rule out our leptogenesis scenario. In Sec. VI, we present the Boltzmann equations relevant for the evolution of the leptonic asymmetry in the effective model, and give numerical estimates and comparisons for the BAU generated via ε - and/or ε' -type CP violation. We also discuss the implications of finite temperature effects for the resonant phenomenon of CP violation and find that such a phenomenon can still be viable. We draw our conclusions in Sec. VII.

II. HEAVY MAJORANA NEUTRINO MODELS

Heavy Majorana neutrinos may naturally be realized in certain GUT's, such as SO(10) [27,28] and/or E_6 [29] models. Nevertheless, these models will also predict several other particles, e.g., leptiquarks, additional charged and neutral gauge bosons (W_R^\pm and Z_R), which may deplete the number density of heavy neutrinos, N_i , through processes of the type $N_i \bar{e}_R \rightarrow W^{+\ast} \rightarrow u_R \bar{d}_R$ and so render the whole analysis very involved. If these particles are sufficiently heavier than the lightest heavy Majorana neutrino and/or the temperature of the Universe [12], this problem may be completely avoided. Since we wish to simplify our analysis without sacrificing any of the essential features involved in the study of the BAU, we shall consider a minimal model with isosinglet neutrinos, which is invariant under the SM gauge group $SU(2)_L \otimes U(1)_Y$. Then, we will present the relevant Lagrangians that govern the interactions of the heavy Majorana neutrinos with the Higgs fields and the ordinary leptons. Also, we will identify the nontrivial CP-violating phases of the model and pay special attention to the one-loop renormalization of the Yukawa couplings.

As has been mentioned above, certain SO(10) [27,28] and/or E_6 [29] models naturally predict the existence of heavy Majorana neutrinos. In SO(10) models, an attractive breaking pattern down to the SM may be given schematically in the following way:

$$\begin{aligned} \text{SO}(10) &\rightarrow G_{422} = SU(4)_{\text{PS}} \otimes SU(2)_R \otimes SU(2)_L \\ &\rightarrow G_{3221} = SU(3)_c \otimes SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)} \\ &\rightarrow \text{SM} = G_{321} = SU(3) \otimes SU(2)_L \otimes U(1)_Y, \end{aligned} \quad (2.1)$$

where the subscript ‘‘PS’’ refers to the Pati-Salam gauge group [30]. The spinor representation of SO(10) is 16 dimensional and its decomposition under G_{422} is given by

$$G_{422}: \mathbf{16} \rightarrow (4,1,2) \oplus (\bar{4},2,1). \quad (2.2)$$

As can be seen from Eq. (2.2), it is evident that SO(10) can accommodate right-handed neutrinos, since it contains the left-right symmetric gauge group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)}$. There are several Higgs-boson representations that can give rise to the breakdown of G_{422} and G_{3221} down to the SM gauge group G_{321} [28,31]. In E_6 models [29], the **27** spinor representation decomposes into $\mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1}$ under SO(10), which leads to four singlet neutrinos per SM family: one neutrino as isodoublet member in **16**, two neutrinos as isodoublet members in **10**, and one singlet neutrino in **1**. In these models, two of the four isosinglets can have Majorana masses of few TeV [29], depending on the representation of the E_6 Higgs multiplets, whereas the other two are very heavy with masses of the order of the unification scale. Possible flavor structures for the isosinglet neutrino mass matrix resulting from the above two representative unified models will be discussed below.

The minimal model under consideration extends the SM field content of the three lepton and quark families by adding a number n_R right-handed neutrinos ν_{Ri} , with $i=1,2,\dots,n_R$. Even though in E_6 models the active isosinglet neutrinos may be more than three, in the SO(10) models mentioned above the symmetric case of having one right-handed neutrino per family turns out to be quite natural, i.e., $n_R=3$. Therefore, we shall not specify the number of ν_{Ri} in the following. To be specific, the leptonic sector of our minimal model consists of the fields

$$\begin{pmatrix} \nu_{iL} \\ l_L \end{pmatrix}, \quad l_R, \quad \nu_{Ri},$$

with $l=e,\mu,\tau$. Since for temperatures $T \gg T_c \gtrsim v$, one has $v(T)=0$, where $v(T)$ is the vacuum expectation value (VEV) of the SM Higgs doublet Φ at temperature T [with $v=v(0)$], the only admissible mass terms are those of the Majorana type and are given by the Lagrangian

$$-\mathcal{L}_M = \frac{1}{2} \sum_{i,j=1}^{n_R} (\bar{\nu}_{Ri}^C M_{ij}^v \nu_{Rj} + \bar{\nu}_{Ri} M_{ij}^{v\ast} \nu_{Rj}^C). \quad (2.3)$$

Here, the superscript C denotes the operation of charge conjugation, which acts on the four-component chiral spinors ψ_L and ψ_R as $(\psi_L)^C = P_R C \bar{\psi}^T$ and $(\psi_R)^C = P_L C \bar{\psi}^T$, where $P_{L(R)} = [1 - (+)\gamma_5]/2$ is the chirality projection operator. In Eq. (2.3), M^v is a $n_R \times n_R$ dimensional symmetric matrix, which is in general complex. The isosinglet mass matrix M^v can be diagonalized by the unitary transformation $U^T M^v U = \hat{M}^v$, where U is a $n_R \times n_R$ dimensional unitary matrix and \hat{M}^v is a positive diagonal matrix containing the n_R heavy Majorana masses. Furthermore, the n_R mass eigenstates N_i are related to the flavor states ν_{Ri} through $\nu_{Ri} = P_R \sum_{j=1}^{n_R} U_{ij} N_j$. In the basis, in which the isosinglet neutrino mass matrix is diagonal and equals \hat{M}^v , the Yukawa sector describing the interaction of the heavy neutrinos with the Higgs doublet and the ordinary leptons reads

$$\mathcal{L}_Y = - \sum_{l=1}^{n_L} \sum_{j=1}^{n_R} h_{lj} (\bar{\nu}_{lL}, \bar{l}_L) \begin{pmatrix} (H - i\chi^0)/\sqrt{2} \\ -\chi^- \end{pmatrix} N_j + \text{H.c.} \quad (2.4)$$

The CP -even Higgs field H , the CP -odd Higgs scalar χ^0 , and the charged Higgs scalars χ^\pm given in Eq. (2.4) are all massless at high temperatures. In the limit $T \rightarrow 0$, H represents the massive SM Higgs boson, whereas χ^0 and χ^\pm are the massless would-be Goldstone bosons eaten by the longitudinal degrees of freedom of the gauge bosons Z and W^\pm , respectively. If the SM Higgs field H is very heavy with mass of order 1 TeV at $T=0$ and also close to the mass of the decaying heavy neutrinos, then mass Higgs effects may not be negligible. Therefore, we shall initially keep the full M_H dependence in our calculations and then present analytic results for the limiting case $M_H=0$.

Before counting all the nontrivial CP -violating phases for the most general case, it may be more instructive to discuss first some simple models that could predict degenerate or almost degenerate heavy Majorana neutrinos and CP violation. To this end, we consider a minimal scenario in which the fermionic matter of the SM is extended by adding two right-handed neutrinos per family, e.g., ν_{lR} and $(S_{lL})^C$ with $l=e, \mu, \tau$. Such a scenario may be derived from certain $SO(10)$ [28] and/or E_6 [29] models. For our illustrations, we neglect possible interfamily mixings. Imposing lepton-number conservation on the model gives rise to the Lagrangian

$$-\mathcal{L} = \frac{1}{2} (\bar{S}_L, (\bar{\nu}_R)^C) \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \begin{pmatrix} (S_L)^C \\ \nu_R \end{pmatrix} + h_R (\bar{\nu}_L, \bar{l}_L) \tilde{\Phi} \nu_R + \text{H.c.}, \quad (2.5)$$

where $\tilde{\Phi} = i\sigma_2 \Phi$ is the isospin conjugate Higgs doublet and σ_2 is the usual Pauli matrix. Even though the mass and coupling parameters may be complex in such a scenario, the phase redefinitions of the fields,

$$\nu_L \rightarrow e^{i\phi_\nu} \nu_L, \quad l_L \rightarrow e^{i\phi_l} l_L, \quad \nu_R \rightarrow e^{i\phi_R} \nu_R, \quad S_L \rightarrow e^{i\phi_S} S_L, \quad (2.6)$$

can, however, make them all real. This model is CP invariant, unless one allows for a nontrivial mixing among generations [32]. Moreover, the model preserves the lepton number and hence cannot produce any excess in L through heavy neutrino decays.

There are two equivalent ways to break the L and CP invariance of the Lagrangian in Eq. (2.5): One has to either (i) introduce two complex L -violating mass terms of the kind $\mu_R \bar{\nu}_R \nu_R^C$ and $\mu_L \bar{S}_L^C S_L$, where both μ_R and μ_L are complex, or equivalently (ii) add the L -violating coupling $h_R (\bar{\nu}_L, \bar{l}_L) \tilde{\Phi} (S_L)^C$ and include the L -violating mass parameter, e.g., $\mu_R \bar{\nu}_R \nu_R^C$. These are the minimal enlargements that can assure L and CP violation on the same footing in this simple two-isosinglet neutrino model. In fact, the necessary conditions for CP invariance in these two scenarios are written down:

$$(i) \quad |h_R|^2 \text{Im}(M^{*2} \mu_L \mu_R) = 0,$$

$$(ii) \quad \text{Im}(h_L h_R^* \mu_R M^*) = 0. \quad (2.7)$$

It can easily be checked that the two equalities in Eq. (2.7) are invariant under the phase redefinitions of the fields given in Eq. (2.6). We must remark that the L -breaking parameters μ_L and μ_R are generally much smaller than M within E_6 scenarios. The origin of these parameters is usually due to residual effects of high-dimensional operators involving superheavy neutrinos [29]. The typical size of the L -violating parameters is $\mu_L, \mu_R \sim M^2/M_{\text{Planck}}$, M^2/M_X or M^2/M_S , where $M_S \approx 10^{-3} M_X$ is some intermediate seesaw scale. As a consequence, such effective minimal models derived from E_6 theories can naturally predict small mass splittings for the heavy neutrinos N_1 and N_2 . To a good approximation, this small mass difference may be determined from the parameter $x_N = m_{N_2}/m_{N_1} - 1 \sim \mu_L/M$ or μ_R/M . For instance, if $M = 10$ TeV and $\mu_L = \mu_R = M^2/M_X$, one then finds $x_N \approx 10^{-12} - 10^{-11}$. As we will see in Sec. IV, these small values for the mass difference x_N can produce large CP asymmetries in the heavy neutrino decays.

In order to deduce the sufficient and necessary conditions for the most general structure of the two right-handed neutrino model, one must consider the systematic approach presented first in [33], which is slightly different from the procedure outlined above. In this approach [32,33], one looks for all possible weak-basis independent combinations that can be formed by Yukawa couplings and the neutrino mass matrix M^ν , and are simultaneously invariant under generalized CP transformations of the fields. These generalized CP transformations of the fermion fields may include unitary flavor rotations, apart from the phase redefinitions mentioned above. Further details may be found in Ref. [32]. Thus, for the model at hand, we find that the sufficient and necessary condition for CP invariance is

$$\text{Im Tr}(h^\dagger h M^{\nu\dagger} M^\nu M^{\nu\dagger} h^T h^* M^\nu) = m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2) \text{Im}(h_{11} h_{12}^*)^2 = 0. \quad (2.8)$$

The (1×2) -dimensional matrix h in Eq. (2.8) contains the Higgs Yukawa couplings, which are defined as h_{lj} in Eq. (2.4), i.e., in the physical mass basis where $M^\nu = \hat{M}^\nu$. One can show that Eq. (2.8) is consistent with the conditions in Eq. (2.7) for the two special cases discussed above. From Eq. (2.8), one readily sees that only one physical CP -violating combination is possible in this minimal model and CP invariance is restored if $m_{N_1} = m_{N_2}$ provided none of the isosinglet neutrinos is massless. The above considerations may be extended to models with more than two right-handed neutrinos and more than one lepton families. In this case, there may be more conditions analogous to Eq. (2.8), which involve high-order terms in the Yukawa-coupling matrix h . However, not all of the conditions are sufficient and necessary for CP invariance. Instead of undertaking the rather difficult task to derive all CP -invariant conditions, we note in passing that the total number \mathcal{N}_{CP} of all nontrivial CP -violating phases in a model with n_L weak isodoublets and n_R neutral isosinglets is $\mathcal{N}_{CP} = n_L(n_R - 1)$ [34].

Since CP violation in N_i decays will necessitate nonzero one-loop absorptive parts of vertex and N_i self-energy graphs

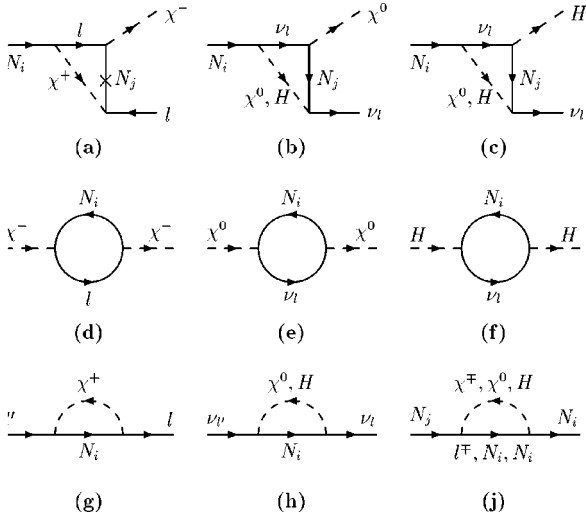


FIG. 1. One-loop graphs contributing to the renormalization of the couplings $\chi^- l N_i$, $\chi^0 \nu_l N_i$, and $H \nu_l N_i$.

as will be seen in Sec. IV, one should also have to address the issue of renormalization of the dispersive counterparts (see also Fig. 1). In order to check explicitly that our minimal model leads indeed to consistent renormalizable results, we shall adopt the following strategy in our analysis. First, we fix the renormalization of all Higgs Yukawa couplings h_{lj} from the decay mode $N_i \rightarrow l^+ \chi^-$. In this way, we determine the counterterms (CT's) of h_{lj} and δh_{lj} . Then, we show that all ultraviolet (UV) divergences cancel in the partial decays $N_i \rightarrow \nu_l \chi^0$ and $N_i \rightarrow \nu_l H$. For this purpose, we first express all bare quantities in terms of renormalized ones as follows:

$$\begin{aligned}
 \nu_{iL}^0 &= \sum_{l'=1}^{n_L} \left(\delta_{ll'} + \frac{1}{2} \delta Z_{ll'}^{\nu} \right) \nu_{l'L}, \\
 l_L^0 &= \sum_{l'=1}^{n_L} \left(\delta_{ll'} + \frac{1}{2} \delta Z_{ll'}^l \right) l_{l'}^L, \\
 N_i^0 &= \sum_{j=1}^{n_R} \left(\delta_{ij} + \frac{1}{2} \delta Z_{ij}^N \right) N_j, \quad \tilde{\Phi} = \left(1 + \frac{1}{2} \delta Z_{\Phi} \right) \tilde{\Phi}, \\
 h_{ij}^0 &= h_{ij} + \delta h_{ij}. \tag{2.9}
 \end{aligned}$$

The superscript ‘‘0’’ in Eq. (2.9) indicates that the field or coupling parameter is unrenormalized, whereas quantities without this superscript are considered to be renormalized. In addition, the CT δZ_{Φ} collectively denotes the wave-function renormalization constants of all components of the Higgs doublet $\tilde{\Phi}$ (or Φ), i.e., the fields χ^{\pm} , χ^0 , and H . The divergent part of all the Higgs wave-function renormalizations, $\delta Z_{\Phi}^{\text{div}}$, has been found to be universal. Expressions showing the universality of $\delta Z_{\Phi}^{\text{div}}$ together with other relevant one-loop analytic results are relegated to the Appendix. Taking

the relations in Eq. (2.9) into account, we find that the renormalized Lagrangian in Eq. (2.4) gets shifted by an amount

$$\begin{aligned}
 -\delta \mathcal{L}_Y &= \frac{1}{2} \sum_{l=1}^{n_L} \sum_{j=1}^{n_R} \left(2 \frac{\delta h_{lj}}{h_{lj}} + \delta Z_{\Phi} + \sum_{l'=1}^{n_L} \delta Z_{l'l}^L \right. \\
 &\quad \left. + \sum_{k=1}^{n_R} \delta Z_{jk}^N \right) \bar{L}_l \tilde{\Phi} N_k + \text{H.c.}, \tag{2.10}
 \end{aligned}$$

where $L_l = (\nu_{lL}, l_L)^T$ and $\delta Z^L = (\delta Z^l, \delta Z^{\nu})$. From Fig. 1(a), it is easy to see that the one-loop correction to the coupling $\chi^+ N l$ can only occur via a $\Delta L = 2$ Majorana mass insertion, owing to charge conservation on the vertices. Naive power counting may then convince oneself that the one-loop irreducible vertex $\chi^+ N l$ is UV finite. Less obvious is the UV finiteness for the proper couplings $\chi^0 N \nu$ and $H N \nu$, which is shown in the Appendix.

As has been discussed above, we shall now determine the Yukawa coupling CT's δh_{lj} from the renormalization of the coupling $\chi^+ N l$. Requiring that all UV terms be absorbed into the definition of h_{lj} , we obtain

$$\delta h_{lj} = -\frac{1}{2} \left(h_{lj} \delta Z_{\chi^-} + \sum_{l'=1}^{n_L} h_{l'l} \delta Z_{l'l}^{*} + \sum_{k=1}^{n_R} h_{lk} \delta Z_{kj}^N \right). \tag{2.11}$$

We observe that δh_{lj} may be separated into two terms: The wave-function term δZ_{χ^-} , which is flavor independent, and the rest, which depends on the flavor and the wave-function CT's $\delta Z_{l'l}^l$ and δZ_{ij}^N . If we had renormalized the Higgs Yukawa couplings from the decays $N \rightarrow \nu H$, the only difference in Eq. (2.11) would have been the appearance of the CT's δZ_H and $\delta Z_{l'l}^{\nu}$ in place of δZ_{χ^-} and $\delta Z_{l'l}^l$, respectively. Also, for the decay $N \rightarrow \nu \chi^0$, one has to include the wave-function renormalization of χ^0 , δZ_{χ^0} , instead of δZ_H in the decay $N \rightarrow \nu H$. Therefore, consistency of Yukawa coupling renormalization requires that differences of the kind $\delta Z_{l'l}^l - \delta Z_{l'l}^{\nu}$, $\delta Z_{\chi^0} - \delta Z_H$ and $\delta Z_{\chi^-} - \delta Z_H$ must be UV safe. In the Appendix, it is shown that all these CT differences are indeed UV finite and vanish in the limit of $M_H \rightarrow 0$. This completes our discussion concerning the one-loop renormalization of heavy Majorana neutrino decays. In the next section, we shall explicitly demonstrate how the renormalization presented here gets implemented within our resummation formalism for unstable particle mixing.

III. RESUMMATION APPROACH FOR TWO-FERMION MIXING

If a Lagrangian contains unstable particles, then these fields cannot be described by free plane waves at times $t \rightarrow \pm \infty$ and hence cannot formally appear as asymptotic states in the conventional perturbation field theory. Within a simple scalar theory with one unstable particle, Veltman [35] showed that, even if one removes the unstable particle from the initial and final states and substitutes it in terms of

asymptotic states, the so-truncated S -matrix theory will still maintain the field-theoretic properties of unitarity and causality. Our main concern in this section will be to present an approach to decay amplitudes that describes the dynamics of unstable particle mixing. Hence, such a formulation, finite width effects in the mixing and decay of nonasymptotic states must be taken into account. This will be done in an effective manner, such that the decay amplitude derived with this method can be embedded in an equivalent form to a transition element [26,25] in agreement with Veltman's S -matrix approach. This effective field-theoretic approach is equivalent to that of the decay of an initial pure state, such as the state K^0 or \bar{K}^0 , which is initially produced by some asymptotic states in kaon experiments, e.g., in $p\pi^-$ or $p\bar{p}$ collisions [17]. Since the time evolution of the decaying system is effectively integrated out over all times, the resummed decay amplitudes derived with this field-theoretic method will not display any explicit time dependence.

The discussion in this section is organized as follows. First, we briefly review the theoretical description of the mixing between stable particles in a simple scalar theory within the framework of the Lehmann-Symanzik-Zimmermann (LSZ) formalism [36]. After gaining some insight, we extend our considerations to the mixing between two unstable scalars. The effective field-theoretic method developed for the scalar case can then carry over to the case of mixing of two unstable fermions, with the help of which ε -type CP violation will be calculated in Sec. IV.

Let us now consider a field theory with N real scalars S_i^0 , with $i=1,2,\dots,N$. We shall assume that the scalars are stable to a good approximation and neglect possible finite width effects of the particles. The bare (unrenormalized) fields S_i^0 and their respective masses M_i^0 may then be expressed in terms of renormalized fields S_i and masses M_i in the following way:

$$S_i^0 = Z_{ij}^{1/2} S_j = \left(\delta_{ij} + \frac{1}{2} \delta Z_{ij} \right) S_j, \quad (3.1)$$

$$(M_i^0)^2 = M_i^2 + \delta M_i^2. \quad (3.2)$$

Here and in the following, summation is understood over repeated indices that do not appear on both sides of an equation. In Eqs. (3.1) and (3.2), $Z_{ij}^{1/2}$ and δM_i are the wavefunction and mass renormalization constants, respectively, which can be determined from renormalization conditions imposed on the two-point correlation functions $\Pi_{ij}(p^2)$ for the transitions $S_j \rightarrow S_i$ in some physical scheme, such as the on-mass-shell (OS) renormalization scheme [37].

It will prove useful for the discussion that follows to give the relation of the pole parts between the unrenormalized

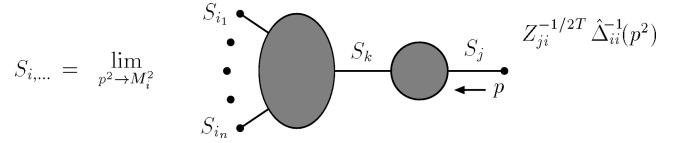


FIG. 2. Diagrammatic representation of the renormalized $(n-1)$ -nonamputated amplitude, $S_{i,\dots}$, and the LSZ reduction formalism.

scalar propagators $\Delta_{ij}(p^2)$ and the renormalized ones $\hat{\Delta}_{ij}(p^2)$. The two pole parts are related through [37]

$$\Delta_{ij}(p^2)|_{p^2 \rightarrow M_i^2, M_j^2} = Z_{im}^{1/2} \frac{\delta_{mn}}{p^2 - M_n^2} Z_{nj}^{1/2T}. \quad (3.3)$$

Using the LSZ reduction formalism shown schematically in Fig. 2, one can deduce the renormalized $(n-1)$ -nonamputated amplitude $S_{i,\dots}$, for a fixed given external line i , from the corresponding unrenormalized n -point Green function $G_{i,\dots}$, where n is the total number of external lines. In this way, we have

$$\begin{aligned} S_{i,\dots} &= \lim_{p^2 \rightarrow M_i^2} G_{j,\dots} Z_{ji}^{-1/2T} (p^2 - M_i^2) \\ &= \lim_{p^2 \rightarrow M_i^2} \mathcal{T}_{k,\dots}^{\text{amp}} Z_{km}^{1/2} \frac{\delta_{mn}}{p^2 - M_n^2} Z_{nj}^{1/2T} Z_{ji}^{-1/2T} (p^2 - M_i^2) \\ &= \lim_{p^2 \rightarrow M_i^2} \mathcal{T}_{k,\dots}^{\text{amp}} Z_{ki}^{1/2}, \end{aligned} \quad (3.4)$$

where $\mathcal{T}_{k,\dots}^{\text{amp}}$ denotes the amplitude amputated at the k external leg. Clearly, the LSZ reduction procedure outlined above can be generalized to all external legs, thus leading to the physical (renormalized) S -matrix element $S_{i_1 \dots i_n}$, which governs the transition amplitude of n asymptotic states.

Let us now consider the mixing of two neutral unstable scalars [24,25], e.g., S_1 and S_2 . Since we are interested in studying the width effects of these particles, we have first to calculate all the $S_i S_j$ Green functions, with $i, j=1,2$. After summing up a geometric series of the self-energies $\Pi_{ij}(p^2)$, the full propagators may be obtained by inverting the following inverse propagator matrix:

$$\Delta_{ij}^{-1}(p^2) = \begin{bmatrix} p^2 - (M_1^0)^2 + \Pi_{11}(p^2) & \Pi_{12}(p^2) \\ \Pi_{21}(p^2) & p^2 - (M_2^0)^2 + \Pi_{22}(p^2) \end{bmatrix}. \quad (3.5)$$

The result of inverting the matrix in Eq. (3.5) may be given by

$$\Delta_{11}(p^2) = \left[p^2 - (M_1^0)^2 + \Pi_{11}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_2^0)^2 + \Pi_{22}(p^2)} \right]^{-1}, \quad (3.6)$$

$$\Delta_{22}(p^2) = \left[p^2 - (M_2^0)^2 + \Pi_{22}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_1^0)^2 + \Pi_{11}(p^2)} \right]^{-1}, \quad (3.7)$$

$$\Delta_{12}(p^2) = \Delta_{21}(p^2) = -\Pi_{12}(s) \{ [p^2 - (M_2^0)^2 + \Pi_{22}(p^2)] \times [p^2 - (M_1^0)^2 + \Pi_{11}(p^2)] - \Pi_{12}^2(p^2) \}^{-1}, \quad (3.8)$$

where $\Pi_{12}(p^2) = \Pi_{21}(p^2)$. Moreover, we find the useful factorization property for the off-diagonal ($i \neq j$) resummed scalar propagators

$$\begin{aligned} \Delta_{ij}(p^2) &= -\Delta_{ii}(p^2) \frac{\Pi_{ij}(p^2)}{p^2 - (M_j^0)^2 + \Pi_{jj}(p^2)} \\ &= -\frac{\Pi_{ij}(p^2)}{p^2 - (M_i^0)^2 + \Pi_{ii}(p^2)} \Delta_{jj}(p^2). \end{aligned} \quad (3.9)$$

The resummed unrenormalized scalar propagators $\Delta_{ij}(p^2)$ are related to the respective renormalized ones $\hat{\Delta}_{ij}(p^2)$ through the expression

$$\Delta_{ij}(p^2) = Z_{im}^{1/2} \hat{\Delta}_{mn}(p^2) Z_{nj}^{1/2T}, \quad (3.10)$$

where $\hat{\Delta}_{ij}(p^2)$ may be obtained from Eqs. (3.6)–(3.8), just by replacing M_i^0 with M_i and $\Pi_{ij}(p^2)$ with $\hat{\Pi}_{ij}(p^2)$. Note that the property given in Eq. (3.9) will also hold true for the renormalized scalar propagators $\hat{\Delta}_{ij}(p^2)$. Taking expressions (3.9) and (3.10) into account, we can derive the resummed and renormalized transition amplitude, denoted here as $\hat{S}_{i,\dots}$, for the external leg i which now represents an unstable particle. This can be accomplished in a way analogous to Eq. (3.4): viz.,

$$\begin{aligned} \hat{S}_{i,\dots} &= \lim_{p^2 \rightarrow M_i^2} \mathcal{T}_{k,\dots}^{\text{amp}} Z_{km}^{1/2} \hat{\Delta}_{mn}(p^2) Z_{nj}^{1/2T} Z_{ji}^{-1/2T} \hat{\Delta}_{ii}^{-1}(p^2) \\ &= \lim_{p^2 \rightarrow M_i^2} \left[\mathcal{T}_{k,\dots}^{\text{amp}} Z_{ki}^{1/2} - \mathcal{T}_{k,\dots}^{\text{amp}} Z_{km}^{1/2} \frac{\hat{\Pi}_{mi}(p^2)(1 - \delta_{mi})}{p^2 - M_m^2 + \hat{\Pi}_{mm}(p^2)} \right] \\ &= S_{i,\dots} - S_{j,\dots} \frac{\hat{\Pi}_{ji}(M_i^2)(1 - \delta_{ij})}{M_i^2 - M_j^2 + \hat{\Pi}_{jj}(M_i^2)}, \end{aligned} \quad (3.11)$$

where $S_{i,\dots}$ and $S_{j,\dots}$ are the renormalized transition elements evaluated from Eq. (3.4) in the stable-particle approximation. One should bear in mind that the OS-renormalized self-energies $\hat{\Pi}_{ji}(M_i^2)$ in Eq. (3.11) have no vanishing absorptive parts, as renormalization can only modify the dispersive (real) part of these self-energies. The reason is that the CT Lagrangian must be Hermitian as opposed to the absorptive parts which are anti-Hermitian. In fact, these additional width mixing effects are those which we wish to include in our formalism for decay amplitudes and are absent in the conventional perturbation theory. It is also important to observe that our approach to decays is not singular; i.e., $\hat{S}_{i,\dots}$ displays an analytic behavior in the degenerate limit $M_i^2 \rightarrow M_j^2$, because of the appearance of the imaginary term $i\text{Im}\hat{\Pi}_{jj}(M_i^2)$ in the denominator of the mixing factor present in the last equality of Eq. (3.11). Finally, we must stress that the inclusion of these phenomena has been performed in an effective manner. Since the decaying unstable particle cannot appear in the initial state [35], the resummed decay amplitude must be regarded as being a part which can effectively be embedded into a resummed S -matrix element [26]. This resummed S -matrix element describes the dynamics of the very same unstable particle, which is produced by some asymptotic states, resides in the intermediate state, and subsequently decays either directly or indirectly, through mixing, into the observed final states.

It is now straightforward to extend our considerations to the case of mixing between two unstable fermions. Following a line of arguments similar to those presented above, we consider a system with two unstable fermions, call them f_1 and f_2 . As usual, we express the bare left- and right-handed chiral fields f_{Li}^0 and f_{Ri}^0 (with $i=1,2$) in terms of renormalized fields as follows:

$$f_{Li}^0 = Z_{Lij}^{1/2} f_{Lj}, \quad f_{Ri}^0 = Z_{Rij}^{1/2} f_{Rj}, \quad (3.12)$$

where $Z_{Lij}^{1/2}$ ($Z_{Rij}^{1/2}$) is the wave-function renormalization constant for the left- (right-) handed chiral fields, which may be determined from the fermionic self-energy transitions $f_j \rightarrow f_i$, $\Sigma_{ij}(\not{p})$, e.g., in the OS renormalization scheme [38]. Analogously with Eq. (3.5), the resummed fermion propagator matrix may be obtained from

$$S_{ij}(\not{p}) = \left[\begin{array}{cc} \not{p} - m_1^0 + \Sigma_{11}(\not{p}) & \Sigma_{12}(\not{p}) \\ \Sigma_{21}(\not{p}) & \not{p} - m_2^0 + \Sigma_{22}(\not{p}) \end{array} \right]^{-1}, \quad (3.13)$$

where $m_{1,2}^0$ are the bare fermion masses, which can be decomposed into the OS-renormalized masses $m_{1,2}$ and the CT mass terms $\delta m_{1,2}$ as $m_{1,2}^0 = m_{1,2} + \delta m_{1,2}$. Inverting the matrix-valued 2×2 matrix in Eq. (3.13) yields

$$S_{11}(\not{p}) = \left[\begin{array}{c} \not{p} - m_1^0 + \Sigma_{11}(\not{p}) \\ -\Sigma_{12}(\not{p}) \frac{1}{\not{p} - m_2^0 + \Sigma_{22}(\not{p})} \Sigma_{21}(\not{p}) \end{array} \right]^{-1}, \quad (3.14)$$

$$S_{22}(\boldsymbol{p}) = \left[\begin{array}{c} \boldsymbol{p} - m_2^0 + \Sigma_{22}(\boldsymbol{p}) \\ -\Sigma_{21}(\boldsymbol{p}) \frac{1}{\boldsymbol{p} - m_1^0 + \Sigma_{11}(\boldsymbol{p})} \Sigma_{12}(\boldsymbol{p}) \end{array} \right]^{-1}, \quad (3.15)$$

$$\begin{aligned} S_{12}(\boldsymbol{p}) &= -S_{11}(\boldsymbol{p}) \Sigma_{12}(\boldsymbol{p}) [\boldsymbol{p} - m_2^0 + \Sigma_{22}(\boldsymbol{p})]^{-1} \\ &= -[\boldsymbol{p} - m_1^0 + \Sigma_{11}(\boldsymbol{p})]^{-1} \Sigma_{12}(\boldsymbol{p}) S_{22}(\boldsymbol{p}), \end{aligned} \quad (3.16)$$

$$\begin{aligned} S_{21}(\boldsymbol{p}) &= -S_{22}(\boldsymbol{p}) \Sigma_{21}(\boldsymbol{p}) [\boldsymbol{p} - m_1^0 + \Sigma_{11}(\boldsymbol{p})]^{-1} \\ &= -[\boldsymbol{p} - m_2^0 + \Sigma_{22}(\boldsymbol{p})]^{-1} \Sigma_{21}(\boldsymbol{p}) S_{11}(\boldsymbol{p}). \end{aligned} \quad (3.17)$$

From Eqs. (3.16) and (3.17), it is now easy to see that the resummed propagator matrix is endowed with a factorization property analogous to Eq. (3.9). There is also an analogous connection between the renormalized and unrenormalized resummed propagators, which may be cast into the form

$$S_{ij}(\boldsymbol{p}) = (Z_{Lim}^{1/2} P_L + Z_{Rim}^{1/2} P_R) \hat{S}_{mn}(\boldsymbol{p}) (Z_{Lnj}^{1/2\dagger} P_R + Z_{Rnj}^{1/2\dagger} P_L), \quad (3.18)$$

where the caret on $S_{ij}(\boldsymbol{p})$ refers to the fact that the resummed fermionic propagators have been OS renormalized. By analogy, the renormalized propagators $\hat{S}_{ij}(\boldsymbol{p})$ may be recovered from $S_{ij}(\boldsymbol{p})$ in Eqs. (3.14)–(3.17), if one makes the obvious replacements $m_i^0 \rightarrow m_i$ and $\Sigma_{ij}(\boldsymbol{p}) \rightarrow \hat{\Sigma}_{ij}(\boldsymbol{p})$.

Employing the LSZ reduction formalism, one can derive the resummed decay amplitude, $\hat{S}_{i\dots}$ of the unstable fermion $f_i \rightarrow X$, in a way similar to what has been done for the scalar case. More explicitly, we have

$$\begin{aligned} \hat{S}_{i\dots} u_i(p) &= \mathcal{T}_{k\dots}^{\text{amp}} (Z_{Lkm}^{1/2} P_L + Z_{Rkm}^{1/2} P_R) \hat{S}_{mn}(\boldsymbol{p}) (Z_{Lnj}^{1/2\dagger} P_R \\ &\quad + Z_{Rnj}^{1/2\dagger} P_L) (Z_{Lji}^{-1/2\dagger} P_R + Z_{Rji}^{-1/2\dagger} P_L) \hat{S}_{ii}^{-1}(\boldsymbol{p}) u_i(p) \\ &= S_{i\dots} u_i(p) - (1 - \delta_{ij}) S_{j\dots} \hat{\Sigma}_{ji}(\boldsymbol{p}) [\boldsymbol{p} - m_j \\ &\quad + \hat{\Sigma}_{jj}(\boldsymbol{p})]^{-1} u_i(p). \end{aligned} \quad (3.19)$$

Again, $S_{i\dots}$ represent the respective renormalized transition amplitudes evaluated in the stable-particle approximation. The amplitudes $S_{i\dots}$ also include all high-order n -point functions, such as vertex corrections. On the basis of the formalism presented here, we shall calculate the CP asymmetries in the decays of heavy Majorana neutrinos in Sec. IV.

Finally, we wish to offer a comment on other approaches, which are used to analyze the phenomenon of CP violation through particle mixing [21,16]. Recently, this issue has been studied in [25], within a S -matrix amplitude formalism related to the one discussed in this section. Obviously, the approach based on the diagonalization of the effective Hamiltonian [21,16] is very helpful to describe ε -type CP violation, if the effective Hamiltonian is diagonalizable

through a similarity transformation, as is the case for the known $K^0 \bar{K}^0$ system. However, if the effective Hamiltonian has mathematically the Jordan form, when expressed in a $K^0 \bar{K}^0$ -like basis, then it can be shown to be nondiagonalizable via a similarity transformation. In this case, the complex mass eigenvalues of the two mixed *non-free* particles are exactly equal and, most importantly, CP violation through particle mixing reaches its maximum attainable value [25].

To give a specific example, let us consider the following effective Hamiltonian for the mixing system of two nearly degenerate heavy neutrinos N_1 and N_2 :

$$\begin{aligned} \mathcal{H}(\boldsymbol{p}) &= \begin{bmatrix} m_1 - \hat{\Sigma}_{11}(\boldsymbol{p}) & -\hat{\Sigma}_{12}(\boldsymbol{p}) \\ -\hat{\Sigma}_{21}(\boldsymbol{p}) & m_2 - \hat{\Sigma}_{22}(\boldsymbol{p}) \end{bmatrix} \\ &\approx \begin{bmatrix} m_N + a - i|b| & -ib \\ -ib^* & m_N - a - i|b| \end{bmatrix}, \end{aligned} \quad (3.20)$$

in the approximation $\boldsymbol{p} \rightarrow m_N \approx m_1 \approx m_2$. In Eq. (3.20), the parameters a and b are real and complex, respectively, and $m_1 = m_N + a$, $m_2 = m_N - a$. The complex parameter b represents the absorptive part of the one-loop neutrino transitions $N_i \rightarrow N_j$. Unitarity requires that the determinant of the absorptive part of $\mathcal{H}(\boldsymbol{p})$ be non-negative. For the effective Hamiltonian (3.20), the corresponding determinant is zero. Such an absorptive effective Hamiltonian naturally arises in the one-lepton doublet model with two right-handed neutrinos. In the limit $a \rightarrow |b|$, the two complex mass eigenvalues of $\mathcal{H}(\boldsymbol{p})$ are exactly degenerate and equal to $m_N - i|b|$. As has been shown in [25], the effective Hamiltonian cannot be diagonalized by a nonunitary similarity transformation in this limit; i.e., the respective diagonalization matrices become singular.

Since our effective field-theoretic method does not involve diagonalization of the effective Hamiltonian through a similarity transformation, such singular situations are completely avoided, leading to well-defined analytic expressions for the decay amplitudes. Furthermore, whenever referring to heavy Majorana neutrino masses in the following, we shall always imply the OS-renormalized masses within the conventional perturbation field theory, which differ from the real parts of the complex mass eigenvalues of the effective Hamiltonian. In the presence of a large particle mixing, the corresponding eigenstates of the latter are generally nonunitary among themselves, whereas the eigenvectors of the former form a well-defined unitary basis, upon which perturbation theory is based. In this context, it is important to note that these field-theoretic OS-renormalized masses are those that enter the condition of CP invariance given in Eq. (2.8). Therefore, we can conclude that, if the two complex mass eigenvalues of the effective Hamiltonian for the mixed heavy Majorana neutrinos are equal, this does not necessarily entail an equality between their respective OS-renormalized masses and, hence, absence of CP violation as well [25].

IV. CP ASYMMETRIES

It is now interesting to see how the resummation formalism presented in Sec. III is applied to describe ε -type CP violation in heavy Majorana neutrino decays. The same formalism can be used for the inverse decays, which occur in

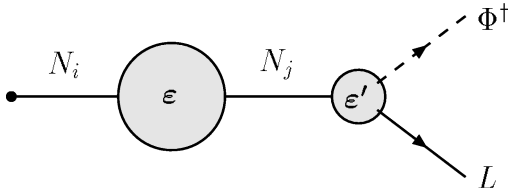


FIG. 3. ε - and ε' -type CP violation in the decays of heavy Majorana neutrinos.

the formulation of the Boltzmann equations (see also Sec. VI). For completeness, we shall include ε' -type CP violation in our analysis, which originates entirely from the one-loop $\Phi l N$ irreducible vertex, and display plots with numerical comparisons between the two kinds of CP -violating contributions mentioned above. Moreover, we wish to address briefly the issue pertaining to the problem of flavor-basis invariance of the CP asymmetries.

Let us consider the decay $N_1 \rightarrow l^- \chi^+$ in a model with two-right handed neutrinos, shown in Fig. 3. The inclusion of all other decay modes will then be straightforward. To make our resummation formalism more explicit, we shall first write down the transition amplitude responsible for ε -type CP violation, denoted as $\mathcal{T}_{N_1}^{(\varepsilon)}$, and then take CP -violating vertex corrections into account. Applying Eq. (3.19) to the heavy neutrino decays, we have

$$\mathcal{T}_{N_1}^{(\varepsilon)} = h_{l1} \bar{u}_l P_R u_{N_1} - i h_{l2} \bar{u}_l P_R [\not{p} - m_{N_2} + i \Sigma_{22}^{\text{abs}}(\not{p})]^{-1} \Sigma_{21}^{\text{abs}}(\not{p}) u_{N_1}. \quad (4.1)$$

In Eq. (4.1), the absorptive part of the one-loop transitions $N_j \rightarrow N_i$, with $i, j = 1, 2$, has the general form

$$\Sigma_{ij}^{\text{abs}}(\not{p}) = A_{ij}(p^2) \not{p} P_L + A_{ij}^*(p^2) \not{p} P_R, \quad (4.2)$$

where

$$A_{ij}(p^2) = \frac{h_{l'i} h_{l'j}^*}{32\pi} \left[\frac{3}{2} + \frac{1}{2} \left(1 - \frac{M_H^2}{p^2} \right)^2 \right]. \quad (4.3)$$

In the limit $M_H \rightarrow 0$, one finds the known result [15,16] $A_{ij} = h_{l'i} h_{l'j}^* / (16\pi)$. On the other hand, the CP -transform resummed amplitude describing the decay $N_1 \rightarrow l^+ \chi^-$, $\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}$, is written down:

$$\begin{aligned} \bar{\mathcal{T}}_{N_1}^{(\varepsilon)} &= h_{l1}^* \bar{v}_{N_1} P_L v_l - i h_{l2} \bar{v}_{N_1} \Sigma_{12}^{\text{abs}}(-\not{p}) [-\not{p} - m_{N_2} \\ &\quad + i \Sigma_{22}^{\text{abs}}(-\not{p})]^{-1} P_L v_l \\ &= h_{l1}^* \bar{u}_l P_L u_{N_1} - i h_{l2}^* \bar{u}_l P_L [\not{p} - m_{N_2} \\ &\quad + i \bar{\Sigma}_{22}^{\text{abs}}(\not{p})]^{-1} \bar{\Sigma}_{21}^{\text{abs}}(\not{p}) u_{N_1}, \end{aligned} \quad (4.4)$$

where the charge-conjugate absorptive self-energy is given by

$$\bar{\Sigma}_{ij}^{\text{abs}}(\not{p}) = A_{ij}(p^2) \not{p} P_R + A_{ij}^*(p^2) \not{p} P_L. \quad (4.5)$$

In deriving the last step of Eq. (4.4), we have made use of the known identities $u(p, s) = C \bar{v}^T(p, s)$ and $C \gamma_\mu C^{-1} = -\gamma_\mu^T$. The expressions in Eqs. (4.1) and (4.4) may be simplified even further if the Dirac equation of motion is employed for the external spinors. Then, the two resummed decay amplitudes $\mathcal{T}_{N_1}^{(\varepsilon)}$ and $\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}$ take the simple form

$$\mathcal{T}_{N_1}^{(\varepsilon)} = \bar{u}_l P_R u_{N_1} \left[h_{l1} - i h_{l2} \frac{m_{N_1}^2 (1 + i A_{22}) A_{21}^* + m_{N_1} m_{N_2} A_{21}}{m_{N_1}^2 (1 + i A_{22})^2 - m_{N_2}^2} \right], \quad (4.6)$$

$$\bar{\mathcal{T}}_{N_1}^{(\varepsilon)} = \bar{u}_l P_L u_{N_1} \left[h_{l1}^* - i h_{l2}^* \frac{m_{N_1}^2 (1 + i A_{22}) A_{21} + m_{N_1} m_{N_2} A_{21}^*}{m_{N_1}^2 (1 + i A_{22})^2 - m_{N_2}^2} \right]. \quad (4.7)$$

The two CP -conjugate matrix elements differ from one another in having complex conjugate Yukawa couplings to each other and scalar currents with opposite chirality. Furthermore, the respective transition amplitudes involving the decays $N_2 \rightarrow l^- \chi^+$, $\mathcal{T}_{N_2}^{(\varepsilon)}$, and $N_2 \rightarrow l^+ \chi^-$, $\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}$, may be obtained from Eqs. (4.6) and (4.7), just by interchanging the indices ‘‘1’’ and ‘‘2’’ everywhere in the above two formulas.

We shall now focus our attention on studying the ε - and ε' -type mechanisms of CP violation in heavy Majorana neutrino decays. For this purpose, we define the following CP -violating parameters:

$$\varepsilon_{N_i} = \frac{|\mathcal{T}_{N_i}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_i}^{(\varepsilon)}|^2}{|\mathcal{T}_{N_i}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_i}^{(\varepsilon)}|^2} \quad \text{for } i = 1, 2, \quad (4.8)$$

$$\varepsilon_N = \frac{|\mathcal{T}_{N_1}^{(\varepsilon)}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}|^2}{|\mathcal{T}_{N_1}^{(\varepsilon)}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}|^2}. \quad (4.9)$$

Correspondingly, one can define the CP -violating parameters ε'_{N_i} and ε'_N , which may quantify CP violation coming exclusively from the one-loop irreducible vertices. In Eqs. (4.8) and (4.9), the parameters ε_{N_i} and ε_N share the common property that do not depend on the final state that N_i decays, despite the fact that the individual squared matrix elements do. In general, both ε - and ε' -type contributions may not be directly distinguishable in the decay widths $\Gamma(N_i \rightarrow l^\pm \chi^\pm)$, unless one has $\varepsilon_{N_i} \gg \varepsilon'_{N_i}$ and vice versa, for some range of the kinematic parameters. Evidently, the physical CP -violating observables we are mainly interested in are

$$\delta_{N_i} = \frac{\Gamma(N_i \rightarrow L \Phi^\dagger) - \Gamma(N_i \rightarrow L^C \Phi)}{\Gamma(N_i \rightarrow L \Phi^\dagger) + \Gamma(N_i \rightarrow L^C \Phi)} \quad \text{for } i = 1, 2, \quad (4.10)$$

$$\delta_N = \frac{\sum_{i=1}^2 \Gamma(N_i \rightarrow L \Phi^\dagger) - \sum_{i=1}^2 \Gamma(N_i \rightarrow L^C \Phi)}{\sum_{i=1}^2 \Gamma(N_i \rightarrow L \Phi^\dagger) + \sum_{i=1}^2 \Gamma(N_i \rightarrow L^C \Phi)}, \quad (4.11)$$

where L refers to all fermionic degrees of freedom of the leptonic isodoublet that heavy Majorana neutrinos can decay. Nevertheless, the parameters ε_{N_i} , ε_N , and ε'_{N_i} defined above are helpful to better appreciate the significance of the two different mechanisms of CP violation.

To elucidate our resummation formalism further, it may be useful to calculate the analytic form of the parameters ε_{N_i} for the interesting case of nearly degenerate heavy Majorana neutrinos. Therefore, we shall consider the approximations $\Delta m_N^2 = m_{N_1}^2 - m_{N_2}^2 \ll m_{N_1}^2 \sim m_{N_2}^2$ and define the parameter $r_N = \Delta m_N^2 / (m_{N_1} m_{N_2})$. Moreover, we neglect high order Yukawa couplings of $O(h_{ij}^4)$ at the amplitude level. It is then not difficult to find the approximate expressions

$$\varepsilon_{N_1} \approx \frac{\text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})}{8\pi |h_{l1}|^2} \frac{r_N}{r_N^2 + 4A_{22}^2}, \quad (4.12)$$

$$\varepsilon_{N_2} \approx \frac{\text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})}{8\pi |h_{l2}|^2} \frac{r_N}{r_N^2 + 4A_{11}^2}. \quad (4.13)$$

The difference between the above expressions and those obtained within the framework of the ordinary perturbation theory is that the regulating terms A_{11}^2 and A_{22}^2 are absent in the latter. Clearly, if these finite width terms were not consistently taken into account, this would cause a singular behavior when the degeneracy between the two heavy Majorana neutrinos is exact [19,15,16]. On physical grounds, however, this should not be very surprising, since the only natural parameter that can regulate such a singularity is the finite width of the heavy Majorana neutrinos. Therefore, one of the main advantages of our approach is that the dynamics of CP violation through heavy-neutrino mixing can be properly described by giving rise to physically well-behaved analytic expressions.

Another important point, also reported in [39], is the fact that both parameters ε_{N_1} and ε_{N_2} contribute constructively to CP violation. Technically speaking, this can be seen as follows. As has been mentioned above, the expression ε_{N_2} in Eq. (4.13) may be obtained from Eq. (4.12) if one replaces the heavy-neutrino index ‘‘1’’ with ‘‘2’’ everywhere in that formula. As a result, the CP -violating combination of Yukawa couplings $\text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})$ flips sign, which gets compensated by a similar sign flip in the parameter r_N . Another significant feature of our analytic results in Eqs. (4.12) and (4.13) is that both ε_{N_1} and ε_{N_2} vanish in the limit $m_{N_1} \rightarrow m_{N_2}$, which is consistent with the requirement of CP invariance given in Eq. (2.8). We have checked that this property of CP invariance persists even if one calculates ε_{N_1} and ε_{N_2} exactly from the expressions (4.6) and (4.7). In addition, we have verified that ε_{N_2} (ε_{N_1}) vanishes in the limit m_{N_1} (m_{N_2}) $\rightarrow 0$, as is also prescribed by Eq. (2.8). As a consequence, our analytic expressions for ε -type CP violation are indeed proportional to the flavor-basis invariant combination of CP noninvariance [cf. Eq. (2.8)], $m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2) \text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})$.

In order to make our analysis complete, it is important to include the contributions from ε' -type CP violation, since they may be significant for very large differences of heavy-neutrino masses, e.g., for $m_{N_1} - m_{N_2} \sim m_{N_1}$ or m_{N_2} . In this regime, the ε -type terms are suppressed by the large virtuality of the heavy-neutrino propagator and so become comparable to the ε' -type terms [15]. It is now useful to define the function

$$F(x, \alpha) = \sqrt{x} \left[1 - \alpha - (1+x) \ln \left(\frac{1 - \alpha + x}{x} \right) \right]. \quad (4.14)$$

If one sets $\alpha=0$ in Eq. (4.14), then $F(x, \alpha)$ reduces to the known function $f(x) = \sqrt{x} [1 - (1+x) \ln(1+1/x)]$, found in [11]. Here, we are only interested in the L -violating absorptive parts of the one-loop vertices $\chi^+ l N_i$, $\chi^0 \nu_l N_i$, and $H \nu_l N_i$, shown in Figs. 1(a)–1(c). The complete analytic expressions are calculated in the Appendix. For later convenience, we also assume that the external decaying heavy Majorana neutrinos are off shell. Having this in mind, we find the off-shell absorptive couplings

$$\mathcal{V}_{\chi^+ l N_i}^{\text{abs}}(\not{p}) = - \frac{h_{l'1}^* h_{l'j} h_{lj}}{16\pi \sqrt{p^2}} \not{p} P_L F \left(\frac{m_{Nj}^2}{p^2}, 0 \right), \quad (4.15)$$

$$\begin{aligned} \mathcal{V}_{\chi^0 \nu_l N_i}^{\text{abs}}(\not{p}) = \mathcal{V}_{H \nu_l N_i}^{\text{abs}}(\not{p}) = & - \frac{h_{l'1}^* h_{l'j} h_{lj}}{32\pi \sqrt{p^2}} \not{p} P_L \left[F \left(\frac{m_{Nj}^2}{p^2}, 0 \right) \right. \\ & \left. + F \left(\frac{m_{Nj}^2}{p^2}, \frac{M_H^2}{p^2} \right) \right]. \end{aligned} \quad (4.16)$$

To make contact with the ε'_{N_i} expressions existing in the literature [11–14], we compute the ε' -type CP asymmetry in the conventional perturbation theory, using Eqs. (4.15) and (4.16) and neglecting wave-function contributions. Considering all decay channels for the decaying heavy Majorana neutrino, e.g., N_1 , we obtain

$$\begin{aligned} \varepsilon'_{N_1} = & \frac{\text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})}{16\pi |h_{l1}|^2 \left[\frac{3}{4} + \frac{1}{4} (1 - M_H^2/m_{N_1}^2)^2 \right]} \left\{ \frac{5}{4} F \left(\frac{m_{N_2}^2}{m_{N_1}^2}, 0 \right) \right. \\ & + \frac{1}{4} F \left(\frac{m_{N_2}^2}{m_{N_1}^2}, \frac{M_H^2}{m_{N_1}^2} \right) + \frac{1}{4} \left(1 - \frac{M_H^2}{m_{N_1}^2} \right)^2 \left[F \left(\frac{m_{N_2}^2}{m_{N_1}^2}, 0 \right) \right. \\ & \left. \left. + F \left(\frac{m_{N_2}^2}{m_{N_1}^2}, \frac{M_H^2}{m_{N_1}^2} \right) \right] \right\}. \end{aligned} \quad (4.17)$$

In the vanishing limit of the Higgs-boson mass, the above formula simplifies to [11–14]

$$\varepsilon'_{N_1} = \frac{\text{Im}(h_{l1}^* h_{l2} h_{l'1}^* h_{l'2})}{8\pi |h_{l1}|^2} f \left(\frac{m_{N_2}^2}{m_{N_1}^2} \right). \quad (4.18)$$

Unlike ε_{N_1} , ε'_{N_1} does not vanish in the degenerate limit of the two heavy Majorana neutrinos N_1 and N_2 . However,

when the value of m_{N_1} approaches that of m_{N_2} , the ε' -type part of the transition amplitude squared for the N_1 decay becomes equal but opposite in sign with the respective one of the N_2 decay. Thus, these two ε' -type terms cancel one another, leading to a vanishing result for the CP -violating parameter ε'_N ($\neq \varepsilon'_{N_1} + \varepsilon'_{N_2}$), which may be defined analogously to Eq. (4.9).

We are now in a position to implement the vertex corrections of the Yukawa couplings to the resummed amplitudes $\mathcal{T}_{N_1}^{(\varepsilon)}$ and $\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}$ given in Eqs. (4.1) and (4.4), respectively. Taking Eqs. (4.15) and (4.16) into account, we find

$$\mathcal{T}_{N_1} = \bar{u}_l P_R \{ h_{l1} + i\mathcal{V}_{l1}^{\text{abs}}(\not{p}) - i[h_{l2} + i\mathcal{V}_{l2}^{\text{abs}}(\not{p})][\not{p} - m_{N_2} + i\bar{\Sigma}_{22}^{\text{abs}}(\not{p})]^{-1} \Sigma_{21}^{\text{abs}}(\not{p}) \} u_{N_1}, \quad (4.19)$$

$$\bar{\mathcal{T}}_{N_1} = \bar{u}_l P_L \{ h_{l1}^* + i\bar{\mathcal{V}}_{l1}^{\text{abs}}(\not{p}) - i[h_{l2}^* + i\bar{\mathcal{V}}_{l2}^{\text{abs}}(\not{p})][\not{p} - m_{N_2} + i\bar{\Sigma}_{22}^{\text{abs}}(\not{p})]^{-1} \bar{\Sigma}_{21}^{\text{abs}}(\not{p}) \} u_{N_1}, \quad (4.20)$$

where we have simplified the notation of the off-shell one-loop vertices as $\mathcal{V}_{li}^{\text{abs}}(\not{p})$. The vertex functions $\bar{\mathcal{V}}_{li}^{\text{abs}}(\not{p})$ are the charge conjugates of $\mathcal{V}_{li}^{\text{abs}}(\not{p})$ and may hence be recovered from Eqs. (4.15) and (4.16) by taking the complex conjugate for the Yukawa couplings and replacing P_R with P_L . It may not be very convenient to present analytic expressions for the CP -violating observables δ_{N_i} , defined in Eq. (4.10), in a rather compact form, although their derivation from Eqs. (4.19) and (4.20) is quite straightforward. Instead, we shall compare the numerical results obtained from our resummation approach with those found with different methods.

Before we proceed with our numerical analysis, it is crucial to take the out-of-equilibrium constraints on heavy-neutrino decays into account. Detailed study of the latter will be performed by solving numerically the Boltzmann equations in Sec. VI. To a good approximation however, Sakharov's third condition imposes a lower bound on the lifetime of the decaying heavy Majorana neutrino, $1/\Gamma_{N_i}$, which may qualitatively be given by the inequality

$$\Gamma_{N_i}(T = m_{N_i}) \leq 2H(T = m_{N_i}), \quad (4.21)$$

where $H(T)$ is the Hubble parameter:

$$H(T) = 1.73 g_*^{1/2} \frac{T^2}{M_{\text{Planck}}}. \quad (4.22)$$

In Eq. (4.22), $g_* \approx 100\text{--}400$ represents the number of active degrees of freedom in usual extensions of the SM and $M_{\text{Planck}} = 1.2 \times 10^{19}$ GeV is the Planck mass scale. Considering the total decay width of the heavy neutrino, $\Gamma_{N_i} = |h_{li}|^2 m_{N_i} / (8\pi)$, the out-of-equilibrium constraint in Eq. (4.21) yields

$$|h_{li}|^2 \leq 7.2 \times 10^{-14} \left(\frac{m_{N_i}}{1 \text{ TeV}} \right). \quad (4.23)$$

In order to have the baryon-to-entropy density ratio in the observed ballpark, i.e., $Y_B \approx 10^{-10} \approx |\delta_{N_i}|/g_*$, one must al-

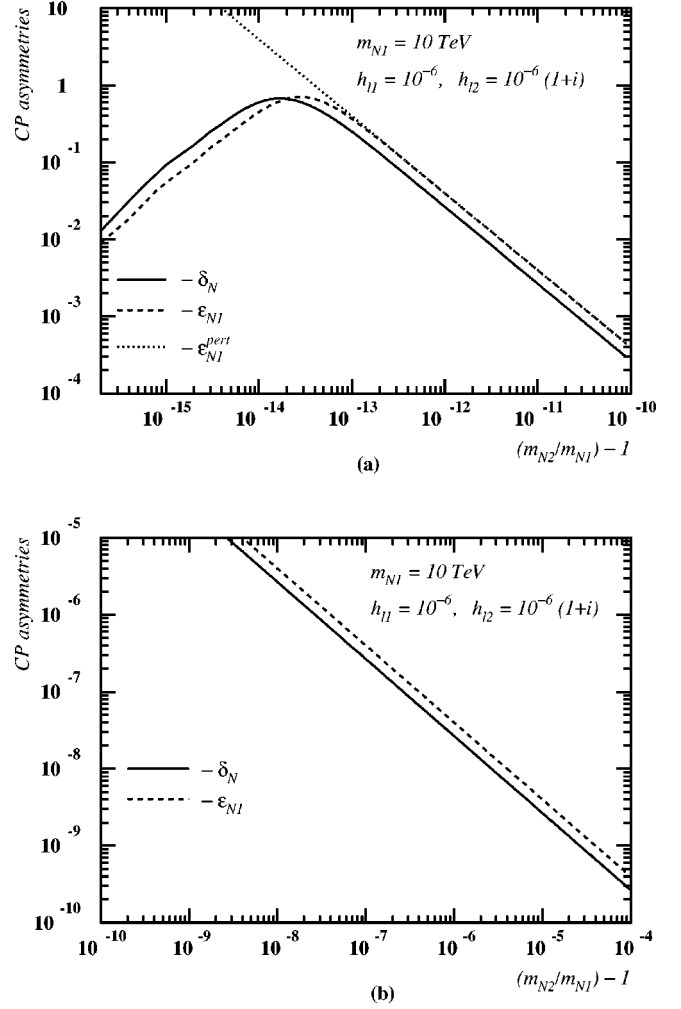


FIG. 4. Numerical estimates of CP asymmetries in scenario (I).

low for CP asymmetries $|\delta_{N_i}|$ of order $10^{-7}\text{--}10^{-6}$. This is practically independent of the heavy-neutrino mass, for masses $m_{N_i} \gtrsim 1$ TeV, provided the out-of-equilibrium constraint on the Yukawa coupling in Eq. (4.23) is satisfied.

We shall give numerical estimates of CP asymmetries in two heavy-neutrino scenarios, which are in compliance with the out-of-equilibrium limits derived above. For our illustrations, we analyze models in which the two right-handed neutrinos mix actively with one lepton family l only. Despite their simplicity, such models exhibit all the essential features of CP violation through heavy-neutrino mixing. Specifically, we consider the following two scenarios:

- (I) $m_{N_1} = 10$ TeV, $h_{l1} = 10^{-6}$, $h_{l2} = 10^{-6}(1+i)$,
 - (II) $m_{N_1} = 10^9$ TeV, $h_{l1} = 10^{-2}$, $h_{l2} = 10^{-2}(1+i)$,
- (4.24)

and assume that N_2 is always heavier than N_1 , i.e., $m_{N_1} \leq m_{N_2}$.

In Fig. 4, we display numerical estimates of the CP asymmetries defined above as a function of the parameter $x_N = m_{N_2}/m_{N_1} - 1$ for the scenario (I). We have divided the range of values for the parameter x_N into two regions: The first region is plotted in Fig. 4(a) and pertains to the kin-

matic domain, where resonant CP violation due to heavy-neutrino mixing occurs. The second one, shown in Fig. 4(b), represents the kinematic range, far away from the resonant CP -violating phenomenon. The dotted line in Fig. 4(a) gives the prediction of ε_{N_1} , obtained from Eq. (4.12) in the conventional perturbation theory. Obviously, $\varepsilon_{N_1}^{\text{pert}}$ diverges for sufficiently small values of x_N , e.g., $x_N < 10^{-13}$. If resummation of the relevant fermionic self-energy graphs is considered, the prediction for ε_{N_1} is given by the dashed lines in Fig. 4, which shows a maximum for $x_N \approx 10^{-13}$. In such a case, CP violation may resonantly increase up to order of unity [24,25]. As has been mentioned above, the physical regulating parameter of the singularity in $\varepsilon_{N_1}^{\text{pert}}$ is the finite width of the heavy neutrino N_2 , which arises naturally within our field-theoretic approach. Thus, the condition for resonant enhancement of CP violation reads

$$m_{N_1} - m_{N_2} \sim \pm A_{22} m_{N_2} = \frac{\Gamma_{N_2}}{2} \quad \text{and/or} \quad A_{11} m_{N_1} = \frac{\Gamma_{N_1}}{2}. \quad (4.25)$$

Clearly, Fig. 4(a) satisfies the above condition. However, the magnitude of the CP asymmetries is governed by the expression

$$\delta_{CP} = \frac{|\text{Im}[(h_{11}^* h_{12})^2]|}{|h_{11}|^2 |h_{12}|^2}, \quad (4.26)$$

which is always $\delta_{CP} \leq 1$. Evidently, both scenarios (I) and (II) given above represent maximal cases of CP violation with $\delta_{CP} = 1$. Therefore, results for any other model may readily be read off from Figs. 4, 5, and 6, by multiplying them with the appropriate model-dependent factor δ_{CP} . The solid line in Fig. 4 gives the numerical estimate for the CP -violating parameter δ_N in Eq. (4.11), where ε' -type contributions are included. The latter are very small in this scenario so as to potentially account for the BAU e.g., $\varepsilon'_{N_1} \approx 10^{-16}$. Furthermore, it may be important to stress that δ_N vanishes in the CP -invariant limit $x_N \rightarrow 0$, as it should be on account of Eq. (2.8).

In Figs. 5 and 6, we give numerical estimates of the CP asymmetries in scenario (II). The difference in this model with scenario I is that the ε' -type effects may not be negligible in the off-resonant region, as can be seen from Figs. 5(a) and 6. In particular, for values of the parameter $x_N < 10^{-11}$ or $x_N > 1$, the individual ε'_{N_1} - and ε'_{N_2} -type contributions may dominate over those of the ε type. Models with $x_N > 1$ have extensively been discussed in the literature [11–14]. Numerical estimates for such models are displayed in Fig. 6. Our attention will now be focused on the domain with $x_N < 10^{-2}$. In Fig. 5(a), we observe that ε'_{N_1} and ε'_{N_2} , represented by the dotted lines, do not vanish in the CP -invariant limit $x_N \rightarrow 0$, as opposed to ε_{N_1} . As a consequence, the CP asymmetry δ_N in Eq. (4.10), in which both ε_{N_1} - and ε'_{N_1} -type terms are considered within our formalism, does not vanish either. The reason is that the physical CP -violating parameter in this highly degenerate mass regime for N_1 and N_2 is the observable δ_N defined in Eq.

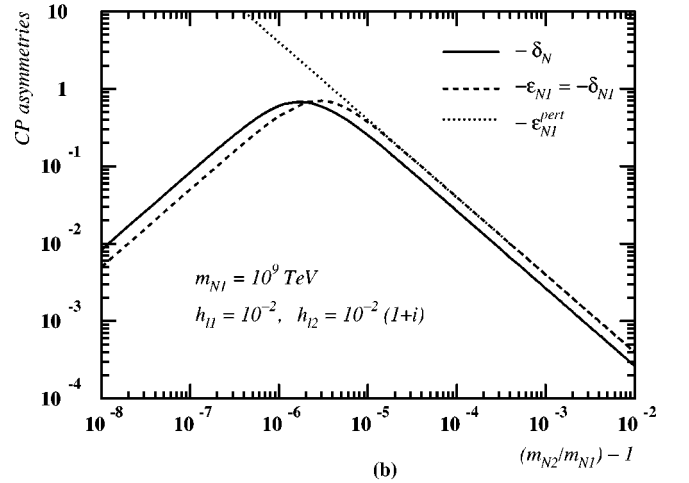
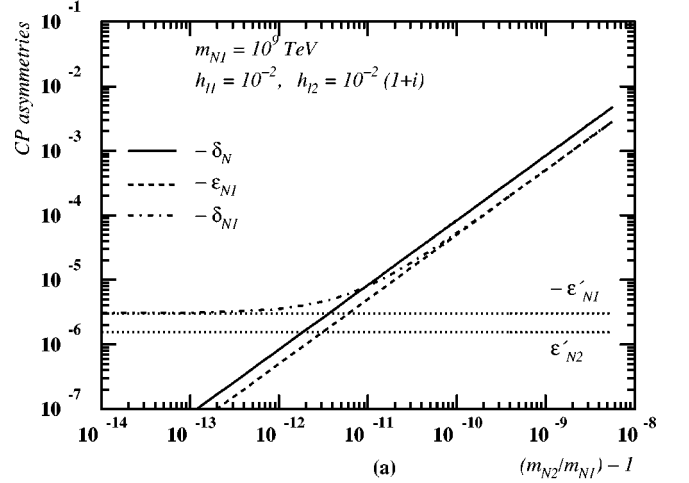


FIG. 5. Numerical estimates of CP asymmetries versus $m_{N_2}/m_{N_1} - 1$ in scenario (II).

(4.11). In fact, the quantity δ_N shares the common feature with ε_{N_1} and tends consistently to zero as $x_N \rightarrow 0$. This fact must be considered to be one of the successes of our resummation approach. Again, CP violation is resonantly amplified, when the condition in Eq. (4.25) is satisfied, as can be seen from Fig. 5(b). Finally, we must remark that $-\delta_N$ has a zero point and eventually becomes negative for $x_N \gg 1$, as is plotted in Fig. 6. Nevertheless, this result should be viewed with great caution. The actual reason is that the effect of the different dissipative Boltzmann factors multiplying the decay rates of the heavy Majorana neutrinos N_1 and N_2 must be considered in the definition of δ_N in Eq. (4.11). These phenomena will be taken into account in Sec. VI, in which we shall write down and solve numerically the relevant Boltzmann equations.

Our computation of the CP asymmetries has been carried out in the physical basis, in which the heavy-neutrino mass matrix is non-negative and diagonal. One might, however, raise the question of whether our analytic results would have been modified if we had chosen a different basis other than the mass basis. The best way to address this question is to discuss first how our expressions would change under a basis transformation. Let us assume a model with two right-handed neutrinos for simplicity and imagine that we wish to

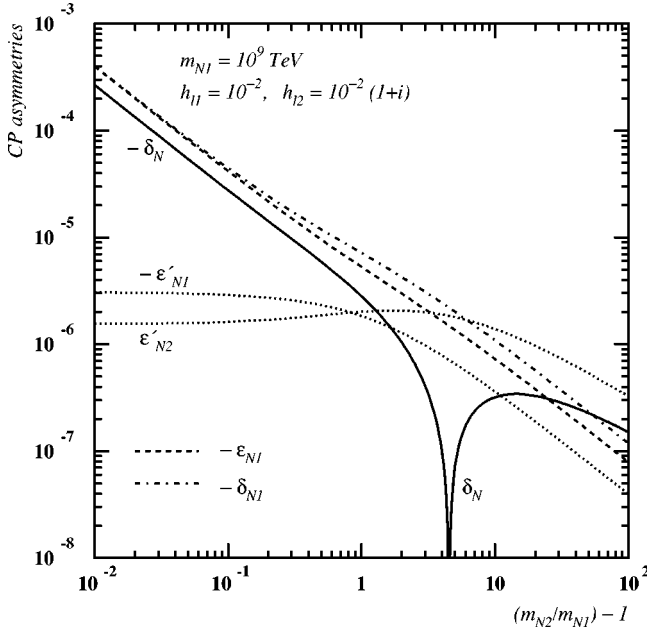


FIG. 6. Numerical estimates of CP asymmetries as a function of $m_{N_2}/m_{N_1} - 1$ in scenario (II).

perform our calculations in a basis in which the two heavy Majorana neutrinos, n_1 and n_2 , say, span a nondiagonal mass matrix. The heavy Majorana neutrinos n_1 and n_2 are then related to the neutrinos N_1 and N_2 , defined in the mass basis, through the unitary transformation

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_R = U^n \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_R, \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_L = U^{n*} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_L. \quad (4.27)$$

The 2×2 unitary matrix U^n relates the physical and nondiagonal mass matrices in the following way:

$$\hat{M}^v = U^{nT} M^n U^n. \quad (4.28)$$

It is now helpful to see how the various kinematic parameters transform under the action of U^n . In particular, the inverse propagator matrix $S_N^{-1}(\not{p})$ undergoes a change depending on $S_n^{-1}(\not{p})$, which is given by

$$S_N^{-1}(\not{p}) = (U^{nT} P_R + U^{n\dagger} P_L) S_n^{-1}(\not{p}) (U^n P_R + U^{n*} P_L). \quad (4.29)$$

This in turn implies that

$$S_N(\not{p}) = (U^{n\dagger} P_R + U^{nT} P_L) S_n(\not{p}) (U^{n*} P_R + U^n P_L), \quad (4.30)$$

where $S_N(\not{p})$ and $S_n(\not{p})$ are the 2×2 propagator matrices, evaluated in the two different heavy neutrino bases. Finally, the Yukawa couplings $h = (h_{11}, h_{12})$, defined in the mass basis, are related to the Yukawa couplings $h^n = (h_{11}^n, h_{12}^n)$ of the nondiagonal basis, via the unitary rotation $h = h^n U^n$.

Given the aforementioned transformation of the Yukawa couplings and that of the resummed propagators in Eq. (4.30) under a basis rotation, it is not difficult to show that the propagator part of the S -matrix amplitude for the process, e.g., $l^- \chi^+ \rightarrow l^+ \chi^-$,

$$\mathcal{T}(l^- \chi^+ \rightarrow l^+ \chi^-) \propto \text{Tr}[h P_R S_N(\not{p}) P_R h^T],$$

is rotational invariant under U^n , i.e.,

$$\text{Tr}[h P_R S_N(\not{p}) P_R h^T] = \text{Tr}[h^n P_R S_n(\not{p}) P_R h^{nT}],$$

where the trace should be taken over the product of spinor and flavor matrices as well. In fact, this avenue has been followed in [25]. If decays of particles are considered, however, such as heavy Majorana neutrino decays, the above flavor-basis invariance is not manifest. The reason is that, within the LSZ formalism, the S -matrix amplitude is obtained by amputating the external legs of the Green functions with inverse propagators defined in the mass basis and any basis transformation will affect the S -matrix expression for the decaying particle.

Therefore, it is essential to introduce a method that always makes reference to the decay of the neutrinos in the diagonal mass basis. To accomplish this, one has to truncate the Green function, e.g., $\mathcal{T}_n^{\text{amp}} S_n(\not{p})$, for the decay $n_i \rightarrow l^- \chi^+$, in the following scheme:

$$\begin{aligned} \hat{S}_i u_{N_i}(p) &= \mathcal{T}_n^{\text{amp}} S_n(\not{p}) (U^{n*} P_R + U^n P_L) (Z_L^{-1/2\dagger} P_R \\ &\quad + Z_R^{-1/2\dagger} P_L)_i (\hat{S}_N)_{ii}^{-1}(\not{p}) u_{N_i}(p), \end{aligned} \quad (4.31)$$

where $Z_L = Z_R^*$ due to the Majorana property of the heavy neutrinos and summation over not displayed indices is implied. Note that $\mathcal{T}_n^{\text{amp}}$ in Eq. (4.31) is calculated in the non-diagonal basis. We must remark that our field-theoretic approach to solving the problem of flavor-basis invariance leads to nonvanishing CP asymmetries proportional to the flavor-basis-independent combination of CP invariance given in Eq. (2.8). In particular, in the limit $(m_{N_2} - m_{N_1}) \rightarrow 0$, the vanishing of δ_N in Figs. 4(a) and 5(a) explicitly demonstrates that our numerical estimates describe genuine effects of CP violation.

V. LOW-ENERGY CONSTRAINTS

If heavy Majorana neutrinos are not much heavier than few TeV [40], these novel particles may then be produced at high-energy ee [41], ep [42], and pp colliders [43], whose subsequent decays can give rise to distinct like-sign dilepton signals. If heavy Majorana neutrinos are not directly accessed at high-energy machines, they may have significant nondecoupling quantum effects on lepton-flavor-violating decays of the Z boson [44,45], the Higgs particle (H) [46], and the τ and μ leptons [47]. Their presence may cause breaking of universality in leptonic diagonal Z -boson [48] and π decays [38] or influence [49] the size of the electroweak oblique parameters S , T , and U [50]. In fact, there exist many observables [51], including the τ -polarization asymmetries and neutrino-counting experiments at the CERN Large Electron Positron Collider (LEP1) or at the SLAC Linear Collider (SLC), to which Majorana neutrinos may have sizable contributions. However, if the out-of-equilibrium constraints are imposed on all lepton families of the model [cf. Eq. (4.23)], all these non-SM effects mentioned above are estimated to be extremely small at the tree or one-loop level. For example, typical tree-level terms

breaking charged-current universality in π decays due to heavy Majorana masses depend linearly on the ratio $|h_{li}v|^2/m_{N_i}^2$ and are therefore very suppressed. Moreover, one-loop-induced flavor-changing decays of the Z boson are also very small, of order

$$\frac{\alpha_w}{4\pi} \frac{|h_{li}v|^2}{m_{N_i}^2} \frac{v^2}{M_W^2} \lesssim 10^{-15} \times \left(\frac{1 \text{ TeV}}{m_{N_i}} \right)^2, \quad (5.1)$$

at the amplitude level, with α_w being the $SU(2)_L$ electroweak fine-structure constant.

It is known that below the critical temperature T_c of the first-order electroweak phase transition, the Higgs boson acquires a nonvanishing VEV [6]. This leads to nonzero light-neutrino masses, which may be obtained from the five-dimensional operator

$$L^T \langle \Phi^T \Phi \rangle L h^T (\hat{M}^\nu)^{-1} h \approx \bar{\nu}_{iL}^C \nu_{iL} \sum_{i=1}^2 h_{li}^2 \frac{v^2}{m_{N_i}}. \quad (5.2)$$

However, for the scenarios (I) and (II), the light-neutrino masses generated at temperatures $T \ll T_c$ are much below the cosmological limit, i.e., $m_\nu \ll 10$ eV. Obviously, all the aforementioned limits cannot jeopardize the viability of our minimal new-physics scenarios.

New-physics interactions may also give large contributions to the EDM of electron, whose experimental upper

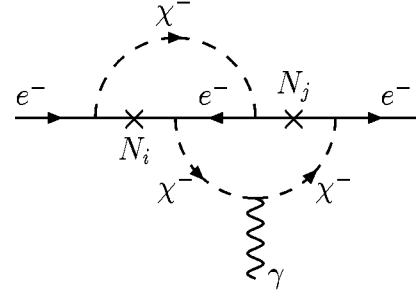


FIG. 7. Typical two-loop diagram contributing to the EDM of electron.

bound is $(d_e/e) < 10^{-26}$ cm [52]. In particular, this bound is crucial, since it may impose a constraint on CP -violating operators similar to those that induce CP violation in heavy Majorana neutrino decays. In our SM extension with right-handed neutrinos, the contribution to the EDM of electron arises at two loops [53]. A typical diagram that gives rise to an EDM for the electron,

$$\mathcal{L}_d = ie \left(\frac{d_e}{e} \right) \bar{e} \sigma_{\mu\nu} \gamma_5 e \partial^\mu A^\nu, \quad (5.3)$$

is shown in Fig. 7. Following the semiquantitative prescription in [53] by making naive dimensional counting for $m_{N_2}, m_{N_1} \gg M_W$, we find

$$\begin{aligned} \left(\frac{d_e}{e} \right) &\sim \text{Im}(h_{1e} h_{2e}^* h_{1l} h_{2l}^*) \frac{\alpha_w^2}{16\pi^2} \frac{m_e}{M_W^2} \frac{v^4}{M_W^4} \frac{m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2)}{(m_{N_1}^2 + m_{N_2}^2)^2} \ln \left(\frac{m_{N_1}}{M_W} \right) \\ &\sim 10^{-24} \text{ cm} \text{ Im}(h_{1e} h_{2e}^* h_{1l} h_{2l}^*) \frac{m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2)}{(m_{N_1}^2 + m_{N_2}^2)^2} \ln \left(\frac{m_{N_1}}{M_W} \right), \end{aligned} \quad (5.4)$$

where the remaining factor depending on the masses of the heavy Majorana neutrinos only in Eq. (5.4) is usually less than 1. It is then obvious that the above EDM limit may be important, if $|h_{li}| > 0.1$, i.e., for superheavy Majorana neutrinos with $m_{N_i} > 10^{11}$ TeV. On the other hand, stability of the Higgs potential under radiative corrections requires that $|h_{li}| = O(1)$ [40]. Nevertheless, the EDM bound derived above gets less restrictive when the mass difference between N_1 and N_2 is much smaller than the sum of their masses, i.e., $(m_{N_1} - m_{N_2})/(m_{N_1} + m_{N_2}) \ll 1$, and is practically absent for values of the parameter $x_N < 10^{-3}$. As a consequence, in our analysis, we have only considered scenarios with $|h_{l1}|, |h_{l2}| \lesssim 10^{-2}$, on which the EDM constraint in Eq. (5.4) is still weak.

In the above discussion of low-energy effects, we have assumed that the out-of-equilibrium constraint given in Eq. (4.23) applies to all lepton families. As we have seen in Sec. IV, one lepton family is sufficient to get the CP asymmetries required for the BAU. Furthermore, sphaleron interactions

preserve the individual quantum numbers $B/3 - L_i$, e.g., $B/3 - L_e$. Also, an excess in L_e will be converted into the observed asymmetry in B . Most interestingly, the so-generated BAU will not be washed out, even if operators that violate L_μ and L_τ are in thermal equilibrium provided that possible $(L_e - L_\mu)$ - and $(L_e - L_\tau)$ -violating interactions are absent [54]. For instance, this can be realized if there are two right-handed neutrinos that mix with the electron family only and produce the BAU, and the remaining isosinglet neutrinos strongly mix with the μ and τ families. This class of heavy Majorana neutrino models may predict sizable new-physics phenomena, which have been mentioned above and can be probed in laboratory experiments.

VI. BOLTZMANN EQUATIONS

In this section, we write down the relevant Boltzmann equations [3,4,55,56], which determine the time evolution of the lepton-number asymmetries. Then, we solve these equations numerically and present results for the expected BAU

within the two different democratic-type scenarios (I) and (II) discussed in Sec. IV. Last, we give estimates of the finite-temperature effects, which may have an impact on the resonant phenomenon of CP violation due to mixing.

The Boltzmann equations describing the time evolution of the lepton asymmetry for a system with two heavy Majorana neutrinos are given by [55,12]

$$\frac{dn_{N_i}}{dt} + 3Hn_{N_i} = - \left(\frac{n_{N_i}}{n_{N_i}^{\text{eq}}} - 1 \right) \gamma_{N_i}, \quad (6.1)$$

$$\frac{dn_L}{dt} + 3Hn_L = \sum_{i=1}^2 \left[\delta_{N_i} \left(\frac{n_{N_i}}{n_{N_i}^{\text{eq}}} - 1 \right) - \frac{n_L}{2n_l^{\text{eq}}} \right] \gamma_{N_i} - \frac{n_L}{n_l^{\text{eq}}} \gamma_\sigma, \quad (6.2)$$

where n_{N_i} , $n_L = n_l - n_{\bar{l}}$ are the densities of the number of N_i and the lepton-number asymmetry, respectively, and $n_{N_i}^{\text{eq}}$ and n_l^{eq} are their values in thermal equilibrium. The Hubble parameter $H = (dR/dt)/R$ determines the expansion rate of the Universe and also depends on the temperature T , through the relation in Eq. (4.22). In Eqs. (6.1) and (6.2), γ_{N_i} and γ_σ are the collision terms given by

$$\gamma_{N_i} = n_{N_i}^{\text{eq}} \frac{K_1(m_{N_i}^2/T)}{K_2(m_{N_i}^2/T)} \Gamma_{N_i}, \quad (6.3)$$

$$\gamma_\sigma = \frac{T}{8\pi^4} \int_0^\infty ds s^{3/2} K_1(\sqrt{s}/T) \sigma'(s). \quad (6.4)$$

Here, $K_1(z)$ and $K_2(z)$ are the modified Bessel functions defined in [57]. In addition, Γ_{N_i} and $\sigma'(s)$ are, respectively, the usual $T=0$ expressions for the total decay width of N_i and the cross section of the $2 \rightarrow 2$ scatterings, involving the L and Φ states, which are taken here to be massless. The latter comprises the scatterings, i.e., $L^C \Phi \rightarrow L \Phi^\dagger$ and its CP -conjugate process $L \Phi^\dagger \rightarrow L^C \Phi$. In fact, the cross section $\sigma'(s)$ is calculated by subtracting all those real intermediate contributions that have already been taken into account in the direct and inverse decays of heavy Majorana neutrinos [4]. Therefore, γ_σ may be regarded as an additional CP -conserving depletion term, which can be shown to be of order h_{ii}^4 in the Yukawa couplings; i.e., one formally finds that $\gamma_\sigma \sim \gamma_{N_i}^2$ in the narrow width approximation [55].

In writing down Eqs. (6.1) and (6.2), several applicable assumptions have been made, which are also reviewed in Ref. [55]. First, we have considered the Friedmann-Robertson-Walker model in the nonrelativistic limit. Second, we have adopted the Maxwell-Boltzmann statistics, which is a good approximation in the absence of effects that originate from Bose condensates or arise due to degeneracy of many Fermi degrees of freedom. Third, we have assumed that the lepton and Higgs weak isodoublets L and Φ are practically in thermal equilibrium, and neglected high orders in n_L/n_l^{eq} and δ_{N_i} . In this context, it has also been assumed that the different particle species are in kinetic equilibrium; i.e., the particles may rapidly change their kinetic energy through

elastic scatterings but the processes responsible for a change of the number of particles are out of equilibrium. These out-of-equilibrium reactions are described by the Boltzmann equations (6.1) and (6.2). Finally, there may exist additional contributions to the Boltzmann equations [12], coming from processes such as $N_i L \rightarrow \Phi^* \rightarrow Q_i \bar{t}_R$, $N_i Q_i \rightarrow L \bar{t}_R$, where Q_i ($i=1,2,3$) denotes the usual quark isodoublets in the SM. These reactions as well as those of the kind $\Phi \Phi^\dagger \rightarrow LL^C$ are still very weak to wash out the BAU generated by the direct heavy Majorana neutrino decays [58], as long as the out-of-equilibrium constraint on the Yukawa couplings in Eq. (4.23) is imposed. Hence, we have neglected these small depletion terms.

Before we evaluate numerically the Boltzmann equations written above, it will prove helpful to make the following substitutions:

$$x = \frac{m_{N_1}}{T}, \quad t = \frac{1}{2H(T)} = \frac{x^2}{2H(x=1)}, \quad (6.5)$$

which is a good approximation for the radiation dominated phase of the Universe. We assume the heavy-neutrino mass hierarchy $m_{N_1} \leq m_{N_2}$ for the two right-handed neutrino scenarios (I) and (II), given in Sec. IV. Furthermore, we introduce the quantities $Y_{N_i} = n_{N_i}/s$ and $Y_L = n_L/s$, where s is the entropy density. In an isentropically expanded universe, the entropy density has the time dependence $s(t) = \text{const} \times R^{-3}(t)$ and may be related to the number density of photons, n_γ , as $s = g_* n_\gamma$, where g_* is given after Eq. (4.22). For our discussion, it will be more convenient to define the parameters

$$K = \frac{K_1(x)}{K_2(x)} \frac{\Gamma_{N_1}}{H(x=1)}, \quad \gamma = \frac{K_2(x)K_1(\xi x)}{K_1(x)K_2(\xi x)} \frac{\Gamma_{N_2}}{\Gamma_{N_1}}, \quad (6.6)$$

with $\xi = m_{N_2}/m_{N_1}$. Making use of the above definitions and relations among the parameters, we obtain the Boltzmann equations for the new quantities Y_{N_1} , Y_{N_2} , and Y_L , viz.

$$\frac{dY_{N_1}}{dx} = -(Y_{N_1} - Y_{N_1}^{\text{eq}}) K x^2, \quad (6.7)$$

$$\frac{dY_{N_2}}{dx} = -(Y_{N_2} - Y_{N_2}^{\text{eq}}) \gamma K x^2, \quad (6.8)$$

$$\begin{aligned} \frac{dY_L}{dx} = & \left[(Y_{N_1} - Y_{N_1}^{\text{eq}}) \delta_{N_1} + (Y_{N_2} - Y_{N_2}^{\text{eq}}) \gamma \delta_{N_2} \right. \\ & \left. - \frac{1}{2} g_* Y_L (Y_{N_1}^{\text{eq}} + \gamma Y_{N_2}^{\text{eq}}) - g_* Y_L Y_{N_1}^{\text{eq}} \frac{\gamma_\sigma}{\gamma_{N_1}} \right] K x^2. \end{aligned} \quad (6.9)$$

In our numerical analysis, we shall neglect the Yukawa coupling suppressed term in Eq. (6.9), which is proportional to γ_σ , since $\gamma_\sigma \ll \gamma_{N_i}$. Moreover, the heavy-neutrino number-entropy densities in equilibrium, $Y_{N_i}^{\text{eq}}(x)$, are given by [4]

$$Y_{N_1}^{\text{eq}}(x) = \frac{3}{8g_*} \int_x^\infty dz z \sqrt{z^2 - x^2} e^{-z} = \frac{3}{8g_*} x^2 K_2(x), \quad (6.10)$$

and $Y_{N_2}^{\text{eq}}(x) = Y_{N_1}^{\text{eq}}(\xi x)$. The differential equations (6.7)–(6.9) are solved numerically, using the initial conditions

$$Y_{N_1}(0) = Y_{N_2}(0) = Y_{N_1}^{\text{eq}}(0) = Y_{N_2}^{\text{eq}}(0) \quad \text{and} \quad Y_L(0) = 0. \quad (6.11)$$

These initial conditions merely reflect the fact that our Universe starts evolving from a lepton symmetric state, in which the heavy Majorana neutrinos are originally in thermal equilibrium. After the evolution of the Universe until temperatures much below m_{N_1} , a net lepton asymmetry has been created. This lepton asymmetry will then be converted into the BAU via the sphalerons. During a first-order electroweak phase transition, the produced excess in L will lead to an excess in B , which is given by [8,56]

$$Y_B = \frac{8N_g + 4N_H}{22N_g + 13N_H} Y_{B-L} \approx -\frac{1}{3} Y_L, \quad (6.12)$$

where $Y_B = n_B/s$, N_g is the number of generations, and N_H is the number of Higgs doublets. The observed BAU is $Y_B^{\text{obs}} = (0.6 - 1) \times 10^{-10}$ [55], which corresponds to an excess of leptons $-Y_L^{\text{obs}} \approx 10^{-9} - 10^{-10}$. In the latter estimate, other alternatives for generating the BAU are also considered, which may arise from the conversion of an individual lepton asymmetry [54] only, e.g., L_e , into the BAU.

In Fig. 8, the observed range for Y_L , denoted as Y_L^{obs} , is indicated with two confining horizontal dotted lines. Furthermore, we display our numerical estimates of $Y_L(x)$ as a function of the parameter $x = m_{N_1}/T$, for selected heavy-Majorana-neutrino scenarios, stated in Eq. (4.24). Specifically, Fig. 8(a) shows explicitly the dependence of Y_L on x , for the three different values of the parameter $x_N = m_{N_2}/m_{N_1} - 1 = 10^{-8}$, 10^{-9} , and 10^{-10} in scenario (I). In this scenario, the lightest heavy Majorana neutrino N_1 has a mass $m_{N_1} = 10$ TeV and the values of the Yukawa couplings are $h_{11} = 10^{-6}$ and $h_{12} = 10^{-6}(1+i)$. The parameter x_N is a measure of the degree of mass degeneracy for N_1 and N_2 . For comparison, it is worth mentioning that the degree of mass degeneracy between K_L and K_S is of order 10^{-15} , which is by far smaller than the one considered here. Since ε' -type CP violation is very small in scenario (I), as has already been discussed in Sec. IV, one has to rely on CP violation through heavy-Majorana-neutrino mixing. We find that for small heavy-neutrino mass splittings determined by x_N , with x_N being in the range between 10^{-9} and 10^{-8} , a sufficiently large lepton (baryon) asymmetry can be generated. The significance of our ε -type CP -violating mechanism may be seen from the fact that in democratic-type scenarios, i.e., in models with all Yukawa couplings being of the same order as those considered here, heavy Majorana neutrinos with masses as low as 1 TeV can still be responsible for the excess of baryons, found by observational measurements. Of course, for larger x_N values, e.g., $x_N > 10^{-8}$, the BAU is getting much smaller than Y_L^{obs} . Furthermore, we

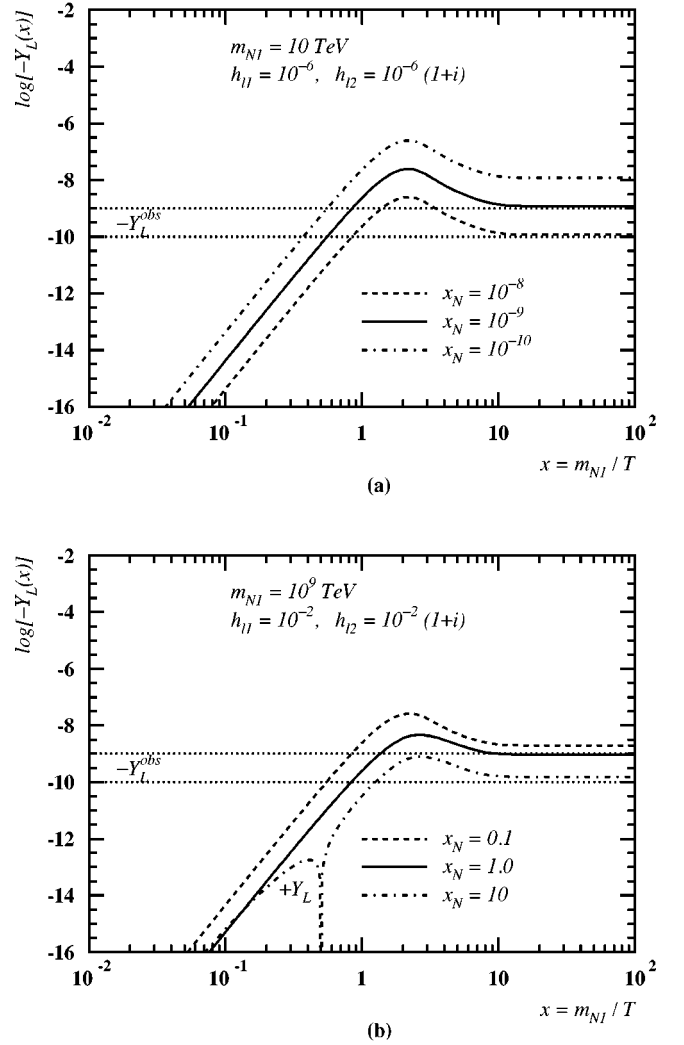


FIG. 8. Lepton asymmetries for selected heavy Majorana neutrino scenarios.

have also checked that CP violation and hence BAU vanishes in the limit $x_N \rightarrow 0$, i.e., when the two OS-renormalized heavy-neutrino masses N_1 and N_2 are exactly equal, as it should be on account of the CP invariance condition in Eq. (2.8).

Figure 8(b) gives numerical estimates of Y_L as a function of x , for scenario (II). In this model, we have chosen $m_{N_1} = 10^9$ TeV, and $h_{11} = 10^{-2}$ and $h_{12} = 10^{-2}(1+i)$. We also present results for three different values of the parameter x_N , $x_N = 0.1$, 1, and 10. In this large- m_{N_1} scenario, a high degree of degeneracy for N_1 and N_2 is not required in order to get sufficient CP violation for the BAU. In fact, the ε - and ε' -type contributions to the decays of heavy Majorana neutrinos are of comparable order and should both be taken into account. Again, one finds numerically an appreciable excess of leptons, Y_L , within the observed range Y_L^{obs} . For the scenario with $x_N = 10$, we obtain positive values for Y_L up to $x \approx 0.5$. This small excess of leptons is rapidly erased by the N_1 heavy neutrino decays, which are almost in thermal equilibrium. At lower temperatures, i.e., for $x \gg 0.5$, the heaviest heavy Majorana neutrino N_2 gets decoupled from the system, and out-of-equilibrium N_1 decays will eventually produce a nonzero value for Y_L at the observable level. Since

the ε - and ε' -type contributions are formally of order h_{li}^4 in this highly nondegenerate scenario, other collision terms of order h_{li}^4 may also be significant, such as the scatterings $\Phi L^C \rightarrow \Phi^\dagger L$ and $\Phi \Phi^\dagger \rightarrow LL^C$ [12,58]. However, the inclusion of these additional effects will not quantitatively affect our numerical results much, as long as the constraint in Eq. (4.23) is valid.

In our numerical analysis presented above, we have not taken into account other effects, which might, to some extent, affect the resonant condition, given in Eq. (4.25). Apart from the intrinsic width of a particle resonance, there may be an additional broadening at high temperatures, due to collisions among particles. Such effects will contribute terms of order h_{li}^4 to the total N_i widths and are small in general [4,59]. Of the most importance are, however, finite-temperature effects on the $T=0$ masses of the particles. Since SM gauge interactions are in kinetic equilibrium in the heat bath, they can give rise to thermal masses to the leptons and the Higgs fields [60–62]. These thermal masses are given by

$$\frac{m_L^2(T)}{T^2} = \frac{1}{32}(3g^2 + g'^2) \approx 0.044,$$

$$\frac{M_\Phi^2(T)}{T^2} = 2d \left(1 - \frac{T_c^2}{T^2} \right), \quad (6.13)$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings at the running scale M_Z , respectively, and $d = [2M_W^2 + M_Z^2 + 2m_t^2 + M_H^2]/(8v^2)$. The critical temperature T_c calculated at one loop may be obtained from [62]

$$T_c^2 = \frac{1}{4d} \left[M_H^2 - \frac{3}{8\pi^2 v^2} (2M_W^4 + M_Z^4 - 4m_t^4) - \frac{1}{8d\pi^2 v^4} (2M_W^3 + M_Z^3)^2 \right]. \quad (6.14)$$

Although the isosinglet heavy neutrinos do not have tree-level couplings to the SM gauge bosons, the difference between the thermal mass of N_i and its respective zero-temperature mass will proceed through Yukawa interactions [60], i.e.,

$$\frac{m_{N_i}^2(T) - m_{N_i}^2(0)}{T^2} = \frac{1}{16} |h_{li}|^2. \quad (6.15)$$

Such a T -dependent shift in the masses of N_i is very small and may be safely neglected in the mass difference $m_{N_1}^2(T) - m_{N_2}^2(T)$, which enters the analytic expressions for resonant CP violation [see, e.g., Eqs. (4.12) and (4.13)]. Making now use of Eqs. (6.13) and (6.14), the authors in [62] find that $0.5 < M_\Phi(T)/T < 2$, when M_H varies from 60 GeV up to 1 TeV, for $T \gg T_c \approx 200$ GeV. In particular, one obtains $M_\Phi(T)/T \approx 0.6$, for $M_H < 200$ GeV. In this Higgs boson mass range, the effective decay widths of the heavy neutrinos $\Gamma_{N_i}(T)$ will be reduced relative to $\Gamma_{N_i}(0)$ by 70–80%, because of considerable phase-space corrections

[61]. If $M_H > 350$ GeV, $M_\Phi(T)/T$ is getting larger than 1, which signals the onset of a nonperturbative regime and pure perturbative methods may not be sufficient to deal with very heavy Higgs bosons. For temperatures T near to the critical temperature T_c , $M_\Phi(T)/T$ will be smaller because of the suppression factor $(1 - T_c^2/T^2)$ in Eq. (6.13). It appears that low-scale leptogenesis is less affected by finite-temperature effects, even though one can always choose larger Yukawa couplings to enhance $\Gamma_{N_i}(T)$ in the light-Higgs-boson scenario. In either case, the resonant phenomenon of mixing-induced CP violation plays a crucial rôle to generate sufficiently large CP asymmetries.

VII. CONCLUSIONS

We have studied the impact of the ε - and ε' -type mechanisms for CP violation on generating the excess of baryons detected in the Universe. As for the scenario of baryogenesis, we have considered that out-of-equilibrium L -violating decays of heavy Majorana neutrinos produce an excess in L , which is converted into the observed asymmetry in B , through the $(B+L)$ -violating sphaleron interactions. In Sec. II, we have described minimal extensions of the SM with right-handed neutrinos, which can predict nearly degenerate heavy Majorana neutrinos without resorting to a fine-tuning of the mass parameters. Such models may naturally occur in certain subgroups of $SO(10)$ [28] or E_6 theories [29]. In a one-lepton family model, the presence of two right-handed neutrinos is sufficient to give rise to the nontrivial CP -violating combination of Yukawa couplings, given in Eq. (2.8). As has been demonstrated in Sec. IV, our physical CP asymmetries depend indeed on this CP -odd invariant. In addition, particular emphasis has been laid on the renormalization of the Yukawa couplings in the minimally extended model.

In the conventional perturbation theory, the wave-function amplitude becomes singular, whenever the degenerate limit of the two mixed heavy neutrinos is considered. Several effective methods have been proposed to solve this problem, such as diagonalizing the effective Hamiltonian of the two-heavy-neutrino system [16,21]. The results obtained with these methods show a resonant enhancement of CP violation, when the two heavy-neutrino masses are getting closer. Such a resonant CP -violating phenomenon is in line with earlier studies in [22]. Unfortunately, the methods based on diagonalizing the effective Hamiltonian are not analytic, if the effective Hamiltonian itself is not diagonalizable [25]. Therefore, it is important to compare the results obtained for the CP asymmetries with a more rigorous field-theoretic approach. In Sec. III, we have extended, in an effective manner, the resummation formalism for particle mixing, which was applied to scatterings in [25], to that of the decays of particles. The resummed decay amplitudes possess all the desirable field-theoretic properties and exhibit an analytic behavior in the mass degenerate limit.

In Sec. IV, we have used the aforementioned field-theoretic approach to perform a systematic analysis of the ε and ε' types of CP violation in the L -violating decays of heavy Majorana neutrinos. For illustration, we have considered minimal extensions of the SM with two right-handed neutrinos. We have found that ε -type CP violation is reso-

nantly enhanced up to order of unity, if the mass splitting of the heavy Majorana neutrinos is comparable to their widths, as is stated in Eq. (4.25), and if the parameter δ_{CP} defined in Eq. (4.26) has a value close to 1. In our view, these two necessary and sufficient conditions for resonant CP violation of order unity constitute a novel aspect in the leptogenesis scenario, which have not been pointed out in their most explicit form before. Taking full advantage of the mechanism of resonant CP violation, one may consider scenarios with nearly degenerate heavy Majorana neutrinos at the TeV scale and all Yukawa couplings being of the same order, which can still be responsible for the BAU. In this kinematic range, the ε' -type contributions are extremely suppressed. Of course, the higher the isosinglet Majorana mass is, the less the above degeneracy is required in order to get sufficiently large CP violation. However, even in the weak-mixing limit, i.e., $m_{N_1} \ll m_{N_2}$, ε -type CP violation is equally important with the ε' -type one and therefore should not be ignored [15]. In contrast, only ε -type CP violation is important in the strong mixing regime, i.e., when $m_{N_1} - m_{N_2} \approx \Gamma_{N_i}$. Another alternative of having sufficiently large CP violation for TeV leptogenesis, with $m_{N_1} \sim 1$ TeV, is to assume an hierarchic pattern for the heavy Majorana masses and the Yukawa couplings, e.g., $m_{N_2} \gg m_{N_1}$ and $h_{12} \gg h_{11}$. Such scenarios were thoroughly investigated in [12,13], and therefore we have not repeated this analysis here. On the other hand, a wide range of heavy-neutrino masses, $T_c \lesssim m_{N_i} \lesssim 10^9$ TeV, is still able to account for the BAU, even if all Yukawa couplings are of the same order. We can hence conclude that the two CP -violating mechanisms under consideration are, to a great extent, determined from the flavor structure of the neutrino mass matrix and the flavor hierarchy of the Yukawa couplings in the model.

In Sec. V, we have presented estimates of low-energy constraints on our minimal model, coming mainly from the electron EDM. The two-loop EDM bound derived is found to be not very severe in order to rule out the leptogenesis scenario and is practically absent if the heavy Majorana neutrinos are nearly degenerate. In Sec. VI, we have briefly discussed the implications of finite-temperature effects for our resonantly enhanced CP violation. Although temperature effects on the heavy-Majorana-neutrino masses are very small, because isosinglet neutrinos interact quite feebly with the Higgs fields in the thermal bath, the final leptons and Higgs fields acquire appreciable nonzero thermal masses. As a consequence, there will be a reduction of the widths of the heavy Majorana neutrinos, relative to their respective values at $T=0$, due to a phase-space suppression. The size of the reduction of the N_i widths depends crucially on the zero-temperature Higgs-boson mass M_H . For $M_H < 200$ GeV, the effective decay width of N_i , $\Gamma_{N_i}(T)$, is estimated to be smaller than $\Gamma_{N_i}(0)$ by 80% at most. This will roughly lead to an 80% decrease of the CP asymmetries, calculated at zero temperature. In this respect, the resonant phenomenon of CP violation through mixing of heavy neutrinos plays a very important rôle for leptogenesis, since ε -type CP violation can still be large for heavy-neutrino mass differences comparable to the effective decay widths $\Gamma_{N_i}(T)$. Finally, further support for the viability of the resonantly enhanced

CP -violating phenomenon is obtained from solving numerically the Boltzmann equations. Numerical estimates reveal that E_6 -type scenarios, which naturally predict a certain degree of degeneracy between the heavy Majorana neutrinos, are able to account for the present excess of baryons. In conclusion, even if all heavy Majorana neutrinos have masses as low as 1 TeV and all couple to the lepton and Higgs isodoublets with a universal Yukawa strength, they can still be responsible for the BAU observed in our epoch, by means of the resonant mechanism of CP violation presented in this paper.

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APPENDIX: ONE-LOOP ANALYTIC EXPRESSIONS

In this appendix, we list the analytic expressions for the one-loop self-energies of the Higgs and fermion fields as well as the one-loop vertex couplings $\chi^+ l N_i$, $\chi^0 \nu_l N_i$, and $H \nu_l N_i$. Detailed discussion of mixing renormalization for Dirac and Majorana fermion theories may be found in [38] and will not be repeated here. Instead, we present the relations between the wave-function CT's and unrenormalized self-energies. Our analytic results will be expressed in terms of standard loop integrals presented in [63], adopting the signature for the Minkowskian metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ (see also Appendix A of Ref. [64]).

The Feynman rules used in our calculations may be read off from the Lagrangian (2.4). We first compute the Higgs self-energies $\chi^- \chi^-$, $\chi^0 \chi^0$, and HH , shown in Figs. 1(d)–1(f). They all found to be equal, viz.

$$\begin{aligned} \Pi_{\chi^- \chi^-}(p^2) &= \Pi_{\chi^0 \chi^0}(p^2) = \Pi_{HH}(p^2) \\ &= \sum_{l=1}^{n_L} \sum_{i=1}^{n_R} \frac{|h_{li}|^2}{8\pi^2} [m_{N_i}^2 B_0(p^2, m_{N_i}^2, 0) \\ &\quad + p^2 B_1(p^2, m_{N_i}^2, 0)]. \end{aligned} \quad (\text{A1})$$

Thus, the universality of the divergent parts of the wave functions δZ_{χ^-} , δZ_{χ^0} , and δZ_H is evident if one calculates $\delta Z_{\Phi}^{\text{div}} = -\text{Re} \Pi'_{\Phi}^{\text{div}}(0)$ from Eq. (A1), for all field components of the Higgs doublet Φ .

The individual contributions to the one-loop fermionic transitions $l' \rightarrow l$, $\nu_{l'} \rightarrow \nu_l$, and $N_j \rightarrow N_i$ are displayed in Figs. 1(g), 1(h), and 1(j), respectively. Explicit calculation of the fermion self-energy transitions gives

$$\Sigma_{ll'}(\not{p}) = - \sum_{i=1}^{n_R} \frac{h_{li} h_{l'i}^*}{16\pi^2} \not{p} P_L B_1(p^2, m_{N_i}^2, 0), \quad (\text{A2})$$

$$\begin{aligned} \Sigma_{\nu_l\nu_{l'}}(\not{p}) = & -\sum_{i=1}^{n_R} \frac{1}{32\pi^2} \{ (h_{li}^* h_{l'i} \not{p} P_R + h_{li} h_{l'i}^* \not{p} P_L) \\ & \times [B_1(p^2, m_{N_i}^2, 0) + B_1(p^2, m_{N_i}^2, M_H^2)] \\ & + m_{N_i} (h_{li}^* h_{l'i}^* P_R + h_{li} h_{l'i} P_L) [B_1(p^2, m_{N_i}^2, 0) \\ & - B_1(p^2, m_{N_i}^2, M_H^2)] \}, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \Sigma_{N_i N_j}(\not{p}) = & -\sum_{i=1}^{n_L} \frac{1}{16\pi^2} (h_{li}^* h_{lj} \not{p} P_R + h_{li} h_{lj}^* \not{p} P_L) \\ & \times \left(\frac{3}{2} B_1(p^2, 0, 0) + \frac{1}{2} B_1(p^2, 0, M_H^2) \right). \end{aligned} \quad (\text{A4})$$

Note that the light-neutrino self-energies $\Sigma_{\nu_l\nu_{l'}}(\not{p})$ contain nonvanishing mass terms in the limit $\not{p} \rightarrow 0$. Even though these contributions vanish when $M_H \rightarrow 0$, they are nonzero at $T=0$, because $M_H \neq 0$, and light neutrinos may hence receive small radiative masses [65]. However, the latter are generally controlled by the mass differences of the heavy neutrinos and/or the Higgs Yukawa couplings h_{li} . If the range of parameters relevant for the BAU is considered, as has been derived in Sec. VI, possible experimental limits on light neutrino masses are estimated to be not very restrictive.

The wave-function renormalization constants can now be expressed in terms of unrenormalized self-energies [38]. Before doing so, we first notice that the one-loop $f_j \rightarrow f_i$ transitions calculated above between the fermions $f_i = l, \nu_l, N_i$ have the generic form

$$\begin{aligned} \Sigma_{ij}(\not{p}) = & \Sigma_{ij}^L(p^2) \not{p} P_L + \Sigma_{ij}^R(p^2) \not{p} P_R + \Sigma_{ij}^M(p^2) P_L \\ & + \Sigma_{ji}^{M*}(p^2) P_R, \end{aligned} \quad (\text{A5})$$

where only dispersive parts are considered. If the transitions are between Majorana fermions in Eq. (A5), one then has the

extra properties $\Sigma_{ij}^L(p^2) = \Sigma_{ij}^{R*}(p^2)$ and $\Sigma_{ij}^M(p^2) = \Sigma_{ji}^M(p^2)$. Adapting the results of [38] to our model, we obtain the wave-function CT's

$$\begin{aligned} \delta Z_{ii}^f = & -\Sigma_{ii}^L(m_i^2) - 2m_i^2 \Sigma_{ii}^{L'}(m_i^2) - m_i [\Sigma_{ii}^{M'}(m_i^2) \\ & + \Sigma_{ii}^{M*'}(m_i^2)] + \frac{1}{2m_i} [\Sigma_{ii}^M(m_i^2) - \Sigma_{ii}^{M*}(m_i^2)] \end{aligned} \quad (\text{A6})$$

and, for $i \neq j$,

$$\begin{aligned} \delta Z_{ij}^f = & \frac{2}{m_i^2 - m_j^2} [m_j^2 \Sigma_{ij}^L(m_j^2) + m_i m_j \Sigma_{ij}^{L'}(m_j^2) + m_i \Sigma_{ij}^M(m_j^2) \\ & + m_j \Sigma_{ij}^{M*}(m_j^2)]. \end{aligned} \quad (\text{A7})$$

The wave-function renormalization of charged leptons may be recovered from Eqs. (A6) and (A7) if one drops all terms depending on $\Sigma_{ij}^M(p^2)$ and its derivative. At this point, it is important to remark that there will be additional contributions to our wave-function CT's of the Higgs and fermion fields from the gauge sector of the SM [64]. For example, there are nonzero thermal mass effects on the particles involved in the loop, which may further break the wave-function universality of the different components of the Higgs doublet [60,61]. As has been discussed in Sec. II, this should not pose any problem to the Yukawa coupling renormalization, as long as all differences of the kind $\delta Z_{\chi^-} - \delta Z_H$ or $\delta Z_{\chi^-} - \delta Z_{\chi^0}$ are UV finite.

We will now present analytic results for the one-loop corrections to the vertices $\chi^\pm l^\mp N_i$, $\chi^0 \nu_l N_i$, and $H \nu_l N_i$, keeping all M_H -dependent mass terms. From Figs. 1(a)–1(c), we observe that the one-loop coupling $\chi^\pm l^\mp N_i$ proceeds via a $\Delta L = 2$ mass insertion only, whereas the couplings $\chi^0 \nu_l N_i$ and $H \nu_l N_i$ can occur through both L -conserving and L -violating interactions. More explicitly, we have

$$i\mathcal{V}_{\chi^+ l^- N_i} = -i \bar{u}_l P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \frac{m_{N_i} m_{N_j}}{16\pi^2} h_{l'i}^* h_{l'j} h_{lj} [C_0(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0)], \quad (\text{A8})$$

$$\begin{aligned} \mathcal{V}_{\chi^0 \nu_l N_i} = & -\bar{u}_l P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \left\{ \frac{m_{N_i} m_{N_j}}{32\sqrt{2}\pi^2} h_{l'i}^* h_{l'j} h_{lj} [C_0(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) + C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) \right. \\ & + C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0)] + \frac{1}{32\sqrt{2}\pi^2} h_{l'i} h_{l'j}^* h_{lj} \{ m_{N_i} m_{N_j} [C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) - C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0)] \\ & \left. - M_H^2 C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \} \right\}, \end{aligned} \quad (\text{A9})$$

$$\begin{aligned}
-i\mathcal{V}_{H\nu_i N_i} = & i\bar{u}_i P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \left\{ \frac{m_{N_i} m_{N_j}}{32\sqrt{2}\pi^2} h_{l'i}^* h_{l'j} h_{lj} [C_0(0,0,m_{N_i}^2,0,m_{N_j}^2,0) + C_0(0,0,m_{N_i}^2,M_H^2,m_{N_j}^2,0) + C_{12}(0,0,m_{N_i}^2,0,m_{N_j}^2,0) \right. \\
& + C_{12}(0,0,m_{N_i}^2,M_H^2,m_{N_j}^2,0)] + \frac{1}{32\sqrt{2}\pi^2} h_{l'i} h_{l'j}^* h_{lj} \{m_{N_i} m_{N_j} [C_{12}(0,0,m_{N_i}^2,0,m_{N_j}^2,0) - C_{12}(0,0,m_{N_i}^2,M_H^2,m_{N_j}^2,0)] \\
& \left. - M_H^2 C_0(0,0,m_{N_i}^2,M_H^2,m_{N_j}^2,0) \} \right\}. \tag{A10}
\end{aligned}$$

As can be readily seen from Eqs. (A9) and (A10), the part of the couplings $\chi^0 \nu_l N_i$ and $H \nu_l N_i$ that conserves lepton number is UV finite and vanishes identically in the massless limit of the Higgs boson.

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