Pre-big-bang inflation requires fine-tuning

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The pre-big-bang cosmology inspired by superstring theories has been suggested as an alternative to slow-roll inflation. We analyze, in both the Jordan and Einstein frames, the effect of spatial curvature on this scenario and show that too much curvature — of either sign — reduces the duration of the inflationary era to such an extent that the flatness and horizon problems are not solved. Hence, a fine-tuning of initial conditions is required to obtain enough inflation to solve the cosmological problems. [S0556-2821(97)01020-5]

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I. INTRODUCTION

The pre-big-bang cosmology inspired by superstring theories has been suggested as a possible implementation of the inflationary-universe scenario [1]. This cosmology is based on a spatially flat solution in which the kinetic energy of a massless dilaton drives an accelerated expansion toward a singularity at time $t_{\rm sing}$, with the scale factor $a(t)\propto(t_{\rm sing}-t)^{-1/\sqrt{3}}$. Before the singularity is reached, stringy and/or nonperturbative effects bring an end to the inflationary phase and, by mechanisms that are not yet completely understood, effectuate a transition to a standard Friedmann-Robertson-Walker (FRW) epoch of decelerating expansion with the dilaton fixed at its present value.

In previous investigations of this scenario considerable effort has been focused on the details of the graceful exit from the inflationary era [2]; it is still not at all clear that this can be done. In this paper we will assume, for the sake of argument, that mechanisms for accomplishing this actually exist and will concentrate instead on the initial conditions. These have received little attention in previous discussions, in large part because the flat-space solution on which these discussions have been based has an inflationary epoch that extends infinitely far back in time.

The situation is quite different once one admits the possibility of even a small amount of spatial curvature, as generality considerations certainly require. Although pre-big-bang inflationary solutions still arise, the inflationary epoch has a finite duration which depends upon the initial curvature. Furthermore, it does not even appear that the preinflationary era can be extended arbitrarily far back. For a closed $(k=1)$ universe this is prevented by the existence of an initial singularity, while the open universe $(k=-1)$ solution, although remaining nonsingular, becomes increasing implausible as $t\to-\infty$. For both cases, as well as for their limiting $k=0$ case, one is thus led to view the scenario as beginning with the appearance (e.g., by a quantum fluctuation) of a sufficiently large, smooth region at an initial time t_0, with the subsequent evolution of this region being determined by the classical field equations and the initial data at t_0. We will find that, in contrast [3] to slow-roll inflation, the pre-big-bang scenario is quite sensitive to these initial conditions.

In Sec. II we obtain the pre-big-bang solutions with arbitrary spatial curvature, and discuss the requirements that must be satisfied in order that there be enough inflation to solve the horizon and flatness problems of the standard cosmology. We then show how these requirements can be phrased as constraints on initial conditions. We carry out this discussion in the Jordan frame, in which the Planck mass is a time-dependent quantity depending on the dilaton field and the fundamental string length $\ell_{\rm st}$ is fixed. In Sec. III we describe the somewhat different, but equivalent, picture that results if one works in the Einstein frame, where the Planck mass is fixed and $\ell_{\rm st}$ varies with time. Section IV contains some concluding remarks.

II. PRE-BIG-BANG COSMOLOGY WITH CURVATURE

A. Flat-space solutions

The evolution of the universe during the pre-big-bang phase of this scenario is governed by the tree-level, low-energy effective action

$$S_{\rm eff}=\frac{1}{2}\int d^4x\sqrt{-g}e^{-\sigma}[\ell_{\rm st}^{-2}(R+\partial_\mu\sigma\partial^\mu\sigma)+R^2+\text{matter terms}+O(e^\sigma)]. \quad (1)$$

If we assume a Robertson-Walker metric with scale factor $a(t)$, the Friedmann equation takes the form

$$\left(H+\frac{1}{2}\frac{\dot\Phi}{\Phi}\right)^2=\frac{1}{12}\left(\frac{\dot\Phi}{\Phi}\right)^2+\frac{8\pi}{3}\frac{\rho}{\Phi}-\frac{k}{a^2}, \quad (2)$$

where a dot indicates differentiation with respect to time, $H=\dot a/a$, and the Brans-Dicke field Φ and the associated time-dependent Planck length $\ell_{\rm Pl}$ are related to the dilaton field by

$$\Phi = \ell_{\rm Pl}^{-2} = \ell_{\rm st}^{-2} e^{-\sigma}. \quad (3)$$

Equation (2) must be supplemented by the equation of motion for the dilaton field

$$\frac{d}{dt}(\dot{\Phi} a^3) = 8\pi a^3(\rho - 3p), \quad (4)$$

as well as the equations governing the fields responsible for the energy density ρ. If the latter is entirely due to radiation, as we assume henceforth, $\rho \propto 1/a^4$ and the right-hand side of Eq. (4) vanishes, implying that

$$B = -\dot{\Phi} a^3 \quad (5)$$

is a constant. (If $k=0$, the freedom to make an overall time-independent rescaling of a means that B has no invariant meaning. For the cases with nonzero spatial curvature we will eliminate this freedom by adopting the convention that $|k|=1$, so that a is the curvature radius of space.) In order to obtain a solution in which the string coupling e^{σ} evolves from weak to strong, we require that B be positive. Using these results, we may rewrite the Friedmann equation as

$$\left(\dot{a} - \frac{B}{2a^2\Phi}\right)^2 = \frac{B^2}{12(a^2\Phi)^2} + \frac{bB}{\sqrt{3}a^2\Phi} - k, \quad (6)$$

where

$$b \equiv 8\pi\sqrt{\frac{3}{3B}}\rho a^4 \quad (7)$$

is a constant.

The solution of these equations is particularly simple when $k=b=0$. With $\dot{\Phi}>0$ there are two solutions, with $a(t) \sim (\pm t - \text{const})^{\pm 1/\sqrt{3}}$. The lower signs give the inflationary solution underlying the pre-big-bang scenario; this solution may be written as

$$a(t) = A(t_{\rm sing} - t)^{-1/\sqrt{3}},$$

$$\Phi(t) = \frac{BA^{-3}}{1+\sqrt{3}}(t_{\rm sing} - t)^{1+\sqrt{3}} \quad (8)$$

with $t_{\rm sing}$ and A being arbitrary constants.

As we will see, this flat-space solution is also a good approximation to the final, inflationary stages of the $k=\pm 1$ solutions. We may view the inflationary era of these solutions as beginning at a time t_i, when they are well approximated by Eq. (8), and ending at a time $t_f < t_{\rm sing}$ when the solution ceases to be reliable, either because the coupling e^{σ} has become large enough that higher-order loop corrections to the low-energy effective action can no longer be neglected or because the space-time curvature has become so great that stringy and/or quantum gravity effects are significant. The amount of inflation during the interval between these can be measured by the factor Z by which the comoving Hubble length $(Ha)^{-1}$ decreases. Using Eq. (8), we find that

$$Z = \frac{H(t_f)a(t_f)}{H(t_i)a(t_i)} = \left(\frac{\Phi(t_i)}{\Phi(t_f)}\right)^{1/\sqrt{3}} = \left(\frac{t_{\rm sing} - t_i}{t_{\rm sing} - t_f}\right)^{(1+\sqrt{3})/\sqrt{3}}. \quad (9)$$

If the presently observed universe is contained within a region that had a size $H^{-1}(t_i)$ at time t_i, then solution of the horizon problem by this inflation requires that $Z > e^{60}$ (see, e.g., Ref. [4]). Hence, the effective Planck mass must change by a very large amount, $\sqrt{\Phi(t_i)/\Phi(t_f)} \gtrsim e^{104}$ and, because the growth of the scale factor is a power law and not exponential, the inflationary period must be of long duration, $(t_{\rm sing} - t_i)/(t_{\rm sing} - t_f) \gtrsim e^{38}$.

Since inflation ends if the coupling becomes strong, $\Phi(t_f) \gtrsim \ell_{\rm st}^{-2}$. Similarly, the fact that the classical equations are reliable only if the curvature is less than the string scale implies that $H^{-1}(t_f) \sim (t_{\rm sing} - t_f) \gtrsim \ell_{\rm st}$. Taken together, these inequalities imply the bound

$$Z \lesssim \mathrm{Min}\left\{[\Phi(t_i)\ell_{\rm st}^2]^{1/\sqrt{3}}, \left(\frac{t_{\rm sing} - t_i}{\ell_{\rm st}}\right)^{(1+\sqrt{3})/\sqrt{3}}\right\}. \quad (10)$$

As we shall see, this bound makes it very difficult to obtain a Z large enough to solve the horizon problem. One could, of course, hope for additional inflation in the nonperturbative and stringy era between t_f and $t_{\rm sing}$. However, mechanisms for realizing this hope remain to be demonstrated.

B. Solutions with spatial curvature

Let us now turn to the solutions with nonzero spatial curvature.[1] We begin by solving Eq. (6) for $\dot{a}$ and then substituting that result in Eq. (5). Introducing the variable

$$\psi = \sqrt{\frac{12}{B}} a^2 \Phi, \quad (11)$$

which is proportional to the square of the Einstein-frame scale factor, we can write the resulting equations as

$$\dot{a} = \frac{1}{\psi}[\sqrt{3} \pm \sqrt{1 + 2b\psi - k\psi^2}], \quad (12)$$

$$\dot{\psi} = \pm\frac{2}{a}\sqrt{1 + 2b\psi - k\psi^2}. \quad (13)$$

The choice of signs in these equations is determined by the requirement that $\psi \sim a^2\Phi$ eventually tend toward zero, so that the curvature term will become negligible and the solution can approach the inflationary $k=0$ solution (8). For $k=-1$ the sign of $\dot{\psi}$ can never change, and so the lower signs in Eqs. (12) and (13) must be chosen throughout. For $k=1$, where $\dot{\psi}$ changes sign, the upper signs apply at early times and the lower ones at late times.

Rather than solve these equations directly, we introduce a parameter η that satisfies

$$\dot{\eta} = \frac{1}{a}. \quad (14)$$

[1]For another treatment of these equations, see Ref. [5]; some related solutions involving dilaton fields in models with spatial curvature are discussed in Ref. [6].

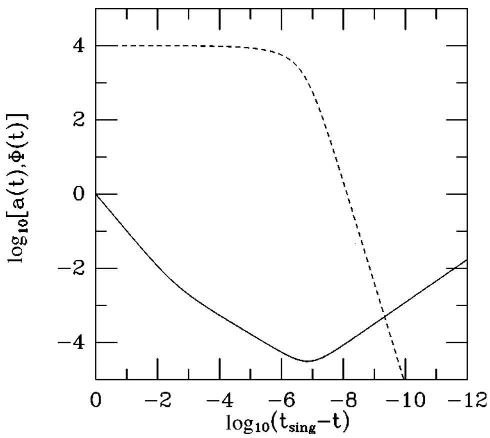

FIG. 1. Evolution of the cosmic scale factor $a(t)$ (solid curve) and the Brans-Dicke field $\Phi(t)$ (broken curve) for $k=-1$ and $b \gg 1$; the scales for a and Φ are arbitrary. Curvature-dominated and radiation-dominated phases precede the inflationary phase; the Brans-Dicke field remains constant during the preinflationary phases. For the case of $b \ll 1$ there is no intermediate radiation-dominated phase and hence no change of slope in the contracting portion of the $a(t)$ curve.

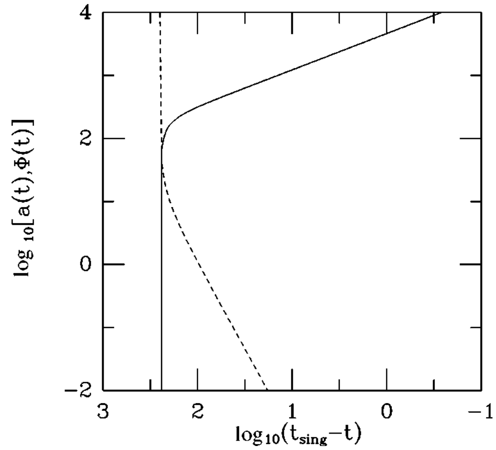

FIG. 2. Same as Fig. 1 for $k=1$ and $b=0$. An initial singularity precedes the inflationary phase (both the scale factor and the Brans-Dicke field are singular).

Examining Eq. (13), we see that $d\psi/d\eta=\dot{\psi}/\dot{\eta}$ depends only on ψ. Hence, ψ can be obtained as a function of η by straightforward integration. After this result is substituted into Eq. (12), a second integration yields $a(\eta)$. The solutions thus obtained are

$$a(\eta)=3^{-1/4}\sqrt{B}Q^{-1}[C(\eta)-bS(\eta)]^{(1+\sqrt{3})/2} \times[-S(\eta)]^{(1-\sqrt{3})/2},$$

$$\Phi(\eta)=Q^2\left[\frac{-S(\eta)}{C(\eta)-bS(\eta)}\right]^{\sqrt{3}}, \tag{15}$$

with Q an arbitrary constant. For $k=-1$, $C(\eta)$ and $S(\eta)$ denote $\cosh\eta$ and $\sinh\eta$, respectively, while for $k=1$ they denote $\cos\eta$ and $\sin\eta$.

For small negative values of η, both the $k=1$ and the $k=-1$ solutions approach the $k=b=0$ solution (8), with $\eta=0$ corresponding to $t_{\rm sing}$. At earlier times, however, the behavior of these solutions is quite different. For the moment, we concentrate on the case $b \lesssim O(1)$, for which radiation is never dominant.

The $k=-1$ solution (see Fig. 1) remains nonsingular for all $\eta<0$. At very early times (large negative values of η), a decreases linearly with time, while the dilaton field remains approximately constant at a value $\Phi(-\infty)\sim Q^2$. The scale factor reaches a minimum $a_{\rm min}$ when $-\eta$ is of order unity, and then begins to grow as the universe goes over into the dilaton-dominated inflationary epoch. By contrast, the $k=1$ solution (Fig. 2) has an initial singularity, with vanishing a and diverging Φ, a finite time $t_{\rm total}$ before the final singularity.

In either case, the solution begins to approximate the inflationary flat space solution when $-\eta$ is of order unity, implying that $\Phi(t_i)\equiv[\ell_{\rm Pl}(t_i)]^{-2}\sim Q^2$ and hence that

$$a(t_i)\sim a_{\rm min}\sim\sqrt{B}\ell_{\rm Pl}(t_i), \tag{16}$$

and [from integration of Eq. (14)]

$$t_{\rm sing}-t_i\sim\frac{1}{2}t_{\rm total}\sim\sqrt{B}\ell_{\rm Pl}(t_i). \tag{17}$$

Substituting these results into Eq. (10) and using Eq. (16), we find that

$$Z \lesssim {\rm Min}\left\{[\Phi(t_i)\ell_{\rm st}^2]^{1/\sqrt{3}},\left(\frac{B}{\Phi(t_i)\ell_{\rm st}^2}\right)^{(1+\sqrt{3})/(2\sqrt{3})}\right\} < B^{1/3}\sim\left(\frac{a(t_i)}{\ell_{\rm Pl}(t_i)}\right)^{2/3}. \tag{18}$$

For $b \gg 1$, the $k=-1$ solution (Fig. 1) is qualitatively rather similar, with Φ again remaining essentially constant until the onset of inflation, but with the scale factor decreasing as $(C-t)^{1/2}$. On the other hand, the $k=1$ solution (Fig. 3) is quite different, with a period of radiation-dominated expansion and contraction (at a roughly constant value of Φ) preceding the final inflationary expansion. In either case, dilaton-dominated inflation begins when $-\eta\sim 1/b$, with $\Phi(t_i)\sim Q^2/b^{\sqrt{3}}$. One finds that

$$a(t_i)\sim b^{-1/2}\sqrt{B}\ell_{\rm Pl}(t_i),$$

$$t_{\rm sing}-t_i\sim b^{-3/2}\sqrt{B}\ell_{\rm Pl}(t_i), \tag{19}$$

and (for $k=1$)

$$t_{\rm total}\sim a_{\rm max}\sim b^{1/2}\sqrt{B}\ell_{\rm Pl}(t_i), \tag{20}$$

while Eq. (18) is replaced by

$$Z \lesssim B^{1/3}b^{-1}\sim\left(\frac{a(t_i)}{b\ell_{\rm Pl}(t_i)}\right)^{2/3}\sim\left(\frac{t_{\rm sing}-t_i}{\ell_{\rm Pl}(t_i)}\right)^{2/3}. \tag{21}$$

C. Constraints on initial conditions

The $k=0$ solution, Eq. (8), can be extended indefinitely far back into the past without encountering either a singularity or a point where the underlying physical assumptions clearly break down (in contrast, say, to the radiation-

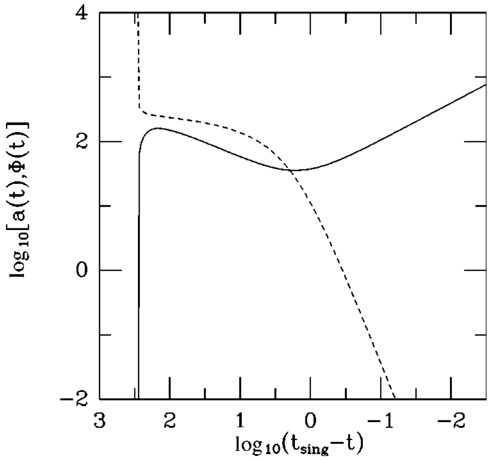

FIG. 3. Same as Fig. 2 for $b \gg 1$. In this case, the initial singularity is followed by a radiation-dominated phase during which the scale factor decreases and the Brans-Dicke field is approximately constant.

dominated FRW solutions, which clearly cannot be trusted once the temperature approaches the Planck scale). Because of this, one can to a certain degree sidestep the question of initial conditions.

This is no longer the case once there is spatial curvature. For $k=1$, where there is an initial singularity, this is obvious. For $k=-1$ there is neither an initial singularity nor a fixed time at which the solution must break down. However, as $t \to -\infty$ the physical size of the region corresponding to the presently observed universe diverges. Unless we want to have an infinitely large homogeneous region in the far past, we must assume the appearance (e.g., by some stringy or quantum gravitational mechanism) at some initial time t_0 of a smooth region that is sufficiently homogeneous and isotropic to be described by a Robertson-Walker metric. The subsequent development of this region will be determined by the initial values[2] a_0, Φ_0, and $\dot{\Phi}_0$. The bounds obtained in the previous subsections place constraints on these initial conditions.

We consider separately the cases where inflation does not begin until some time after t_0 and that where it begins immediately at t_0; these correspond to $a_0|\dot{\Phi}_0|/\Phi_0$ being less than or greater than $\max(1,b)$, respectively. In the former case, the relation between Z and B in Eqs. (18) and (21) implies that

$$Z \lesssim \frac{(-a_0^3 \dot{\Phi}_0)^{1/3}}{\max(1,b)}. \tag{22}$$

If, instead, inflation begins immediately at t_0, Eq. (9) for Z must be corrected to take into account the fact that the portion of the classical solution in the interval between t_i and t_0 does not correspond to actually realized inflation; the actual amount of inflation (i.e., from t_0 until t_f) is reduced by a factor of

$$\frac{a_0^2 \Phi_0}{a^2(t_i)\Phi(t_i)} \sim \frac{\max(1,b)}{|\dot{\Phi}_0|/a_0\Phi_0}. \tag{23}$$

The bound corresponding to Eq. (22) for this case is

$$Z \lesssim \left(\frac{\Phi_0^3}{\dot{\Phi}_0^2} \right)^{1/3}. \tag{24}$$

(The fact that a_0 does not explicitly enter the bounds in this case was to be expected, since in the inflationary epoch the curved space solutions approximate the flat space solution, for which there is an overall scale ambiguity in the definition of a.)

Furthermore by combining the relation between Z and $\Phi(t_i)$ in Eq. (10) with the fact that Φ is monotonically decreasing, one obtains the bound

$$Z \lesssim (\Phi_0 \ell_{\mathrm{st}}^2)^{1/\sqrt{3}}; \tag{25}$$

in fact, this bound can be strengthened somewhat for certain choices of parameters with $k=1$.

These bounds on the amount of inflation, Eqs. (22), (24), make clear the sensitivity of pre-big-bang inflation to initial conditions. In the case where inflation does not begin until sometime after $t=t_0$, either an increase in the initial curvature, i.e., decreasing a_0, or an increase in the amount of radiation, i.e., larger b, with all other quantities held fixed, can reduce Z to the point where it is insufficient to solve the horizon and flatness problems. In the other case, when inflation begins immediately at $t=t_0$, an increase in $\dot{\Phi}_0$ or a decrease in Φ_0 can defeat inflation.

Recently, Veneziano has studied another, not unrelated, aspect—the effect of initial inhomogeneity and anisotropy on pre-big-bang inflation [7]. Making the assumption of small initial curvature, he showed that small amounts of inhomogeneity and anisotropy do not prevent the ultimate transition to the pre-big-bang inflationary phase and concluded that pre-big-bang inflation is robust. The first statement is consistent with our results—we find that radiation and spatial curvature only *postpone* the inflationary phase—and extends these to small levels of anisotropy and inhomogeneity. However, our interpretation is less rosy than Veneziano's. Since the end of pre-big-bang inflation is fixed by other considerations, postponing the onset of the dilaton-dominated phase can severely limit the beneficial effects of pre-big-bang inflation. We speculate that anisotropy and inhomogeneity may also be able to defeat pre-big-bang inflation by postponing the onset of the dilaton-dominated phase.

III. THE VIEW FROM THE EINSTEIN FRAME

The theory defined by action of Eq. (1) can be recast in a number of different, but equivalent, forms by conformal rescalings of the metric. In particular, the conformal transformation $\tilde{g}_{\mu\nu}=(\Phi/m_{\mathrm{Pl}}{}^2)g_{\mu\nu}$ [and hence $\tilde{a}/a=d\tilde{t}/dt=\Phi^{1/2}/m_{\mathrm{Pl}}$] takes the Jordan frame description that we have used thus far into the Einstein frame description in which the gravitational part of the action takes the standard Einstein-Hilbert form.

The behavior in this frame is qualitatively quite different

[2]There is also a discrete choice for $\dot{a}_0$ corresponding to the sign ambiguity in Eqs. (12) and (13); we will assume that the value corresponding to the inflationary solution is chosen.

than that in the Jordan frame. For $k=-1$, $\tilde{a}$ (which, as noted earlier, is proportional to $\sqrt{\psi}$) decreases monotonically, while for $k=1$, $\tilde{a}$ vanishes at both the initial and final singularities, with only a single maximum in between, no matter what the value of b. The inflationary epoch itself, instead of being a period of accelerated expansion, is one of ever more rapid contraction, with $\tilde{a}(t)\propto(\tilde{t}_{\rm sing}-\tilde{t})^{1/3}$, so that $\dot{\tilde{a}}$ and $\ddot{\tilde{a}}$ are both negative. Of course, the answers to physical questions cannot be changed by a field redefinition, so if the horizon problem is solved in one frame it must be solved in the other [8]. Indeed, the ratio in Eq. (9) that we used to characterize the amount of inflation is the same in either frame:

$$\frac{H(t_f)a(t_f)}{H(t_i)a(t_i)}=\left(\frac{t_{\rm sing}-t_i}{t_{\rm sing}-t_f}\right)^{(1+\sqrt{3})/\sqrt{3}}=\left(\frac{\tilde{t}_{\rm sing}-\tilde{t}_i}{\tilde{t}_{\rm sing}-\tilde{t}_f}\right)^{2/3}$$
$$=\frac{\tilde{H}(t_f)\tilde{a}(t_f)}{\tilde{H}(t_i)\tilde{a}(t_i)}. \qquad (26)$$

Here, we have used the fact that during inflation $\tilde{H}\tilde{a}\propto 1/\tilde{a}^2$.

In the Einstein frame, the sensitivity of the amount of inflation to the amount of curvature (or radiation) can be understood as follows. The terms in the Friedmann equation corresponding to curvature, radiation energy density, and Brans-Dicke-field energy density vary as $1/\tilde{a}^2$, $1/\tilde{a}^4$, and $1/\tilde{a}^6$, respectively. Inflation only begins when $\tilde{a}$ has become small enough that the last of these terms is dominant. Hence, by increasing the curvature or the amount of radiation, the duration of inflation is made shorter.

Although the two frames are mathematically equivalent, they do suggest different levels of ''naturalness.'' The initial-condition constraints that we obtained in the previous section arise (in different forms) in both frames. However, in the Einstein frame the picture of inflation is far less compelling: A big, smooth region emerges at the end of inflation because an even bigger smooth region was present at the beginning of inflation. These considerations can be rephrased in terms of fundamental length scales. If one views the dilaton theory as being an effective theory based on an underlying string theory with a fundamental length scale $\ell_{\rm st}$, then the Jordan-frame picture, with inflation taking a small smooth region into a large smooth one, is perhaps more natural. However, nothing that we have done has relied on any underlying string physics; a similar scenario (up to the imple mentation of the graceful exit) could be obtained from any generalized Brans-Dicke theory.[3] In this latter case, there is no fundamental length. The only natural way to characterize distances as ''large'' or ''small'' is relative to the time-dependent Planck length. But this leads immediately to a difficulty. The bounds in Eqs. (18) and (21) tell us that significant inflation during the dilaton-dominated era is possible only in a universe whose characteristic cosmological scales — $a(t_i)$, $(t_{\rm sing}-t_i)$, $a_{\rm min}$ [for $k=-1$], and $t_{\rm total}$ [for $k=1$] — are all enormous when measured in units of its characteristic Planck length $\ell_{\rm Pl}(t_i)$. The flatness problem is just such a mismatch in scales. Thus, the price of curing one set of naturalness problems is the reintroduction of an earlier naturalness problem.

[3]Some of the difficulties associated with achieving sufficient inflation in such theories have been discussed in Ref. [9].

IV. CONCLUDING REMARKS

One measure of the naturalness of a cosmological scenario is its sensitivity to initial conditions. Indeed, the primary motivation for the inflationary paradigm was to solve the naturalness problems that arose because the standard cosmology appeared to require a finely tuned initial state. A characteristic of previous implementations of inflation is that they are rather insensitive to the initial curvature [3,10]. For example, in slow-roll inflation

$$\ln Z=\frac{8\pi}{m_{\rm Pl}{}^2}\int_{\phi_i}^{\phi_{\rm end}}\frac{V(\phi)d\phi}{V'(\phi)}, \qquad (27)$$

is determined by the shape of the inflationary potential, the initial value ϕ_i of the inflaton field, and the value $\phi_{\rm end}$ of the inflaton at which the slow-roll approximation breaks down; the only constraint on the initial curvature is that it not have such a large positive value that recollapse would occur before inflation could commence. Viewed in the context of inhomogeneous cosmological models, this means that all sufficiently large regions of space with either negative or not-too-large positive curvature will inflate [11,12].

In pre-big-bang inflation the end of the inflationary era is fixed, while its beginning is delayed by curvature. Too much curvature — of either sign — shortens the duration of the inflationary era to the point that the flatness and horizon problems are not solved. Thus, in the absence of a mechanism that would naturally cause a large region of space to materialize with tiny curvature, pre-big-bang inflation requires fine-tuning of initial conditions to solve these cosmological problems. This makes it less robust, and therefore less attractive as an implementation of the inflationary paradigm.

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