Maintaining a wormhole with a scalar field

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It is well known that it takes matter that violates the averaged weak energy condition to hold the throat of a wormhole open. The production of such ''exotic'' matter is usually discussed within the context of quantum field theory. In this paper I show that it is possible to produce the exotic matter required to hold a wormhole open classically. This is accomplished by coupling a scalar field to matter that satisfies the weak energy condition. The energy-momentum tensor of the scalar field and the matter separately satisfy the weak energy condition, but there exists an interaction energy-momentum tensor that does not. It is this interaction energy-momentum tensor that allows the wormhole to be maintained. [S0556-2821(97)07720-5]

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INTRODUCTION

A wormhole is a handle which connects two different space-times or two distant regions in the same space-time [1–3]. To keep a wormhole open it is necessary to thread its throat with matter that violates the averaged weak energy condition [1,2]. In other words, there exist null geodesics passing through the wormhole, with tangent vectors $k^\mu = dx^\mu/d\sigma$, which satisfy $\int_0^\infty T_{\mu\nu}k^\mu k^\nu d\sigma < 0$. Such matter obviously violates the weak energy condition which states that $T^{\mu\nu}U_\mu U_\nu \geqslant 0$ for all nonspacelike vectors U^μ. The weak energy condition ensures that all observers will see a positive energy density. Matter that violates the weak energy condition is called exotic. Thus it takes exotic matter to hold a wormhole open.

Most discussions of exotic matter involve quantum field theory effects, such as the Casimir effect [4]. In this paper I show that it is possible to generate the exotic matter required to maintain a wormhole classically. This is accomplished by coupling a scalar field to matter which satisfies the weak energy condition. The energy-momentum tensor of the scalar field and the matter separately satisfy the weak energy condition, but there exists an interaction energy-momentum tensor that does not. It is this interaction energy-momentum tensor that allows the wormhole to remain open.

To create a wormhole I take two static, spherically symmetric, scalar-vac solutions of the Einstein field equations and join them together. A surface energy-momentum tensor will exist on the surface where these two manifolds are joined. This surface energy-momentum tensor violates the weak energy condition. However, if its source is a scalar field coupled to matter, I show that the energy-momentum tensor of the matter and of the field can both satisfy the weak energy condition. The violation of the weak energy condition for the total energy-momentum tensor is produced by the interaction energy-momentum tensor. In addition to satisfying the weak energy condition I show that the matter and scalar field also satisfy the dominant energy condition. The dominant energy condition ensures that the four velocity associated with the local flow of energy and momentum is nonspacelike. Thus a wormhole can be maintained classically by coupling a scalar field to matter that satisfies the weak and dominant energy conditions.

EQUATIONS OF MOTION AND THE ENERGY-MOMENTUM TENSOR

Consider a collection of timelike particles interacting with a scalar field ϕ. The action will be taken to be [5]

$$S = -\sum_n m_n \int \sqrt{-g_{\mu\nu}U_n^\mu U_n^\nu}\, d\tau_n + \frac{1}{2}\sum_n \int \lambda_n(\tau_n)[g_{\mu\nu}U_n^\mu U_n^\nu + 1]\, d\tau_n - \sum_n \alpha_n \int \phi(x_n(\tau_n))\, d\tau_n, \tag{1}$$

$$-\frac{1}{2}\int \nabla^\mu\phi\nabla_\mu\phi\sqrt{g}\; d^4x, \tag{2}$$

where $x_n^\mu(\tau_n)$ and U_n^μ are the position and four velocity of the nth particle, τ_n is the proper time along its world line, m_n is its rest mass, $\lambda_n(\tau_n)$ are Lagrange multipliers, and the α_n are coupling constants.

The scalar field equations are found by varying the action with respect to $\phi(x)$ and are given by

$$\Box^2\phi = \frac{1}{\sqrt{g}}\sum_n \alpha_n \int \delta^4(x^\mu - x_n^\mu(\tau_n))\, d\tau_n, \tag{3}$$

where $\Box^2 = \nabla^\mu\nabla_\mu$. The equations of motion for the particles are found by varying the action with respect to $x_n^\mu(\tau_n)$ and are given by

$$(m_n + \lambda_n)\left(\frac{dU_n^\mu}{d\tau_n} + \Gamma^\mu_{\alpha\beta}U_n^\alpha U_n^\beta\right) + \frac{d\lambda_n}{d\tau_n}U_n^\mu = -\alpha_n\nabla^\mu\phi. \tag{4}$$

Contracting with U_n^μ gives

$$\frac{d\lambda_n}{d\tau_n} = \alpha_n\frac{d\phi}{d\tau_n}. \tag{5}$$

Thus

$$\lambda_n = \alpha_n \phi. \tag{6}$$

The equations of motion for the particles are then given by

$$\frac{d}{d\tau_n}[(m_n+\alpha_n\phi)U_n^\mu]+(m_n+\alpha_n\phi)\Gamma^\mu_{\alpha\beta}U_n^\alpha U_n^\beta = -\alpha_n \nabla^\mu \phi. \tag{7}$$

The energy-momentum tensor of the field and particles is given by

$$T^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}}. \tag{8}$$

From Eq. (1) the energy-momentum tensor is given by

$$T^{\mu\nu} = \sum_n \frac{1}{\sqrt{g}} \int (m_n+\alpha_n\phi)U_n^\mu U_n^\nu \delta^4(x-x_n(\tau_n))d\tau_n$$
$$+\nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi. \tag{9}$$

There is therefore an interaction energy-momentum tensor given by

$$T^{\mu\nu}_{(I)} = \sum_n \frac{\alpha_n}{\sqrt{g}} \int \phi(x_n)U_n^\mu U_n^\nu \delta^4(x-x_n(\tau_n))d\tau_n. \tag{10}$$

This interaction energy-momentum tensor is necessary if

$$\nabla_\mu T^{\mu\nu} = 0 \tag{11}$$

is to give the correct equations of motion for the particles.

Now consider a collection of particles which all have the same value of α/m (the simplest possibility would be to take $\alpha_n = \pm m_n$). From Eqs. (3) and (9) it can be seen that it is the trace of the particle energy-momentum tensor which acts as the source of the scalar field. In the continuum limit Eqs. (3) and (9) become

$$\Box^2\phi = -\alpha^* T_m \tag{12}$$

and

$$T^{\mu\nu} = (1+\alpha^*\phi)[(\rho+P)U^\mu U^\nu + Pg^{\mu\nu}] + \nabla^\mu\phi\nabla^\nu\phi$$
$$-\frac{1}{2}g^{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi, \tag{13}$$

where

$$T_m = 3P - \rho_m \tag{14}$$

is the trace of the matter energy-momentum tensor, ρ is the rest mass density, $\alpha^* = \alpha/m$, and P is the pressure.

WORMHOLE SOLUTIONS

In this section exact wormhole solutions with matter energy-momentum tensors that satisfy the weak and dominant energy conditions will be found. The first step is to obtain static spherically symmetric solutions to the scalar-vac Einstein field equations. Two such solutions will then be joined at the radial coordinate $r=b$. In the process of joining these manifolds together, surface energy and stresses will be produced at $r=b$. The properties of the surface energy-momentum tensor will then be examined in relation to the various energy conditions.

The metric will be taken to be of the form

$$ds^2 = -e^\alpha dt^2 + e^\beta[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{15}$$

The Einstein field equations

$$R_{\mu\nu} = -8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \tag{16}$$

give the three independent equations

$$\frac{1}{2}\alpha'' + \frac{1}{4}\alpha'(\alpha'+\beta') + \frac{\alpha'}{r} = 0, \tag{17}$$

$$\frac{1}{2}\alpha'' + \beta'' - \frac{1}{4}\alpha'(\beta'-\alpha') + \frac{\beta'}{r} = -8\pi G\phi'^2, \tag{18}$$

and

$$\frac{1}{2}\beta'' + \frac{1}{4}\beta'(\alpha'+\beta') + \frac{1}{2r}(3\beta'+\alpha') = 0. \tag{19}$$

The scalar field equation is

$$\frac{\partial}{\partial r}\left(r^2 e^{(\alpha+\beta)/2}\frac{\partial\phi}{\partial r}\right) = 0. \tag{20}$$

Equation (17) plus Eq. (19) gives

$$\frac{1}{2}(\alpha+\beta)'' + \frac{1}{4}(\alpha+\beta)'^2 + \frac{3}{2r}(\alpha+\beta)' = 0. \tag{21}$$

The general solution to this differential equation, which satisfies the boundary condition $\alpha+\beta\to 0$ as $r\to\infty$, is

$$\alpha+\beta = 2\ln\left(1-\frac{A}{r^2}\right), \tag{22}$$

where A is an integration constant. Using this to eliminate β' in Eq. (17) gives

$$\frac{1}{2}\alpha'' + \frac{A\alpha'}{r(r^2-A)} + \frac{\alpha'}{r} = 0. \tag{23}$$

The solution of this equation is

$$\alpha = AD\int \frac{dr}{r^2-A}, \tag{24}$$

where D is an integration constant. The solution for α depends on whether $A>0$, $A<0$, or $A=0$. It turns out that $A=0$ gives a flat space-time and that there is no consistent solution for $A<0$. Thus I will only consider the case $A>0$. The solution to Eq. (23) which satisfies the boundary condition $\alpha\to 0$ as $r\to\infty$ is

$$\alpha = a\ \ln\left(\frac{1-\frac{Gm}{ar}}{1+\frac{Gm}{ar}}\right), \tag{25}$$

where $a = \frac{1}{2}D\sqrt{A}$ and $Gm/a = \sqrt{A}$. The solution for β can then be found from Eq. (22) and the line element is given by

$$ds^2 = -\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^a dt^2 + \left(1+\frac{Gm}{ar}\right)^4\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^{(2-a)} \times[dr^2 + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)]. \tag{26}$$

In the weak field limit, $|Gm/ar| \ll 1$,

$$g_{00} \simeq -\left(1-\frac{2Gm}{r}\right). \tag{27}$$

Thus m is the gravitational mass.

The scalar field can be found from Eq. (20). The solution which satisfies the boundary condition $\phi \to 0$ as $r \to \infty$ and is consistent with Eq. (18) is given by

$$\phi = \pm\sqrt{\frac{4-a^2}{16\pi G}}\ \ln\left(\frac{1-Gm/ar}{1+Gm/ar}\right) \tag{28}$$

with $-2 \leqslant a \leqslant 2$. Since Eqs. (26) and (28) are invariant under $a \to -a$ only $0 \leqslant a \leqslant 2$ needs to be considered. Note that for $a=2$ the above solution reduces to the Schwarzschild solution in isotropic coordinates.

The space-time geometry appears to be badly behaved at $r = Gm/a$ for $a \neq 2$. The Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ is given by

$$R = \frac{2G^2m^2r^4}{a^2}(4-a^2)\left(r+\frac{Gm}{a}\right)^{-(4+a)}\left(r-\frac{Gm}{a}\right)^{(a-4)}, \tag{29}$$

which diverge as $r \to Gm/a$. The curvature scalar $I = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ also diverges as $r \to Gm/a$. Thus $r = Gm/a$ is a physical singularity in the space-time. In fact, it is a naked singularity since it can be seen by distant observers. Thus, when joining the manifolds at $r=b$ it is necessary to take $b > Gm/a$.

Before joining the two scalar-vac manifolds together it is convenient to change coordinates. Let l be a new radial coordinate with $l=0$ at $r=b$ and

$$dl^2 = \left(1+\frac{Gm}{ar}\right)^4\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^{(2-a)} dr^2. \tag{30}$$

On one manifold take $0 \leqslant l < \infty$ and on the other take $-\infty < l \leqslant 0$. The manifold that consists of these manifolds joined at $l=0$ has the line element

$$ds^2 = -\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^a dt^2 + dl^2 + r^2\left(1+\frac{Gm}{ar}\right)^4 \times\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^{(2-a)}(d\theta^2 + \sin^2\theta\ d\phi^2), \tag{31}$$

where $r = r(l)$ and $-\infty < l < \infty$. The coordinates r and l are related via

$$\frac{dr}{dl} = \pm\left(1-\frac{Gm}{ar}\right)^{-2}\left(\frac{1-Gm/ar}{1+Gm/ar}\right)^{(a/2-1)}, \tag{32}$$

where the plus sign corresponds to the manifold with $l > 0$ and the minus sign corresponds to the manifold with $l < 0$.

To find the surface energy-momentum tensor I will use the method developed by Israel [6,7]. The surface energy-momentum tensor

$$S^\mu{}_\nu = \lim_{\epsilon \to 0}\int_{-\epsilon}^{\epsilon} T^\mu{}_\nu dl \tag{33}$$

is given by

$$8\pi G S^\mu{}_\nu = \gamma^\mu{}_\nu - \delta^\mu{}_\nu\gamma \quad (\mu,\nu = 0,2,3), \tag{34}$$

where

$$\gamma^\mu{}_\nu = K^{+\mu}{}_\nu - K^{-\mu}{}_\nu, \tag{35}$$

$K^{+\mu}{}_\nu$ is the extrinsic curvature of the surface $r=b$ on the manifold with $l \geqslant 0$, and $K^{-\mu}{}_\nu$ is the extrinsic curvature of the surface $r=b$ on the manifold with $l \leqslant 0$. For μ or $\nu = 1$, $S^\mu{}_\nu = 0$. Using $K_{\mu\nu} = -\frac{1}{2}g_{\mu\nu,l}$ gives

$$S^t{}_t = \left(\frac{1}{2\pi Gb}\right)\left(1-\frac{G^2m^2}{a^2b^2}\right)^{-1}\left(1+\frac{Gm}{ab}\right)^{-2} \times\left(\frac{1-Gm/ab}{1+Gm/ab}\right)^{(a/2-1)}\left(1-\frac{Gm}{b}+\frac{G^2m^2}{a^2b^2}\right) \tag{36}$$

and

$$S^\theta{}_\theta = S^\phi{}_\phi = \left(\frac{1}{4\pi Gb}\right)\left(1-\frac{G^2m^2}{a^2b^2}\right)^{-1}\left(1+\frac{Gm}{ab}\right)^{-2} \times\left(\frac{1-Gm/ab}{1+Gm/ab}\right)^{(a/2-1)}\left(1+\frac{G^2m^2}{a^2b^2}\right). \tag{37}$$

In the previous section it was shown that the energy-momentum tensor for an ideal fluid coupled to a scalar field is given by

$$T^{\mu\nu} = (1+\alpha^*\phi)T_m^{\mu\nu} + \nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi, \tag{38}$$

where $T_m^{\mu\nu}$ is the usual energy-momentum tensor for an ideal fluid. The scalar field equation was also shown to be

$$\Box^2\phi = -\alpha^* T_m. \tag{39}$$

The surface energy momentum tensor does not have the form of an ideal fluid energy-momentum tensor because it contains only surface stresses (i.e., $T^{\theta\theta}, T^{\phi\phi} \neq 0$ but $T^{ll}=0$). For matter which produces surface stresses I will take Eqs. (38) and (39) with

$$T^\mu_{m\nu} = [\sigma U^\mu U_\nu + S(\delta^\mu{}_\theta\delta^\theta{}_\nu + \delta^\mu{}_\phi\delta^\phi{}_\nu)]\delta(l), \tag{40}$$

where σ is the surface density and S is the surface stress. Using Eqs. (31) and (33)–(35) to find $S_\nu{}^\mu$ and equating it to Eqs. (36) and (37) gives

$$(1+\alpha^*\phi)\sigma=-\left(\frac{1}{2\pi Gb}\right)\left(1-\frac{G^2m^2}{a^2b^2}\right)^{-1}\left(1+\frac{Gm}{ab}\right)^{-2}\times\left(\frac{1-Gm/ab}{1+Gm/ab}\right)^{(a/2-1)}\left(1-\frac{Gm}{b}+\frac{G^2m^2}{a^2b^2}\right) \tag{41}$$

and

$$(1+\alpha^*\phi)S=\left(\frac{1}{4\pi Gb}\right)\left(1-\frac{G^2m^2}{a^2b^2}\right)^{-1}\left(1+\frac{Gm}{ab}\right)^{-2}\times\left(\frac{1-Gm/ab}{1+Gm/ab}\right)^{(a/2-1)}\left(1+\frac{G^2m^2}{a^2b^2}\right). \tag{42}$$

For $0\leqslant a\leqslant 2$ it is easy to show that $(1+\alpha^*\phi)\sigma<0$ and $(1+\alpha^*\phi)S>0$.

Now consider the various energy conditions. An energy-momentum tensor $T^{\mu\nu}$ will satisfy the weak energy condition if $T^{\mu\nu}U_\mu U_\nu\geqslant 0$ for all nonspacelike vectors U^μ. If the weak energy condition is satisfied all observers will measure a positive energy density. An energy-momentum tensor will satisfy the dominant energy condition if it satisfies the weak energy condition and if $T^{\mu\nu}U_\mu$ is nonspacelike for all nonspacelike vectors U^μ. If the dominant energy condition is satisfied all observers will measure the four vector associated with the local flow of energy and momentum to be nonspacelike. It is easy to show that the scalar field energy-momentum tensor satisfies the weak and dominant energy conditions.

The matter energy-momentum tensor $T_m^{\mu\nu}$ will satisfy the weak energy condition iff [8,9]

$$\sigma\geqslant 0 \quad \text{and} \quad \sigma+S\geqslant 0. \tag{43}$$

Both of these conditions will be satisfied iff $1+\alpha^*\phi\leqslant 0$. From Eq. (28) this gives

$$1\pm\alpha^*\sqrt{\frac{(4-a^2)}{16\pi G}}\ \ln\left(\frac{1+Gm/ab}{1-Gm/ab}\right)\leqslant 0. \tag{44}$$

For a solution to exist the lower sign must be chosen if $\alpha^*m>0$ and the upper sign must be chosen if $\alpha^*m<0$.

$T_m^{\mu\nu}$ will satisfy the dominant energy condition iff it satisfies the weak energy condition and [8,9]

$$\sigma\geqslant S. \tag{45}$$

This will be satisfied iff $1+\alpha^*\phi\leqslant 0$ and

$$\left(1-\frac{Gm}{ab}\right)^2+\frac{2Gm}{ab}(1-a)\geqslant 0. \tag{46}$$

For $0\leqslant a\leqslant 1$ this will be satisfied for all Gm/b. For $1<a\leqslant 2$ this will be satisfied for

$$-a<\frac{Gm}{b}\leqslant a^2-a\sqrt{a^2-1}. \tag{47}$$

The final energy condition, which appears in some of the singularity theorems, is the strong energy condition. For $\sigma>0$ the matter energy momentum tensor will satisfy the dominant energy condition iff [8,9]

$$\sigma+2S\geqslant 0. \tag{48}$$

From Eqs. (41) and (42) it can be shown that the strong energy condition will be satisfied iff $m\leqslant 0$. Since the strong energy condition does not have a strong physical motivation, unlike the weak and dominant energy conditions, I will not impose it on $T_m^{\mu\nu}$ (in fact the strong energy condition can be violated by a massive classical scalar field [8]).

Now consider the scalar field equation

$$\Box^2\phi=\alpha^*(\sigma-2S)\delta(l). \tag{49}$$

Integrating from $l=-\epsilon$ to $l=\epsilon$ gives

$$\alpha^*(\sigma-2S)=\mp\sqrt{\frac{4-a^2}{\pi G}}\frac{Gm}{ab^2}\left(1-\frac{G^2m^2}{a^2b^2}\right)^{-1}\left(1+\frac{Gm}{ab}\right)^{-2}\times\left(\frac{1-Gm/ab}{1+Gm/ab}\right)^{(a/2-1)}. \tag{50}$$

As before, if the weak energy condition is to be satisfied by $T_m^{\mu\nu}$, the lower sign must be chosen if $\alpha^*m<0$ and the upper sign must be chosen if $\alpha^*m<0$. Equation (50) is a constraint on σ and S, which are given by Eqs. (41) and (42). Combining this with Eqs. (41) and (42) gives

$$(1+\alpha^*\phi)=\pm\frac{\alpha^*(1-Gm/2b+G^2m^2/a^2b^2)(Gm/ab)^{-1}}{\sqrt{\pi G(4-a^2)}}. \tag{51}$$

Since $1-Gm/2b+G^2m^2/a^2b^2>0$ (for $0\leqslant a\leqslant 2$) any solution of this equation will automatically satisfy the weak energy condition (choosing the upper or lower sign as discussed above). This equation can be written as

$$f(x)=2x^2-ax+2\mp\Lambda\lambda x-\frac{1}{2}\Lambda^2x\ln\left(\frac{1+x}{1-x}\right)=0, \tag{52}$$

where $\Lambda=\sqrt{4-a^2}$, $\lambda=\sqrt{4\pi G}/\alpha^*$, and $x=Gm/ab$. For $0\leqslant a<2$, $\lim_{x\to\pm 1}f(x)=-\infty$ and $f(0)=2$. Thus for $0\leqslant a<2$ there is at least one positive and one negative value of x, in the interval $(-1,1)$, which satisfies $f(x)=0$. For $a=2$, $f(x)=2(x^2-x+1)$ which has no real zeros. Thus, for $a=2$ it is not possible to join the two manifolds, independent of whether the energy conditions hold or not. This might seem strange at first since the manifolds are Schwarzschild space-times, which can be joined with the appropriate surface energy and stress. However, in the usual Schwarzschild case there is no scalar field. In the case examined here the vanishing of the scalar field gives the additional constraint $\sigma-2S=0$, which is inconsistent with the energy and stress produced by joining the Schwarzschild space-times. Thus for $0\leqslant a<2$ there exists at least one value of Gm/ab that corresponds to a space-time in which $T_m^{\mu\nu}$ satisfies the weak energy condition. If the range of a is restricted to $0 \leqslant a \leqslant 1$ the corresponding $T_m^{\mu\nu}$ will also satisfy the dominant energy condition. For $1 < a < 2$ Eq. (47) must be checked to see if $T_m^{\mu\nu}$ satisfies the dominant energy condition. For $a=2$ it is not possible to join the manifolds together.

Before leaving this section I want to show that it is not possible for $T_m^{\mu\nu}$ to satisfy the weak energy condition if the gravitational field described by Eq. (31) is weak. In the weak field limit $x \ll 1$ and Eq. (52) reduces to

$$(a \pm \lambda\Lambda)x \simeq 2. \tag{53}$$

For x to be small it is necessary that $|\lambda\Lambda| \gg 1$. Thus $\pm\lambda\Lambda x > 0$, which implies that

$$\pm \alpha^* m > 0. \tag{54}$$

If $T_m^{\mu\nu}$ satisfies the weak energy condition the lower sign must be chosen if $\alpha^* m > 0$ and the upper sign must be chosen if $\alpha^* m < 0$. This is clearly inconsistent. Thus Eq. (31) cannot describe a weak gravitational field if $T_m^{\mu\nu}$ satisfies the weak energy condition. In fact, it can be shown numerically that the smallest value of $|x|$ that satisfies Eq. (52) (with the appropriate sign) is $|x| \simeq 0.75$.

CONCLUSION

The energy-momentum tensor for a scalar field coupled to an ideal fluid was derived. In addition to the energy-momentum tensor for the matter and the scalar field there exists an interaction energy-momentum tensor. The interaction energy-momentum tensor can violate the weak and dominant energy conditions even if the matter and scalar field energy-momentum tensors do not. It is the interaction energy-momentum tensor that allows the wormhole to be maintained.

A wormhole was created by joining two static, spherically symmetric, scalar-vac solutions of the Einstein field equations. A surface energy-momentum tensor that violates the weak energy condition exists on the surface where the two space-times are joined. If the source of the energy-momentum tensor is taken to be a scalar field coupled to matter I showed that the energy-momentum tensor of the matter and scalar field can satisfy the weak and dominant energy conditions. The violation of the weak energy condition is produced by the interaction energy-momentum tensor. Thus a wormhole can be maintained classically by coupling a scalar field to matter that satisfies the weak and dominant energy conditions. Finally, I showed that it is not possible for the matter energy-momentum tensor to satisfy the weak energy condition if the gravitational field is weak.

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