Supersymmetry enhancement of *D*-*p*-branes and *M*-branes

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We examine the supersymmetry of classical D-brane and M-brane configurations and explain the dependence of Killing spinors on coordinates. We find that one-half supersymmetry is broken in the bulk and that supersymmetry near the D-brane horizon is restored for $p \leqslant 3$, for solutions in the stringy frame, but only for $p=3$ in the ten-dimensional canonical frame. We study the enhancement for the case of four intersecting D-3-branes in ten dimensions and the implication of this for the size of the infinite throat of the near horizon geometry in noncompactified theory. We find some indications of universality of near horizon geometries of various intersecting brane configurations. [S0556-2821(97)07316-5]

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I. INTRODUCTION

The implications of the enhancement of supersymmetry of certain classical solutions in supergravity near the horizon have been studied mostly in four dimensions (4D) and in 5D [1–7]. In these cases, one finds that some supersymmetries are broken in the bulk, but that the breaking becomes weaker as one approaches the horizon. In these dimensions the extreme black holes with nonvanishing entropy, proportional to the area of the horizon, are available. The enhancement of supersymmetry near the horizon of such black holes was instrumental for the study of the entropy of such black holes [4] and of a tension of a magnetic string [8]. In higher dimensions some p-branes and M-branes are known to have an enhancement of supersymmetry near the horizon [3]. No such study has been performed for the D-branes [9]. One of the purposes of this paper is to fill in this gap.

The enhancement of supersymmetry near the horizon of intersecting and/or overlapping branes was also not studied before. In principle, one may expect to find some form of supersymmetric attractors there as suggested in [10]. As the first step in this direction we will find out what happens for the four intersecting D-3-branes near the horizon. We will find out that near the horizon the geometry of 3-3-3-3 solution is the same as of 6-2-2-2 and of 0-4-4-4 solutions.

The understanding of enhancement has been based largely on the behavior of the geometry and the dilaton near the horizon. In all known cases of *enhancement of supersymmetry* the near horizon geometry is regular (in any frame) and of the form $\mathrm{adS}_{p+2}\times S^{d-p-2}$ and the dilaton is regular [1–7].

In all known cases of *absence of enhancement of supersymmetry* near the horizon the geometry is regular in stringy frame and of the form $M_{p+2}\times S^{d-p-2}$ with the dilaton blowing up linearly near the horizon in the inertial frame of the Minkowski space M_{p+2} [2,3].

We will find out that for the D-p-branes the situation is not as simple since in the stringy frame the near horizon geometry is not regular apart from $p=3$. It is actually conformal to a regular geometry of the type $\mathrm{adS}_{p+2}\times S^{8-p}$ for all cases but $p=5$ (where the conformal geometry is $M_7\times S^3$). The conformal factor which brings $p\neq 3$ branes from the stringy frame to the one with regular geometry is proportional to r^{p-3} and, therefore, is not a canonical ten-dimensional frame either. Also, the dilaton is not regular at the horizon except for $p=3$. In view of this the issue of enhancement of supersymmetry near the horizon for the D-branes is not on the same footing as in all cases studied before. Therefore, we will simply proceed with explicit evaluation of the supersymmetry transformations near the horizon in two natural frames: the stringy one and the canonical one.

Thus our main goal is to examine the supergravity transformations of various classical solutions, to determine whether or not they exhibit restoration of supersymmetry at the horizon. It may be useful to mention here the following general feature of Killing spinors. One can predict their dependence on coordinates for static solutions, based on supersymmetry algebra. Assume that we have a geometry with some time component of the metric $g_{tt}(x)$ where x are space coordinates. Killing spinors of unbroken supersymmetry usually are found as a product of the function of space times the constant spinor ϵ_0:

$$\epsilon(x)=K(x)\epsilon_0 . \tag{1}$$

This can be understood as follows. The commutator of two supersymmetries has to produce a translation. For static configurations the translation in time direction has to be a Killing vector. Introducing a vielbein $e_a{}^{\mu}$ we have

$$(\overline{\epsilon}\Gamma^{\mu}\epsilon')\frac{\partial}{\partial x^{\mu}}=(\overline{\epsilon}\Gamma^{0}\epsilon')e_0{}^{t}\frac{\partial}{\partial t}=(\overline{\epsilon_o}\Gamma^{0}\epsilon_o')K^2e_0{}^{t}\frac{\partial}{\partial t}=\frac{\partial}{\partial t}. \tag{2}$$

Thus,

$$K^2e_0{}^{t}=K^2g_{tt}^{-1/2}=1\Rightarrow K=(g_{tt})^{1/4} \tag{3}$$

and, therefore, the dependence of the Killing spinor has to be

$$\epsilon(x)=(g_{tt})^{1/4}\epsilon_0 . \tag{4}$$

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This indeed is the case for all static configurations.

In Sec. II, we will explicitly calculate the supersymmetry transformations of the classical fields of a D-p-brane background in the string metric of $N=2D=10$ supergravity. We expect to find that, away from the horizon, the classical solution is invariant under $\frac{1}{2}$ of all supersymmetry transformations. We will then see if any of the supersymmetries are restored near the horizon. In Sec. III, we will perform the same calculations for the ten-dimensional canonical frame solutions for a D-p-brane background. In Sec. IV, we will investigate M-brane classical backgrounds of 11-dimensional supergravity and demonstrate the enhancement of supersymmetry near the horizon of the two-brane and the five-brane. In Sec. V, we will study the effect of T duality on supersymmetry in the bulk and near the horizon. We will study the enhancement of supersymmetry for solutions corresponding to intersecting D-branes in ten dimensions in Sec. VI. In Sec. VII we will present some near horizon geometries of the different configurations of branes and discuss the universality issues.

II. *D-p*-BRANES IN THE STRING FRAME

The supersymmetry transformations of the dilatino and gravitino fields in the presence of a $p+2$ form gauge field strength in $N=2D=10$ supergravity (IIA or IIB) are given by [11]

$$\delta\psi_\mu=\partial_\mu\epsilon-\tfrac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon + \frac{(-1)^p}{8(p+2)!}e^{\phi}F_{\mu_1\cdots\mu_{p+2}}\gamma^{\mu_1\cdots\mu_{p+2}}\gamma_\mu\epsilon'_{(p)}, \quad (5)$$

$$\delta\lambda=\gamma^\mu(\partial_\mu\phi)\epsilon+\frac{3-p}{4(p+2)!}e^{\phi}F_{\mu_1\cdots\mu_{p+2}}\gamma^{\mu_1\cdots\mu_{p+2}}\epsilon'_{(p)}, \quad (6)$$

$$\epsilon'_{(0,4.8)}=\epsilon, \quad \epsilon'_{(2,6)}=\gamma_{11}\epsilon, \quad \epsilon'_{(-1,3,7)}=\imath\epsilon, \quad \epsilon'_{(1,5)}=\imath\epsilon^*, \quad (7)$$

where ϵ is a 32-component spinor, and ω is the spin connection given by

$$\omega_\mu^{ab}=-e^{\nu[a}(\partial_\mu e_\nu^{b]}-\partial_\nu e_\mu^{b]})-e^{\rho[a}e^{\sigma b]}(\partial_\sigma e_{c\rho})e_\mu^c,$$

$$g_{\mu\nu}=e_\mu^a e_\nu^b\eta_{ab}. \quad (8)$$

The classical solution for the metric and fields of D-p-branes in the string metric is given by

$$ds^2=H^{-1/2}dx_{(p+1)}^2-H^{1/2}dx_{(9-p)}^2, \quad F_{01\ldots pi}=\partial_i H^{-1},$$

$$e^{2\phi}=H^{-(p-3)/2}, \quad (9)$$

$$H=1+\left(\frac{c}{r}\right)^{7-p}, \quad r^2=x_{(p+1)}^2+\cdots+x_9^2, \quad (10)$$

where the fermionic fields vanish (and consequently, so do the variations in the graviton, dilaton, and gauge field strength).

For this solution, the spin connection is then given by

$$\omega_r^{\hat{i}\hat{s}}=\frac{\partial_i H}{4H^{3/2}}\delta_{r\hat{s}}, \quad \omega_k^{\hat{i}\hat{j}}=\frac{\partial_j H}{4H}\delta_{k\hat{i}}, \quad r,s\in\{0\ldots p\},$$

$$i,j\in\{p+1\ldots 9\}, \quad (11)$$

where $\hat{s}$ is an index in the flat tangent space.

From this, we see that the supersymmetry transformations are now given by

$$\delta\lambda=\frac{(3-p)(\partial_i H)\gamma^i}{4H^{5/4}}[\epsilon+\gamma_0\cdots\gamma_p\epsilon'], \quad (12)$$

$$\delta\psi_r=\partial_r\epsilon+\frac{(\partial_i H)}{8H^{3/2}}\gamma^i\gamma_r[\epsilon+\gamma_0\cdots\gamma_p\epsilon']=\hat{\nabla}_r\epsilon, \quad (13)$$

$$\delta\psi_i=\partial_i\epsilon-\sum_{i\neq j}\left(\frac{(\partial_j H)}{8H}\gamma^i\gamma^j\right)[\epsilon+\gamma_0\cdots\gamma_p\epsilon'] -\left(\frac{\partial_i H}{8H}\right)\gamma_0\cdots\gamma_p\epsilon' =\hat{\nabla}_i\epsilon. \quad (14)$$

Clearly, these vanish for all r if the following conditions are satisfied [11]:

$$\epsilon+\gamma_0\cdots\gamma_p\epsilon'=0, \quad (15)$$

$$\epsilon=H^{-1/8}\epsilon_0. \quad (16)$$

The solutions ϵ are Killing spinors. The dependence on space coordinates here is found in agreement with the prediction in Eq. (4), since here $H^{-1/8}=(g_{tt})^{1/4}$. The spinor condition projects out one-half of the degrees of freedom of the Killing spinor for all D-p-brane backgrounds. This amounts to saying that the classical solutions to D-p-branes are preserved under one-half of the supersymmetry transformations. Thus, D-p-branes break one-half of supersymmetry away from the brane.

The next task is to examine the transformations near the horizon. We know that for small r and for $p<7$,

$$H\propto r^{p-7}. \quad (17)$$

Therefore, the transformations (viewed in the flat tangent space) will vary with r as

$$\delta\lambda,\delta\psi_r,\delta\psi_i\propto r^{(3-p)/4}. \quad (18)$$

Thus, for $p<3$, the classical solutions of the D-p-brane near the horizon will be invariant under all supersymmetry transformations near the brane. For $7>p>3$, the dilatino field is not invariant unless Eq. (15) is satisfied. Thus, supersymmetry is one-half broken even at the horizon. We find that for $p=3$, the dilatino field is invariant, while the gravitino field is not. But the gravitino field is not gauge invariant, so we must examine the transformation of the gauge-invariant gravitino field strength in the flat tangent space. This transformation is given by the generalized curvature tensor $(R_{ab})_\alpha{}^\beta$:

$$\psi_{\mu\nu}=\hat{\nabla}_{[\mu}\psi_{\nu]}, \quad \delta\psi_{\mu\nu}=\hat{\nabla}_{[\mu}\delta\psi_{\nu]}=[\hat{\nabla}_{\mu},\hat{\nabla}_{\nu}]\epsilon, \tag{19}$$

$$(\delta\psi_{ab})_{\alpha}=e^{\mu}_{a}e^{\nu}_{b}[\hat{\nabla}_{\mu},\hat{\nabla}_{\nu}]\epsilon=[\hat{\nabla}_{a},\hat{\nabla}_{b}]\epsilon=(R_{ab})_{\alpha}{}^{\beta}\epsilon_{\beta}. \tag{20}$$

Note that the supercovariant derivative $\hat{\nabla}$ is the sum of the covariant derivative and a term involving the gauge field strength. Thus, the generalized curvature tensor is the sum of the Riemann curvature tensor and terms involving the gauge field strength. By plugging in the supersymmetry transformations given above, we find the integrability condition

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{s}}]\epsilon=0, \tag{21}$$

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{i}}]\epsilon=\left\{-\partial_i\left(\frac{\partial_k H}{8H^{3/2}}\right)\gamma^k\gamma_r+\frac{(\partial_k H)^2}{32H^{5/2}}\gamma_i\gamma_r\right\}\times(\epsilon+\gamma_0\cdots\gamma_p\epsilon'_{(p)}), \tag{22}$$

$$[\hat{\nabla}_{\hat{i}},\hat{\nabla}_{\hat{j}}]\epsilon=\frac{1}{8}\left\{-H^{-1/2}\left[\partial_i\left(\frac{\partial_l H}{H}\right)\gamma^j\gamma^l-\partial_j\left(\frac{\partial_k H}{H}\right)\gamma^i\gamma^k\right]+\frac{(\partial_k H)^2}{4H^{5/2}}[\gamma^i\gamma^j]-\frac{(\partial_i H)(\partial_l H)}{2H^{5/2}}\gamma^l\gamma^j+\frac{(\partial_j H)(\partial_k H)}{2H^{5/2}}\gamma^k\gamma^i\right\}(\epsilon+\gamma_0\cdots\gamma_p\epsilon'_{(p)}). \tag{23}$$

Again, this vanishes for all r if Eq. (15) is satisfied. We also find that each term in the generalized curvature tensor is proportional to $r^{(3-p)/2}$. We can see this more easily by noting that

$$\hat{\nabla}_r,\hat{\nabla}_i\propto r^{(3-p)/4}, r\rightarrow 0. \tag{24}$$

Thus, the generalized curvature tensor vanishes, term by term, as $r\rightarrow 0$ for $p<3$. In particular, the Riemann tensor in the tangent space vanishes as $r\rightarrow 0$ for $p<3$. Since the tangent space is flat, this implies that the curvature scalar (which is found by contracting the Riemann tensor with the metric) also vanishes. Note that this supersymmetry enhancement occurs for all of the cases where the dilaton blows up as $r\rightarrow 0$, since

$$e^{2\phi}\sim r^{(7-p)(p-3)/2}, \phi\sim(7-p)(p-3)/4\ln r. \tag{25}$$

For $p=3$ in the limit as $r\rightarrow 0$, we find that the curvature does not vanish. However, the various terms in the field strength transformations cancel each other. Thus, the generalized curvature vanishes, and full supersymmetry is restored in the $p=3$ case as well. This matches our geometric understanding of the situation, as shown in [3]. As $r\rightarrow 0$, the D-3-brane metric tends to $\mathrm{adS}_5\times S_5$. This calculation was performed in the string frame, and thus enhancement only occurs when $r\ll l_s$. It is not clear how one interprets this from a string-theoretic point of view.

III. SUPERSYMMETRY OF *D*-*p*-BRANES IN THE TEN-DIMENSIONAL CANONICAL METRIC

In the ten-dimensional canonical metric, the supersymmetry transformations of the dilatino and gravitino fields are given by [12]

$$\delta\lambda=\frac{1}{2\sqrt{2}}(\nabla_M\phi)\gamma^M\gamma^{11}\epsilon+\frac{(3-p)}{8\sqrt{2}(p+2)!}e^{[(3-p)/4]\phi}\gamma^{M_1\cdots M_{p+2}}F_{M_1\cdots M_{p+2}}\epsilon'_{(p)}, \tag{26}$$

$$\delta\psi_M=\nabla_M\epsilon+\frac{(3-p)3^{p-1}}{32(p+2)!}e^{[(3-p)/4]\phi}\left(\gamma_M^{M_1\cdots M_{p+2}}-\frac{(7-p)(p+2)}{p+1}\delta_M^{M_1}\gamma^{M_2\cdots M_{p+2}}\right)F_{M_1\cdots M_{p+2}}\epsilon'_{(p)}, \tag{27}$$

$$\epsilon'_{(0)}=\imath\epsilon, \quad \epsilon'_{(1,2)}=\epsilon, \tag{28}$$

and the classical solution for D-p-branes is given by

$$ds^2=H^{(p-7)/8}(dt^2-dx_1^2-\cdots-dx_p^2)-H^{(p+1)/8}(dx_{p+1}^2+\cdots+dx_9^2),$$
$$H=1+\left(\frac{c}{r}\right)^{7-p}, \tag{29}$$

$$F_{01\ldots pi}=-\frac{\partial_i H}{H^2}, \quad e^{2\phi}=H^{(3-p)/2}. \tag{30}$$

We will find that, in this case, we can determine all of the information we need about enhancement from the dilatino variation, which reduces to

$$\delta\lambda=\left(\frac{3-p}{8\sqrt{2}}\right)\frac{(\partial_i H)}{H}H^{-(p+1)/16}\gamma^i\gamma^{11}[\epsilon+\gamma^{11}\gamma_0\cdots\gamma_p\epsilon'_{(p)}]. \tag{31}$$

As expected, we find that, away from the horizon, the solution is preserved only when ϵ obeys the spinor condition

$$\epsilon+\gamma^{11}\gamma_0\cdots\gamma_p\epsilon'_{(p)}=0. \tag{32}$$

As $r\rightarrow 0$, we find that

$$\delta\lambda\rightarrow r^{-[(3-p)^2/16]}. \tag{33}$$

Thus, we find that, if $p\neq 3$, supersymmetry is not enhanced in the ten-dimensional canonical frame. For $p=3$, the dilaton is regular and the ten-dimension canonical frame is the same as the string frame, wherein we have already determined that supersymmetry is enhanced at the horizon.

It seems at first strange that supersymmetry should appear in some cases to be enhanced in the string metric, while not in the ten-dimensional canonical metric. But this should not be too surprising, since each metric measures the supersymmetry breaking on a different scale. One notes that the integrability condition is very closely related to the curvature, which is clearly different depending on whether one uses the string metric or canonical metric. It may be that the extent to which some supersymmetries are broken goes to zero when measured on the string scale, but not when measured on the ten-dimensional Planck scale. One must then ask which scale is appropriate for asking questions regarding supersymmetry enhancement. This may be a question whose answer depends on M-theoretic considerations.

In any case, these ambiguities do not affect us when dealing with D-3-branes, for which the string frame and canonical frame are identical. We will see in Sec. VII that we can find a class of solutions which exhibits near horizon adS geometry. These solutions are all equivalent to a configuration involving only intersecting three-branes, for which there is no ambiguity.

IV. SUPERSYMMETRY OF CLASSICAL SOLUTIONS OF M-BRANES

Using the 11-dimensional canonical metric, we find the supersymmetry transformation [13,14]

$$\delta\psi_\mu=\partial_\mu\epsilon-\tfrac{1}{4}\omega_\mu^{ab}\gamma_a\gamma_b\epsilon+\frac{i}{288}(\gamma_{[\mu}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^{\delta]}-8\,\delta_\mu^\alpha\gamma^{[\beta}\gamma^\gamma\gamma^{\delta]})F_{\alpha\beta\gamma\delta}\epsilon. \tag{34}$$

The supermembrane classical field configuration is given by

$$ds^2=H^{-2/3}(dt^2-dx_1^2-dx_2^2)-H^{1/3}(dx_3^2+\cdots+dx_{10}^2),$$

$$H=1+\left(\frac{c}{r}\right)^{8-p}, \tag{35}$$

$$F_{012i}=-\frac{\partial_i H}{H^2}. \tag{36}$$

We thus find the following transformations for the gravitino field strength

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{s}}]=0, \tag{37}$$

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{i}}]=-\left[H^{1/6}\partial_i\left(\frac{\partial_k H}{6H^{3/2}}\right)\gamma^k-\frac{(\partial_k H)^2}{36H^{7/3}}\gamma_i+\frac{(\partial_k H)(\partial_i H)}{36H^{7/3}}\gamma^k\right]\gamma_r(1+i\gamma^0\gamma^1\gamma^2), \tag{38}$$

$$[\hat{\nabla}_{\hat{i}},\hat{\nabla}_{\hat{j}}]=-\left[H^{-1/3}\left\{\partial_i\left(\frac{\partial_l H}{12H}\right)\gamma^l\gamma_j-\partial_j\left(\frac{\partial_k H}{12H}\right)\gamma^k\gamma_i\right\}+\frac{(\partial_k H)^2}{72H^{7/3}}[\gamma^i,\gamma^j]+\frac{(\partial_k H)(\partial_j H)}{36H^{7/3}}\gamma_i\gamma^k-\frac{(\partial_i H)(\partial_l H)}{36H^{7/3}}\gamma_j\gamma^l\right](1+i\gamma^0\gamma^1\gamma^2). \tag{39}$$

In the bulk, these variations vanish only when the spinor condition is satisfied. As $r\to 0$, we see that the variations vanish for any ϵ. Thus supersymmetry is half-broken in the bulk, but is enhanced at the horizon of the M-2-brane in the 11-dimensional canonical frame.

The classical solution for the M-5-brane is given by

$$ds^2=H^{-1/3}(dt^2+dx_1^2+\cdots+dx_5^2)-H^{2/3}(dx_6^2+\cdots+dx_{10}^2),$$

$$H=1+\left(\frac{c}{r}\right)^3, \tag{40}$$

$$F^{\alpha\beta\gamma\delta}=-\epsilon^{012345\alpha\beta\gamma\delta\epsilon}\frac{\partial_\epsilon H}{H^2}. \tag{41}$$

Using Eq. (34), and plugging in as before, we find

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{s}}]=0, \tag{42}$$

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{i}}]=\left[-H^{1/6}\partial_i\left(\frac{\partial_k H}{12H^{3/2}}\right)\gamma^k+\frac{(\partial_k H)^2}{36H^{8/3}}\gamma^i+\frac{(\partial_k H)(\partial_i H)}{72H^{8/3}}\gamma^k\right]\gamma_r(1+i\gamma^6\cdots\gamma^{10}), \tag{43}$$

$$[\hat{\nabla}_{\hat{i}},\hat{\nabla}_{\hat{j}}]=H^{-2/3}\left[\partial_i\left(\frac{\partial_l H}{6H}\right)\gamma^j\gamma^l-\partial_j\left(\frac{\partial_k H}{6H}\right)\gamma^i\gamma^k+\frac{(\partial_k H)^2}{18H^2}[\gamma^i,\gamma^j]+\frac{(\partial_j H)(\partial_k H)}{9H^2}\gamma^i\gamma^k-\frac{(\partial_l H)(\partial_i H)}{9H^2}\gamma^j\gamma^l\right](1+i\gamma^6\cdots\gamma^{10}). \tag{44}$$

Again, this yields one-half supersymmetry breaking in the bulk, but as $r\to 0$, the variations vanish for all ϵ. Thus, we see supersymmetry enhancement for both the 2-brane and 5-brane of M theory in the 11-dimensional canonical frame. On the basis of the properties of the geometry $\mathrm{adS}_4\times S^7$ for the 2-brane and $\mathrm{adS}_7\times S^4$ for the 5-brane near the horizon, the enhancement of supersymmetry was studied in [3]. Here, we have in addition checked that the generalized curvature vanishes near the horizon and, therefore, there are no constraints on Killing spinors near the horizon.

V. T DUALITY

It may at first seem odd that supersymmetry is enhanced for $p=3$ but not for $p=4$. One might expect that configurations T dual to the D-3-brane will exhibit the same supersymmetry at the horizon. In particular, one might expect the D-4-brane solution to also exhibit enhancement. But one must first note that the 4-brane and 3-brane solutions given above are not T dual. The T dual of the 4-brane solution [15] is given by

$$ds^2 = H^{-1/2}(dt^2 - dx_1^2 - \cdots - dx_3^2) - H^{1/2}(dx_4^2 + \cdots + dx_9^2),$$

$$H = 1 + \left(\frac{c}{r}\right)^3, \tag{45}$$

which depends on the harmonic function of the 4-brane. But in any case, [16] showed that T duality does not necessarily respect supersymmetry, even in the bulk. They found that the supersymmetry of a configuration is preserved only if one dualizes along a direction on which the Killing spinor does not depend. From Eq. (16) we see that for a D-brane solution, the graviton field is only preserved if the Killing spinor depends on the transverse coordinates. When we dualize the 3-brane solution, we find that we break supersymmetries at the horizon only if we increase the D-brane dimension, which occurs precisely when we dualize along directions upon which the Killing spinor depends.

VI. NEAR HORIZON SUPERSYMMETRY OF INTERSECTING *D*-*p*-BRANES

We consider the classical solution for four D-3-branes, pairwise intersecting on one-branes. For simplicity, we make the particular choice of orientations (1 2 3), (3 4 5), (5 6 1), and (2 4 6). We then have the classical solution [17–20]

$$ds^2 = H^{-2}dt^2 - H^2(dx_7^2 + dx_8^2 + dx_9^2) - (dx_1^2 + \cdots + dx_6^2),$$

$$H = 1 + \frac{c}{r}, \tag{46}$$

$$\hat{F}_{0123i} = \hat{F}_{0345i} = \hat{F}_{0561i} = \hat{F}_{0246i} = -\frac{\partial_i H}{H^2},$$

$$F_{\alpha\beta\gamma\delta} = \tfrac{1}{2}(\hat{F}_{\alpha\beta\gamma\delta} + {}^*\hat{F}_{\alpha\beta\gamma\delta}). \tag{47}$$

We have chosen the four three-brane charges to be the same for simplicity, although the argument holds for arbitrary charges. When compactified down to four dimensions, this solution is known to form a black hole with $\frac{1}{8}$ supersymmetry in the bulk, but $\frac{1}{4}$ supersymmetry near the horizon. Using the previous methods, one derives the supersymmetry transformations

$$\delta\lambda = 0, \tag{48}$$

$$[\hat{\nabla}_{\hat{0}}, \hat{\nabla}_{\hat{i}}] = \left[\partial_i\left(\frac{\partial_k H}{2H^3}\right)\gamma_k + \frac{(\partial_k H)^2}{2H^4}\gamma_i\right]\gamma_0 \Delta_1, \tag{49}$$

$$[\hat{\nabla}_{\hat{i}}, \hat{\nabla}_{\hat{j}}] = \left[\partial_i\left(\frac{\partial_l H}{2H}\right)\frac{\gamma_l\gamma_j}{H} - \partial_j\left(\frac{\partial_k H}{2H}\right)\frac{\gamma_k\gamma_i}{H} + \frac{1}{2}\frac{(\partial_k H)^2}{H^4}[\gamma_i,\gamma_j] - \frac{(\partial_l H)(\partial_i H)}{H^4}\gamma_l\gamma_j + \frac{(\partial_k H)(\partial_j H)}{H^4}\gamma_k\gamma_i\right]\Delta_1 - \frac{3}{8}\left[\frac{(\partial_k H)^2}{2H^4}[\gamma_i,\gamma_j] - \frac{(\partial_l H)(\partial_i H)}{H^4}\gamma_l\gamma_j + \frac{(\partial_k H)(\partial_j H)}{H^4}\gamma_k\gamma_i\right]\Delta_2, \tag{50}$$

$$[\hat{\nabla}_{\hat{r}}, \hat{\nabla}_{\hat{s}}] = 0, \tag{51}$$

$$[\hat{\nabla}_{\hat{0}}, \hat{\nabla}_{\hat{r}}] = -\frac{(\partial_k H)^2}{16H^4}\gamma_0\gamma_r[\pm(1+\gamma_1\gamma_2\gamma_4\gamma_5) \pm(1+\gamma_3\gamma_4\gamma_6\gamma_1) \pm(1+\gamma_5\gamma_6\gamma_2\gamma_3)], \tag{52}$$

$$[\hat{\nabla}_{\hat{r}}, \hat{\nabla}_{\hat{i}}] = -\left[\partial_i\left(\frac{\partial_k H}{2H^2}\right)\frac{\gamma^k}{H} - \frac{(\partial_k H)^2}{2H^4}\gamma_i + \frac{(\partial_i H)(\partial_l H)}{2H^4}\gamma_l\right]\Gamma\gamma_r\left(\pm\frac{i\Gamma}{4}\gamma_0\gamma_1\gamma_2\gamma_3 \pm\frac{i\Gamma}{4}\gamma_0\gamma_3\gamma_4\gamma_5 \pm\frac{i\Gamma}{4}\gamma_0\gamma_5\gamma_6\gamma_1 \pm\frac{i\Gamma}{4}\gamma_0\gamma_2\gamma_4\gamma_6\right) + \left[\frac{(\partial_k H)^2}{16H^4}\gamma_i\gamma_r - \frac{(\partial_i H)(\partial_l H)}{8H^4}\gamma_l\gamma_r\right]\Delta_2, \tag{53}$$

$$\Delta_1 = 1 + \frac{i\Gamma}{4}\gamma_0\gamma_1\gamma_2\gamma_3 + \frac{i\Gamma}{4}\gamma_0\gamma_3\gamma_4\gamma_5 + \frac{i\Gamma}{4}\gamma_0\gamma_5\gamma_6\gamma_1 + \frac{i\Gamma}{4}\gamma_0\gamma_2\gamma_4\gamma_6,$$

$$\Gamma = \frac{1+\Gamma_{11}}{2}, \tag{54}$$

$$\Delta_2 = 1 + \tfrac{1}{3}\gamma_1\gamma_2\gamma_4\gamma_5 + \tfrac{1}{3}\gamma_3\gamma_4\gamma_6\gamma_1 + \tfrac{1}{3}\gamma_5\gamma_6\gamma_2\gamma_3. \tag{55}$$

These transformations involve two different types of spinor projector combinations. The first is a sum of four projectors

$$4\Delta_1 = (1+i\Gamma_\alpha) + (1+i\Gamma_\beta) + (1+i\Gamma_\gamma) + (1+i\Gamma_\delta), \tag{56}$$

$$\Gamma_\alpha = \gamma_0\gamma_a\gamma_b\gamma_c, \tag{57}$$

whose γ matrix indices are in the directions along the three-branes. Only three of these projectors are independent, however, since

$$\epsilon = -i\Gamma_\alpha\epsilon = -i\Gamma_\beta\epsilon = -i\Gamma_\beta\epsilon \rightarrow \epsilon = -i(-\Gamma_\alpha\Gamma_\beta\Gamma_\gamma)\epsilon = -i\Gamma_\delta\epsilon. \tag{58}$$

Thus, this combination of spinor projectors breaks supersymmetry to $\frac{1}{8}$. The second projector combination is a sum of three projectors of the form

$$\pm(1+\Gamma_{12}) \pm (1+\Gamma_{13}) \pm (1+\Gamma_{23}), \quad \Gamma_{\alpha\beta} = \Gamma_\alpha\Gamma_\beta, \tag{59}$$

where the term with γ matrices is a product of any two of the analogous terms in the first projector combination. By noting that

$$\Gamma_\alpha\Gamma_\beta = \Gamma_\beta\Gamma_\alpha, \quad \Gamma_\alpha\Gamma_\beta\Gamma_\gamma = -\Gamma_\delta, \tag{60}$$

we see that there are only three possible ways to form a term of a projector of this type. We can in fact write this projector in the more symmetric form

$$1+\frac{1}{6}\sum_{i<j}\Gamma_{ij}. \tag{61}$$

We find that of the three projectors, only two are independent. Therefore, this projector combination preserves $\frac{1}{4}$ of supersymmetry. We see

$$\epsilon=-i\Gamma_{\alpha}\epsilon, \epsilon=-i\Gamma_{\beta}\rightarrow\epsilon=-\Gamma_{\alpha\beta}. \tag{62}$$

Thus, if ϵ survives a projector combination of the first type, it also satisfies the second. Therefore, the $\frac{1}{8}$ supersymmetry preserved by the first projector is a subset of the $\frac{1}{4}$ supersymmetry preserved by the second.

The transformations in the bulk preserve $\frac{1}{8}$ of the total supersymmetry. As $r\rightarrow 0$,

$$[\hat{\nabla}_{\hat{0}},\hat{\nabla}_{\hat{i}}]\rightarrow 0, \tag{63}$$

$$[\hat{\nabla}_{\hat{i}},\hat{\nabla}_{\hat{j}}]\rightarrow -\frac{3}{8c^2}\left(\frac{[\gamma_i,\gamma_j]}{2}-\frac{x^l x^i}{r^2}\gamma_l\gamma_j+\frac{x^j x^k}{r^2}\gamma_k\gamma_i\right)\Delta_2, \tag{64}$$

$$[\hat{\nabla}_{\hat{r}},\hat{\nabla}_{\hat{i}}]\rightarrow\left(\frac{\gamma_i}{16c^2}-\frac{x^i x^l}{8r^2c^2}\gamma_l\right)\gamma_r\Delta_2, \tag{65}$$

which preserve $\frac{1}{4}$ supersymmetry, as expected. Thus, we are able to recover the supersymmetry of a four-dimensional black hole, both in the bulk and at the horizon, using a ten-dimensional supergravity calculation of intersecting D-brane solutions.

VII. UNIVERSALITY OF THE NEAR HORIZON GEOMETRIES

We may consider the 3-3-3-3 solution of intersecting D-branes with harmonic functions $H_\alpha=1+r_\alpha/r$ with four different parameters r_α for each harmonic function. We then find the following geometry [19,18,20]

$$\begin{aligned}ds^2=&(H_\alpha H_\beta H_\gamma H_\delta)^{-1/2}dt^2-(H_\alpha H_\beta H_\gamma H_\delta)^{-1/2}\\&\times(dx_7^2+\cdots+dx_8^2)-(H_\alpha H_\beta H_\gamma H_\delta)^{1/2}\\&\times\left(\frac{dx_1^2}{H_\alpha H_\beta}+\cdots+\frac{dx_6^2}{H_\gamma H_\delta}\right).\end{aligned} \tag{66}$$

The meaning of the r_α is that it measures the size of the throat and the volume of the sphere of $\mathrm{adS}_5\times S^5$, which is the near horizon geometry of a single three-brane.

For the intersecting solution of four three-branes the near horizon canonical geometry has a nice form: the geometry is that of an anti–de Sitter space times a circle and times an Euclidean space: $\mathrm{adS}_2\times S^2\times E^6$:

$$\begin{aligned}ds^2=&\frac{r^2}{\sqrt{r_\alpha r_\beta r_\gamma r_\delta}}dt^2-\frac{\sqrt{r_\alpha r_\beta r_\gamma r_\delta}}{r^2}dr^2-\sqrt{r_\alpha r_\beta r_\gamma r_\delta}d^2\Omega_2\\&-\sqrt{r_\alpha r_\beta r_\gamma r_\delta}\left(\frac{dx_1^2}{r_\alpha r_\beta}+\cdots+\frac{dx_6^2}{r_\gamma r_\delta}\right).\end{aligned} \tag{67}$$

The size of the infinite throat is now given by the inverse scalar curvature of the adS_2 geometry [6]

$$2\pi\sqrt{r_\alpha r_\beta r_\gamma r_\delta}=\frac{4\pi}{R}. \tag{68}$$

The flat six-dimensional Euclidean geometry is not of the standard form $ds^2=\delta_{ij}d\tilde{x}d\tilde{x}$, as each direction has to be rescaled with different constant parameters to bring the coordinates x^i to $\tilde{x}^i$. This expression upon compactification will become the entropy of the black hole. From the point of view of the ten-dimensional geometry it is of the size of the near horizon throat of adS_2.

However, we also find evidence that there is in fact a class of solutions which exhibits the same near horizon geometry. For example, consider the 0-4-4-4 solution, with the three four-branes pairwise intersecting on two-branes. The metric is given by [21]

$$\begin{aligned}ds^2=&(H_0H_{4\alpha}H_{4\beta}H_{4\gamma})^{-1/2}dt^2-(H_0H_{4\alpha}H_{4\beta}H_{4\gamma})^{-1/2}\\&\times(dx_7^2+\cdots+dx_8^2)-(H_0H_\alpha H_\beta H_\gamma H_\delta)^{1/2}\\&\times\left(\frac{dx_1^2}{H_{4\alpha}H_{4\beta}}+\cdots+\frac{dx_6^2}{H_{4\beta}H_{4\gamma}}\right),\end{aligned} \tag{69}$$

with $H_0=1+r_0/r$ the harmonic function associated with the zero-brane, and $H_{4\alpha}=1+r_{4\alpha}/r$ associated with the three four-branes. Near the horizon, this geometry approaches

$$\begin{aligned}ds^2=&\frac{r^2}{\sqrt{r_0r_{4\alpha}r_{4\beta}r_{4\gamma}}}dt^2-\frac{\sqrt{r_0r_{4\alpha}r_{4\beta}r_{4\gamma}}}{r^2}dr^2\\&-\sqrt{r_0r_{4\alpha}r_{4\beta}r_{4\gamma}}d\Omega_2-\sqrt{r_0r_{4\alpha}r_{4\beta}r_{4\gamma}}\\&\times\left(\frac{dx_1^2}{r_{4\alpha}r_{4\beta}}+\cdots+\frac{dx_6^2}{r_\beta r_\gamma}\right).\end{aligned} \tag{70}$$

As before, the size of the adS_2 throat is given by

$$2\pi\sqrt{r_0r_{4\alpha}r_{4\beta}r_{4\gamma}}=\frac{4\pi}{R}. \tag{71}$$

Another example is the 6-2-2-2 solution, with the two-branes intersecting at a point, and all two-branes embedded within the six-brane. Its geometry is given by [21]

$$\begin{aligned}ds^2=&(H_6H_{2\alpha}H_{2\beta}H_{2\gamma})^{-1/2}-(H_6H_{2\alpha}H_{2\beta}H_{2\gamma})^{1/2}(dx_7^2+\cdots\\&+dx_9^2)-\left(\frac{H_{2\alpha}H_{2\beta}H_{2\gamma}}{H_6}\right)^{1/2}\left(\frac{dx_1^2}{H_{2\alpha}}+\cdots+\frac{dx_6^2}{H_{2\gamma}}\right),\end{aligned} \tag{72}$$

where $H_6=1+r_6/r$ is the harmonic function for the six-brane and $H_{2\alpha}=1+r_{2\alpha}/r$ are associated with the three two-branes. Near the horizon, this geometry tends to

$$ds^2=\frac{r^2}{\sqrt{r_6 r_{2\alpha} r_{2\beta} r_{2\gamma}}}dt^2-\frac{\sqrt{r_6 r_{2\alpha} r_{2\beta} r_{2\gamma}}}{r^2}dr^2 -\sqrt{r_6 r_{2\alpha} r_{2\beta} r_{2\gamma}}d\Omega_2-\sqrt{\frac{r_{2\alpha} r_{2\beta} r_{2\gamma}}{r_6}} \times\left(\frac{dx_1^2}{r_{2\alpha}}+\cdots+\frac{dx_6^2}{r_{2\gamma}}\right). \quad (73)$$

This is an $\mathrm{adS}_2\times S^2\times E^6$ geometry with adS_2 throat volume given by

$$2\pi\sqrt{r_6 r_{2\alpha} r_{2\beta} r_{2\gamma}}=\frac{4\pi}{R}. \quad (74)$$

It is possible that this indicates a universality of near horizon black hole geometries in 10 and 11 dimensions. Specifically, it may be that all classical solutions for intersecting branes in 10 and 11 dimensions which preserve $\frac{1}{8}$ supersymmetry in the bulk and which have constant (but nonzero) curvature at the horizon (in the tangent space) exhibit a near horizon geometry of the form $\mathrm{adS}_l\times S^m\times E^n$. Imagine that we have a metric of the form

$$ds^2=H^y dx_{(t)}^2+H^{x_1}dx_1^2+\cdots+H^{x_n}dx_n^2, \qquad H\rightarrow\left(\frac{q}{r}\right)^{-z}, \quad r\rightarrow 0, \quad (75)$$

where $z=d-2$, d is the number of overall transverse directions, and the x_t are the coordinates in those directions. By examining the spin connection and covariant derivatives in a straightforward manner, one can show that

$$\hat{\nabla}_\mu \propto r^{yz/2-1}. \quad (76)$$

Thus, curvature is only constant when $zy=2$. This (and the demand of $\frac{1}{8}$ supersymmetry) forms powerful constraints on the solutions which can be examined. Suppose we only consider conventional solutions, by which we mean solutions for intersecting branes such that each term of the metric is the product of the appropriate terms in the metric solutions of the individual branes. One-eighth supersymmetry requires the presence of at least three branes. In ten dimensions, we may assume that $e^\phi=1$, since that seems typical when the curvature at the horizon is constant but nonzero. In that case, the string frame is the same as the canonical frame, and we find that for a (conventional) solution with three branes, $y=\frac{3}{2}$. This does not allow for an integer value of z, and is thus unacceptable. For a solution with four branes, we find we must have $y=2, z=1$, and we have three overall transverse directions. In order to have $\frac{1}{8}$ supersymmetry from four branes, we must demand that any direction have either zero, two, or four branes extend along it. Quite clearly, this will give us a near horizon geometry of $\mathrm{adS}_{2+x}\times S^2\times E^{6-x}$.

For solutions in 11 dimensions involving the intersection of three branes, we find that the solutions given in [20] for the $2\perp 2\perp 5$ and $2\perp 5\perp 5$ are nonregular at the horizon. The solution for $2\perp 2\perp 2$ yields a near horizon geometry (after a simple coordinate transformation) of

$$ds^2=\left[\frac{2\delta}{(r_\alpha r_\beta r_\gamma)^{1/3}}\right]^2 dt^2-\left[\frac{(r_\alpha r_\beta r_\gamma)^{1/3}}{2\delta}\right]^2 dr^2 -(r_\alpha r_\beta r_\gamma)^{2/3}d\Omega_3-(r_\alpha r_\beta r_\gamma)^{1/3}\left(\frac{dx_1^2}{r_\alpha}+\cdots+\frac{dx_6^2}{r_\gamma}\right). \quad (77)$$

This is $\mathrm{adS}_2\times S^3\times E^6$. The solution for $5\perp 5\perp 5$ can be transformed to a metric of the form

$$ds^2=\left[\frac{r}{2(r_\alpha r_\beta r_\gamma)^{1/3}}\right]^2(dt^2-dx_{10}^2)-\left[\frac{2(r_\alpha r_\beta r_\gamma)^{1/3}}{r}\right]^2 dr^2 +(r_\alpha r_\beta r_\gamma)^{2/3}d\Omega_2-(r_\alpha r_\beta r_\gamma)^{1/3}\left(\frac{dx_1}{r_\alpha r_\beta}+\cdots+\frac{dx_6}{r_\beta r_\gamma}\right). \quad (78)$$

This is $\mathrm{adS}_3\times S^2\times E^6$, which, in five dimensions, is the dual of $\mathrm{adS}_2\times S^3\times E^6$.

We also examine the intersections of the two two-branes and two five-branes in 11D. There are two parameters in the harmonic functions for the two-branes r_1, r_2 and two parameters in the harmonic functions for the five-branes $\hat{r}_1, \hat{r}_2$. For the two-branes the parameters measure the size of the adS_4 throat of the near horizon geometry of a single two-brane. For the five-branes the parameters measure the size of the adS_7 throat of the near horizon geometry of a single five-brane. These parameters appear in the near horizon intersecting solution on an unequal footing, as opposed to the cases examined above.

The near horizon geometry of $2\perp 2\perp 5\perp 5$ [17,18] in the canonical 11-dimensional frame is a product space of the type an anti–de Sitter space times a circle and times an Euclidean space: $\mathrm{adS}_2\times S^2\times E^6\times E^1$. Universality appears if one performs a conformal rescaling of the 11-dimensional metric with a constant parameter

$$\left(\frac{\hat{r}_1\hat{r}_2}{r_1 r_2}\right)^{1/6} ds_{11}^2 =\left[\frac{r^2}{\sqrt{r_1 r_2 \hat{r}_1 \hat{r}}}dt^2-\frac{\sqrt{r_\alpha r_\beta r_\gamma r_\delta}}{r^2}dr^2-\sqrt{r_\alpha r_\beta r_\gamma r_\delta}d^2\Omega_2\right] -\sqrt{r_\alpha r_\beta r_\gamma r_\delta}\left[\frac{dx_1^2}{r_\alpha r_\beta}+\cdots+\frac{dx_6^2}{r_\gamma r_\delta}\right]+\left(\frac{\hat{r}_1\hat{r}_2}{r_1 r_2}\right)^{1/2}(dx_7)^2. \quad (79)$$

In this form again we may recognize the size of the anti–de Sitter throat which will measure the entropy of the black hole in 4D upon compactification.

Thus some indication of universality comes out from this analysis of geometries even before compactification.

VIII. CONCLUSION

We have found that, in the string frame, classical D-brane supergravity solutions preserve $\frac{1}{2}$ supersymmetry in the bulk, but preserve full supersymmetry at the horizon for $p \leqslant 3$. In the ten-dimensional canonical frame, however, supersymmetry is enhanced only for $p=3$. It seems that the next step is to understand this supersymmetry enhancement from the string theory point of view.

We have found supersymmetry enhancement for *M*-theory two-branes and five-branes, as expected. We have also found enhancement for the configuration of four *D*-3-branes, pairwise intersecting on one-branes. This configuration, when compactified to four dimensions, is known to give a black hole which exhibits the same enhancement. We described the near horizon geometry of different configurations in 10D and 11D and found some signatures of universal behavior near the horizon before compactification. In particular, it appears that all solutions in 10 and 11 dimensions which preserve $\frac{1}{8}$ supersymmetry in the bulk and which have a regular (but nonzero) Riemann tensor at the horizon exhibit a near horizon geometry of the form $\mathrm{adS}_l \times S^m \times E^n$. The size of the anti–de Sitter throat then gives the entropy of the corresponding configuration, when compactified.

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