

Polarization measurements and T violation in exclusive semileptonic B decays

Guo-Hong Wu,^{1,*} Ken Kiers,^{2,†} and John N. Ng^{1,‡}

¹TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

²Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000

(Received 15 May 1997)

We provide a general analysis of time reversal invariance violation in the exclusive semileptonic B decays $B \rightarrow D \ell \bar{\nu}$ and $B \rightarrow D^* \ell \bar{\nu}$. Measurements of the lepton and D^* polarizations can be used to search for and identify nonstandard model sources of T violation. Upper limits are placed on the T -odd polarization observables in both the supersymmetric R -parity-conserving and R -parity-breaking theories, as well as in some nonsupersymmetric extensions of the standard model, including multi-Higgs-doublet models, leptoquark models, and left-right symmetric models. It is noted that many of these models allow for large T -violating polarization effects which could be within the reach of the planned B factories. [S0556-2821(97)06721-0]

PACS number(s): 11.30.Er, 12.60.Jv, 13.20.He, 13.88.+e

I. INTRODUCTION

The origin of CP violation remains one of the mysteries of elementary particle physics today, although the observed CP -violating phenomena in the kaon system are consistent with the standard model Cabibbo-Kobayashi-Maskawa (CKM) [1] paradigm. One of the principal goals of the planned B factories is to test the standard model (SM) parametrization of CP violation through precision measurements in several of the hadronic decay modes of the B meson [2]. Any deviation from the SM prediction would be a signal of new physics. Such a signal would of course be welcome, since the gauge hierarchy problem of the SM has led to a widely held belief that the SM is actually a low-energy approximation to some more complete theory. A generic feature of many extensions of the SM is the presence of new CP -violating phases. Given the large number of B 's expected at the B factories, it is clearly important to examine the various CP -odd observables in the B system in order to identify those which are sensitive to new physics. Of particular interest are those observables which receive negligible contributions from SM sources.

In this work we present a detailed analysis of several of the T -odd¹ observables which are available in the exclusive semileptonic decays of B mesons to D and D^* mesons. Measurements of these observables would complement the studies of CP violation in the hadronic decay modes and could serve as valuable tools in order to identify the Lorentz structure of any observed new effects. In a previous paper [3] we have shown that one can define T -odd polarization observables (TOPO's) in the decays $B \rightarrow D^{(*)} \ell \bar{\nu}$ ($\ell = e, \mu, \tau$) which are sensitive separately to effective scalar, pseudoscalar, and right-handed current interactions. In the present work we will provide a more comprehensive analysis of

these observables in addition to considering the prospects in various models for measuring a positive signal.

It has long been known that the semileptonic decays of pseudoscalar mesons provide an ideal place in which to search for non-SM T -violating signals [4]. One of the best studied of these T -odd observables is the muon transverse polarization in the decay $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ ($K_{\mu 3}^+$), defined by

$$P_\mu^\perp \equiv \frac{d\Gamma(\vec{n}) - d\Gamma(-\vec{n})}{d\Gamma(\vec{n}) + d\Gamma(-\vec{n})}, \quad (1)$$

where \vec{n} is the projection of the muon spin normal to the decay plane. Experiments at the Brookhaven National Laboratory give the combined result [5]

$$P_\mu^\perp = (-1.85 \pm 3.60) \times 10^{-3}, \quad (2)$$

which translates into an upper bound of 0.9% at the 95% confidence level. Current efforts at the ongoing KEK E246 experiment [6] and at a recently proposed BNL experiment [7] are expected to reduce the error on this quantity by factors of 10 and 100, respectively. This optimistic experimental outlook has generated much theoretical interest in the muon transverse polarization in both the $K_{\mu 3}^+$ [8–12] and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ ($K_{\mu 2 \gamma}^+$) [13–16] decays.

The muon transverse polarization defined above is proportional to $\vec{s}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)$, which is the only T -odd quantity available in that decay. One can define analogous quantities for the leptons in the decays $B \rightarrow D^{(*)} \ell \bar{\nu}$ and one finds that, neglecting tensor effects, they are sensitive to non-SM scalar and pseudoscalar effective interactions in the D and D^* cases, respectively [3]. The τ lepton polarization in these decays has been studied in multi-Higgs models [17–20] and, more recently, in R -parity conserving supersymmetric (SUSY) models with large intergenerational squark mixing [12,3]. In the latter case the effect arises at one loop. In both types of models the transverse τ polarization can be rather large (from 10's of percent to order unity) compared to the muon transverse polarization in $K_{\mu 3}$ decay. One of the reasons for these seemingly large numbers is that the polarization effects in these models are proportional to the lepton

*Electronic address: gwu@alph02.triumf.ca

†Electronic address: kiers@bnl.gov

‡Electronic address: misery@triumf.ca

¹We assume CPT invariance throughout and so will use “ CP odd” and “ T odd” interchangeably.

mass. Choosing $\ell = \tau$ can thus give a substantial enhancement compared to the $\ell = \mu$ case. From an experimental point of view this means that polarization measurements in $B_{\tau 3}$ decays can achieve the same ‘‘new physics reach’’ as analogous measurements in $K_{\mu 3}$, with far fewer events. One recent study suggests that HERA-B could achieve an eventual sensitivity to the transverse τ polarization on the order of a few percent [21], which would then be competitive — in terms of reach — with that expected in the current $K_{\mu 3}$ experiments. One could in principle also study the transverse polarization of the electrons or muons in semileptonic B decays. Since these lighter leptons are highly energetic, however, it is in practice very difficult to measure their polarizations. For this reason the electron and muon transverse polarizations will not be considered here, although these quantities need not be small in some extensions of the SM.

As we have noted previously [3], the semileptonic B decays have a novel feature compared to the analogous K decays in that the B can decay to both pseudoscalar and vector mesons. The polarization vector of the D^* , which is odd under T , may thus also be used to construct TOPO’s. There are in fact two distinct TOPO’s which may be constructed using the D^* polarization and they are both sensitive to effective right-handed current interactions. For $\ell = \tau$, one of the TOPO’s can also depend (to a lesser extent) on effective pseudoscalar interactions. Combining the lepton polarization measurements and D^* polarization measurements would thus allow one to probe separately the different Lorentz structures of non-SM sources of T violation.

The outline of this paper is as follows. In Sec. II we provide a model-independent analysis of the lepton and D^* polarization based on an effective Lagrangian approach. The T -odd D^* polarization observables can be related to T -odd triple-momentum correlations [22,23] in the four-body final state of the decay $B \rightarrow D^*(D\pi)\ell\bar{\nu}$. This connection is made explicit in Appendix B. In Sec. III the maximal sizes of these T -odd polarization observables are estimated in several classes of models. In R -parity conserving SUSY, T violation occurs at the loop level and its effect is negligible in the absence of squark family mixings. We demonstrate that large enhancements can occur in the presence of squark generational mixings, giving rise to observable T -odd polarization effects while escaping the flavor-changing neutral current (FCNC) bounds. We also consider R -parity-violating SUSY models. In this case, the present data place stringent limits on these TOPO’s. We then consider several non-SUSY models, giving estimates for the maximal sizes of the TOPO’s in multi-Higgs models, leptoquark models, and left-right symmetric models. We conclude in Sec. IV with a brief discussion and a summary of our results.

II. GENERAL ANALYSIS

In this section we provide a general analysis of the T -odd polarization observables available in semileptonic B decays. The effects of new physics may be conveniently parametrized by an effective Lagrangian written in terms of the SM fields. For definiteness, we will always consider the decays $B^- \rightarrow D^{(*)0}\ell^-\bar{\nu}$, with $l = e, \mu, \tau$. The analogous TOPO’s for the charge conjugates of these decays may always be

obtained simply by changing the sign [24]. One could in principle also consider the decays of neutral B ’s. In these decays the electromagnetic final state interactions (FSI’s) could mimic the T -odd observables which we will be studying. This effect is, however, small on the scale of the experimental sensitivity expected at the upcoming experiments and could probably be ignored. Furthermore, even in the presence of such FSI’s, one could measure a ‘‘true’’ T -odd observable by measuring the TOPO in both the B and \bar{B} modes and then taking the difference in order to subtract out the FSI effects [24]. Ideally, one would measure both neutral and charged B decays in order to maximize the statistics.

A. Form factors

Let us begin by establishing some notation. The relevant hadronic matrix elements may be parametrized by the following form factors:

$$\langle D(p') | \bar{c} \gamma_\mu b | B(p) \rangle = f_+(p+p')_\mu + f_-(p-p')_\mu, \quad (3a)$$

$$\langle D(p') | \bar{c} \gamma_\mu \gamma_5 b | B(p) \rangle = 0, \quad (3b)$$

$$\langle D^*(p', \epsilon) | \bar{c} \gamma^\mu b | B(p) \rangle = i \frac{F_V}{m_B} \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* (p+p')_\alpha q_\beta, \quad (3c)$$

$$\begin{aligned} \langle D^*(p', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p) \rangle \\ = -F_{A0} m_B \epsilon_\mu^* - \frac{F_{A+}}{m_B} (p+p')_\mu \epsilon^* \cdot q - \frac{F_{A-}}{m_B} q_\mu \epsilon^* \cdot q, \end{aligned} \quad (3d)$$

where p and p' are the four-momenta of the B and D (D^*), respectively, ϵ is the polarization vector of the D^* , $q = p - p'$, and the form factors are functions of q^2 . We use the convention $\epsilon_{0123} = 1$. In the SM these form factors are relatively real to a good approximation, but their functional dependences on q^2 are, *a priori*, unknown. Note that the expression in Eq. (3b) is equal to zero since one cannot form an axial vector using only p and p' .

In order to derive the corresponding expressions for the scalar and pseudoscalar hadronic matrix elements, we apply the Dirac equation [9], yielding

$$\langle D(p') | \bar{c} b | B(p) \rangle = \frac{m_B^2}{m_b - m_c} \left[f_+(1 - r_D) + f_- \frac{q^2}{m_B^2} \right], \quad (4a)$$

$$\langle D(p') | \bar{c} \gamma_5 b | B(p) \rangle = 0, \quad (4b)$$

$$\langle D^*(p', \epsilon) | \bar{c} b | B(p) \rangle = 0, \quad (4c)$$

$$\begin{aligned} & \langle D^*(p', \epsilon) | \bar{c} \gamma_5 b | B(p) \rangle \\ &= \frac{m_B}{m_b + m_c} (\epsilon^* \cdot q) \left[F_{A0} + F_{A+} (1 - r_{D^*}) + F_{A-} \frac{q^2}{m_B^2} \right], \end{aligned} \quad (4d)$$

where m_b and m_c are the masses of the b and c quarks, $r_D = m_D^2/m_B^2$ and $r_{D^*} = m_{D^*}^2/m_B^2$.

There has been considerable progress in the past few years in understanding the functional forms and interdependence of the above form factors. Isgur and Wise made the key observation in 1989 [25] that in the infinite mass limit for the heavy quarks, all of the form factors are proportional to each other and so may be expressed in terms of one universal function, now called the Isgur-Wise function. Corrections to this picture due to the finite masses of the quarks, as well as perturbative QCD effects, can be incorporated in a systematic way in what has come to be known as heavy quark effective theory (HQET) [26]. In our numerical work, we will use the leading order results of HQET. Our analytical results, however, will be written in terms of the form factors themselves, with no assumptions about heavy quark symmetry. In the heavy quark symmetry limit we have

$$f_{\pm} = \pm \frac{1 \pm \sqrt{r_D}}{2\sqrt{r_D}} \xi(w), \quad (5a)$$

$$F_V = F_{A+} = -F_{A-} = \frac{1}{2\sqrt{r_{D^*}}} \xi(w), \quad (5b)$$

$$F_{A0} = -\sqrt{r_{D^*}} (w+1) \xi(w), \quad (5c)$$

where ξ denotes the Isgur-Wise function and where $w = (m_B^2 + m_{D^*}^2 - q^2)/(2m_B m_{D^*})$. The Isgur-Wise function is normalized to unity at zero recoil, $\xi(1) = 1$.

It is convenient to parametrize the physics of semileptonic B decays in terms of effective four-Fermi interactions as follows

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_{\alpha} (1 - \gamma_5) b \bar{\ell} \gamma^{\alpha} (1 - \gamma_5) \nu \\ & + G_S \bar{c} b \bar{\ell} (1 - \gamma_5) \nu + G_P \bar{c} \gamma_5 b \bar{\ell} (1 - \gamma_5) \nu \\ & + G_V \bar{c} \gamma_{\alpha} b \bar{\ell} \gamma^{\alpha} (1 - \gamma_5) \nu \\ & + G_A \bar{c} \gamma_{\alpha} \gamma_5 b \bar{\ell} \gamma^{\alpha} (1 - \gamma_5) \nu + \text{H.c.}, \end{aligned} \quad (6)$$

where G_F is the Fermi constant and V_{cb} is the relevant CKM matrix element. The first term in the effective Lagrangian is due to the SM W -exchange diagram and the remaining terms characterize contributions coming from new physics, with G_S , G_P , G_V , and G_A denoting the strengths of the new effective scalar, pseudoscalar, vector, and axial-vector interactions, respectively. The effects of effective tensor interactions are negligible in most models and they will be omitted from the present discussion for simplicity. Note that since T violation arises from the interference between the SM amplitude, which contains a left-handed neutrino, and the non-SM amplitude, we do not need to consider four-Fermi operators involving a right-handed neutrino.

The new physics contributions to the decay amplitude may be taken into account by the following replacement of the form factors:

$$f_{+} \rightarrow f'_{+} = f_{+} (1 + \delta_{+}), \quad (7)$$

$$f_{-} \rightarrow f'_{-} = f_{-} (1 + \delta_{-}), \quad (8)$$

$$F_V \rightarrow F'_V = F_V (1 + \delta_V), \quad (9)$$

$$F_{A0} \rightarrow F'_{A0} = F_{A0} (1 + \delta_{A0}), \quad (10)$$

$$F_{A+} \rightarrow F'_{A+} = F_{A+} (1 + \delta_{A+}), \quad (11)$$

$$F_{A-} \rightarrow F'_{A-} = F_{A-} (1 + \delta_{A-}). \quad (12)$$

The δ parameters are given by

$$\delta_{+} = -\Delta_V, \quad (13)$$

$$\delta_{-} = -\Delta_V - \Delta_S \cdot \left(\frac{f_{+}}{f_{-}} (1 - r_D) + \frac{q^2}{m_B^2} \right), \quad (14)$$

$$\delta_V = -\Delta_V, \quad (15)$$

$$\delta_{A0} = \Delta_A, \quad (16)$$

$$\delta_{A+} = \Delta_A, \quad (17)$$

$$\delta_{A-} = \Delta_A - \Delta_P \cdot \left(\frac{F_{A0}}{F_{A-}} + \frac{F_{A+}}{F_{A-}} (1 - r_{D^*}) + \frac{q^2}{m_B^2} \right), \quad (18)$$

where

$$\Delta_S = \frac{\sqrt{2} G_S}{G_F V_{cb}} \frac{m_B^2}{(m_b - m_c) m_{\ell}}, \quad (19)$$

$$\Delta_P = \frac{\sqrt{2} G_P}{G_F V_{cb}} \frac{m_B^2}{(m_b + m_c) m_{\ell}}, \quad (20)$$

$$\Delta_V = \frac{\sqrt{2} G_V}{G_F V_{cb}}, \quad (21)$$

$$\Delta_A = \frac{\sqrt{2} G_A}{G_F V_{cb}}. \quad (22)$$

These δ (Δ) parameters could in general be complex and could then give rise to observable CP -violating effects. Since it is typically true that the TOPO's which we will describe are insensitive to new (SM-like) $V-A$ quark-current interactions, it is also convenient to introduce one more parameter,

$$\Delta_R = \frac{1}{2} (\Delta_V + \Delta_A), \quad (23)$$

which measures the strength of an effective right-handed quark-current interaction.

B. τ polarization in $B \rightarrow D \tau \bar{\nu}$ decay

Let us begin by deriving the expression for the τ lepton transverse polarization in the semileptonic decay

$$B(p) \rightarrow D(p') \tau(p_\tau) \bar{\nu}(p_\nu). \quad (24)$$

The τ transverse polarization in this decay is perfectly analogous to the muon transverse polarization in $K_{\mu 3}$ decay. The amplitude arising from the general effective Lagrangian of Eq. (6) can be written as

$$\begin{aligned} \mathcal{M} = & -\frac{G_F}{\sqrt{2}} V_{cb} \bar{u}(p_\tau) \gamma^\mu (1 - \gamma_5) v(p_\nu) [f'_+(p+p')_\mu \\ & + f'_-(p-p')_\mu], \end{aligned} \quad (25)$$

which has the same form as the SM amplitude except for the replacement $f_\pm \rightarrow f'_\pm$.

The polarization observable may be written in terms of two independent kinematical variables, which we will take to be the energies of the D meson and the τ lepton. This choice is not unique, but is convenient for our purposes. Working in the B rest frame, we introduce dimensionless quantities x and y which are proportional to these energies, but which are normalized to half the B mass, $x = 2p \cdot p' / p^2 = 2E_D / m_B$ and $y = 2p \cdot p_\tau / p^2 = 2E_\tau / m_B$. The differential partial width is then given by

$$\lambda_D(x, y) = \frac{\sqrt{r_\tau}}{\rho_D(x, y)} \sqrt{(x^2 - 4r_D)(y^2 - 4r_\tau) - 4(1 - x - y + \frac{1}{2}xy + r_D + r_\tau)^2}, \quad (30)$$

where $r_\tau = m_\tau^2 / m_B^2$.

The expression for $P_\tau^{\perp(D)}$ can now be written explicitly in terms of the effective four-Fermi interactions of Eq. (6) and then simplified by keeping only the linear terms in the Δ parameters. (The terms quadratic in Δ can easily be included if they are not negligible in a specific model.) This gives

$$P_\tau^{\perp(D)}(x, y) = -\sigma_D(x, y) \text{Im} \Delta_S, \quad (31a)$$

$$\sigma_D(x, y) = h_D(x) \lambda_D(x, y), \quad (31b)$$

$$h_D(x) = 2f_+^2(1 - r_D) + 2f_+ f_- (1 - x + r_D). \quad (31c)$$

To leading order in HQET the function $h_D(x)$ has a very simple form, given by

$$h_D(x) \rightarrow (1 - r_D) \left(1 + \frac{x}{2\sqrt{r_D}} \right) \xi^2. \quad (32)$$

There are three features of these expressions which are of interest. First of all, note that the τ transverse polarization in this decay is proportional to the effective scalar four-Fermi interaction, as was claimed above. This feature is well known in the analogous $K_{\mu 3}$ decay. A second observation is that the polarization function $\sigma_D(x, y)$ is explicitly propor-

$$\frac{d^2\Gamma(B \rightarrow D \tau \bar{\nu})}{dx dy} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{128\pi^3} \rho_D(x, y), \quad (26)$$

with

$$\begin{aligned} \rho_D(x, y) = & |f'_+|^2 g_1(x, y) + 2 \text{Re}(f'_+ f'^*_-) g_2(x, y) \\ & + |f'_-|^2 g_3(x). \end{aligned} \quad (27)$$

The kinematical functions $g_i(x, y)$ are defined in Appendix A.

The transverse polarization of the τ lepton is then defined as in Eq. (1),

$$P_\tau^{\perp(D)} = \frac{d\Gamma(\vec{n}) - d\Gamma(-\vec{n})}{d\Gamma_{\text{total}}}, \quad (28)$$

where $\vec{n} \equiv (\vec{p}_D \times \vec{p}_\tau) / |\vec{p}_D \times \vec{p}_\tau|$ is a unit vector perpendicular to the decay plane, and $d\Gamma(\pm\vec{n})$ is the differential partial width with the τ spin vector along $\pm\vec{n}$. $d\Gamma_{\text{total}}$ denotes the partial width after summing over the lepton spins. The above expression may be written in terms of f'_\pm as follows:

$$P_\tau^{\perp(D)}(x, y) = -\lambda_D(x, y) \text{Im}(2f'_+ f'^*_-), \quad (29)$$

with

tional to the mass of the lepton involved, and is therefore largest for the τ lepton. The transverse polarization of the lepton will then be largest for the τ mode in models for which Δ_S is independent of the lepton mass,² including multi-Higgs-doublet models and R -parity conserving SUSY models with large intergenerational squark mixing. If Δ_S depends on the lepton mass (as in, e.g., R -parity-breaking SUSY models and leptoquark models), then the lepton polarization need not be largest for the case $\ell = \tau$. Our final observation is that, to leading order in HQET, the Dalitz density $\rho_D(x, y)$ is proportional to ξ^2 , so that the polarization function $\sigma_D(x, y)$ is independent of $\xi(w)$. The average polarization [defined below in Eq. (33)] does have a mild dependence on the form of the Isgur-Wise function. This latter remark applies in general to polarization observables. The contour plots for $\rho_D(x, y)$ and $\sigma_D(x, y)$ are given in Fig. 1, taking $\xi(w) = 1.0 - 0.75 \times (w - 1)$, which is representative of the current experimental data [27].

The average polarization over a region of phase space S can be defined as follows:

²Note that the definition of Δ_S includes a factor of $1/m_\ell$ which must be canceled in order for Δ_S to be independent of the lepton mass [see Eq. (19)].

$$\overline{P_\tau^{(D)}} \equiv \frac{\int_S dx dy \rho_D(x, y) P_\tau^{\perp(D)}(x, y)}{\int_S dx dy \rho_D(x, y)}. \quad (33)$$

This average is a measure of the difference between the number of τ leptons with their spins pointing above and below the decay plane divided by the total number of τ leptons in the same region of phase space S . In terms of the four-Fermi interactions, we have

$$\overline{P_\tau^{(D)}} = -\overline{\sigma_D} \text{Im} \Delta_S. \quad (34)$$

Since we are only keeping contributions to the polarization which are first order in Δ_S , $\overline{\sigma_D}$ is independent of Δ_S and we may carry out the integration numerically. Averaging over the whole phase space gives

$$\overline{P_\tau^{(D)}} = -0.22 \times \text{Im} \Delta_S. \quad (35)$$

C. τ polarization in $B \rightarrow D^* \tau \bar{\nu}$ decay

The τ transverse polarization in the decay

$$B(p) \rightarrow D^*(p') \tau(p_\tau) \bar{\nu}(p_\nu) \quad (36)$$

is defined in complete analogy with that for the decay to the D . The general effective four-Fermi interactions of Eq. (6) contribute to this decay with an amplitude given by

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{cb} \bar{u}(p_\tau) \gamma_\mu (1 - \gamma_5) v(p_\nu) \epsilon_\rho^* \mathcal{M}^{\rho\mu}, \quad (37)$$

$$\begin{aligned} \mathcal{M}^{\rho\mu} = & F'_{A0} m_B g^{\rho\mu} + \frac{F'_{A+}}{m_B} (p + p')^\mu q^\rho + \frac{F'_{A-}}{m_B} q^\mu q^\rho \\ & + i \frac{F'_V}{m_B} \epsilon^{\mu\rho\alpha\beta} (p + p')_\alpha q_\beta. \end{aligned} \quad (38)$$

Working again in the B rest frame, we define $x = 2p \cdot p' / p^2 = 2E_{D^*} / m_B$ and $y = 2p \cdot p_\tau / p^2 = 2E_\tau / m_B$. Summing over the spins of the final states, we find the following expression for the differential partial width:

$$\frac{d^2\Gamma(B \rightarrow D^* \tau \bar{\nu})}{dx dy} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{128\pi^3} \rho_{D^*}(x, y), \quad (39)$$

with

$$\begin{aligned} \rho_{D^*}(x, y) = & |F'_{A0}|^2 f_1(x, y) + |F'_{A+}|^2 f_2(x, y) + |F'_{A-}|^2 f_3(x) \\ & + |F'_V|^2 f_4(x, y) + 2 \text{Re}(F'_{A0} F'_{A+}^*) f_5(x, y) \\ & + 2 \text{Re}(F'_{A0} F'_{A-}^*) f_6(x, y) \\ & + 2 \text{Re}(F'_{A+} F'_{A-}^*) f_7(x, y) \\ & + 2 \text{Re}(F'_{A0} F'_V)^* f_8(x, y). \end{aligned} \quad (40)$$

The subscripts of the eight functions denote the corresponding contributions from the different form factors. These functions are collected in Appendix A.

After a kinematic analysis, it is found that only interference terms between the axial form factors contribute to $P_\tau^{\perp(D^*)}$, so that

$$\begin{aligned} P_\tau^{\perp(D^*)} = & -\lambda_{D^*}(x, y) \left[\text{Im}(F'_{A0} F'_{A+}^*) \left(\frac{x}{2r_{D^*}} + 1 \right) \right. \\ & + \text{Im}(F'_{A0} F'_{A-}^*) \left(\frac{x}{2r_{D^*}} - 1 \right) \\ & \left. + \text{Im}(F'_{A+} F'_{A-}^*) \left(\frac{x^2}{2r_{D^*}} - 2 \right) \right], \end{aligned} \quad (41)$$

with

$$\lambda_{D^*}(x, y) = \frac{\sqrt{r_\tau}}{\rho_{D^*}(x, y)} \sqrt{(x^2 - 4r_{D^*})(y^2 - 4r_\tau) - 4(1 - x - y + \frac{1}{2}xy + r_{D^*} + r_\tau)^2}. \quad (42)$$

The τ transverse polarization may now be written in terms of the effective four-Fermi interactions of Eq. (6). Keeping the leading, linear terms in the Δ parameters, we find

$$P_\tau^{\perp(D^*)} = -\sigma_{D^*}(x, y) \text{Im} \Delta_P \quad (43a)$$

$$\sigma_{D^*}(x, y) = h_{D^*}(x) \lambda_{D^*}(x, y) \quad (43b)$$

$$\begin{aligned} h_{D^*}(x) = & [F_{A0} + F_{A+}(1 - r_{D^*}) + F_{A-}(1 - x + r_{D^*})] \\ & \times \left[F_{A0} \left(\frac{x}{2r_{D^*}} - 1 \right) + F_{A+} \left(\frac{x^2}{2r_{D^*}} - 2 \right) \right]. \end{aligned} \quad (43c)$$

The expression for $h_{D^*}(x)$ again has a very simple form in the heavy quark symmetry limit:

$$h_{D^*}(x) \rightarrow (1 - r_{D^*}) \left(1 + \frac{x}{2\sqrt{r_{D^*}}} \right) \xi^2. \quad (44)$$

Comparison with Eq. (32) shows that this expression may be obtained from the analogous expression for $h_D(x)$ by taking $r_D \rightarrow r_{D^*}$.

The τ lepton polarization in the $B \rightarrow D^* \tau \bar{\nu}$ decay is sensitive only to effective pseudoscalar four-Fermi interactions [3]. This observable is thus complementary to its analogue in

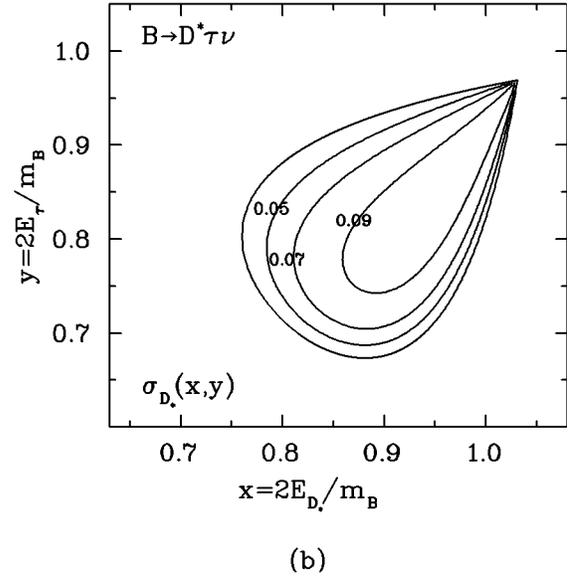
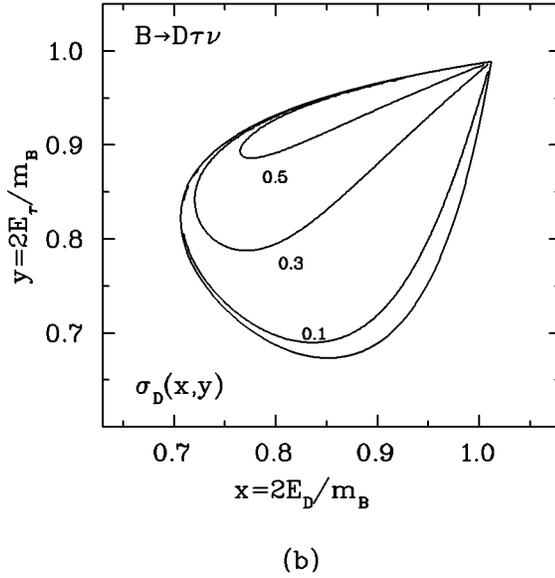
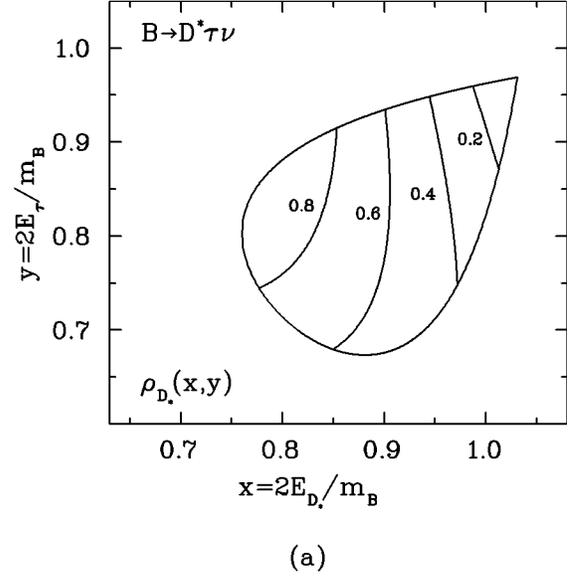
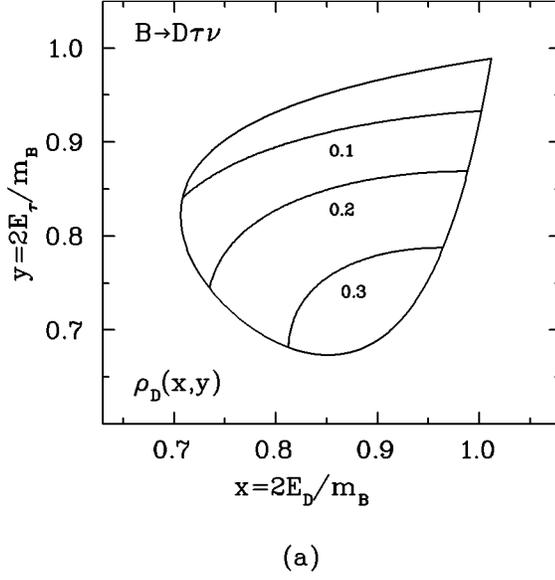


FIG. 1. Contour plots for the semileptonic decay $B \rightarrow D\tau\bar{\nu}$, using $\xi = 1 - 0.75 \times (w - 1)$ for the Isgur-Wise function: (a) the Dalitz density function $\rho_D(x,y)$; (b) the transverse τ polarization function $\sigma_D(x,y)$.

the decay $B \rightarrow D\tau\bar{\nu}$, which is sensitive to effective scalar interactions. We note in passing that Garisto [18] has found the transverse tau polarization in $B \rightarrow D^*\tau\bar{\nu}$ to have an additional dependence on effective right-handed quark-current interactions. There is no discrepancy with our results, however, since the effect which Garisto discusses only arises when one fixes the polarization state of the D^* , instead of summing over polarizations as we have done. The right-handed current effect cancels in the sum. In the next subsection we will discuss an observable which is sensitive to such right-handed interactions, but which requires a measurement

FIG. 2. Contour plots for the semileptonic decay $B \rightarrow D^*\tau\bar{\nu}$, using $\xi = 1 - 0.75 \times (w - 1)$ for the Isgur-Wise function: (a) the Dalitz density function $\rho_{D^*}(x,y)$; (b) the transverse τ polarization function $\sigma_{D^*}(x,y)$.

of *only* the D^* polarization (not that of both the D^* and the τ).

As noted in Sec. II B, the Dalitz density $\rho_{D^*}(x,y)$ is quadratically dependent on $\xi(w)$, whereas the polarization function $\sigma_{D^*}(x,y)$ is to a good approximation independent of $\xi(w)$. The average polarization varies slightly with $\xi(w)$. The contour plots for $\rho_{D^*}(x,y)$ and $\sigma_{D^*}(x,y)$ are given in Fig. 2, taking again $\xi(w) = 1.0 - 0.75 \times (w - 1)$ [27].

The average transverse polarization of the τ lepton can be defined as in Eq. (33). Averaging over the whole phase space gives

$$\overline{P_\tau^{(D^*)}} = -\overline{\sigma_{D^*}} \text{Im}\Delta_P = -0.068 \times \text{Im}\Delta_P. \quad (45)$$

Note that $\overline{\sigma_{D^*}}$ is about a factor of 3 smaller than $\overline{\sigma_D}$. This is because effectively only one of the three polarization states of the D^* , the longitudinal polarization, contributes to the transverse τ polarization [3].

D. D^* polarization in $B \rightarrow D^* \ell \bar{\nu}$ decay

In the previous two subsections we have looked at TOPO's constructed using the spin of the tau in the decays $B \rightarrow D \ell \bar{\nu}$ and $B \rightarrow D^* \ell \bar{\nu}$. Since the D^* is a vector meson, however, the latter channel offers additional TOPO's which may be constructed by using the projection of the D^* polarization transverse to the decay plane. As we have already noted, these new observables will be sensitive to effective right-handed current interactions, making them complementary to the lepton transverse polarization observables discussed above.

Let us denote the three-momenta of the D^* and ℓ in the B rest frame by \vec{p}_{D^*} and \vec{p}_ℓ , respectively. We may then define three orthogonal vectors \vec{n}_1 , \vec{n}_2 , and \vec{n}_3 by

$$\vec{n}_1 \equiv \frac{(\vec{p}_{D^*} \times \vec{p}_\ell) \times \vec{p}_{D^*}}{|\vec{p}_{D^*} \times \vec{p}_\ell| |\vec{p}_{D^*}|}, \quad (46a)$$

$$\vec{n}_2 \equiv \frac{\vec{p}_{D^*} \times \vec{p}_\ell}{|\vec{p}_{D^*} \times \vec{p}_\ell|}, \quad (46b)$$

$$\vec{n}_3 \equiv \frac{\vec{p}_{D^*}}{|\vec{p}_{D^*}|} \frac{m_{D^*}}{E_{D^*}}. \quad (46c)$$

The unusual normalization of \vec{n}_3 is due to the boost from the D^* rest frame to the B rest frame. The constraint $\epsilon^2 = -1$ can now be written in a symmetric form,

$$(\vec{\epsilon} \cdot \vec{n}_1)^2 + (\vec{\epsilon} \cdot \vec{n}_2)^2 + (\vec{\epsilon} \cdot \vec{n}_3)^2 = 1. \quad (47) \quad \text{with}$$

Note that \vec{n}_1 and \vec{n}_3 lie in the decay plane, whereas \vec{n}_2 is perpendicular to the decay plane.

The polarization vector of the D^* can be taken to be real. It is then clear from Eqs. (46a)–(46c) that the D^* polarization projection transverse to the decay plane, $\vec{\epsilon} \cdot \vec{n}_2$, is T odd, and that the D^* polarization projections inside the decay plane, $\vec{\epsilon} \cdot \vec{n}_1$ and $\vec{\epsilon} \cdot \vec{n}_3$, are T even. Since the polarization vector always comes up quadratically in the differential width, the pieces which are odd under time reversal must be proportional to $(\vec{\epsilon} \cdot \vec{n}_2)(\vec{\epsilon} \cdot \vec{n}_1)$ or to $(\vec{\epsilon} \cdot \vec{n}_2)(\vec{\epsilon} \cdot \vec{n}_3)$. For the moment we will define observables explicitly in terms of the D^* polarization vector. At the end of this subsection we will comment on how one could measure these quantities by measuring the angular distributions of the decay products of the D^* .

Let us then formally define a measure of the T -odd correlation involving the D^* polarization as follows

$$P_{D^*}^{(\ell)} \equiv \frac{d\Gamma - d\Gamma'}{d\Gamma_{\text{total}}} = \frac{2d\Gamma_{T \text{ odd}}}{d\Gamma_{\text{total}}}, \quad (48)$$

where $d\Gamma'$ is obtained by performing a T transformation on $d\Gamma$, $d\Gamma_{T \text{ odd}}$ is the T -odd piece in the partial width, and $d\Gamma_{\text{total}}$ is the partial width after summing over polarizations in the final state. Note that there is also an implicit sum over the spin of the final state charged lepton in Eq. (48), so that this observable depends *only* on the D^* polarization. We may then express this observable in terms of the two independent kinematical variables x and y , yielding

$$\begin{aligned} P_{D^*}^{(\ell)}(x, y) = & -(\vec{\epsilon} \cdot \vec{n}_1)(\vec{\epsilon} \cdot \vec{n}_2) \lambda_1(x, y) \text{Im}(F'_{A0} F'_{V^*}) + (\vec{\epsilon} \cdot \vec{n}_3) \\ & \times (\vec{\epsilon} \cdot \vec{n}_2) \lambda_2(x, y) [\text{Im}(F'_{A0} F'_{A^*}) + \text{Im}(F'_{A^+} F'_{V^*}) \\ & \times (x + 2y - 2 - r_\ell) + \text{Im}(F'_{A^-} F'_{V^*}) r_\ell \\ & + \text{Im}(F'_{A0} F'_{V^*}) d_\ell(x, y)], \end{aligned} \quad (49)$$

$$\lambda_1(x, y) = \frac{4[(x^2 - 4r_{D^*})(y^2 - 4r_\ell) - 4(1 - x - y + \frac{1}{2}xy + r_{D^*} + r_\ell)^2]}{\rho_{D^*}(x, y) \sqrt{x^2 - 4r_{D^*}}}, \quad (50a)$$

$$\lambda_2(x, y) = \frac{4\sqrt{(x^2/4r_{D^*}) - 1}}{\rho_{D^*}(x, y)} \sqrt{(x^2 - 4r_{D^*})(y^2 - 4r_\ell) - 4(1 - x - y + \frac{1}{2}xy + r_{D^*} + r_\ell)^2}, \quad (50b)$$

$$d_\ell(x, y) = (y - 1) - \frac{2x(1 + r_{D^*} + r_\ell - x - y + \frac{1}{2}xy)}{x^2 - 4r_{D^*}}, \quad (50c)$$

where $r_\ell = m_\ell^2/m_B^2$, with $\ell = e, \mu, \tau$.

These expressions may be simplified by writing them in terms of the effective four-Fermi interactions of Eq. (6) and neglecting terms quadratic in the Δ parameters. This gives

$$P_{D^*}^{(\ell)}(x,y) = (\vec{\epsilon} \cdot \vec{n}_1)(\vec{\epsilon} \cdot \vec{n}_2)\sigma_1^\ell(x,y)\text{Im}\Delta_R + (\vec{\epsilon} \cdot \vec{n}_3)(\vec{\epsilon} \cdot \vec{n}_2) \times [\sigma_2^\ell(x,y)\text{Im}\Delta_R + \sigma_3^\ell(x,y)\text{Im}\Delta_P], \quad (51)$$

where

$$\sigma_1^\ell(x,y) = -2\lambda_1(x,y)F_{A0}F_V, \quad (52a)$$

$$\sigma_2^\ell(x,y) = 2\lambda_2(x,y)F_V[F_{A+}(x+2y-2-r_\ell) + F_{A-r_\ell} + F_{A0}d_\ell(x,y)], \quad (52b)$$

$$\sigma_3^\ell(x,y) = -\lambda_2(x,y)r_\ell F_V[F_{A0} + F_{A+}(1-r_{D^*}) + F_{A-}(1+r_{D^*}-x)]. \quad (52c)$$

The T -odd D^* polarization observable can thus receive contributions from both right-handed current and effective pseudoscalar interactions. The pseudoscalar contribution is suppressed by r_ℓ , however, so that the decay modes $B \rightarrow D^* e \bar{\nu}$ and $B \rightarrow D^* \mu \bar{\nu}$ may be used to isolate and measure the right-handed current effect. As we have noted above in Secs. II B and II C, the transverse polarization of the τ lepton is sensitive to an effective scalar four-Fermi interaction in the decay $B \rightarrow D \tau \bar{\nu}$, and to a pseudoscalar interaction in the decay $B \rightarrow D^* \tau \bar{\nu}$. Combining all three polarization measurements, it is thus possible to probe separately the three different sources of nonstandard model T violation which we have included in the effective Lagrangian of Eq. (6).³

For the remainder of this section we will concentrate on the $\ell = e$ and μ modes, studying their sensitivity to an effective right-handed current interaction. Aside from the fact that these two channels naturally isolate the effective right-handed interactions, they are also favored by virtue of their larger branching fractions compared to $\ell = \tau$. Given the small masses of the electron and muon compared to the other energy scales in the problem, we may safely set $r_\ell = m_\ell^2/m_B^2 = 0$. We will subsequently also drop the superscript ℓ . The expression for the T -odd D^* polarization observable then becomes

$$P_{D^*}(x,y) = [(\vec{\epsilon} \cdot \vec{n}_1)\sigma_1(x,y) + (\vec{\epsilon} \cdot \vec{n}_3)\sigma_2(x,y)] \times (\vec{\epsilon} \cdot \vec{n}_2)\text{Im}\Delta_R, \quad (53)$$

where, to leading order in HQET, the two polarization functions are given by

$$\sigma_1(x,y) \rightarrow \lambda_1(x,y)(x+2\sqrt{r_D})\frac{\xi^2}{2\sqrt{r_D}}, \quad (54a)$$

$$\sigma_2(x,y) \rightarrow \lambda_2(x,y)(2-x-2y)\frac{1-\sqrt{r_D}}{x-2\sqrt{r_D}}\xi^2. \quad (54b)$$

Note that both $\lambda_1(x,y)$ and $\lambda_2(x,y)$ are proportional to $1/\rho_{D^*}$, and therefore to $1/\xi^2$. The polarization functions $\sigma_i(x,y)$ ($i=1,2$) are then independent of the Isgur-Wise function $\xi(w)$ as noted above. The contour plots for the Dalitz density function $\rho_{D^*}(x,y)$ and the polarization functions $\sigma_1(x,y)$ and $\sigma_2(x,y)$ are shown in Fig. 3, assuming $\xi(w) = 1.0 - 0.75 \times (w-1)$ [27].

We have previously analyzed the two different polarization structures present in the expression for $P_{D^*}(x,y)$ [3]. As was noted there, the term proportional to σ_1 involves only transverse polarization components, while that proportional to σ_2 requires a nonzero longitudinal projection of the D^* polarization in order to be nonvanishing.⁴ In addition to multiplying distinct polarization structures, however, the two functions σ_1 and σ_2 themselves have quite different symmetry properties in the two-dimensional phase space spanned by x and y . In principle, then, there are at least two distinct ways in which to differentiate between the two contributions to $P_{D^*}(x,y)$. The first is to devise a method which can pick out one or the other polarization structure, and the second is to make use of the symmetries of σ_1 and σ_2 in order to differentiate between them. The latter of these two has been discussed in some detail in Ref. [3], so let us first recapitulate those results and then discuss how one can get at the polarization structures themselves.

It is straightforward to demonstrate that $\rho_{D^*}(x,y)\sigma_2(x,y)$ is antisymmetric under the exchange of lepton and antineutrino energies, and that the allowed phase space region is symmetric under the same exchange. Thus, integrating over all of phase space — or over any region which is symmetric under the exchange — eliminates the σ_2 term and leaves only the piece due to the σ_1 term. In order to pick out σ_2 , we note that $\rho_{D^*}\sigma_1$ is symmetric under $y \rightarrow 2-x-y$ and $x \rightarrow x$, so that an asymmetric average over phase space may be used to eliminate the σ_1 term. In both cases, these properties are independent of the functional forms of the form factors. Performing these averages over all phase space then yields for the nonvanishing piece in the two cases

$$\overline{P_{D^*}^{(1)}} \approx 0.51 \times (\vec{\epsilon} \cdot \vec{n}_1)(\vec{\epsilon} \cdot \vec{n}_2)\text{Im}\Delta_R, \quad (55)$$

$$\overline{P_{D^*}^{(2)}} \equiv \frac{\int dx \left(\int_{y_{\min}}^{y_{\text{mid}}} dy - \int_{y_{\text{mid}}}^{y_{\max}} dy \right) \rho_{D^*}(x,y) P_{D^*}(x,y)}{\int dx dy \rho_{D^*}(x,y)} \approx 0.40 \times (\vec{\epsilon} \cdot \vec{n}_2)(\vec{\epsilon} \cdot \vec{n}_3)\text{Im}\Delta_R, \quad (56)$$

where $y_{\text{mid}} \equiv (y_{\min} + y_{\max})/2$. The asymmetric-average approach used to pick out the σ_2 term in Eq. (56) also works when the lepton is not massless. In fact, this method also eliminates the extra pseudoscalar term which is present in Eq. (51), so that even for $\ell = \tau$ it is possible to isolate the

³We note again that T -odd observables are typically insensitive to new *left*-handed interactions since the interference of such diagrams with the SM diagram does not lead to observable phases.

⁴This latter term would be absent for on-shell massless vector bosons such as the photon.

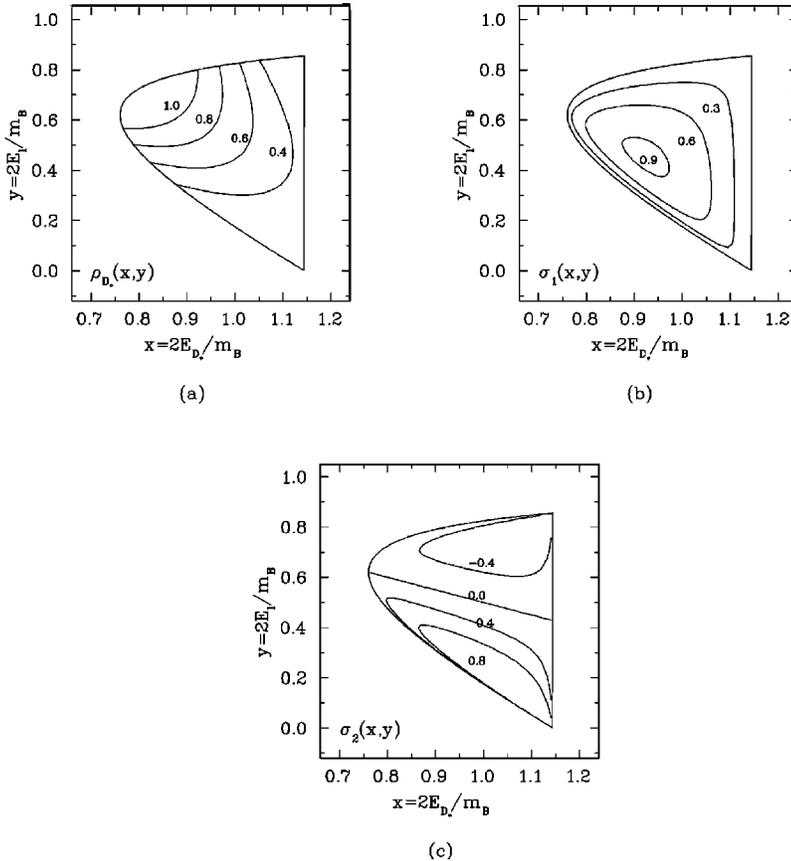


FIG. 3. Contour plots for the semileptonic decay $B \rightarrow D^* \ell \bar{\nu}$ ($\ell = e, \mu$), using $\xi = 1 - 0.75 \times (w - 1)$ for the Isgur-Wise function: (a) the Dalitz density function $\rho_{D^*}^{(\ell)}(x, y)$; (b) the D^* polarization function $\sigma_1(x, y)$; (c) the D^* polarization function $\sigma_2(x, y)$. The masses of the leptons are neglected in these plots.

right-handed current contribution. Numerically, however, the average D^* polarization found using this prescription is about a factor of 3 smaller for the tau compared to the electron and muon channels.

An alternative method for differentiating between the two polarization structures is to examine the angular distributions of the decay products of the vector meson. It is straightforward to demonstrate that the resulting asymmetries (integrated appropriately over the momenta of the final state particles) have the same structure as the terms which define $P_{D^*}^{(\ell)}(x, y)$ in Eq. (51), up to the replacement of the factors $(\vec{\epsilon} \cdot \vec{n}_i)(\vec{\epsilon} \cdot \vec{n}_j)$ by a numerical factor of $1/\pi$. In Appendix B, we demonstrate this explicitly for the decay mode⁵ $D^* \rightarrow D\pi$.

Before we turn to the section on model estimates, it is worth pointing out that the numerical coefficients in Eqs. (35), (45), (55), and (56), evaluated at leading order in the heavy quark expansion, will be modified when the effects due to finite quark masses and QCD corrections are included. The uncertainty in the Isgur-Wise function can also affect these coefficients but to a much lesser extent, as mentioned earlier. In order to get a feel for the size of these corrections, we have reevaluated the coefficients using the QCD sum rule estimates for $\xi(w)$ and for the form factors given in Table

5.1 of Ref. [26]. The estimates in Ref. [26] correspond to next-to-leading order in the $1/m_Q$ expansion of HQET. Our findings are that the correction to the τ polarization in $B \rightarrow D\tau\bar{\nu}$ is less than one percent of the value quoted in Eq. (35), while the analogous correction for $B \rightarrow D^*\tau\bar{\nu}$ leads to an increase of about 15% in the magnitude of the polarization. For the D^* polarization in $B \rightarrow D^*\ell\bar{\nu}$ ($\ell = e, \mu$), $P_{D^*}^{(1)}$ and $P_{D^*}^{(2)}$ were found to increase, respectively, by about 20% and 25% relative to the values quoted in Eqs. (55) and (56). Considering the uncertainties in our current knowledge of the form factors, we will simply use the leading order results obtained in this section when making our model estimates. It should be understood, however, that more precise knowledge of the form factors could change our estimates (generally increasing them) by up to about 25%. The main results of our general analysis are listed in Table I.

III. MODEL ESTIMATES

In this section we examine the prospects for the various T -odd observables, both in supersymmetric models and in some nonsupersymmetric models. We start by looking at SUSY models that conserve R parity. In this case, there are no CP -violating contributions to our observables at tree level. As we have noted elsewhere, however [3], there can be rather large effects (even though they occur at one loop) in SUSY models with intergenerational squark mixings. In this case both the τ polarization and D^* polarization observables can receive sizable contributions. We then examine models in which R parity is explicitly violated. In such models,

⁵In the case of the neutral D^* , and depending on the experimental setup, it might be easier to use the $D^* \rightarrow D\gamma$ mode, since the two photons from the π^0 decay could be quite soft. The charged pions produced in the case of charged D^* decays, however, should be easier to detect [28].

TABLE I. T -odd polarization observables (TOPO's) for exclusive semileptonic B decays in terms of effective scalar (Δ_S), pseudoscalar (Δ_P), right-handed quark current (Δ_R), and left-handed quark current (Δ_L) four-Fermi interactions. $P_\tau^{(D)}$ and $P_\tau^{(D^*)}$ denote the transverse τ polarization in the $B \rightarrow D\tau\bar{\nu}$ and $B \rightarrow D^*\tau\bar{\nu}$ decays, respectively; $P_{D^*}^{(\ell)}$ ($\ell = e, \mu, \tau$) denotes the T -odd D^* polarization observable in the $B \rightarrow D^*\ell\bar{\nu}$ decay.

	$\text{Im}\Delta_S$	$\text{Im}\Delta_P$	$\text{Im}\Delta_R$	$\text{Im}\Delta_L$
$P_\tau^{(D)}$	✓	0	0	0
$P_\tau^{(D^*)}$	0	✓	0	0
$P_{D^*}^{(e, \mu)}$	0	0	✓	0
$P_{D^*}^{(\tau)}$	0	✓	✓	0

T -violating scalar and pseudoscalar interactions can arise at tree level, leading to nonzero values for the transverse τ polarization in the decays $B \rightarrow D(*)\tau\bar{\nu}$. The sizes of these observables are subject to stringent experimental constraints. We next consider some non-SUSY extensions of the SM. We first examine the multi-Higgs-doublet and leptoquark models, which can both induce effective scalar and pseudoscalar four-Fermi interactions at tree level. Then we look at left-right symmetric models, where we focus on the effects due to the extra gauge bosons only, and give an estimate of the size of the T -odd D^* polarization observable. The results obtained in these models are summarized in Table II.

A. SUSY with intergenerational squark mixing

The notion of squark family mixings comes from the observation that the mass matrices of the quarks and squarks are generally expected to be diagonalized by different unitary transformations in generation space [29–31]. The relative flavor rotations between the $\tilde{u}_L, \tilde{u}_R, \tilde{d}_L$, and \tilde{d}_R squarks and their corresponding quark partners are denoted by the three by three unitary matrices $V^{U_L}, V^{U_R}, V^{D_L}$, and V^{D_R} , respectively. The significance of these mixings for

TABLE II. Contributions to the effective four-Fermi interactions and to the various TOPO's from SUSY with squark intergenerational mixing, SUSY with R -parity violation, the three Higgs-doublet model (3HDM), leptoquark models, and left-right symmetric models (LRSM's). We have neglected the effects due to charged Higgs bosons in LRSM's, assuming that the Higgs bosons are sufficiently heavy to decouple. The numbers in the table are the maximal polarization effects and are meant mainly for the purpose of illustration. Their actual sizes in particular models will depend on the details of the models.

	squark mixing	R SUSY	3HDM	Leptoquarks	LRSM
Δ_S	✓	✓	✓	✓	0
Δ_P	✓	✓	✓	✓	0
Δ_R	✓	0	0	0	✓
$ P_\tau^{(D)} $	0.35	0.05	~ 1	~ 1	0
$ P_\tau^{(D^*)} $	0.05	0.008	0.3	0.2	0
$ P_{D^*}^{(1)} $	0.02	0	0	0	0.08
$ P_{D^*}^{(2)} $	0.016	0	0	0	0.06

T -violating semileptonic meson decays was noted in a previous work [12] and discussed in some detail for the transverse muon polarization in $K_{\mu 3}^+$ [12] and $K_{\mu 2\gamma}^+$ [16] decays. In Sec. III A, we focus on the various TOPO's in different exclusive semileptonic B decay channels [3].

To estimate the maximal T -violation effects in semileptonic B decays, we consider the one-loop diagrams [12,16] with a gluino (\tilde{g}) and top and bottom squarks (\tilde{t}, \tilde{b}) in the loop, and with W or charged Higgs exchange. The relevant mixing matrix elements involved are V_{32}^U and V_{33}^D . When the mixing is large, we can have doubly-enhanced T -violation effects — due to mixing and to the large top quark mass. Note that flavor changing neutral current processes only constrain the combinations $V^U V^{U*}$ and $V^D V^{D*}$. For example, D - \bar{D} mixing can put nontrivial constraints on the product $V_{32}^U V_{31}^{U*}$. We will assume maximal mixing between the $(\tilde{t}_R, \tilde{c}_R)$ squarks and thus take $|V_{32}^{UR}| = 1/\sqrt{2}$ to estimate the maximal polarization effects.

1. H^+ exchange and τ lepton polarization

Charged Higgs exchange can give rise to effective scalar and pseudoscalar but not vector and axial-vector interactions, as can be seen from Lorentz invariance of the amplitude. It could thus contribute to the transverse polarization of the τ lepton in both $B \rightarrow D\tau\bar{\nu}$ and $B \rightarrow D^*\tau\bar{\nu}$ decays, but it does not contribute to the D^* polarization in the e, μ modes. Furthermore, in the large $\tan\beta$ limit, the induced effective scalar and pseudoscalar interactions from W -boson exchange are suppressed by $1/\tan\beta$ relative to the charged Higgs exchange. To estimate the maximal size of P_τ^\perp , we need only consider charged Higgs contributions.

The m_t -enhanced effective scalar-pseudoscalar four-Fermi interaction can be estimated from the diagram that contains a $\tilde{g}-\tilde{t}-\tilde{b}$ loop and the $H^- \tilde{t}_R \tilde{b}_L^*$ vertex. It is given by [16,3]

$$\mathcal{L}_H = \frac{4G_F}{\sqrt{2}} C_H (\bar{c}_R b_L) (\bar{\tau}_R \nu_L) + \text{H.c.}, \quad (57)$$

with

$$C_H = -\frac{\alpha_s}{3\pi} I_H \tan\beta \frac{m_t m_\tau}{m_H^2} \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} V_{33}^H V_{33}^{D_L} V_{32}^{U_R*}, \quad (58)$$

where $\alpha_s \approx 0.1$ is the QCD coupling evaluated at the mass scale of the sparticles in the loop, A_t is the soft SUSY breaking A term for the top squark, μ denotes the two Higgs superfields mixing parameter, $\tan\beta$ is the ratio of the two Higgs VEVs, $m_{\tilde{g}}$ is the mass of the gluino and V_{ij}^H is the mixing matrix in the charged-Higgs-squark coupling $H^+ \tilde{u}_{iR}^* \tilde{d}_{jL}$. The integral function I_H is given by

$$I_H = \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \times \frac{2}{(m_{\tilde{t}}^2/m_{\tilde{g}}^2) z_1 + (m_{\tilde{b}}^2/m_{\tilde{g}}^2) z_2 + (1-z_1-z_2)}, \quad (59)$$

which is equal to one at $m_{\tilde{t}}=m_{\tilde{b}}=m_{\tilde{g}}$ and varies slowly away from this degenerate point. The contributions to Δ_S and Δ_P from charged Higgs exchange are then given by

$$\Delta_S = -\frac{\alpha_s}{3\pi} I_H \tan\beta \frac{m_B}{(m_b - m_c)} \frac{m_B m_t}{m_H^2} \times \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} \times \frac{[V_{33}^H V_{33}^{DL} V_{32}^{UR*}]}{V_{cb}}, \quad (60)$$

$$\Delta_P = \frac{\alpha_s}{3\pi} I_H \tan\beta \frac{m_B}{(m_b + m_c)} \frac{m_B m_t}{m_H^2} \times \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} \times \frac{[V_{33}^H V_{33}^{DL} V_{32}^{UR*}]}{V_{cb}}. \quad (61)$$

To estimate the maximal τ polarization effects, we assume $|V_{33}^{DL}|=|V_{33}^H|\sim 1$, $m_H=100$ GeV and $\tan\beta=50$ [32]. With maximal squark mixings, $|V_{32}^{UR}|=1/\sqrt{2}$. Setting $|\mu|=A_t=m_{\tilde{g}}$, $m_t=180$ GeV, $m_b=4.5$ GeV, $m_c=1.5$ GeV, $V_{cb}=0.04$, and $I_H=1$, we find

$$|\Delta_S| \leq 1.6, \quad (62)$$

$$|\Delta_P| \leq 0.8., \quad (63)$$

Averaging over the whole phase space gives, for $B \rightarrow D \tau \bar{\nu}$,

$$|\overline{P_\tau^{(D)}}| = 0.22 \times |\text{Im}\Delta_S| \leq 0.35, \quad (64)$$

and, for $B \rightarrow D^* \tau \bar{\nu}$,

$$|\overline{P_\tau^{(D^*)}}| = 0.068 \times |\text{Im}\Delta_P| \leq 0.05. \quad (65)$$

Both limits scale as

$$(100 \text{ GeV}/m_H)^2 (\tan\beta/50) [\text{Im}(V_{33}^H V_{33}^{DL} V_{32}^{UR*})/(1/\sqrt{2})].$$

In the absence of squark family mixing, the polarization effects are suppressed by a factor of $m_t V_{32}^{UR}/m_b V_{cb} \sim 10^3$.

2. W exchange and D^* polarization

As has been shown in Sec. II D, the T -odd polarization correlation of the D^* in the decay $B \rightarrow D^* \ell \bar{\nu}$ (with $\ell = e, \mu$) is only sensitive to an effective right-handed (RH) quark current interaction. With squark generational mixing, an effective RH interaction can be induced at one loop by the W -boson exchange diagram with left-right mass insertions in both the top and bottom squark propagators. This leads to a term in the effective Lagrangian given by [16,3]

$$\mathcal{L}_W = -\frac{4G_F}{\sqrt{2}} C_0 (\bar{c}_R \gamma^\alpha b_R) (\bar{\ell}_L \gamma_\alpha \nu_L) + \text{H.c.}, \quad (66)$$

with

$$C_0 = \frac{\alpha_s}{36\pi} I_0 \frac{m_t m_b (A_t - \mu \cot\beta) (A_b - \mu \tan\beta)}{m_{\tilde{g}}^4} \times V_{33}^{SKM} V_{32}^{UR*} V_{33}^{DR}, \quad (67)$$

where A_b is the soft SUSY breaking A term for the bottom squark, V_{ij}^{SKM} is the super CKM matrix associated with the W -squark coupling $W^+ \tilde{u}_{iL}^* \tilde{d}_{jL}$, and the integral function I_0 is given by

$$I_0 = \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \times \frac{24z_1 z_2}{[(m_{\tilde{t}}^2/m_{\tilde{g}}^2) z_1 + (m_{\tilde{b}}^2/m_{\tilde{g}}^2) z_2 + (1-z_1-z_2)]^2}. \quad (68)$$

Note that $I_0=1$ for $m_{\tilde{t}}/m_{\tilde{g}}=m_{\tilde{b}}/m_{\tilde{g}}=1$, but it increases rapidly to ~ 8 as the squark-to-gluino mass ratios decrease to $m_{\tilde{t}}/m_{\tilde{g}}=m_{\tilde{b}}/m_{\tilde{g}}=\frac{1}{2}$.

The Δ_R parameter of Eq. (23) is then given by

$$\Delta_R = -\frac{\alpha_s}{36\pi} I_0 \times \frac{m_t m_b (A_t - \mu \cot\beta) (A_b - \mu \tan\beta)}{m_{\tilde{g}}^4} \frac{V_{33}^{SKM} V_{32}^{UR*} V_{33}^{DR}}{V_{cb}}. \quad (69)$$

To estimate the maximal size of Δ_R from the W -exchange diagram, we take $I_0=5$, $\tan\beta=50$, $A_t=A_b=|\mu|=m_{\tilde{g}}=200$ GeV, and $|V_{33}^{DR}|=|V_{33}^{SKM}|=1$. With maximal squark mixing ($|V_{32}^{UR}|=1/\sqrt{2}$), we have the upper limit

$$|\Delta_R| \leq 0.08. \quad (70)$$

The averages of the two TOPO's related to the D^* polarization are given in Eqs. (55) and (56). Choosing the optimal orientations of the polarization vector in the two cases and inserting the above bound on $|\Delta_R|$ yields the following upper limits

$$|\overline{P_{D^*}^{(1)}}| \leq 0.02, \quad (71)$$

$$|\overline{P_{D^*}^{(2)}}| < 0.016. \quad (72)$$

These limits for the D^* polarization scale as $(200 \text{ GeV}/M_{\text{SUSY}})^2 (\tan\beta/50) (I_0/5) [\text{Im}(V_{33}^{SKM} V_{32}^{UR*} V_{33}^{DR})/(1/\sqrt{2})]$, where M_{SUSY} is the SUSY breaking scale. In the absence of squark intergenerational mixing, the D^* polarization effect will be suppressed by a factor of $m_t V_{32}^{UR}/m_c V_{cb} \sim 10^3$.

B. R -parity violating theories

The requirement of gauge invariance does not uniquely specify the form of the superpotential in a generic supersymmetric model. In addition to the terms which are usually present, one could also add the following terms:

$$\lambda_{ijk}L_iL_jE_k^c + \bar{\lambda}_{ijk}L_iQ_jD_k^c + \lambda''_{ijk}U_i^cD_j^cD_k^c + \mu_iL_iH', \quad (73)$$

where the coefficients could in general be complex and where i, j and k are generation indices. Note that we have omitted the implicit sum over $SU(2)_L$ and $SU(3)_C$ indices and that $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. Of the four types of terms listed above, the last one may be rotated away by a redefinition of the L and H fields [33].

The above λ and $\bar{\lambda}$ terms violate lepton number whereas the λ'' term violates baryon number. All three terms may be forbidden by imposing a discrete symmetry called R parity [34]. Alternatively, one can use the experimental data to place constraints on these R -parity-breaking couplings. The most stringent constraints are on the $\bar{\lambda}\lambda''$ combinations and come from the nonobservation of proton decay [35]. To satisfy the proton stability requirement, one can also invoke a discrete Z_3 symmetry called baryon parity which naturally allows for the lepton number violating terms while forbidding the baryon number violating λ'' term [36]. For this reason, we will simply set $\lambda''_{ijk} = 0$ in our analysis. The R -parity-violating interactions in the Lagrangian may then be written in the mass basis of the component fields as

$$\begin{aligned} \mathcal{L}^R = & -2\lambda_{ijk}[(v_L^i)^c e_L^j \bar{e}_R^{k*} + \bar{e}_R^k e_L^j \bar{\nu}_L^i + \bar{e}_R^k \nu_L^i \bar{e}_L^j] \\ & -\lambda'_{ijk}\{(V_{KM})_{j\ell}[(v_L^i)^c d_L^\ell \bar{d}_R^{k*} + \bar{d}_R^k d_L^\ell \bar{\nu}_L^i + \bar{d}_R^k \nu_L^i \bar{d}_L^\ell] \\ & -[(e_L^i)^c u_L^j \bar{d}_R^{k*} + \bar{d}_R^k u_L^j \bar{e}_L^i + \bar{d}_R^k e_L^i \bar{u}_L^j]\} + \text{H.c.} \end{aligned} \quad (74)$$

The $\bar{\lambda}$ and λ' parameters are related by unitary rotations in generation space [37]. Note that while the above parametrization is not unique (one could, for example, put V_{KM}^\dagger in the ‘‘up’’ sector rather than V_{KM} in the ‘‘down’’ sector) the physics itself is parametrization independent.

Integrating out the relevant supersymmetric particles of Eq. (74) gives rise to two types of contributions to the quark-level transition $b \rightarrow c \ell \bar{\nu}$. The first type of contribution has the SM $V-A$ structure and cannot interfere with the SM W -exchange diagram in order to give rise to observable T -violating effects. The second type of contribution can induce scalar and pseudoscalar effective interactions. The relevant effective interaction for the τ mode is given by

$$\mathcal{L}_{\text{eff}}^R = -\frac{1}{2} \frac{\lambda_{3j3} \lambda'_{j23}}{(m_{\tilde{e}_L^j})^2} \bar{c}(1 + \gamma^5) b \bar{\tau}(1 - \gamma^5) \nu_\tau + \text{H.c.}, \quad (75)$$

where summation over $j=1,2$ is implied. The resulting expressions for the corresponding Δ parameters are then

$$\Delta_S = -\frac{1}{2} \frac{\lambda_{3j3} \lambda'_{j23}}{(m_{\tilde{e}_L^j})^2} \left(\frac{\sqrt{2}}{G_F V_{cb}} \right) \frac{m_B^2}{(m_b - m_c) m_\tau}, \quad (76)$$

$$\Delta_P = -\frac{1}{2} \frac{\lambda_{3j3} \lambda'_{j23}}{(m_{\tilde{e}_L^j})^2} \left(\frac{\sqrt{2}}{G_F V_{cb}} \right) \frac{m_B^2}{(m_b + m_c) m_\tau}. \quad (77)$$

Setting the slepton masses to 100 GeV we obtain the following estimates

$$\Delta_S \approx -8 \times 10^2 \frac{\lambda_{3j3} \lambda'_{j23}}{(m_{\tilde{e}_L^j}/100 \text{ GeV})^2}, \quad (78)$$

$$\Delta_P \approx -4 \times 10^2 \frac{\lambda_{3j3} \lambda'_{j23}}{(m_{\tilde{e}_L^j}/100 \text{ GeV})^2}. \quad (79)$$

The tau polarization is subject to constraints from present experimental data. The rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ gives the bound $|\lambda'_{j23}| < 0.01$ [37], whereas $|\lambda_{133}| < 0.001$ from bounds on the neutrino mass [38] and $|\lambda_{233}| < 0.03$ from leptonic tau decays [39]. We have assumed in each case a mass of 100 GeV for the sparticles. We thus arrive at the following 90% confidence level upper bounds on the transverse τ polarizations,

$$|\overline{P_\tau^{(D)}}| < 0.05, \quad (80)$$

$$|\overline{P_\tau^{(D^*)}}| < 0.008. \quad (81)$$

In the limit of degenerate sparticle masses, these bounds are independent of the sparticle mass scale.

We noted above that there are actually two types of R -parity violating processes which could contribute to the quark-level transition $b \rightarrow c \ell \bar{\nu}$. The first of these was ignored since it has the SM $V-A$ structure and thus cannot interfere with the SM W -exchange diagram, while the second was seen to give rise to an effective scalar-pseudoscalar interaction. It is interesting to note, however, that the SM-like term can *also* give rise to a T -odd transverse τ polarization if it interferes with the tree-level charged-Higgs diagram which is generically present in supersymmetric models. This effect is technically of second order in the Δ parameters, yet it need not be small if we take the current upper limit on $\tan\beta/m_H$, which is approximately 0.5 GeV^{-1} [32]. In this limit the magnitude of the effect could be comparable to the limits quoted in Eqs. (80) and (81). Note also that while R -parity violating interactions can give rise to scalar and pseudoscalar interactions, there is no tree-level induced right-handed current interaction which could contribute to the T -odd D^* polarization.

C. Nonsupersymmetric models

Let us now estimate the contributions to the TOPO's in some non-SUSY models. We will consider in turn the three-Higgs-doublet model (3HDM), leptoquark models and left-right symmetric models (LRSM's).

1. Multi-Higgs-doublet model

An effective scalar-pseudoscalar four-Fermi interaction can be induced by tree-level charged Higgs exchange with CP -violating complex couplings. To be specific, let us consider the three Higgs-doublet model [40,41]. The charged Higgs couplings to the fermions are given by

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i=1}^2 (\alpha_i \overline{U}_L V_{KM} M_D D_R + \beta_i \overline{U}_R M_U V_{KM} D_L + \gamma_i \overline{\nu}_L M_E E_R) H_i^+ + \text{H.c.}, \quad (82)$$

where M_U , M_D , and M_E are the diagonal mass matrices for the up-type quarks, down-type quarks, and charged leptons, respectively. The complex couplings α_i , β_i , and γ_i appear in the unitary mixing matrix between the mass eigenstates and gauge eigenstates of the charged Higgs boson. They satisfy six constraints, three of which are

$$\frac{\text{Im}(\alpha_1 \beta_1^*)}{\text{Im}(\alpha_2 \beta_2^*)} = \frac{\text{Im}(\alpha_1 \gamma_1^*)}{\text{Im}(\alpha_2 \gamma_2^*)} = \frac{\text{Im}(\beta_1 \gamma_1^*)}{\text{Im}(\beta_2 \gamma_2^*)} = -1. \quad (83)$$

It is clear from these relations that the CP -violating effective scalar and pseudoscalar interactions will always be proportional to $(1/m_{H_1^+}^2 - 1/m_{H_2^+}^2)$. Assuming that H_2^+ is much heavier than H_1^+ , we find that the scalar and pseudoscalar Δ parameters are given by

$$\Delta_S = \frac{(\alpha_1 \gamma_1^* m_b + \beta_1 \gamma_1^* m_c) m_B^2}{m_{H_1^+}^2 (m_b - m_c)}, \quad (84)$$

$$\Delta_P = \frac{(\alpha_1 \gamma_1^* m_b - \beta_1 \gamma_1^* m_c) m_B^2}{m_{H_1^+}^2 (m_b + m_c)}. \quad (85)$$

Current data place a more stringent bound on $\text{Im}(\beta_1 \gamma_1^*)$ than on $\text{Im}(\alpha_1 \gamma_1^*)$ [41]. For $m_{H_1^+} < 440$ GeV, the inclusive process $B \rightarrow X \tau \bar{\nu}$ gives the strongest limit of $|\text{Im}(\alpha_1 \gamma_1^*)|/m_{H_1^+}^2 < 0.2$ GeV $^{-2}$ at the 95% C.L. [19]. This limit in turn constrains the Δ 's by $|\text{Im}\Delta_S| < 8$ and $|\text{Im}\Delta_P| < 4$. Therefore, the τ transverse polarizations in $B \rightarrow D \tau \bar{\nu}$ and $B \rightarrow D^* \tau \bar{\nu}$ decays are given by

$$|\overline{P}_\tau^{(D)}| \leq \sim 1, \quad (86)$$

$$|\overline{P}_\tau^{(D^*)}| < 0.3, \quad (87)$$

which is in agreement with a previous estimate [18]. Qualitatively similar results have been found in the inclusive case [17,19].

2. Leptoquarks

Both scalar and vector leptoquark models [42] can give rise to effective scalar and pseudoscalar interactions for the semileptonic B decays. The calculation of the τ transverse polarization in these models is similar to the analysis of the muon transverse polarization in $K^+ \rightarrow \pi^0 \mu^+ \nu$ decay [43]. Unlike in that case, however, the current experimental data allow for a rather large τ polarization in B decays. The difference compared to $K_{\mu 3}$ is that the bound on the couplings for B decay comes mainly from $t \rightarrow c \tau^+ \tau^-$ and is much weaker than that for the $K_{\mu 3}$ decay, which comes from $D \rightarrow \mu^+ \mu^-$.

Let us consider, as an example, the following $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant leptoquark interaction,

$$\mathcal{L} = (\lambda_{ij} \overline{Q}_i e_{Rj} + \lambda'_{ij} \overline{u}_{Ri} L_j) \phi + \text{H.c.}, \quad (88)$$

where Q and L denote the usual quark and lepton doublets, respectively, ϕ is a color-triplet, weak-doublet scalar leptoquark, and i, j are the family indices. An effective scalar-pseudoscalar four-Fermi interaction is then induced by the exchange of the scalar leptoquark, giving⁶

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \frac{\lambda_{33}^* \lambda'_{23}}{m_\phi^2} (\overline{c_R} b_L) (\overline{\tau_R} \nu_{L\tau}). \quad (89)$$

The resulting expressions for Δ_S and Δ_P are given by

$$\Delta_S = -50 \times \lambda_{33}^* \lambda'_{23} \times \left(\frac{200 \text{ GeV}}{m_\phi} \right)^2 \quad (90)$$

$$\Delta_P = -\frac{m_b - m_c}{m_b + m_c} \Delta_S, \quad (91)$$

so that the transverse τ polarization in $B \rightarrow D \tau \bar{\nu}$ and $B \rightarrow D^* \tau \bar{\nu}$ decays can be, respectively, of order unity and 0.2 if we take $|\text{Im}(\lambda_{33}^* \lambda'_{23})| \sim 0.1$. Note that leptoquark exchange does not give rise to a right-handed current at tree level.

3. Left-right symmetric models

An effective right-handed quark current can be induced at tree level in left-right symmetric models (LRSM's) [44]. We will concentrate on this effect and neglect the effective scalar and pseudoscalar interactions by assuming that the charged Higgs decouple. Consider the most general class of models with gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The charged gauge boson mass eigenstates are related to the weak eigenstates by the following two by two unitary matrix,

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = \begin{pmatrix} \cos \zeta & -\sin \zeta \\ e^{i\omega} \sin \zeta & e^{i\omega} \cos \zeta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}, \quad (92)$$

where ζ is the W_L - W_R mixing angle and ω is a CP -violating phase. The bounds on m_{W_2} and ζ depend on the relation between the CKM mixing matrix for the left-handed quarks, $V^L = V_{KM}$, and the analogous mixing matrix V^R for the right-handed quarks. In any case, m_{W_2} is at least heavier than several hundred GeV [45,46], and we can safely neglect its effect for the purposes of our estimate.

The presence of the off-diagonal term in the W_L - W_R mixing matrix means that the lighter mass eigenstate W_1 can induce an effective right-handed current interaction of the form $(\overline{c_R} \gamma_\mu b_R) (\overline{\ell}_L \gamma^\mu \nu_L)$. The resulting expression for Δ_R has the simple form

⁶We neglect for simplicity the effective tensor interaction which is also induced by this exchange.

$$\Delta_R = -e^{i\omega} \zeta \frac{g_R V_{cb}^R}{g_L V_{cb}^L}, \quad (93)$$

where g_L and g_R are the gauge couplings for $SU(2)_L$ and $SU(2)_R$, respectively. We will assume $g_L = g_R$ for our estimate.

Stringent bounds on the W_L - W_R mixing have been derived by assuming manifest left-right symmetry ($V^L = V^R$) or pseudo-manifest left-right symmetry [$V^R = K_1(V^L) * K_2$, where K_1 and K_2 are diagonal phase matrices]. Thus, for example, $|\zeta| < 4\%$ from μ decay experiments [47], $|\zeta| < 4 \times 10^{-3}$ from the analysis of $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decays (subject to some theoretical hadronic uncertainties) [48], and $|\zeta| < 5 \times 10^{-3}$ from semileptonic d and s decays [49]. The upper bound on $|\text{Im}\Delta_R|$ is then in the range

$$|\text{Im}\Delta_R| \leq |\zeta| < (0.004 \sim 0.04), \quad (94)$$

and the D^* polarization in these scenarios is smaller than $10^{-3} - 10^{-2}$.

If one does not impose manifest or pseudo-manifest left-right symmetry, the constraints on ζ tend to become less stringent. Thus, for example, it is possible to have $|V_{cb}^R| = 1$ and $|\zeta| \leq 0.013$ at the 90% C.L. [46]. The induced right-handed current can be significantly enhanced in this case, since

$$|\text{Im}\Delta_R| \leq 25 \times |\zeta| \leq 0.32 \quad (95)$$

and the T -odd D^* polarization in the $B \rightarrow D^* \ell \bar{\nu}$ ($\ell = e, \mu$) decays could be as large as 8%.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have examined several of the T -odd polarization observables in the exclusive semileptonic decays $B \rightarrow D^{(*)} \ell \bar{\nu}$. We have provided a model-independent analysis of these observables, concentrating on the τ transverse polarization in $B \rightarrow D^{(*)} \tau \bar{\nu}$ and on the T -odd D^* polarization in the decays $B \rightarrow D^* \ell \bar{\nu}$, with $\ell = e, \mu$. These observables provide an attractive place in which to look for effects coming from new physics. As is known, they receive negligible contributions from standard model sources. Furthermore, they are quite clean theoretically, depending only on a small number of q^2 -dependent form factors which are in principle measurable or calculable on the lattice or within the context of heavy quark effective theory. We have also noted that the three types of observables under consideration are sensitive separately to three different types of quark-level effective interactions: the τ polarization in the decay to the D (D^*) probes effective scalar (pseudoscalar) interactions, and the T -odd D^* polarization depends only on effective right-handed current interactions. This observation is independent of the functional forms of the form factors. A final general remark concerning these observables is that the branching ratios for these decays should be quite accessible at the planned B factories. Using the leading order results of HQET and taking $\xi(w) = 1.0 - 0.75 \times (w - 1)$ (as we have in our numerical work), we find that

$$\begin{aligned} & B(B \rightarrow D \tau \bar{\nu}) : B(B \rightarrow D^* \tau \bar{\nu}) : B(B \rightarrow D \ell \bar{\nu}) : B(B \rightarrow D^* \ell \bar{\nu}) \\ & \sim \frac{1}{10} : \frac{1}{4} : \frac{1}{3} : 1, \end{aligned} \quad (96)$$

with $\ell = e$ or μ . While these ratios should be taken as being only approximate, they do indicate that one can expect branching ratios for the first two decays (which are currently unmeasured) to be of order one percent. They also show that, all else being equal, the experimental sensitivity to a T -violating effective right-handed current interaction is much greater than that to a scalar or pseudoscalar interaction. This is particularly true if one combines the measurements in the electron and muon modes.

In this work we have not included the effects of possible tensor interactions. In all of the models which we have considered — with the possible exception of the leptoquark models — such effects are either not present or are quite small. It is worth noting, however, that a model-independent analysis of tensor effects may also be performed along the same lines as followed here [50]. It is also straightforward to derive the tensor form factors for both the $B \rightarrow D$ and $B \rightarrow D^*$ transitions in HQET.

It is interesting to compare the sensitivity of the tau transverse polarization in $B_{\tau 3}$ to that of the muon in $K_{\mu 3}^+$. *A priori* one expects the polarization effect to be larger for $B_{\tau 3}$ than for $K_{\mu 3}^+$ due to the larger quark and lepton masses in the B case. The lepton polarization in these two cases may generically be written as $P_\ell \sim \sigma_\ell \times \text{Im}\Delta_S^\ell$, where the kinematical polarization function σ_ℓ contains a helicity suppression factor, $\sigma_\ell \propto m_\ell / m_M$ (m_M is the mass of the decaying meson), and where Δ_S^ℓ is a model-dependent parameter which measures the strength of the effective scalar interaction. The relative sizes of Δ_S^τ and Δ_S^μ are model dependent, so let us consider the 3HDM as an example. In this case the ratio $\Delta_S^\tau / \Delta_S^\mu$ is enhanced roughly by the factor m_B^2 / m_K^2 . Thus, up to numerical factors of order unity, the transverse lepton polarization is enhanced by $P_\tau / P_\mu \sim m_B m_\tau / m_K m_\mu \sim 10^2$. Similar qualitative analyses can be performed for the other models which we have considered. The rather large enhancement which one generically finds implies that in order to reach a given sensitivity to new physics, one requires far fewer B decays than K decays. The B system, as we have noted above, has the added advantage that there are several semileptonic B decay channels which have no analogue in the K system and which may in principle be used to identify separately the various possible sources of T violation.

Although we have considered here only the decays $B \rightarrow D^{(*)} \ell \bar{\nu}$, our results may also be applied to the related decays $B \rightarrow \pi(\rho, \omega) \ell \bar{\nu}$. The results of HQET are not applicable to these decays, so that the form factors need to be obtained using phenomenological models and/or experimental data. It is expected, however, that the T -odd polarization effects in these modes could be just as large as for the $b \rightarrow c$ transitions. The usefulness of these decays as probes for T -odd signals of new physics may be limited, however, since their branching ratios are expected to be smaller by one to two orders of magnitude.

In conclusion, we have presented a general analysis of several T -odd polarization observables in the semileptonic B

decays to D and D^* mesons. We have given numerical estimates of these observables in both supersymmetric R -parity-conserving and R -parity-breaking models as well as in some nonsupersymmetric extensions of the SM, namely the three-Higgs-doublet model, leptoquark models, and left-right symmetric models. The results of these model estimates have been summarized in Table II. It is encouraging that the polarization effects in many of these models can be in the range of a few percent to several tens of percent and could thus be accessible to the planned B factories.

ACKNOWLEDGMENTS

We would like to thank M. Atiya, D. Coward, S. Dawson, M. Diwan, R. Garisto, F. Goldhaber, Y. Grossman, Y. Kuno, K. Lau, W. Marciano, F. Paige, M. Pospelov, and A. Soni for useful conversations. This work was partially supported by the Natural Sciences and Engineering Research Council of Canada. K.K. is also grateful to the High Energy Theory Group at Brookhaven National Laboratory for support provided under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy.

APPENDIX A

In this appendix we define the kinematical functions $g_i(x,y)$ and $f_i(x,y)$ which arise in the definitions of $\rho_D(x,y)$ and $\rho_{D^*}(x,y)$. They are given, for lepton ℓ , by

$$g_1(x,y) = (3-x-2y+r_\ell-r_D)(x+2y-1-r_\ell-r_D) - (1+x+r_D)(1-x+r_D-r_\ell), \quad (\text{A1a})$$

$$g_2(x,y) = r_\ell(3-x-2y-r_D+r_\ell), \quad (\text{A1b})$$

$$g_3(x) = r_\ell(1-x+r_D-r_\ell), \quad (\text{A1c})$$

and

$$f_1(x,y) = (1-x+r_{D^*}-r_\ell) + \frac{1}{r_{D^*}}(x+y-1-r_{D^*}-r_\ell) \times (1-y+r_\ell-r_{D^*}), \quad (\text{A2a})$$

$$f_2(x,y) = [(x+2y-1-r_{D^*}-r_\ell)(3-x-2y-r_{D^*}+r_\ell) - (1-x+r_{D^*}-r_\ell)(1+x+r_{D^*})] \left(\frac{x^2}{4r_{D^*}} - 1 \right), \quad (\text{A2b})$$

$$f_3(x) = r_\ell(1-x+r_{D^*}-r_\ell) \left(\frac{x^2}{4r_{D^*}} - 1 \right), \quad (\text{A2c})$$

$$f_4(x,y) = 2xy(1-y+r_\ell-r_{D^*}) + 2x(2-x-y) \times (x+y-1-r_{D^*}-r_\ell) - 4(1-y+r_\ell-r_{D^*}) \times (x+y-1-r_{D^*}-r_\ell) - 4r_{D^*}y(2-x-y), \quad (\text{A2d})$$

$$f_5(x,y) = \frac{1}{r_{D^*}}x(1-y)(x+y-1) - \frac{r_\ell}{2r_{D^*}}x(3-2x-3y - r_{D^*}+r_\ell) + 2(1-y)(1-x-y) - x + 2r_{D^*} - r_\ell(x+y), \quad (\text{A2e})$$

$$f_6(x,y) = \frac{r_\ell}{2r_{D^*}}[x(1-y+r_\ell-r_{D^*}) - 2r_{D^*}(2-x-y)], \quad (\text{A2f})$$

$$f_7(x,y) = r_\ell(3-x-2y-r_{D^*}+r_\ell) \left(\frac{x^2}{4r_{D^*}} - 1 \right), \quad (\text{A2g})$$

$$f_8(x,y) = 2y(1-y+r_\ell-r_{D^*}) - 2(2-x-y) \times (x+y-1-r_{D^*}-r_\ell). \quad (\text{A2h})$$

APPENDIX B: FOUR-BODY FINAL STATES

In this appendix we demonstrate how the two T -odd D^* polarization observables defined in the text [see Eqs. (49), (55), and (56)] may be related to T -odd momentum correlations in the four-body final state of the decay $B \rightarrow D^*(D\pi)\ell\bar{\nu}$. The two observables have different structures in terms of the D^* polarization vector and may be separately extracted by employing suitable integration prescriptions in the integration over the momentum of the final state pion. We will examine two different types of prescriptions and calculate the statistical error in each case. A previous analysis of T -odd asymmetries in the four-body final state may be found in Refs. [22,23], where it was noted that the final state interaction effects on the T -odd observables are probably negligible. One could similarly study the T -odd momentum correlations in the channel $B \rightarrow D^*(D\gamma)\ell\bar{\nu}$, but this channel will not be examined here.

Let us then calculate the differential partial width for $B \rightarrow D^*(D\pi)\ell\bar{\nu}$. The Feynman rule for the effective $D^{*\mu}-\pi-D$ vertex is simply given by fp_π^μ [51], where the constant f may be inferred from the partial width of the decay $D^* \rightarrow \pi D$. This width is given by

$$\Gamma(D^* \rightarrow \pi D) = \left(\frac{1}{3} \right) \frac{|f|^2 (p_\pi)^3}{8\pi m_{D^*}^2}, \quad (\text{B1})$$

where

$$p_\pi = \frac{1}{2m_{D^*}} \lambda^{1/2}(m_{D^*}^2, m_\pi^2, m_D^2) \quad (\text{B2})$$

denotes the magnitude of the pion momentum in the D^* rest frame and $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. In order to calculate the decay rate for $B \rightarrow D^*(D\pi)\ell\bar{\nu}$, we need to sum over the intermediate states of the D^* , which may be done either by using a Breit-Wigner propagator for the D^* or by employing a density matrix approach. The resulting expression for the partial differential width in the B rest frame is given by

$$\frac{d^2\Gamma(B \rightarrow D^*(D\pi)\ell\bar{\nu})}{dx dy} \Big|_S = \frac{3m_B G_F^2 |V_{cb}|^2}{512\pi^4 (p_\pi^*)^2} \left(\int_S d\Omega_\pi^* |\tilde{\mathcal{M}}|^2 \right) \times B(D^* \rightarrow \pi D), \quad (\text{B3})$$

where

$$\tilde{\mathcal{M}} = \mathcal{M}^{\rho\alpha} \left(p_{\pi\rho} - p_{D^*\rho} \frac{p_\pi \cdot p_{D^*}}{m_{D^*}^2} \right) \bar{u}_L(p_\ell) \gamma_\alpha v_L(p_\nu), \quad (\text{B4})$$

and where $\mathcal{M}^{\rho\alpha}$ has been defined above in Eq. (38). The angular integral in Eq. (B3) is to be performed *in the rest frame of the decaying D^** using some prescription ‘‘S.’’ This prescription may be designed such that it picks out the T -odd contributions.

The angles in the D^* rest frame may be defined as follows

$$\vec{p}_B = |\vec{p}_B| (0, 0, -1), \quad (\text{B5})$$

$$\vec{p}_\ell = |\vec{p}_\ell| (\sin\theta_\ell, 0, \cos\theta_\ell), \quad (\text{B6})$$

$$\vec{p}_\pi = |\vec{p}_\pi| (\sin\theta_\pi \cos\phi_\pi, \sin\theta_\pi \sin\phi_\pi, \cos\theta_\pi), \quad (\text{B7})$$

where \vec{p}_B , \vec{p}_ℓ , and \vec{p}_π are the momenta *in the rest frame of the D^** . There are then in principle three T -odd structures which one may construct in terms of the pion momentum. These are

$$\vec{p}_\pi \cdot (\vec{p}_B \times \vec{p}_\ell) \sim \sin\theta_\pi \sin\phi_\pi, \quad (\text{B8})$$

$$\begin{aligned} & (\vec{p}_\pi \cdot \vec{p}_\ell) \vec{p}_\pi \cdot (\vec{p}_B \times \vec{p}_\ell) \\ & \sim \sin\theta_\pi \cos\theta_\pi \sin\phi_\pi, \quad \sin^2\theta_\pi \sin\phi_\pi \cos\phi_\pi, \end{aligned} \quad (\text{B9})$$

$$(\vec{p}_\pi \cdot \vec{p}_B) \vec{p}_\pi \cdot (\vec{p}_B \times \vec{p}_\ell) \sim \sin\theta_\pi \cos\theta_\pi \sin\phi_\pi. \quad (\text{B10})$$

Only the latter two structures are present in the partial width since, in the D^* rest frame,

$$\left(p_{\pi\rho} - p_{D^*\rho} \frac{p_\pi \cdot p_{D^*}}{m_{D^*}^2} \right) \rightarrow g_{\rho i} p_\pi^i, \quad (\text{B11})$$

so that all terms in the squared amplitude are bilinear in the pion momentum. The observable T -odd functional forms are then given by

$$T_1(\theta_\pi, \phi_\pi) = \sin^2\theta_\pi \sin\phi_\pi \cos\phi_\pi, \quad (\text{B12})$$

$$T_2(\theta_\pi, \phi_\pi) = \sin\theta_\pi \cos\theta_\pi \sin\phi_\pi. \quad (\text{B13})$$

There are several integration prescriptions which may be used to extract the terms in the width which are proportional to T_1 and T_2 . In general these reduce to weighting the differential width by some function $f(\theta_\pi, \phi_\pi)$ in such a way that only the desired piece survives the angular integration. We shall examine two such prescriptions in this appendix. The first approach (prescription ‘‘A’’) is closely related to

that used in Ref. [23] and amounts to weighting the angular integral by ± 1 , depending on the angle. In the second approach (prescription ‘‘B’’), the integrand is weighted by the functional form itself, which also has the effect of eliminating all but the desired piece. As we shall show, prescription ‘‘B’’ is statistically more efficient than prescription ‘‘A.’’

Let us first consider prescription ‘‘A.’’ In this case the integrand is weighted by ± 1 as a function of the angle. Two different such prescriptions may be used to pick out separately the terms proportional to T_1 and T_2 , while eliminating all other terms. It is straightforward to verify that the following two prescriptions do the job:

$$\begin{aligned} T_1(\theta_\pi, \phi_\pi): \int_{A_1} d\Omega_\pi^* & \equiv \int_0^\pi \sin\theta_\pi d\theta_\pi \left(\int_0^{\pi/2} - \int_{\pi/2}^\pi \right. \\ & \left. + \int_\pi^{3\pi/2} - \int_{3\pi/2}^{2\pi} \right) d\phi_\pi, \end{aligned} \quad (\text{B14})$$

$$\begin{aligned} T_2(\theta_\pi, \phi_\pi): \int_{A_2} d\Omega_\pi^* & \equiv \left(\int_0^{\pi/2} - \int_{\pi/2}^\pi \right) \sin\theta_\pi d\theta_\pi \\ & \times \left(\int_0^\pi - \int_\pi^{2\pi} \right) d\phi_\pi. \end{aligned} \quad (\text{B15})$$

We may then define the following normalized asymmetries

$$\begin{aligned} \mathcal{A}_{A_1}(x, y) & \equiv \left(\frac{d^2\Gamma_{A_1}^{4\text{-bdy}}}{dx dy} \right) \left(\frac{d^2\Gamma^{4\text{-bdy}}}{dx dy} \right)^{-1} \\ & = -\frac{1}{\pi} \lambda_1(x, y) \text{Im}(F'_{A_0} F_V'^*) \end{aligned} \quad (\text{B16})$$

$$(\text{B17})$$

and

$$\begin{aligned} \mathcal{A}_{A_2}(x, y) & \equiv \left(\frac{d^2\Gamma_{A_2}^{4\text{-bdy}}}{dx dy} \right) \left(\frac{d^2\Gamma^{4\text{-bdy}}}{dx dy} \right)^{-1} \\ & = \frac{1}{\pi} \lambda_2(x, y) [\text{Im}(F'_{A_0} F_{A_+}^*) + \text{Im}(F'_{A_+} F_V'^*) \\ & \quad \times (x + 2y - 2 - r_\ell) + \text{Im}(F'_{A_-} F_V'^*) r_\ell \\ & \quad + \text{Im}(F'_{A_0} F_V'^*) d_\ell(x, y)]. \end{aligned} \quad (\text{B18})$$

The above two asymmetries are proportional to the two terms in the expression given for the polarization of the D^* in Eq. (49), that is,

$$P_{D^*}^{(\ell)}(x, y) = (\vec{\epsilon} \cdot \vec{n}_2) \pi [(\vec{\epsilon} \cdot \vec{n}_1) \mathcal{A}_{A_1}(x, y) + (\vec{\epsilon} \cdot \vec{n}_3) \mathcal{A}_{A_2}(x, y)]. \quad (\text{B20})$$

We have thus confirmed our assertion that the two polarization structures in Eq. (49) may be measured separately by following the decay of the D^* and studying the T -odd momentum correlations in the resulting four-body final state.

We now turn to prescription ‘‘B.’’ In this case the differential width is weighted by the functional form itself in the angular integration. One may easily verify that weighting the

width by T_i picks out the term proportional to T_i and eliminates all other terms. Prescription ‘‘B’’ is then defined by:

$$T_1(\theta_\pi, \phi_\pi): \int_{B_1} d\Omega_\pi^* \equiv \int d\Omega_\pi^* \left(\frac{10}{\pi} \right) T_1(\theta_\pi, \phi_\pi), \quad (\text{B21})$$

$$T_2(\theta_\pi, \phi_\pi): \int_{B_2} d\Omega_\pi^* \equiv \int d\Omega_\pi^* \left(\frac{10}{\pi} \right) T_2(\theta_\pi, \phi_\pi). \quad (\text{B22})$$

The normalizing factor of $10/\pi$ has been included so that the resulting asymmetries [defined in analogy with Eqs. (B16) and (B18)] have the same numerical value using either method; that is, $\mathcal{A}_{B_i}(x, y) = \mathcal{A}_{A_i}(x, y)$.

In order to compare prescriptions ‘‘A’’ and ‘‘B,’’ it is useful to calculate the statistical uncertainties which would be expected in a measurement of the two asymmetries, \mathcal{A}_{A_i} and \mathcal{A}_{B_i} , given some number of events N . In particular, we will calculate the uncertainties of the averaged quantities $\overline{\mathcal{A}_{A_i}}$ and $\overline{\mathcal{A}_{B_i}}$, in which the averages over x and y are performed as prescribed in Eqs. (55) and (56), for $i=1$ and 2, respectively. The numerical calculations will be carried out for the electron and muon channels, since these are the modes which we have concentrated on in the text.

We first define the expectation value of some operator \mathcal{O} as follows:

$$\langle \mathcal{O} \rangle \equiv \frac{\int dx dy \int d\Omega_\pi^* |\mathcal{M}|^2 \mathcal{O}}{\int dx dy \int d\Omega_\pi^* |\mathcal{M}|^2}. \quad (\text{B23})$$

This expectation value corresponds to a ‘‘measurement’’ of the operator \mathcal{O} in the probability distribution defined by $|\mathcal{M}|^2$. The statistical error for this observable, given N events, is then

$$\sigma_{\mathcal{O}} = \frac{\sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}}{\sqrt{N}}. \quad (\text{B24})$$

The four averaged asymmetries may be expressed in terms of this compact notation by writing

$$\overline{\mathcal{A}_{A_i}} = \langle \mathcal{O}_{A_i} \rangle, \quad \overline{\mathcal{A}_{B_i}} = \langle \mathcal{O}_{B_i} \rangle, \quad (\text{B25})$$

where the four operators are given by

$$\mathcal{O}_{A_i} = \pm 1, \quad \mathcal{O}_{B_i} = \pm \left(\frac{10}{\pi} \right) T_i(\theta_\pi, \phi_\pi). \quad (\text{B26})$$

The appropriate sign to choose in the above expressions depends in general on θ_π , ϕ_π , and y .

It is now straightforward to calculate the statistical uncertainties associated with the averaged asymmetries in the prescriptions ‘‘A’’ and ‘‘B.’’ In order to evaluate these numerically, we may safely neglect the term $\langle \mathcal{O} \rangle^2$ in Eq. (B24), since it is the square of the averaged asymmetry and is typically quite small compared to $\langle \mathcal{O}^2 \rangle$, which is of order unity. Taking $\xi(w) = 1.0 - 0.75 \times (w - 1)$ and setting the Δ 's to zero in $|\mathcal{M}|^2$, we find

$$\sigma_{A_1} \approx \frac{\sqrt{\langle \mathcal{O}_{A_1}^2 \rangle}}{\sqrt{N}} = \frac{1}{\sqrt{N}}, \quad (\text{B27})$$

$$\sigma_{A_2} \approx \frac{\sqrt{\langle \mathcal{O}_{A_2}^2 \rangle}}{\sqrt{N}} = \frac{1}{\sqrt{N}}, \quad (\text{B28})$$

$$\sigma_{B_1} \approx \frac{\sqrt{\langle \mathcal{O}_{B_1}^2 \rangle}}{\sqrt{N}} = \frac{0.75}{\sqrt{N}}, \quad (\text{B29})$$

$$\sigma_{B_2} \approx \frac{\sqrt{\langle \mathcal{O}_{B_2}^2 \rangle}}{\sqrt{N}} = \frac{0.89}{\sqrt{N}}. \quad (\text{B30})$$

We could, alternatively, calculate the number of events required to achieve a given statistical uncertainty. In this case, the ratio of the number of events required in prescriptions ‘‘B’’ and ‘‘A’’ is given by

$$\frac{N_{B_1}}{N_{A_1}} \approx 0.57, \quad (\text{B31})$$

$$\frac{N_{B_2}}{N_{A_2}} \approx 0.79. \quad (\text{B32})$$

Thus prescription ‘‘B’’ is more efficient than prescription ‘‘A,’’ as we have asserted.

-
- [1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)
 [2] For a review, see for example, *B Decays*, edited by S. Stone (World Scientific, Singapore, 1994).
 [3] G.-H. Wu, K. Kiers, and J. N. Ng, Phys. Lett. B **402**, 159 (1997).
 [4] J. J. Sakurai, Phys. Rev. **109**, 980 (1958); N. Cabibbo and A. Maksymowicz, Phys. Lett. **9**, 352 (1964); **11**, 360(E) (1964); **14**, 72(E) (1966).
 [5] M. K. Campbell *et al.*, Phys. Rev. Lett. **47**, 1032 (1981); S. R.

- Blatt *et al.*, Phys. Rev. D **27**, 1056 (1983).
 [6] Y. Kuno, in *CP Violation, It's Implications to Particle Physics and Cosmology*, Proceedings of the Topical Conference, Tsukuba, Japan, 1993, edited by Y. Kuno and Y. Okada [Nucl. Phys. B (Proc. Suppl.) **37A**, 87 (1994)].
 [7] M. V. Diwan *et al.*, AGS Proposal — Search for T Violating Muon Polarization in $K^+ \rightarrow \mu^+ \pi^0 \nu$ Decay, 1996 (unpublished).
 [8] A. R. Zhitnitskii, Yad. Fiz. **31**, 1024 (1980) [Sov. J. Nucl. Phys. **31**, 529 (1980)].

- [9] M. Leurer, Phys. Rev. Lett. **62**, 1967 (1989); P. Castoldi, J. M. Frere, and G. Kane, Phys. Rev. D **39**, 2633 (1989); H. Y. Cheng, *ibid.* **28**, 150 (1983).
- [10] H. Y. Cheng, Phys. Rev. D **26**, 143 (1982); R. Garisto and G. Kane, *ibid.* **44**, 2038 (1991); G. Belanger and C. Q. Geng, *ibid.* **44**, 2789 (1991).
- [11] E. Christova and M. Fabbrichesi, Phys. Lett. B **315**, 113 (1993); M. Fabbrichesi and F. Vissani, Phys. Rev. D **55**, 5334 (1997).
- [12] G.-H. Wu and J. N. Ng, Phys. Lett. B **392**, 93 (1997).
- [13] W. J. Marciano, in *Future Directions in Particle and Nuclear Physics at Multi-GeV Hadron Beam Facilities*, Proceedings of the Workshop, Upton, New York, 1983, edited by D. Geesaman (BNL Report No. 52389, Upton, 1993).
- [14] C. Q. Geng and S. K. Lee, Phys. Rev. D **51**, 99 (1995).
- [15] M. Kobayashi, T.-T. Lin, and Y. Okada, Prog. Theor. Phys. **95**, 361 (1996).
- [16] G.-H. Wu and J. N. Ng, Phys. Rev. D **55**, 2806 (1997).
- [17] D. Atwood, G. Eilam, and A. Soni, Phys. Rev. Lett. **71**, 492 (1993).
- [18] R. Garisto, Phys. Rev. D **51**, 1107 (1995).
- [19] Y. Grossman and Z. Ligeti, Phys. Lett. B **347**, 399 (1995).
- [20] Y. Kuno, Chin. J. Phys. **32**, 1015 (1994).
- [21] K. Lau (private communication).
- [22] E. Golowich and G. Valencia, Phys. Rev. D **40**, 112 (1989).
- [23] J. G. Körner, K. Schilcher, and Y. L. Wu, Phys. Lett. B **242**, 119 (1990).
- [24] L. B. Okun and I. B. Khriplovich, Yad. Fiz. **6**, 821 (1967) [*Sov. J. Nucl. Phys.* **6**, 598 (1968)].
- [25] N. Isgur and M. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
- [26] For a review, see for example, M. Neubert, Phys. Rep. **245**, 259 (1994).
- [27] A. Ryd, contribution to the *Proceedings of the 2nd International Conference on B Physics and CP Violation*, Honolulu, Hawaii, 1997, edited by T. E. Browder, F. A. Harris, and S. Pakvasa (World Scientific, Singapore, in press).
- [28] D. Coward (private communication).
- [29] B. A. Campbell, Phys. Rev. D **28**, 209 (1983); M. J. Duncan, Nucl. Phys. **B221**, 285 (1983); J. F. Donoghue, H. P. Nilles, and D. Wyler, Phys. Lett. **128B**, 55 (1983); L. J. Hall, V. A. Kostelecky, and S. Raby, Nucl. Phys. **B267**, 415 (1986); F. Gabbiani and A. Masiero, *ibid.* **B322**, 235 (1989); J. S. Hagelin, S. Kelley, and T. Tanaka, *ibid.* **B415**, 293 (1994); F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, *ibid.* **B477**, 321 (1996).
- [30] M. Dine, R. Leigh, and A. Kagan, Phys. Rev. D **48**, 4269 (1993); Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993); M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B420**, 468 (1994).
- [31] N. Arkani-Hamed, H.-C. Cheng, and L. J. Hall, Phys. Rev. D **53**, 413 (1996); **54**, 2242 (1996).
- [32] Y. Grossman, H. Haber, and Y. Nir, Phys. Lett. B **357**, 630 (1995); J. A. Coarasa, R. A. Jimenez, and J. Sola, hep-ph/9701392.
- [33] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).
- [34] P. Fayet, Phys. Lett. **69B**, 489 (1977); G. Farrar and P. Fayet, *ibid.* **76B**, 575 (1978).
- [35] A. Yu. Smirnov and F. Vissani, Phys. Lett. B **380**, 317 (1996).
- [36] L. E. Ibanez and G. G. Ross, Nucl. Phys. **B386**, 3 (1992).
- [37] K. Agashe and M. Graesser, Phys. Rev. D **54**, 4445 (1996).
- [38] R. M. Godbole, P. Roy, and X. Tata, Nucl. Phys. **B401**, 67 (1993).
- [39] V. Barger, G. F. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989); A. Pich, in *Tau 96*, Proceedings of the 4th Workshop, Estes Park, Colorado, edited by J. Smith and W. Toki [Nucl. Phys. B (Proc. Suppl.) **55C**, 3 (1997)], hep-ph/9612308.
- [40] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).
- [41] Y. Grossman, Nucl. Phys. **B426**, 355 (1994).
- [42] For a recent review of leptoquark phenomenology see, for example, J. L. Hewett and T. G. Rizzo, this issue Phys. Rev. D **56**, 5709 (1997), and references therein.
- [43] See, for example, G. Belanger and C. Q. Geng in Ref. [10].
- [44] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); **11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, *ibid.* **12**, 1502 (1975); G. Senjanovic, Nucl. Phys. **B153**, 334 (1979).
- [45] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982).
- [46] P. Langacker and S. U. Sankar, Phys. Rev. D **40**, 1569 (1989).
- [47] J. Carr *et al.*, Phys. Rev. Lett. **51**, 627 (1983); A. E. Jodidio *et al.*, Phys. Rev. D **34**, 1967 (1986); **37**, 237(E) (1988).
- [48] J. F. Donoghue and B. R. Holstein, Phys. Lett. **113B**, 382 (1982).
- [49] L. Wolfenstein, Phys. Rev. D **29**, 2130 (1984).
- [50] K. Kiers and G.-H. Wu (in preparation).
- [51] V. M. Belyaev, V. M. Braun, A. Khodjamirian, and R. Rückl, Phys. Rev. D **51**, 6177 (1995); L. Angelos and G. P. Lepage, *ibid.* **45**, R3021 (1992).