Closed inhomogeneous string cosmologies

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(Received 24 April 1997)

We present a general algorithm which permits us to construct solutions in string cosmology for heterotic and type-IIB superstrings in four dimensions. Using a chain of transformations applied in sequence—conformal, T duality, and SL(2,**R**) rotations, along with the usual generating techniques associated with Geroch transformations in Einstein frame—we obtain solutions with all relevant low-energy remnants of the string theory. To exemplify our algorithm we present an inhomogeneous string cosmology with S^3 topology of spatial sections, discuss some properties of the solution, and point out some subtleties involved in the concept of homogeneity and isotropy in string cosmology. [S0556-2821(97)06420-5]

PACS number(s): 11.25.Mj, 04.20.Jb, 04.50.+h

I. INTRODUCTION

The cosmological implications of string theory are receiving quite broad attention these days (for a necessarily incomplete list of recent and not-so-recent references see [1,2,3,4]). This is not by chance since, given the lack of traces of stringy effects in ''low-energy'' particle physics, the cosmological scenario appears as a perfect arena for the search of string blueprints in the early Universe.

In spite of the impressive progress in the study of string theory beyond the perturbative regime (for a review see, for example, [5]), we still lack a full-fledged nonperturbative formulation which allows a description of the early Universe at Planck time, where the classical concepts of space and time cannot be used any longer. In the meantime, one is bound to study classical cosmology using the low-energy effective action induced by string theory, which generalizes general relativity by including other massless fields. As a consequence, in string cosmology one continues to ask essentially the same questions earlier formulated in the framework of Einstein relativity, but now posed in the presence of those extra degrees of freedom. The central puzzle of theoretical cosmology remains as well the same, and is concerned mainly with the question of why the present Universe looks as isotropic and homogeneous at large scales. The desire to answer this last question leads one to consider the initial conditions and the evolution of the Universe at the beginning of expansion.[1]

Apart from a few exceptions [3,4], most of the work in string cosmology is focused on homogeneous solutions. However, since in general relativity the generic solution near the cosmological singularity is neither isotropic nor homogeneous, one is led to study cosmological models with less symmetry, leaving aside not only the principle of isotropy but homogeneity as well. The ideas to study such a general behavior were inspired by Landau [9] and worked out in details in classical papers by Belinski, Lifshitz, and Khalatnikov[2] (BLK) [10]. The techniques due to BLK are strongly rooted in physical intuition, but lack, however, rigor when approximations are used in a highly nonlinear regime. Although the conclusions of this analysis are probably correct at large, one must necessarily confront them against the analytic behavior of exact solutions.

Because of the mathematical complexity of the generic inhomogeneous models, one usually deals with solutions in which homogeneity is broken in only one direction. The space-times obtained this way are usually referred to as G_2 or Einstein-Rosen cosmologies [11–13] and are thought to provide the leading approximation to a general solution near the initial singularity [14]. It is important to mention here the relation of the Einstein-Rosen cosmologies to the interesting problem of scattering of plane waves in general relativity [15,16]. This relation, considered first in [17], consists just in inverting the arrow of time of the vacuum, or vacuum with massless fields, inhomogeneous solutions. After all, the approach to the focusing two-surface in the collision of self-gravitating plane waves (not necessarily gravitational) is just the time inverse of the behavior near the singularity in a G_2 cosmology. This being so, the study of the behavior of string-induced G_2 cosmologies may lead to a better understanding of such a highly nonlinear process as plane wave scattering.

The main purpose of this paper is to describe a sufficiently general algorithm which permits the construction of new solutions of string cosmology starting from vacuum solutions of the Einstein field equations. Vacuum G_2 space-times will be used as the basic building blocks of our string solutions. Before going any further in the description of our generating technique, it will be useful to briefly mention some facts about the low-energy effective theory of the different superstring models and their symmetries.

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[1]We will not dwell here on different approaches and views as to how to solve this problem, but just mention some central ideas such as chaotic cosmology program [6], C-gravitational entropy [7], and the inflationary scenario [8].

[2]In string cosmology the BLK approach has been recently discussed in the context of the pre-big-bang scenario by Veneziano [4].

A. String low-energy effective actions

On general grounds, the massless bosonic sector of superstrings includes the gravitational field $G_{\mu\nu}$, the dilaton ϕ, with vacuum expectation value determining the string coupling constant, and the antisymmetric rank-two tensor $B^{(1)}_{\mu\nu}$, in addition to other fields depending on the particular superstring model under study. The lowest order effective action for the massless fields can be written as

$$S_{\rm eff}=\frac{1}{(\alpha')^{(D-2)/2}}\int d^Dx\sqrt{G}e^{-2\phi}\left[R+4(\partial\phi)^2-\frac{1}{12}(H^{(1)})^2\right]+S_{\rm md}, \tag{1.1}$$

where $H^{(1)}=dB^{(1)}$ is the field strength associated with the Neveu-Schwarz–Neveu-Schwarz (NS-NS) two-form and $S_{\rm md}$ is a model-dependent part which includes other massless degrees of freedom. When $D<10$ some of these massless fields correspond to gauge and moduli fields associated with the specific compactification chosen, and the dilaton ϕ appearing in (1.1) is related to the ten-dimensional dilaton ϕ_{10} by $2\phi=2\phi_{10}-\ln V_{10-D}$, where V_{10-D} is the volume of the internal manifold measured in units of $\sqrt{\alpha'}$.

We restrict our attention to generic degrees of freedom, leaving aside the internal components of the ten-dimensional fields. In the heterotic string case, $S_{\rm md}$ contains the Yang-Mills action for the background gauge fields A^a_μ, which we set to zero in the following. In the case of the model-dependent part of the type-IIB superstring things are slightly more involved; among the massless degrees of freedom in the Ramond-Ramond (R-R) sector we find, along with a pseudoscalar χ (the axion) and a rank-two antisymmetric tensor $B^{(2)}_{\mu\nu}$, a rank four self-dual form $A^{\rm sd}_{\mu\nu\sigma\lambda}$. The presence of this self-dual form spoils the covariance of the effective action for the massless fields, since there is no way of imposing the self-duality condition in a generally covariant way. But, if we set $A_{\rm sd}$ to zero, we can write a covariant action for the remaining fields with

$$S^{\rm IIB}_{\rm md}=-\frac{1}{(\alpha')^{(D-2)/2}}\times\int d^Dx\sqrt{G}\left[\frac{1}{2}(\partial\chi)^2+\frac{1}{12}(H^{(1)}\chi+H^{(2)})^2\right].$$

Notice that the R-R fields do not couple directly to the dilaton. Thus, the lower dimensional ($D<10$) R-R fields χ and $B^{(2)}_{\mu\nu}$ are obtained from the ten-dimensional ones through $\chi=\sqrt{V_{10-D}}\chi_{10}$ and $B^{(2)}=\sqrt{V_{10-D}}B^{(2)}_{10}$.

By combining the dilaton and the axion of the ten-dimensional type-IIB superstring into a single complex field $\lambda=\chi_{10}+ie^{-\phi_{10}}$, it is possible to check that the bosonic effective action is invariant under the SL(2,**R**) transformation[3] $\lambda\to(a\lambda+b)/(c\lambda+d)$, $G_{\mu\nu}\to|c\lambda+d|G_{\mu\nu}$ [18]. Writing this transformation in terms of four-dimensional fields ($D=4$) we find that the action is invariant under the following field redefinitions (cf. [19]):

$$\begin{aligned}
\chi_4'&=\frac{bd+ace^{-2\phi_4}}{[(c\chi_4+d)^2+c^2e^{-2\phi_4}]^{1/4}}+\frac{ac\chi_4^2+(ad+bc)\chi_4}{[(c\chi_4+d)^2+c^2e^{-2\phi_4}]^{1/4}},\\
e^{-\phi_4'}&=\frac{e^{-\phi_4}}{[(c\chi_4+d)^2+c^2e^{-2\phi_4}]^{1/4}},\\
H^{(1)\prime}&=dH^{(1)}-cH^{(2)},\\
H^{(2)\prime}&=[(c\chi_4+d)^2+c^2e^{-2\phi_4}]^{3/4}(-bH^{(1)}+aH^{(2)}),\\
G'_{\mu\nu}&=[(c\chi_4+d)^2+c^2e^{-2\phi_4}]^{1/2}G_{\mu\nu}.
\end{aligned}\tag{1.2}$$

Comparing with the corresponding transformations for the ten-dimensional fields we find that the four-dimensional ones acquire an ''anomalous weight'' due to the fact that V_6, as measured in the string frame, does transform under SL(2,**R**).

The algorithm to be described in the following uses as input diagonal vacuum solutions to Einstein equations with two commuting spacelike Killing vectors. Infinite dimensional families of solutions of this kind are known [11,12,17,13]. After transforming these diagonal vacuum solutions to generate off-diagonal terms in the metric, we generalize them to include a minimally coupled massless scalar field. A conformal rescaling of the Einstein metric gives solutions to four-dimensional dilaton gravity and a T-duality transformation generates an antisymmetric rank-two tensor $B^{(1)}_{\mu\nu}$ from the off-diagonal components of the metric. This leaves us with string cosmology solutions of both the heterotic and type-IIB superstring. In the latter case, we still can generate nontrivial background R-R fields by performing a SL(2,**R**) rotation of the solution (cf. [2]). In the next section, we will detail the algorithm and in Sec. III a concrete application will be worked out. Finally, in Sec. IV we will summarize our conclusions.

II. STRING COSMOLOGY FROM GENERAL RELATIVITY

After having outlined the key steps to be followed in the construction of solutions, we now turn to the generating algorithm itself. In order not to get too involved with the details we shall omit some technicalities, referring in cases to the original papers. We present here only those details and expressions necessary to get the final solution and to keep the presentation self-consistent. As mentioned above our starting point is a globally diagonalizable G_2 line element which may be put into the following convenient form:

$$ds^2=e^{f(t,z)}(-dt^2+dz^2)+\gamma_{ab}(t,z)dx^adx^b,\quad a,b=1,2. \tag{2.1}$$

The local behavior of the spacetime is defined by the gradient of the transitivity surface area $K(t,z)\equiv\sqrt{\det\gamma_{ab}}$, which can be globally timelike, spacelike, null or vary from point to point. We will not restrict our attention to any of these specific cases keeping the determinant arbitrary; just mention that in cosmology one is most interested in a globally time-

[3]The two antisymmetric tensors $B^{(1)}_{\mu\nu}$ and $B^{(2)}_{\mu\nu}$ also transform as a doublet under SL(2,**R**). Notice that here we are using the string frame in which the metric is not invariant.

like case, which includes anisotropic Bianchi type I–VII models and their inhomogeneous generalizations, and a general case where the gradient of the transitivity area may vary from point to point. The latter includes the most general Bianchi VIII and IX types with local rotational symmetry with or without inhomogeneities [12], precisely the case which will serve us as example in this paper. If one of the Killing vector fields $\xi_1=\partial_1$ and $\xi_2=\partial_2$ is orthogonal to the hypersurface obtained by dragging the surface t-z along the other Killing vector field then the spacetime is globally diagonalizable. We thus start from such a spacetime in vacuum and notice that we can write

$$\gamma_{ab}(t,z)dx^a dx^b=K(t,z)[e^{p(t,z)}(dx^1)^2+e^{-p(t,z)}(dx^2)^2], \tag{2.2}$$

where the function $p(t,z)$ satisfies the linear wave equation

$$\frac{d}{dt}(K\dot{p})-\frac{d}{dz}(Kp')=0$$

and the other metric function $f(t,z)$ may be found by a quadrature [11–13].

One now applies one of the standard techniques to generate the nondiagonal solution starting from the diagonal seed (2.2). This can be done in multiple ways: either using inverse scattering method of Ref. [14], the solution generating algorithm described in [20], or an appropriately adapted Hoenselaers-Kinnersley-Xanthopoulos (HKX) transformation [21]. Actually, even a simple Ehlers rotation of the Killing vectors [22] will do the job. Although this last procedure does not generate a genuinely nondiagonal solution (one may globally rediagonalize the metric leading to a new solution), combined with a T-duality transformation in the string frame it will produce the desired nonvanishing components for the $B^{(1)}_{\mu\nu}$ field.

Assuming that the nondiagonal vacuum solution has been constructed, we now have to solve Einstein equations for the metric (2.1) with a minimally coupled massless scalar field (our proto-dilaton). Fortunately, these solutions may be quite easily obtained, since a massless scalar field has the same characteristics of propagation as gravity, and does not introduce any extra degree of nonlinearity into the problem. In fact, if $\phi(t,z)$ is the scalar field, the new solution of the Einstein equations may be written as [23]

$$\gamma_{ab}(t,z)=\gamma^{\text{vac}}_{ab}(t,z),$$

$$f(t,z)=f^{\text{vac}}(t,z)+f^{\text{sc}}(t,z), \tag{2.3}$$

where $f^{\text{vac}}(t,z)$ is the vacuum solution of the line element (2.1), $f^{\text{sc}}(t,z)$ is determined by

$$\dot{f}^{\text{sc}}=\frac{2K}{K'^2-\dot{K}^2}[2K'\dot{\phi}\phi'-\dot{K}(\dot{\phi}^2+\phi'^2)],$$

$$f'^{\text{sc}}=\frac{2K}{K'^2-\dot{K}^2}[K'(\dot{\phi}^2+\phi'^2)-2\dot{K}\dot{\phi}\phi'] \tag{2.4}$$

and the scalar field $\phi(t,z)$ verifies the following linear differential equation:

$$\frac{d}{dt}(K\dot{\phi})-\frac{d}{dz}(K\phi')=0. \tag{2.5}$$

If the gradient of the transitivity surface area is globally timelike one may choose $K\sim t$, and the solutions of the equation may be presented as combinations of the Bessel functions of the first and second kind [17]:

$$\begin{aligned}\phi=\beta\ \ln t&+\mathcal{L}\{A_\omega\cos[\omega(z+z_0)]J_0(\omega t)\}\\&+\mathcal{L}\{B_\omega\cos[\omega(z+z_0)]N_0(\omega t)\}\\&-\sum_i d_i\text{arccosh}\left(\frac{z+z_i}{t}\right),\end{aligned}$$

where $\mathcal{L}$ indicates linear combinations of the terms in curly brackets, and ω can have a discrete or continous spectrum. On the other hand, in the more general case when the gradient varies from point to point and the spatial sections have S^3 topology, as it happens in the case of Bianchi IX models, $K\sim\sin t\sin z$ [24] and the general solution of the equation (2.5) can be expanded in Legendre polynomials of the first and second kind:

$$\begin{aligned}\phi=\alpha_1\ \ln\left|\tan\frac{t}{2}\right|&+\alpha_2\ \ln\left|\tan\frac{z}{2}\right|+\alpha_3\ \ln|\sin t\sin z|\\&+\sum_{l=0}^{\infty}[A_lP_l(\cos t)+B_lQ_l(\cos t)]\\&\times[C_lP_l(\cos z)+D_lQ_l(\cos z)].\end{aligned} \tag{2.6}$$

Equations (2.3), (2.4), and (2.6) summarize the preliminary construction in the Einstein frame. Now we transform the solution into the string frame by the conformal transformation [25,26]

$$ds^2\to e^{2\phi(t,z)}ds^2.$$

This provides us with a solution to four-dimensional dilaton gravity. We are, nevertheless, interested in having nontrivial values for other background fields, mainly the two-form potential $B^{(1)}_{\mu\nu}$ appearing in (1.1). To accomplish this we can use a T-duality transformation. Taking adapted coordinates in which x^0 denotes the coordinate along the Killing vector chosen to dualize, we find new values for $(G_{\mu\nu},\phi,B^{(1)}_{\mu\nu})$ given by [27]:

$$\begin{gathered}\widetilde{G}_{00}=\frac{1}{G_{00}},\quad \widetilde{G}_{0\mu}=\frac{B^{(1)}_{0\mu}}{G_{00}},\\ \widetilde{G}_{\mu\nu}=G_{\mu\nu}-\frac{G_{0\mu}G_{0\nu}+B^{(1)}_{\mu 0}B^{(1)}_{0\nu}}{G_{00}},\\ \widetilde{B}^{(1)}_{0\mu}=\frac{G_{0\mu}}{G_{00}},\quad \widetilde{B}^{(1)}_{\mu\nu}=B^{(1)}_{\mu\nu}-\frac{G_{0\mu}B^{(1)}_{0\nu}+G_{0\nu}B^{(1)}_{\mu 0}}{G_{00}},\\ \widetilde{\phi}=\phi-\ln\sqrt{G_{00}}.\end{gathered} \tag{2.7}$$

The new background fields so obtained automatically satisfy the field equations derived from the effective action (1.1) [27]. Since we have two commuting Killing vectors we can dualize with respect to both of them to end up with nonzero values for several components of $B^{(1)}_{\mu\nu}$. At this point it is important to stress that we are using Buscher's formula (2.7) just as a formal procedure to generate new solutions of the low-energy field equations.

So far, the application of our algorithm leads to solutions to the low-energy equations for the ''universal massless spectrum'' of the four dimensional heterotic and type-II superstring. In the latter case we would be interested in getting solutions including also background values for the R-R fields. In the case of the type-IIB superstring this can be done using the invariance of the whole effective action $S_{\rm univ}+S^{IIB}_{\rm md}$ under the four-dimensional SL(2,**R**) transformation (1.2) [2]. In particular, starting with a solution in which $\chi=B^{(2)}_{\mu\nu}=0$, we can generate nontrivial values for these R-R fields, thus completing our generating algorithm. The important point here is to realize that, in this last step, we are doing more than just performing a S-duality transformation. Although SL(2,**R**) is a symmetry of the low-energy effective type-IIB supergravity, only a subgroup SL(2,**Z**) is an actual symmetry of the full underlying string theory. Therefore in performing a generic SL(2,**R**) rotation we will end up with solutions which are not equivalent to the original one at the level of the string theory.

III. INHOMOGENEOUS MIXMASTER STRING COSMOLOGIES

To illustrate the use of the algorithm we suggest to look at the following example. It is well known in cosmology that Bianchi type IX models (the famous Mixmaster universes [6]), let alone their inhomogeneous generalizations, represent a very interesting class of models to study. Because of the presence of spatial curvature and the nontrivial S^3 topology one may investigate their effects on the initial expansion. We consider here the locally rotationally symmetric (LRS) case which may be incorporated within the Einstein-Rosen spacetimes [12]. To shorten the procedure we pass over the first two steps and start directly with the solution to Einstein-Klein-Gordon equations obtained previously by one of us [12]. We can write it in the string frame as

$$ds^2=\left(\tan\frac{t}{2}\right)^{2M/k}e^{2\phi_1(t,\theta)}\{e^{f(t,\theta)}(-dt^2+d\theta^2)+[I_1^2(t)\sin^2\theta+I_3^2(t)\cos^2\theta]d\varphi^2+I_3^2(t)d\psi^2+2I_3^2(t)\cos\theta d\varphi d\psi\} \tag{3.1}$$

where the functions $f(t,\theta)$, $I_1(t)$, and $I_3(t)$ are defined by

$$f(t,\theta)=2\ln I_1(t)+C^2\sin^2 2t(1+3\cos^2\theta)\sin^2\theta+2C^2\sin^2 t(1-3\cos^2 t)\sin^4\theta-16\frac{M}{k}C\cos t\sin^2\theta,$$

$$I_1^2(t)=\frac{k^2}{2A}\frac{(\tan t/2)^{2A/k}+(\cot t/2)^{2A/k}}{(\tan t/2+\cot t/2)^2},$$

$$I_3^2(t)=\frac{2A}{(\tan t/2)^{2A/k}+(\cot t/2)^{2A/k}},$$

and the coordinates $0\leqslant\theta\leqslant\pi$, $0\leqslant\phi\leqslant 2\pi$, and $0\leqslant\psi\leqslant 4\pi$ are taken to be Euler angles. For the sake of simplicity, we have chosen a scalar dilation field $\phi(t,\theta)=\phi_0(t)+\phi_1(t,\theta)$ containing just two modes, the homogeneous one

$$\phi_0(t)=\frac{M}{k}\ln\tan\frac{t}{2},$$

and the inhomogeneous growing mode $\phi_1(t,\theta)$ compatible with the boundary conditions at $\theta=0,\pi$ [12],

$$\phi_1(t,\theta)=\frac{C}{4}(1-3\cos^2 t)(1-3\cos^2\theta).$$

The S^3 topology plays an important role near the initial singularity by imposing strong restrictions on the allowed modes of the scalar field, excluding for example strictly decreasing inhomogeneous solutions.

After T-dualizing with respect to the Killing vector $\xi=\partial_\varphi$ we trade the off-diagonal term in the metric, $G_{\varphi\psi}$, for a nonvanishing component $B^{(1)}_{\varphi\psi}$ of the rank-two tensor potential. The dual metric has finally the form

$$d\tilde{s}^2=\left(\tan\frac{t}{2}\right)^{2M/k}e^{2\phi_1(t,\theta)}\left[e^{f(t,\theta)}(-dt^2+d\theta^2)+\frac{I_1^2(t)I_3^2(t)\sin^2\theta}{I_1^2(t)\sin^2\theta+I_3^2(t)\cos^2\theta}d\psi^2\right]+\left(\tan\frac{t}{2}\right)^{-2M/k}\frac{e^{-2\phi_1(t,\theta)}}{I_1^2(t)\sin^2\theta+I_3^2(t)\cos^2\theta}d\varphi^2, \tag{3.2}$$

while the new dilaton field $\tilde{\phi}(t,\theta)$ turns out to be

$$\tilde{\phi}=-\frac{1}{2}\ln[I_1^2(t)\sin^2\theta+I_3^2(t)\cos^2\theta]. \tag{3.3}$$

The newly generated two-form field can be expressed as

$$\tilde{B}(t,\theta)=\frac{I_3^2(t)\cos\theta}{I_1^2(t)\sin^2\theta+I_3^2(t)\cos^2\theta}d\varphi\wedge d\psi.$$

It is remarkable that the expression (3.3) for the transformed dilaton is independent of the particular dilaton we began with. However, the latter leaves its imprint on the geometry, as can be seen from Eq. (3.2).

Several interesting things happen with this solution. When the homogeneous part of the dilaton ϕ_0 is absent the solution is everywhere regular in the sense of curvature invariants and fields, whatever the value of k, except for the points located along the axis $\theta=0,\pi$ at $t=0$. This can be traced back to the fact that those points are on singular orbits of the Killing vector field ∂_φ. When $\phi_0(t)\neq 0$ we get a big-bang singularity as t goes to zero, as well as a big crunch in $t=\pi$. With respect to the homogeneous character of the initial singularity, we find that for the metric before T-duality (3.1) the singularity is approached homogeneously, whereas Eq. (3.2) gives rise to an inhomogeneous big-bang. As will be discussed later this is not unexpected after all, since some types of inhomogeneities are generated by the T-duality transformation.

Suppose now the inhomogeneous part of the dilaton field is switched off, $C\sim 0$ and $\phi_0(t)\neq 0$. The solution before T-duality describes then a Bianchi IX LRS cosmology which becomes inhomogeneous after the transformation. Had we started with the isotropic Friedmann-Robertson-Walker solution ($2I_1^2=2I_3^2=k\sin t$) before T dualizing, we would have finished with anisotropic Kantowski-Sachs-type cosmology,

$$d\tilde{s}^2=\left(\tan\frac{t}{2}\right)^{\pm\sqrt{3}} I_1^2(t)(-dt^2+d\theta^2+\sin^2\theta d\psi^2)$$
$$+\frac{1}{I_1^2(t)}\left(\tan\frac{t}{2}\right)^{\mp\sqrt{3}} d\varphi^2,$$

which falls into the class of solutions recently studied by Barrow and Dąbrowski in [1].

One thing to be learned from the above examples is that inhomogeneous and/or anisotropic spacetimes can be related to homogeneous and isotropic backgrounds via T-duality. Since this is a symmetry of string theory, and therefore strings cannot distinguish between dual spacetimes, it seems that isotropy and homogeneity, and even some types of curvature singularities, become less intuitive concepts in string theory than they are in classical general relativity. This is not really a surprise, since it is well known that string theory forces us to revise the physical role of other classical concepts, such as the topology of the spacetime or the value of the cosmological constant [28]. Imagine now an inhomogeneous spacetime T-dual to a homogeneous one, so the physics at the string scale is the same in both backgrounds.[4] This seems to pose an apparent puzzle, for in the field theory limit ($\alpha'\rightarrow 0$) we are left with point particles which could, in principle, probe these inhomogeneities. Looking more carefully at the problem it turns out that this is not really the case, because of the hidden presence of the Planck length $\sqrt{\alpha'}$ in Buscher's formulae. This makes the typical scale of these T-duality-generated inhomogeneities to be of the order of the Planck length, so they cannot be detected using low-energy (particle) probes. Thus the dual spacetime looks as homogeneous as the original one when looked at large distances. The moral of the story is then that there are certain types of inhomogeneities, those that can be removed by a T-duality transformation, which in a sense are ''pure gauge'' from the string theory point of view. In the same vein, this generalizes to the case when we start with an inhomogeneous background: now we have, in addition to the long-wavelength inhomogeneities inherited from the original geometry, those generated by T-duality. Since their typical scale is of the order of the Planck length they look like ripples on a smooth manifold.

[4]It is important to stress that physics in dual backgrounds looks different *only* when described using momentum modes in both of them. If one, however, uses the winding states associated with the compact isometry in the dual background, the physical equivalence manifests itself explicitly.

IV. CONCLUSIONS

Before closing let us summarize the main results obtained. We have designed a general algorithm which allows the construction of cosmological solutions to the low-energy effective theory of the heterotic and type-IIB superstring, starting with vacuum solutions to Einstein equations. This provides us with a powerful tool to construct new solutions in string cosmology, given the huge amount of vacuum spacetimes with two or more commuting spacelike Killing vectors known in general relativity. Starting from either homogeneous or inhomogeneous seed metrics, we pass to Einstein-Klein-Gordon solutions, to end up after a conformal rescaling with solutions to dilaton gravity. Using T-duality and the S-duality of the type-IIB superstring we finally get cosmological spacetimes for both the heterotic and type-IIB theories, in the latter case also including nontrivial R-R fields.

As an illustrative example we have constructed the string theory version of the inhomogeneous Mixmaster-type cosmology obtained in Ref. [12]. We have also discussed the role played by inhomogeneities in string cosmology and argued that there is a class of those, the ones generated by T-duality, that have no physical relevance at any energy scale. Thus, T-duality classifies inhomogeneous spacetimes into physically equivalent pairs. As it happens with many other issues, string theory seems to challenge our physical intuition about the real meaning of a homogeneous geometry.

ACKNOWLEDGMENTS

We would like to thank A. Achúcarro, J. L. F. Barbón, I. L. Egusquiza, J. L. Mañes, and M. A. Valle-Basagoiti for interesting discussions and suggestions. This work has been supported in part by a Spanish Ministry of Education Grant (CICyT) PB93-0507 and a Basque Country University Grant UPV/EHU/72.310EBO36/95. R.L. and M.A.V.-M. acknowledge financial support from the Basque Government under Grant Nos. BFI94-094 and BFI96-071, respectively.

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