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Long distance $c \to u\gamma$ effects in weak radiative decays of D mesons

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We present a detailed analysis of the $D \to V\gamma$ transitions, using a model which combines heavy quark effective theory and the chiral Lagrangian approach and includes symmetry breaking. We notice that in addition to the previously considered s-channel annihilation and t-channel W exchange, there is a long-distance penguinlike $c \to u\gamma$ contribution in the t-channel of Cabibbo-suppressed modes. Its magnitude is determined by the size of symmetry breaking, which we calculate with a vector dominance approach. Although smaller in magnitude, the penguinlike contribution would lead to sizable effects in case of cancellations among the other contributions to the amplitude. Thus, it may invalidate suggested tests for beyond the standard model effects in these decays. We also indicate the range of expectations for the branching ratios of various $D \to V\gamma$ modes. [S0556-2821(97)08019-3]

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I. INTRODUCTION

The study of weak radiative decays of charmed mesons is still in its early developing stage. No such events have been observed so far and there is no published upper limits yet [1] for weak decays of D mesons involving emission of real photons. On the theoretical front, the treatment of these decays faces a different situation than encountered in the amply studied weak radiative decays of B and K mesons. In the former, flavor-changing radiative decays can be interpreted at the quark level as dominated by the short-distance electromagnetic penguin, i.e., the $b \to s\gamma$ transition [2]. In K-meson decays, both short-distance and long-distance contributions may compete in various transitions [3]. In the weak radiative decays of charmed particles, the short-distance $c \to u\gamma$ process has been shown to give a negligible contribution [4,5]; as a result, these decays present the challenge and opportunity of developing the required theoretical treatment for the long-distance dynamics involved.

The importance of a reliable description for these long-distance transitions is enhanced by the observation [6,7] that these decays provide also the interesting possibility for testing physics beyond the standard model, particularly nonminimal supersymmetric models.

The first comprehensive phenomenological analysis of all possible $D \to V\gamma$ weak decays has been presented only recently [4]; several other papers have considered [7–10] specific modes, using various models. On the basis of the standard model weak Hamiltonian with QCD corrections [11], it emerges [7,8] that in the quark picture without symmetry breaking the neutral charmed meson radiative decays can be viewed as due to W exchange in the $u(t)$ channel, their amplitude being proportional to the a_2 Wilson coefficient. The charged decays on the other hand, evolve from annihilation diagrams, and their amplitude is proportional to the a_1 Wilson coefficient.

In the present paper we calculate the decay amplitudes for nine $D \to V\gamma$ transitions, by using a hybrid model which is a combination of heavy quark effective theory (HQET) and chiral Lagrangian [8]. We include systematically SU(3) breaking into the amplitudes derived from it with particular attention paid to the coupling of vector mesons to photons as determined from experiment.

An additional contribution to the radiative decays is due to the long-distance $c \to u\gamma$ transition. The similar contributions involved in the radiative decays of b and s quarks have been analyzed recently in several papers. The basic idea is to consider [12] the long-distance penguinlike quark process $Q \to q + V$, in which the pairs of quark-antiquark produced in the weak process materialize into vector mesons. These vector mesons are then allowed to convert into photons via the usual vector dominance process. Using this procedure, the long-distance effects in $B \to s(d)\gamma$ have been estimated [12,13] with improved accuracy, as well as the effect in the $B \to Xl^+l^-$ decays [14]. In Ref. [13] it was shown that the long-distance $s \to d\gamma$ transition is likely to be significantly larger than the short-distance one. It was also pointed out in this paper that the size of this long-distance contribution, estimated with vector meson dominance, is determined by flavor symmetry breaking.

In this paper we consider the effect of the ‘‘long-distance penguin’’ embodied in the $c \to u\gamma$ transition, in the D mesons weak radiative decays. (The role of this transition in charmed baryon decays is discussed in Ref. [15].) When we include this contribution which is proportional to the a_2 Wilson coefficient, with SU(3) symmetry breaking, we find that this newly considered contribution is present in the Cabibbo-suppressed weak radiative decays of charm mesons. The inclusion of the long-distance $c \to u\gamma$ affects certain simple relations among the decay amplitudes $(D^0 \to \rho^0\gamma)/(D^0 \to \overline{K^{*0}}\gamma)$ and $(D_s^+ \to K^{*+}\gamma)/D_s^+ \to \rho^+\gamma)$ which were noted previously [6,7] and were suggested as possible tests for new physics. The effect is not present in the amplitudes of the Cabibbo-allowed decays $D^0\to\overline{K^{*0}}\gamma$ and $D_s^+\to\rho^+\gamma$. We point out that as a result of the Glashow-Iliopoulos-Maiani (GIM) mechanism, the long-distance $c\to u\gamma$ contribution will vanish in case of exact SU(3) symmetry.

Using a hybrid model [8] and including systematically SU(3) breaking into amplitudes derived from it, with particular attention paid to the coupling of vector mesons to photons as derived from experiment, we calculate all the $D\to V\gamma$ transitions. The numerical values of these amplitudes are displayed in Tables II and III. Since the relative phase of different contributions is unknown, we cannot make firm predictions for the expected rates. Nevertheless, taking this uncertainty into account and after fixing some of the constants of the model, we are able to indicate a fairly limited expected range for branching ratios of certain modes. Thus, we show that the Cabibbo-allowed $D_s^+\to\rho^+\gamma$ is expected to have a branching ratio of $(3-4.5)\times10^{-4}$, while the Cabibbo-suppressed decays $D_s^+\to K^{*+}\gamma$, $D^+\to\rho^+\gamma$ are expected to occur with branching ratios in the $(2-4)\times10^{-5}$ range.

In Sec. II we present the theoretical framework for our calculation. In Sec. III we display the explicit expressions of all calculated decay amplitudes and in Sec. IV we summarize and compare with previous calculations.

II. THE THEORETICAL FRAMEWORK

In this section we present in detail the theoretical basis needed for the calculation of the $D\to V\gamma$ amplitudes, which evolve from long-distance dynamics. This basis covers the strong and weak interaction sectors and throughout the presentation we explain our considerations for the choice of relevant numerical parameters.

A. Chiral lagrangians, heavy quark limit, and vector meson dominance

We incorporate in our Lagrangian [16] both the heavy flavor SU(2) symmetry [17,18] and the $SU(3)_L\times SU(3)_R$ chiral symmetry, spontaneously broken to the diagonal $SU(3)_V$ [19], which can be used for the description of heavy and light pseudoscalar and vector mesons. A similar Lagrangian, but without the light vector octet, was first introduced by Wise [20], Burdman and Donoghue [21], and Yan *et al.* [22]. It was then generalized with the inclusion of light vector mesons by Casalbuoni *et al.* [23].

The light degrees of freedom are described by the 3×3 Hermitian matrices

$$\Pi=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} \end{pmatrix} \tag{1}$$

and

$$\rho_\mu=\begin{pmatrix} \frac{\rho_\mu^0+\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{-\rho_\mu^0+\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \overline{K_\mu^{*0}} & \Phi_\mu \end{pmatrix} \tag{2}$$

for the pseudoscalar and vector mesons, respectively. They are usually expressed through the combinations

$$u=\exp\left(\frac{i\Pi}{f}\right), \tag{3}$$

where $f\simeq f_\pi=132$ MeV is the pion pseudoscalar decay constant, and

$$\hat{\rho}_\mu=i\frac{\tilde{g}_V}{\sqrt{2}}\rho_\mu, \tag{4}$$

where $\tilde{g}_V=5.9$ was fixed in the case of exact flavor symmetry [19]. In the following we will also use gauge field tensor $F_{\mu\nu}(\hat{\rho})$:

$$F_{\mu\nu}(\hat{\rho})=\partial_\mu\hat{\rho}_\nu-\partial_\nu\hat{\rho}_\mu+[\hat{\rho}_\mu,\hat{\rho}_\nu]. \tag{5}$$

It is convenient to introduce two currents $\mathcal{V}_\mu=\frac{1}{2}(u^\dagger D_\mu u+uD_\mu u^\dagger)$ and $\mathcal{A}_\mu=\frac{1}{2}(u^\dagger D_\mu u-uD_\mu u^\dagger)$. The covariant derivative of u and $u^\dagger$ is defined as $D_\mu u=(\partial_\mu+\hat{B}_\mu)u$ and $D_\mu u^\dagger=(\partial_\mu+\hat{B}_\mu)u^\dagger$, with $\hat{B}_\mu=ieB_\mu Q$, $Q=\mathrm{diag}(2/3,-1/3,-1/3)$, B_μ being the photon field.

The light meson part of the strong Lagrangian can be written as [19]

$$\mathcal{L}_{\text{light}}=-\frac{f^2}{2}\{\mathrm{tr}(\mathcal{A}_\mu\mathcal{A}^\mu)+a\,\mathrm{tr}[(\mathcal{V}_\mu-\hat{\rho}_\mu)^2]\}+\frac{1}{2\tilde{g}_V^2}\mathrm{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})]. \tag{6}$$

The constant a in Eq. (6) is in principle a free parameter. In the case of exact vector meson dominance (VMD) $a=2$ [19,24]. However, the photoproduction and decay data indicate [13] that the SU(3) breaking modifies the VMD in

$$\mathcal{L}_{V-\gamma}=-e\tilde{g}_Vf^2B_\mu\left(\rho^{0\mu}+\frac{1}{3}\omega^\mu-\frac{\sqrt{2}}{3}\Phi^\mu\right). \tag{7}$$

Instead of the exact SU(3) limit ($\tilde{g}_V=m_V/f$), we shall use the measured values, defining

$$\langle V(\epsilon_V,q)|V_\mu|0\rangle=\epsilon_\mu^*(q)g_V(q^2). \tag{8}$$

The couplings $g_V(m_V^2)$ are obtained from the leptonic decays of these mesons. In our calculation we use $g_\rho(m_\rho^2)\simeq g_\rho(0)=0.17\ \mathrm{GeV}^2$, $g_\omega(m_\omega^2)\simeq g_\omega(0)=0.15\ \mathrm{GeV}^2$, and $g_\Phi(m_\Phi^2)\simeq g_\Phi(0)=0.24\ \mathrm{GeV}^2$.

Both the heavy pseudoscalar and the heavy vector mesons are incorporated in a 4×4 matrix:

$$H_a=\frac{1}{2}(1+\not{v})(P^*_{a\mu}\gamma^\mu-P_a\gamma_5), \tag{9}$$

where $a=1,2,3$ is the $SU(3)_V$ index of the light flavors, and $P^*_{a\mu}$, P_a, annihilate a spin 1 and spin 0 heavy meson $Q\overline{q_a}$ of velocity v, respectively. They have a mass dimension 3/2 instead of the usual 1, so that the Lagrangian is in the heavy quark limit $m_Q\to\infty$ explicitly mass independent. Defining moreover

$$\overline{H_a}=\gamma^0 H_a^\dagger\gamma^0=(P^{*\dagger}_{a\mu}\gamma^\mu+P_a^\dagger\gamma_5)\frac{1}{2}(1+\not{v}), \tag{10}$$

we can write the strong Lagrangian as

$$\begin{aligned}\mathcal{L}_{\rm even}=&\mathcal{L}_{\rm light}+i{\rm Tr}(H_a v_\mu D^\mu\overline{H_a})+ig{\rm Tr}[H_b\gamma_\mu\gamma_5(\mathcal{A}^\mu)_{ba}\overline{H_a}]\\&+i\beta{\rm Tr}[H_b v_\mu(\mathcal{V}^\mu-\hat{\rho}^\mu)_{ba}\overline{H_a}]\\&+\frac{\beta^2}{4f^2}{\rm Tr}(\overline{H_b}H_a\overline{H_a}H_b).\end{aligned} \tag{11}$$

where $D^\mu\overline{H_a}=(\partial_\mu+\mathcal{V}_\mu-ieQ'B_\mu)\overline{H_a}$, with $Q'=2/3$ for c quark [8].

It contains two unknown parameters, g and β, which cannot be determined by symmetry arguments, but must be fitted by experiment. It is the most general even-parity Lagrangian in the leading heavy quark mass ($m_Q\to\infty$) and chiral symmetry ($m_q\to 0$ and the minimal number of derivatives) limit.

The electromagnetic field can couple to the mesons also through the anomalous interaction, i.e., through the odd parity Lagrangian. The contributions to this Lagrangian arise from terms of the Wess-Zumino-Witten kind, given by [24–26]

$$\mathcal{L}^{(1)}_{\rm odd}=-4\frac{C_{VV\Pi}}{f}\epsilon^{\mu\nu\alpha\beta}{\rm Tr}(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Pi). \tag{12}$$

The coupling $C_{VV\Pi}$ can be determined in the case of the exact SU(3) flavor symmetry following the hidden symmetry approach of [19,24] and it is found to be $C_{VV\Pi}=3\tilde{g}_V^2/32\pi^2=0.33$. In the following we use the VMD (7), however we allow for SU(3) symmetry breaking in the couplings of vector mesons to photon, which is expressed by the physical values of g_V and m_V. The decay width for $V\to P\gamma$ can be written as

$$\Gamma(V\to P\gamma)=\frac{\alpha}{24}\frac{(m_V^2-m_P^2)^3}{m_V^3}|g_{VP\gamma}|^2. \tag{13}$$

Using Eqs. (12) and (7) modified as explained above, and taking the experimental value for the K-meson decay coupling $f_K=0.160$ GeV we find

$$g_{\omega\pi\gamma}=4\frac{g_\rho}{m_\rho^2}\frac{C_{VV\Pi}}{f_\pi}, \tag{14}$$

$$g_{\rho\pi\gamma}=4\frac{g_\omega}{3m_\omega^2}\frac{C_{VV\Pi}}{f_\pi}, \tag{15}$$

$$g_{K^{*+}K^+\gamma}=2\left(\frac{g_\omega}{3m_\omega^2}+\frac{g_\rho}{m_\rho^2}-\frac{2}{3}\frac{g_\Phi}{m_\Phi^2}\right)\frac{C_{VV\Pi}}{f_K}, \tag{16}$$

$$g_{K^{*0}K^0\gamma}=2\left(\frac{g_\omega}{3m_\omega^2}-\frac{g_\rho}{m_\rho^2}-\frac{2}{3}\frac{g_\Phi}{m_\Phi^2}\right)\frac{C_{VV\Pi}}{f_K}. \tag{17}$$

At this point, we have the choice of using the symmetry value $C_{VV\Pi}=0.33$, or using a best fit by comparing Eqs. (14)–(17) to the experimental values [1]: $\Gamma(\omega\to\pi\gamma)=(7.25\pm0.5)\times10^{-4}$ GeV, $\Gamma(\rho^+\to\pi^+\gamma)=(6.8\pm0.6)\times10^{-5}$ GeV, $\Gamma(\rho^0\to\pi^0\gamma)=(1.2\pm0.3)\times10^{-4}$ GeV, $\Gamma(K^{*+}\to K^+\gamma)=(5.0\pm0.5)\times10^{-5}$ GeV, $\Gamma(K^{*0}\to K^0\gamma)=(1.2\pm0.1)\times10^{-4}$ GeV. We choose as a best fit the value $C_{VV\Pi}=0.31$, which reproduces the observed width of $K^{*+}\to K^+\gamma$ and gives $\Gamma(\omega\to\pi\gamma)=9.8\times10^{-4}$ GeV, $\Gamma(\rho\to\pi\gamma)=7.7\times10^{-5}$ GeV, $\Gamma(K^{*0}\to K^0\gamma)=1.42\times10^{-4}$ GeV. Comparing these figures with the experimental results, it is obvious that the inclusion of VMD with SU(3) breaking improves the results obtained [24] for exact SU(3). We remark that the inclusion of SU(3) symmetry breaking effects for these decays has been suggested often, including its inclusion in $C_{VV\Pi}$ [25–27]. Our approach here takes into account VMD with the observed values of g_V and m_V, without additional symmetry breaking parameters, while the approaches of [25,26] make fits using available experimental data on $\Gamma(V\to P\gamma)$.

We will also need the odd-parity Lagrangian in the heavy sector. There are two contributions [8,28] in it, characterized by coupling strenghts λ and λ'. The first is given by

$$\mathcal{L}_1=i\lambda{\rm Tr}[H_a\sigma_{\mu\nu}F^{\mu\nu}(\hat{\rho})_{ab}\overline{H_b}]. \tag{18}$$

In this term the interactions of light vector mesons with heavy pseudoscalar or heavy vector mesons is described. The light vector meson can then couple to the photon by the standard VMD prescription. This term is of the order $1/\lambda_\chi$ with λ_χ being the chiral perturbation theory scale [29].

The second term gives the direct heavy quark-photon interaction and is generated by the Lagrangian

$$\mathcal{L}_2=-\lambda'{\rm Tr}[H_a\sigma_{\mu\nu}F^{\mu\nu}(B)\overline{H_a}]. \tag{19}$$

The parameter λ' can be approximately related to the charm quark magnetic moment via $\lambda'\simeq 1/(6m_c)$ [17,22,29,30] and it should be considered as a higher order term in $1/m_Q$ expansion [29,30].

In order to gain information on these couplings we turn to an analysis of $D^{*0}\to D^0\gamma$, $D^{*+}\to D^+\gamma$, and $D_s^{*+}\to D_s^+\gamma$ decays. Experimentally, only the ratios $R_\gamma^0=\Gamma(D^{*0}\to D^0\gamma)/\Gamma(D^{*0}\to D^0\pi^0)$ and $R_\gamma^+=\Gamma(D^{*+}\to D^+\gamma)/\Gamma(D^{*+}\to D^+\pi^0)$ are known [1]. Using VMD with Eqs. (11), (18), (19) we calculate

$$R_\gamma^0=64\pi\alpha\frac{f^2}{g^2}\left(\frac{p_\gamma^+}{p_\pi^0}\right)^3\left[\lambda'+\lambda\frac{\tilde{g}_V}{2\sqrt{2}}\left(\frac{g_\rho}{m_\rho^2}+\frac{g_\omega}{3m_\omega^2}\right)\right]^2 \tag{20}$$

and

$$R_{\gamma}^{+}=64\pi\alpha\frac{f^2}{g^2}\left(\frac{p_{\gamma}^{+}}{p_{\pi}^{0}}\right)^3\left[\lambda'-\lambda\,\frac{\widetilde{g}_V}{2\sqrt{2}}\left(\frac{g_\rho}{m_\rho^2}-\frac{g_\omega}{3m_\omega^2}\right)\right]^2. \tag{21}$$

Numerically, the inclusion of SU(3) breaking changes $|\lambda'+\frac{2}{3}\lambda|$ in R_{γ}^{0} to become $|\lambda'+0.77\lambda|$ and $|\lambda'-\frac{1}{3}\lambda|$ in R_{γ}^{+}, becomes $|\lambda'-0.427\lambda|$. Taking the $R_{\gamma}^{0}=0.616$ and $R_{\gamma}^{+}=0.036$ [1], we obtain two sets of solutions for $|\lambda'/g|$ and $|\lambda/g|$. The first is $|\lambda/g|=0.533\ \mathrm{GeV}^{-1}$, $|\lambda'/g|=0.411\ \mathrm{GeV}^{-1}$ and the second is $|\lambda/g|=0.839\ \mathrm{GeV}^{-1}$, $|\lambda'/g|=0.175\ \mathrm{GeV}^{-1}$. In our calculation we have the combinations $|\lambda'+0.77\lambda|=0.821|g|\ \mathrm{GeV}^{-1}$ and $|\lambda'-0.427\lambda|=0.183|g|\ \mathrm{GeV}^{-1}$. For $D_s^{+*}D_s^{+}\gamma$ one derives the coupling $|\lambda'-\lambda\widetilde{g}_V/\sqrt{2}g_\Phi/3m_\Phi^2|=|\lambda'-0.32\lambda|$. Unfortunately, R_{γ}^{+} is poorly known, essentially within a factor of 3 [1], which could induce rather large errors in our determination of these constants. On the other hand, one should mention that the values we use for $R_{\gamma}^{+}, R_{\gamma}^{0}$ fit well the theoretical expectations for these ratios, as determined in rather different models [28–31].

In addition to strong and electromagnetic interaction, we have to specify the weak one. The nonleptonic weak Lagrangian on the quark level can be written as usual [11]:

$$\mathcal{L}_{SD}^{\mathrm{eff}}(\Delta c=\Delta s=1)=-\frac{G_F}{\sqrt{2}}V_{uq_i}V_{cq_j}^{*}[a_1(\overline{u}q_i)_{V-A}^{\mu}(\overline{q_j}c)_{V-A,\mu}+a_2(\overline{u}c)_{V-A,\mu}(\overline{q_j}q_i)_{V-A}^{\mu}], \tag{22}$$

where V_{ij} are the Cabibbo-Kaboyashi-Maskowa (CKM) matrix elements, G_F is the Fermi constant and $(\overline{\Psi}_1\Psi_2)^{\mu}\equiv\overline{\Psi}_1\gamma^{\mu}(1-\gamma^5)\Psi_2$. In our calculation we use $a_1=1.26$ and $a_2=-0.55$ as found in [11].

At the hadronic level, the weak current transforms as $(\overline{3}_L,1_R)$ under chiral $SU(3)_L\times SU(3)_R$ being linear in the heavy meson fields D^a and D_{μ}^{*a} and is taken as [16]

$$\begin{aligned}J_{Qa}^{\mu}=&\tfrac{1}{2}i\alpha\mathrm{Tr}[\gamma^{\mu}(1-\gamma_5)H_b u_{ba}^{\dagger}]\\&+\alpha_1\mathrm{Tr}[\gamma_5 H_b(\hat{\rho}^{\mu}-\mathcal{V}^{\mu})_{bc}u_{ca}^{\dagger}]\\&+\alpha_2\mathrm{Tr}[\gamma^{\mu}\gamma_5 H_b v_{\alpha}(\hat{\rho}^{\alpha}-\mathcal{V}^{\alpha})_{bc}u_{ca}^{\dagger}]+\cdots,\end{aligned} \tag{23}$$

where $\alpha=f_H\sqrt{m_H}$ [20], α_1 was first introduced by Casalbuoni *et al.* [23,32], while α_2 was introduced in [16]. It has to be included, since it is of the same order in the $1/m_Q$ and chiral expansion as the term proportional to α_1 [16].

B. The $c\rightarrow u\gamma$ long distance contribution

In addition to the photon interaction discussed in the previous subsection it was noticed [12,13] that the SU(3) breaking causes a long-distance penguinlike contribution proportional to the a_2 Wilson coefficient as shown in Fig. 1. Knowing that $V_{ud}V_{cd}^{*}=-V_{us}V_{cs}^{*}$ and using factorization one derives [12–15] the effective Hamiltonian for $c\rightarrow u\rho(\omega,\Phi)$ transition

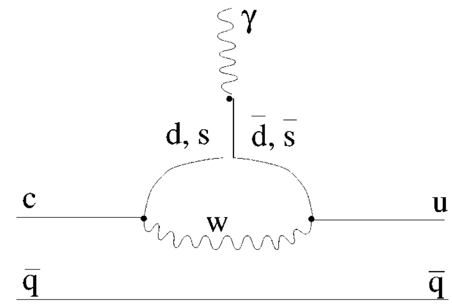

FIG. 1. The long-distance penguinlike $c\rightarrow u\gamma$ transition.

$$\mathcal{H}[c\rightarrow u\rho(\omega,\Phi)]=\frac{G_F}{\sqrt{2}}a_2V_{ud}V_{cd}^{*}\overline{u}\gamma^{\mu}(1-\gamma_5)c\times\left[-\epsilon_{\mu}^{\rho}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\epsilon_{\mu}^{\omega},-\epsilon_{\mu}^{\Phi}\right)\right]. \tag{24}$$

This equation is not in a gauge invariant form as necessary for replacement of V by γ using VMD in order to calculate the $c\rightarrow u\gamma$ from $c\rightarrow uV$. We employ a procedure which was advanced by Deshpande, Trampetić, and He [12] and by Golowich and Pakvasa [33] in treating the similar $b\rightarrow s\gamma$ via J/Ψ (see also [34]). Thus, one applies the Gordon identity [12] to extract the transverse part needed in the $V\rightarrow\gamma$ transition

$$m_c\overline{u}\gamma^{\mu}(1-\gamma_5)c=(2P^{\mu}-q^{\mu})\overline{u}(1+\gamma_5)c-i\overline{u}\sigma^{\mu\nu}q_{\nu}(1+\gamma_5)c, \tag{25}$$

where the term proportional to m_u has been neglected. P^{μ} is the c quark momentum and q^{μ} is the V momentum. If c quark is at rest, then the $(2P^{\mu}-q^{\mu})\epsilon_{T\mu}^{V}=0$, giving the following transverse part of the $c\rightarrow uV$ amplitude:

$$\begin{aligned}\mathcal{A}[c\rightarrow u\rho(\omega,\Phi)]_T=&\frac{G_F}{\sqrt{2}}a_2V_{ud}V_{cd}^{*}\frac{\overline{u}\sigma^{\mu\nu}(1+\gamma_5)c}{m_c}q_{\nu}\frac{1}{2}\\&\times\{-g_\rho(m_\rho^2)\epsilon_{\mu}^{*\rho}[g_\omega(m_\omega^2)\\&\times\epsilon_{\mu}^{*\omega},-\sqrt{2}g_\Phi(m_\Phi^2)\epsilon_{\mu}^{*\Phi}]\}.\end{aligned} \tag{26}$$

For the calculation of long-distance penguin contribution to $D\rightarrow V\gamma$ decay amplitude, one can to use the $D\rightarrow VV'$ decay amplitudes allowing then the transition $V'\rightarrow\gamma$. The simplest prescription for the gauge invariant VMD was given by Sakurai [35], extending the standard VMD Lagrangian in Eq. (7)

$$\mathcal{L}_{V-\gamma}^{\mathrm{ext}}=e\frac{g_V}{m_V^2}\left[-\frac{1}{2}F_{\mu\nu}V^{\mu\nu}-J_{\mu}^{V}B^{\mu}\right], \tag{27}$$

where J_{μ}^{V} is a conserved electromagnetic current given by Eq. (7), $V^{\mu\nu}$ is the vector meson field strength tensor $V^{\mu\nu}=\partial^{\mu}V^{\nu}-\partial^{\nu}V^{\mu}$, and g_V is defined in Eq. (8). This approach has been used by [4,33,34]. Here we follow their procedure and we derive the VMD amplitude for $c\rightarrow u\gamma$ to be given by

$$\mathcal{A}(c\to u\gamma)=\frac{G_F}{\sqrt{2}}a_2V_{ud}V_{cd}^*\frac{\overline{u}\sigma^{\mu\nu}(1+\gamma_5)c}{m_c}\times q_\nu\epsilon_\mu^{*\gamma}\left[-\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2(0)}{m_\Phi^2}\right]. \tag{28}$$

This expression (28) is now in a gauge invariant form. In the case of exact SU(3) symmetry the expression in square brackets vanishes as a result of GIM cancellations. This effect was found to be significantly larger than the short distance one in the $s\to d\gamma$ [13] and $c\to u\gamma$ [15] cases. Then the long distance penguin contribution in $D\to V\gamma$ amplitude is given by

$$A_{LD}(D\to V\gamma)=\frac{G_F}{\sqrt{2}}a_2V_{ud}V_{cd}^*\left[-\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2(0)}{m_\Phi^2}\right]\Bigg\{-\frac{2V(0)}{m_D+m_V}\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{*\gamma}\epsilon_\nu^{*V}\times p_\alpha p_{V\beta}+i(m_D+m_V)A_1(0)\left[\epsilon^{*\gamma}\cdot\epsilon^{*V}-\epsilon^{*\gamma}\cdot p_V\epsilon^{*V}\cdot q\frac{1}{p_V\cdot q}\right]\Bigg\}. \tag{29}$$

where the relevant form factors are defined in the matrix element $\langle V(p_V,\epsilon_V)|(V-A)^\mu|D(p)\rangle$, parametrized usually in $D\to Vl\nu_l$ semileptonic decay as [4,11,34,16,36]

$$\langle V(p_V,\epsilon_V)|(V-A)^\mu|D(p)\rangle=\frac{2V(q^2)}{m_D+m_V}\epsilon^{\mu\nu\alpha\beta}\epsilon_{V\nu}^*p_\alpha p_{V\beta}+i\epsilon_V^*\cdot q\frac{2m_V}{q^2}q^\mu[A_3(q^2)-A_0(q^2)]+i(m_D+m_V)\times\left[\epsilon_V^{\mu*}A_1(q^2)-\frac{\epsilon_V^*\cdot q}{(m_D+m_V)^2}(p+p_V)^\mu A_2(q^2)\right], \tag{30}$$

where $q=p-p_V$. In order that these matrix elements be finite at $q^2=0$, the form factors satisfy the relation [11]

$$A_3(q^2)-\frac{m_D+m_V}{2m_V}A_1(q^2)+\frac{m_D-m_V}{2m_V}A_2(q^2)=0, \tag{31}$$

and $A_3(0)=A_0(0)$.

Now, in order to obtain $V(0)$, $A_1(0)$ appearing in Eq. (29) we rely on the knowledge of form factors $|V^{DV}(0)|$ and $|A_1^{DV}(0)|$, determined in the semileptonic decays. Experimentally these form factors were extracted from the $D^+\to\overline{K}^{0*}l\nu_l$ and $D_s^+\to\Phi l\nu_l$ decays, assuming the pole behavior [1]. The hybrid model of [23,32], described in the previous section, works well for small recoil momentum in semileptonic decays, or equivalently in the case of $q_{\rm max}^2$. We follow their approach, since the experimental extrapolation of form factors at $q^2=0$ assumes their pole behavior. They have found [32]

$$V^{DV}(q_{\rm max}^2)=-\frac{\tilde{g}_V}{\sqrt{2}}\lambda f_{D'*}\frac{m_D+m_V}{v\cdot q_{\rm max}+m_{D'*}-m_D}, \tag{32}$$

where $m_{D'*}$ denotes the mass of the corresponding vector meson pole. The monopole assumption leads to

$$V^{DV}(0)=-\frac{\tilde{g}_V}{\sqrt{2}}\lambda f_{D'*}\frac{(m_D+m_V)(m_{D'*}+m_D-m_V)}{m_{D'*}^2}. \tag{33}$$

For the A_1 form factor the authors of [32] found

$$A_1^{DV}(q_{\rm max}^2)=-\frac{\tilde{g}_V}{\sqrt{2}}2\alpha_1\frac{\sqrt{m_D}}{m_D+m_V}, \tag{34}$$

and at $q^2=0$

$$A_1^{DV}(0)=-\tilde{g}_V\sqrt{2}\alpha_1\frac{\sqrt{m_D}}{m_D+m_V}\left[1-\frac{(m_D-m_V)^2}{m_{D_{1^+}}{}^2}\right], \tag{35}$$

where $m_{D_{1^+}}$ is the mass of the $\overline{q}c$ $J^P=1^+$ bound state. (We use the masses of $\overline{s}c$ and $\overline{d}c$ bound states to be 2.53 GeV and 2.42 GeV as in [32].) In [1] there are listed data on form factors at $q^2=0$ obtained from $D^+\to\overline{K}^{*0}l\nu_l$ and $D_s^+\to\Phi l\nu_l$ decays. From $D^+\to\overline{K}^{*0}l\nu_l$ decay the form factors are $|V^{DK^*}(0)|=1.0\pm0.2$, $|A_1^{DK^*}(0)|=0.55\pm0.03$ and $|A_2^{DK^*}(0)|=0.40\pm0.08$. From $D_s^+\to\Phi l\nu_l$ decay data it was extracted [1] $|V^{D_s\Phi}(0)|=0.9\pm0.3$, $|A_1^{D_s\Phi}(0)|=0.62\pm0.06$ and $|A_2^{D_s\Phi}(0)|=1.0\pm0.3$. Using the values $|V(0)|$ from both decays and taking the average of the two values derived from Eq. (33) for $|\lambda|$, we obtain $|\lambda|=0.479\ \mathrm{GeV}^{-1}$. This gives $|g|=0.58$, found from $|\lambda/g|=0.839\ \mathrm{GeV}^{-1}$. The value $|\lambda/g|=0.533\ \mathrm{GeV}^{-1}$ leads to somewhat higher value of g than expected [1].

Using again the average of two experimental $A_1(0)$ values, we obtain $|\alpha_1|=0.171\ \mathrm{GeV}^{1/2}$.

III. THE DECAY AMPLITUDES

The amplitudes for $D\to V\gamma$ can be written in the gauge-invariant form

$$A[D(p)\to V(\epsilon_V,p_V)\gamma(\epsilon_\gamma,q)]=e\frac{G_F}{\sqrt{2}}V_{uq_i}V_{cq_j}^*\{\epsilon_{\mu\nu\alpha\beta}q^\mu\epsilon_\gamma^{*\nu}p^\alpha\epsilon_V^{*\beta}A_{PC}+i(\epsilon_V^*\cdot q\epsilon_\gamma^*\cdot p_V-p_V\cdot q\epsilon_V^*\cdot\epsilon_\gamma^*)A_{PV}\}. \tag{36}$$

The A_{PC} and A_{PV} denote the parity conserving and parity violating parts of the amplitude [4]. The different contributions to the decay amplitude arising in our model are displayed in schematic form in Fig. 2. The photon can be first emitted from the D meson which becomes D^*, which then weakly decays into vector meson V. Their vertices are pro-

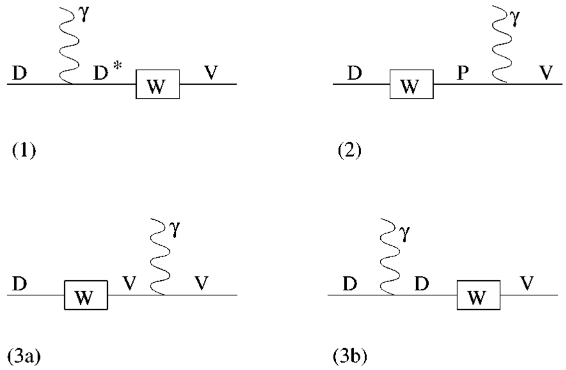

FIG. 2. Skeleton graphs for the various contributions to the decay amplitudes $D\to V\gamma$. Graph (1) contributes to $A_{PC}^{(I)}$, graph (2) to $A_{PC}^{(II)}$ part and graphs (3a), (3b) to A_{PV} in Table II.

portional to a combination of λ' and λ, calculated in the analysis of $D^*\to D\gamma$ decay amplitudes. We denote this part of amplitude as $A_{PC}^{(I)}$. When calculating these decay amplitudes, we have to remark that the $1/m_c$ corrections coming from light-quark current, effectively included into the λ' parameter, are not necessarily the same as in the case of $D^*\to D\gamma$. This uncertainty unfortunately increases present theoretical and experimental uncertainty already present in the calculation of $D\to V\gamma$. The second contribution comes from the weak decay of D meson, firstly into off-shell light pseudoscalar, which then decays into $V\gamma$. We denote this part of amplitudes as $A_{PC}^{(II)}$.

The charged charm meson can radiate a real photon from the term $-ev^\mu B_\mu\ \mathrm{Tr}[H_a(Q-2/3)_{ab}\overline{H}_b]$ given in Eq. (11), while the charged light vector meson can radiate through the last term of Eq. (6). Both contributions are present in A_{PV}. The SU(3) breaking effects are accounted in f_D, f_{D_s}, g_{K^*} and g_ρ [we take $g_{K^*}=(m_{K^*}/m_\rho)g_\rho$].

The long distance penguinlike contribution is present in the Cabibbo-suppressed charm meson decays. It contributes to both the parity-conserving and the parity-violating parts of the decay amplitude. We denote its contribution as $A_{PC}^{(III)}$ and $A_{PV}^{(III)}$.

The Cabibbo-allowed decay amplitudes are proportional to the product $|V_{ud}V_{cs}^*|$.

$$A_{PC}(D^0\to\overline{K^{*0}}\gamma)=4a_2\left|\lambda'+\lambda\,\frac{\tilde g_V}{2\sqrt2}\left(\frac{g_\rho}{m_\rho^2}+\frac{g_\omega}{3m_\omega^2}\right)\right| \times\frac{f_D g_{K^*}m_{D^*}}{m_{D^*}^2-m_{K^*}^2}\sqrt{\frac{m_{D^*}}{m_D}}+2a_2|C_{VV\Pi}| \times\left(\frac{g_\omega}{3m_\omega^2}-\frac{g_\rho}{m_\rho^2}-\frac{2}{3}\frac{g_\Phi}{m_\Phi^2}\right)\frac{f_D m_D^2}{m_D^2-m_K^2}, \tag{37}$$

$$A_{PV}(D^0\to\overline{K^{*0}}\gamma)=0, \tag{38}$$

$$A_{PC}(D_s^+\to\rho^+\gamma)=4a_1\left|\lambda'-\lambda\,\frac{\tilde g_V}{3\sqrt2}\frac{g_\Phi}{m_\Phi^2}\right| \times\frac{f_{D_s}g_\rho m_{D_s^*}}{m_{D_s^*}^2-m_\rho^2}\sqrt{\frac{m_{D_s^*}}{m_{D_s}}} +4a_1|C_{VV\Pi}|\frac{g_\omega}{3m_\omega^2}\frac{f_{D_s}m_{D_s}^2}{m_{D_s}^2-m_\pi^2}, \tag{39}$$

$$A_{PV}(D_s^+\to\rho^+\gamma)=2a_1\,\frac{f_{D_s}g_\rho}{m_{D_s}^2-m_\rho^2}. \tag{40}$$

The long-distance contribution of the penguinlike operators appears in the Cabibbo-suppressed ($V_{ud}V_{cd}^*$) decay amplitudes and has the a_2 Wilson constant as coefficent:

$$A_{PC}(D^+\to\rho^+\gamma)=4a_1\left|\lambda'-\lambda\,\frac{\tilde g_V}{2\sqrt2}\left(\frac{g_\rho}{m_\rho^2}-\frac{g_\omega}{3m_\omega^2}\right)\right| \times\frac{f_D g_\rho m_{D^*}}{m_{D^*}^2-m_\rho^2}\sqrt{\frac{m_{D^*}}{m_D}}+4a_1|C_{VV\Pi}| \times\left(\frac{g_\omega}{3m_\omega^2}\right)\frac{f_D m_D^2}{m_D^2-m_\pi^2}+2a_2\frac{|V^{D\rho}(0)|}{m_D+m_\rho} \times\left[-\frac12\frac{g_\rho^2}{m_\rho^2}+\frac16\frac{g_\omega^2}{m_\omega^2}+\frac13\frac{g_\Phi^2}{m_\Phi^2}\right], \tag{41}$$

$$A_{PV}(D^+\to\rho^+\gamma)=2a_1\,\frac{f_D g_\rho}{m_{D_s}^2-m_\rho^2}+2a_2\frac{1}{m_D-m_\rho}|A_1^{D\rho}(0)| \times\left[-\frac12\frac{g_\rho^2}{m_\rho^2}+\frac16\frac{g_\omega^2}{m_\omega^2}+\frac13\frac{g_\Phi^2}{m_\Phi^2}\right], \tag{42}$$

$$A_{PC}(D_s^+\to K^{*+}\gamma)=4a_1\left|\lambda'-\lambda\,\frac{\tilde g_V}{3\sqrt2}\frac{g_\Phi}{m_\Phi^2}\right|\frac{f_{D_s}g_{K^*}m_{D_s^*}}{m_{D_s^*}^2-m_{K^*}^2} \times\sqrt{\frac{m_{D_s^*}}{m_{D_s}}}+2a_1|C_{VV\Pi}| \times\left(\frac{g_\omega}{3m_\omega^2}+\frac{g_\rho}{m_\rho^2}-\frac23\frac{g_\Phi}{m_\Phi^2}\right)\frac{f_{D_s}m_{D_s}^2}{m_{D_s}^2-m_K^2} +2a_2\frac{|V^{D_sK^*}(0)|}{m_{D_s}+m_{K^*}} \times\left[-\frac12\frac{g_\rho^2}{m_\rho^2}+\frac16\frac{g_\omega^2}{m_\omega^2}+\frac13\frac{g_\Phi^2}{m_\Phi^2}\right], \tag{43}$$

TABLE I. The B^{VP_i} coefficients defined in relation (19), where $s=\sin\theta$, $c=\cos\theta$, and θ is the η-η' mixing angle.

B^{VP_i}	π	η	η'
ρ^0	$\frac{1}{3\sqrt{2}}\frac{g_\omega}{m_\omega^2}$	$-\frac{1}{\sqrt{2}}c(c-\sqrt{2}s)\frac{g_\rho}{m_\rho^2}$	$-\frac{1}{\sqrt{2}}s(\sqrt{2}c+s)\frac{g_\rho}{m_\rho^2}$
ω	$\frac{1}{\sqrt{2}}\frac{g_\rho}{m_\rho^2}$	$-\frac{1}{3\sqrt{2}}c(c-\sqrt{2}s)\frac{g_\omega}{m_\omega^2}$	$-\frac{1}{3\sqrt{2}}s(\sqrt{2}c+s)\frac{g_\omega}{m_\omega^2}$
Φ	0	$-\frac{\sqrt{2}}{3}c(\sqrt{2}c+s)\frac{g_\Phi}{m_\Phi^2}$	$-\frac{\sqrt{2}}{3}s(\sqrt{2}c-s)\frac{g_\Phi}{m_\Phi^2}$

$$A_{PV}(D_s^+\to K^{*+}\gamma)=2a_1\frac{f_{D_s}g_{K*}}{m_{D_s}^2-m_{K*}^2}+2a_2\left[-\frac{1}{2}\frac{g_\rho^2}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2}{m_\Phi^2}\right]\times\frac{|A_1^{D_sK*}(0)|}{m_{D_s}-m_{K*}}. \quad (44)$$

The Cabibbo-suppressed decays of D^0 meson involve the contribution from the η-η' mixing and we take the mixing angle $\theta=-20°$ [1]. We present the decay amplitudes for $D^0\to V^0\gamma$ ($V^0=\rho,\omega,\Phi$) as

$$A_{PC}(D^0\to V^0\gamma)=4a_2b_V^0\left|\lambda'+\lambda\frac{\tilde{g}_V}{2\sqrt{2}}\left(\frac{g_\rho}{m_\rho^2}+\frac{g_\omega}{3m_\omega^2}\right)\right|\times\frac{f_Dg_Vm_{D*}}{m_{D*}^2-m_V^2}\sqrt{\frac{m_{D*}}{m_D}}+4a_2|C_{VV\Pi}|\times f_Dm_D^2b^V+2a_2\left[-\frac{1}{2}\frac{g_\rho^2}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2}{m_\Phi^2}\right]\frac{|V^{DV^0}(0)|}{m_D+m_V}, \quad (45)$$

where $b_\rho^0=-1/\sqrt{2}$, $b_\omega^0=1/\sqrt{2}$, and $b_\Phi^0=1$. The coefficients b^V are obtained

$$b^V=\sum_{i=1}^{3}\frac{B^{VP_i}}{m_D^2-m_{P_i}^2}, \quad (46)$$

where P_i is π for ρ and ω and K for Φ. The coefficients B^{VP_i} are given in the Table I.

$$A_{PV}(D^0\to\rho^0/\omega\gamma)=2a_2\frac{|A_1^{D\rho}(0)|}{m_D-m_\rho}\left[-\frac{1}{2}\frac{g_\rho^2}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2}{m_\Phi^2}\right]. \quad (47)$$

For completeness, we give also the decay amplitudes of doubly suppressed decays $D^+\to K^{*+}\gamma$, $D^0\to K^{*0}\gamma$ in which case the amplitudes are proportional to $|V_{us}V_{cd}^*|$:

TABLE II. The parity conserving and parity violating amplitudes for Cabibbo-allowed charm meson decays in units 10^{-8} GeV^{-1}.

$D\to V\gamma$	$\|\mathcal{A}_{PC}^{I}\|$	$\|\mathcal{A}_{PC}^{II}\|$	$\|\mathcal{A}_{PV}\|$
$D^0\to\overline{K^{*0}}\gamma$	6.4	6.2	0
$D_s^+\to\rho^+\gamma$	1.4	7.3	7.4

$$A_{PC}(D^+\to K^{*+}\gamma)=4a_1\left|\lambda'-\lambda\frac{\tilde{g}_V}{2\sqrt{2}}\left(\frac{g_\rho}{m_\rho^2}-\frac{g_\omega}{3m_\omega^2}\right)\right|\frac{f_Dg_{K*}m_{D*}}{m_{D*}^2-m_{K*}^2}\times\sqrt{\frac{m_{D*}}{m_D}}+2a_1|C_{VV\Pi}|\left(\frac{g_\omega}{3m_\omega^2}+\frac{g_\rho}{m_\rho^2}-\frac{2}{3}\frac{g_\Phi}{m_\Phi^2}\right)\times\frac{f_Dm_D^2}{m_D^2-m_K^2}, \quad (48)$$

$$A_{PV}(D^+\to K^{*+}\gamma)=2a_1\frac{f_Dg_{K*}}{m_D^2-m_{K*}^2}, \quad (49)$$

$$A_{PC}(D^0\to K^{*0}\gamma)=4a_2\left|\lambda'+\lambda\frac{\tilde{g}_V}{2\sqrt{2}}\left(\frac{g_\rho}{m_\rho^2}+\frac{g_\omega}{3m_\omega^2}\right)\right|\frac{f_Dg_{K*}m_{D*}}{m_{D*}^2-m_{K*}^2}\sqrt{\frac{m_{D*}}{m_D}}+2a_2|C_{VV\Pi}|\left(\frac{g_\omega}{3m_\omega^2}-\frac{g_\rho}{m_\rho^2}-\frac{2}{3}\frac{g_\Phi}{m_\Phi^2}\right)\frac{f_Dm_D^2}{m_D^2-m_K^2}, \quad (50)$$

$$A_{PV}(D^0\to K^{*0}\gamma)=0. \quad (51)$$

We present numerical results for the parity conserving and parity violating amplitudes in Table II for the Cabibbo-allowed decays and for the Cabibbo-suppressed decays in Table III, where we denote $\mathcal{A}_{PC}=eG_F/\sqrt{2}V_{uq_i}V_{cq_j}^*A_{PC}$ and $\mathcal{A}_{PV}=eG_F/\sqrt{2}V_{uq_i}V_{cq_j}^*A_{PV}$. In our numerical calculation we use $f_{D_s}=0.240$ GeV [1], $f_D=0.2$ GeV, and we take $|\lambda'-0.32\lambda|=0.052$ GeV^{-1}, since we take $\lambda=0.479$ GeV^{-1} and $g=0.58$. We use also $|C_{VV\Pi}|=0.31$. In our estimation we did not analyze the errors arising from the experimental data. In the case of $D^*\to D\gamma$ decays they can

TABLE III. The parity conserving and parity violating amplitudes for Cabibbo-suppressed charm meson decays in units 10^{-9} GeV^{-1}. The last two decays are doubly Cabibbo-suppressed.

$D\to V\gamma$	$\|\mathcal{A}_{PC}^{I}\|$	$\|\mathcal{A}_{PC}^{II}\|$	$\|\mathcal{A}_{PC}^{III}\|$	$\|\mathcal{A}_{PV}\|$	$\|\mathcal{A}_{PV}^{III}\|$
$D^0\to\rho^0\gamma$	8.2	10.7	0.2	0	0.3
$D^0\to\omega\gamma$	7.3	10.7	0.2	0	0.3
$D^0\to\Phi\gamma$	18.8	13.4	0	0	0
$D^+\to\rho^+\gamma$	5.9	13.9	0.2	15.9	0.3
$D_s^+\to K^{*+}\gamma$	4.1	23.2	0.2	20.8	0.4
$D^+\to K^{*+}\gamma$	1.6	4.2	0	4.3	0
$D^0\to K^{*0}\gamma$	3.3	3.2	0	0	0

be as large as 100% [1]. An additional uncertainty is coming from the couplings taken rather far from their mass-shell values, although in $D\to V\gamma$ decays we expect that these deviations are still quite small. Unfortunately, the sign of λ', λ, $C_{VV\Pi}$, and g_V, cannot be determined from the present experimental data and therefore, we are not able to make concrete predictions for the decay rates. However, we notice that the penguinlike long-distance contribution (III) is quite small when compared to the dominant contributions and it amounts to a few percent to the decay amplitudes. Nevertheless, in the case of neutral charm meson decays it can be rather important due to possible cancellation of the contributions I and II. This contribution is the only source of parity violating amplitudes for $D^0\to\rho^0(\omega)\gamma$ decay.

We note that the long-distance $c\to u\gamma$ contribution has a coefficient

$$C_V=-\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}+\frac{1}{3}\frac{g_\Phi^2(0)}{m_\Phi^2}, \tag{52}$$

which we calculate assuming there is no q^2 dependence in the g_{V_i} values between $m_{V_i}^2$ and 0. Should such a dependence occur, it would obviously affect the value of C_V in view of sensitive GIM cancellation involved.

We come now to the relevance of this $c\to u\gamma$ contribution with respect to possible tests for new physics in D decays.

The ratio of decay widths $R_K=\Gamma(D_s^+\to K^{*+}\gamma)/\Gamma(D_s^+\to\rho^+\gamma)$ was suggested recently to be used as a test of the physics beyond the standard model [7]. It was noticed that in the case of exact SU(3) symmetry $R_K=\tan^2\theta_c$ [up to the phase space factor $(q_{K*}/q_\rho)^3$]. In addition to SU(3) breaking effects coming from different masses and couplings, we notice that the presence of the penguinlike contribution modifies the ratio R_K. If there is no cancellation among the other contributions to the amplitudes, the modification may be several percent only, in the same range or even smaller than SU(3) breaking. However, before we gain enough knowledge from experiment on the size of the amplitudes, it is rather difficult to expect that the sign of new physics can be seen from the deviations from this ratio. In any case, it is instructive to note here that a typical figure for the amount of SU(3) breaking can be obtained, e.g., by comparing the calculated ratio $\mathcal{A}_{PV}(D_s^+\to K^{*+}\gamma)/\mathcal{A}_{PV}(D_s^+\to\rho^+\gamma)=[g_{K*}(m_{D_s}^2-m_\rho^2)]/[g_\rho(m_{D_s}^2-m_{K*}^2)]\tan\theta_c=1.24\tan\theta_c$, to the symmetry value of $\tan\theta_c$. The ratio $R_\rho=\Gamma(D^0\to\rho^0\gamma)/\Gamma(D^0\to\overline{K^{*0}}\gamma)$ offers the same possibility [6] to look for a deviation from $R_\rho=\frac{1}{2}\tan^2\theta_c$. We point out that the same conclusion is valid for R_ρ as for R_K.

IV. SUMMARY

We have reinvestigated charm meson weak radiative decays into light vector mesons, systematically including SU(3) symmetry breaking effects using a VMD approximation. The coupling of charm vector, charm pseudoscalar mesons and photons are changed due to this symmetry breaking, as well as Wess-Zumino-Witten couplings in the light sector.

In addition to these known contributions, we have found that the long-distance penguinlike contribution, proportional to a_2 Wilson coefficient appears in the charm meson radiative weak Cabibbo-suppressed decays. The parity conserving and parity violating decay amplitudes obtain typically a few percent contribution of the $c\to u\gamma$ long distance penguin operator.

Although this effect is not very large, in the case of neutral charm meson decay it might play an important role, due to possible cancellation of the dominant contributions.

At this point, we would like to select and summarize those of our results which have smaller uncertainties and to compare them with previous calculations. Among the Cabibbo-allowed decays, $D_s^+\to\rho^+\gamma$ is less affected by interference and we expect it to occur with the significant branching ratio $B(D_s^+\to\rho^+\gamma)=(3-5)\times10^{-4}$. Among the Cabibbo-suppressed decays, those involving less uncertainties are $D^+\to\rho^+\gamma$ and $D_s^+\to K^{*+}\gamma$ and we expect their occurrence with $B(D^+\to\rho^+\gamma)=(1.8-4.1)\times10^{-5}$ and $B(D_s^+\to K^{*+}\gamma)=(2.1-3.2)\times10^{-5}$. These results are fairly close to those of Ref. [4], but of a more precise range and present an interesting challenge for experiment. We remark that the prediction of Ref. [9] is one order of magnitude smaller.

Concerning the other decays we calculate, our range of predictions is considerably weaker. Thus, $D^0\to\overline{K^{*0}}\gamma$ could have a branching ratio as high as 3×10^{-4}, but it also might be orders of magnitude smaller as a result of interference between A_{PC}^I and A_{PC}^{II}. Its measurement is therefore of great interest. Likewise, the decays $D^0\to(\rho^0,\omega^0,\Phi^0)\gamma$ could have branching ratios as large as 2×10^{-5}, though in case of large negative interference among various contributions the branching ratio may be reduced by two orders of magnitude.

A main difference between our results and those of Ref. [4] is the amount of parity-violation in various decays. These authors treat $D\to V\gamma$ as driven by $D\to VV'\to V\gamma$ [33]. However, the D-meson nonleptonic decays are rather difficult for any theoretical description [4] and the experimental errors on $D\to VV'$ are rather large [1]. In any case, this issue will be clarified by the awaited experiments.

Finally, we note that the suggestion that the ratios $R_K=\Gamma(D_s^+\to K^{*+}\gamma)/\Gamma(D_s^+\to\rho^+\gamma)$ [7] or $R_\rho=\Gamma(D^0\to\rho^0\gamma)/\Gamma(D^0\to\overline{K^0}\gamma)$ [6] might be useful in a search of a signal for physics beyond the standard model, could be invalidated by the presence of long-distance penguinlike $c\to u\gamma$ contributions in case of large cancellations among various contributions to the amplitudes. This effect would affect more the R_ρ ratio.

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