Rare top quark decays $t \to bW^+Z$ and $t \to cW^+W^-$

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The large value of the top quark mass implies that the rare top quark decays $t \to bW^+Z$, sW^+Z, and dW^+Z, and $t \to cW^+W^-$ and uW^+W^-, are kinematically allowed so long as $m_t \geq m_W + m_Z + m_{d_i} \approx 171.5$ GeV$+m_{d_i}$ or $m_t \geq 2m_W + m_{u,c} \approx 160.6$ GeV$+m_{u,c}$, respectively. The partial decay widths for these decay modes are calculated in the standard model. The partial widths depend sensitively on the precise value of the top quark mass. The branching ratio for $t \to bW^+Z$ is as much as 1×10^{-5} for $m_t = 200$ GeV, and could be observable at the CERN Large Hadron Collider. The rare decay modes $t \to cW^+W^-$ and uW^+W^- are highly Glashow-Iliopoulos-Maiani (GIM) suppressed, and thus provide a means for testing the GIM mechanism for three generations of quarks in the u, c, t sector. [S0556-2821(97)01113-2]

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Now that the top quark mass is known to be quite large, it is possible to examine the question of which rare decay modes of the top quark are kinematically allowed processes. The current average value of the top quark mass $m_t = 175 \pm 8$ GeV [1,2] from the Collider Detector at Fermilab (CDF) and D0 Collaborations implies that the decays $t \to bW^+Z$, sW^+Z, and dW^+Z are allowed decay modes of the top quark so long as $m_t \geq m_W + m_Z + m_{d_i} \approx 171.5$ GeV$+m_{d_i}$. The rare decays $t \to cW^+W^-$ and uW^+W^- also are allowed if $m_t \to 2m_W + m_{c,u} \approx 160.6$ GeV$+m_{c,u}$. For the present central value of the measured top quark mass, all of these processes are occurring at or near threshold, and are highly phase space suppressed. The decays $t \to cW^+W^-$ and uW^+W^- are also highly Glashow-Iliopoulos-Maiani (GIM) suppressed, and thus are not likely to be seen at standard model rates. The decays $t \to d_i W^+Z$, however, are not GIM suppressed and are potentially observable at the CERN Large Hadron Collider (LHC). The partial decay widths for these rare decay modes rapidly increase for larger values of the top quark mass, and thus are very sensitive to the precise value of the top quark mass. Since the decay widths are proportional to $|V_{td_i}|^2$, $i = 1,2,3$, the rare decay $t \to bW^+Z$ (with $|V_{tb}|^2 \approx 1$) will dominate unless the value of the top quark is below or nearly at threshold for this process.

We begin with the calculation of the partial decay width $\Gamma(t \to bW^+Z)$ in the standard model. The branching ratio for this decay process has been computed previously by Decker, Nowakowski, and Pilaftsis [3] and Mahlon and Parke [4].[1] The authors of Ref. [4] included the finite widths of the W and Z in their calculation, and found a significant enhancement in the decay width near threshold due to finite width effects. There is some disagreement in the numerical results of Refs. [3] and [4]. The numerical results presented here are basically consistent with the published results of Ref. [4] in the narrow width approximation. There is some numerical difference with Ref. [4] which probably stems from the inclusion of finite width effects in that calculation. In addition, explicit analytic formulas for the squared amplitude of $t \to bW^+Z$ are presented in this work. These formulas do not appear elsewhere in the literature, and are useful for more detailed studies of the decay mode. Finally, the decay widths for the other rare decay modes $t \to cW^+W^-$ and uW^+W^- also are computed. A search for these decay modes directly tests Cabbibo-Kobayashi-Maskawa (CKM) unitarity in the u-quark sector.

[1] The decay process $Q \to qWZ$ also was considered in Ref. [5] for very heavy fourth generation quarks and exotics with mass ≥ 240 GeV.

I. $t \to bW^+Z$

The rare decay $t \to bW^+Z$ proceeds via the three tree-level graphs drawn in Fig. 1. The amplitudes for these Feynman diagrams are

$$\mathcal{A}_1 = V_{tb}\left(\frac{ig}{\sqrt{2}}\right)\left(\frac{ig}{\cos\theta_W}\right)\epsilon_W^\mu \epsilon_Z^\nu \overline{u(p_b)}\Bigg[\gamma^\mu P_L\left(\frac{i}{\not k_1 - m_t}\right) \times \{g_{t_L}\gamma^\nu P_L + g_{t_R}\gamma^\nu P_R\}\Bigg] u(p_t), \tag{1}$$

$$\mathcal{A}_2 = V_{tb}\left(\frac{ig}{\sqrt{2}}\right)\left(\frac{ig}{\cos\theta_W}\right)\epsilon_W^\mu \epsilon_Z^\nu \overline{u(p_b)}\Bigg[\{g_{b_L}\gamma^\nu P_L + g_{b_R}\gamma^\nu P_R\} \times\left(\frac{i}{\not k_2 - m_b}\right)\gamma^\mu P_L\Bigg] u(p_t), \tag{2}$$

$$\mathcal{A}_3 = V_{tb}\left(\frac{ig}{\sqrt{2}}\right)(ig\cos\theta_W)\epsilon_W^\mu \epsilon_Z^\nu \left(\frac{-i}{k_3^2 - m_W^2}\right) \times\left(g^{\lambda\rho} - \frac{k_3^\lambda k_3^\rho}{m_W^2}\right)\overline{u(p_b)}\gamma^\lambda P_L u(p_t)[-g^{\mu\rho}(k_3+p_W)^\nu + g^{\nu\rho}(p_Z+k_3)^\mu + g^{\mu\nu}(-p_Z+p_W)^\rho], \tag{3}$$

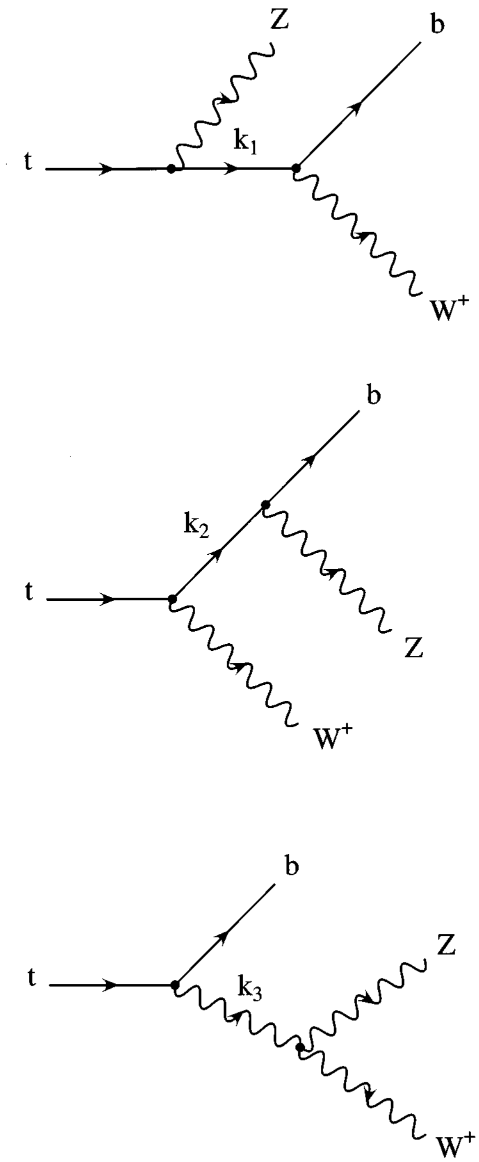

FIG. 1. $t\to bW^+Z$. Feynman diagrams correspond to the amplitudes $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$.

where the four-momenta k_1, k_2, and k_3 are given by

$$k_1=p_t-p_Z=p_b+p_W,$$

$$k_2=p_t-p_W=p_b+p_Z, \tag{4}$$

$$k_3=p_t-p_b=p_W+p_Z,$$

and the couplings of the Z boson to the left- and right-handed top and bottom quarks are

$$g_{t_L}=\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right),$$

$$g_{t_R}=\left(-\frac{2}{3}\sin^2\theta_W\right),$$

$$g_{b_L}=\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right), \tag{5}$$

$$g_{b_R}=\left(\frac{1}{3}\sin^2\theta_W\right).$$

In the above amplitudes, $P_{L,R}$ stand for the left- and right-handed projectors $P_{L,R}=(1\mp\gamma_5)/2$, and θ_W is the weak mixing angle. The amplitude $\mathcal{A}_3$ depends on the triple gauge vertex W^+W^-Z. This amplitude has been written in unitary gauge, where there is a contribution to the W gauge-boson propagator proportional to $k_3^\lambda k_3^\rho/m_W^2$. The amplitude also can be written in 't Hooft–Feynman gauge ($\xi=1$), where this contribution is replaced by the exchange of the would-be Goldstone boson of the W.

The total amplitude is given by $\mathcal{A}=\mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3$, and the amplitude squared is

$$|\mathcal{A}|^2=|\mathcal{A}_1|^2+|\mathcal{A}_2|^2+|\mathcal{A}_3|^2+2\mathcal{A}_1\mathcal{A}_2^*+2\mathcal{A}_1\mathcal{A}_3^*+2\mathcal{A}_2\mathcal{A}_3^*, \tag{6}$$

where the identities

$$\mathcal{A}_1\mathcal{A}_2^*=\mathcal{A}_2\mathcal{A}_1^*,$$

$$\mathcal{A}_1\mathcal{A}_3^*=\mathcal{A}_3\mathcal{A}_1^*, \tag{7}$$

$$\mathcal{A}_2\mathcal{A}_3^*=\mathcal{A}_3\mathcal{A}_2^*,$$

have been used.

The square amplitude $|\mathcal{A}_1|^2$ is

$$|\mathcal{A}_1|^2=|V_{tb}|^2\left(\frac{g^4}{2\cos^2\theta_W}\right)\left(\frac{1}{k_1^2-m_t^2}\right)^2\Bigg(4g_{t_L}^2\Bigg\{\left[(k_1\cdot p_t)(k_1\cdot p_b)-\frac{1}{2}k_1^2(p_t\cdot p_b)\right]+\frac{2}{m_Z^2}(p_t\cdot p_Z)\Bigg[(k_1\cdot p_b)(k_1\cdot p_Z)$$
$$-\frac{1}{2}k_1^2(p_b\cdot p_Z)\Bigg]+\frac{2}{m_W^2}(p_b\cdot p_W)\left[(k_1\cdot p_t)(k_1\cdot p_W)-\frac{1}{2}k_1^2(p_t\cdot p_W)\right]+\frac{4}{m_W^2m_Z^2}(p_t\cdot p_Z)(p_b\cdot p_W)\Bigg[(k_1\cdot p_Z)(k_1\cdot p_W)$$
$$-\frac{1}{2}k_1^2(p_W\cdot p_Z)\Bigg]\Bigg\}-12m_t^2g_{t_L}g_{t_R}\left\{(k_1\cdot p_b)+\frac{2}{m_W^2}(k_1\cdot p_W)(p_b\cdot p_W)\right\}+2m_t^2g_{t_R}^2\Bigg\{(p_t\cdot p_b)+\frac{2}{m_Z^2}(p_t\cdot p_Z)(p_b\cdot p_Z)$$
$$+\frac{2}{m_W^2}(p_t\cdot p_W)(p_b\cdot p_W)+\frac{4}{m_W^2m_Z^2}(p_t\cdot p_Z)(p_b\cdot p_W)(p_W\cdot p_Z)\Bigg\}\Bigg). \tag{8}$$

The square amplitude $|\mathcal{A}_2|^2$ is related to $|\mathcal{A}_1|^2$ by the interchanges $g_{t_{L,R}}\leftrightarrow g_{b_{L,R}}$, $m_t\leftrightarrow m_b$, $p_t\leftrightarrow p_b$, and $k_1\leftrightarrow k_2$:

$$|\mathcal{A}_2|^2=|V_{tb}|^2\left(\frac{g^4}{2\cos^2\theta_W}\right)\left(\frac{1}{k_2^2-m_b^2}\right)^2\Bigg(4g_{b_L}^2\Bigg\{\left[(k_2\cdot p_b)(k_2\cdot p_t)-\frac{1}{2}k_2^2(p_t\cdot p_b)\right]+\frac{2}{m_Z^2}(p_b\cdot p_Z)\left[(k_2\cdot p_t)(k_2\cdot p_Z)-\frac{1}{2}k_2^2(p_t\cdot p_Z)\right]$$
$$+\frac{2}{m_W^2}(p_t\cdot p_W)\left[(k_2\cdot p_b)(k_2\cdot p_W)-\frac{1}{2}k_2^2(p_b\cdot p_W)\right]+\frac{4}{m_W^2m_Z^2}(p_b\cdot p_Z)(p_t\cdot p_W)\left[(k_2\cdot p_Z)(k_2\cdot p_W)-\frac{1}{2}k_2^2(p_W\cdot p_Z)\right]\Bigg\}$$
$$-12m_b^2g_{b_L}g_{b_R}\left\{(k_2\cdot p_t)+\frac{2}{m_W^2}(k_2\cdot p_W)(p_t\cdot p_W)\right\}+2m_b^2g_{b_R}^2\Bigg\{(p_t\cdot p_b)+\frac{2}{m_Z^2}(p_b\cdot p_Z)(p_t\cdot p_Z)+\frac{2}{m_W^2}(p_b\cdot p_W)(p_t\cdot p_W)$$
$$+\frac{4}{m_W^2m_Z^2}(p_b\cdot p_Z)(p_t\cdot p_W)(p_W\cdot p_Z)\Bigg\}\Bigg). \tag{9}$$

The square amplitude $|\mathcal{A}_3|^2$ is

$$|\mathcal{A}_3|^2=|V_{tb}|^2\left(\frac{g^4\cos^2\theta_W}{2}\right)\left(\frac{1}{k_3^2-m_W^2}\right)^2\Bigg(4(p_t\cdot p_b)\Bigg\{-3(m_W^2+m_Z^2)+2\gamma(m_W^2-m_Z^2)-\gamma^2(m_W^2+m_Z^2)+2(1-\gamma^2)(p_W\cdot p_Z)$$
$$+\left[\frac{3}{2}\left(\frac{1}{m_W^2}+\frac{1}{m_Z^2}\right)+\gamma\left(\frac{1}{m_Z^2}-\frac{1}{m_W^2}\right)-\frac{1}{2}\gamma^2\left(\frac{1}{m_W^2}+\frac{1}{m_Z^2}\right)\right](p_W\cdot p_Z)^2+\frac{1}{m_W^2m_Z^2}(1-\gamma^2)(p_W\cdot p_Z)^3\Bigg\}+4(p_t\cdot p_W)(p_b\cdot p_W)$$
$$\times\left\{-2-4\gamma+2\gamma^2+\frac{4}{m_W^2}(1+\gamma)(p_W\cdot p_Z)+\frac{1}{m_W^2m_Z^2}(1+2\gamma+\gamma^2)(p_W\cdot p_Z)^2\right\}+4(p_t\cdot p_Z)(p_b\cdot p_Z)$$
$$\times\left\{-2+4\gamma+2\gamma^2+\frac{4}{m_Z^2}(1-\gamma)(p_W\cdot p_Z)+\frac{1}{m_W^2m_Z^2}(1-2\gamma+\gamma^2)(p_W\cdot p_Z)^2\right\}+4[(p_t\cdot p_W)(p_b\cdot p_Z)+(p_b\cdot p_W)(p_t\cdot p_Z)]$$
$$\times\left\{-6+2\gamma^2+\left[2\left(\frac{1}{m_Z^2}+\frac{1}{m_W^2}\right)-2\gamma\left(\frac{1}{m_Z^2}-\frac{1}{m_W^2}\right)\right](p_W\cdot p_Z)-\frac{1}{m_W^2m_Z^2}(1-\gamma^2)(p_W\cdot p_Z)^2\right\}\Bigg), \tag{10}$$

where $\gamma=(1-m_Z^2/m_W^2)$ in unitary gauge. In 't Hooft–Feynman gauge, one obtains the same expression with $\gamma=\sin^2\theta_W$ and $\cos^2\theta_W$ replaced by $(m_Z/m_W)^2$. The terms in $|\mathcal{A}_3|^2$ proportional to γ are antisymmetric under $p_t\leftrightarrow p_b$, $p_W\leftrightarrow p_Z$ and $m_W\leftrightarrow m_Z$ while the terms which are independent of γ or proportional to γ^2 are invariant under this interchange.

The interference term $\mathcal{A}_1\mathcal{A}_2^*$ is

$$\mathcal{A}_1\mathcal{A}_2^* = |V_{tb}|^2\left(\frac{g^4}{2\cos^2\theta_W}\right)\left(\frac{1}{k_1^2-m_t^2}\right)\left(\frac{1}{k_2^2-m_b^2}\right)\Bigg(g_{t_L}g_{b_L}\Bigg\{-6(k_1\cdot k_2)(p_t\cdot p_b)-2(k_1\cdot p_t)(k_2\cdot p_b)-2(k_1\cdot p_b)(k_2\cdot p_t)$$
$$-\frac{4}{m_Z^2}[(k_1\cdot k_2)(p_b\cdot p_Z)(p_t\cdot p_Z)+(k_1\cdot p_Z)(k_2\cdot p_Z)(p_t\cdot p_b)+(k_2\cdot p_Z)(k_1\cdot p_t)(p_b\cdot p_Z)+(k_1\cdot p_Z)(p_t\cdot p_Z)(k_2\cdot p_b)$$
$$-2(p_t\cdot p_Z)(k_1\cdot p_b)(k_2\cdot p_Z)-2(k_1\cdot p_Z)(p_b\cdot p_Z)(k_2\cdot p_t)]-\frac{4}{m_W^2}[(k_1\cdot k_2)(p_b\cdot p_W)(p_t\cdot p_W)+(k_1\cdot p_W)(k_2\cdot p_W)(p_t\cdot p_b)$$
$$+(k_2\cdot p_W)(k_1\cdot p_b)(p_t\cdot p_W)+(k_1\cdot p_W)(p_b\cdot p_W)(k_2\cdot p_t)-2(p_b\cdot p_W)(k_1\cdot p_t)(k_2\cdot p_W)-2(k_1\cdot p_W)(p_t\cdot p_W)(k_2\cdot p_b)]$$
$$+\frac{4}{m_W^2m_Z^2}[(k_1\cdot k_2)(p_W\cdot p_Z)\{-(p_t\cdot p_b)(p_W\cdot p_Z)+(p_b\cdot p_Z)(p_t\cdot p_W)+(p_b\cdot p_W)(p_t\cdot p_Z)\}+(p_W\cdot p_Z)^2\{(k_1\cdot p_t)(k_2\cdot p_b)$$
$$+(k_1\cdot p_b)(k_2\cdot p_t)\}+(p_t\cdot p_b)(p_W\cdot p_Z)\{(k_1\cdot p_Z)(k_2\cdot p_W)+(k_1\cdot p_W)(k_2\cdot p_Z)\}+2(k_1\cdot p_Z)(k_2\cdot p_Z)(p_b\cdot p_W)(p_t\cdot p_W)$$
$$+2(k_1\cdot p_W)(k_2\cdot p_W)(p_b\cdot p_Z)(p_t\cdot p_Z)-(k_1\cdot p_b)(k_2\cdot p_Z)(p_t\cdot p_W)(p_W\cdot p_Z)-(k_2\cdot p_t)(k_1\cdot p_Z)(p_b\cdot p_W)(p_W\cdot p_Z)$$
$$-(k_1\cdot p_t)(k_2\cdot p_Z)(p_b\cdot p_W)(p_W\cdot p_Z)-(k_2\cdot p_b)(k_1\cdot p_Z)(p_t\cdot p_W)(p_W\cdot p_Z)-(k_1\cdot p_t)(k_2\cdot p_W)(p_b\cdot p_Z)(p_W\cdot p_Z)$$
$$-(k_2\cdot p_b)(k_1\cdot p_W)(p_t\cdot p_Z)(p_W\cdot p_Z)-(k_1\cdot p_b)(k_2\cdot p_W)(p_W\cdot p_Z)(p_t\cdot p_Z)-(k_2\cdot p_t)(k_1\cdot p_W)(p_b\cdot p_Z)(p_W\cdot p_Z)]\Bigg\}$$
$$+2m_b^2g_{t_L}g_{b_R}\Bigg\{3(k_1\cdot p_t)+\frac{2}{m_Z^2}(k_1\cdot p_Z)(p_t\cdot p_Z)-\frac{4}{m_W^2}(p_t\cdot p_W)(k_1\cdot p_W)+\frac{2}{m_W^2m_Z^2}(p_W\cdot p_Z)[(k_1\cdot p_W)(p_t\cdot p_Z)$$
$$+(p_t\cdot p_W)(k_1\cdot p_Z)-(p_W\cdot p_Z)(k_1\cdot p_t)]\Bigg\}+2m_t^2g_{t_R}g_{b_L}\Bigg\{3(k_2\cdot p_b)+\frac{2}{m_Z^2}(k_2\cdot p_Z)(p_b\cdot p_Z)-\frac{4}{m_W^2}(p_b\cdot p_W)(k_2\cdot p_W)$$
$$+\frac{2}{m_W^2m_Z^2}(p_W\cdot p_Z)[(k_2\cdot p_W)(p_b\cdot p_Z)+(p_b\cdot p_W)(k_2\cdot p_Z)-(p_W\cdot p_Z)(k_2\cdot p_b)]\Bigg\}$$
$$+2m_t^2m_b^2g_{t_R}g_{b_R}\Bigg\{-5+\frac{2}{m_W^2m_Z^2}(p_W\cdot p_Z)^2\Bigg\}\Bigg). \tag{11}$$

$\mathcal{A}_1\mathcal{A}_2^*$ is invariant under the simultaneous interchanges $g_{t_{L,R}}\leftrightarrow g_{b_{L,R}}$, $m_t\leftrightarrow m_b$, $p_t\leftrightarrow p_b$, and $k_1\leftrightarrow k_2$.

The two interference terms $\mathcal{A}_1\mathcal{A}_3^*$ and $\mathcal{A}_2\mathcal{A}_3^*$ are

$$\mathcal{A}_1\mathcal{A}_3^* = |V_{tb}|^2\left(\frac{g^4}{2}\right)\left(\frac{1}{k_1^2-m_t^2}\right)\left(\frac{1}{k_3^2-m_W^2}\right)\Bigg(g_{t_L}\Bigg\{-4[(2-\gamma)(k_1\cdot p_W)(p_t\cdot p_b)+(k_1\cdot p_b)(p_t\cdot p_W)-3(k_1\cdot p_t)(p_b\cdot p_W)]$$
$$+4[(2+\gamma)(k_1\cdot p_Z)(p_t\cdot p_b)+(k_1\cdot p_t)(p_b\cdot p_Z)-3(k_1\cdot p_b)(p_t\cdot p_Z)]-\frac{4}{m_W^2}(1+\gamma)(k_1\cdot p_W)[2(p_t\cdot p_W)(p_b\cdot p_W)$$
$$+(p_t\cdot p_W)(p_b\cdot p_Z)-(p_W\cdot p_Z)(p_t\cdot p_b)+(p_b\cdot p_W)(p_t\cdot p_Z)]+\frac{4}{m_Z^2}(1-\gamma)(k_1\cdot p_Z)[2(p_t\cdot p_Z)(p_b\cdot p_Z)$$
$$+(p_t\cdot p_Z)(p_b\cdot p_W)-(p_W\cdot p_Z)(p_t\cdot p_b)+(p_b\cdot p_Z)(p_t\cdot p_W)]-\frac{2}{m_W^2}(3+\gamma)(p_W\cdot p_Z)[(k_1\cdot p_W)(p_t\cdot p_b)$$
$$-(p_t\cdot p_W)(k_1\cdot p_b)+(p_b\cdot p_W)(k_1\cdot p_t)]+\frac{2}{m_Z^2}(3-\gamma)(p_W\cdot p_Z)[(k_1\cdot p_Z)(p_t\cdot p_b)-(p_b\cdot p_Z)(k_1\cdot p_t)$$
$$+(p_t\cdot p_Z)(k_1\cdot p_b)]+\frac{4}{m_W^2m_Z^2}(1+\gamma)(p_W\cdot p_Z)(p_b\cdot p_W)[(k_1\cdot p_W)(p_t\cdot p_Z)-(p_W\cdot p_Z)(k_1\cdot p_t)+(p_t\cdot p_W)(k_1\cdot p_Z)]$$
$$-\frac{4}{m_W^2m_Z^2}(1-\gamma)(p_W\cdot p_Z)(p_t\cdot p_Z)[(k_1\cdot p_W)(p_b\cdot p_Z)-(p_W\cdot p_Z)(k_1\cdot p_b)+(p_b\cdot p_W)(k_1\cdot p_Z)]\Bigg\}$$
$$+4m_t^2g_{t_R}\Bigg\{(-4+\gamma)(p_b\cdot p_W)+(-2+\gamma)(p_b\cdot p_Z)+\frac{1}{m_W^2}\left(\frac{7}{2}+\frac{1}{2}\gamma\right)(p_W\cdot p_Z)(p_b\cdot p_W)$$

$$+\frac{1}{m_Z^2}\left(\frac{3}{2}-\frac{1}{2}\gamma\right)(p_W\cdot p_Z)(p_b\cdot p_Z)+\frac{1}{m_W^2m_Z^2}(1+\gamma)(p_W\cdot p_Z)^2(p_b\cdot p_W)\Bigg\}\Bigg) \tag{12}$$

and

$$\mathcal{A}_2\mathcal{A}_3^*=|V_{tb}|^2\left(\frac{g^4}{2}\right)\left(\frac{1}{k_2^2-m_b^2}\right)\left(\frac{1}{k_3^2-m_W^2}\right)\Bigg(g_{b_L}\Bigg\{-4[(2-\gamma)(k_2\cdot p_W)(p_t\cdot p_b)+(k_2\cdot p_t)(p_b\cdot p_W)-3(k_2\cdot p_b)(p_t\cdot p_W)]$$
$$+4[(2+\gamma)(k_2\cdot p_Z)(p_t\cdot p_b)+(k_2\cdot p_b)(p_t\cdot p_Z)-3(k_2\cdot p_t)(p_b\cdot p_Z)]-\frac{4}{m_W^2}(1+\gamma)(k_2\cdot p_W)[2(p_b\cdot p_W)(p_t\cdot p_W)$$
$$+(p_b\cdot p_W)(p_t\cdot p_Z)-(p_W\cdot p_Z)(p_t\cdot p_b)+(p_t\cdot p_W)(p_b\cdot p_Z)]+\frac{4}{m_Z^2}(1-\gamma)(k_2\cdot p_Z)[2(p_b\cdot p_Z)(p_t\cdot p_Z)$$
$$+(p_b\cdot p_Z)(p_t\cdot p_W)-(p_W\cdot p_Z)(p_t\cdot p_b)+(p_t\cdot p_Z)(p_b\cdot p_W)]-\frac{2}{m_W^2}(3+\gamma)(p_W\cdot p_Z)[(k_2\cdot p_W)(p_t\cdot p_b)$$
$$-(p_b\cdot p_W)(k_2\cdot p_t)+(p_t\cdot p_W)(k_2\cdot p_b)]+\frac{2}{m_Z^2}(3-\gamma)(p_W\cdot p_Z)[(k_2\cdot p_Z)(p_t\cdot p_b)-(p_t\cdot p_Z)(k_2\cdot p_b)$$
$$+(p_b\cdot p_Z)(k_2\cdot p_t)]+\frac{4}{m_W^2m_Z^2}(1+\gamma)(p_W\cdot p_Z)(p_t\cdot p_W)[(k_2\cdot p_W)(p_b\cdot p_Z)-(p_W\cdot p_Z)(k_2\cdot p_b)+(p_b\cdot p_W)(k_2\cdot p_Z)]$$
$$-\frac{4}{m_W^2m_Z^2}(1-\gamma)(p_W\cdot p_Z)(p_b\cdot p_Z)[(k_2\cdot p_W)(p_t\cdot p_Z)-(p_W\cdot p_Z)(k_2\cdot p_t)+(p_t\cdot p_W)(k_2\cdot p_Z)]\Bigg\}$$
$$+4m_b^2g_{b_R}\Bigg\{(-4+\gamma)(p_t\cdot p_W)+(-2+\gamma)(p_t\cdot p_Z)+\frac{1}{m_W^2}\left(\frac{7}{2}+\frac{1}{2}\gamma\right)(p_W\cdot p_Z)(p_t\cdot p_W)$$
$$+\frac{1}{m_Z^2}\left(\frac{3}{2}-\frac{1}{2}\gamma\right)(p_W\cdot p_Z)(p_t\cdot p_Z)+\frac{1}{m_W^2m_Z^2}(1+\gamma)(p_W\cdot p_Z)^2(p_t\cdot p_W)\Bigg\}\Bigg), \tag{13}$$

where $\gamma=1-m_Z^2/m_W^2$ or $\sin^2\theta_W$. The interference terms $\mathcal{A}_2\mathcal{A}_3^*$ and $\mathcal{A}_1\mathcal{A}_3^*$ are related by the interchanges $g_{t_{L,R}}\leftrightarrow g_{b_{L,R}}$, $m_t\leftrightarrow m_b$, $p_t\leftrightarrow p_b$, and $k_1\leftrightarrow k_2$.

The above square amplitudes have been written in terms of k_1 and k_2, and p_t and p_b in order to exhibit the symmetries of the square amplitudes explicitly. The total square amplitude can be rewritten in terms of the three dot products $(p_b\cdot p_W)$, $(p_b\cdot p_Z)$, and $(p_W\cdot p_Z)$, by eliminating p_t, k_1, and k_2 in the above formulas.

The partial width for the decay mode $t\to bW^+Z$ is given by the three-body phase space integral

$$\Gamma(t\to bW^+Z)=\frac{1}{(2\pi)^3}\frac{1}{32m_t^3}\int dm_{23}^2dm_{12}^2\overline{|\mathcal{A}|^2}, \tag{14}$$

where the invariant square masses $m_{ij}^2=(p_i+p_j)^2$ are defined in terms of the momenta of the final particles, and the spin-averaged square amplitude

$$\overline{|\mathcal{A}|^2}=\frac{1}{2}|\mathcal{A}|^2, \tag{15}$$

since one averages rather than sums over the top quark spin.

The partial decay width $\Gamma(t\to bW^+Z)$ is plotted in Fig. 2 as a function of the top quark mass. The phase space integral

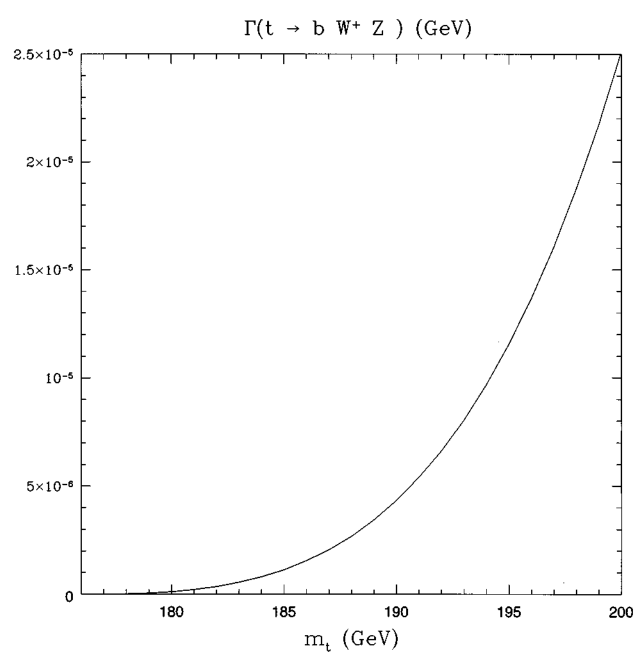

FIG. 2. $\Gamma(t\to bW^+Z)$ as a function of the top quark mass for $m_W=80.3$ GeV, $m_Z=91.2$ GeV, $m_b=4.5$ GeV, $\sin^2\theta_W=0.23$, and $|V_{tb}|^2=1$. The partial decay width vanishes at threshold, where $m_t=m_b+m_W+m_Z$.

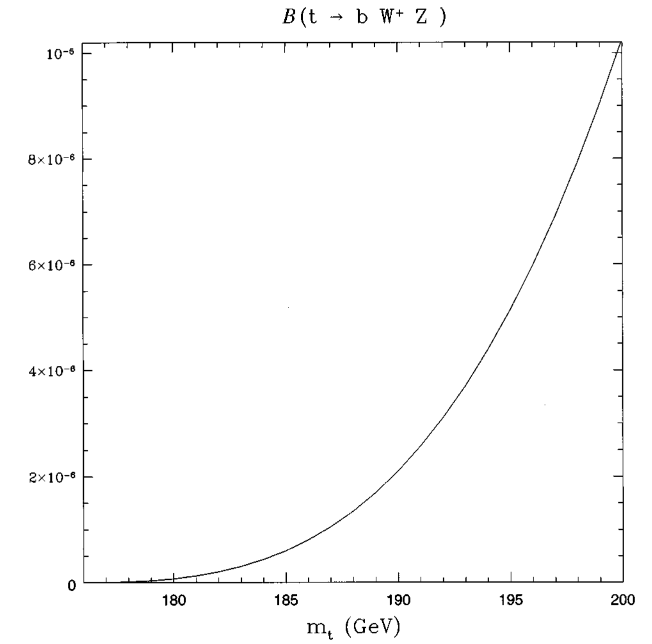

FIG. 3. $B(t\to bW^+Z)$ as a function of the top quark mass for $m_W=80.3$ GeV, $m_Z=91.2$ GeV, $m_b=4.5$ GeV, and $\sin^2\theta_W=0.23$. The branching ratio vanishes at threshold, where $m_t=m_b+m_W+m_Z$.

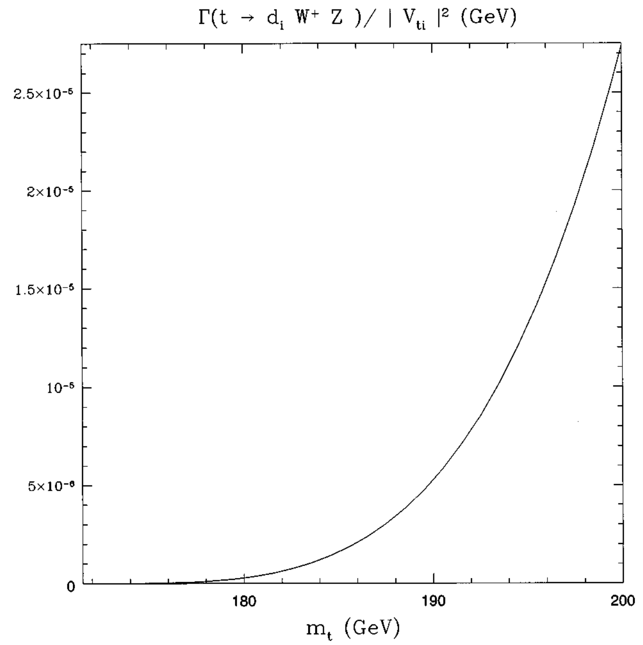

FIG. 4. $\Gamma(t\to d_iW^+Z)/|V_{ti}|^2$ as a function of the top quark mass for $m_W=80.3$ GeV, $m_Z=91.2$ GeV, $\sin^2\theta_W=0.23$, and $m_{d_i}=0$. The partial decay width vanishes at threshold, where $m_t=m_W+m_Z$. This graph is relevant for the decays $t\to dW^+Z$ and $t\to sW^+Z$.

was performed numerically for the parameter values $m_W=80.3$ GeV, $m_Z=91.2$ GeV, $m_b=4.5$ GeV, $\sin^2\theta_W=0.23$, and $|V_{tb}|=1$. The partial width is plotted over the range from $m_t=176$ GeV, where the partial width vanishes, to $m_t=200$ GeV, where the partial decay width is 2.5×10^{-5} GeV. The branching ratio $B(t\to bW^+Z)$ also is plotted as a function of the top quark mass in Fig. 3, assuming that the total width of the top quark is dominated by $t\to bW^+$:

$$\Gamma(t\to bW^+)=|V_{tb}|^2\frac{g^2}{64\pi}\frac{1}{m_W^2m_t^3}\lambda^{1/2}(m_t^2,m_W^2,m_b^2)\times\{m_t^4+m_b^4-2m_W^4+m_t^2m_W^2+m_b^2m_W^2-2m_t^2m_b^2\}, \tag{16}$$

where

$$\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2xz. \tag{17}$$

The CKM matrix element $|V_{tb}|^2$ cancels out of the branching ratio. The branching ratio increases from zero for $m_t=176$ GeV to 1.0×10^{-5} for $m_t=200$ GeV.[2] Although this branching ratio is too small to be observed at the Tevatron, it is large enough to be interesting for the LHC which is expected to yield about a million fully reconstructed top quark events per year [6]. The observability of this decay mode depends on the precise value of the top quark mass. The branching ratio is greater than 10^{-6} for $m_t\gtrsim187$ GeV. The extreme sensitivity of the branching ratio to the top quark mass implies that the decay mode could be used to extract or bound the top quark mass. The decay mode also is sensitive to the presence of the triple gauge vertex W^+W^-Z with the standard model coupling.

The rare decay $t\to bW^+Z$ is at threshold for the present central value of the top quark mass, so the decays $t\to sW^+Z$ and $t\to dW^+Z$ could be more important if $t\to bW^+Z$ is kinematically forbidden or just allowed. Alternatively, it might be possible to look at these modes by applying a tight cut on the invariant mass of the W^+ and Z momenta to exclude $t\to bW^+Z$ but not $t\to sW^+Z$ and dW^+Z. The partial decay widths for the s and d final states can be obtained from the partial decay width for the b mode by replacing m_b by m_s or m_d, and $|V_{tb}|^2$ by $|V_{ts}|^2$ or $|V_{td}|^2$. The partial decay width $\Gamma(t\to d_iW^+Z)/|V_{ti}|^2$ is plot-

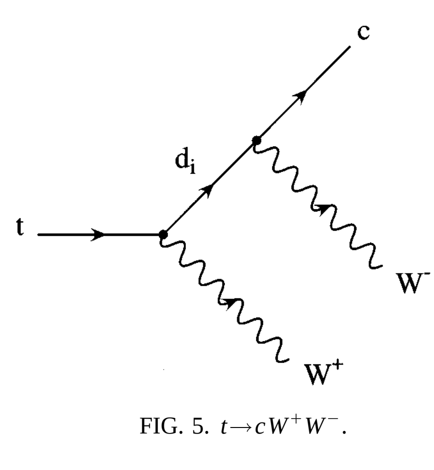

FIG. 5. $t\to cW^+W^-$.

[2]The value of the branching ratio for this value of m_t is consistent with the result of Mahlon and Parke [4] in the narrow width approximation. There is some numerical difference with their narrow width results for smaller values of m_t, which probably stems from the inclusion of finite width effects proportional to Γ_W/m_W and Γ_Z/m_Z in their calculation.

ted as a function of the top quark mass for vanishing m_{d_i} in Fig. 4. The partial width divided by the CKM matrix element squared is zero at threshold where $m_t=m_W+m_Z$ and increases to 2.7×10^{-5} GeV at $m_t=200$ GeV. For canonical values of $|V_{ts}|^2$ and $|V_{td}|^2$, these partial widths will be too small to be observed at LHC.

II. $t\to cW^+W^-$

The rare decay $t\to cW^+W^-$ proceeds through tree-level graphs with intermediate d, s, and b quarks, as depicted in Fig. 5. The amplitude for the decay is

$$\mathcal{A}=\sum_{j=d,s,b} V_{tj}V_{cj}^*\left(\frac{ig}{\sqrt{2}}\right)^2 \epsilon^{\mu}_{W^-}\epsilon^{\nu}_{W^+}\overline{u(p_c)} \times\left[\gamma^{\mu}P_L\left(\frac{i}{\not k-m_j}\right)\gamma^{\nu}P_L\right]u(p_t), \quad (18)$$

where $k=p_c+p_{W^-}=p_t-p_{W^+}$. The spin-averaged amplitude squared is given by

$$\overline{|\mathcal{A}|}^2=\frac{1}{2}\sum_{j,k} V_{tj}V_{cj}^*V_{tk}^*V_{ck}\left(\frac{g}{\sqrt{2}}\right)^4\left(\frac{1}{k^2-m_j^2}\right)\left(\frac{1}{k^2-m_k^2}\right)4\Bigg\{\left[(k\cdot p_t)(k\cdot p_c)-\frac{1}{2}k^2(p_c\cdot p_t)\right]$$
$$+\frac{2}{m_W^2}\Bigg((p_t\cdot p_{W^+})\left[(k\cdot p_c)(k\cdot p_{W^+})-\frac{1}{2}k^2(p_c\cdot p_{W^+})\right]+(p_c\cdot p_{W^-})\left[(k\cdot p_t)(k\cdot p_{W^-})-\frac{1}{2}k^2(p_t\cdot p_{W^-})\right]\Bigg)$$
$$+\frac{4}{m_W^4}(p_t\cdot p_{W^+})(p_c\cdot p_{W^-})\left[(k\cdot p_{W^+})(k\cdot p_{W^-})-\frac{1}{2}k^2(p_{W^+}\cdot p_{W^-})\right]\Bigg\}, \quad (19)$$

where the factor of 1/2 comes from averaging over the top quark spin. Note that this square amplitude can be derived from Eq. (8) or (9). The amplitude squared can be rewritten in terms of the three dot products $(p_c\cdot p_{W^+})$, $(p_c\cdot p_{W^-})$, and $(p_{W^+}\cdot p_{W^-})$ of the final particle momenta by eliminating p_t and k in the above formula.

The partial width for the decay mode $t\to cW^+W^-$ is given by the three-body phase space integral

$$\Gamma(t\to cW^+W^-)=\frac{1}{(2\pi)^3}\frac{1}{32m_t^3}\int dm_{23}^2dm_{12}^2\overline{|\mathcal{A}|}^2, \quad (20)$$

where the invariant square masses $m_{ij}^2=(p_i+p_j)^2$ are defined in terms of the momenta of the final particles. It is possible to perform the $m_{W^+W^-}^2$ integration explicitly, so that the partial width is given by an integral over $x=m_{cW^-}^2=(p_c+p_{W^-})^2=(p_t-p_{W^+})^2=k^2$:

$$\Gamma(t\to cW^+W^-)=\frac{1}{(2\pi)^3}\frac{1}{32m_t^3}\frac{1}{2}\left(\frac{g}{\sqrt{2}}\right)^4\sum_{j,k} V_{tj}V_{cj}^*V_{tk}^*V_{ck}\int_{(m_c+m_W)^2}^{(m_t-m_W)^2}dx\left(\frac{1}{x-m_j^2}\right)\left(\frac{1}{x-m_k^2}\right)$$
$$\times\frac{1}{2x}\lambda^{1/2}(x,m_c^2,m_W^2)\lambda^{1/2}(x,m_t^2,m_W^2)\Bigg\{(x+m_t^2-m_W^2)(x+m_c^2-m_W^2)$$
$$+\frac{1}{m_W^2}[(x-m_t^2-m_W^2)(x+m_c^2-m_W^2)(x-m_t^2+m_W^2)+(x-m_c^2-m_W^2)(x+m_t^2-m_W^2)(x-m_c^2+m_W^2)]$$
$$+\frac{1}{m_W^4}[(x-m_t^2+m_W^2)(x-m_t^2-m_W^2)(x-m_c^2+m_W^2)(x-m_c^2-m_W^2)]\Bigg\}. \quad (21)$$

The integrand of Eq. (21) is symmetric under the interchange $m_t^2\leftrightarrow m_c^2$, but this symmetry is broken by the limits of integration of the remaining phase space integral. An important observation about the decay width is that the width vanishes for $m_j^2=0$ or $m_k^2=0$ by CKM unitarity:

$$\sum_{j=d,s,b} V_{tj}V_{cj}^*=0. \quad (22)$$

This GIM suppression can be made manifest by replacing the two d-quark propagators by

$$\left(\frac{1}{x-m_j^2}\right)\to\left[\left(\frac{1}{x-m_j^2}\right)-\frac{1}{x}\right]=\frac{m_j^2}{x(x-m_j^2)}, \tag{23}$$

which implies that the integrand is multiplied by

$$\frac{m_j^2 m_k^2}{x^2}. \tag{24}$$

Thus, the final formula for the partial width is

$$\Gamma(t\to cW^+W^-)=\frac{1}{(2\pi)^3}\frac{1}{32m_t^3}\frac{1}{2}\left(\frac{g}{\sqrt{2}}\right)^4\sum_{j,k} V_{tj}V_{cj}^*V_{tk}^*V_{ck}\ I(m_j^2,m_k^2,m_c^2,m_t^2,m_W^2), \tag{25}$$

where the integral equals

$$\begin{aligned} I(m_j^2,m_k^2,m_c^2,m_t^2,m_W^2)=m_j^2m_k^2\int_{(m_c+m_W)^2}^{(m_t-m_W)^2}dx\left(\frac{1}{x-m_j^2}\right)\left(\frac{1}{x-m_k^2}\right)\frac{1}{2x^3}\lambda^{1/2}(x,m_c^2,m_W^2)\lambda^{1/2}(x,m_t^2,m_W^2)\\ \times\Big\{(x+m_t^2-m_W^2)(x+m_c^2-m_W^2)+\frac{1}{m_W^2}[(x-m_t^2-m_W^2)(x+m_c^2-m_W^2)(x-m_t^2+m_W^2)\\ +(x-m_c^2-m_W^2)(x+m_t^2-m_W^2)(x-m_c^2+m_W^2)]+\frac{1}{m_W^4}[(x-m_t^2+m_W^2)(x-m_t^2-m_W^2)\\ \times(x-m_c^2+m_W^2)(x-m_c^2-m_W^2)]\Big\}. \end{aligned} \tag{26}$$

Numerical integration of Eq. (26) (which assumes that GIM suppression is operative) shows that the decay width is completely dominated by the contribution with $m_j^2=m_k^2=m_b^2$. The partial width is plotted in Fig. 6 as a function of the top quark mass for $m_W=80.3$ GeV, $m_c=1.5$ GeV, $\sin^2\theta_W=0.23$, and $V_{cb}=0.036-0.046$. The two curves correspond to the lower and upper values of the CKM matrix element V_{cb}. The partial width vanishes at threshold where $m_t=m_c+2m_W$, and is at most $\approx10^{-12}$ GeV for $m_t=200$ GeV. This extremely small partial width is a direct consequence of three-family unitarity of the CKM matrix in the u-quark sector. If the GIM suppression condition Eq. (22) is relaxed, the integral appearing in Eq. (21) is a factor of 2×10^5 larger than $I(m_b^2,m_b^2,m_c^2,m_t^2,m_W^2)$ for each value of m_j^2 and m_k^2. Thus, it is quite possible that the rare decay $t\to cW^+W^-$ occurs at an observable level in non-standard model theories. A search for this rare decay mode would directly test CKM unitarity of the tc rows of the CKM matrix.

The partial width for the rare decay $t\to uW^+W^-$ can be obtained from the above with the replacement $c\leftrightarrow u$. The partial width for the up mode is even smaller than for the charm mode due to smaller CKM matrix elements. This decay mode can be used to test CKM unitarity of the tu rows of the CKM matrix.

III. CONCLUSIONS

The partial widths for the rare top quark decay modes $t\to bW^+Z$, sW^+Z, dW^+Z, cW^+W^-, and uW^+W^- have been calculated in the standard model. The decay mode $t\to bW^+Z$ is potentially observable at LHC rates for top quark masses above 187 GeV, and could be used to accurately determine the top quark mass. The decay amplitude also depends on the triple decay vertex W^+W^-Z, and therefore tests for the presence of this coupling and its value. The

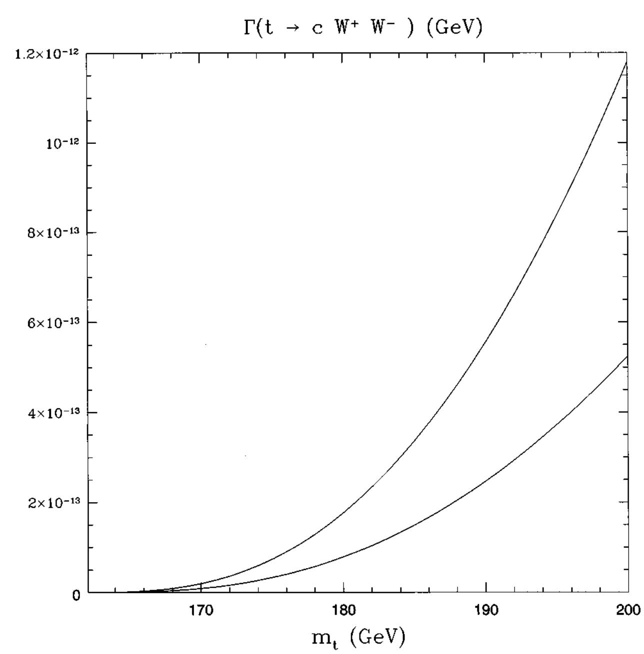

FIG. 6. $\Gamma(t\to cW^+W^-)$ as a function of the top quark mass for $m_W=80.3$ GeV, $m_c=1.5$ GeV, $\sin^2\theta_W=0.23$, and $V_{cb}=0.036-0.046$. The partial decay width vanishes at threshold, where $m_t=m_c+2m_W$.

decays $t \to cW^+W^-$ and uW^+W^- are extremely GIM suppressed in the standard model, but may be much larger in nonstandard scenarios. A search for these rare decay modes tests CKM unitarity of the tc and tu rows of the CKM matrix,

$$\sum_{j=d,s,b} V_{tj} V^*_{u_i j} = 0. \tag{27}$$

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