

## Effects of the magnetic dipole moment of charged vector mesons in their radiative decay distribution

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We consider the effects of anomalous magnetic dipole moments of vector mesons in the decay distribution of photons emitted in two-pseudoscalar decays of charged vector mesons. By choosing a kinematical configuration appropriate to isolate these effects from model-dependent and dominant bremsstrahlung contributions, we show that this method can provide a valid alternative for a measurement of the unknown magnetic dipole moments of charged vector mesons. [S0556-2821(97)01119-3]

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Electromagnetic multipole moments are important static properties that characterize particles and nuclei. While the electromagnetic current conservation imposes that the total electric charge must be conserved in a given reaction, its higher multipoles are not fixed in general by theoretical requirements and must be determined from experimental measurements. For elementary particles, the magnetic dipole moments of  $e^-$ ,  $\mu^-$  are measured with high precision [1] whereas better constraints on the magnetic dipole and electric quadrupole moments of  $W^\pm$  gauge bosons are becoming available from the CERN  $e^+e^-$  collider LEP 2 and Fermilab Tevatron colliders. In the case of hadrons, only the magnetic dipole moments of quasistable baryons have been measured, while those of hadronic resonances remain unknown [1].

The spin precession technique [2] used to measure the magnetic moments of octet baryons and  $\Omega^-$  is not applicable in the case of hadronic resonances due to their very short lifetimes ( $\leq 10^{-18}$  sec). An alternative method based on photon emission off hadrons [3] can be used in the latter case, because the photon carries information on higher multipoles of emitting particles.<sup>1</sup> As an application of this method, the angular distribution of soft photons emitted in  $\Omega^- \rightarrow \Lambda K^-$  decays has been computed in order to study the sensitivity to the anomalous magnetic moment of  $\Omega^-$  [4]. The results obtained with this method, however, are not competitive with the precision attained [5] using the usual spin precession technique [2].

In a previous paper [6] we have analyzed the effects of anomalous magnetic moments of charged vector mesons ( $\rho^+$ ,  $K^{*+}$ ) in the decay rates of two-pseudoscalar radiative decays. These decay rates are almost insensitive to the effects of magnetic dipole moments unless high values are used for the infrared cutoff photon energies. However, this reduction of photon phase space strongly suppresses the decay rates and makes difficult their accurate measurement using this method.

Following a similar approach, in this work we analyze the effects of  $\rho^+$  and  $K^{*+}$  anomalous magnetic moments in the decay distributions of photons emitted in the two-pseudoscalar decays of these vector mesons. In order to improve the sensitivity on these effects, we consider the photon energy spectrum for photons emitted at small angles with respect to charged pseudoscalar mesons.

Besides the possible experimental difficulties for reconstruction of these particular configurations, there are two limitations of the present approach. First, one should bear in mind that the decays of these unstable particles cannot be separated from its production process as required in our calculations. In fact, when considering the production and decay mechanisms of a charged resonance in a radiative process, some care must be taken [7] to maintain electromagnetic gauge invariance of the amplitude in the presence of the finite width of the resonance.<sup>2</sup> On the other hand, we neglect the vector meson decay widths appearing in the propagators after photon emission off vector mesons. While the first difficulty could be overcome by imposing appropriate cuts to suppress photon radiation in the production mechanism of the vector mesons, we expect that the second approximation accounts for neglecting terms of  $O(\Gamma/M)$  as far as the photon energy is not taken as very low. The importance of other reasonable approximations made in our calculations are discussed at the end of the paper.

As is known, the total magnetic moment for a positively charged ( $e > 0$ ) vector meson of mass  $M$  is given by

$$\mu_V = (1 + \kappa) \frac{e}{2M}, \quad (1)$$

where  $\kappa$  is the anomalous piece of the magnetic moment. In analogy with the  $W^\pm$  magnetic dipole moment in the standard electroweak theory,  $\kappa = 1$  can be considered as the natu-

<sup>1</sup>In fact, this method is used in the measurements of the  $W^\pm$  boson multipoles at the Tevatron collider.

<sup>2</sup>A corresponding analysis for the  $\rho^-$  resonance in the process  $\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \gamma$  is underway [8].

ral or canonical value for the vector mesons [9]. However, substantial deviations from this canonical value can be expected and in fact, some available calculations of  $\Delta\kappa \equiv \kappa - 1$  in the context of phenomenological quark models indicate

values as large as  $\Delta\kappa \sim 2.6$  [10] for the  $\rho$  meson.

Let us start with the structure of the gauge-invariant amplitude for the  $V^+ \rightarrow P^+ P^0 \gamma$  decay [ $V^+$  is the charged vector meson and  $P^+$  ( $P^0$ ) is a charged (neutral) pseudoscalar]:

$$\begin{aligned} \mathcal{M} = i e g_{VPP'} \left\{ \left( \frac{p \cdot \epsilon^*}{p \cdot k} - \frac{d \cdot \epsilon^*}{d \cdot k} \right) (p - p') \cdot \eta + \left( \frac{p \cdot \epsilon^*}{p \cdot k} - \frac{d \cdot \epsilon^*}{d \cdot k} \right) k \cdot \eta \right. \\ \left. + \left[ 2 + \frac{\Delta\kappa}{2} \left( 1 + \frac{\Delta^2}{M^2} \right) \right] \left( \frac{d \cdot \epsilon^*}{d \cdot k} k \cdot \eta - \epsilon^* \cdot \eta \right) - (2 + \Delta\kappa) \left( \frac{p \cdot \epsilon^*}{p \cdot k} k \cdot \eta - \epsilon^* \cdot \eta \right) \frac{p \cdot k}{d \cdot k} \right\} + \mathcal{O}(k) = \mathcal{M}_{\text{Low}} + \mathcal{O}(k). \end{aligned} \quad (2)$$

In the above expression,  $d$ ,  $p$ ,  $p'$ , and  $k$  denote, respectively, the four-momenta of  $V^+$ ,  $P^+$ ,  $P^0$ , and the photon,  $\eta$  ( $\epsilon^*$ ) is the polarization four-vector of  $V^+$  ( $\gamma$ ),  $\Delta^2 \equiv m_{P^+}^2 - m_{P^0}^2$ , and  $g_{VPP'}$  denotes the strength of the  $V^+ P^+ P^0$  interaction. The term in curly brackets in Eq. (2) corresponds to Low's amplitude [11], i.e., to the leading terms in the expansion of the amplitude for soft photons. The terms of order  $k^{-1}$  arise exclusively from photon emission off the charges of  $V^+$  and  $P^+$  and the terms of order  $k^0$  include also the photon emission amplitudes from the magnetic moment of  $V^+$  and a contact term which is necessary for gauge invariance.

The residual terms of order  $k$  in Eq. (2) contain contributions from the electric quadrupole moment of  $V^+$  and other possible model-dependent pieces. In the following we will neglect these contributions and discuss their relative importance at the end of this paper.

A straightforward calculation gives the squared amplitude (with the sum over vector meson polarizations)

$$\begin{aligned} \sum_{V^+ \text{ pols}} |\mathcal{M}_{\text{Low}}|^2 = e^2 g_{VPP'}^2 \left\{ \left| \frac{p \cdot \epsilon^*}{p \cdot k} - \frac{d \cdot \epsilon^*}{d \cdot k} \right|^2 \left( M^2 - 2\Sigma^2 + \frac{\Delta^4}{M^2} \right) + \frac{(\Delta\kappa)^2}{M^2} (p \cdot k)^2 \left| \frac{p \cdot \epsilon^*}{p \cdot k} - \frac{d \cdot \epsilon^*}{d \cdot k} \right|^2 \right. \\ \left. - \epsilon \cdot \epsilon^* \left[ 2 + \frac{\Delta\kappa}{2} \left( 1 + \frac{\Delta^2}{M^2} \right) - (2 + \Delta\kappa) \frac{p \cdot k}{d \cdot k} \right]^2 \right\}, \end{aligned} \quad (3)$$

where we have defined  $\Sigma^2 \equiv m_{P^+}^2 + m_{P^0}^2$ . The previous result is in agreement with the Burnett-Kroll's theorem [12] (see also [3]), which establishes the absence of terms of  $\mathcal{O}(k^{-1})$  in the probability for polarized photons. The terms of order  $k^{-1}$  appear only if we consider the squared amplitude for polarized photons *and* vector mesons.

In order to choose the decay distributions suitable to observe the effects of  $\Delta\kappa \neq 0$ , we set in the rest frame of the vector meson. In this case, the infrared factor in the previous result becomes

$$\sum_{\gamma \text{ pols}} \left| \frac{p \cdot \epsilon^*}{p \cdot k} - \frac{d \cdot \epsilon^*}{d \cdot k} \right|^2 = \frac{|\vec{p}|^2 \sin^2 \theta}{\omega^2 (E - |\vec{p}| \cos \theta)^2}, \quad (4)$$

where  $E$  and  $\omega$  are, respectively, the energies of the charged pseudoscalar and the photon in the rest frame of  $V^+$  ( $|\vec{p}| = \sqrt{E^2 - m_{P^+}^2}$ ). The angle  $\theta$  defines the direction of photon emission with respect to the charged pseudoscalar in the same frame.

Since the  $\Delta\kappa$ -dependent terms in Eq. (3) start at order  $\omega^0$ , we can expect, according to Eq. (4), that the differential decay distribution for photons of low energy would be more sensitive to  $\Delta\kappa \neq 0$  if we cut the large values of  $\theta$ . Using this property, in Figs. 1–3 we show the energy decay distributions of photons [normalized to the corresponding nonradiative rates, i.e.,  $(1/\Gamma_{nr}) d\Gamma/dx d\cos\theta$ , where  $x = 2\omega/M$ ] in the  $\rho^+ \rightarrow \pi^+ \pi^0 \gamma$ ,  $K^{*+} \rightarrow K^0 \pi^+ \gamma$ , and  $K^{*+} \rightarrow K^+ \pi^0 \gamma$  decays. The short-dashed lines in all these plots correspond to the

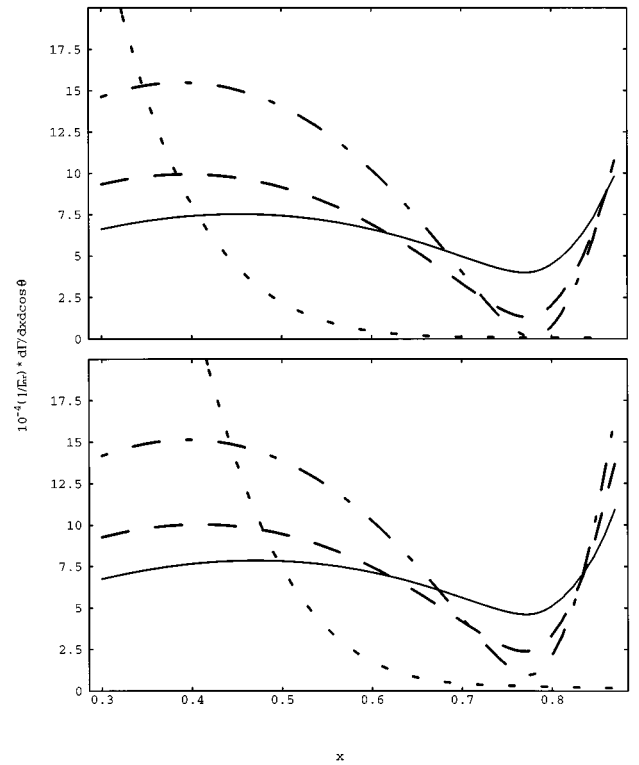


FIG. 1. Differential decay distribution of photons in the decay  $\rho^+ \rightarrow \pi^+ \pi^0 \gamma$ . The short-dashed plot corresponds to the term of order  $\omega^{-2}$  and the solid, long-dashed, and long-short-dashed plots are the terms of order  $\omega^0$  when  $\Delta\kappa = -1, 0, \text{ and } 1$ , respectively. The upper and lower parts are for  $\theta = 10^\circ$  and  $20^\circ$ , respectively.

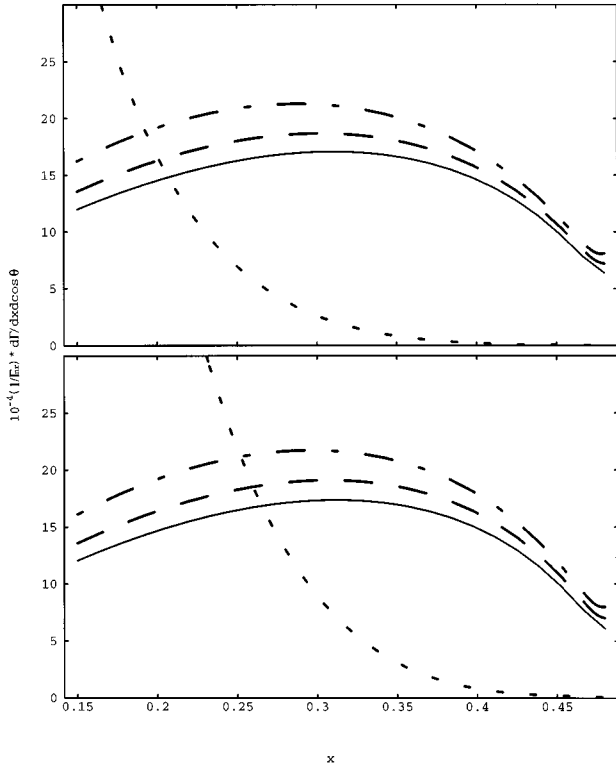


FIG. 2. Differential decay distribution of photons in the  $K^{*+} \rightarrow K^0 \pi^+ \gamma$  decay. The description is the same as in Fig. 1.

terms of order  $\omega^{-2}$  [first term in Eq. (3)] and arise exclusively from bremsstrahlung. The terms of order  $\omega^0$  in Eq. (3) are plotted for three different values of the anomalous magnetic moment:  $\Delta\kappa = -1$  (solid line),  $\Delta\kappa = 0$  (long-dashed), and  $\Delta\kappa = 1$  (long-short-dashed). The upper and lower parts in Figs. 1–3 correspond, respectively, to  $\theta = 10^\circ$  and  $20^\circ$ . Note that the only unknown parameter in the plotted distributions is  $\Delta\kappa$ .

As expected, the contributions of order  $\omega^0$  in Eq. (3) dominate over the terms of order  $\omega^{-2}$  except for very low values of the photon energy. Thus, the terms dependent on  $\Delta\kappa$  can be safely isolated by removing the—model-independent—contributions of order  $\omega^{-2}$ . On the other hand, while the energy distribution in  $K^{*+} \rightarrow K^0 \pi^+ \gamma$  is largely independent of the precise value of  $\Delta\kappa$ , the best sensitivity to the effects of anomalous magnetic moments in  $K^{*+}$  decays is observed in the  $K^{*+} \rightarrow K^+ \pi^0 \gamma$  channel. The physical reason for this is that the radiation emitted by a moving charge decreases with its velocity as observed in Eq. (4) ( $v = |\vec{p}|/E$  is smaller for  $K^+$  than for  $\pi^+$  in  $K^{*+}$  decays).

Another interesting property of these plots is the dip observed in the  $\rho^+$  and  $K^{*+} \rightarrow K^+ \pi^0 \gamma$  decays near the end of the photon energy spectrum. This dip (which corresponds to a vanishing distribution in the case  $\Delta\kappa = 1$ ) looks similar to the null radiation amplitudes observed in the angular distribution of some radiative processes [13]. We think, however, that its origin is not the same since this dip appears for the two values of  $\theta$  considered here and it is absent in the  $K^0 \pi^+$  mode of  $K^{*+}$  decay.

Based on Eq. (3), we can give a rough estimate of the accuracy expected for the measurement of  $\Delta\kappa$  using this method. For instance, if we assume  $\Delta\kappa = 0$  and  $\theta = 15^\circ$ , then

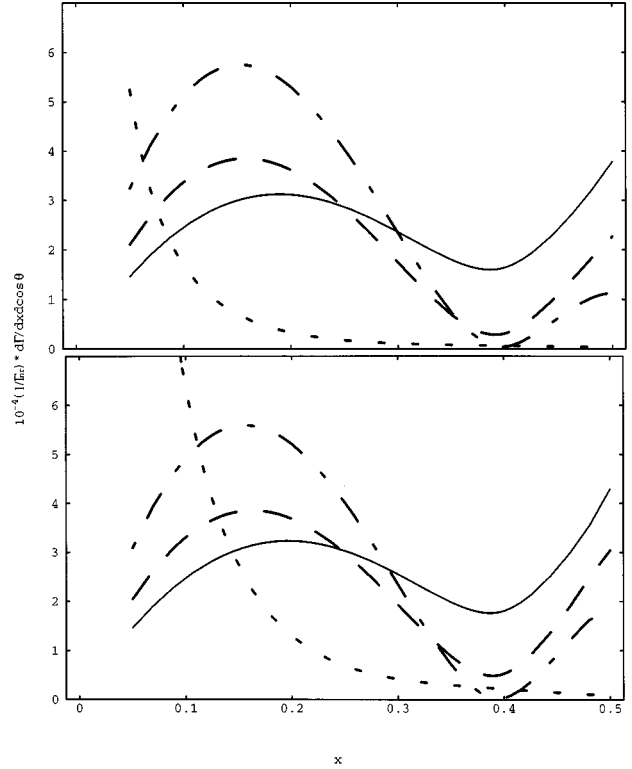


FIG. 3. Differential decay distribution of photons in the  $K^{*+} \rightarrow K^+ \pi^0 \gamma$  decay. The description is the same as in Fig. 1.

the decay distributions of  $\rho^+ \rightarrow \pi^+ \pi^0 \gamma$  and  $K^{*+} \rightarrow K^+ \pi^0 \gamma$  are required to be measured with a 25% error in order to achieve an accuracy of  $\delta|\Delta\kappa| = 0.5$ . This precision is almost independent of the full range of photon energies where the terms of order  $\omega^0$  clearly dominate over the terms of order  $\omega^{-2}$ .

Before concluding let us discuss the relative size of the contributions neglected in our calculation. As it was shown in [6], the model-dependent contributions of the type  $\rho^+ \rightarrow \pi^+ \omega \rightarrow \pi^+ \pi^0 \gamma$  and the analogous contributions to  $K^{*+}$  decays are negligible in the decay rate. We can expect negligible model-dependent contributions also in the decay distributions considered in this paper. Indeed, model-dependent contributions will start at order  $\omega^0$  and arise from the interference between model-dependent amplitudes (of order  $\omega$ ) and the term of order  $\omega^{-1}$ , i.e., their effects will be suppressed at small values of  $\theta$ . Since a similar argument holds for the contribution of the electric quadrupole moment, we can expect that our results for the photon spectra would not be sizably affected when we consider photons emitted at small angles with respect to charged pseudoscalar mesons.

In conclusion, the decay distributions of photons emitted in two-pseudoscalar radiative decays of charged vector mesons offer a valid alternative for a determination of their magnetic dipole moments. For this purpose, a measurement of the spectra of soft photons emitted at small angles with respect to charged pseudoscalar mesons is required. The model-dependent contributions and the one due to electric quadrupole moment of vector mesons are expected to be negligible for these kinematical configurations.

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