

Pion-nucleon phase shifts in heavy baryon chiral perturbation theory

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We calculate the phase shifts in pion-nucleon scattering using the heavy baryon formalism. We consider phase shifts for the pion energy range of 140 to 200 MeV. We employ two different methods for calculating the phase shifts—the first using the full third order calculation of the pion-nucleon scattering amplitude and the second by including the resonances Δ and N^* as explicit degrees of freedom in the Lagrangian. We compare the results of the two methods with phase shifts extracted from fits to the pion-nucleon scattering data. We find good to fair agreement between the calculations and the phase shifts from scattering data. [S0556-2821(97)07919-8]

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The amplitudes for nonleptonic decays of hyperons are modulated by the final state strong scattering in the manner of the Fermi-Aidzu-Watson theorem [1]. These final state phase shifts are crucial in calculating the various CP -violating asymmetries in hyperon decays [2]. Some of the asymmetries depend on $\sin \delta$ where δ is some combination of the final state scattering phase shifts and a knowledge of δ is necessary to make predictions for CP violations in hyperon decays. Recently there have been new calculations [3] of $\Lambda-\pi$ scattering phase shifts in the framework of heavy baryon chiral perturbation theory (HBCHPT), with much smaller results than some earlier dispersive estimates [5]. Calculations of $\Lambda-\pi$ phase shifts are relevant to the measurement of CP violation in the hyperon decay $\Xi \rightarrow \Lambda \pi$ [2]. An experiment to measure the combined asymmetry $\Delta\alpha = \Delta\alpha_\Lambda + \Delta\alpha_\Xi$ will be carried out in the near future at Fermilab [4]. Here, for example, $\Delta\alpha_\Xi = \alpha_\Xi + \overline{\alpha_\Xi}$ and $\Delta\alpha_\Xi$ is proportional to $\tan(\delta_s - \delta_p)$. As a test of how reliable these phase shift calculations might be, we apply the same techniques to calculate the pion-nucleon phase shift in the energy range $E_\pi = 140-200$ MeV, where experimental data exist and can be compared to the predictions. It is important to point out that the calculations in Ref. [3] were done in a $SU(2)_L \times SU(2)_R$ HBCHPT and so we believe that a calculation in the $\pi-N$ sector should be a good test of the reliability of the calculations in Ref. [3].

The pion-nucleon Lagrangian in the heavy baryon has been written down to $O(p^3)$ (see Ref. [6] and the references therein) and has been used to calculate the S -wave scattering lengths for pion-nucleon scattering at threshold [7]. Recently the pion nucleon scattering has been calculated to $O(p^3)$ in Ref. [8]. We will use the Lagrangian to calculate the phase shifts away from threshold. Our purpose in this paper is to check that calculations of the phase shifts based on the HBCHPT are in reasonable agreement with the experimental data. We will not be aiming for precise matching of the theoretical and the experimental numbers but rather we will be satisfied if our calculations agree with the data to a factor of about 2. In the next section we describe the pion-nucleon

Lagrangian and describe our calculations while in the following section we discuss our results and summarize our calculations.

I. PION-NUCLEON LAGRANGIAN

The basic framework that we will employ to calculate the phase shifts is the heavy baryon chiral perturbation theory [9]. As is well known, the relativistic formalism of the chiral Lagrangian with baryons suffers from some severe problems which arise because the baryon mass does not vanish in the chiral limit and is not small relative to the scale of chiral symmetry breaking scale Λ_χ . The baryon four momentum is, therefore, not small relative to Λ_χ which results in the loss of the one to one correspondence between the loop and the small momentum expansion in CHPT. In the heavy baryon formalism, the baryons are treated as static sources and only the baryon momentum relative to the rest mass is important. The Lagrangian in this limit is constructed by taking the extreme nonrelativistic limit of the relativistic Lagrangian and expanding in inverse powers of the heavy baryon masses. In this formalism the one to one correspondence between the loops and the small momentum expansion is restored.

The lowest order $SU(2)_L \times SU(2)_R$ invariant pion nucleon Lagrangian can be written as [6]

$$\mathcal{L}_{\text{relativistic}} = \bar{\psi} \left[i \gamma \cdot D - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right] \psi$$

$$\xrightarrow{\text{static limit}} \bar{\psi} [i \gamma \cdot D + g_A S \cdot u] \psi, \quad (1)$$

where

$$S_\mu = \frac{i \gamma_5 \sigma_{\mu\nu} v^\nu}{2},$$

$$D_\mu = \partial_\mu + V_\mu,$$

$$V_\mu = \frac{1}{2} [\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger],$$

$$u_\mu = i [\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger]$$

$$\xi = e^{i\pi_a T_a / F_\pi}$$

(π_a represents the pion with isospin index a),

v^ν is the nucleon four velocity and in the rest frame of the nucleon S_μ is the spin operator. $F_\pi = 93$ MeV is the pion decay constant, $T_a = 1/2\tau^a$ are the isospin generators, and $g_A \sim 1.26$ is the axial vector coupling measured in the neutron β decay. This Lagrangian generates the vertices

$$\text{vector coupling: } \frac{1}{4F_\pi^2} \varepsilon^{abc} (v \cdot k_1 + v \cdot k_2) \tau^c,$$

$$\text{axial vector coupling: } \frac{g_A}{F_\pi} S \cdot k_2 \tau^a,$$

where k_1 is the incoming pion momentum and k_2 is the outgoing pion momentum. To the Lagrangian above we have to add the pion Lagrangian $\mathcal{L}_{\pi\pi}$ whose form is well known (for a recent review see Ref. [6]). We can organize the Lagrangian in terms of the small momentum in the calculation

$$\mathcal{L}(\pi, N) = \mathcal{L}^1(\pi, N) + \mathcal{L}^2(\pi, N) + \mathcal{L}^3(\pi, N), \quad (2)$$

where $\mathcal{L}^i(\pi, N)$ generates terms of $\sim p^i$, p being the small momentum. As already mentioned the scattering amplitude has been calculated to third order in the small momentum p in Ref. [8]. The low energy constants that appear in the chiral Lagrangian were fixed by fitting to the available pion-nucleon data on the threshold parameters like scattering lengths, volume, effective ranges, etc., the pion-nucleon σ term, and the Goldberger-Treiman discrepancy. It was noted in Ref. [8] and also verified by us that the full third order calculation results in a better description of the data. However, the expansion in the small momentum appears to be slowly converging with contributions from the various orders being sometimes of the same magnitude indicating the importance of higher order calculations. The calculations are expected to give good agreement with data near threshold but we will use these results to calculate the phase shifts away from threshold in the energy range $1080 \leq \sqrt{s} \leq 1130$ MeV. As mentioned in the introduction we are only interested in a reasonable agreement, by about a factor of 2, of our calculations with the experimental data.

The low energy constants in $\mathcal{L}^2(\pi, N) + \mathcal{L}^3(\pi, N)$ have also been estimated from the resonance exchange approximation. The basic idea is to start from a fully relativistic Lagrangian involving the pion and the nucleon resonances. The resonances are then integrated out from the theory to produce the higher dimensional terms in the pion-nucleon Lagrangian. In this scenario the Δ and the $N^*(1440)$ and the other resonances are not included as dynamical degrees of freedom in the effective theory for pion-nucleon scattering. The resonance approximation is clearly expected to work well close to threshold so that the resonances are far enough to be integrated out of the theory. The fact that the $\Delta(1232)$

is sufficiently close to the energy region we are interested in can be a motivation to include the Δ as an explicit degree of freedom in the chiral Lagrangian. We therefore include the Δ in our calculations and write the total Lagrangian as

$$\mathcal{L} = \mathcal{L}(\pi, N, \Delta) = \mathcal{L}(\pi, N) + \mathcal{L}(\pi, N, \Delta). \quad (3)$$

The disadvantage of including the Δ as an explicit degree of freedom is that the consistent power counting in HBCHPT is destroyed. This problem can be solved by treating $m_\Delta - m_N = \Delta$ as a small parameter and then considering a chiral Lagrangian expansion in the small momentum p which now includes the Δ [10]. Moreover, the full third order calculation should also include the additional loop contributions due to the Δ . In this paper, however, we will not follow the approach of Ref. [10] but for the purpose of the paper it will be sufficient to include only the tree level contribution from $\mathcal{L}(\pi, N) + \mathcal{L}(\pi, N, \Delta)$. A calculation based on this approximation may give a better description of the data for the P_{33} phase shift, where the Δ shows up as a resonance, compared to a calculation without the Δ included in the Lagrangian. However, we would like to stress that a full consistent third order calculation including the Δ should be performed before drawing any conclusion about the inclusion of the Δ in the Lagrangian. We will also include the $N^*(1440)$ in the Lagrangian whose effect may be important for the P_{11} phase shift. We will also expand the tree level contribution of the Δ and the N^* resonances in powers of the small momentum and try to understand the dominant contributions [11].

Following Jenkins and Manohar [12] one can incorporate the Δ by writing the Lagrangian

$$\mathcal{L}(\pi N \Delta) = \frac{3g_A}{2\sqrt{2}F_\pi} [\bar{T}^{\mu a} u_\mu^a N + \bar{N} u_\mu^a T^{\mu a}], \quad (4)$$

$$T^{\mu 1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \Delta^{++} - \frac{\Delta^0}{\sqrt{3}} \\ \frac{\Delta^+}{\sqrt{3}} - \Delta^- \end{bmatrix}, \quad (5)$$

$$T^{\mu 2} = \frac{i}{\sqrt{2}} \begin{bmatrix} \Delta^{++} + \frac{\Delta^0}{\sqrt{3}} \\ \frac{\Delta^+}{\sqrt{3}} + \Delta^- \end{bmatrix}, \quad (6)$$

$$T^{\mu 3} = -\sqrt{\frac{2}{3}} \begin{bmatrix} \Delta^+ \\ \Delta^0 \end{bmatrix}, \quad (7)$$

where

$$u_\mu^a = i \text{Tr}(T^a \xi^\dagger \partial_\mu U \xi^+),$$

$$U = \xi^2.$$

We can include the $N^*(1440)$ in our calculation also and write the interaction Lagrangian involving the N^* , pion, and the nucleon as

$$\mathcal{L}(\pi NN^*) = g_{N^*N\pi} \overline{\psi_{N^*}} S \cdot u \psi_N. \quad (8)$$

The coupling $g_{N^*N\pi}$ can be fixed from the branching fraction of $N^* \rightarrow N\pi$ [13].

In the static limit the momentum of off-shell N, Δ and $N^*(1440)$ are written as

$$P_N = m_N v + k,$$

$$P_\Delta = m_\Delta v + k,$$

$$P_{N^*} = m_{N^*} v + k,$$

where

$$k \sim p \sim k_\pi \ll m_{N, \Delta, N^*}.$$

The propagators for the various fields in the heavy baryon limit are

$$S_F(N) = \frac{i}{v \cdot k} = S_F(N^*),$$

$$S_F(\Delta) = -\frac{i}{v \cdot k - \Delta} \left[g^{\mu\nu} - v^\mu v^\nu + \frac{4}{3} S^\mu S^\nu \right],$$

while the pion propagator is

$$\Delta(\pi) = \frac{i}{q^2 - m_\pi^2}.$$

Calculation procedure

The pion-nucleon scattering amplitude with the outgoing (incoming) pion carrying the isospin index $b(a)$ can be written as

$$T^{ba} = T^+ \delta^{ab} + i \varepsilon^{bac} T^- \tau^c. \quad (9)$$

Our job is to calculate T^+, T^- . Having obtained these we can isolate the $T(\text{isospin}) = \frac{1}{2}$ and $T(\text{isospin}) = \frac{3}{2}$ amplitudes in the following manner. We first calculate

$$T(p\pi^- \rightarrow p\pi^-) = T^+ + T^-,$$

$$T(p\pi^- \rightarrow n\pi^0) = -\sqrt{2} T^-.$$

We can then use these amplitudes to extract

$$\begin{aligned} a_{T=1/2} &= T(p\pi^- \rightarrow p\pi^-) - \frac{1}{\sqrt{2}} T(p\pi^- \rightarrow n\pi^0) \\ &= T^+ + 2T^-, \end{aligned} \quad (10)$$

$$\begin{aligned} a_{T=3/2} &= T(p\pi^- \rightarrow p\pi^-) + \sqrt{2} T(p\pi^- \rightarrow n\pi^0) \\ &= T^+ - T^-. \end{aligned} \quad (11)$$

Next we expand the isospin amplitudes in partial waves (S, P waves) as

$$a_{T=1/2} = S_{11} + (2P_{13} + P_{11}) \cos \theta$$

$$+ i \sin \theta \sigma \cdot \hat{\mathbf{n}} (P_{13} - P_{11}), \quad (12)$$

$$\begin{aligned} a_{T=3/2} &= S_{31} + (2P_{33} + P_{31}) \cos \theta \\ &+ i \sin \theta \sigma \cdot \hat{\mathbf{n}} (P_{33} - P_{31}), \end{aligned} \quad (13)$$

where the subscripts refer to twice the I (isospin) and J (angular momentum) values, $\hat{\mathbf{n}} = (\mathbf{k}_1 \times \mathbf{k}_2) / |\mathbf{k}_1 \times \mathbf{k}_2|$, and θ is the scattering angle.

For a given partial wave amplitude $A_{2I, 2J}$ the phase shift $\delta_{2I, 2J}$, for small phase shift, is

$$A_{2I, 2J} \sim e^{i\delta_{2I, 2J}} \sin \delta_{2I, 2J} \approx \delta_{2I, 2J}, \quad (14)$$

where s is the center-of-mass (c.m.) energy. The scattering amplitude is in general complex and it develops an imaginary part at order p^3 . The above approximation assumes that the imaginary part of the scattering amplitude is small for small phase shifts. The contributions from \mathcal{L}^1 to the amplitudes are

$$\begin{aligned} T^+ &= \frac{g_A^2}{F_\pi^2 \omega} i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma k_{1\mu} k_{2\nu}, \\ T^- &= \frac{\omega}{2F_\pi^2} \left[1 - \frac{g_A^2 (\omega^2 - k_1 \cdot k_2)}{\omega^2} \right], \end{aligned} \quad (15)$$

where $\omega = v \cdot k$. The contributions from the Δ exchange are

$$\begin{aligned} &T(p\pi^- \rightarrow p\pi^- \text{ with } \Delta \text{ exchange}) \\ &= \left[\frac{3g_A}{2\sqrt{2}F} \frac{2}{3} \right]^2 i \bar{u}_p \left[(\omega^2 - k_1 \cdot k_2) \frac{\omega - 2\Delta}{\omega^2 - \Delta^2} \right. \\ &\quad \left. + i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma k_{1\mu} k_{2\nu} \frac{\Delta - 2\omega}{\omega^2 - \Delta^2} \right] u_p, \end{aligned} \quad (16)$$

$$\begin{aligned} T(p\pi^- \rightarrow n\pi^0) &= \left[\frac{3g_A}{2\sqrt{2}F} \frac{2}{3} \right]^2 \sqrt{2} i (-\bar{u}_n) \\ &\quad \times \left[(\omega^2 - k_1 \cdot k_2) \frac{\omega}{\omega^2 - \Delta^2} \right. \\ &\quad \left. + i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma k_{1\mu} k_{2\nu} \frac{\Delta}{\omega^2 - \Delta^2} \right] u_p, \end{aligned} \quad (17)$$

where

$$\Delta = m_\Delta - m_N.$$

From these expressions one can check that there is a pole only for the P_{33} channel where the Δ shows up as a resonance.

Following Ref. [11] we can, for $\omega < \Delta$, expand the Δ contribution in powers of ω/Δ . We can think of the various terms in the expansion as coming from higher order terms, beginning at order p^2 , of the chiral Lagrangian. We will

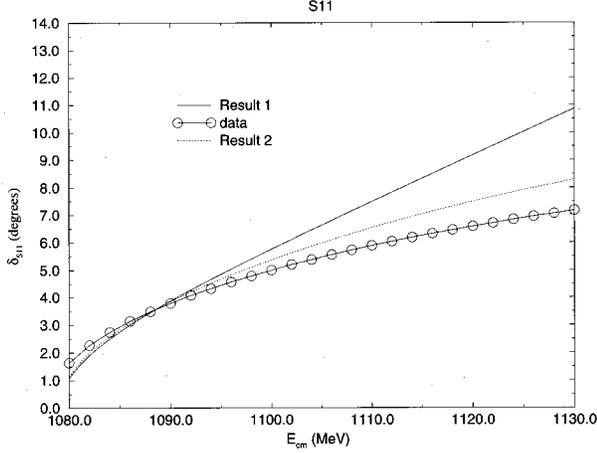


FIG. 1. S_{11} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data.

truncate the expansion at order p^4 and compare the results with the calculation done with including the full Δ contribution.

The contributions from N^* are

$$\begin{aligned}
 T^+ &= \frac{g_{N^*N\pi}^2}{F_\pi^2 \omega} i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma k_{1\mu} k_{2\nu} \frac{\omega}{\omega^2 - M^2} \\
 &\quad - \frac{g_{N^*N\pi}}{2F_\pi^2} (\omega^2 - k_1 \cdot k_2) \frac{M}{\omega^2 - M^2}, \\
 T^- &= \frac{g_{N^*N\pi}^2}{F_\pi^2 \omega} i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma k_{1\mu} k_{2\nu} \frac{M}{\omega^2 - M^2} \\
 &\quad - \frac{g_{N^*N\pi}}{2F_\pi^2} (\omega^2 - k_1 \cdot k_2) \frac{\omega}{\omega^2 - M^2}, \quad (18)
 \end{aligned}$$

where

$$M = m_{N^*} - m_N.$$

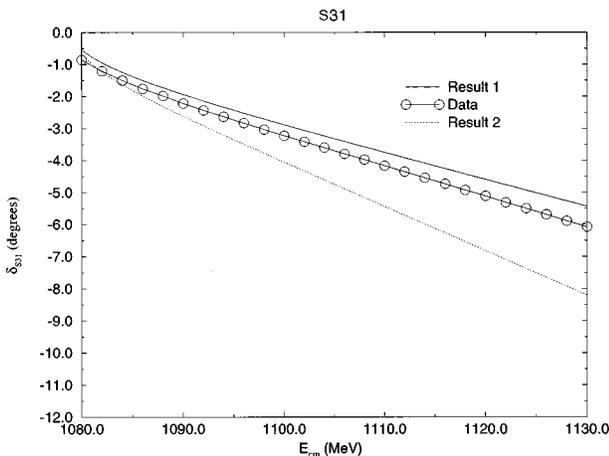


FIG. 2. S_{31} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data.

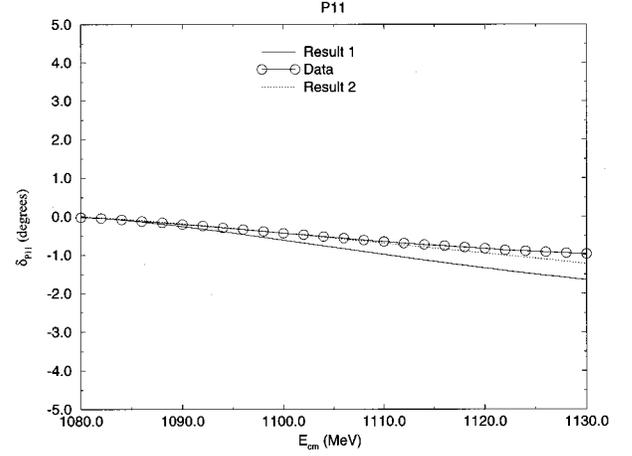


FIG. 3. P_{11} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data.

In this case the pole shows up in the P_{11} channel at the N^* mass. Like in the case for the Δ , we will also do an expansion to order p^4 by expanding the denominator in the above equations in powers of ω/M .

We therefore consider two cases in our calculations. In the first case we include the Δ and the $N^*(1440)$ in the effective Lagrangian and consider only the tree level contributions. For the second case we will use the full third order calculation without the Δ and the N^* given in Ref. [8]. All the details of the calculation can be found in Ref. [8] and we do not repeat them here. We will call the results of the calculations of the two cases as Result 1 and Result 2.

II. RESULTS AND DISCUSSIONS

In this section we present and discuss our results. We show our results for the S_{11} , S_{31} , P_{11} , P_{13} , P_{31} , and P_{33} partial waves. We compare our result to phase shifts extracted from fits to pion nucleon scattering data which were obtained from the SAID program [14]. In each figure we show generally three curves. Two curves show the calculation for

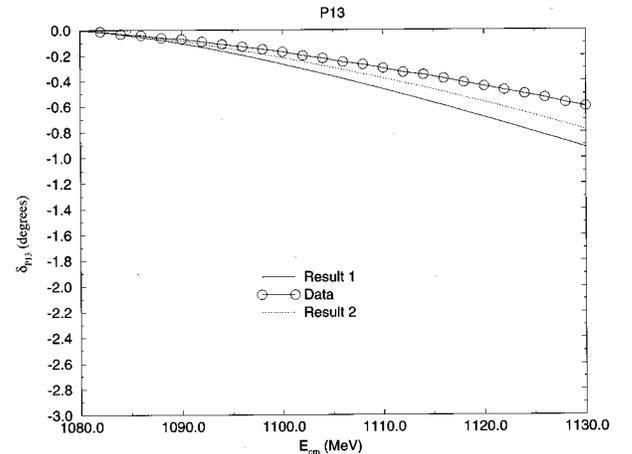


FIG. 4. P_{13} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data.

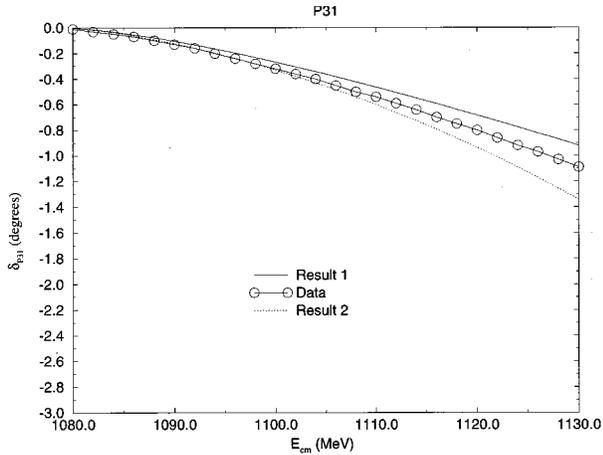


FIG. 5. P_{31} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data.

the two cases described above and the the third curve shows the phase shifts obtained from fits to experimental data from the SAID program. The errors for the phase shifts are available from the SAID program for certain single energy values. They vary typically from 0.1 to 0.3 degrees. In the energy range we are interested in there are no error calculations available for the phase shifts for the P_{13} and the P_{31} partial waves. We, therefore, do not show the error bars in our graphs. We also limit ourselves to small phase shifts, typically, < 10 degrees or in the energy range 1080–1130 MeV, corresponding to a pion energy range $E_\pi \approx 140$ –200 MeV, which includes the $\Lambda \rightarrow N\pi$ region.

In Fig. 1 we show the S_{11} phase shifts. We find the agreement with data is good to about 1100 MeV for calculations 1 and 2 beyond which the calculated phase shifts are larger than the experimental ones. The results of calculation 2 based on the calculations of Ref. [8] are in better agreement with the data compared to calculation 1. In Fig. 2 we show the S_{31} phase shifts. The agreement between the results of calculation 1 and experiment is reasonably good, with the result of calculation 1 being in better agreement with data than calculation 2, though near threshold the result of calculation 2 is in better agreement with data. In Fig. 3 we show the P_{11} phase shifts. The agreement between data and the calculation is better for Result 2 which describes the data quite well for most of the energy range. The tree level contribution from the N^* included in Result 1 improves agreement with data, but as already mentioned the result of calculation 2 describes the data better than calculation 1, indicating perhaps the importance of the other resonances whose effects are included in the low energy constants of the chiral Lagrangian if one believes in the resonance approximation. We also note that the $N^*(1440)$ may be far enough in mass from our region of interest to give the dominant contribution to the phase shift. In Fig. 4 we show the P_{13} phase shifts. There is moderate agreement with data. The agreement of calculation 2 with data is much better than that of calculation 1 again. In Fig. 5 we show the P_{31} phase shifts. Like P_{13} the agreement is moderately good with data, with results of calculation 1 and calculation 2 being less and more than the data. The result of calculation 2 is in good

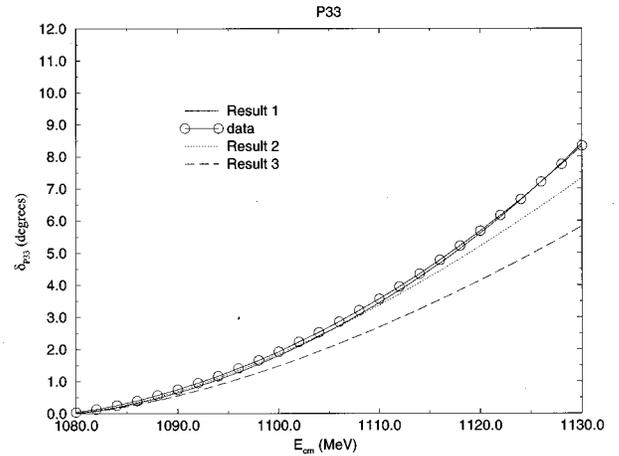


FIG. 6. P_{33} phase shifts. The solid, dotted lines correspond to calculation 1, 2 while the symbol ‘‘circle’’ represents the phase shifts extracted from fits to the pion-nucleon scattering data. In this figure we also show the result of calculation 1 (Result 3) with the Δ and the N^* contributions expanded to $O(p^4)$, represented by the long-dashed line.

agreement with the data to the center-of-mass energy of 1100 MeV. In Fig. 6 we show the P_{33} phase shifts. The agreement here with experiment is quite good. In this case calculation 1 is in better agreement with experiment than the results of calculation 2. We also perform calculation 1 but expand the Δ and N^* contributions to $O(p^4)$. The result of this calculation is close to those of calculation 1 to about $E_{c.m.} = 1100$ MeV, where the effects of the higher order terms (higher than p^4) in the expansion are not significant. Similar agreements between the results of this calculation and calculation 1 are also found for the other P - and the S -wave phase shifts for $E_{c.m.} \leq 1100$ MeV.

In the light of our results it appears that the results of calculation 2 are in better agreement with data than the results of calculation 1 except for the S_{31} and the P_{33} phase shifts. For the P_{33} phase shift this is probably due to the fact that the Δ contribution dominates the phase shift. However, as mentioned earlier, the expansion in small momentum is slowly converging and higher order terms are important. The inclusion of higher order terms in calculation 2 may improve the agreement with data for the P_{33} phase shift.

In summary we have calculated the S and P phase shifts in pion-nucleon scattering in heavy baryon chiral perturbation theory. We find that HBCHPT can give reasonable description of the phase shifts away from threshold in the energy range 1080–1130 MeV. The inclusion of the Δ may be necessary for the P_{33} phase shift but no final conclusion can be reached until higher order calculations are done. Based on these results, the results obtained earlier for $\Lambda - \pi$ phase shifts are probably correct within a factor of 2 and thus confirm the smallness of the $\Lambda - \pi$ phase shifts and in turn the smallness of $\Delta\alpha_\Xi$.

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