

Electrical conductivity in the early universe

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We calculate the electrical conductivity in the early universe at temperatures below as well as above the electroweak vacuum scale, $T_c \approx 100$ GeV. Debye and dynamical screening of electric and magnetic interactions leads to a finite conductivity, $\sigma_{el} \sim T/\alpha \ln(1/\alpha)$, at temperatures well below T_c . At temperatures above, W^\pm charge-exchange processes—analogueous to color exchange through gluons in QCD—effectively stop left-handed charged leptons. However, right-handed leptons can carry current, resulting in σ_{el}/T being only a factor $\sim \cos^4 \theta_W$ smaller than at temperatures below T_c . [S0556-2821(97)06620-4]

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I. INTRODUCTION

The strong magnetic fields measured in many spiral galaxies, $B \sim 2 \times 10^{-6}$ G [1], are conjectured to be produced primordially; proposed mechanisms include fluctuations during an inflationary universe [2] or at the grand unified theory (GUT) scale [3], and plasma turbulence during the electroweak transition [4–6] or in the quark-gluon hadronization transition [7,8]. The production and later diffusion of magnetic fields depends crucially on the electrical conductivity σ_{el} of the matter in the universe; typically, over the age of the universe, t , fields on length scales smaller than $L \sim (t/4\pi\sigma_{el})^{1/2}$ are damped. In this paper we calculate σ_{el} in detail below and above the electroweak transition scale.

The electrical conductivity was estimated in [2] in the relaxation time approximation as $\sigma_{el} \sim n\alpha\tau_{el}/m$ with $m \sim T$ and relaxation time $\tau_{el} \sim 1/(\alpha^2 T)$, where $\alpha = e^2/4\pi$. In Refs. [9] and [10] the relaxation time was corrected with the Coulomb logarithm. A deeper understanding of the screening properties in QED and QCD plasmas in recent years has made it possible to calculate a number of transport coefficients including viscosities, diffusion coefficients, momentum stopping times, etc., exactly in the weak coupling limit [11–13]; also [14]. However, calculation of processes that are sensitive to very singular forward scatterings remain problematic. For example, the calculated color diffusion and conductivity [15], even with dynamical screening included, remain infrared divergent due to color exchange in forward scatterings. Also the quark and gluon damping rates at non-zero momenta calculated by resumming ring diagrams exhibit infrared divergences [16] whose resolution requires more careful analysis including higher order diagrams as, e.g., in the Bloch-Nordseick calculation of Ref. [17]. Charge exchanges through W^\pm exchange, processes similar to gluon color exchange in QCD, are important in forward scatterings at temperatures above the W mass, M_W . While, as we show, such processes lead negligible transport by left-handed charged leptons, they do not prevent transport by right-handed charged leptons. As a consequence, electrical conduction at temperatures above the electroweak transition is large, and does not inhibit generation of magnetic fields [10]; the observed magnetic fields in galaxies could then be gen-

erated earlier than the electroweak transition [4,5] and have survived until today. More generally, we find that the electrical conductivity is sufficiently large that it does not lead to destruction of large-scale magnetic flux over time scales of the expansion of the universe.

In Sec. II we calculate the electrical conductivity for $T \ll M_W$. In this regime the dominant momentum transfer processes in electrical transport are electrodynamic, and one can ignore weak interactions. We turn then in Sec. III to the very early universe, $T \gg T_c$, where the W^\pm are effectively massless and their effects on electrical conduction must be taken into account.

II. ELECTRICAL CONDUCTIVITIES IN HIGH TEMPERATURE QED

We first calculate the electrical conductivity in the electroweak symmetry-broken phase at temperatures well below the electroweak boson mass scale, $T \ll M_W$. As we argue below, charged leptons $\ell = e^-, \mu^-, \tau^-$ and anti-leptons $\bar{\ell} = e^+, \mu^+, \tau^+$ dominate the currents in the regime in which $m_\ell \ll T$. In the broken-symmetry phase weak interactions between charged particles, which are generally smaller by a factor $\sim (T/M_W)^4$ compared with photon-exchange processes, can be ignored. The primary effect of strong interactions is to limit the drift of strongly interacting particles, and we need consider only electromagnetic interactions between charged leptons and quarks.

Transport processes are most simply described by the Boltzmann kinetic equation for the distribution functions, $n_i(\mathbf{p}, \mathbf{r}, t)$, of particle species i , of charge e :

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{r}} + e \mathbf{E} \cdot \nabla_1 \right) n_1 \\ &= -2\pi\nu_2 \sum_{234} |M_{12 \rightarrow 34}|^2 [n_1 n_2 (1 \pm n_3)(1 \pm n_4) \\ & \quad - n_3 n_4 (1 \pm n_1)(1 \pm n_2)] \\ & \quad \times \delta_{\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_3 + \mathbf{p}_4} \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4), \end{aligned} \quad (1)$$

where \mathbf{E} is the external electric field driving the charges, and the right side describes the two-particle collisions $12 \leftrightarrow 34$ slowing them down. The \pm signs refer to bosons and fermions. The sums are over momenta \mathbf{p}_2 , \mathbf{p}_3 , and \mathbf{p}_4 , and the statistical factor ν_2 accounts for the number of leptons and spin projections that scatter with particle 1. We suppress a factor inverse volume squared on the right side. Massless lepton-lepton or antilepton-antilepton scattering conserves the electrical current, and affects the conductivity only in higher order. In lowest order we need to take into account only lepton-antilepton scattering. The square of the matrix element for scattering of a charged lepton from initial momentum \mathbf{p}_1 to final momentum \mathbf{p}_3 with an antilepton from \mathbf{p}_2 to \mathbf{p}_4 is

$$|M_{12 \rightarrow 34}|^2 = \frac{2e^4}{16\epsilon_1\epsilon_2\epsilon_3\epsilon_4} \frac{s^2 + u^2}{t^2}, \quad (2)$$

where s, t, u are the usual Mandelstam variables formed from the incoming and outgoing momenta.

To solve the kinetic equation for a weak driving field, we write $n_i = n_i^0 + n_i^0(1 \pm n_i^0)\Phi_i$, where $n_i^0 = [\exp(\epsilon_i/T) \mp 1]^{-1}$ is the global equilibrium distribution (chemical potentials are taken to vanish in the early universe), and linearize Eq. (1) in the deviations Φ_i of the distribution functions from equilibrium (see Refs. [18], [19], and [13] for details); thus Eq. (1) reduces to

$$\begin{aligned} e\mathbf{E} \cdot \mathbf{v}_1 \frac{\partial n_1}{\partial \epsilon_1} &= -2\pi\nu_2 \sum_{234} |M_{12 \rightarrow 34}|^2 n_1^0 n_2^0 (1 \pm n_3^0) (1 \pm n_4^0) \\ &\quad \times (\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4) \delta_{\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_3 + \mathbf{p}_4} \\ &\quad \times \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4). \end{aligned} \quad (3)$$

We set the stage by considering the simple situation of conduction by massless charged leptons (component 1) and antileptons (component 2) limited by their scattering together; later we include effects of quarks. The electric field drives the charged leptons and antileptons into flow with steady but opposite fluid velocities $\mathbf{u}_1 = -\mathbf{u}_2$ in the center-of-mass system (see Fig. 1). Assuming that collisions keep the drifting components in local thermodynamic equilibrium, we approximate the quasiparticle distribution functions by those of relativistic particles in equilibrium with a local flow velocity \mathbf{u} ,

$$n_i(\mathbf{p}_i) = \frac{1}{\exp[(\epsilon_i - \mathbf{u}_i \cdot \mathbf{p}_i)/T] \mp 1}; \quad (4)$$

the deviation Φ_i is thus

$$\Phi_i = \mathbf{u}_i \cdot \mathbf{p}_i / T. \quad (5)$$

The ansatz (4) for the distribution function is an excellent approximation; a more elaborate variational calculation which we have carried out (see [13] for analogous calculations for the viscosity) gives almost the same distribution function, and a corresponding electrical conductivity only 1.5% smaller than our result, Eq. (9) below. Equation (5)

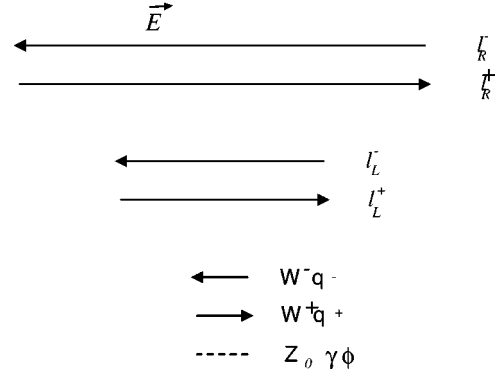


FIG. 1. The particle currents generated by an electric field in the early universe at $T \gg T_c$. Right-handed charged leptons, $l = e, \mu, \tau$, interact only through γ and Z^0 exchanges while left-handed leptons also interact by W^\pm exchanges, which drag neutrinos along and decrease the current. The vector and Higgs bosons, γ, Z^0, W^\pm, ϕ , cannot flow due to W^\pm exchanges, and $q\bar{q}g$ are stopped by strong interactions.

gives the total electric current from lepton-antilepton pairs with N_ℓ effectively massless species present at temperature T :

$$\mathbf{j}_{\ell\bar{\ell}} = en_{\text{ch}}\mathbf{u}_1 = e\mathbf{u}_1 \frac{3\zeta(3)}{\pi^2} N_\ell T^3, \quad (6)$$

where n_{ch} is the number density of electrically charged leptons plus antileptons. Note that since photon exchange does not transfer charge (as do W^\pm), particle 3 in Eq. (1) has the same charge as particle 1, and particle 4 the same as 2, and $\mathbf{u}_3 = \mathbf{u}_1$ and $\mathbf{u}_4 = \mathbf{u}_2 = -\mathbf{u}_1$.

To calculate σ_{el} we multiply Eq. (3) by $\nu_1 \mathbf{v}_1$ and sum over \mathbf{p}_1 , to find an equation of the form

$$-e\mathbf{E}N_\ell T^2/18 = -\xi\mathbf{u}_1 \quad (7)$$

where ξ results from the right side of Eq. (3). Since QED interactions are dominated by very singular forward scattering arising from massless photon exchange, the sum on the right side in ξ diverges unless we include Debye screening of longitudinal interactions and dynamical screening of transverse interactions due to Landau damping. This is done by including (in the Coulomb gauge) the Coulomb self-energy, Π_L , and the transverse photon self-energy, Π_T , in the longitudinal and transverse electromagnetic propagators. The inverse of the longitudinal propagator becomes $q^2 + \Pi_L(\omega, q)$, and the inverse of the transverse propagator, $t = \omega^2 - q^2$ becomes $\omega^2 - q^2 - \Pi_T(\omega, q)$, where ω and q are the energy and momentum transferred by the electromagnetic field in the scattering. The quantity ξ can be calculated to leading logarithmic order in the coupling constant by expanding for small momentum transfers (see Ref. [13] for details). Small momentum transfer processes are screened by $\Pi_L \sim q_D^2 = 4\pi\alpha N_\ell T^2/3$, and $\Pi_T \sim i\pi q_D^2 \omega/4q$. (Large momentum transfers, $q \gtrsim \langle p \rangle \sim 3T$, are cut off by the distribution functions.) The resulting integrals $\int q^2 d^2q / |q^2 + \Pi_{L,T}|^2$ give characteristic logarithms $\ln(T/q_D)$ and we find

$$\xi = \frac{2 \ln 2}{9\pi} N_\ell^2 \alpha^2 \ln(C/\alpha N_\ell) T^4. \quad (8)$$

The constant $C \sim 1$ in the logarithm gives the next to leading order terms (see [13] for the calculation of second-order contributions to the viscosity). The electrical conductivity for charged leptons is thus [12,19]

$$\sigma_{\text{el}}^{(\ell)} \equiv j_{\ell} / E = \frac{3\zeta(3)}{\ln 2} \frac{T}{\alpha \ln(1/\alpha N_l)}, \quad m_e \ll T \ll T_{\text{QCD}}. \quad (9)$$

Note that the number of lepton species drops out except in the logarithm. The above calculation taking only electrons as massless leptons ($N_l=1$) gives a first approximation to the electrical conductivity in the temperature range $m_e \ll T \ll T_{\text{QGP}}$, below the hadronization transition, $T_{\text{QGP}} \sim 150$ GeV, at which hadronic matter undergoes a transition to a quark-gluon plasma. Thermal pions and muons in fact also reduce the conductivity by scattering electrons, but they do not become significant current carriers because their masses are close to T_{QGP} .

For temperatures $T > T_{\text{QGP}}$, the matter consists of leptons and deconfined quarks. The quarks themselves contribute very little to the current, since strong interactions limit their drift velocity. The quark drift velocity can be estimated by replacing α by the strong interaction fine structure constant, α_s , in ξ , Eqs. (7) and (8), which yields $\mathbf{u}_q \sim \mathbf{u}_\ell (\alpha^2 \ln \alpha^{-1}) / (\alpha_s^2 \ln \alpha_s^{-1})$.

Even though quarks do not contribute significantly to the currents they are effective scatterers, and thus modify the conductivity (an effect ignored in the recent numerical analysis of Ref. [20]). To calculate the quark contribution to the lepton conductivity, we note that the collision term between leptons (1), (3) and quarks (2), (4) includes the following additional factors compared with lepton-lepton scattering: a factor 1/2, because the quark velocity, \mathbf{u}_2 , is essentially zero, a factor 3 from colors, and a factor 2 because the charged leptons collide on both q and \bar{q} ; finally we must sum over flavors with a weight Q_q^2 , where $Q_q e$ is the charge of quark flavor $q = u, d, s, c, b, t$. We must also divide by the number of leptons, N_l , to take into account the number of quark scatterings relative to lepton scatterings. Including ℓq and $\ell \bar{q}$ collisions on the right side of Eq. (9) we find the total electrical conductivity of the early universe [12,19]:

$$\begin{aligned} \sigma_{\text{el}} &= \frac{N_l}{N_l + 3 \sum_q N_q Q_q^2} \sigma_{\text{el}}^{(\ell)} \\ &= \frac{N_l}{N_l + 3 \sum_q N_q Q_q^2} \frac{3\zeta(3)}{\ln 2} \frac{T}{\alpha \ln(1/\alpha N_l)}, \quad T_{\text{QGP}} \ll T \ll M_W. \end{aligned} \quad (10)$$

The charged lepton and quark numbers N_l and N_q count only the species present in the plasma at a given temperature, i.e., those with masses $m_i \lesssim T$. Figure 2 illustrates the conductivities (9), (10). For simplicity this figure assumes that the quarks and leptons make their appearance abruptly when T becomes $> m_i$; in reality they are gradually produced thermally as T approaches their masses. Since a range of particle energies in the neighborhood of the temperature is included, possible resonance effects in scatterings are smeared out.

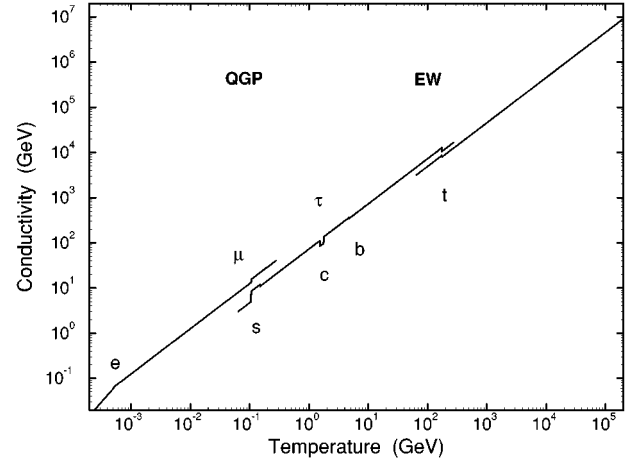


FIG. 2. Electrical conductivity vs temperature. The temperatures, where the transitions from hadronic to quark-gluon plasma and electroweak symmetry breaking occur, are indicated by QGP and EW, respectively. The conductivity σ_{el} is given by Eqs. (9), (10), (20) in the three regions and are extrapolated into the regions of the phase transitions. The quark and lepton masses in the figure indicate the temperatures at which they are thermally produced and thus affect the conductivity [see Eq. (10) and discussion in text].

We will not attempt to calculate the electrical conductivity in the range $M_W \lesssim T \lesssim T_c$ below the critical temperature. Recent lattice calculations [21] predict a relatively sharp transition at a temperature $T_c \sim 100$ GeV from the symmetry broken phase at low temperatures, $T \ll T_c$, to the symmetric phase at $T \gg T_c$. The transition is sensitive, however, to the unknown Higgs mass, with a first order transition predicted only for Higgs masses below ~ 90 GeV. The calculations of the conductivity are technically more involved when masses are comparable to the temperature. Furthermore one must begin to include contributions of the W^\pm to currents and scatterings, as the thermal suppression of their density decreases near the transition.

III. THE SYMMETRY-RESTORED PHASE

To calculate the conductivity well above the electroweak transition, $T \gg T_c$, where the electroweak symmetries are fully restored, we describe the electroweak interactions by the standard model Weinberg-Salam Lagrangian with a minimal Higgs boson:

$$\begin{aligned} \mathcal{L}_{\text{MSM}} &= -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{L} \gamma^\mu \left(i \partial_\mu - \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{W}_\mu \right. \\ &\quad \left. - \frac{g'}{2} Y B_\mu \right) L + \bar{R} \gamma^\mu \left(i \partial_\mu - \frac{g'}{2} Y B_\mu \right) R \\ &\quad + \left| \left(i \partial_\mu - \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{W}_\mu - \frac{g'}{2} Y B_\mu \right) \phi \right|^2 - \mu^2 \phi^\dagger \phi \\ &\quad - \lambda (\phi^\dagger \phi)^2 - G_1 \bar{L} \phi R + i G_2 \bar{L} \tau_2 \phi^* R + \text{H.c.} \end{aligned} \quad (11)$$

Here L denotes left-handed doublet and R right-handed singlet leptons or quarks, $e = g \sin \theta_W = g' \cos \theta_W$ and electrical

charge $Q = T_3 + Y/2$. The last terms provide masses for leptons and quarks in the low temperature phase, $T < T_c$, where the Higgs field has a nonvanishing vacuum expectation value $\langle \phi \rangle = (0, v)/\sqrt{2}$; at zero temperature $v^2 = -\mu^2/\lambda = 1/(G_F\sqrt{2}) = 4M_W^2/g^2 = (246 \text{ GeV})^2$.

At temperatures below T_c , where $\mu^2 < 0$, the Higgs mechanism naturally selects the representation W^\pm, Z^0 , and γ of the four intermediate vector bosons. At temperatures above the transition—where $\langle \phi \rangle$ vanishes for a sharp transition, or tends to zero for a crossover—we consider driving the system with external vector potentials $A^a = B, W^\pm, W^3$, which give rise to corresponding “electric” fields \mathbf{E}_a , where

$$E_i^a \equiv F_{i0}^a = \partial_i A_0^a - \partial_0 A_i^a, \quad A^a = B \quad (12)$$

$$\equiv F_{i0}^a = \partial_i A_0^a - \partial_0 A_i^a - g \epsilon_{abc} A_i^b A_0^c,$$

$$A^a = W^1, W^2, W^3. \quad (13)$$

One can equivalently drive the system with the electromagnetic and weak fields derived from A, Z^0 , and W^\pm , as when $T \ll T_c$, or any other rotated combination of these. We consider here only the weak field limit and ignore the nonlinear driving terms in Eq. (13). The self-couplings between gauge bosons are important, however, in the scattering processes in the plasma determining the conductivity, as we discuss below.¹

The electroweak fields A^b act on the matter to generate currents J_a of the various particles present in the plasma, such as left and right-handed leptons and their antiparticles, and quarks, vector bosons, and Higgs bosons. The Higgs and vector boson contributions are, as we shall see, negligible. Therefore the significant terms in the currents are

$$J_B^\mu = \frac{g'}{2} (\bar{L} \gamma^\mu Y L + \bar{R} \gamma^\mu Y R), \quad (14)$$

$$J_{W^i}^\mu = \frac{g}{2} \bar{L} \gamma^\mu \tau_i L. \quad (15)$$

We define the conductivity tensor σ_{ab} in general by

$$\mathbf{J}_a = \sigma_{ab} \mathbf{E}^b. \quad (16)$$

¹Pure SU(2) gauge fields undergo a confining first order phase transition [22] at a critical temperature, T_c , where the fields are intrinsically strongly interacting. In an academic universe without Higgs fields the electroweak phase transition is nonexistent. If, in this case, we run the temperature from far above M_W to low temperatures the electroweak running coupling constants diverge at a very small temperature $\sim 10^{-23}$ GeV, signalling that the interactions of the fields have become strong, and that one is in the neighborhood of the SU(2) phase transition. For our purposes we are therefore safe in ignoring such nonlinear effects and the SU(2) phase transition. However, the nature of electrical conduction in the confined state by charged “mesons” formed, e.g., by $e^+ \nu_e$ or $u\bar{d}$, remains an interesting problem in principle. We thank Eduardo Fradkin for calling this issue to our attention.

Equation (16) with the equations of motion in a gauge with $\partial^\mu A_\mu^a = 0$ yields the weak field flux diffusion equation for the transverse component of the fields, as in QED,

$$(\partial_t^2 - \nabla^2) \mathbf{A}_a = \sigma_{ab} \partial_t \mathbf{A}_b, \quad (17)$$

describing the decay of weak fields in terms of the the conductivity.

The electroweak U(1) × SU(2) symmetry implies that the conductivity tensor, σ_{ab} , in the high temperature phase is diagonal in the representation $a, b = B, W^1, W^2, W^3$, as can be seen directly from the (weak field) Kubo formula

$$\sigma_{ab} = - \lim_{\omega \rightarrow 0} \lim_{k \rightarrow 0} \frac{1}{\omega} \text{Im} \langle J_a J_b \rangle_{\text{irr}}, \quad (18)$$

which relates the conductivity to (one-boson irreducible) current-current correlation functions.² The construction of the conductivity in terms of the Kubo formula assures that the conductivity and hence the related entropy production in electrical conduction are positive. Then

$$\sigma = \begin{pmatrix} \sigma_{BB} & 0 & 0 & 0 \\ 0 & \sigma_{WW} & 0 & 0 \\ 0 & 0 & \sigma_{WW} & 0 \\ 0 & 0 & 0 & \sigma_{WW} \end{pmatrix}. \quad (19)$$

Due to isospin symmetry of the W interactions the conductivities $\sigma_{W^i W^i}$ are the same, $\equiv \sigma_{WW}$, but differ from the B -field conductivity, σ_{BB} .

The calculation of the conductivities σ_{BB} and σ_{WW} in the weak field limit parallels that done for $T \ll T_c$. The main difference is that weak interactions are no longer suppressed by a factor $(T/M_W)^4$ and the exchange of electroweak vector bosons must be included. The conductivity, σ_{BB} , for the Abelian gauge field B can be calculated similarly to the electrical conductivity at $T \ll T_c$. Taking into account the fact that both left-handed neutrinos and charged leptons couple to the B field with the same sign, and that they scatter the same way, their flow velocities are equal. Consequently, in the scatterings $12 \leftrightarrow 34$, $\mathbf{u}_1 = \mathbf{u}_3$ and $\mathbf{u}_2 = \mathbf{u}_4$, whether or not the interaction is by charge exchange. The situation is thus similar to electrodynamic case.

Although the quarks and W^\pm are charged, their drifts in the presence of an electric field do not significantly contribute to the electrical conductivity. Charge flow of the quarks is stopped by strong interactions, while similarly flows of the W^\pm are effectively stopped by $W^+ + W^- \rightarrow Z^0$, via the triple boson coupling. Charged Higgs bosons are likewise stopped via $W^\pm \phi^\dagger \phi$ couplings. These particles do, however, affect the conductivity by scattering leptons.

²Since the linearized currents are proportional to the Pauli spin matrices τ^a for the W^a ($a=1,2,3$) fields and the identity matrix $\tau_0 = 1$ for the B field, one finds explicitly in one-loop order that $\sigma_{ab} \propto \text{Tr}\{\tau_a \tau_b\} = 2 \delta_{ab}$ ($a=0,1,2,3$). Including the dominant interactions in the current-current loop is basically equivalent to solving the Boltzmann equation, which produces no off-diagonal elements.

The lepton and quark mass terms in the Weinberg-Salam Lagrangian provide masses only when the Higgs field has a nonzero expectation value. For $T \gg T_c$ the quarks and leptons have thermal masses, which, for the longitudinal (electric) degrees of freedom, are of order the plasma frequency, $m_{\text{pl}} \sim gT$, and of likely order $m_{\text{mag}} \sim g^2 T$ [23] for the transverse (magnetic) mass. These small masses give rise to spin-flip interactions changing the helicity; such interactions are, however, suppressed by factors of m/T , and can therefore be neglected here. The mass terms also provide a small coupling, $G_l = \sqrt{2}m_l/v$, between the Higgs and leptons, proportional to the ratio of the lepton mass to the vacuum expectation value, which leads to a negligibly small contribution to the conductivity. Even the coupling of τ mesons with the Higgs is a factor $G_\tau^2/e^2 \sim 10^{-3}$ smaller than their coupling to the B field. Charge transfer scatterings via Higgs exchange are more singular than scatterings via B exchange, and are enhanced by a factor $1/e^2$; nonetheless such processes are negligible.

These considerations imply that the B current consists primarily of right-handed e^\pm , μ^\pm , and τ^\pm , interacting only through exchange of uncharged vector bosons B , or equivalently γ and Z^0 . Because the left-handed leptons interact through \mathbf{W} as well as through B , they give only a minor contribution to the current. They are, however, effective scatterers of right-handed leptons. The resulting conductivity is

$$\begin{aligned} \sigma_{BB} &= \frac{1}{2} \frac{N_l \cos^2 \theta_W}{\left[\frac{1}{8}(Y_R^2 + 2Y_L^2)N_l + \frac{1}{8} \sum_q N_q (Y_{q_R}^2 + Y_{q_L}^2) \right]} \sigma_{\text{el}}^{(\prime\prime)} \\ &= \frac{9}{19} \cos^2 \theta_W \sigma_{\text{el}}^{(\prime\prime)}, \quad T \gg T_c, \end{aligned} \quad (20)$$

where the $Y_{q_{R,L}}$ are the right and left-handed quark hypercharges, and the $Y_{R,L}$ are the right and left-handed charged lepton hypercharges. The terms entering the prefactor are (i) a factor 1/2 because only the right-handed leptons contribute

significantly to the conductivity; (ii) a net factor of $\cos^2 \theta_W$ because, on the left side of the kinetic equation, the B field couples to right-handed leptons with a factor $g' = e/\cos \theta_W$, and the square of the matrix element, entering the right side of the kinetic equation, contains a factor g'^4 , while the current J_B is proportional to g' times the deviation of the distribution function; and (iii) a factor $(Y_R^2 + 2Y_L^2)/8 = 3/4$ in the scatterings of the right-handed charged leptons with right and left-handed leptons, and (iv) a factor $\sum_q N_q (Y_{q_R}^2 + Y_{q_L}^2)/8 = 11/12$ in the scatterings of the right-handed charged leptons with right and left-handed quarks. The factor 9/19 holds in the limit that the temperature is much greater than the top quark mass, m_t ; excluding the top quark for $T_c < T < m_t$ gives 108/211 instead.

Applying a W^3 field to the electroweak plasma drives the charged leptons and neutrinos oppositely since they couple through $g\tau_3 W_3$. In this case, exchanges of W^\pm dominate the interactions as charge is transferred in the singular forward scatterings, so that $\mathbf{u}_3 = \mathbf{u}_2 = -\mathbf{u}_1$. The collision term is then weighted by a factor $(\mathbf{p}_1 + \mathbf{p}_2)$ instead of a factor $(\mathbf{p}_1 - \mathbf{p}_2) = \mathbf{q}$ and one ends up with an integral $\int p^2 dq^2 / (q^2 + q_D^2)^2 \approx T^2/q_D^2 \sim \alpha^{-1}$ for the longitudinal part of the interaction. For the transverse part of the interaction one encounters a logarithmic singularity; while Landau damping is not sufficient to screen the interaction, a QCD magnetic (gluon) mass, $m_{\text{mag}} \sim g^2 T$, will provide an infrared cutoff. Besides the logarithms, the factor α^{-1} remains and we expect that

$$\sigma_{WW} \sim \alpha \sigma_{BB}. \quad (21)$$

This effect of W^\pm exchange is analogous to the way gluon exchange in QCD gives strong stopping and reduces the ‘‘color conductivity’’ significantly [15]; similar effects are seen in spin diffusion in Fermi liquids [18].

The electrical conductivity is found from σ_{BB} and σ_{WW} by rotating the B and W^3 fields and currents by the Weinberg angle; using Eq. (16) we obtain

$$\begin{pmatrix} J_A \\ J_{Z^0} \end{pmatrix} = \mathcal{R}(\theta_W) \sigma \mathcal{R}(-\theta_W) \begin{pmatrix} A \\ Z^0 \end{pmatrix} = \begin{pmatrix} \sigma_{BB} \cos^2 \theta_W + \sigma_{WW} \sin^2 \theta_W & (\sigma_{BB} - \sigma_{WW}) \cos \theta_W \sin \theta_W \\ (\sigma_{WW} - \sigma_{BB}) \cos \theta_W \sin \theta_W & \sigma_{BB} \sin^2 \theta_W + \sigma_{WW} \cos^2 \theta_W \end{pmatrix} \begin{pmatrix} A \\ Z^0 \end{pmatrix}. \quad (22)$$

Thus the electrical conductivity is given by

$$\sigma_{AA} = \sigma_{BB} \cos^2 \theta_W + \sigma_{WW} \sin^2 \theta_W; \quad (23)$$

σ_{el}/T above the electroweak transition differs from that below mainly by a factor $\sim \cos^4 \theta_W \approx 0.6$.

In the wide temperature range we are considering the coupling constants in fact run as $\alpha_i(Q) = \alpha_i(\mu) + b_i \ln(Q/\mu)$ where the coefficients b_i are found by renormalization group calculations [24,25]. In a high temperature plasma typical momentum transfers Q are of order $q_D \sim eT$. The exact values employed for Q is not important as the couplings only increase logarithmically with temperature. In the temperature range 1 to 10⁶ GeV, α^{-1} varies from 130 to 123 and $\sin^2 \theta_W$ from 0.21 to 0.28.

IV. SUMMARY AND OUTLOOK

We have calculated the electrical and electroweak conductivities in the early universe over a wide range of temperatures. Typically, $\sigma_{\text{el}} \approx T/\alpha \ln(1/\alpha)$, where the logarithmic dependence on the coupling constant arises from Debye and dynamical screening of small momentum-transfer interactions. In the quark-gluon plasma, at $T \gg T_{\text{QGP}} \sim 150$ MeV, the additional stopping on quarks reduces the electrical conductivity from that in the hadronic phase. In the electroweak symmetry-restored phase, $T \gg T_c$, interactions between leptons and W^\pm and Z^0 bosons reduce the conductivity further. The electrical conductivity does not vanish (as one might have imagined to result from singular unscreened W^\pm exchanges), and is larger than previous estimates, within an

order of magnitude. The current is carried mainly by right-handed leptons since they interact only through exchange of γ and Z^0 .

From the above analysis we can infer the qualitative behavior of other transport coefficients. The characteristic electrical relaxation time, $\tau_{el} \sim [\alpha^2 \ln(1/\alpha)T]^{-1}$, defined from $\sigma \approx e^2 n \tau_{el} / T$, is a typical ‘‘transport time’’ which determines relaxation of transport processes when charges are involved. Right-handed leptons interact through Z^0 exchanges only, whereas left-handed leptons may change into neutrinos by W^\pm exchanges as well. Since Z^0 exchange is similar to photon exchange when $T \gg T_c$, the characteristic relaxation time is similar to that for electrical conduction, $\tau_\nu \sim [\alpha^2 \ln(1/\alpha)T]^{-1}$ (except for the dependence on the Weinberg angle). Thus the viscosity is $\eta \sim \tau_\nu \sim T^3 / [\alpha^2 \ln(1/\alpha)]$. For $T \ll M_W$ the neutrino interaction is suppressed by a factor $(T/M_W)^4$; in this regime neutrinos have longest mean free paths and dominate the viscosity [19].

The electrical conductivity of the plasma in the early universe is sufficiently large that large-scale magnetic flux

present in this period does not diffuse significantly over timescales of the expansion of the universe. The time for magnetic flux to diffuse on a distance scale L is $\tau_{diff} \sim \sigma_{el} L^2$. Since the expansion time scale t_{exp} is $\sim 1/(t_{Planck} T^2)$, where $t_{Planck} \sim 10^{-43}$ s is the Planck time, one readily finds that

$$\frac{\tau_{diff}}{t_{exp}} \sim \alpha x^2 \frac{\tau_{el}}{t_{Planck}} \gg 1, \quad (24)$$

where $x = L/ct_{exp}$ is the diffusion length scale in units of the distance to the horizon. As described in Refs. [10] and [5], sufficiently large domains with magnetic fields in the early universe would survive to produce the primordial magnetic fields observed today.

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