Cosmic strings in an open universe: Quantitative evolution and observational consequences

P. P. Avelino*
Centro de Astrofísica, Universidade do Porto, Rua do Campo Alegre 823, PT-4150 Porto, Portugal

R. R. Caldwell†
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104

C. J. A. P. Martins‡
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, United Kingdom
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The cosmic string scenario in an open universe is developed—including the equations of motion, a model of network evolution, the large angular scale cosmic microwave background (CMB) anisotropy, and the power spectrum of density fluctuations produced by cosmic strings with dark matter. We first derive the equations of motion for a cosmic string in an open Friedmann-Robertson-Walker (FRW) space-time. With these equations and the cosmic string stress-energy conservation law, we construct a quantitative model of the evolution of the gross features of a cosmic string network in a dust-dominated, $\Omega<1$ FRW space-time. Second, we apply this model of network evolution to the results of a numerical simulation of cosmic strings in a dust-dominated, $\Omega=1$ FRW space-time, in order to estimate the rms temperature anisotropy induced by cosmic strings in the CMB. By comparing to the COBE-DMR observations, we obtain the normalization for the cosmic string mass per unit length μ as a function of Ω. Third, we consider the effects of the network evolution and normalization in an open universe on the large scale structure formation scenarios with either cold or hot dark matter (CDM, HDM). The string+HDM scenario for $\Omega<1$ appears to produce too little power on scales $k\gtrsim 1\ \Omega h^2$/Mpc. In a low density universe the string+CDM scenario is a better model for structure formation. We find that for cosmological parameters $\Gamma=\Omega h\sim 0.1$–0.2 in an open universe the string+CDM power spectrum fits the shape of the linear power spectrum inferred from various galaxy surveys. For $\Omega\sim 0.2$–0.4, the model requires a bias $b\gtrsim 2$ in the variance of the mass fluctuation on scales $8\ h^{-1}$ Mpc. In the presence of a cosmological constant, the spatially flat string+CDM power spectrum requires a slightly lower bias than for an open universe of the same matter density. [S0556-2821(97)05920-1]

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I. INTRODUCTION

Cosmic strings are topological defects which may have formed in the very early universe and may be responsible for the formation of large scale structure observed in the universe today [1–3]. In order to test the hypothesis that the inhomogeneities in our universe were induced by cosmic strings one must compare observations of our universe with the predictions of the cosmic string model. To date, most work on the cosmic string scenario has been carried out with a background cosmological model which is a spatially flat, $\Omega=1$ Friedmann-Robertson-Walker (FRW) space-time. (See [4–7] for other work on open scenarios with defects.)

Observational evidence indicates that to within 95% confidence, the present-day cosmological density parameter lies in the range $0.2<\Omega<2$, and is most likely less than or equal to unity [8]. This reason alone is enough motivation to investigate open cosmologies. We are further compelled when we recognize that the string+cold dark matter (CDM) linear power spectrum, while producing too much small scale power in the case $\Omega=1$ [9], appears to fit the the shape of the power spectrum estimated from the various three-dimensional galaxy redshift surveys for $\Omega<1$ [5,6]. Hence, we aim to develop the tools necessary to study cosmic strings in an open universe.

The outline of this paper is as follows. In Sec. II we construct a background cosmology composed of a dust-dominated, $\Omega<1$ FRW space-time. We derive the cosmic string equations of motion and the energy conservation equation in an open universe, and discuss the effects of the spatial curvature and rapid, curvature-dominated expansion on the density of strings through a simple solution of these equations.

We next construct an analytic model of the long string evolution in an open universe. While not as sophisticated as the model developed by Austin, Copeland, and Kibble [10], we improve on earlier work [11] by following the procedure of Martins and Shellard (MS) [12], which treats the mean string velocity as a dynamical variable. We should point out that the MS model provides an accurate description of the behavior of a cosmic string network seen in numerical simulations of radiation- through matter-dominated expansion in the case $\Omega=1$. Since no such simulations exist in the case of an open universe, the model presented in this paper is an

*Electronic address: pedro @ pulsar.astro.up.pt
†Electronic address: caldwell @ dept.physics.upenn.edu
‡Also at C.A.U.P., Rua do Campo Alegre 823, 4150 Porto, Portugal. Electronic address: C.J.A.P.Martins@damtp.cam.ac.uk

extrapolation. Nevertheless, the onset of curvature domination in the present case is qualitatively similar to the radiation-matter transition; in both cases, the dominant dynamical effect during the transition is due to the shift in the time dependence of the scale factor. Hence, we expect that the model developed in this section, which includes the effects of curvature-driven expansion and spatial curvature on the the string equations of motion, will be sufficient to provide a good description of cosmic string evolution in an open universe—although future numerical work will undoubtedly be required to test this model.

In Sec. III, by numerically solving the evolution equations, we find that with the onset of curvature domination, the mean string velocity and energy density decay rapidly. We also note that there does not appear to be a scaling solution for the gross features of the string network, as occurs in a spatially flat, $\Omega=1$ cosmology. Similar work has also been carried out recently by Martins [13]. In Sec. IV we construct a semianalytic model of the CMB anisotropy induced by strings in an open cosmology. We obtain the normalization of the string mass per unit length μ, as a function of Ω, by comparing with the Cosmic Background Explorer (COBE) Differential MIcrowave Radiometer (DMR) observations. Next, we consider the effect of the new normalization on the large scale structure power spectrum when $\Omega<1$ by adapting the Albrecht-Stebbins [9] semianalytic model for the string+CDM and hot dark matter (HDM) scenarios. While the power spectrum does not completely specify the non-Gaussian fluctuation patterns generated by cosmic string wakes, it serves as a useful gauge of the viability of the scenario. We find that in an open universe, the string+HDM spectrum suffers from a lack of power on small scales, compared to the linear power spectrum estimated from various galaxy redshift surveys. However, the string+CDM spectrum in an open universe with bias $b\gtrsim 2$ appears to fit the observed power spectrum. Finally, in Sec. VI, we consider the case of a spatially flat, low matter density universe with a cosmological constant. Applying the same tools that we have developed for the study of an open universe, we obtain the CMB normalization of the mass per unit length. We find that the string+CDM power spectrum requires a slightly lower bias than for an open universe with the same matter density. We conclude in Sec. VII.

Throughout this paper we have set the speed of light to unity, $c=1$, and adopted the convention $H_0\equiv 100h$ km sec^{-1} Mpc^{-1}.

II. OPEN UNIVERSE

To begin our investigation of cosmic strings in an open FRW space-time, we must construct a background cosmology. We find it convenient to use the metric

$$ds^2=a^2(\tau)[d\tau^2-(1+Kr^2)^{-2}(dx^2+dy^2+dz^2)], \tag{2.1}$$

where $r^2=(x^2+y^2+z^2)/R_c^2$, $K=-1$ for an open FRW space-time, and the coordinates lie in the range $\{x,y,z\}\in(-R_c,R_c)$. Note that for $K=-1,0,+1$ the spatial sections are H^3, R^3, S^3, respectively. The radius of spatial curvature is given by

$$R=a(\tau)R_c=H^{-1}|1-\Omega|^{-1/2}. \tag{2.2}$$

Thus, a universe with a density close to critical has a very large radius of curvature and is very flat, whereas in a low density universe the curvature radius is comparable to the Hubble radius. The cosmological time t is related to the conformal time by $t=\int d\tau a(\tau)$.

In this space-time, the induced metric on the cosmic string world sheet is

$$\gamma_{ab}=g_{\mu\nu}x^{\mu}{}_{,a}x^{\nu}{}_{,b}\rightarrow\gamma_{\tau\tau}=a^2(\tau)\left[1-\frac{\dot{\mathbf{x}}^2}{(1+Kr^2)^2}\right],$$

$$\gamma_{\sigma\sigma}=-a^2(\tau)\frac{\mathbf{x}'^2}{(1+Kr^2)^2}. \tag{2.3}$$

The indices a,b denote coordinates τ,σ on the string world sheet, with $\cdot=\partial_\tau$ and $'=\partial_\sigma$, where σ is a parameter along the string. We have chosen the gauge such that the time parameter along the string coincides with the conformal time τ, and $\dot{\mathbf{x}}\cdot\mathbf{x}'=0$.

Referring to [2] (Eq. 6.1.15) for the string stress-energy tensor, we find that the energy density in string is given by

$$\begin{aligned}\rho&=\frac{\mu}{a^2}\int d\sigma\sqrt{-\gamma}\gamma^{ab}x^{\tau}{}_{,a}x^{\tau}{}_{,b}\delta^3[\mathbf{x}-\mathbf{x}(\tau,\sigma)]\\&=\frac{\mu}{a^2}\int d\sigma\frac{\mathbf{x}'}{\sqrt{(1+Kr^2)^2-\dot{\mathbf{x}}^2}}\delta^3[\mathbf{x}-\mathbf{x}(\tau,\sigma)]\\&=\frac{\mu}{a^2}\int d\sigma\tilde{\epsilon}\,\delta^3[\mathbf{x}-\mathbf{x}(\tau,\sigma)]\Rightarrow\tilde{\epsilon}\equiv\frac{\mathbf{x}'}{\sqrt{(1+Kr^2)^2-\dot{\mathbf{x}}^2}}.\end{aligned} \tag{2.4}$$

Here, $\tilde{\epsilon}$ is the comoving coordinate length of string per unit σ. The integral above tells us that the total energy in string is not simply the total length of string, weighted by a relativistic γ factor, times the mass per unit length μ. The spatial curvature also has an effect, as there is a contribution to the energy by the Kr^2 term for strings with large spatial extent. This coupling of curvature to the string energy will have an important effect on the evolution of very large strings.

The equations of motion for cosmic string in the space-time, Eq. (2.1), are given by (generalizing Eq. 6.1.12 of [2])

$$x^{\mu;a}_{,a}+\Gamma^{\mu}_{\nu\rho}\gamma^{ab}x^{\nu}_{,a}x^{\rho}_{,b}=0,$$

$$\mu=\tau\rightarrow\dot{\tilde{\epsilon}}=-2\frac{\dot{a}}{a}\tilde{\epsilon}\frac{\dot{\mathbf{x}}^2}{(1+Kr^2)^2}, \tag{2.5}$$

$$\begin{aligned}\mu=i\rightarrow\ddot{x}^i+2\frac{\dot{a}}{a}\dot{x}^i\left(1-\frac{\dot{\mathbf{x}}^2}{(1+Kr^2)^2}\right)-\frac{1}{\tilde{\epsilon}}\left(\frac{x'^i}{\tilde{\epsilon}}\right)'\\=\frac{2Ka^2H^2|1-\Omega|}{1+Kr^2}[x^i(\dot{\mathbf{x}}^2-\tilde{\epsilon}^{-2}\mathbf{x}'^2)-2\dot{x}^i(\mathbf{x}\cdot\dot{\mathbf{x}})\\+2\tilde{\epsilon}^{-2}x'^i(\mathbf{x}\cdot\mathbf{x}')].\end{aligned} \tag{2.6}$$

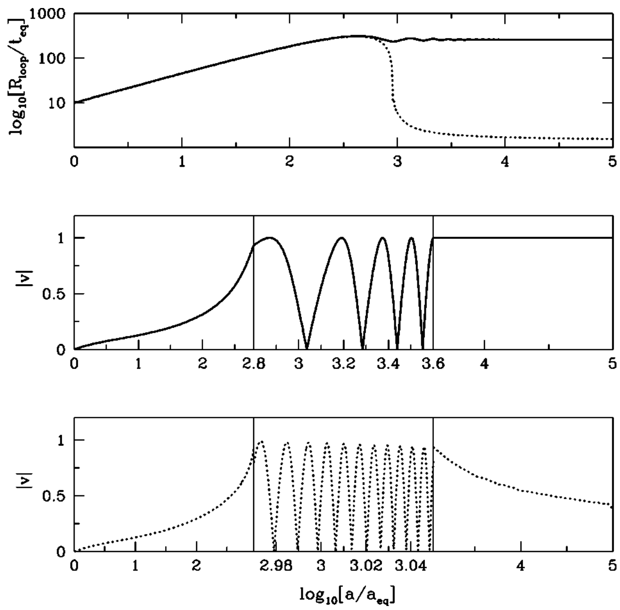

FIG. 1. The evolution of a circular cosmic string loop formed at $t=t_{eq}$ with an initial radius $R_{loop}=10t_{eq}$, in a universe with $\Omega=1$ and 0.2 given by the solid and dotted lines, respectively. The top panel shows the evolution of the radius in units of t_{eq} versus $\log_{10}(a/a_{eq})$. The bottom two panels show the evolution of the velocity. An expanded scale shows the first oscillations as the loop enters the horizon, after which we show only the maximum velocity in each period of oscillation.

The above equations represent the first main result of this paper; by setting $K=-1$, Eqs. (2.5) and (2.6) give the equations of motion for cosmic strings in an open FRW space-time. Similarly, $K=+1$, 0 give the equations of motion in closed and flat FRW space-times. Hereafter we will only consider the case of an open universe.

We can get a better idea of the effects of the rapid expansion and space-time curvature by studying a simple solution of these microscopic equations. For our purposes it is sufficient to consider dust-dominated expansion only, for which the scale factor is given by

$$a(\tau)=\frac{\Omega\sinh^2(\tau/2)}{H_0(1-\Omega)^{3/2}}. \tag{2.7}$$

Here H_0 and Ω are the present-day Hubble constant and cosmological density parameter. We have solved Eq. (2.6) for the case of a circular loop with an initial physical radius $R_{loop}(t_{eq})=10t_{eq}$ and velocity $v(t_{eq})=0$ at the time of radiation-matter equality. The evolution of the loop radius and velocity, for the cases $\Omega=1$, 0.2, is shown in Fig. 1. At early times the evolution is indistinguishable, as the loop is conformally stretched by the expansion and picks up speed. As the loop falls inside the Hubble horizon, it begins to oscillate. We see that the frequency of oscillation, while constant relative to H, is higher relative to t^{-1} in an open universe owing to the faster expansion rate. This is crucial because oscillating loops in an expanding universe lose a constant fraction of their energy in each period [12,14]. Hence, the average velocity at late times is smaller than the spatially flat value $v^2_{flat}=1/2$. Incidentally, the velocity damping guarantees that the denominator of $\tilde{\epsilon}$, $(1-r^2)^2-\dot{x}^2$, is always positive; for large r, $\dot{x}$ must decrease. In short, the curvature-driven expansion serves to damp the string motion.

We will move on to concentrate on Eq. (2.5), the conservation equation, for the present. Making the definition

$$v^2\rho=\frac{\mu}{a^2}\int d\sigma\,\tilde{\epsilon}\dot{\mathbf{x}}^2\delta^3[\mathbf{x}-\mathbf{x}(\tau,\sigma)], \tag{2.8}$$

for the mean-squared string velocity, we obtain the energy density conservation equation

$$\dot{\rho}+2\frac{\dot{a}}{a}\rho=-2\frac{\dot{a}}{a}\frac{\mu}{a^2}\int d\sigma\;\tilde{\epsilon}\frac{\dot{\mathbf{x}}^2}{(1-r^2)^2}\delta^3[\mathbf{x}-\mathbf{x}(\tau,\sigma)]. \tag{2.9}$$

For $r\rightarrow 1$ $(0<r^2<1)$ with fixed $\dot{\mathbf{x}}^2$ the term on the right-hand side (RHS) of Eq. (2.9) becomes large and negative. Hence, this confirms that the effect of the spatial curvature will be to enhance the dilution of the energy density in long strings due to the expansion.

Let us examine Eq. (2.9) more closely. The integrand on the RHS is just the coordinate energy $\tilde{\epsilon}$ times the velocity squared, weighted by a factor of the ratio r of the string length to the curvature scale. By averaging over strings of length scale λ, we may rewrite Eq. (2.9) as

$$\dot{\rho}_\lambda+2\frac{\dot{a}}{a}\rho_\lambda=-2\frac{\dot{a}}{a}\rho_\lambda\langle v^2\rangle[1-(\lambda/R)^2]^{-2}. \tag{2.10}$$

The ratio λ/R determines the importance of the curvature contribution. For strings much smaller than the curvature scale, $\lambda\ll R$, we obtain the usual flat space energy density conservation equation. For very large strings, $\lambda\rightarrow R$. Hence, string with support on very large scales samples more of, or is more tightly coupled to, the curvature. Crudely, the effect is that the long string energy density is dissipated more rapidly as the space-time expands.

III. QUANTITATIVE STRING EVOLUTION

We now build on the work of Martins and Shellard [12] to construct an analytic model of long string network evolution in an open universe. As carried out by MS, we treat the average string velocity as well as the characteristic string length scale as dynamical variables. For long strings, the characteristic length scale is related to the network density of long strings by $\rho_L=\mu L^{-2}$. Hence, we obtain, from Eq. (2.10),

$$\frac{dL}{dt}=LH\{1+v^2[1-(1-\Omega)(LH)^2]^{-2}\}+\frac{1}{2}\tilde{c}v. \tag{3.1}$$

The phenomenological loop chopping efficiency parameter $\tilde{c}$ models the transfer of energy from the long strings to loops. Next, an evolution equation for the velocity may be obtained by differentiating Eq. (2.8) and using Eq. (2.6):

$$\frac{dv}{dt}=\{[1-(1-\Omega)(LH)^2]^2-v^2\}\frac{\kappa}{L} -2Hv\{1-v^2[1-(1-\Omega)(LH)^2]^{-2}\}. \quad (3.2)$$

As in MS [cf. Eqs. (2.40) and (2.41) of [12]] the parameter κ (MS use k) has been introduced to describe the presence of small scale structure on the long strings. Equations (3.1) and (3.2) are the second main result of this paper. Again, in the limit $\Omega\to 1$, the flat space evolution equations [Eqs. (2.20) and (2.38) of [12]] are obtained. (Note that the above equations are equally valid in a closed, $\Omega>1$ FRW space-time.) We have omitted the friction damping terms due to the interaction of the cosmic strings with the hot cosmological fluid, which are important only near the time the strings were formed. Similar equations are given in Ref. [13], although it was assumed that the contribution of the curvature terms is negligible.

In a spatially flat, $\Omega=1$ FRW space-time, scaling solutions may be found for which $\dot{v}=0$ and $L/t=$const. In a dust-dominated era, the Allen-Shellard numerical simulation suggests the values $\kappa=0.43$ and $\tilde{c}=0.15$ (note that MS used $\kappa=0.49$ and $\tilde{c}=0.17$—these values give the same mean velocity, but the string density is closer to the Bennett-Bouchet value, though is consistent within the quoted error bars).

In an open, $\Omega<1$ FRW space-time, for normal types of matter (i.e., dust or radiation), H does not decay like t^{-1}. Ignoring the curvature terms, the solution of Eq. (3.2) for which $v=$const is inconsistent with $L\propto t$ from Eq. (3.2), and inconsistent with $L\propto H^{-1}$ from Eq. (3.1). As noted in Ref. [13], it does not appear possible to find a scaling solution in an open universe.

We have numerically solved the evolution equations (3.1) and (3.2) with the expansion scale factor given by Eq. (2.7). We choose the initial conditions for L/t and v to be given by the $\Omega=1$ dust era scaling solution of $v=0.61$ and $L/t=0.53$. Our reasoning is that at early times, when the scale factor behaves as $a\propto t^{2/3}$, the evolution is indistinguishable from an $\Omega=1$ space-time. Only at late times is the effect of the curvature-dominated expansion important. Similarily, we assume that the coefficients κ and $\tilde{c}$, describing the small scale structure and chopping efficiency, are unchanged from their dust-era values. This is somewhat unrealistic, since these parameters differ even between the radiation and dust eras. Up to radiation-matter equality, however, work by MS has shown that the effect on the evolution is dominated by the change in the expansion rate rather than the shift in the parameters. Hence, we expect our model to be reliable for the observationally allowed values of Ω as we do not follow the evolution too far beyond matter-curvature equality.

Sample results are displayed in Fig. 2. We see that the rms velocity and string density decrease rapidly at late times when the curvature begins to dominate the expansion. For $\Omega=0.2$, the velocity drops to $v=0.41$, and the length scale grows to $L/t=0.63$. By letting $\Omega\to 1$ in Eqs. (3.1) and (3.2), we find final values $v=0.44$ and $L/t=0.64$. Hence, the rapid, curvature-dominated expansion is the main cause of the departure from scaling, since the spatial curvature terms contribute only a $\lesssim 10\%$ effect to the evolution. Consequently, our results depend only very weakly on the Hubble parameter h. As pointed out by Martins, the energy density in long strings is actually growing relative to the background cosmological fluid. At sufficiently late times, the strings will come to dominate the energy density of the universe. This deviation from the scaling solutions should have an important effect on the large angle CMB anisotropy due to cosmic strings.

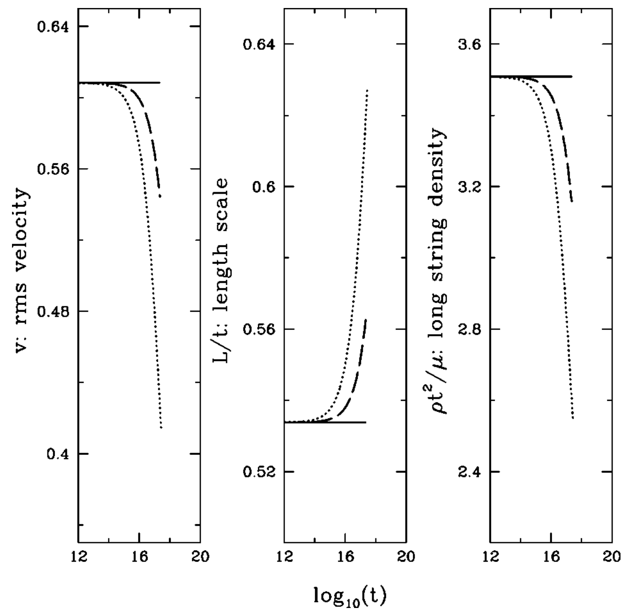

FIG. 2. The evolution of the average string velocity and the characteristic length scale of long strings in an open FRW space-time with $\Omega=1.0$, 0.6, and 0.2 given by the solid, dashed, and dotted curves, respectively. The horizontal axis is the logarithm of the cosmological time t. As the expansion becomes curvature dominated, the average velocity decays and the characteristic string length scale grows. As a result, the number of long strings in a box of linear dimension t decreases, although the string energy density relative to the background energy density grows.

IV. CMB ANISOTROPY

While it is beyond our means to simulate the evolution of cosmic strings in an $\Omega<1$, dust-dominated FRW space-time at present, we may nevertheless adapt our model for the quantitative evolution of a string network to estimate the amplitude of CMB temperature anisotropy induced by cosmic strings.

We would like to determine the COBE-smoothed rms temperature anisotropy due to cosmic strings in an $\Omega<1$ cosmology. Hence, we must compute $C(0°,10°)$, the 0° angular separation correlation function smoothed over 10° in the manner of COBE. To do so, we will make the following simplifying assumptions

(1) The large angle CMB anisotropy is due to the gravitational perturbations caused by cosmic strings along the line of sight out to the surface of last scattering.

(2) The mean, observer-averaged angular correlation function may be written as the sum of the contributions by strings located in the time interval $[t,t+\delta t]$; the contribution due to strings separated by an interval larger than the characteristic time scale, $\delta t\gtrsim L$, is negligible.

(3) The effect of the negative spatial curvature in the open

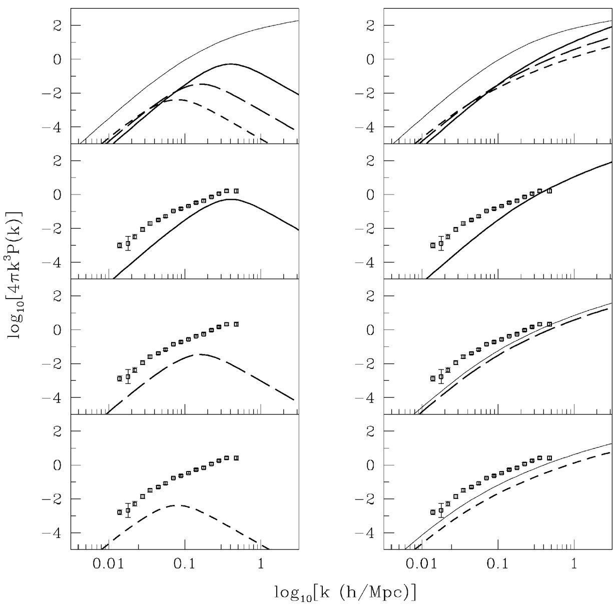

FIG. 3. The CMB-normalized power spectrum $P(k)$ of density fluctuations produced by cosmic strings with HDM (left) and CDM (right) are presented for $\Omega = 1.0$, 0.4, and 0.2, given by the thick solid, long-dashed, and short dashed curves, respectively. For all cases, we have used $h = 0.7$. In the top panels, the thin solid line is the standard CDM spectrum normalized to COBE following [34]. In the lower panels, the data points are the PD reconstruction of the linear power spectrum, with the amplitude rescaled $\propto \Omega^{-0.3}$. In the bottom two $\Omega < 1$ string+CDM panels, the thin solid line shows the CMB-normalized power spectrum for the case of a cosmological constant with the same matter density. A bias $b \sim 2-4$ is necessary to obtain $\sigma_8 \sim 1$. In the presence of a cosmological constant, a smaller bias is required.

universe is to shift temperature anisotropy correlations to smaller angular scales than in a spatially flat universe.

(4) The mean rms temperature anisotropy contributed in a time interval δt is proportional to the density of strings present and the mean string velocity during that interval. This is similar to Perivolaropoulos' model [15] in which the CMB anisotropy is a superposition of random impulses due to the Kaiser-Stebbins effect [16], for which $\delta T \propto 8\pi G \mu v$, for each long string present.

Given the first two assumptions, the correlation function may be written as

$$C(\theta, z_{ls}) = \int_0^{z_{ls}} dz \; C(\theta)_{,z}. \tag{4.1}$$

Here, $C(\theta, z_{ls})$ is the temperature correlation function contributed by strings out to the redshift of last scattering, z_{ls}. The function $C(\theta, z)$ has been tabulated from the numerical simulation of CMB anisotropy induced by cosmic strings in an $\Omega = 1$, dust-dominated FRW space-time (see Fig. 3 of [17] where we observe that the dominant contribution to the rms anisotropy for $\Omega = 1$ occurs within a redshift $z \lesssim 10$). The function $C(\theta)_{,z}$ is obtained empirically by differentiating $C(\theta, z)$. This procedure does not rely on assumption (2) above. However, in order to interpret $C(\theta)_{,z}$ as the contribution to the angular power spectrum due to strings in the interval $[z, z+\delta z]$, we must restrict our use to a time resolution $\delta t \gtrsim L$, greater than the characteristic time scale.

The negatively curved spatial sections of the open FRW space-time lead to a generic suppression of large-angle correlations. (See Ref. [18] and references therein for more discussion.) We may understand this effect by considering that an object with angular size θ_{-1} at a redshift z from an observer in an open FRW space-time subtends a smaller angle $\theta_{-1} < \theta_0$ than the angle of the same object from the same redshift in a spatially flat FRW space-time. (The subscripts 0, -1 refer to the sign of the spatial curvature.) We may express this relationship between the angles subtended as

$$\begin{aligned}\theta_{-1} &\equiv f(\theta_0, z, \Omega) \\ &= 2 \arcsin\left[\sin\frac{\theta_0}{2} \frac{\Omega^2(1+z-\sqrt{1+z})}{\Omega z + (2-\Omega)(1-\sqrt{1+\Omega z})} \right].\end{aligned} \tag{4.2}$$

As a result, $\theta_{-1} \leqslant \theta_0$ for all $z \geqslant 0$ and $\Omega \leqslant 1$. In the limit $\Omega \to 1$ or $z \to 0$, Eq. (4.2) reduces to the identity, with $\theta_0 = \theta_{-1}$. In order to include the effect of the geometry on the temperature anisotropy correlation function in an open universe, we write

$$\frac{dC_\ell}{dz}(\Omega) = 2\pi \int_0^\pi d(\cos\theta_{-1}) P_\ell(\cos\theta_{-1}) C(\theta_0)_{,z}. \tag{4.3}$$

By shifting the argument of the Legendre polynomial to smaller angles, correlations on a particular angular scale are associated with a larger ℓ mode in an open than in a flat FRW space-time.

To implement our final assumption above, we model the effect of the curvature-dominated expansion on the correlation function by weighting the contribution at different redshifts using our model of quantitative string evolution:

$$C_\ell(\Omega) = \int_0^{z_{ls}} dz \left[\frac{\rho_L(\Omega, z)}{\rho_L(\Omega=1, z)} \frac{v^2(\Omega, z)}{v^2(\Omega=1, z)} \right] \frac{dC_\ell}{dz}(\Omega). \tag{4.4}$$

Hence, the moments of the correlation function, which is proportional to $(\delta T)^2$, are weighted by two powers of the string velocity relative to the $\Omega = 1$ value. We model the contribution of the N long strings in each volume to the temperature amplitude as $\sqrt{N}$, so that only one factor of the string density relative to the $\Omega = 1$ value is included above. The functional dependence of ρ_L and v on the redshift for a given open cosmology is obtained by integrating Eqs. (3.1) and (3.2). Because these weights change on a time scale comparable to or slower than L, assumption (2) is satisfied.

We foresee that the CMB anisotropy will be diminished due to the dilution of the string density, the decrease in mean velocity, and the negative spatial curvature in an $\Omega < 1$ universe. The geometric effect due to the negative spatial curvature, in Eq. (4.3), will lead to a decrease in the amplitude of the anisotropies generated at distances to the observer which are large compared to the curvature length scale. The dynamical effect due to the late time evolution of the string network, in Eq. (4.4), will lead to a decrease in the amplitude of the anisotropies generated at late times. The result is an overall decrease in the amplitude of the CMB anisotropy spectrum for a given mass per unit length.

We may estimate the normalization of the cosmic string mass per unit length μ in an open universe by comparing the observations of COBE-DMR [19] with our predictions. We carry out a procedure similar to that given in [17], computing the smoothed autocorrelation function

$$C(0°,10°,\Omega)\equiv\sum_{\ell=2}^{\infty}\frac{2\ell+1}{4\pi}|G_\ell|^2|W_\ell(7°)|^2C_\ell(\Omega). \tag{4.5}$$

Here, we smooth the temperature pattern first with the average DMR beam model window function G_ℓ (tabulated values are given in [20]) which is approximately a 7° beam, and second with a 7° full width at half maximum (FWHM) Gaussian window function $W_\ell(7°)$ for an effective smoothing of 10°. Thus, we find for the case $\Omega=0.2$, $G\mu=1.7^{+0.6}_{-0.3}\times10^{-6}$. The effect of the spatial geometry on the smoothed autocorrelation function is only ~20% for $\Omega=0.2$; the dilution of the string density and the decrease in the mean velocity due to the rapid expansion are the main causes of the change in the rms anisotropy amplitude. We have rescaled the error bars assessed in [17], assigning no errors due to the crudeness of our model. For $\Omega\sim1$ this seems reasonable; for low Ω we underestimate our uncertainty in the normalization. The empirical formula for the CMB normalization of the string mass per unit length,

$$G\mu(\Omega)=G\mu(\Omega=1)\Omega^{-0.3}=1.05\,^{+0.35}_{-0.20}\times10^{-6}\Omega^{-0.3}, \tag{4.6}$$

fits our results to within 5% for $0.1\leqslant\Omega\leqslant1$. We stress that our estimate of the normalization is valid, within the abovementioned error bars, insofar as Allen *et al.* [17] have accurately simulated the large angle CMB anisotropy induced by realistic cosmic strings. This is the third main result of this paper.

We take this opportunity to comment on the effect of an open universe on the small angular scale CMB anisotropy induced by cosmic strings. Although no firm predictions of the high-ℓ C_ℓ spectrum have been made to present, recent work [21,22] has shed light on the qualitative features of the spectrum. Based on numerical simulations, they observe a feature near $\ell\sim100$ attributed to the decay of vector perturbations smaller than the horizon scale on the surface of last scattering. For higher ℓ, there is a single, low, broad feature (as opposed to the secondary oscillations predicted in inflationary scenarios) in the range $\ell\sim400$–600, as conjectured by Magueijo *et al.* [23]. In an open universe, the apparent size of fluctuations near the surface of last scattering shift to smaller angles as $\theta\propto\Omega^{1/2}$. Hence, we expect the location of the feature due to the decay of the vector perturbations to shift as $\ell\sim100\Omega^{-1/2}$ towards smaller angular scales.

We end these comments on the small angular scale spectrum by adding that MS have shown that the transient in the evolution of the long string density and velocity across the radiation-matter transition, observed in the Bennett-Bouchet numerical simulations [24,25], may last as late as $\sim10^3t_{\rm eq}$ (see Figs. 18(c) and 18(d) of [12]). In particular, the ratio $\rho_L/\rho_{\rm crit}$, which is higher in the radiation era, does not settle down to the matter era scaling value until $\sim10^3t_{\rm eq}$, and the evolution of the mean velocity displays a peak near $\sim30t_{\rm eq}$ before reaching the matter-era value. For low values of Ω and h, the redshift of radiation-matter equality approaches last scattering, so that this transient may have an important effect on the small angle CMB anisotropy generated near the surface of last scattering [26].

V. LARGE SCALE STRUCTURE

Finally, we consider the large scale structure formation scenario with cosmic strings. We will examine both the HDM and CDM scenarios, by adapting the methods of Ref. [9] to estimate the power spectrum of density fluctuations produced by cosmic strings. While the effect of low Ω on these string scenarios has been examined previously by Mahonen *et al.* [27] and Ferreira [6], our contribution will be the effect of the quantitative string evolution and the normalization of μ on the power spectrum.

In the semianalytic model of Albrecht and Stebbins, the power spectrum of density perturbations induced by cosmic strings in an $\Omega=1$ universe is approximated by

$$P(k)=16\pi^2(1+z_{\rm eq})^2\mu^2\int_{\tau_i}^{\infty}|T(k;\tau')|^2\mathcal{F}(k\xi/a)d\tau',$$

$$\mathcal{F}(k\xi/a)=\frac{2}{\pi^2}\beta^2\Sigma\frac{\chi^2}{\xi^2}[1+2(k\chi/a)^2]^{-1}. \tag{5.1}$$

In these equations, a is the scale factor which evolves smoothly from radiation- to dust-dominated expansion, τ_i is the conformal time at which the string network formed, and $T(k,\tau')$ is the transfer function for the evolution of the causally compensated perturbations [see Eq. (2) of [9] and Eqs. 5.23 and 5.45 of [28]], specific to either CDM or HDM. In the case of HDM, $T(k,\tau')$ includes a term fit to numerical calculations of the damping of perturbations by nonrelativistic neutrinos.

The parameters used in the Albrecht-Stebbins estimate of the cosmic string power spectrum are given by

$$\xi\equiv(\rho_L/\mu)^{-1/2},\quad \beta\equiv\langle v^2\rangle^{1/2},$$

$$\Sigma\equiv\frac{\mu_r}{\mu}\gamma_b\beta_b+\frac{1}{2\gamma_b\beta_b}\left(\frac{\mu_r^2-\mu^2}{\mu\mu_r}\right), \tag{5.2}$$

where χ is the curvature scale of wakes, β_b is the macroscopic bulk velocity of string, $\gamma_b=(1-\beta_b^2)^{-1/2}$, and μ_r is the renormalized mass per unit length, which reflects the accumulation of small scale structure on the string. The "I model" developed by Albrecht and Stebbins uses the following values of the parameters:

era	$\xi/(a\tau)$	χ/ξ	β	β_b	μ_r/μ
radiation	0.16	2.0	0.65	0.30	1.9
dust	0.16	2.0	0.61	0.15	1.4

We note that the values of β, β_b, and μ_r were taken from the Allen-Shellard (AS) simulation [29,30]. The values of ξ

and χ, however, reflect an estimate based on the Bennett-Bouchet (BB) [24,25] and AS simulations. The radiation-era values were used for the I model to determine the power spectrum in a spatially flat, $\Omega=1$ universe.

While the I model closely resembles the AS and BB simulations, we might hope to make an improvement by including the effect of the evolution of the string network parameters across the transition from radiation- to dust-dominated expansion [26]. As investigated by MS, the ratio L/t interpolates between the radiation- and dust-dominated scaling values, whereas the mean velocity displays a short burst during which the string network rapidly sheds loops. [See Figs. 18(c) and (d) of [12].] Hence, we make the identifications $\xi=L$ and $\chi=2\xi$, and use the evolution of L/t to interpolate between radiation- and dust-era values of μ_r and κ, and use β to guide β_b. Note that the radiation-era values are $(\tilde{c},\kappa)=(0.24,0.18)$ for the BB simulation and (0.22,0.16) for the AS simulation. Applying the model of realistic network evolution to the power spectrum, we find that for the same μ, the only change is a $\sim 30\%$ boost in the power for the AS values, which is consistent with the quoted uncertainties on the parameters measured in the simulations.

To adapt the power spectrum for an open universe, we would like to use the transfer function $T(k,\tau')$ appropriate for $\Omega<1$. In the present work, however, we will use the $\Omega=1$ transfer function, which should be satisfactory on the scales of interest, $\lambda \lesssim 10^2$ Mpc. Because perturbations do not grow as fast as $\delta\rho/\rho \propto a$ in a low density universe, we use the factor $g(\Omega)$ [defined in Eq. (5.5)] to modify the amplitude of present-day perturbations [31]. Hence, we obtain the power spectrum (adapted from [9])

$$4\pi k^3 P(k)=\frac{4\pi\Omega^2 h^4 \theta_1^2 \mu_6(\Omega)^2 k^4 g(\Omega)^2}{1+(\theta_2 k)+(\theta_3 k)^2+(\theta_4 k)^3+(\theta_8 k)^4+(\theta_6 k)^{\theta_7}}\left[\frac{1}{1+1/(\theta_5 k)^2}\right]^2, \tag{5.3}$$

model	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8
HDM	6.8	4.7	4.4	1.55	2198	2.46	6.6	3.2
CDM	6.8	4.7	4.4	1.55	2198	0	0	0

where k is measured in units Ωh^2/Mpc and $\mu_6(\Omega) \equiv G\mu(\Omega)\times 10^6$ obtained from Eq. (4.6). Numerical simulations of string seeded structure formation by Avelino [32], based on the Allen-Shellard simulation, find agreement with Eq. (5.3) in a flat universe on the limited range of scales accessible to the simulation.

Sample power spectra for various cosmological parameters, constructed using Eq. (5.3), are shown in Fig. 3. The string mass per unit length μ in each of the curves has been determined by the CMB normalization obtained from Eq. (4.6) in Sec. IV. In the top panels, the power spectra for $h=0.7$ and $\Omega=1.0$, 0.4, and 0.2 are shown. For reference, the standard CDM power spectrum [33] is also displayed. In the three descending panels, the individual spectra are shown with the Peacock-Dodds [34] (PD) reconstruction of the linear power spectrum. For $\Omega<1$ the reconstructed spectrum has been scaled as $\propto \Omega^{-0.3}$ (see Eq. 41 of [34]) for comparison.

We first consider structure formation by strings with HDM. Based on the normalization of μ obtained by [17], we see from Fig. 3 that the power spectrum approximately fits the shape of the PD spectrum on large scales. As a gauge of the string+HDM model for low Ω, we have computed the variance of the excess mass fluctuation in a ball of radius $R=8h^{-1}$ Mpc,

$$\sigma_8^2=\int |w(kR)|^2 4\pi k^2 P(k)dk,$$

$$w(x)=3(\sin x - x\cos x)/x^3, \tag{5.4}$$

which is observed to be around unity [34–36]. An excellent fit to our results is given by the empirical formula

$$\sigma_8(\Omega,h)=0.25(\pm 0.1)\times\left(\mu_6(\Omega)\frac{g(\Omega)}{\Omega}\,\frac{\Gamma(1+2.6\Gamma-1.6\Gamma^2)}{1+(10\Gamma)^{-2}}\right), \tag{5.5}$$

with

$$g(\Omega)\equiv \tfrac{5}{2}\Omega/[1+\tfrac{1}{2}\Omega+\Omega^{4/7}],$$

which is valid to within $\sim 10\%$ for $0.1 \leqslant \Omega \leqslant 1$ and $0.4<h<0.8$. The error bars on σ_8 are estimated based on the quoted uncertainties in the string parameters [24,25,29,30] and the uncertainty in the CMB normalization of μ [17] included in $G\mu(\Omega)$, which we repeat are probably too small for low Ω. Evaluating Eq. (5.5) for various values of the cosmological parameters, we predict $\sigma_8(1.0,0.5)=0.25\pm 0.1$ and $\sigma_8(0.2,0.5)=0.05\pm 0.02$. For $\Omega=1$ the string+HDM scenario requires a modest boost or bias in the power in order to achieve $\sigma_8 \sim 0.57$–0.75 [34,36]. These results are in agreement with past work by Colombi [37], based on the Bennett-Bouchet simulations. We pause to note that the nonlinear dynamics of wakes and filaments [9,38–49] may produce such a bias sufficient to reproduce the observed clustering of objects on large scales. However, in an open universe the peak amplitude of $k^3P(k)$ drops and shifts to larger scales, so that some sort of scale-dependent boost would be required to produce more power for $k \gtrsim 1\ \Omega h^2$/Mpc. Hence, string+HDM in an open universe does not appear to be a viable model for structure formation.

Structure formation by strings with CDM in a flat, $\Omega=1$ universe, when normalized on large scales, suffers from producing too much power on small scales. As pointed out by

[4–6] this problem may be overcome, as for standard CDM, in a low density, $\Omega<1$ universe. Examining Fig. 3, we see that the string+CDM power spectrum ''bends over'' on small scales as we lower Ω. Hence, for $\Gamma\equiv\Omega h\sim 0.1$–0.2, the spectrum approximately fits the shape of the PD reconstruction. The variance of the mass fluctuation is given by the empirical formula

$$\sigma_8(\Omega,h)=0.9(\pm 0.5)\left(\mu_6(\Omega)\frac{g(\Omega)}{\Omega}\frac{\Gamma(1-0.36\Gamma)}{1+(50\Gamma)^{-2}}\right), \tag{5.6}$$

which is valid to within $\sim 10\%$ for $0.1\leqslant\Omega\leqslant 1$ and $0.4<h<0.8$. Evaluating Eq. (5.6), we predict $\sigma_8(0.4,0.7)=0.4\pm 0.2$ and $\sigma_8(0.2,0.5)=0.2\pm 0.1$. Hence, for $\Gamma\sim 0.1$–0.2, the range of values of the mass fluctuation excess falls well below the estimate of $\sigma_8=0.6^{+32\%}_{-24\%}\exp[(-0.36-0.31\Omega+0.28\Omega^2)\ln\Omega]$ [50] by a factor of ~ 2–4. Within the uncertainties quoted in Eq. (5.6), a bias as low as $b\sim 1.5$ may be needed. Recent work by Sornborger *et al.* [44] on the structure of cosmic string wakes has shown that the ratio of the baryon to CDM density in wakes is enhanced. For a single wake formed near radiation-matter equality, the baryon enhancement at late times is ~ 2.4 in a region of thickness ~ 0.3 Mpc. These results, which suggest that structure formation by strings is biased, allow our conclusion that the string+CDM model may be a viable candidate for the formation of large scale structure in an open universe.

VI. COSMOLOGICAL CONSTANT

In this section we briefly consider the effect of a cosmological constant on the cosmic string scenario. The background cosmology in this case is a spatially flat, FRW spacetime with a cosmological fluid composed of vacuum and matter components such that $\Omega_m+\Omega_\Lambda=1$. The expansion scale factor is given by the expression

$$a(t)=\left[\frac{1-\Omega_\Lambda}{\Omega_\Lambda}\sinh^2\!\left(\frac{3}{2}H_0 t\sqrt{\Omega_\Lambda}\right)\right]^{1/3}, \tag{6.1}$$

where H_0, and Ω_Λ are the present-day Hubble constant and vacuum-matter density parameter. We may now follow a similar procedure as outlined in Sec. III to study the evolution of the long string length scale L and velocity v by taking the spatially flat, $\Omega\to 1$ limit in Eqs. (3.1) and (3.2). In this case we find that for a comparable matter density as in an open universe, the dilution of the string energy density and the damping of string motion are much weaker in the cosmological constant universe. Note that the argument of the sinh in the scale factor, evaluated at the present-day, is $\frac{1}{2}\ln|(1+\sqrt{\Omega_\Lambda})/(1-\sqrt{\Omega_\Lambda})|$. Hence, for small Ω_Λ the scale factor behaves to leading order as $a(t)\sim t^{2/3}$, just as for matter-dominated expansion. Only when $\Omega_\Lambda\to 1$ are the effects of the exponential expansion important, damping the string motion. For example, in the case of $\Omega_m=0.3$, the ratio L/t is only $\sim 5\%$ larger and the velocity is only $\sim 5\%$ smaller than the $\Omega_m=1$, spatially flat value. For the open universe with $\Omega=0.3$, the ratio L/t has grown by $\sim 15\%$ and the velocity has dropped by $\sim 30\%$ from their $\Omega=1$ values.

We may estimate the CMB normalization of the mass per unit length as a function of Ω_m following the methods of Sec. IV. However, there is no correction for the geometry, since the spatial sections are flat. Hence, we find the empirical formula

$$G\mu(\Omega_m)_\Lambda=1.05^{+0.35}_{-0.20}\times 10^{-6}\Omega_m^{-0.05} \tag{6.2}$$

fits our results to within 5% for $0.1\leqslant\Omega_m\leqslant 1$. The subscript Λ is used to differentiate the above normalization from the case of an open universe, in Eq. (4.6). We see that the normalization is relatively insensitive to the presence of a cosmological constant.

Finally, we may consider the properties of the cosmic string scenario for structure formation with CDM in the presence of a cosmological constant. We may adapt Eq. (5.3) for the string+CDM power spectrum by setting $\Omega=\Omega_m$ and using the appropriate growth factor [31]. The variance of the mass excess on length scales $R=8h^{-1}$ Mpc is fit by the empirical formula

$$\sigma_8(\Omega_m,h)=0.9(\pm 0.5)\times\left(\mu_6(\Omega)\frac{g(\Omega_m,\Omega_\Lambda)}{\Omega_m}\frac{\Gamma(1-0.36\Gamma)}{1+(50\Gamma)^{-2}}\right), \tag{6.3}$$

with

$$g(\Omega_m,\Omega_\Lambda)\equiv\tfrac{5}{2}\Omega_m/[\Omega_m^{4/7}-\Omega_\Lambda+(1+\tfrac{1}{2}\Omega_m)(1+\tfrac{1}{70}\Omega_\Lambda)],$$

which is valid to within $\sim 10\%$ for $0.1\leqslant\Omega\leqslant 1$ and $0.4<h<0.8$. We find that the amplitude of the string+CDM power spectrum with a cosmological constant is higher than in an open universe with the same matter density, as demonstrated in Fig. 3. Evaluating Eq. (6.3), we predict $\sigma_8(0.2,0.5)=0.3\pm 0.2$ and $\sigma_8(0.4,0.7)=0.5\pm 0.3$. Comparing to observations, based on the estimate $\sigma_8=0.6^{+32\%}_{-24\%}\exp[(-0.59-0.16\Omega+0.06\Omega^2)\ln\Omega]$ [50] for a spatially flat universe, we find that a slightly lower bias than in an open universe, $b\sim 1.5$–4, is required. Hence, the string+ΛCDM scenario may be viable if the strings generate a sufficient bias to explain the clustering on $8h^{-1}$ Mpc scales.

VII. CONCLUSION

In this paper we have laid out many of the tools necessary to study cosmic strings in an open universe. We have first derived the equations of motion and energy conservation in an $\Omega<1$ FRW space-time. We have extended the MS model of quantitative string evolution [12] to the case of an open, $\Omega<1$ universe. We believe this extrapolation is reasonable for the range of values of Ω of interest. We have found that with the onset of curvature dominated expansion, the long string energy density and mean velocity decay rapidly. We have shown that the resulting effect on the large angle CMB temperature fluctuations induced by cosmic strings is a lower level of anisotropy than in a critical, $\Omega=1$ universe, for the same μ. Constructing a semianalytic model for the generation of CMB anisotropy in an open universe, based in part on the AS numerical simulation [29,30], we found that comparison with the COBE-DMR observations [19] leads to a higher normalization of the cosmic string mass per unit length. To the extent that the CMB anisotropy induced by realistic cosmic strings has been accurately simulated in Ref. [17], we believe our results, Eqs. (4.6) and (6.2), are reliable within the errors discussed. The new normalization of μ, the first estimate of the normalization of μ in a low density universe (as far as we are aware), is consistent with all other observational constraints on cosmic strings, including the bound on a stochastic gravitational wave background arising from pulsar timing [51].

Finally, we have demonstrated the effect of an open, $\Omega<1$ universe on the power spectrum of density fluctuations produced by cosmic strings with HDM and CDM. As we mentioned in Sec. I, the power spectrum $P(k)$ does not completely specify the cosmic string structure formation scenario. Fluctuations generated by string wakes and filaments are non-Gaussian, so that knowledge of $P(k)$ alone is insufficient to specify all the properties of the density field. Although the linear power spectrum (5.3) is in agreement with the results of Avelino [32] and Colombi [37] on a limited range of scales, we are unable to make finely detailed comparisons with observations without more knowledge of the distribution of cosmic string seeded density perturbations. For example, it is not clear whether the estimates of the rms linear fluctuation in the mass distribution [34,36] obtained from the various galaxy redshift surveys, which depend strongly on the Gaussianity of the initial density field, are directly applicable to a theory with a non-Gaussian fluctuation spectrum. Nevertheless, we have found that the string +CDM spectrum fits the shape of the PD reconstruction of the linear power spectrum [34] for cosmological parameters in the range $\Gamma\sim 0.1$–0.2. We have computed the variance of the mass fluctuation in a sphere of radius $R=8\ h^{-1}$ Mpc, requiring a bias $b\gtrsim 2$ for consistency with the inferred σ_8 of the linear density field. In the case of a cosmological constant, a slightly lower bias is required than for an open universe string+CDM spectrum with the same matter density. These findings are similar to Ref. [6], in which the product $bG\mu$ was estimated in order to fit the string+CDM spectrum to the 1-in-6 Infrared Astronomy Satellite (IRAS) QDOT survey [52], and to Ref. [5], in which the effects of an open universe on global defects, including global strings and textures, were considered. The results of Ref. [44] indicate that the density of baryonic matter is enhanced in CDM wakes by a factor of ~ 2.4, suggesting that a bias $b\sim 2$ may be possible. It is clear that high resolution simulations, as Ref. [53], are necessary to further develop the cosmic string structure formation scenario.

The results presented in this paper provide excellent motivation to continue investigation of the cosmic string scenario, which should be possible with the equations of motion for strings and the normalization of μ for $\Omega<1$.

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