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Spectrum of scalar-scalar bound states

W. Mödritsch*
Institut für Theoretische Physik, Technische Universität Wien, A-1040 Wien, Wiedner Hauptstraße 8-10, Wien, Austria
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An exactly solvable, Barbieri-Remiddi-like equation for bound states of two scalar constituents interacting with massless vector particles is presented for both stable and unstable particles. With the help of this equation the bound state spectrum is calculated to $O(\alpha^4)$ for a SU(N) non-Abelian gauge theory. The result for the Abelian case reproduces the known result from previous calculations. It is shown how different graphs as in the fermionic theory contribute to the spectrum to this order. Furthermore the bound state correction to the decay width for a weakly decaying system is calculated. This result is equal to its fermionic counterpart. Thus the theorem on bound state corrections for weakly decaying particles, formulated previously for fermions only, has been extended to the scalar theory. [S0556-2821(97)06021-9]

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I. INTRODUCTION

While the discussion of fermionic bound states has a long history [1], much less attention has been paid to the similar problem with scalar constituents. Only the ladder approximation with scalar interaction is a well-known example and has already been discussed in the 1950s and 1960s [2] in the framework of the Bethe-Salpeter (BS) equation. Indeed, to this day the only known fundamental matter fields are fermionic. But in supersymmetric theories for each fermion two scalar partners are required. Since some of them, probably stop or sbottom, could have masses within the reach of the next generation of e^+e^- accelerators, even the observation of bound states of those particles seems possible. These objects and systems built of scalar composite particles in atomic physics underline the need for an equally clear and transparent approach as the one developed for the fermionic case [3]. A recent attempt in this direction [4] splits the boson propagator into a particle and antiparticle propagator in order to be able to treat them like fermions. The spectrum is then obtained by constructing the Hamiltonian via a Foldy-Wouthuysen transformation and a perturbation theory in the manner of Salpeter. Using the perturbation theory derived in the first two works of [4] could avoid the divergencies due to higher powers of the spatial momentum that would appear in a pure Foldy-Wouthuysen approach [5]. Furthermore Ref. [4] introduces a real two-body formalism opposite to [5] where the Coulomb field appears as an external field that makes this formalism appear to be not very reliable. All these drawbacks can be circumvented by developing an exactly solvable zero-order equation and subsequently using a systematic perturbation theory.

To the best of our knowledge there exists no attempt in the literature to construct a solvable zero-order equation for the BS equation containing two charged scalars interacting via a vector field. This goal will be achieved in Sec. II.

In Sec. III we will review briefly the BS perturbation theory and use it to calculate the spectrum of bound states for scalar particles with equal mass for both the Abelian and non-Abelian case to $O(\alpha^4)$. This will be of importance if the stop has a narrow width. If the width becomes comparable to the level splittings this consideration can be understood as a determination of the scalar-antiscalar potential.

The decay width is also subject of the second application we present in Sec. IV. We calculate the bound state correction to the decay width Γ of system of scalar constituents to $O(\alpha^2\Gamma)$.

Finally Sec. V is devoted to the conclusions and to the discussion of our results.

II. A BOUND STATE EQUATION FOR SCALAR PARTICLES

A. Stable particles

As a starting point we present here an exactly solvable equation for a stable scalar particle and its antiparticle, which interact via a vector field. The extension to the unequal mass case is given in Appendix B.

We start from the BS equation for a bound state wave function χ:

$$\chi^{\mathrm{BS}}_{ij}(p) = -iS_{ii'}\left(\frac{P}{2}+p\right)S_{j'j}\left(-\frac{P}{2}+p\right)\times\int\frac{d^4p'}{(2\pi)^4}K_{i'j',i''j''}(P,p,p')\chi^{\mathrm{BS}}_{i''j''}(p'), \quad (1)$$

where S is the exact scalar propagator, and K is the sum of all two scalar irreducible graphs. Both are normalized to be Feynman amplitudes. Furthermore, we have introduced relative momenta p and p', a total momentum $P=p_1-p_2$ (where p_1 is the momentum of the particle and p_2 of the antiparticle in opposite direction), and we choose the center of mass (CM) frame where $P=(P_0,\vec{0})=(2m+E,\vec{0})$.

As a first approximation to Eq. (1) we would like to use, in addition to the free relativistic scalar propagators, the kernel due to the Coulomb interaction

$$K_C(p,p') = 4\pi\alpha\,\frac{(P_0+p_0+p_0')(P_0-p_0-p_0')}{(\vec{p}-\vec{p}\,')^2}. \quad (2)$$

*Electronic address: wmoedrit@tph16.tuwien.ac.at

For a non-Ablelian theory with gauge group SU(N) we use

$$\alpha = \frac{N^2-1}{2N}\frac{g^2}{4\pi}. \tag{3}$$

In this case χ has to be a singlet in order for K_C to represent an attractive force. The kernel (2) has the drawback that it is p_0 dependent and the exact solution of Eq. (1) with Eq. (2) is not known. However, in the nonrelativistic regime by the scaling argument [6]

$$p_0 = O(m\alpha^2), \quad |\vec{p}| = O(m\alpha), \tag{4}$$

$$P_0 = 2m - O(m\alpha^2),$$

we can start with an instantaneous approximation to the kernel since p_0 is of $O(\alpha^2 m)$ in this region and may be included in the corrections afterwards. Doing this, we may write an equation for $\chi(\vec{p}) := \int (dp_0/2\pi)\chi(p)$ by performing zero-component integrations on both sides of Eq. (1). On the right-hand side (RHS) we get by using the relativistic free propagator ($E_p = \sqrt{m^2+\vec{p}^2}$)

$$-i\int \frac{dp_0}{2\pi}\frac{1}{[(P_0/2+p_0)^2-E_p^2+i\epsilon][(-P_0/2+p_0)^2-E_p^2+i\epsilon]} = \frac{1}{2E_pP_0}\left[\frac{1}{2E_p-P_0}-\frac{1}{2E_p+P_0}\right] = \frac{1}{E_p(4E_p^2-P_0^2)} \tag{5}$$

and it is quite easy to show that a zero-order Kernel K_0 defined by

$$K_0(p,p') := 4\pi\alpha\frac{4m\sqrt{E_pE_{p'}}}{\vec{q}^2} \tag{6}$$

gives the solvable equation

$$\chi(\vec{p}) = -\frac{1}{E_p(P_0^2-4E_p^2)}\int\frac{d^3p}{(2\pi)^3}4\pi\alpha\frac{4m\sqrt{E_pE_{p'}}}{\vec{q}^2}\chi(\vec{p}\,'). \tag{7}$$

This equation is exactly solvable since $\chi(\vec{p}) = \phi(\vec{p})/\sqrt{E_p}$ transforms it into a Schrödinger equation with Coulomb potential.

Thus we have, for $\chi(p)_{nlm}$ (the eigenfunctions to the quantum numbers nlm),

$$\chi_{nlm}(p) = i\frac{\sqrt{E_p}[(M_n^{(0)})^2-4E_p^2]}{\sqrt{2M_n^{(0)}}[(M_n^{(0)}/2+p_0)^2-E_p^2+i\epsilon][(-M_n^{(0)}/2+p_0)^2-E_p^2+i\epsilon]}\phi_{nlm}(\vec{p}), \tag{8}$$

$$\overline{\chi(p)} = \chi(p) \quad \text{for } \phi \text{ real} \tag{9}$$

to the eigenvalues for P_0:

$$M_n^{(0)} = 2m\sqrt{1-\sigma_n^2}, \quad \sigma_n = \frac{\alpha}{2n}. \tag{10}$$

Here $\phi_{nlm}(\vec{p})$ denotes the Coulomb wave function in momentum space. Equation (9) is dictated by the requirement that $\overline{\chi}$ should acquire the same analytic properties as the underlying field correlators (Φ denotes the scalar field operator):

$$\chi(p) = \int e^{ipx}\left\langle 0\left|T\Phi^\dagger\left(\frac{x}{2}\right)\Phi\left(-\frac{x}{2}\right)\right|P_n\right\rangle, \tag{11}$$

$$\overline{\chi(p)} = \int e^{-ipx}\left\langle P_n\left|T\Phi\left(\frac{x}{2}\right)\Phi^\dagger\left(-\frac{x}{2}\right)\right|0\right\rangle. \tag{12}$$

Using the integral representation for the step function that is included in the time-ordered product, one derives Eq. (9). The normalization condition is obtained by observing that the four-point Green function G and its inverse obey

$$GG^{-1} = 1, \tag{13}$$

and that in the vicinity of a bound state pole at $P_0 = P_n$ [cf. Eq. (23) below] this implies

$$\overline{\chi_n}\left[\frac{\partial}{\partial P_0}(-iD_0^{-1}+K_0)\right]_{P_0\to P_n}\chi_n = 1. \tag{14}$$

Taking the equation for the (zero-order) Green function

$$iG_0 = -D_0 + D_0K_0G_0, \tag{15}$$

with

$$D_0 = \frac{(2\pi)^4\delta^4(p-p')}{[(P_0/2+p_0)^2-E_p^2+i\epsilon][(-P_0/2+p_0)^2-E_p^2+i\epsilon]}, \tag{16}$$

instead of that for the BS wave function and using again Eq. (6) we find

$$G_0 = -F(p)\frac{G_C(\hat{E},\vec{p},\vec{p}\,')}{4m}F(p'), \tag{17}$$

with

$$\hat{E}=\frac{P_0^2-4m^2}{4m} \tag{18}$$

and

$$F(p)=\frac{\sqrt{E_p}(P_0^2-4E_p^2)}{[(P_0/2+p_0)^2-E_p^2+i\epsilon][(-P_0/2+p_0)^2-E_p^2+i\epsilon]}. \tag{19}$$

G_C denotes the well-known Coulomb Green function in momentum space. These solutions can be used for a systematic BS perturbation theory for scalar constituents, as will be demonstrated in the next section.

B. Unstable particles

As has been shown recently by Kummer and Mödritsch [7] for the fermionic case, an important simplification can be achieved in some calculations if the width of the bound state is already included in the zero-order equation. Furthermore, if the width becomes comparable to the level shifts, this approach even becomes indispensible. For the scalar case this can be done by the replacement

$$E_p\rightarrow\sqrt{E_p^2-i\Gamma m}. \tag{20}$$

While Eq. (20) leads to expressions for the BS wave functions that contain unpleasant expressions for the particle poles it has the advantage that the propagator has the form as expected from the phase space of an unstable particle. Furthermore the above calculation remains essentially unchanged if we define the square root in Eq. (20) to be that with the negative imaginary part (clearly we demand $\Gamma>0$ and $m>0$). Only the energy in the resulting equation for the Green function and thus in Eq. (17) changes to

$$\hat{E}=\frac{P_0^2-4m^2}{4m}+i\Gamma. \tag{21}$$

The eigenvalues for P_0 are

$$\begin{aligned}P_{0,n}&=2m\left(1-\sigma_n^2-i\,\frac{\Gamma}{m}\right)^{1/2}\\&\approx 2m-m\sigma^2-\frac{m\sigma_n^4}{4}+\frac{\Gamma^2}{4m}-i\Gamma-i\,\frac{\sigma_n^2\Gamma}{2}.\end{aligned} \tag{22}$$

In the case of the fermions we managed to construct wave functions independent of Γ. This was possible because the small components of the propagator containing $P_0-i\Gamma$ instead of $P_0+i\Gamma$ were projected away by the choice of an appropriate kernel K. This cannot be achieved in the scalar case and thus, surprisingly enough, the scalar wave functions look more complicated than the fermionic ones. A version for a zero-order equation for decaying particles where the propagator is chosen in close analogy to the fermionic case has been developed in [8]. In our present work we, instead, proceed in the spirit of our generalized approach.

III. PERTURBATION THEORY

Perturbation theory for the BS equation starts from the equation for the Green function G_0 [Eq. (15)] of the scattering of two scalars, which is exactly solvable. This is in close analogy to the fermionic case [9]. D_0 is the product of two zero-order propagators and K_0 is the corresponding kernel. The exact Green function G may be represented as

$$G=\sum_l \chi_{\rm nl}^{\rm BS}\frac{1}{P_0-P_n}\overline{\chi_{\rm nl}^{\rm BS}}+G_{\rm reg}=G_0\sum_{\nu=0}^{\infty}(HG_0)^\nu, \tag{23}$$

where the corrections are contained in the insertions H and $G_{\rm reg}$ is the part of G regular at $P_0=P_n$. It is easy to show that H can be expressed by the full kernel K and the full propagators D:

$$H=-K+K_0+iD^{-1}-iD_0^{-1}. \tag{24}$$

Thus the perturbation kernel is essentially the negative difference of the exact BS kernel and of the zero-order approximation.

Expanding both sides of Eq. (23) in powers of P_0-P_n, the mass shift is obtained [6,10]:

$$\Delta M-i\,\frac{\Delta\Gamma}{2}=\langle h_0\rangle(1+\langle h_1\rangle)+\langle h_0 g_1 h_0\rangle+O(h^3). \tag{25}$$

Here the BS expectation values are defined as, e.g.,

$$\langle\langle h\rangle\rangle\equiv\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\,\overline{\chi_{ij}}(p)h_{ii'jj'}(p,p')\chi_{i'j'}(p'). \tag{26}$$

We emphasize the four-dimensional p integrations, which correspond to the generic case, rather than the usual three-dimensional ones in a completely nonrelativistic expansion. We distinguish these two cases by introducing the notation $\langle\langle\ \rangle\rangle$ for a four-dimensional expectation value and $\langle\ \rangle$ for the usual nonrelativistic expectation value:

$$\langle V(\vec{p},\vec{p}\,')\rangle=\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\,\phi^*(\vec{p}\,')V(\vec{p},\vec{p}\,')\phi(\vec{p}). \tag{27}$$

Of course, Eq. (26) reduces to an ordinary ''expectation value'' involving d^3p and $\Phi(\vec{p})$, whenever h does not depend on p_0 and p_0'.

In Eq. (25) h_i and g_i represent the expansion coefficients of H and G_0 near the pole at P_n, respectively: i.e.,

$$H=\sum_{m=0}^{\infty}h_m(P_0-P_n)^m, \tag{28}$$

$$G_0=\sum_{m=0}^{\infty}g_m(P_0-P_n)^{m-1}. \tag{29}$$

Similar corrections arise for the wave functions [6,10]:

$$\chi^{(1)}=(g_1h_0+\tfrac{1}{2}\langle h_1\rangle)\chi^{(0)}. \tag{30}$$

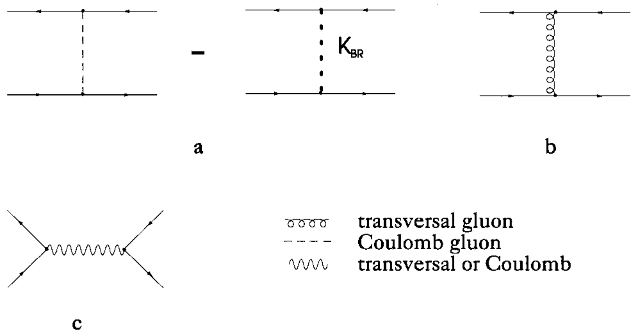

FIG. 1. Tree graph corrections.

A. Fine structure

As an application of this perturbation theory as well as of the new zero-order equation for scalar particles developed in the last section, we will present here the calculation of the fine structure of two stable scalar particles interacting via a vector particle. Existing calculations [4] outline the possibility of a perturbation theory similar to that of Salpeter, but the actual calculations for the scalar-scalar case are done with the help of the Fouldy-Wouthuysen transformation and the iterated Salpeter perturbation theory. Our present approach is much more transparent and allows in principle the inclusion of any higher-order effect in a straightforward manner. First we will calculate the fine structure for two scalars of equal mass interacting by an Abelian vector field. Then we consider also the non-Abelian case, which could be of interest for the stop-antistop system. In this case we will calculate the spectrum up to order α_s^4.

Since in the zero-order equation we have replaced the exact one Coulomb exchange (2) by K_0 as given in Eq. (6) we have now to calculate the contribution of $-K_C+K_0$ to the energy levels. This is shown in Fig. 1(a). With

$$\begin{aligned}\langle\langle -K_C\rangle\rangle &= -4\pi\alpha\int \frac{d^4p}{(2\pi)^4}\frac{d^4p'}{(2\pi)^4}\,\overline{\chi(p)} \\ &\quad\times\frac{(P_0+p_0+p_0')(P_0-p_0-p_0')}{(\vec p-\vec p')^2}\,\chi(p') \\ &= -\left\langle \frac{P_0^2+2E_p^2+2E_{p'}^2}{4P_0\sqrt{E_pE_{p'}}}\,\frac{4\pi\alpha}{\vec q^2}\right\rangle \\ &= -\left\langle\left(\frac{2m}{P_0}-\frac{\sigma_n^2}{2}\right)\frac{4\pi\alpha}{\vec q^2}\right\rangle \end{aligned} \tag{31}$$

$$\langle\langle K_0\rangle\rangle = \frac{2m}{P_0}\left\langle\frac{4\pi\alpha}{\vec q^2}\right\rangle, \tag{32}$$

we obtain

$$\Delta M_C := \langle\langle -K_C+K_0\rangle\rangle = \frac{\sigma_n^2}{2}\left\langle\frac{4\pi\alpha}{\vec q^2}\right\rangle = \frac{m\alpha^4}{16n^4}. \tag{33}$$

The fact that the p integrations are well behaved and the result is of $O(\alpha^4)$ prove the usefulness of our zero-order kernel.

The transverse gluon of Fig. 1(b) gives rise to a kernel:

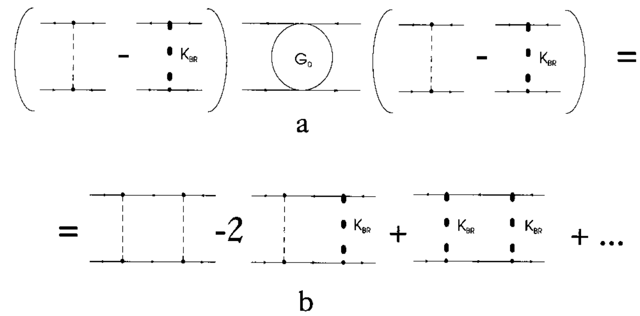

FIG. 2. Second order Coulomb corrections.

$$H_T = \frac{4\pi\alpha}{q^2}\left((\vec p+\vec p\,')^2 - \frac{(\vec p^{\,2}-\vec p\,'^2)^2}{\vec q^{\,2}}\right). \tag{34}$$

Performing the zero-component integrations exactly and expanding in terms of the spatial momenta one obtains to leading order (cf. [11])

$$\Delta M_T = \langle\langle H_T\rangle\rangle = -\frac{4\pi\alpha}{m^2}\left\langle\frac{\vec p^{\,2}}{\vec q^{\,2}} - \frac{(\vec p\vec q)^2}{\vec q^{\,4}}\right\rangle \tag{35}$$

$$= m\alpha^4\left(\frac{1}{8n^4}+\frac{\delta_{l0}}{8n^3} - \frac{3}{16n^3\left(l+\frac{1}{2}\right)}\right). \tag{36}$$

Due to the fact that scalars can only form spin-zero bound states, the annihilation graph into one gauge particle (with spin one) contributes only for p waves and thus is suppressed by two additional powers in α. Furthermore, as in the fermionic case, it vanishes for the non-Abelian theory due to the color trace since the bound states are color singlets. As can be seen from the above results the contribution of the transverse gauge field is equal for fermions and bosons. However, the relativistic correction to the Coulomb exchange appears to be different. Let us therefore check the contribution of this Coulomb correction from second-order perturbation theory [Fig. 2(a)]. These contributions give only rise to $O(\alpha^5 \ln\alpha)$ effects in the fermionic theory. Since the leading Coulomb singularity is canceled we may hope that we can replace the Green function by the free propagator. Indeed it can be shown that the next terms of the Green function give only higher-order contributions.

Due to the presence of the zero-component momentum in the scalar Coulomb gluon vertex we observe that the contribution from Fig. 2(b) diverges linearly. However, it is an easy exercise to show that in the sum of graphs [Fig. 2(b) + Fig. 3] this linear divergence cancels. Thus we regularize all the single graphs, sum up, and find a finite result. We have

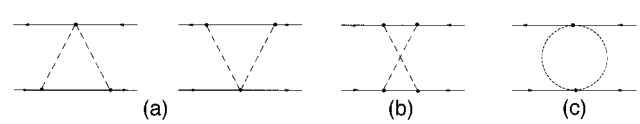

FIG. 3. Abelian scalar box graphs.

used dimensional regularization as well as a one-dimensional Pauli-Villars regularization. Both give the same result for the finite parts of the integrals:

$$\Delta M_{\rm box}=\langle\langle h_0^{(3)}\rangle\rangle+\langle\langle(-K_C+K_0)g_1(-K_C+K_0)\rangle\rangle, \tag{37}$$

$$\langle\langle h_0\rangle\rangle=\left\langle\left\langle -i\int\frac{d^3k}{(2\pi)^3}\frac{I_0}{\vec{k}^2(\vec{q}-\vec{k})^2}\right\rangle\right\rangle, \tag{38}$$

where I_0 is decomposed according to Fig. 3 for a generic SU(N) theory:

$$I_0^{[3(a)]}=\left(C_F^2-\frac{C_F}{2N}\right)\int_{k_0}\left(\frac{(P+p+p'-k_0)(P+2p-k_0)}{[(P/2+p-k)^2-m^2]}+\frac{(-P+2p'+k_0)(-P+p+p'+k_0)}{[(-P/2+p'+k)^2-m^2]}\right)\approx 2\left(C_F^2-\frac{C_F}{2N}\right)\left(\frac{\Lambda}{2}-im\right), \tag{39}$$

$$I_0^{[3(b)]}=\frac{C_F}{2N}\int_{k_0}\frac{(P+2p'+k_0)(-P+2p'+k_0)(P+p+p'-k_0)(P+2p-k_0)}{[(P/2+p-k)^2-m^2][(-P/2+p'+k)^2-m^2]}\approx\frac{C_F}{2N}\left(\frac{\Lambda}{2}-2im\right), \tag{40}$$

$$I_0^{[3(c)]}=-\left(C_F^2-\frac{C_F}{2N}\right)\int_{k_0}=-\left(C_F^2-\frac{C_F}{2N}\right)\frac{\Lambda}{2}. \tag{41}$$

Using the abbreviations

$$\int_{k_0}=\int\frac{dk_0}{2\pi}\frac{\Lambda^2}{k_0^2+\Lambda^2}, \tag{42}$$

$$C_F=\frac{N^2-1}{2N}, \tag{43}$$

we have written the result for Pauli-Villars regularization to make the cancellation of the liner divergent parts obvious.

For the double Coulomb exchange graph from Fig. 2 we obtain for the time component integral

$$I_0^{[2(b)]}=-C_F^2\int_{k_0}\frac{[(p_0'+k_0)^2-P^2+4m\sqrt{E_kE_{p'}}][(p_0+k_0)^2-P^2+4m\sqrt{E_kE_p}]}{[(P/2+k_0)^2-E_k^2][(-P/2+k_0)^2-E_k^2]}\approx -C_F^2\left(\frac{\Lambda}{2}-im\right). \tag{44}$$

Collecting everything from above we have

$$I_0=I_0^{[3(a)]}+I_0^{[3(b)]}+I_0^{[3(c)]}+I_0^{[2(b)]}=-C_F^2 im, \tag{45}$$

which leads with Eqs. (38), (43), and (3) immediately to the result

$$\Delta M_{\rm Abelian\ box}=-\frac{m\alpha^4}{16n^3(l+1/2)}. \tag{46}$$

The net result for the spectrum of two scalars bound by an Abelian gauge field is equal to that of Ref. [4]. However, we showed which graphs contribute in a pure BS approach, which can be used as a basis for any higher-order calculation.

We have also checked the derivative $\partial K/\partial P_0$ contributing to h_1 and the X graphs of Fig. 4(g) with transverse gauge particles for possible contributions. Our estimates only yield contributions to higher order. Due to mass and wave function renormalization we can further assume that the graphs of Figs. 4(e) and 4(f) give only contributions to $O(\alpha^5\ln\alpha)$ as in the fermionic case [12]. Possible large contributions of lighter particles to the vacuum polarization as depicted in Fig. 4(c) can be treated as in the fermionic case [13].

In supersymmetric theories a $|\Phi|^4$ term is part of the Lagrangian. Clearly it can be put in by hand into the Langragian of an ordinary quantum field theory. The contribution from an interaction term of the form $-\lambda/2(\Phi^\dagger T^a\Phi)(\Phi^\dagger T^a\Phi)$ is easily calculated:

$$\Delta M_X=-C_F\lambda\frac{m\alpha^3}{32\pi n^3}\delta_{l0} \tag{47}$$

and gives a contribution of the same form as the Darwin term (usually interpreted as a zitterbewegung contribution), which is suppressed by two orders in α in the scalar theory. There may exist a small chance that this term may be helpful for the determination of the supersymmetry parameters of the theory contained in λ.

For an ordinary quantum field theory without a direct interaction on the tree level it was shown first by Rohrlich [14] that a counter term of this form is needed for the scattering of two scalars [e.g., the graphs of Fig. 3 and the first of Fig. 2(b) with photons in the Feynman gauge]. It is interesting to note that in the Coulomb gauge the divergencies for the Coulomb photons cancel and the only divergent graph is the one of Fig. 3(c) with transverse photons.

The spectrum calculated so far is common for the Abelian and the non-Abelian theory. Collecting all pieces we have

$$\Delta M=\Delta M^{j}_{F,nl}+\frac{m\alpha^4}{8}\left[\frac{5}{4n^4}+\frac{\delta_{l0}}{n^3}\left(1-\frac{C_F\lambda}{4\pi\alpha}\right)-\frac{4}{n^3\left(l+\frac{1}{2}\right)}\right], \tag{48}$$

where $\Delta M^{j}_{F,nl}$ originates in the contribution of j light fermions to the vacuum polarization and can be found in [13].

It was pointed out first in [15] that in the case of a non-Abelian gauge field further corrections may arise due to the gluon splitting vertices. The $O(\alpha^3)$ corrections from Figs. 4(a) and 4(b) as well as the $O(\alpha^4)$ corrections from the corresponding two-loop graphs are obviously the same as in the fermionic case. The vertex correction shown below in Fig. 4(d) has been calculated in [15] for the fermionic case.

Here we will give a calculation of the same contribution for scalar constituents. After performing the color trace the perturbation kernel for the second graph in Fig. 4(d) reads

$$H_{[4(\mathrm{d}),2]}=-8ig^4\int\frac{d^4k}{(2\pi)^4}\frac{(P_0+p_0+p_0'-k_0)(-P_0+p_0+p_0')}{\vec{q}^2(\vec{k}-\vec{q})^2[(P/2+p+k)^2-m^2]k^2}\left(-(\vec{p}\vec{q})+\frac{(\vec{p}\vec{k})(\vec{q}\vec{k})}{\vec{k}^2}\right). \tag{49}$$

Performing the k_0 integration and using the scaling

$$P_0\rightarrow 2m+O(\alpha^2),$$

$$p_0\rightarrow\alpha^2 p_0,$$

$$\vec{k}\rightarrow\alpha\vec{k}$$

to extract the leading contribution in α we find that

$$H_{[4(\mathrm{d}),2]}=-\frac{g^2m}{2}\frac{\vec{p}\vec{q}}{|\vec{q}|^3}. \tag{50}$$

Adding the similar contribution from the first graph in Fig. 4(d) gives

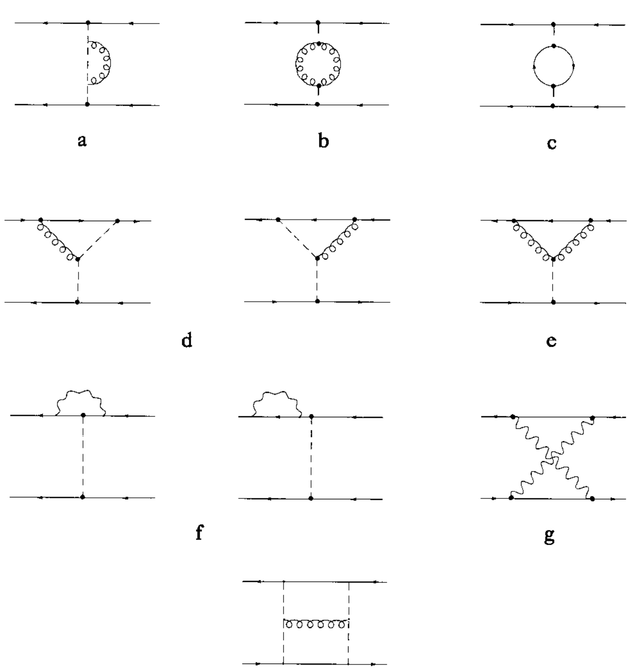

FIG. 4. Graphs contributing equally as in the fermionic case.

$$H_{[4(\mathrm{d})]}=-\frac{9\pi^2\alpha^2 m}{|\vec{q}|}. \tag{51}$$

This result differs by a factor $4m^2$ from the fermionic result, which is compensated by a corresponding difference in the wave functions to give eventually precisely the same result as in the fermionic case

$$\Delta M=\left\langle\frac{9\pi^2\alpha^2}{4m|\vec{q}|}\right\rangle=\frac{9m\alpha^4}{32n^3\left(l+\frac{1}{2}\right)}. \tag{52}$$

In view of the fact that the result depends only on the angular momentum and not on the spin this seems reasonable. However, we have seen in the case of the Darwin term that this kind of reasoning sometimes fails.

Proceeding to the graph of Fig. 4(h) we observe that in contrast to the fermionic case the zero component integration develops Coulomb divergencies like the Abelian contributions. Since these integrations are a little bit cumbersome in dimensional regularization we sketch the calculation in the Appendix. It turns out finally that the box graph contribution in Fig. 4(h) gives the same result as in the fermionic case, which was calculated recently [16]:

$$\langle\langle H_{[4(\mathrm{h})]}\rangle\rangle=-\frac{81}{128}\pi(12-\pi^2)\left\langle\frac{\alpha^3}{\vec{q}^2}\right\rangle. \tag{53}$$

The box graph with two Coulomb lines crossed vanishes due to the color trace. Another box graph with the Coulomb vertices on one scalar line replaced by a ‘‘seagull vertex’’ (two gauge boson–two scalar vertex) can be shown to contribute to $O(\alpha^5)$. However, they are in principle needed to cancel the Coulomb singularities.

Thus the difference in the spectrum of the scalar bound state to $O(\alpha^4)$ compared to the fermionic case is entirely due to the graphs also present in the Abelian theory discussed above.

IV. BOUND STATE CORRECTIONS TO THE DECAY WIDTH

Assuming that the scalar particle under consideration decays into two other particles, the decay width is the imaginary part of the corresponding self-energy function Σ at the mass shell. Focusing on the stop quark a possible scenario could be $\tilde{t}_R \to b + \tilde{\chi}_i$ [17]. We shall be interested in terms of the order $O(\alpha^2\Gamma)$ where Γ is the tree level decay width. The first part of the perturbation kernel due to the exact inverse propagator $p^2 - m^2 - \Sigma(p^2)$ for the bound state corrections to the decay width reads

$$H_1 = iD^{-1} - iD_0^{-1}$$
$$\approx -2i(2\pi)^4\delta^4(p_1 - p_2 - P)\Sigma'(m^2)(p_1^2 - m^2)(p_2^2 - m^2). \tag{54}$$

In Eq. (54), p_1 and p_2 are the four momenta of the particle and the antiparticle, respectively. P is the total momentum. To derive Eq. (54) we expanded the self-energy function around the mass shell

$$\Sigma(p^2) = \Sigma(m^2) + \Sigma'(m^2)(p^2 - m^2) + O[(p^2 - m^2)^2] \tag{55}$$

and we assumed that the decay width used in the zero-order equation (e.g., in D_0) is given by

$$\Gamma = -\,\mathrm{Im}\,\frac{\Sigma(m^2)}{m}. \tag{56}$$

As has been first shown in [18], the gauge dependence contained in the off-shell contribution Σ' is canceled by parts of the vertex correction depicted in Fig. 4(f). The latter give rise to a perturbation kernel,

$$H_2 = \Lambda_0\,\frac{4\pi\alpha}{\vec{q}^2}\,(-P_0 + p_0 + p_0'), \tag{57}$$

with Λ_0 representing the vertex correction. The color trace is already included in α. As in the fermionic case [7] it is possible to derive a Ward identity, which guarantees the cancellation of the gauge-dependent terms [the T^a's are the $\mathrm{SU}(N)$ generators]

$$\Lambda_\mu^a(p, q=0) = -2gT^a p_\mu\,\frac{\partial}{\partial p^2}\,\Sigma(p^2),$$

$$\mathrm{Im}\Lambda_0[p=(m,\vec{0}), q=0] = -2m\ \mathrm{Im}\Sigma'(m^2). \tag{58}$$

But the detailed calculation shows differences to the fermionic case: the sum of the contributions from H_1 and H_2 vanishes to the desired order with the help of the zero-order equation

$$\mathrm{Im}\langle\langle H_1 + H_2\rangle\rangle \approx 0. \tag{59}$$

On the other hand, we observed above that the wave functions for decaying fermions and scalars were very different. While it was possible to obtain the same wave functions for decaying fermions and for stable ones, in the bosonic case we used wave functions explicitly containing the decay width (cf. Sec. II B). Thus we have to reexamine the relativistic corrections to the energy levels. Among the contributions considered in the last section only the relativistic Coulomb correction Fig. 1(a) can produce corrections to $O(\alpha^2\Gamma)$.

It is easy to see that the only difference comes from the fact that the perturbation has to be taken at the position of the pole (22), which leads to the replacement

$$\sigma_n^2 \to \sigma_n^2 + i\,\frac{\Gamma}{m} \tag{60}$$

in Eq. (31). We thus get a relativistic correction to the decay width of the bound state

$$\Delta\Gamma_{[1(\mathrm{a})]} = -\frac{\Gamma\alpha^2}{2n^2}. \tag{61}$$

This has to be added to the $O(\alpha^2\Gamma)$ term of Eq. (22) to yield the final result for the bound state correction to the decay width:

$$\Delta\Gamma = -\frac{\Gamma\alpha^2}{4n^2}. \tag{62}$$

This result generalizes the result of [7,18] to the bosonic case. We can thus say that the effect of the bound state corrections to the decay width can be interpreted entirely as a time dilatation effect as was first conjectured for the fermionic theory [19].

V. CONCLUSION

We have presented a consistent formalism for the calculation of bound state properties for scalar particles interaction with an Abelian or non-Abelian spin one vector field. This is done by deriving a solvable relativistic zero-order equation similar to that of Barbieri and Remiddi both for stable and unstable scalars. Based on this equation a systematic perturbation theory can be built that allows especially the calculation of the position of the bound state poles to higher orders.

Using this approach the bound state spectrum was calculated to $O(\alpha^4)$. We found that we had to take into account the Abelian box graphs to this order. This is not the case in the fermionic theory. All the relativistic Coulomb corrections only reproduce the $\vec{p}^4$ term from the expansion of $\sqrt{m^2 + \vec{p}^2}$, indicating that a fully relativistic formulation is not really economic for the lowest orders in perturbation theory. However, the advantage of the presented formalism is that it is straightforwardly applicable to any higher-order calculation. We calculated also the non-Abelian contributions to $O(\alpha^4)$. Furthermore our approach makes possible the calculation of the bound state corrections to the decay width of weakly decaying scalar particles. We show that—as in the fermionic case—the inclusion of a finite, constant decay width in the zero-order equation simplifies the problem of the bound state correction to the decay width in a profound way. It is now possible to clearly isolate the underlying cancellation mechanism, which automatically gives a gauge-independent result that can be interpreted as time dilatation alone. We can thus generalize the theorem on the bound state corrections for the decay width to the scalar case: the leading bound state corrections for weakly bound systems of unstable scalars (with decays such as $\tilde{t}_R \rightarrow b + \tilde{\chi}_i$) are *always* of the form (62).

It would be very interesting to observe a particle where the above-mentioned predictions could be tested. Today it seems that the stop-antistop system could be a candidate. It will be heavy enough to allow a perturbative treatment even for the non-Abelian case. Whether the decay width will be small enough to allow a detailed study of the spectrum remains open to speculation at present. But even for a quite large decay width the scalar-scalar potential will provide the basis for interesting threshold calculations for this case [20], analogous to the ones for the top-antitop system [19,21].

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APPENDIX A: ZERO-COMPONENT INTEGRATIONS FOR THE NON-ABELIAN BOX GRAPH

The diagram 4(h) leads to the energy component integrals

$$I_0 := \int \frac{dk_0}{2\pi} \int \frac{dt_0}{2\pi} \frac{(P_0+2p_0-t_0)(P_0+2p_0-q_0-t_0)(-P_0+2p_0-q_0-k_0)(-P_0+2p_0-k_0)}{[(P_0/2+p_0-t_0)^2-E^2_{\vec{p}-\vec{t}}][(-P_0/2+p_0-k_0)^2-E^2_{\vec{p}-\vec{k}}][(t_0-k_0)^2-(\vec{t}-\vec{k})^2]}. \tag{A1}$$

This integral is power counting logarithmic divergent, but it turns out that the first integration is finite, which leads to a linear divergent second integration. After scaling Eq. (A1) reduces to

$$I_0 = -\int \frac{dk_0}{2\pi} \int \frac{dt_0}{2\pi} \times \frac{(2m-t_0)(2m+k_0)}{(t_0-i\epsilon)(k_0+i\epsilon)[(t_0-k_0)^2-(\vec{t}-\vec{k})^2+i\epsilon]} \tag{A2}$$

To make this integral accessible for the methods of dimensional regularization we use the following trick:

$$\int \frac{dt_0}{2\pi} \frac{1}{t_0 \pm i\epsilon} = \lim_{\mu\to 0} \int \frac{dt_0}{2\pi} \frac{t_0 \pm \mu}{t_0-\mu^2+i\epsilon}. \tag{A3}$$

Performing first the t_0 integration we have

$$I_0 = I_{0,1} + I_{0,2},$$

$$I_{0,1} = \int \frac{dk_0}{2\pi} \frac{(2m+k_0)(k_0+\mu)}{k_0^2-\mu^2+i\epsilon} I_{t_0},$$

$$I_{0,2} = \int \frac{dk_0}{2\pi} I_{t_0},$$

$$I_{t_0} = -\frac{2mi\Gamma(2-D/2)}{(4\pi)^{D/2}} \times \int_0^1 dx \frac{(xk_0-\mu)[x(1-x)]^{D/2-2}}{\left[-k_0^2+\frac{\mu^2}{x}+|\vec{t}-\vec{k}|^2/(1-x)\right]^{2-D/2}} - \frac{i\Gamma(1-D/2)}{(4\pi)^{D/2}} |\vec{t}-\vec{k}|^{D-2}.$$

The limit $\mu \rightarrow 0$ has to be performed very carefully to obtain

$$I_{0,1} = \frac{2m^2}{|\vec{t}-\vec{k}|^2} - \frac{m}{2|\vec{t}-\vec{k}|},$$

$$I_{0,2} = -\frac{m}{2|\vec{t}-\vec{k}|}. \tag{A4}$$

Thus we have to the desired accuracy

$$I_0 = \frac{2m^2}{|\vec{t}-\vec{k}|^2}. \tag{A5}$$

It should be noted that dimensional regularization does not show linear divergencies. Instead the use of the regularization (42) leads to a visible liner divergent term (of higher order in α) $i\Lambda/(8|\vec{t}-\vec{k}|)$, which has to be canceled by similar contributions from graphs where the two Coulomb gluon vertices on one or both scalar lines are double Coulomb vertices.

APPENDIX B: ZERO-ORDER EQUATION FOR UNEQUAL MASSES

It is possible to extend the formalism derived in this work to the case of unequal constituent masses. To illustrate this we will give in this Appendix a solvable relativistic zero-order equation for two scalar particles with unequal masses.

We start with Eq. (1) with $S_{ii'}S_{jj'}$ replaced by

$$S^{(1)}_{ii'}\left(\frac{P_0}{2}+p_0\right) S^{(2)}_{jj'}\left(-\frac{P_0}{2}+p_0\right) = \frac{1}{[(P_0/2+p_0)^2-E^2_{1,p}+i\epsilon][(-P_0/2+p_0)^2-E^2_{2,p}+i\epsilon]}, \tag{B1}$$

where

$$E_{1,p}^2=m_1^2+\vec{p}^2,$$

$$E_{2,p}^2=m_2^2+\vec{p}^2.$$

Assuming again a static zero-order kernel and performing the p_0 integration leads to

$$\chi(\vec{p})=-\frac{E_{1,p}+E_{2,p}}{2E_{1,p}E_{2,p}[(E_{1,p}+E_{2,p})^2-P_0^2]}\times\int\frac{d^3p}{(2\pi)^3}K_0(\vec{p},\vec{p}\,',P_0)\chi(\vec{p}\,'). \tag{B2}$$

Since $(E_{1,p}+E_{2,p})^2-P_0^2$ does not lead to a Schrödinger equation we multiply the numerator and denominator of the right-hand side with the nonrelativistic limit of the latter expression: $M^2+M/m_R\vec{p}^2-P_0^2$, where

$$M=m_1+m_2,$$

$$m_R=\frac{m_1m_2}{m_1+m_2}$$

are the mass threshold and the reduced mass, respectively. With a kernel of the form

$$K_0(\vec{p},\vec{p}\,',P_0)=2M\sqrt{R(\vec{p},P_0)R(\vec{p}\,',P_0)}\,\frac{4\pi\alpha}{(\vec{p}-\vec{p}\,')^2},$$

with

$$R(\vec{p},P_0)=\frac{2E_{1,p}E_{2,p}[(E_{1,p}+E_{2,p})^2-P_0^2]}{(E_{1,p}+E_{2,p})[M^2+(M/m_R)\vec{p}^2-P_0^2]}, \tag{B3}$$

the Bethe-Salpeter equation (1) may be solved in the same way as the equal mass case except that $\sqrt{E_p}\rightarrow\sqrt{R}$.

The eigenvalues are obtained from

$$\frac{P_0^2-M^2}{2M}=\frac{m_R\alpha^2}{2n^2},$$

$$P_0=P_n=M+\frac{m_R\alpha^2}{2n^2}+\frac{m_R^2\alpha^2}{8Mn^4}+\cdots .$$

The normalization of the wave function is now more involved. Again we start from

$$GG^{-1}=1,$$

$$\overline{\chi_n}\left[\frac{\partial}{\partial P_0}(-iD_0^{-1}+K_0)\right]_{P_0\rightarrow P_n}\chi_n=1, \tag{B4}$$

but now also K_0 is P_0 dependent. After some calculation we finally abtain for the normalized wave function

$$\chi(p)=-i\,\frac{\sqrt{R(\vec{p},P_n)}[M^2+(M/m_R)\vec{p}^2-P_n^2]}{\sqrt{2P_n}[(P_n/2+p_0)^2-E_p^2+i\epsilon][(-P_n/2+p_0)^2-E_p^2+i\epsilon]}\,\phi(\vec{p}). \tag{B5}$$

The extension of the present work to the unequal mass case is thus possible in a straightforward manner but the calculations become more involved.

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