Extension of the geometrical two-chain model to high energies

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We present an extension of the geometrical two-chain model to higher energies. The agreement with experimental data is achieved using prehadronization chain breakups due to the colored dipole radiation mechanism and the discretization of the soft gluon emission process. [S0556-2821(97)04319-1]

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I. INTRODUCTION

The geometrical two-chain (G2C) model was established to describe relatively low (up to $\sqrt{s}\sim 100$ GeV) multiparticle production processes. The model gives a unified picture of a wide range of processes: from elastic hadron scattering and $e^+e^-\rightarrow$hadrons reactions (Ref. [1]), through diffractive dissociation in hadron-proton interactions (Ref. [2]), to non-single-diffraction hadron-hadron (Ref. [3]) and hadron-nucleus interactions (Ref. [4]). The geometrization of the interaction picture was obtained using the eikonal function formalism in terms of hadronic matter distribution of interacting hadrons (assuming pointlike leptons).

The geometrical two-chain model acts in two steps. The first is soft scattering, which turns incoming particles into two intermediate objects. They can be, to some extent, treated as chains comprised of quarks (diquarks) on their ends similar to the well-known string structures of the LUND (Ref. [5]) or dual parton (Ref. [6]) models. The second stage of the interaction is the hadronization of outgoing chains.

The creation of chains (strings) and their consecutive fragmentation, in a very general sense, is a common feature of all contemporary multiparticle production process descriptions. However, the particular realization differs from model to model. In the G2C model chain creation is not controlled by respective structure functions. In that sense it is a rather conservative and old-fashioned model but, on the other hand, such treatment has some advantages. Our knowledge of the structure function behavior, especially at low x values, is still not perfect. In our model the parametrization used does not pretend to be a fundamental law of nature but is just a *parametrization*, which can be changed and tuned to new data as they will appear. Generally speaking, our geometrization strategy is not in contradiction with the standard structure function approach.

II. GEOMETRICAL TWO-CHAIN MODEL

In the G2C model two chains are created on the first stage of the interaction process. The invariant mass of each of them ($M_{1,2}$) is determined by the impact parameter b of the particular collision and the c.m. system (c.m.s.) available energy ($\sqrt{s}$):

$$M_{1,2}\sim M_0+\left(\frac{\Omega(b)}{\Omega(0)}\right)^{\alpha}\left(\frac{\sqrt{s}}{2}-M_0\right), \qquad (1)$$

where M_0 is a mass of the lightest hadron that can be formed from the particular quark contents of the chain and Ω is related to the mass distributions in the colliding hadrons ($\rho_{1,2}$) by

$$\Omega(b)=\int d\vec{r}\rho_1(\vec{b})\rho_2(\vec{b}-\vec{r}). \qquad (2)$$

The value of the α parameter used in the present work is equal to 0.28.

In the case of the $e^+e^-\rightarrow$hadrons reaction only one chain is created. Thus $\sqrt{s}/2$ is replaced by $\sqrt{s}$ and the impact parameter is set to $0[\Omega(b)/\Omega(0)\equiv 1]$. One chain is created also in the single diffractive case and its mass is chosen randomly from the $1/M^2$ distribution. Two such chains appear in double diffractive events. [In the present paper only the non-singled-diffractive (NSD) events will be discussed.]

The hadronization starts later when all chains are formed. This is especially important for $h-\mathcal{N}$ interactions and for high-energy interactions in the multichain model, as discussed below. Hadrons are created from q-$\overline{q}$ (or diquark-antidiquark) pairs emerging from the chain c.m.s. energy uniformly in the phase space. This is an important point of the model: no additional dynamical constraints are introduced during the hadronization phase; these constraints originated from kinematical restrictions enforced by momentum and energy conservation laws. Thus the G2C model can be considered as a minimal model to study divergences between reality (experiments) and nonreducible kinematic (phase space) background. Such differences can be interpreted as being introduced by some dynamics of the multiparticle production process.

The transverse momentum of created quarks originate due to a tunneling mechanism described by the formula

$$f(p_\perp)\sim\exp\left(-\frac{m_\perp^2}{\kappa}\right) \qquad (3)$$

and it is conserved locally. The value of κ used in our calculations for light quark pair creation is equal to 0.28 while for the strange quarks and diquarks it is 0.38 and 0.6, respectively. Flavors of crated hadrons are given by the very few conventional suppression factors such as s/ud, qq/q, vector/scalar. The mass difference of the hadron under consideration and the lightest hadron that can be built from a particular quark configuration is also taken into account in a way similar to that given in Eq. (3).

The phase space density of the produced hadrons is again parametrized using the geometrical picture. The mean number of hadronization breakups of the chain $\langle n_{q\bar{q}}\rangle$ (q-$\bar{q}$ or qq-$\bar{q}\bar{q}$ pairs) is given by

$$\langle n_{q\bar{q}}\rangle = n_0\left(\frac{\Omega(b)}{\Omega(0)}\right)^{\beta}, \tag{4}$$

where

$$n_0 = A\ \ln(M_{\mathrm{chain}}) + B. \tag{5}$$

The actual number of created pairs $n_{q\bar{q}}$ is distributed due to the Poissonian distribution with the respective mean value. It is directly related to the number of first rank hadrons produced in the event. Values of β, A, and B used are 0.4, 6.7, and -6.2, respectively. The correction for low ($\leqslant 10$ GeV) chain masses is introduced. The smooth change of A to the value of 3.0 at $\leqslant 3$ GeV (B has to be changed respectively) gives a reasonable limit for particle multiplicities at extremely low energies and can be understood, to some extent, as an effect of the resonant hadron production.

The created hadron flavors are ordered in rapidity to preserve the information of the incoming hadron quark contents. For barions the popcorn mechanism from the LUND model (Ref. [7]) was adopted.

III. GEOMETRICAL MULTICHAIN MODEL

The G2C model as described above reproduces very well a number of interaction characteristics such as e.g., mean multiplicities, multiplicity distributions, and main inclusive distributions of produced particles. It has been shown in Refs. [1,3] up to $\sqrt{s}\sim 30$ GeV. For higher energies, of course, this model has to fail. Since initial state radiation (ISR) data have been published, the increase of the height of the plateau in inclusive rapidity distributions [which leads to the faster than ln(s) growth of the mean multiplicity] is a very well experimentally established fact. This was one of the most apparent reasons for investigations of a new physical mechanism responsible for very the high energy interaction picture. In the majority of current models the multistring intermediate state is present. One way of introducing it is developed in dual-parton- (DP-) like models (Ref. [6]) using multi-Pomeron structures. Another way, using the concept of gluon emission from expanding strings, is adapted to the LUND class of models (Refs. [5,8]).

In the framework of a geometrical interaction picture there is a natural place for this second approach. Just after formation of two chains in the ''soft scattering'' phase they can be forced to break up through the ''soft gluon emission'' before exact hadronization starts. Such a picture is a gist of the geometrical multichain model (GMC), which is the subject of this paper.

IV. SOFT GLUON EMISSION

The internal structure of the geometrical chain is in fact quite similar to that one of the LUND string. There are two colored objects of relatively low mass, each carrying invariant mass of M_{chain}. In the quark-antiquark (quark-diquark) rest system both colored ends have to move with almost the velocity of light so due to the existence of the color charge the color dipole radiation has to occur. Thus, it is expected that the soft gluon brehmsstrahlung cascade originates in such a system. Its mechanism is described extensively in the ARIADNE code (Ref. [8]) adopted by general LUND interaction programs. The ARIADNE 4.07 is a particular realization of this idea and it was used in GMC calculations in the present paper.

The probability of the soft gluon emission is well known (e.g., Refs. [9,5]) and can be described with analogy to QED using the Sudakov form factor in the following way:

$$\frac{dP(p_\perp^2,y)}{dp_\perp^2 dy} = \frac{d\sigma(p_\perp^2,y)}{dp_\perp^2 dy}\ \exp\left(-\int_{p_\perp^2}^{p_{\perp\max}^2} d^2k\,\mathcal{I}(k^2)\right), \tag{6}$$

where

$$\mathcal{I}(k^2) = \int_{y_{\min}(k^2)}^{y_{\max}(k^2)} dy'\,\frac{d\sigma(k^2,y')}{dk^2 dy'}, \tag{7}$$

$$d\sigma \sim \alpha_s \frac{dp_\perp^2}{p_\perp^2} dy. \tag{8}$$

Values of the model parameter used in our GMC realization are as the default setting in the ARIADNE 4.07 program. A small difference is connected with the gluon emission suppression related to a space extension of the emission source. This works for diquark chain ends only so it is not important for the $e^+e^-\to$hadrons processes. The parameter interpreted as a transverse source size is set in the present calculations to 1 GeV while in the default ARIADNE its value is equal to 0.6 GeV. This change is related to the analysis of Bose-Einstein correlations in hadronic interactions presented in Ref. [10]. However, it is not crucial for the results of this work. Very similar results could also be obtained with the value of 0.6 GeV and a respective slight change in other model parameters.

The important difference of the GMC model in comparison with standard ARIADNE soft gluon emission is in the implementation of the discretization of the process. The idea is described in the Ref. [11]. In this paper the (logarithmic) phase space [$\ln(p_\perp^2)$,y] is divided into discrete cells and the point is that each cell can be occupied by only one brehmsstrahlung gluon. The theoretical basis of this picture is widely discussed in the original paper. In the most general way it can be expressed as follows: if there are two gluons occasionally emitted too close to one another they combine together to finally form one ''effective gluon.'' The measure of the closeness can be derived, to some extent, from the running coupling constant QCD and for two gluons the minimum distance in rapidity should be of order of 11/6 (Ref. [11]).

In our model this idea is adopted in a way that gluons are produced by the standard ARIADNE colored antenna and then the gluon recombination is initiated. The distance between gluons is defined in three-dimensional [$\ln(p_x^2)$,$\ln(p_y^2)$,y] space. The recombination procedure starts with the pair of gluons closest to each other and finished when all ''effective gluons'' stay apart by at least the value of a parameter denoted by δy_g, which is going to be adjusted to the data.

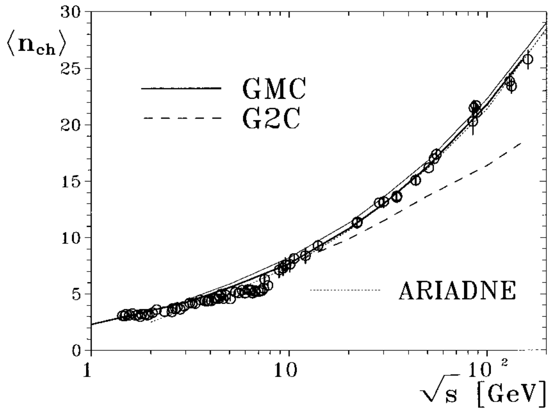

FIG. 1. Mean charged multiplicity in $e^+e^- \rightarrow$ hadrons reactions. The thick solid line represents result of the geometrical multichain model calculations while the long-dashed one is for the same parameter value set but without soft gluon emission process (pure two-chain picture). The thin solid line represents the GMC model results with weaker gluon clustering ($\delta y_g = 0$). The short-dashed line is a result of the LUND interaction model calculations. The data points are from [15].

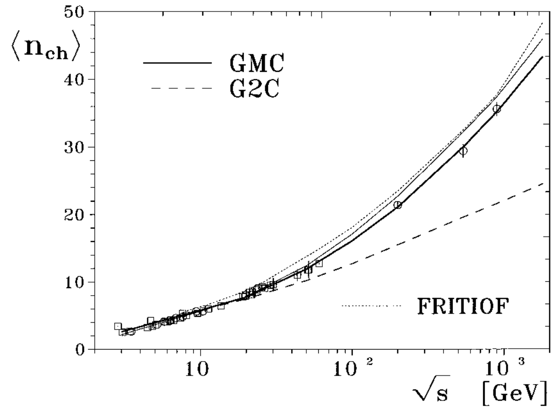

FIG. 2. Mean charged multiplicity in hadron-proton multiparticle production reactions. The thick solid line represents results of the geometrical multichain (GMC) model calculations while the long-dashed one is for the pure two-chain (G2C) interaction picture. The thin solid line represents the GMC model results with weaker gluon clustering ($\delta y_g = 0$). The short-dashed line is a result of the LUND model calculations. The data points are from [16].

After soft gluon emissions the main chain is broken into smaller parts. In the present version of the GMC model each gluon becomes a $q\bar{q}$ pair and its momentum is equally shared between them.

Further, chain pieces [each one containing again a q-$\overline{q}$ (qq-q) pair] of smaller masses and with some additional transverse momenta hadronize according to the G2C model described above.

V. RESULTS AND DISCUSSION

If the mean particle multiplicity produced during the hadronization phase is proportional to the logarithm of the chain mass then the emission of gluons with nonzero transverse masses and consecutive breakup of the chain before hadronization leads to the increase of the average multiplicity. Also the additional transverse momentum originated from the soft gluon brehmsstrahlung should increase the mean secondary hadron $p_\perp$. The question is as follows: is it possible to reach quantitative agreement with measurements preserving the idea described above with acceptable parameter values?

As done in Ref. [1], first we proceed with the mean hadron multiplicities in $e^+e^- \rightarrow$ hadrons reactions. The geometrical model parameters α and β in Eqs. (1) and (4) do not interfere with this $[\Omega \equiv \Omega(0)]$. The same can be said about parameters of the suppression of short wavelength gluon emissions by extended (diquark) chain ends in the ARIADNE model as described in Ref. [8]. Values of A and B in Eq. (5) were first obtained at this point. The parameter δy_g of the discrete QCD approximation used has a small influence on the e^+e^- mean multiplicity data due to the relatively small range of chain masses under study (the results for $\delta y_g = 0$ are shown in Fig. 1 by the thin solid line). The result of the GMC model calculations is presented in Fig. 1 by the thick solid line. It was obtained with the $\delta y_g = 1.45$ adjusted using mainly other data that will be shown below. Results of calculations without the soft gluon emission (pure G2C model with the same A and B parameter values) are given by the dashed line. Expected enhancement is clearly seen and the accuracy achieved is very satisfactory.

Predictions of the LUND interaction model (ARIADNE +JETSET with the set of parameters adjusted to the DELPHI data) are given also by the short-dashed line. At this point it should be clarified why the LUND model leads to results very similar to ours, for which we have used also ARIADNE procedures to describe soft gluon emissions but with the discretization procedure hereafter. There are of course differences in the hadronization description but they are not essential here. The important difference is in the treatment of colored antenna radiated gluons. The discretization process used in our GMC model works on a *gluon* level. Finally each gluon (''effective gluon'') has to form a common end of two chains, which then fragment independently. Thus each soft emission creates one additional chain, which is forced to hadronize to at least one final hadron. In the LUND picture the low mass (below some critical value) *jets* are combined with the nearby ones before they hadronize. Final results, as can be seen, are similar but the underlying physics is quite different.

In the next step parameters α and β in Eqs. (1) and (4) have to be adjusted using the proton-proton multiplicity data.

The mean charged multiplicity, multiplicity moments (defined as $C_k = \langle n_{\mathrm{ch}}^k \rangle / \langle n_{\mathrm{ch}} \rangle^k$) data, and the GMC model results are presented in the Figs. 2 and 3. The result of G2C calculations is also given.

To see the influence of the discretization of the gluon brehmsstrahlung process result of the calculation with the δy_g parameter equal to 0 is given in the Figs. 1 and 2 by the thin solid line. The difference is not very large but it has to be pointed out here that the number of gluons emitted is primarily limited by the threshold value of $p_\perp$ of emitted gluons used in a colored dipole radiation mechanism adopted in the ARIADNE code. Its value was set to 0.6 GeV/c. In principle, such a limit is equivalent to the assumption that all softer gluons are effectively included in harder emissions or hadronization processes. Thus $\delta y_g = 0$ does not mean absence clustering of emitted gluons, but only an omission of

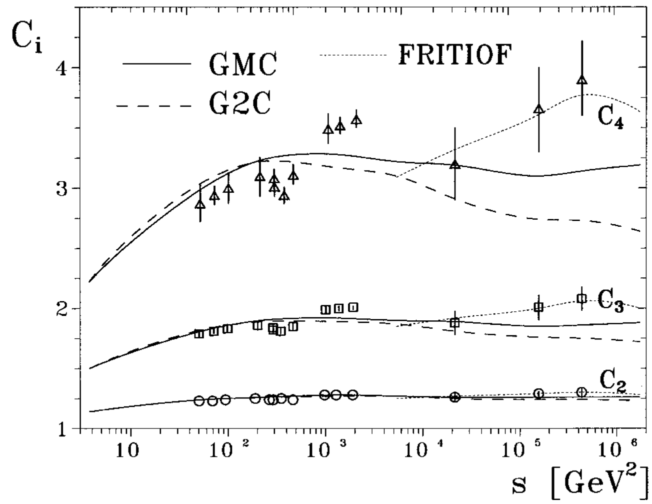

FIG. 3. Normalized charged multiplicity distribution moments for hadron-proton multiparticle production reactions. The solid line represents result of the geometrical multichain (GMC) model calculations while the long-dashed one is for the pure two-chain (G2C) interaction picture. The short-dashed line is a result of the LUND model calculations. The data points are from [16].

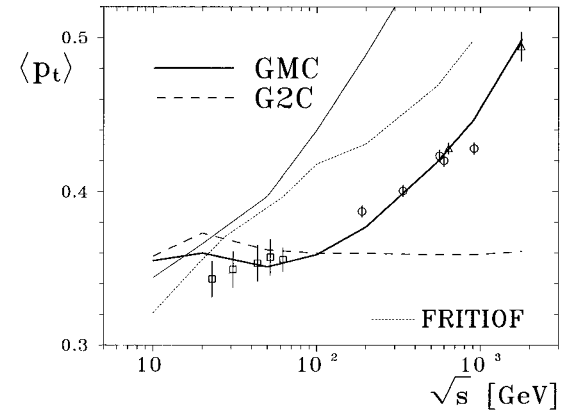

FIG. 4. Mean charged particle transverse momentum for hadron-proton multiparticle production reactions. The line description as in Fig. 2. The data point are from [17].

joining a relatively ''massive'' gluon.

The results of the LUND model calculations with the FRITIOF (Ref. [12]) program are also presented in Fig. 2 by the short-dashed line. For the soft gluon emission there are few different sets of ARIADNE program parameters. Here the same set called ''DELPHI'' was used as for the description of $e^+e^-\rightarrow$hadrons mean multiplicities in Fig. 1.

As it can be seen, mean multiplicities are reproduced by the GMC model very well while obtained multiplicity distributions are slightly narrower than those observed in experiments at the highest energies (Fig. 3). However, the difference is not very significant and, what is more important, the tendency to decrease the higher moments with the increasing interaction energy seen for G2C converts toward experimental results.

Obtained values of the model parameters α, β, A, and B were then used to determine finally the value of δy_g using the data on secondary hadron transverse momenta.

The dependence of the averaged transverse momentum on energy is shown in Fig. 4. The constant value (for high energies) predicted by the G2C model is settled by the value of the κ parameter in Eq. (3). The same value of κ with the soft gluon radiation mechanism gives an increase of average $p_\perp$ in perfect agreement up to the highest available energy accelerator data. The average transverse momenta of produced particles are quite sensitive to the gluon clustering mechanism. Thus they were used to determine the value of δy_g in the GMC model. For example, the very strong increase of the average value of $p_\perp$ for $\delta y_g = 0$ is shown in Fig. 4 (thin solid line). Finally δy_g was found to be equal to 1.45.

The most significant and crucial test of the GMC model is the comparison with inclusive energy [longitudinal momentum or (pseudo)rapidity] data. The increase of the plateau and the breaking of the Feynman scaling at high energies are two features of the highest importance in applications of the model for the hadronic cascade in the thick media.

Rapidity distributions for SPS and Tevatron energies are presented in Fig. 5. The very significant change from G2C to GMC model predictions is seen and the agreement obtained with the GMC model is meaningful. The results of FRITIOF with DELPHI set of parameters for ARIADNE are also given for comparison.

However good the agreement is, it cannot be treated as a proof of the correctness of the proposed model. There are other very well-known models available consistent with the data as well. Two main classes of such models are the one based on the DPM picture (there are many particular realizations of the DPM idea; see, e.g., Ref. [6]) and the relativistic string model of LUND (Ref. [5]). There are many particular realization of the DPM idea. The most complete comparison between some of them and the data is given in Ref. [13]. The very interesting model calculations are presented in Ref. [14]. The multipomeron exchange concept is combined there with the impact parameter description and the standard LUND string fragmentation procedures JETSET are used. The LUND model itself for proton-proton and $e^+e^-\rightarrow$hadrons reactions is available as a package of FRITIOF, ARIADNE, PYTHIA, and

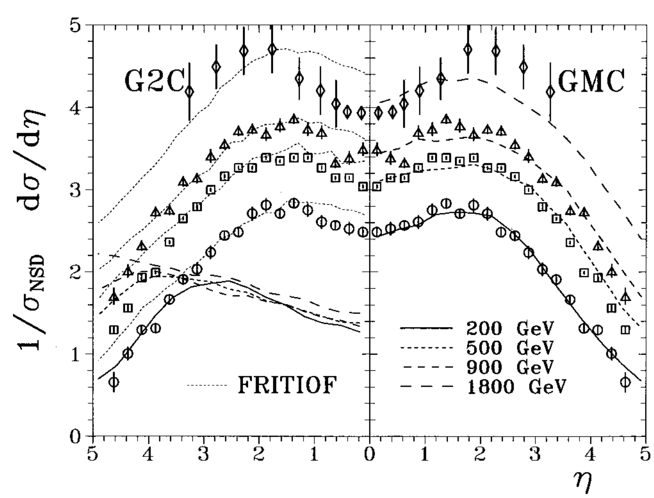

FIG. 5. Inclusive rapidity spectra of charged particles in proton-proton interactions at SPS and Tevatron energies. The results for G2C model are given in the left part of the figure while the present GMC model calculations are in the right part. The short-dashed lines in the left part are results of the LUND model. The data points are from [16].

JETSET routines. Results obtained using these codes are presented for all interaction characteristics discussed in the figures. It should be said here that even if a particular feature is better described by one model it is not an argument against or for any model. The LUND interaction picture is of course much more complete, which was shown in a number of papers. The point is that our GMC model with its physical assumption is *also* able to reproduce the data quite well.

VI. SUMMARY

The extension of the geometrical two-chain multiparticle production mechanism to higher energies is obtained by the introduction of the soft gluon emission process. The framework of the ARIADNE colored antenna radiation with the recent idea of discretization of the process is used to perform high-energy chain pre-hadronization breakups. The default ARIADNE 4.07 parameter values were used. The discrete net step δy_g was found to be equal to 1.45, which is close to the presumed value of 11/6. The difference is not significant according to the slightly different implementation of the $[\ln(p_\perp^2),y]$ cell idea.

The consistency between multiparticle hadronic interactions data and predictions of the GMC model presented in this paper and the solid theoretical basis of the model grant the capability to reasonably extrapolate the model to high energies.

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