

## Formation of topological black holes from gravitational collapse

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We consider the gravitational collapse of a dust cloud in an asymptotically anti-de Sitter spacetime in which points connected by a discrete subgroup of an isometry subgroup of anti-de Sitter spacetime are identified. We find that black holes with event horizons of any topology can form from the collapse of such a cloud. The quasilocal mass parameter of such black holes is proportional to the initial density, which can be arbitrarily small. [S0556-2821(97)00120-3]

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### I. INTRODUCTION

Black hole solutions to the Einstein field equations play a substantial role in our understanding of gravitation. They provide us with an important arena for testing some of our most fundamental ideas about thermodynamics and quantum physics. An increasingly large body of evidence that black holes do indeed exist in our universe as physical entities [1], and are not merely mathematical constructs, motivates the study of such objects even further.

There has been growing interest in recent years concerning more exotic black-hole solutions to the Einstein field equations, particularly those with nontrivial topology. Although it was postulated by Freidmann *et al.* [2] that in a globally hyperbolic, asymptotically flat spacetime topologically nontrivial event-horizon structures cannot be observed (they collapse before light can traverse them), it was later shown that such topologies could be passively observed [3]. In addition to this, it has been shown numerically that black hole event horizons with toroidal topology can form, at least temporarily [4]. More recently Aminneborg *et al.* have shown that by suitably identifying points in  $(3+1)$ -dimensional anti-de Sitter (AdS) spacetime, black hole solutions can result [5]. The black holes they constructed were eternal black holes whose event horizons had nontrivial topology of genus higher than 1. The construction of these AdS black holes are generalizations of the construction in  $2+1$  dimensions of Bañados, Teitelboim, and Zanelli (BTZ) [6], and the resultant  $(3+1)$ -dimensional black holes can be considered higher dimensional analogues of the lower-dimensional case.

It is natural to ask to what extent this type of black hole can arise from known, or at least hypothetically plausible, physical processes. It was recently shown that pair-production of such black holes of arbitrary genus is possible in the presence of a domain wall of suitable topology. In addition to this, solutions with nonzero quasilocal mass and charge were obtained [7], generalizing the constructions given in Ref. [5] to spacetimes with nonconstant curvature.

We investigate here the possibility of forming this type of black hole from the collapse of a cloud of pressureless, uncharged dust. Traditionally, the study of gravitational collapse into black holes has been limited to spheres of collapsing dust and gas. We consider a cloud of dust embedded in a region created by identifying points on a surface of constant

negative curvature. This is a generalization of the collapse process considered in  $2+1$  dimensions [8], in which it was shown that a disk of pressureless dust could collapse into a BTZ black hole provided the initial density was sufficiently large. Although we find that black holes of nontrivial topology can form from gravitational collapse in a nonasymptotically flat spacetime, we do not find an analogous constraint on the initial density; rather we find that arbitrarily small initial densities of pressureless dust will collapse to form such black holes.

The outline of our paper is as follows. We consider in Sec. II the general structure of the exterior and interior metrics. In Sec. III we solve for the metric inside the dust cloud and in Sec. IV we match this solution to the exterior metric. We consider the case of collapse to a ‘‘massless black hole’’ in Sec. V and summarize our results in Sec. VI.

### II. TOPOLOGICAL BLACK HOLE METRICS

A further generalization of the black hole metrics discussed in Ref. [5] was recently obtained in the context of investigating which cosmological  $C$ -metrics could provide suitable instantons for black hole pair production [7]. These metrics included terms for a nonzero mass and charge, thereby yielding spacetimes of varying curvature. We consider here collapse to neutral black holes only, for which the metric outside of the dust reads

$$ds^2 = -\left(\frac{\Lambda}{3}R^2 + b - \frac{2M}{R}\right)dT^2 + \frac{dR^2}{\left(\frac{\Lambda}{3}R^2 + b - \frac{2M}{R}\right)} + R^2[d\hat{\theta}^2 + c \sinh^2(\sqrt{d}\hat{\theta})d\hat{\phi}^2], \quad (1)$$

where  $T$  is the time coordinate,  $R$  is the radial coordinate, and  $\hat{\theta}$  and  $\hat{\phi}$  are coordinates on a two-surface of constant curvature,  $\hat{\phi}$  being an angular coordinate whose range is from  $0$  to  $2\pi$ .  $\Lambda$  refers to the cosmological constant, with positive  $\Lambda$  corresponding to anti-de Sitter spacetime, and  $M$  is a constant of integration corresponding to the quasilocal mass [7].

Solving the Einstein field equations for empty space, it is clear that  $b = -d$  is the only solution. Without loss of gen-

erality we may set the magnitudes of  $b$  and  $d$  to unity (or zero). There are then three possibilities [7].

(1) If  $b = -d = +1$ ,  $c$  is forced to be  $-1$  in order to preserve the signature of the metric. This is suitable for spaces with positive, negative or zero asymptotic curvature. Surfaces of constant  $(R, T)$  are two-spheres, and the topology of spacetime is  $\mathbf{R}^2 \otimes \mathbf{S}^2$ . The cosmological constant may be positive or negative. The resultant spacetime is simply Schwarzschild ( $-$ anti-) de Sitter space.

(2) If  $b = -d = 0$ , the signature forces  $c = +1/d$  in the limit  $d \rightarrow 0$ .  $\Lambda$  in this case is strictly positive. The metric corresponds to a spacetime that is asymptotically anti-de Sitter in the  $(R, T)$  section. The topology is that of a torus  $\mathbf{R}^2 \otimes \mathbf{T}^2$ .

(3) If  $b = -d = -1$ , then  $c = +1$  and  $\Lambda$  is again strictly positive. In this case the topology is  $\mathbf{R}^2 \otimes \mathbf{H}_g^2$ , where  $\mathbf{H}_g^2$  is a two-surface of constant negative curvature and genus  $g > 1$ .

In case (2), without suitable identifications in the  $(\hat{\theta}, \hat{\phi})$  sector, the black hole event horizon would be an infinite sheet. Such ‘‘black plane’’ solutions have been recently noted in the literature [9]. However, a black hole with a compact event horizon may be obtained by considering only a portion of the sheet, and making this section periodic through the suitable identification of points. The shape for the section chosen is a polygon formed from geodesics in the  $(\hat{\theta}, \hat{\phi})$  sector, in this case straight lines, cut out of the plane. The sum of the angles in the polygon must be equal to  $2\pi$  or greater in order to avoid the formation of conical singularities. In addition, the number of sides must be an integer multiple of four [5]. The simplest polygon fulfilling these criteria is a square. Opposite sides of the square are identified, yielding the toroidal topology noted above.

Turning now to case (3), which has the exterior metric

$$ds^2 = - \left( \frac{\Lambda}{3} R^2 - 1 - \frac{2M}{R} \right) dT^2 + \frac{dR^2}{\frac{\Lambda}{3} R^2 - 1 - 2M/R} + R^2 (d\hat{\theta}^2 + \sinh^2 \hat{\theta} d\hat{\phi}^2), \tag{2}$$

where the  $(\hat{\theta}, \hat{\phi})$  sector is now a space with constant negative curvature, also known as a hyperbolic plane or a pseudosphere. These spaces have been discussed in detail by Balazs and Voros [10]. Geodesics on the pseudosphere are formed from intersections of the pseudosphere with planes through the origin, and are the analogues of great circles on a surface of constant positive curvature (a sphere), which are intersections of the sphere and planes through the origin. A projection of the pseudosphere onto the  $(y_1, y_2)$  plane is known as the Poincaré disk. On it, geodesics are segments of circles, orthogonal to the disk boundary at the edges [10]. The pseudosphere, its associated Poincaré disk, and the geodesics are shown in Fig. 1.

A compact surface on the pseudosphere can be obtained by identifying opposite sides of a suitably chosen polygon centered at the origin. In order to avoid conical singularities, we must construct a polygon from geodesics which has angles that sum to  $2\pi$  or more, and a number of sides that is a multiple of four. Since the geodesics on the pseudosphere meet at angles smaller than those for geodesics meeting on a

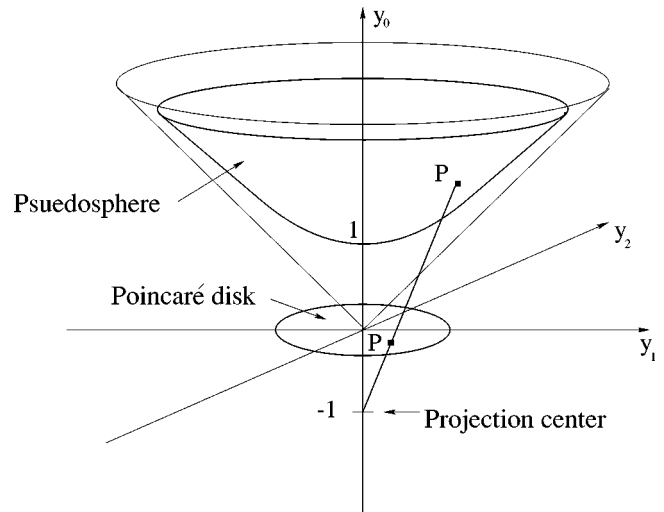


FIG. 1. The pseudosphere is one half of the hyperboloid. The axes do not represent any coordinates in particular. Beneath the pseudosphere is the Poincaré disk, the center of which is the origin. (Balazs and Voros, p. 121).

flat plane, an octagon is the simplest solution, yielding a surface of genus 2. This construction is shown in Fig. 2. In general, a polygon of  $4g$  sides yields a surface of genus  $g$ , where  $g \geq 2$ .

We shall consider in this paper collapse of a cloud of dust to a black hole whose exterior metric is given by Eq. (2), keeping in mind that our procedure easily generalizes to the toroidal case. In order for a black hole to form, two conditions must be satisfied. First, the exterior metric must be matched successfully to a metric for the interior of the dust

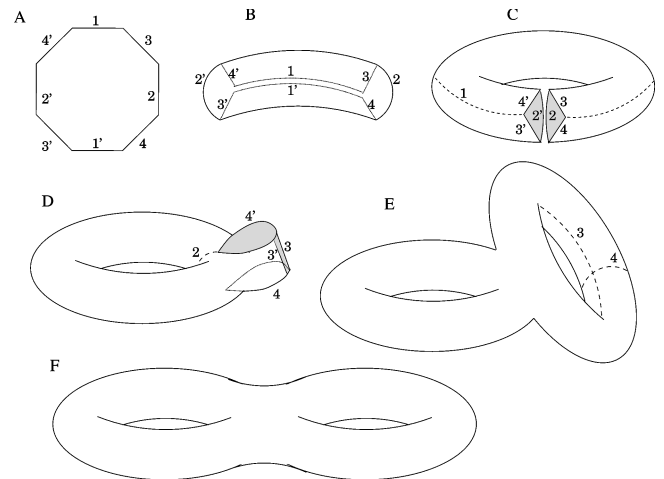


FIG. 2. The identification of the octagon is shown. Opposite sides of the octagon in A will be identified. The sides are drawn straight for clarity. Dashed lines indicate where sides have been sewn together. First, sides 1 and 1' are identified, folding the top and bottom of the octagon away from view (B). Sides 2 and 2' are brought together to form a torus with a diamond shaped hole, as in C. Next, sides 3 and 3' are stretched out and joined for D. The loop is lengthened along the direction of identification 3 and bent until 4 and 4' meet, forming a second torus. Finally the topology is deformed to the preferred shape, seen in F. Identification of a polygon of genus  $g$  will clearly result in  $g$  attached tori or, equivalently, a  $g$ -holed pacifier.

cloud. Secondly, the dust cloud must be shown to collapse in a finite amount of proper time. Simple criteria of homogeneity and topology suggest that we match the above spacetime metric (2) to an interior metric of the form

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{b - \tilde{k}r^2} + r^2 [d\theta^2 + c \sinh^2(\sqrt{d}\theta) d\phi^2] \right), \tag{3}$$

where  $r$ ,  $\theta$ , and  $\phi$  are the comoving radial and angular interior coordinates, respectively.  $t$  is the proper time of the dust cloud and  $a(t)$  is the scale factor.  $\tilde{k}$  refers to the curvature. Again, requiring the above metric to be an exact solution of the Einstein equations with the stress energy tensor being that of pressureless dust yields  $b = -d = -c = -1$  and  $\tilde{k} < 0$ , so that the form of the interior metric becomes

$$ds^2 = -dt^2 + \frac{a(t)^2}{kr^2 - 1} dr^2 + r^2 a(t)^2 (d\theta^2 + \sinh^2 \theta d\phi^2), \tag{4}$$

where we have substituted  $k = |\tilde{k}|$ .

In the next section, we shall solve Einstein's equations inside the dust cloud.

### III. THE INTERIOR SOLUTION

Consider a collapsing dust cloud, surrounded by a vacuum, where the metric for the dust is given by Eq. (4). It remains for us to solve for the scale factor  $a(t)$ , checking to see under what circumstances (if any)  $a(t)$  vanishes in a finite amount of proper time  $t$ .

The stress energy tensor for freely falling dust is given by  $T_{\mu\nu} = \rho u_\mu u_\nu$ , where  $u_\nu$  is the four velocity, defined such that  $g^{\mu\nu} u_\mu u_\nu = -1$ . In comoving coordinates, this means  $u_\nu = (-1, 0, 0, 0)$ . If the initial density is given by  $\rho_0$  and the scale factor is initially  $a_0$ , the conservation of stress energy,  $T^{\mu\nu}{}_{;\nu} = 0$ , will require that  $\rho(t)[a(t)]^3 = \rho_0 a_0^3$ . In addition, requiring that the dust satisfies the standard positive energy criteria implies that  $\rho(t) \geq 0$ .

The relevant form of the Einstein field equations with a nonzero cosmological constant is given by

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{5}$$

Application of this yields two equations: from the temporal component

$$\dot{a}^2 = \frac{-\Lambda}{3} a^2 + \frac{8}{3} \pi G \frac{a_0^3 \rho_0}{a} + k \tag{6}$$

and from the three spatial components

$$2a\ddot{a} + \dot{a}^2 - k + \Lambda a^2 = 0, \tag{7}$$

where the overdot is used to denote the time derivative  $d/dt$ . The second equation is directly derivable from the first, leaving a single independent equation.

From our earlier discussion we know that  $k$  must be strictly greater than zero, implying

$$\Lambda + 3 \frac{\dot{a}_0^2}{a_0^2} > 8\pi G \rho_0, \tag{8}$$

from Eq. (6), where  $\dot{a}_0$  is the initial inward velocity of the cloud. Given Eqs. (6) and (8),  $k$  is guaranteed to be positive for all future times. This somewhat counterintuitive condition reflects the need to maintain negative curvature within the dust cloud. An initial density that is too large could reverse the sign of the curvature of the spatial sections of its spacetime. The more familiar case (1), where  $b = -d = +1$  and  $c = -1$ , has no analogous limits since the form of its metric allows a free choice in the sign of the curvature. No such choice is admissible for our metric in a manner that preserves the spacetime signature.

When the cosmological constant is absorbed into the definition of the stress-energy tensor of the Einstein equations, it becomes

$$\tilde{T}_{\mu\nu} = \tilde{\rho} g_{\mu\nu} + (\tilde{\rho} + \tilde{p}) u_\mu u_\nu, \tag{9}$$

where

$$\tilde{\rho} = \rho - \frac{\Lambda}{8\pi G}, \quad \tilde{p} = p + \frac{\Lambda}{8\pi G}. \tag{10}$$

In other words, we can consider the cosmological constant as in some manner contributing to the pressure in our space and lowering the density therein. If  $\Lambda$  is not of sufficient size, or we do not give the cloud enough initial inward velocity, collapse cannot occur. In fact, for collapse from rest we require that the net effective density from matter and the vacuum energy be negative. If this condition is not satisfied, the metric will change its signature and will no longer properly describe the dust cloud.

Assuming the condition (8) is satisfied, the dust will abide by the equation of motion given by Eq. (6). Solving Eq. (6) yields

$$d\tilde{t} = dx \sqrt{\frac{x}{-x^3 + (1 + v_0^2)[(1 - B)x + B]}}, \tag{11}$$

where  $\tilde{t} = \sqrt{(\Lambda/3)} t$ ,  $x$  is the relative scale factor  $a/a_0$ , and  $v_0 \equiv dx/d\tilde{t}|_{\tilde{t}=0}$ . The parameter  $B$  is given by

$$B = \frac{8\pi G \rho_0}{(1 + v_0^2)\Lambda} \tag{12}$$

and has the range  $1 > B > 0$ . We now have a parametric solution for the scale factor  $a(t)$ . While it is possible to generate an exact solution to this equation, it is irrelevant for our purposes. It is sufficient for us to know that  $a(t)$  exists as some combination of elliptic functions.

Since the interior metric is defined, we can move on to finding the collapse time. If we start from time  $t = 0$ , we can use Eq. (11) to find the time until complete collapse in the interior coordinates. The collapse time  $t_c$  is found by integrating Eq. (11) from  $x = 1$  to  $x = 0$ :

$$\tilde{t}_c = \int_1^0 dx \sqrt{\frac{x}{-x^3 + (1 + v_0^2)[(1 - B)x + B]}}. \tag{13}$$

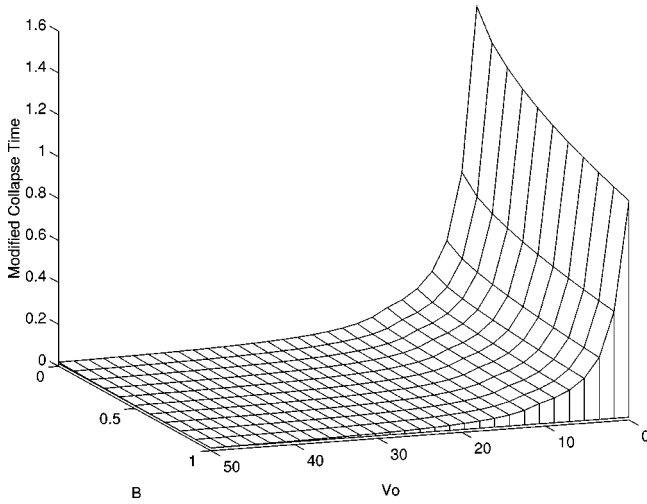


FIG. 3. The collapse time times  $\sqrt{\Lambda/3}$  is shown as a function of  $B$  and  $v_0$ . Increasing either  $B$  or  $v_0$  speeds collapse.

This integral can be evaluated numerically. We find that the collapse time is finite for all allowed  $B$  between 0 and 1, and for all allowed  $v_0$ . The collapse time for a range of  $v_0$  and all allowed values of  $B$  is shown in Fig. 3.

#### IV. MATCHING CONDITIONS

In the exterior coordinates, the stress energy vanishes and the exterior metric (2) provides the correct description of spacetime. Our final condition on the collapse process will be the requirement that the boundary between the two spacetimes be smooth, with no energy shells separating them. In order to make the dust edge a boundary surface with no shell of stress energy present, we will require that [11,12]

$$[g_{ij}] = 0, \quad [K_{ij}] = 0, \quad (14)$$

where  $[\Psi]$  denotes the discontinuity in  $\Psi$  across the edge,  $K_{ij}$  is the extrinsic curvature of the dust edge, and the subscripts  $i, j$  refer to the coordinates on the dust edge.

The metric on the edge of the dust cloud is

$$ds^2 = -d\tau^2 + \mathfrak{R}^2(\tau)(d\bar{\theta}^2 + \sinh^2 \bar{\theta} d\bar{\phi}^2), \quad (15)$$

where  $\tau$ ,  $\bar{\theta}$ , and  $\bar{\phi}$  are the boundary coordinates. In the interior coordinates,  $r = r_0$  at the boundary. In the exterior coordinates, the boundary is at  $R = \mathfrak{R}(\tau)$ . Continuity of the metric immediately implies  $\theta = \hat{\theta} = \bar{\theta}$  and  $\phi = \hat{\phi} = \bar{\phi}$ . It is also possible to choose  $\tau = t$ . The boundary metric then becomes

$$ds^2 = -dt^2 + \mathfrak{R}^2(t)(d\theta^2 + \sinh^2 \theta d\phi^2). \quad (16)$$

Successful matching of the interior and exterior metrics forces yields

$$\left(\frac{\Lambda}{3}\mathfrak{R}^2 - 1 - \frac{2M}{\mathfrak{R}}\right)\dot{\mathfrak{R}}^2 - \frac{\mathfrak{R}^2}{[(\Lambda/3)\mathfrak{R}^2 - 1 - 2M/\mathfrak{R}]} = 1. \quad (17)$$

The overdot refers to the  $d/dt$ . Solving for  $\dot{T}$  yields

$$\dot{T} = \frac{dT}{dt} = \sqrt{\frac{\mathfrak{R}^2 + [(\Lambda/3)\mathfrak{R}^2 - 1 - 2M/\mathfrak{R}]}{[(\Lambda/3)\mathfrak{R}^2 - 1 - 2M/\mathfrak{R}]^2}}, \quad (18)$$

with  $\mathfrak{R}(t) = r_0 a(t)$ . This condition tells us the initial radius of the dust cloud is  $\mathfrak{R}_0 = r_0 a_0$ . It also means that collapse to  $a = 0$  implies collapse to  $\mathfrak{R} = 0$ .

The extrinsic curvature tensor is calculated by the equation [14]

$$K_{ij} = -n_\alpha \frac{\delta e_{(i)}^\alpha}{\delta \xi^j} = -n_\alpha (\partial_j e_i^\alpha + \Gamma_{\mu\nu}^\alpha e_i^\mu e_j^\nu), \quad (19)$$

where  $n_\alpha$  is the unit spacelike normal to the edge,  $e_{(i)}^\alpha$  are the basis vectors on the edge defined by  $g_{edg}^{\alpha\beta} = e_i^\alpha e_j^\beta \eta^{ij}$ , and  $\xi^i$  are the coordinates on the edge. In the interior coordinates these quantities are

$$n_\alpha = \left(0, \frac{a(t)}{\sqrt{kr_0^2 - 1}}, 0, 0\right) \quad (20)$$

and

$$e_t^\alpha = (1, 0, 0, 0), \quad e_\theta^\alpha = (0, 0, 1/r_0 a(t), 0),$$

$$e_\phi^\alpha = \left(0, 0, 0, \frac{1}{r_0 a(t) \sinh(\theta)}\right). \quad (21)$$

The calculations of the extrinsic curvature tensor yield

$$K_{11} = K_{22} = \frac{\sqrt{kr_0^2 - 1}}{r_0 a(t)} \quad (22)$$

as the only nonvanishing components.

Repeating this procedure in the exterior coordinates, we find

$$e_T^\alpha = (\dot{T}, \mathfrak{R}, 0, 0), \quad e_\theta^\alpha = (0, 0, 1/\mathfrak{R}, 0), \quad e_\phi^\alpha = \left(0, 0, 0, \frac{1}{\mathfrak{R} \sinh \theta}\right) \quad (23)$$

for the basis vectors on the edge and

$$n_\alpha = (-\mathfrak{R}, \dot{T}, 0, 0) \quad (24)$$

for the unit normal. The nonzero components of the extrinsic curvature tensor are

$$K_{00} = -\frac{d}{d\mathfrak{R}} \sqrt{\mathfrak{R}^2 + \left(\frac{\Lambda}{3}\mathfrak{R}^2 - 1 - \frac{2M}{\mathfrak{R}}\right)} \quad (25)$$

and

$$K_{11} = K_{22} = \frac{\dot{T}}{\mathfrak{R}} \left(\frac{\Lambda}{3}\mathfrak{R}^2 - 1 - \frac{2M}{\mathfrak{R}}\right). \quad (26)$$

The matching conditions then imply that  $K_{00}$  in Eq. (25) vanishes, yielding

$$C = \mathfrak{R}^2 + \frac{\Lambda}{3}\mathfrak{R}^2 - 1 - \frac{2M}{\mathfrak{R}}, \quad (27)$$

where  $C$  is simply a constant. Using  $\mathfrak{R}(t) = r_0 a(t)$  we find upon matching Eq. (26) to Eq. (22) and comparison of Eq. (27) to Eq. (6) that  $C = kr_0^2 - 1$  and

$$M = \frac{1}{2}r_0^3 a \left( \dot{a}^2 + \frac{\Lambda}{3}a^2 - k \right) = \frac{4}{3}\pi G \rho_0 (r_0 a_0)^3. \quad (28)$$

Hence a dust cloud of arbitrarily small density can collapse; the boundary conditions produce no additional constraints. This is in contrast to (2+1)-dimensional collapse of dust into a black hole, which can only proceed if the initial density is sufficiently large relative to  $r_0 a_0$  [8]. Note also that Eq. (27) implies

$$\begin{aligned} \mathfrak{R}^2 &= \mathfrak{R}_0^2 + \frac{\Lambda}{3}(\mathfrak{R}_0^2 - \mathfrak{R}^2) + 2M \left( \frac{1}{\mathfrak{R}} - \frac{1}{\mathfrak{R}_0} \right) \\ &= r_0^2 \dot{a}_0^2 + \frac{\Lambda}{3}r_0^2(a_0^2 - a^2) + \frac{2M}{r_0^2 a_0 a}, \end{aligned} \quad (29)$$

which defines  $\mathfrak{R}^2$ , and ensures it will always be positive for collapse provided the constraint (8) is satisfied.

As long as our conditions are satisfied, an event horizon  $R_h$  will form around the collapsing dust. This horizon is found by setting

$$\frac{1}{3}\Lambda R_h^2 - 1 - \frac{2M}{R_h} = 0, \quad (30)$$

which produces one real, positive root at

$$R_h = \frac{1}{\Lambda} \frac{(3M\sqrt{\Lambda} + \sqrt{9M^2\Lambda - 1})^{2/3} + 1}{(3M\sqrt{\Lambda} + \sqrt{9M^2\Lambda - 1})^{1/3}} \quad (31)$$

provided  $9M^2\Lambda \geq 1$ . If  $9M^2\Lambda < 1$  then

$$R_h = \sqrt{\frac{4}{\Lambda}} \cos\left(\frac{\cos^{-1}(\sqrt{9M^2\Lambda})}{3}\right). \quad (32)$$

The comoving time  $t_h$ , defined by  $R_h = r_0 a(t_h)$ , is the time at which the dust edge and the event horizon are coincident, and may be found by integrating (see Fig. 4)

$$\tilde{t}_h = \int_1^{x_h} dx \sqrt{\frac{x}{-x^3 + (1+v_0^2)[(1-B)x+B]}}. \quad (33)$$

However, the coordinate time at which an external observer will witness the formation of the event horizon is infinite, since an outgoing radial light ray emitted from the surface of the cloud at time  $T$  obeys the equation

$$\frac{dR}{dT} = \frac{\Lambda}{3}R^2 - 1 - \frac{2M}{R}, \quad (34)$$

and arrives at a point  $R_F$  at time

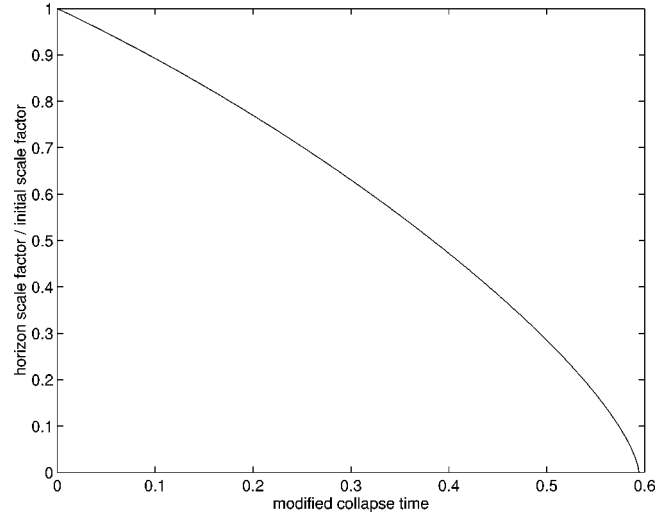


FIG. 4. A sample collapse, showing  $x_h$  shrinking as horizon time  $\tilde{t}_h$  increases. In this case,  $v_0 = 1$  and  $B = 0.5$ . This smooth collapse, with no bouncing, guarantees that  $\tilde{t}_h < \tilde{t}_c$ , where  $0 < x_h \equiv a(t_h)/a_0 < 1$  since  $a(t)$  is a decreasing function of  $t$  and  $R_h$  is nonvanishing. The value of  $\tilde{t}_h$  is finite since the integrand in Eq. (33) is positive for  $0 < x_h < 1$ , and we necessarily have  $\tilde{t}_h < \tilde{t}_c$  upon comparing Eq. (13) with Eq. (33).

$$T_F = T + \int_{r_0 a(T)}^{R_F} \frac{R dR}{(\Lambda/3)R^3 - R - 2M}, \quad (35)$$

which diverges logarithmically as  $r_0 a(T) \rightarrow r_0 a(t_h) = R_h$ . Hence the collapse is unobservable from outside. The proper time for a light source at the edge of the cloud is equal to the comoving time  $t$  and so the emission time between wave crests of wavelength  $\lambda$  is just  $dt$ . The arrival time between observed wavelengths  $\lambda_F$  is just  $dT_F$ , and so the redshift of light from the dust edge is

$$\begin{aligned} z \equiv \frac{\lambda_F}{\lambda} - 1 &= \frac{dT_F}{dt} - 1 = \frac{dT}{dt} - \frac{r_0 \dot{a}}{kr_0^2 - 1 - r_0^2 \dot{a}^2} - 1 \\ &= \frac{1}{\sqrt{kr_0^2 - 1 + r_0 \dot{a}}} - 1, \end{aligned} \quad (36)$$

which diverges as  $t \rightarrow t_h$  since  $r_0 \dot{a}(t_h) = -\sqrt{kr_0^2 - 1}$ . Hence the collapsing fluid fades from sight, analogous to the usual Oppenheimer-Snyder collapse of case (1).

We close this section by noting that the only curvature singularity in the spacetime is at  $R = 0$ . This is easily seen by computing the Kretschmann scalar, which in the exterior coordinates is

$$K = 24 \frac{(\Lambda/3)^2 R^6 + 2M^2}{R^6}. \quad (37)$$

Alternatively, in the interior coordinates

$$K = \frac{320(\pi G a_0^3 \rho_0)^2 - 32\pi G a_0^3 \rho_0 a^3 \Lambda + 8a^6 \Lambda^2}{3a^6}, \quad (38)$$

which diverges when  $t \rightarrow t_c$ .

### V. COLLAPSE OF A MASSLESS PSEUDOSPHERE

The solution for the mass of the pseudosphere is consistent with our previous result that collapse requires sufficient curvature, not density as in traditional black hole models. This begs the question of what will happen if the density, and therefore the mass, vanishes.

The interior metric is unchanged, but the exterior metric is now

$$ds^2 = -\left(\frac{\Lambda}{3}R^2 - 1\right)dT^2 + \frac{dR^2}{\frac{\Lambda}{3}R^2 - 1} + R^2(d\theta^2 + \sinh^2\theta d\phi^2), \quad (39)$$

where the event horizon is  $R_h = \sqrt{3/\Lambda}$ . The solution for the first derivative of the scale factor becomes

$$\dot{a}^2 = -\frac{\Lambda}{3}a^2 + k, \quad (40)$$

where now

$$k = \frac{\Lambda}{3}a_0^2 + \dot{a}_0^2 \quad (41)$$

and the parameter  $B$  is zero. From Eq. (14) then, the ‘‘collapse’’ time for such a ‘‘massless’’ cloud is finite, given by

$$t_c = \sqrt{\frac{3}{\Lambda}} \int_1^0 \frac{dx}{\sqrt{1+v_0^2-x^2}} = \sqrt{\frac{3}{\Lambda}} \arcsin\left(\frac{1}{\sqrt{1+v_0^2}}\right). \quad (42)$$

In spite of these suggestive results, we expect that a massless cloud would be indistinguishable from the surrounding space, and indeed we find the interior massless metric is merely a transformation of the exterior coordinates. The transformation is

$$R = r \cos\left(\sqrt{\frac{\Lambda}{3}}t\right),$$

$$\sinh\left(\sqrt{\frac{\Lambda}{3}}T\right) = \frac{\sin(\sqrt{\Lambda/3}t)}{\sqrt{\Lambda/3[r\cos(\sqrt{\Lambda/3}t)]^2 - 1}}, \quad (43)$$

$$\hat{\theta} = \theta, \quad \hat{\phi} = \phi. \quad (44)$$

Here,  $k = \Lambda/3$  and  $a$  has been transformed to a cosine function. The ‘‘collapse’’ time has become simply the time at which the identification surfaces used to construct the  $(3+1)$ -dimensional AdS black hole (39) merge [5]. The curvature is no longer singular at  $R=0$ , and the singularity is of the Misner type.

### VI. CONCLUSIONS

We have demonstrated that a cloud of pressureless dust can undergo gravitational collapse to a black hole whose event horizon has arbitrary topology. The properties of the collapsing dust are similar to those of the usual [case (1)] Oppenheimer-Snyder collapse, with black holes of arbitrarily small mass being formed from arbitrarily small initial density distributions. Perhaps the most unusual feature of the collapse scenario investigated here is that, for a given value of the cosmological constant, collapse is not possible once the density is sufficiently large unless the dust cloud is given a large enough initial inward velocity. This feature is a consequence of the negative curvature of the  $(\theta, \phi)$  sector in the spacetimes we consider.

Of course the exterior black hole spacetimes to which the dust collapses have nontrivial topology. By enclosing the entire dust cloud and a portion of the vacuum inside a shell of appropriate stress-energy and topology, it might be possible to match these solutions onto background spacetimes whose topology is  $\mathbf{R}^2 \otimes \mathbf{S}^2$ , using methods similar to those discussed in Refs. [13,14]. Work on this is in progress.

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