

Stochastic gravitational background from inflationary phase transitions

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(Received 12 May 1997)

We consider true vacuum bubbles generated in a first order phase transition occurring during the slow rolling era of a two field inflation: it is known that gravitational waves are produced by the collision of such bubbles. We find that the epoch of the phase transition strongly affects the characteristic peak frequency of the gravitational waves, causing an observationally interesting redshift in addition to the post-inflationary expansion. In particular it is found that a phase transition occurring typically 10–20 e -foldings before the reheating at $kT \approx 10^{15}$ GeV may be detected by the next LIGO gravity waves interferometers. Moreover, for recently proposed models capable of generating the observed large scale voids as remnants of the primordial bubbles (for which the characteristic wave lengths are several tens of Mpc), it is found that the level of anisotropy of the cosmic microwave background provides a deep insight upon the physical parameters of the effective Lagrangian. [S0556-2821(97)02120-6]

PACS number(s): 98.80.Cq, 04.30.Db

I. INTRODUCTION

The knowledge of the nature of matter perturbations in the observed universe is crucial for obtaining information about the very early universe and the very high energy physics. The deepest presently available window to the early universe is the cosmic microwave background (CMB), that in the next decade will be deeply investigated by the MAP and Planck missions, after the discovery of its anisotropies by the Cosmic Background Explorer (COBE) [1].

However, the cosmic gravitational background (CGB) will play a crucial role in the near future as the most powerful tool in reconstructing primordial physics (see [2] for an extensive overview). In fact, it consists of the gravitational waves (GWs) generated during the inflationary era; they are carriers of unperturbed physical traces of the very primordial history of the universe, since the GW decoupling probably occurred about 70 e -foldings before the recombination.

The study of the perturbations and defects produced during inflation recently underwent a great revival; it is motivated by the evidence of strong inhomogeneities from the direct reconstruction of the three-dimensional matter distribution traced by galaxies and their peculiar velocities in the modern redshift surveys. The most important conclusions of these observations are that strong underdensities, or voids, appear very prominent in the data [3–6]. In search of the inflationary generation mechanisms for such inhomogeneities, one of the most interesting ideas introduced in cosmology in recent years is the possibility of performing a phase transition *during* inflation. In such scenarios, two fields act on stage: one, say ω , slow rolls, driving enough inflation to solve the standard problems; the second field, say ψ , tunnels from a false vacuum state to an energetically favored true vacuum state, producing bubbles of the new phase embedded in the old one. Both processes are governed by a two-field potential $U(\omega, \psi)$. To avoid the graceful exit problem, the true vacuum state has to allow for a period of inflation on its

own. We can then speak of a true vacuum channel over which the bubbles slow roll until inflation ends, and reheating takes place. Depending on the potential, we can distinguish two different scenarios of first order inflation. The first is the classical extended inflation [7–9]: the bubbles are produced in a copious quantity, so that they eventually fill the space and complete the transition. To avoid too large distortions on the CMB, this scenario must produce very small bubbles [10,11], so that they are rapidly thermalized after inflation. No trace of the bubbles is left in our universe, and from this point of view such scenarios do not lead to new predictions over inflation without bubble production. In the second scenario, proposed in [12] and implemented in [13], the phase transition is completed *before* the end of inflation; the amount of e -foldings between the phase transition and the end of inflation makes the scale of the bubbly perturbations interestingly nonvanishing, and allows them to leave observable traces. In particular, if a phase transition occurred sufficiently early, the bubbles are stretched to cosmological scales, and the present large scale structure is therefore strictly linked to the primordial originating transition, which is therefore observable and testable as first suggested in [14].

In this work we concentrate upon the traces of the above phenomenology on the CGB. The most natural way for the nucleated bubbles to generate gravitational waves is through collision with each other; compared with the ordinary tensor perturbations occurring in slow rolling inflation, this is in fact a very potent source of primordial GW, as first argued in [15]. The problem was analyzed in the context of the ordinary extended inflation, in which the bubbles are completely empty and the phase transition occurs at the end of inflation. The computation of the amplitude and frequency spectrum of the gravitational radiation was performed first in the case of two bubbles [16] and then by considering an envelope of hundreds of bubbles [17], that substantially confirmed the previous results; a work emphasizing the observation possibilities can be found in [18]. Finally in [19] the problem of

gravitational waves from lower energy first order phase transition (like the electroweak transition occurring nearly at 100 GeV) was considered.

Here we want to extend these results to the second kind of inflationary phase transitions mentioned above; the main differences from the treated cases are that (i) the nucleation epoch occurs before the end of inflation, and (ii) generally the bubbles are not empty.

The paper is organized as follows: in Sec. II we recall the main results on the gravitational radiation emitted during the first order phase transition in the scenario of extended inflation, pointing out the approximations for which the computations and the results are valid; in Sec. III we extend these results to recently proposed inflationary models capable of performing a first order phase transition before the end of inflation and we discuss the observation possibilities and the existing constraints; finally, Sec. IV contains the conclusions.

II. COLLISIONS AT THE END OF INFLATION

A detailed analysis of a first order phase transition in the context of extended inflation can be found in [10]. The field ψ that undergoes the transition is the same as that which drives inflation, and its dynamics is assumed to be governed by a potential containing two nondegenerate minima, the true and false vacuum (TV, FV). The central quantity needed to characterize the transition is the bubble nucleation rate for unit volume in the semiclassical limit [20]

$$\Gamma = \mathcal{M}^4 e^{-B}, \quad (1)$$

where B is the Euclidean least action over the bounce minus the action for the external de Sitter spacetime solution [20]. The constant \mathcal{M} (with a dimension of mass) is of the order of the energy T at which the phase transition occurs. Values of \mathcal{M} a few orders of magnitude below the Planck mass m_{Pl} are generally assumed, in order to avoid quantum gravity effects [21].

The transition is completed roughly when at least one bubble per unit Hubble volume is nucleated, and accurate computations [10] show that extended inflation is successful (in the sense that bubbles percolate) if, at a time t_e ,

$$Q = \frac{4\pi}{9} \left(\frac{\Gamma}{H^4} \right)_{t_e} = 1. \quad (2)$$

The bubbles are empty and their walls rapidly approach the light speed. Depending on the intensity of the nucleation rate, the collisions between the bubbles occur when they have comoving spatial dimension less than or equal to the effective horizon H^{-1} at the transition epoch given by $T \simeq \mathcal{M}$. In the context of the standard extended inflation almost all the bubbles are nucleated at the end of inflation, so that if we take $H_0 = 100h$ km/sec/Mpc in an $\Omega = 1$ universe their comoving size is approximately $10^{-21}h^{-1}$ Mpc. Such bubbles soon reenter the horizon and rapidly thermalize without leaving trace in the matter distribution.

On the other hand, a very characteristic GW spectrum is produced during the transition. If the bubbles are perfectly spherical, no spacetime perturbation is seen from the outside.

When two (or more) bubbles collide they become a GW source, and the collection of collisions during the whole phase transition leaves observationally interesting traces on the CGB [15]. The problem of computing the emitted GW spectrum is conceptually simple but computatively difficult. One has to simulate the bubble nucleations with rate given by Eq. (1) and to evolve them with the Klein-Gordon equation. Whenever collisions occur, one must compute the radiation emitted. It has been performed [16] with the aid of two main approximations: linearized gravity and static background. The former is consistent if the fraction of energy that goes in GW is small with respect to unity; the latter is valid if the transition completes within a Hubble time. Following [22], the total energy radiated at frequency ω in the direction \mathbf{k} into the solid angle $d\Omega$ and in the interval $d\omega$ is

$$\frac{dE}{d\omega d\Omega} = 2G\omega^2 \Lambda_{ijlm}(\mathbf{k}) T^{ij*}(\mathbf{k}, \omega) T^{lm}(\mathbf{k}, \omega), \quad (3)$$

where $\Lambda_{ijlm}(\mathbf{k})$ is the projection tensor for gravitational radiation [22] and $T^{\mu\nu}(\mathbf{k}, \omega)$ the Fourier transform of the energy momentum tensor. The computations were first performed in the simplified case of two colliding bubbles [16]. The end of the transition in that case was modeled as a modulating function $T_{ij}(t) \rightarrow T_{ij}(t)C(t)$ that makes the signal vanish after a cutoff time τ of the order of the initial separation between the bubbles. In [17] an ingenious model for $C(t)$ was introduced by excluding from the integration the spatial region where bubbles overlap; at $t \gg t_e$ the bubbles completely fill the space, and the GW emission ends. This also allowed one to perform many bubbles simulations. The results are intuitive and consistent with the approximations. First, the bounce is approximated as [10]

$$B(t) \simeq B(t_e) - \beta(t - t_e), \quad (4)$$

where β^{-1} sets the natural scale for the phase transition. Since this is of the order of the initial average separation between the colliding bubbles, the static background approximation requires that $\beta^{-1} \leq H^{-1}$. The GW spectrum shows a very characteristic peak at $\omega_{\text{GW}} \simeq 1.6\beta$ and the fraction of energy radiated in GW is found $\Omega_{\text{GW}} = \rho_{\text{GW}}/\rho_c \simeq 0.06(H/\beta)^2$; $\rho_c = 3H^2/8\pi G = g\pi^2 T_e^4/30$ is the critical energy density at the time t_e ; g counts the number of relativistic degrees of freedom at the temperature T_e of the phase transition, and is typically of the order of 100. We remark that these results agree with physical expectations: the peak frequency is simply the inverse of the time scale of the process, and the $(H/\beta)^2$ scaling for Ω_{GW} can be naturally inferred in terms of the energy density ρ_c and β^{-1} [16]. To obtain the corresponding present quantities, one has to take into account the cosmic redshift from T_e down to the present 3 K. Ω_{GW} remains unchanged during the radiation epoch, but undergoes a decrease during the matter dominated era; ω_{GW} redshifts via the cosmic expansion. The quantities relevant at the present are therefore

$$\omega_{\text{GW}}(\text{Hz}) \simeq 3 \times 10^7 \left(\frac{\beta}{H} \right) g_{100}^{1/6} T_{15}, \quad (5)$$

$$\Omega_{\text{GW}} h^2 \simeq 10^{-6} \left(\frac{H}{\beta} \right)^2 g_{100}^{-1/3}, \quad (6)$$

where we have defined $T_{15} = T_e / (10^{15} \text{ GeV})$ and $g_{100} = g/100$.

We emphasize once again that the above results hold for the extended inflation scenario, in which bubbles are nucleated at the end of inflation and rapidly thermalize. In the next section we will extend these results to the models of first order inflation capable of performing the nucleation epoch well before the end of inflation, showing how the above results change and discussing the consequences on the observability.

III. COLLISIONS DURING INFLATION

Recent studies on inflation [12] have shown that a not *ad hoc* slow rolling is generated by the gravitational part of the Lagrangian, already at the level of fourth order gravity (FOG). A matter field potential containing false and true vacuum minima generates a first order phase transition, but the latter in general occurs at an epoch characterized by a number N_0 of e -foldings before the end of inflation. Depending on this quantity, the scale of the bubble-like perturbations may be stretched out to cosmological size by exponential growth, thus making the bubbles candidates to be the seeds for the formation of the voids observed today [13,14,23]. We consider the consequences on the CGB spectrum of this FOG model (and we refer to [12] and [13] for an extensive treatment); since a conformal transformation makes the Lagrangian as in ordinary two field inflation, our results hold for any kind of inflation in which one field slow rolls and the other one undergoes a first order phase transition.

Once the conformal transformation is performed, the FOG action takes the form

$$S = \int \sqrt{-g} d^4x \times \left[-\frac{\mathcal{R}}{16\pi} + \frac{3}{4\pi} \omega_{;\mu} \omega^{;\mu} + \frac{1}{2} e^{-2\omega} \psi_{;\mu} \psi^{;\mu} + U(\psi, \omega) \right], \quad (7)$$

where the potential is

$$U(\psi, \omega) = e^{-4\omega} \left[V(\psi) + \frac{3M^2}{32\pi} W(\psi) (1 - e^{2\omega})^2 \right], \quad (8)$$

and generates TV bubbles with $W(\psi) = 1 + (8\lambda/\psi_0^4) \psi^2 (\psi - \psi_0)^2$, a degenerate quartic, and $V(\psi) = (1/2)m^2 \psi^2$, the symmetry breaking term. The slow roll inflation driven by ω takes place at $\omega \gg 1$, and is over when ω approaches zero. At large ω , the potential U is dominated by $W(\psi)$, and thus the false vacuum minimum at $\psi \approx \psi_0$, for which $U_{\text{FV}} = e^{-4\omega} [(3M^2/32\pi)(1 - e^{2\omega})^2 + V(\psi_0)]$ is unstable with respect to tunneling towards the true vacuum $\psi=0$ (for which $U_{\text{TV}} \approx 3M^2/32\pi$). At small ω , instead, U is dominated by $V(\psi)$, and both the true and the false vacua converge to the global zero-energy minimum at $\omega = \psi = 0$, where inflation ends and reheating takes place. The slow-roll solu-

tion in this model for $\omega \gg 1$ can be written very conveniently as [12] $N = (3/4)e^{2\omega}$ where N is the number of e -foldings to the end of inflation.

It has been shown [13] that taking into account the gravitational corrections and dropping the thin wall approximation due to a finite thickness δR for the wall surrounding a bubble of radius R , the nucleation rate Γ defined in Eq. (1) becomes

$$\Gamma = \frac{\mathcal{M}^4 B^2 (\delta R)^4}{16R^4} e^{-B} = \mathcal{M}'^4 e^{-B}, \quad (9)$$

and in terms of N the bounce is

$$B = \frac{N^4}{N_1^4} \left[1 - \left(\frac{N_2}{N} \right)^2 \right] \left[1 - \left(\frac{N}{N_3} \right)^4 \right], \quad (10)$$

where the first set of brackets is the thin wall correction, and the second set of brackets is the gravitational correction; moreover, the N_i are related to the physical quantities by

$$N_1^2 = \frac{3^{3/2}}{4} \frac{\psi_0 m^3}{M^2 \lambda}, \quad N_2^2 = \frac{27\pi}{8} \frac{\psi_0^2 m^2}{M^2 \lambda},$$

$$N_3^2 = \left(\frac{27\pi}{32} \right)^{1/2} \frac{\psi_0 m^2}{M^2 \lambda^{1/2}}. \quad (11)$$

By indicating with N_0 the amount of e -foldings to the end of inflation when the phase transition takes place, the quantity Q defined in Eq. (2) takes the approximate form

$$Q = \frac{4\pi\Gamma}{9H^4} = \exp \left\{ \frac{(N_0^4 - N^4)}{N_1^4} \left[1 - \left(\frac{N_2}{N_0} \right)^2 \right] \left[1 - \left(\frac{N_0}{N_3} \right)^4 \right] \right\}. \quad (12)$$

The two approximations that have been adopted to obtain the above formulas are satisfied by the conditions $N > N_2$ (thin-wall), and for $N < N_3$ (gravitational correction). Also, $N_0 > N_1$ is required to guarantee tunneling through TV bubbles [12].

The central underdensity of a bubble is determined by the shape of the potential (8) at the nucleation epoch, characterized by N :

$$\delta \equiv |\delta\rho/\rho| = \frac{U_{\text{FV}} - U_{\text{TV}}}{U_{\text{FV}}} = [(N/N_4)^2 + 1]^{-1},$$

$$N_4^2 = 3\pi \frac{\psi_0^2 m^2}{M^2} = \frac{64}{9\pi} M^2 N_1^{-4} N_3^8. \quad (13)$$

From the current microwave background measurements, we obtain $M \approx 5 \times 10^{-6}$ (in Planck units) [12], a value that we will adopt in the results below. However, since in our model one should also consider the contribution of the bubbles to the microwave background, this constraint is actually only an upper limit on M . As an intuitive feature, note that the four physical parameters M, m, λ, ψ_0 , and therefore the four e -folding constants N_{1-4} , fully determine the inflationary potential at N_0 (the nucleation epoch): the vacuum energy density, the energy difference between FV and TV, the amplitude of the barrier, and the value ψ_0 of the FV phase.

The linearized gravity approximation involved to obtain the results (5) and (6) is surely satisfied in our case since the energy that goes into the walls and therefore in GW is a fraction δ given by Eq. (13) of the total FV energy in the bubble. The other important approximation is to consider spacetime as static during the transition; in other words, the time scale of the nucleation era must be smaller than the Hubble time. This is satisfied in our case if the condition $\beta^{-1} \leq H^{-1}$ holds at N_0 , where β is defined in Eq. (4); this means $-(\partial B/\partial t)_{N_0} \geq H$, that using Eq. (10) becomes

$$\mathcal{B} = \left(\frac{\partial B}{\partial N} \right)_{N_0} = \frac{4N_0^3}{N_1^4} \left[1 - \frac{1}{2} \left(\frac{N_2}{N_0} \right)^2 - 2 \left(\frac{N_0}{N_3} \right)^4 + \frac{3}{2} \frac{N_0^2 N_2^2}{N_3^4} \right] \geq 1. \quad (14)$$

By defining

$$x = (N_2/N_0)^2, \quad y = (N_0/N_3)^4, \quad (15)$$

the quantity in brackets in Eq. (14) is simply

$$z(x,y) = 1 - \frac{x}{2} - 2y + \frac{3xy}{2}, \quad (16)$$

still taking into account the gravitational and post-thin-wall corrections. The range of interest for both x and y is $[0,1]$ for consistency of our approximations. Since N_1 does not appear in $z(x,y)$, for the validity of Eq. (14) it is enough that $z(x,y)$ assumes some positive value; it is easily seen that this is true in the range $0 < x < 1$, $0 < y < (x-2)/(3x-4)$. By choosing a value of N_0 and $N_2 < N_0$ to fix x , a value of y always exists that yields $z(x,y) > 0$; therefore, our model performs a fast transition for

$$N_1 \leq [4N_0^3 z(x,y)]^{1/4}. \quad (17)$$

We can now extend the results (5) and (6) to the present inflationary models. The peak frequency of the GW spectrum contains an *additional* redshift due to the inflationary expansion between the phase transition and the end of slow rolling. Moreover, in calculating Ω_{GW} we must take into account that the fraction of energy that goes in gravitational radiation is reduced by a fraction δ , Eq. (13), with respect to Eq. (6). Therefore, the results are

$$\omega_{\text{GW}}(\text{Hz}) \approx 3 \times 10^7 \mathcal{B} g_{100}^{1/6} T_{15} \exp(-N_0), \quad (18)$$

$$\Omega_{\text{GW}} h^2 \approx 10^{-6} \mathcal{B}^{-2} g_{100}^{-1/3} \delta(N_0). \quad (19)$$

In the following we will adopt the values $T_{15} \approx 1$ (fixing the epoch of the end of slow rolling at typical GUT energy scales), and $g_{100} \approx 1$.

From the above formulas it is clear that in general any model of two field first order inflation radically changes the frequencies of the GW spectrum to be observed today with respect to the ordinary extended inflation models. The main difference is represented by the shift of the peak frequency towards lower values depending on the amount of e -foldings between the phase transition and the end of inflation. This

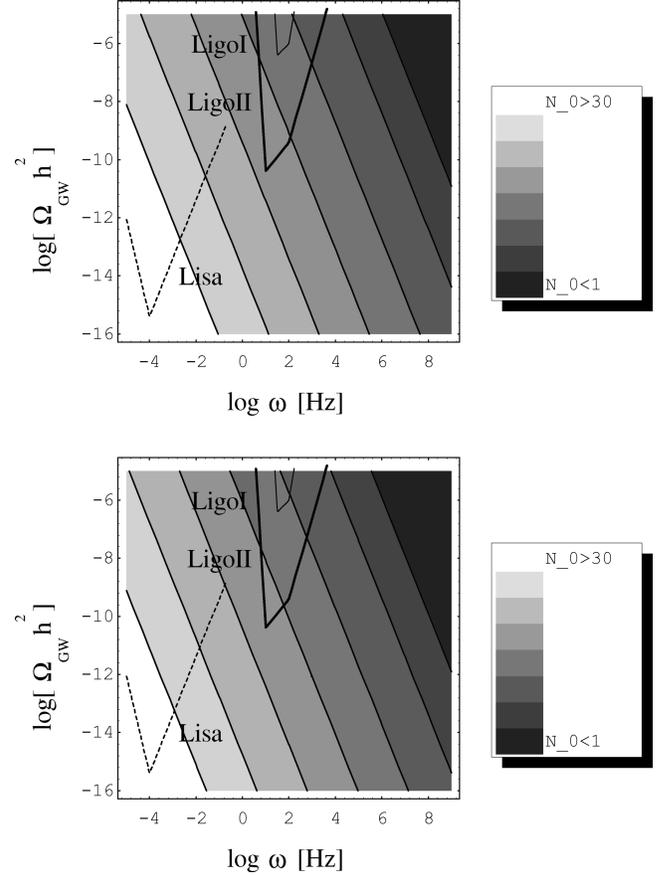


FIG. 1. Amplitude-peak frequency relation for the CGB generated in a first order phase transition occurring N_0 e -foldings before the end of inflation. For clarity, we have exploited the two interesting cases $\delta_{N_0} = 0.1$ (top) and 0.01 (bottom) and we have shown the dependence of the curves on N_0 (the gray tonality change corresponds to a scansion of 5 in N_0 , as indicated in the legend box). Note the shift of the frequencies towards ranges that will be investigated by the next interferometric detectors.

has very interesting consequences for what concerns the search for the present traces of the era driven by very high energy physics. We now make some general considerations about this, referring to the next two subsections to a detailed analysis of two important cases.

Formulas (18) and (19) may be rewritten emphasizing the relation between the physical parameters

$$\Omega_{\text{GW}} h^2 = 9 \times 10^8 \frac{\delta(N_0)}{[\omega_{\text{GW}}(\text{Hz})]^2} \exp(-2N_0), \quad (20)$$

with the constraint $\mathcal{B} = 10^{-3} \sqrt{\delta(N_0)/\Omega_{\text{GW}} h^2} \geq 1$. In Fig. 1 we plot experimentally interesting $\Omega_{\text{GW}} h^2$ as a function of the peak frequency ω_{GW} and for different N_0 (the change in gray tonality corresponds to a scansion of 5 in N_0 , as indicated in the legend box); for clarity, we have set $\delta_{N_0} = 0.1$ (top) and 0.01 (bottom). Each point on the figure corresponds to the GW spectrum generated by a first order phase transition that occurred N_0 e -foldings before the end of inflation. For $N_0 \rightarrow 0$ we obtain the known results in the case of extended inflation [Eqs. (5) and (6)]. The increase of N_0 pushes the peak frequency towards exponentially small values and

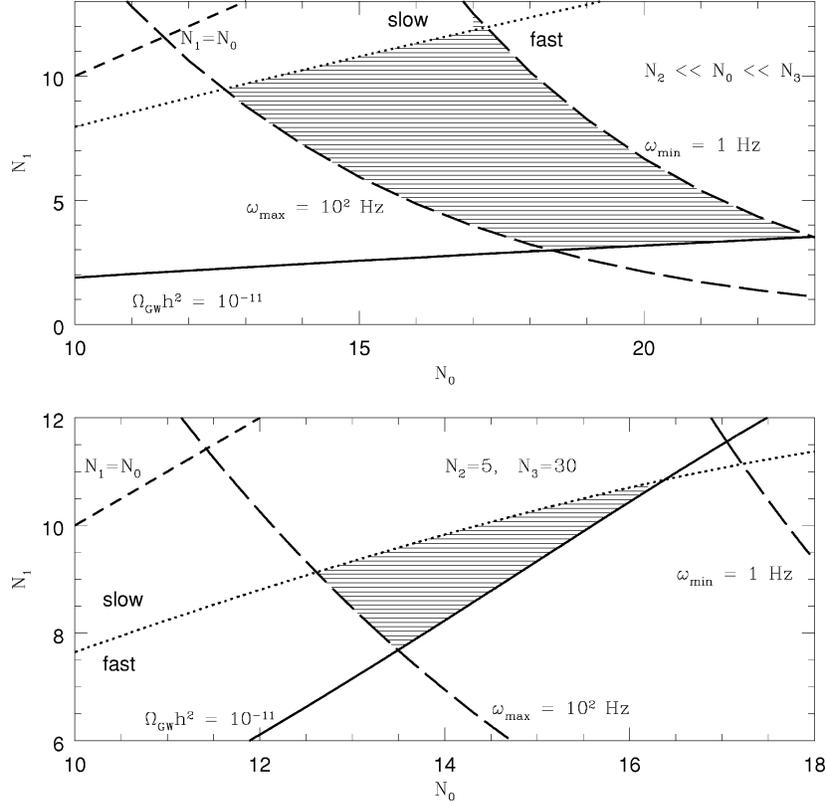


FIG. 2. Advanced LIGO detection region for (N_0, N_1) , in the cases for which the post-thin-wall and gravitational corrections are negligible (upper panel) or not (lower panel). The dashed areas contain physical parameters respecting all the indicated constraints; particularly it yields frequencies among the maximal and minimal limits (long dashed lines) and amplitude above the minimal observable one (solid lines).

intersects very interestingly the plotted expected level of sensitivities of the next generation of GW interferometric detectors: if first order phase transitions occurred at $10 \leq N_0 \leq 30$ during the inflationary era (realistically arising from spontaneous breaking of very high energy symmetries), they should leave near future detectable peaks in the CGB frequency spectrum. By continuing to increase N_0 beyond several decades, we enter the class of phase transitions that could leave traces in the present large scale matter distribution. In particular, at $N_0 \geq 50$, the remnants of the nucleated bubbles are of astrophysically interesting size, and correspond to the large voids detected in the galaxy distribution [4,5,12,14,23,24]. For such models, the GW frequencies correspond to cosmological wavelengths, to be interestingly investigated through their induced CMB anisotropies; as we will show in the sequel, a deep insight already comes from considering the anisotropy amplitude; however, it is interesting to point out that in the future these models could be tested with powerful methods (presently under investigation) capable of extracting the pure CGB signal from the data coming from the next high resolution CMB experiments [25,26].

In the following we consider in more detail the correspondence between the CGB and the parameters of the effective Lagrangian, focusing on two important cases: spectra detectable by the next LIGO interferometric detectors, and spectra from colliding bubbles of astrophysically interesting size.

A. Near future detectable GW from bubbles

The Advanced LIGO project will reach a sensibility of about $\Omega_{\text{GW}} h^2 \approx 10^{-11}$ in the frequency range $\omega_{\text{min}} \approx 1$ Hz, $\omega_{\text{max}} \approx 10^2$ Hz (see [2] and references therein). From Eqs. (18) and (19), we see that the amplitude and the peak frequency fix the products $\mathcal{B} \exp(-N_0)$ and $\mathcal{B}^{-2} \delta(N_0)$, namely two conditions for our physical parameters. Then, by fixing N_2, N_3 so as to yield a reasonable parametric region for N_0, N_1 (we remember that $N_2 \leq N_0 \leq N_3$), we may search for the physical parameter set that could explain the observation of a definite peak at some frequency in the above ranges. Such a set is constrained by Eq. (18),

$$\left[\frac{3 \times 10^7}{\omega_{\text{max}}} 4N_0^3 z(x, y) e^{-N_0} \right]^{1/4} \leq N_1 \leq \left[\frac{3 \times 10^7}{\omega_{\text{min}}} 4N_0^3 z(x, y) e^{-N_0} \right]^{1/4}, \quad (21)$$

to have the peak frequency in the observation range, and by Eq. (19),

$$N_1 \geq \left[\frac{9\pi z^2(x,y)N_0^8}{8M^2N_3^8} \Omega_{\text{GW}} h^2 + \sqrt{\left(\frac{9\pi z^2(x,y)N_0^8}{8M^2N_3^8} \Omega_{\text{GW}} h^2 \right)^2 + 16z^2(x,y)N_0^6 10^6 \Omega_{\text{GW}} h^2} \right]^{1/4}, \quad (22)$$

to have a sufficient amplitude. Moreover Eq. (17) [with $z(x,y) > 0$ of course] must be satisfied to have the completion of the transition in less than a Hubble time and $N_1 < N_0$ to have nucleation of TV bubbles. The dashed areas in Fig. 2 are the regions of the parameters (N_0, N_1) for which both the peak frequency and the amplitude fall into the observation ranges; the upper panel shows the limit in which the post-thin-wall and gravitational corrections are negligible, while in the lower panel such corrections are important. The constraints define the region of observability shaded in Fig. 2; as an interesting and intuitive feature we note that a first order inflationary phase transition that is LIGO detectable occurs at $N_0 = 10-20$ e -foldings before the end of inflation, which is just the amount necessary to redshift the corresponding frequencies in ordinary extended inflation [$\approx 10^7$ Hz, see Eq. (5) and Fig. 1] towards the observation range. From the nucleation epoch it is easily found that the comoving size of these perturbations is approximately $10^{-18} - 10^{-14} h^{-1}$ Mpc; this means that they reenter the horizon early in the radiation dominated era and most likely do not leave traces in the density field. Finally we remark once again that for each set of N_{1-4} there exists a set of physical parameters $\mathcal{M}', \psi_0, m, \lambda$, as it is easily seen from Eqs. (9), (10), (11), and (12).

B. Large scale bubbles

As we already mentioned in the Introduction, the recent analysis on the modern redshift surveys have assessed the large scale voids as the dominant feature of the nearby universe. Several works have been performed in these years on the hypothesis that the primordial origin of the voids is in an inflationary phase transition in the context of models of FOG and in general two field inflation [12–14, 23, 24, 27, 28]. Here we add a new tool. It has been shown [12] that voids of tens of Mpc of diameter would be the relics of bubbles with comoving size expanded by a factor $\exp(50)$ between the phase transition and the end of inflation. This request fixes the product

$$\mathcal{B}^{-1} \exp(N_0) \approx \exp(50), \quad (23)$$

and by requiring $\mathcal{B} \geq 1$ we have $N_0 \geq 50$. The frequency in Eq. (18) is therefore:

$$\omega_{\text{GW}}(\text{Hz}) \approx 10^{-15}. \quad (24)$$

These frequencies are covered by the most stringent presently available constraint upon the amplitude of the CGB, provided by the amount of anisotropy of the CMB (see [2] and references therein). Precisely, this constraint imposes that

$$\Omega_{\text{GW}} h^2 \leq 7 \times 10^{-11} \left(\frac{H_0}{\omega} \right)^2 \quad \text{for } H_0 \leq \omega \leq 30H_0, \quad (25)$$

and consequently our frequencies localize themselves on the high frequency border of this range, where $\Omega_{\text{GW}} h^2 \leq 10^{-13}$ ($H_0 = 100h$ km/sec/Mpc). Note that the corresponding wavelengths determine CMB anisotropies on small angular scales ($\theta < 1^\circ$), that will be deeply investigated by the MAP and Planck missions of the next decade, as mentioned in the Introduction. A great effort is being done in order to develop theoretical analysis instruments capable of extracting the pure CGB signal from the whole spectrum of CMB anisotropies, both for what concerns pure $\delta T/T$ and polarization (see, e.g., [25, 26]); this matter will provide a powerful tool to investigate the whole CGB spectrum, and in particular the traces of early inflationary phase transitions, that are the subject here. Here we concentrate on the overall amplitude constraint given by Eq. (25), since it already provides a deep insight into the physical parameter space; from Eqs. (13) and (19) we see that it must be

$$\delta(N_0) \leq 10^{-7} \mathcal{B}^2, \quad (26)$$

or, alternatively to Eq. (21),

$$N_1 \leq \left[\frac{9\pi z^2(x,y)N_0^8}{8 \times 10^7 M^2 N_3^8} + \sqrt{\left(\frac{9\pi z^2(x,y)N_0^8}{8 \times 10^7 M^2 N_3^8} \right)^2 + \frac{16z^2(x,y)N_0^6}{10^7}} \right]^{1/4}; \quad (27)$$

this sets a very narrow range (at fixed N_2 and N_3) for N_0 and N_1 . Moreover, another condition arises from Eq. (13) for the minimal $\delta(N_0)$ capable of yielding the voids observed today [24],

$$\delta(N_0) \geq 10^{-2}. \quad (28)$$

In Fig. 3 we plot again all the constraints for $N_2 \leq N_0 \leq N_3$ (upper panel, negligible post-thin-wall and gravitational corrections) and $N_2 = 30, N_3 = 70$ (lower panel). In both panels, each curve refers to the indicated constraint: the solid line represents the condition (23), and guarantees that the bubbles are expanded to the observed sizes. The short dashed line draws the condition (28). Finally, the long dashed line represents the strong constraint coming from the CMB (26) and (27). The part of solid line below all the curves satisfies all the constraints, and restricts to $N_0 > 55$ the epoch of the phase transition; also, the gravitational and post-thin-wall corrections (important in the lower panel) allow for a viable parameter set slightly larger than when they are negligible (upper panel). For physical parameters on the allowed set, the presently large scale observed voids correspond, both for size and underdensity, to the TV bubbles nucleated during inflation, without exceeding the constraints coming from the CMB isotropy.

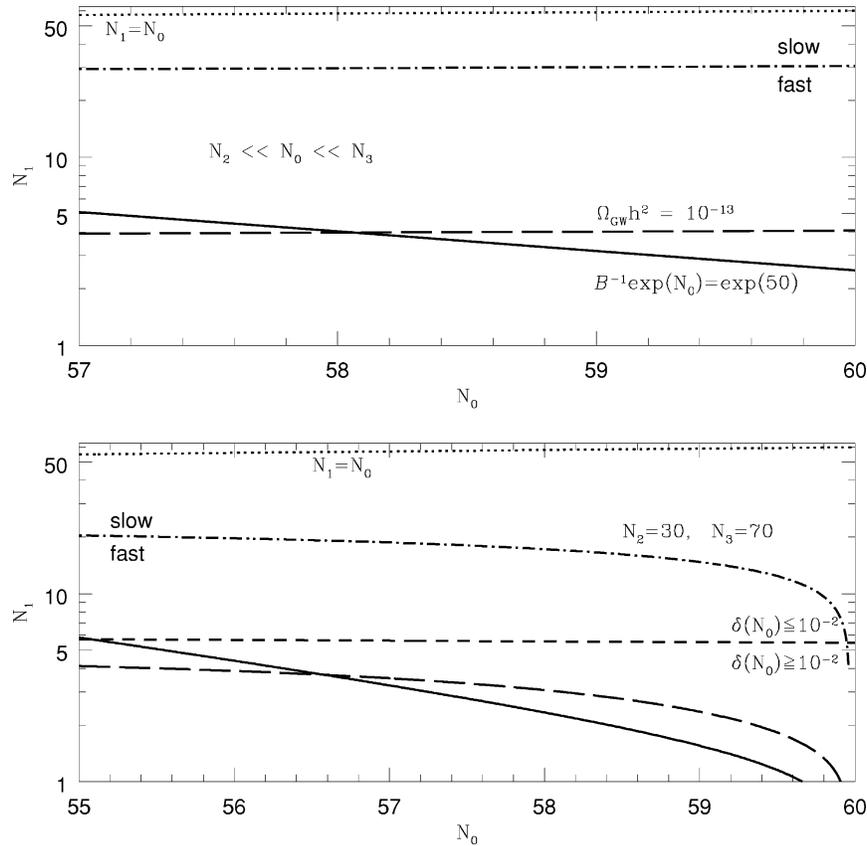


FIG. 3. Insight upon the physical parameters of effective Lagrangian yielding large scale structure from bubbles; the cases for which the post-thin-wall and gravitational corrections are negligible (upper panel) or not (lower panel) are shown. The segments of the solid line below all the other curves sketch the relation among the physical parameters for models capable of explaining the large scale observed voids as the relics of bubbles nucleated more than 55 e -foldings before the end of inflation; all the performed constraints are respected, and the N_1 axis is logarithmic to show all of them; note particularly the request of sufficiently deep bubbles (short dashed lines) and the strong constraint coming from the upper limit to the level of CMB anisotropy (long dashed lines).

IV. CONCLUSION

We have extended the known results on the stochastic gravitational background produced by colliding bubbles in two fields models of first order inflation. In such models, a field performs the first order phase transition and a second field (that in fourth order gravity is of gravitational origin) provides the inflationary slow rolling. The resulting general phenomenology is that the phase transition occurs well before the end of inflation. This has very nontrivial consequences on the gravitational radiation produced by colliding bubbles. It is found that the expansion between the phase transition and the end of inflation causes an additional redshift for the peak frequency of the spectrum with respect to the ordinary models of extended inflation; also, the energy carried by the gravitational waves is reduced by a fraction equal to the density contrast of the (not completely empty) bubbles. The result is that if first order phase transitions occurred during the inflationary era (realistically arising from spontaneous breaking of very high energy symmetries), they should leave near future detectable peaks in the CGB frequency spectrum. That detection would be a first experimental verification of our very high energy physics theoretical investigations. In particular, a peak at some frequency explored by the next LIGO interferometers may be explained in

terms of a first order phase transition occurring typically 10–20 e -foldings before the reheating at grand unified theory (GUT) energy scales ($T_e \simeq 10^{15}$ GeV).

Moreover, the gravitational radiation produced in recently proposed models capable of generating the observed large scale voids has been examined. In these models, the phase transition occurs more than 50 e -foldings before the end of inflation. As expected, the typical frequencies of the spectrum are in the range in which the isotropy of the CMB puts the strongest existing constraint on the amplitude of the gravitational background, and gives a very deep insight into the physical parameter space. We find a precise new relation among the physical parameters in the class of the models in which the phase transition is fast (namely less than a Hubble time); the CMB constraint requires that the phase transition occurs generally more than 55 e -foldings before the end of inflation. These results localize the parameter set of the effective Lagrangian for which the present large scale observed voids correspond, both in size and underdensity, to the TV bubbles nucleated during the inflationary era, without perturbing the CMB. The fraction of CMB anisotropies and polarization due to the phenomena investigated here could be recognized in the high resolution CMB data that will be provided by the experiments of the next decade.

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