## Constraints from $b \rightarrow s \gamma$ on gauge-mediated supersymmetry-breaking models

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We consider the branching ratio of  $b \rightarrow s\gamma$  in gauge-mediated supersymmetry-breaking theories. Useful bounds on the parameter space of these models are derived from the experimental bounds on  $b \rightarrow s\gamma$ . Constraints on masses of the next to lightest supersymmetric particle are presented as a function of  $\tan\beta$  and  $M/\Lambda$  for  $\mu < 0$  and  $\mu > 0$ . [S0556-2821(97)03913-1]

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There has been tremendous recent interest in the phenomenological implications of gauge-mediated supersymmetrybreaking theories [1-5]. These theories are characterized by a gravitino as the lightest supersymmetric particle (LSP) and have signatures for supersymmetry which are distinct from the usual minimal supersymmetric standard model. The interest in these theories is especially heightened because of the possible explanation of the peculiar event seen at the Collider Detector at Fermilab with final state containing  $e^+e^-\gamma\gamma$  [6] and missing transverse energy. A successful explanation of this event requires the lightest neutralino to be the next to lightest supersymmetric particle (NLSP). Whether or not this explanation withstands the test of time, it would seem important to examine in detail the mass constraints on NLSP that ensue in these models from the rare decay  $b \rightarrow s \gamma$  where supersymmetry (SUSY) contributions occur in one-loop diagrams. Although some preliminary work exists in special cases [3], we study the full allowed parameter space.

We shall analyze  $b \rightarrow s \gamma$  in the gauge-mediated SUSYbreaking model in the whole range of combinations of  $\Lambda$  and *M*, the number *n* of  $(5+\overline{5})$  pairs, and a range of tan $\beta$  starting from small values to large values. [Here M is the messenger scale which is related to  $M = \lambda \langle s \rangle$ , where  $\langle s \rangle$  is the vacuum expectation value (VEV) of the scalar component of the hidden sector superfields, and  $\lambda$  is the Yukawa coupling. The parameter  $\Lambda$  is equal to  $\langle F_s \rangle / \langle s \rangle$ , where  $\langle F_s \rangle$  is the VEV of the auxiliary component of s.] We show plots of branching ratio as functions of  $\mu$  to show when  $\mu$  becomes too large to require fine-tuning, where  $\mu$  is the coefficient of the bilinear Higgs term in the superpotential. We also consider both small and large values of  $tan\beta$ , so that we have regions where lighter stau can be the NLSP. We then translate the bounds of  $b \rightarrow s \gamma$  into the bounds on the NLSP masses, since the signals of these models at the CERN  $e^+e^-$  collider LEP 2 and Fermilab Tevatron involve (will involve) the production of these NLSP's and their subsequent decays into LSP and other particles.

We take  $\Lambda \sim 100$  TeV since soft SUSY-breaking scalar masses are then of the order of the weak scale. The parameter is  $M \ge \Lambda$ . There could be a large hierarchy  $M \ge \Lambda$  [5]; however, the upper bound of the gravitino mass  $\sim 1$  keV restricts the  $M/\Lambda < 10^4$  [7]. In our calculation we use n=1 and n=2. The representation 10 can be included by noting that one (10+10) pair corresponds to n=3.

The induced gaugino and scalar masses at the scale M are [8]

$$\widetilde{M}_{i}(M) = ng\left(\frac{\Lambda}{M}\right) \frac{\alpha_{i}(M)}{4\pi} \Lambda, \qquad (1)$$

$$\widetilde{m}^{2}(M) = 2(n) f\left(\frac{\Lambda}{M}\right) \sum_{i=1}^{3} k_{i} C_{i} \left(\frac{\alpha_{i}(M)}{4\pi}\right)^{2} \Lambda^{2}, \qquad (2)$$

where  $k_i$  and  $C_i$  are 1,1,3/5 and 4/3, 3/4, and  $Y^2$  for SU(3), SU(2), and U(1), respectively. The values of  $C_i$  apply only to the fundamental representations of SU(3) and SU(2) and are zero for the gauge singlets.  $\alpha_1$  is the grand unified theory normalized coupling.

We use the exact messenger-scale threshold functions [4]

$$g(x) = \frac{1+x}{x^2} \ln(1+x) + (x \to -x), \qquad (3)$$

$$f(x) = \frac{1+x}{x^2} \left[ \ln(1+x) - 2\operatorname{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\operatorname{Li}_2\left(\frac{2x}{1+x}\right) \right] + (x \to -x),$$
(4)

rather than the limiting values, f(0) = g(0) = 1, and f(1) = 0.7 and g(1) = 1.4. We shall require that electroweak symmetry be radiatively broken. We use  $\alpha_s = 0.120$ ,  $\sin^2 \theta_w = 0.2321$  and  $\alpha = 1/127.9$  at the weak scale as the gauge coupling inputs. We first go up to the messenger scale M with gauge and Yukawa couplings, and fix the sparticle masses with the boundary conditions (1) and (2). We next go down with the  $6 \times 6$  mass matrices for the squarks and sleptons to find the sparticle spectrum for large as well as small  $\tan\beta$ . We use the renormalization group equations given in Ref. [9]. Also, we do not choose a particular model for  $\mu$ . Note that the soft Higgs boson mass parameters  $m_{H_1}^2$  and  $m_{H_2}^2$  from Eq. (2), along with the ratio of the (VEV)  $\tan\beta (\equiv v_2/v_1)$  uniquely specifies  $|\mu|$ . We will use two extreme values of tan $\beta$  equal to 3 and 42 (for n=1) for illustration. It is interesting that in the case of  $\tan\beta = 3$  neutralino is the NLSP (next to lightest SUSY particle) for all values of  $M > \Lambda$ ; however, for tan $\beta = 42$ , either the stau or the neutralino is the NLSP depending on whether  $\mu$  and  $M/\Lambda$  are small or large. For n = 2, however, lighter stau can be NLSP even for the small  $\tan\beta$  for the lower ratios of  $M/\Lambda$ . When stau is the NLSP, these models cannot explain "the" event in Tevatron. However, the scenarios can have other interesting collider signatures both in Tevatron as well as LEP. Calculation of  $b \rightarrow s \gamma$  amplitude involves the coefficients of short distance photonic and gluonic operators  $c_7(M_w)$  and  $c_8(M_w)$  [10]. Effects of QCD corrections to two loops is then carried out. For the standard model these calculations are given in Ref. [11]. Calculations of next-to-leading order (NLO) agree with the previous calculations while reducing the theoretical errors [12]. Contributions from various supersymmetric contributions are given in a generic form in Ref. [13]. We use our calculated mass spectrum and the couplings to calculate the  $b \rightarrow s \gamma$  rate. The results depend on  $\Lambda$ , M,  $\tan\beta$ , *n*, and the sign of  $\mu$ . We shall use more physical variables  $\tan\beta$ ,  $\mu$ , sign of  $\mu$ ,  $M/\Lambda$  and n. Our figures are for the minimal case of n=1, and we shall remark on the situation for higher n. The total amplitude has contribution from the W loop, charged Higgs boson  $(H^{\pm})$  loop, chargino  $(\chi^{\pm})$  loop, neutralino  $(\chi^0)$  loop, and the gluino  $(\tilde{g})$  loop. We find that the neutralino and the gluino contributions to the amplitude are less than 1% in the whole range of parameter space. The charged Higgs boson contribution adds constructively to the W-loop contribution. The chargino contribution can occur with either sign, but is generally much smaller than the Higgs boson contribution. An exception is when  $\tan\beta$  is large and  $\mu < 0$  and chargino interference opens up the allowed parameter space.

We consider branching ratio vs  $\mu$  for either sign of  $\mu$  in two different scenarios: (a)  $\tan\beta=42$  and  $M=1.1\Lambda$  and  $M=10^{4}\Lambda$ ; (b)  $\tan\beta=3$  and  $M=1.1\Lambda$  and  $M=10^{4}\Lambda$  for n= 1. The two different values of M form the boundaries of the envelope of parameter space that would be traced by any relation between  $\Lambda$  and M. We exclude the case  $M=\Lambda$  since it produces a massless scalar in the messenger sector. We have used the particle data values for the Cabibbo-Kobayashi-Maskawa matrix elements and also have imposed the constraint  $|V_{ts}^*V_{tb}|^2/|V_{cb}|^2=0.95\pm0.04$  [14].

In Fig. 1(a) corresponding to scenario (a) we display  $b \rightarrow s \gamma$  branching ratio as a function of  $|\mu|$  for  $\mu > 0$ . Solid lines represent  $M = 1.1\Lambda$  and the dashed lines  $M = 10^4\Lambda$ . CLEO bound  $1 \times 10^{-4} < B(b \rightarrow s \gamma) < 4.2 \times 10^{-4}$  at 95% C.L. clearly rules out the smaller values of  $\mu$ . We find  $\mu > 720$ . When the branching ratio is  $4.2 \times 10^{-4}$  the NLSP (stau in the case of  $M = 1.1\Lambda$ ) mass is 182 GeV. Fig. 1(b) displays scenario (a) for  $\mu < 0$ . The chargino destructive interference for larger values of  $M/\Lambda$  does not yield any useful constraint on the parameter space. In this parameter space, lighter mass NLSP solutions correspond to stau as NLSP. The extreme left ends of the curves correspond to a bound on the lightest slepton mass  $\sim 65$  GeV.

In Figs. 2(a) and 2(b) we consider two cases in scenario (b), for positive and negative  $\mu$ , respectively. The left ends of the curves correspond to the lowest chargino mass bound which we have taken to be around 75 GeV. In these cases, chargino contribution is very small. The variation with *M* is also small. We then find that the CLEO constraint leads to  $\mu > 460$  GeV for positive  $\mu$  and  $|\mu| > 426$  GeV for negative  $\mu$  when  $M = 1.1\Lambda$ .

Constraints on  $\mu$  can be translated into bounds for the masses of supersymmetric particles. We are interested in dis-



FIG. 1. Plots for  $b \rightarrow s \gamma$  branching ratio as a function of  $|\mu|$  for  $\tan\beta=42$ . Solid lines correspond to  $M=1.1\Lambda$ , dashed lines correspond to  $M=10^4\Lambda$ . (a) For  $\mu>0$ , (b) for  $\mu<0$ .

playing these bounds for the masses of NLSP. In Fig. 3(a) we display the lower bound on neutralino mass as a function of tan $\beta$  for  $\mu > 0$  and  $\mu < 0$  for two limiting values of M, i.e.,  $M = 1.1\Lambda$  and  $M = 10^4\Lambda$ . However, to be conservative we add the theoretical uncertainity (around 15%) based on the NLO condition [12] on top of the CLEO bound. As  $\tan\beta$  becomes larger, chargino interference in the  $\mu < 0$  case for  $M = 10^4 \Lambda$  removes any useful bounds. Though the variation of the ratio of  $M/\Lambda$  does not produce much difference in the branching ratio for the lower values of  $\tan\beta$ , it has bigger effects in the bounds for the NLSP. In Fig. 3(b) we display the lower bounds on NLSP when  $\tan\beta \ge 31$ . Four cases considered are similar to Fig. 3(a). However, we do not have any CLEO bound on the NLSP when  $\mu < 0$  and  $M = 10^4 \Lambda$  as well as for  $M = 1.1\Lambda$  throughout the range of tan $\beta$  displayed. The solid curve and the dashed curve correspond to the bounds on lighter stau mass which is the NLSP and the dotdashed curve corresponds to the bounds on the neutralino mass, which is the NLSP in this case. The bound on  $\mu$ , for the positive values of  $\mu$ , is a monotonic function increasing from 460 GeV for  $\tan\beta = 3$  to 720 GeV for  $\tan\beta = 42$  when  $M = 1.1\Lambda$ . For  $M = 10^4\Lambda$ , the bound increases from 480 GeV to 920 GeV in the same range of  $tan\beta$ . These large values of  $\mu$  raise a problem of fine-tuning.

For n=2, the constraint on the NLSP mass is higher, e.g., for tan $\beta = 3$ , lowest mass for the stau (since it is the NLSP) allowed by CLEO data (plus the theoretical uncertainity) would be 91 GeV when  $\mu < 0$  and  $M = 1.1\Lambda$ , for  $\mu > 0$ , the lowest lighter stau mass allowed is 97 GeV for  $M = 1.1\Lambda$  and the lowest neutralino (NLSP) mass is 80 GeV for



FIG. 2. Plots for  $b \rightarrow s \gamma$  branching ratio as a function of  $|\mu|$  for  $\tan\beta=3$ . Solid lines correspond to  $M=1.1\Lambda$ , dashed lines correspond to  $M=10^4\Lambda$ . (a) For  $\mu>0$ , (b) for  $\mu<0$ .

 $M = 10^4 \Lambda$ . There is no bound in the case of  $\mu < 0$  and  $M = 10^4 \Lambda$ . For  $\tan\beta = 42$ , with  $\mu > 0$ , lowest stau mass (NLSP) allowed is 117 GeV for  $M = 1.1\Lambda$  and the lowest stau mass (NLSP) is 124 GeV for  $M = 10^4 \Lambda$ . For  $\mu < 0$  we do not have any bound. We have assumed the value of 175 GeV for the top running mass. The results are rather insensitive to this mass. A variation of 5% in mass results in the change of branching ratio of less than 1%.

In conclusion, we have used the CLEO bound on the branching ratio for  $b \rightarrow s \gamma$  to limit the parameter space of the gauge-mediated supersymmetry-breaking models. We have found useful bounds on masses of NLSP. We also have found that for positive  $\mu$ , irrespective of tan $\beta$ ,  $\mu$  is restricted



FIG. 3. (a) Bound on NLSP (neutralino) mass as a function of  $\tan\beta$ . Curves from the top (around  $\tan\beta=5$ ) correspond to (i)  $\mu>0$  and  $M=1.1\Lambda$ , (ii)  $\mu<0$  and  $M=1.1\Lambda$ , (iii)  $\mu>0$  and  $M=1.0^{4}\Lambda$ . (b) Bound on NLSP (stau for the solid and dashed lines and neutralino for the dot-dashed line) mass as a function of higher range of  $\tan\beta$ . Curves from the top (around  $\tan\beta=32$ ) correspond to (i)  $\mu>0$  and  $M=1.1\Lambda$ , (ii)  $\mu>0$  and  $M=1.0^{4}\Lambda$ .

to large values. Since this raises the problem of fine-tuning, our analysis shows that gauge-mediated model generally favors negative  $\mu$  solutions. When  $\mu$  is negative the available parameter space increases with the ratio of  $M/\Lambda$ . In the near future, with an improved bound on the branching ratio for  $b \rightarrow s \gamma$ , it will be possible to put more severe constraints on the parameter space.

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