Infinite momentum frame calculation of semileptonic heavy $\Lambda_b \to \Lambda_c$ transitions including HQET improvements

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We calculate the transition form factors that occur in heavy Λ-type baryon semileptonic decays such as, e.g., in $\Lambda_b \to \Lambda_c^+ + l^- + \bar{\nu}_l$. We use Bauer-Stech-Wirbel-type infinite momentum frame wave functions for the heavy Λ-type baryons which we assume to consist of a heavy quark and a light spin-isospin zero diquark system. The form factors at $q^2 = 0$ are calculated from the overlap integrals of the initial and final Λ-type baryon states. To leading order in the heavy mass scale the structure of the form factors agrees with the HQET predictions including the normalization at zero recoil. The leading order ω dependence of the form factors is extracted by scaling arguments. By comparing the model form factors with the HQET predictions at $O(1/m_Q)$ we obtain a consistent set of model form factors up to $O(1/m_Q)$. With our preferred choice of parameter values we find that the contribution of the nonleading form factor is practically negligible. We use our form factor predictions to compute rates, spectra, and various asymmetry parameters for the semileptonic decay $\Lambda_b \to \Lambda_c^+ + l^- + \bar{\nu}_l$. [S0556-2821(97)07819-3]

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I. INTRODUCTION

The first evidence of semileptonic Λ_b was reported by the ALEPH and OPAL Collaborations who saw an excess of correlated $\Lambda_s l^-$ pairs over $\Lambda_s l^+$ pairs (with high p_T leptons) from Z decays [1,2]. The $\Lambda_s l^-$ excess was readily interpreted as evidence for semileptonic decays of bottom Λ baryons via the chain $\Lambda_b \to \Lambda_c \to \Lambda_s$ [1,2]. In the meantime some of the Λ_c in the event sample have been fully reconstructed using the decay channel $\Lambda_c^+ \to pK^- \pi^+$ [3]. Most recently, the Collider Detector at Fermilab (CDF) Collaboration [4] at the Tevatron Collider measured the lifetime of the Λ_b using its semileptonic (SL) decay based on an event sample of 197 ± 25 reconstructed semileptonic decays. From the experience with SL bottom meson decays, one expects a significant fraction of the SL $\Lambda_b \to \Lambda_c^+ X l^- \bar{\nu}_l$ transitions to consist of the exclusive mode $\Lambda_b \to \Lambda_c^+ l^- \bar{\nu}_l$. One can be quite hopeful that fully reconstructed SL $\Lambda_b \to \Lambda_c$ events will become available in the near future.

It is therefore important to study theoretical models for the $\Lambda_b \to \Lambda_c$ transition form factors including their velocity transfer (or momentum transfer) dependence. In the heavy meson sector there has been a calculation of the $B \to D(D^*)$ current-induced heavy meson transition form factors in terms of the Bauer-Stech-Wirbel (BSW) form factor model which was improved to $O(1/m_Q)$ by comparison with the heavy quark effective theory (HQET) [5]. It is the purpose of this paper to provide a corresponding form factor calculation for the baryonic $\Lambda_b \to \Lambda_c$ transitions using again BSW-type form factors improved by HQET.

Let us briefly review the order 1 and $O(1/m_Q)$ structure of the $\Lambda_b \to \Lambda_c$ form factors as predicted by HQET [6]. For that purpose we choose to define the three vector and axial vector form factors f_i^V and f_i^A by

$$\langle \Lambda_c(v_2)|V_\mu|\Lambda_b(v_1)\rangle = \bar{u}(v_2)(f_1^V \gamma_\mu + f_2^V v_{1\mu} + f_3^V v_{2\mu})u(v_1),$$

$$\langle \Lambda_c(v_2)|A_\mu|\Lambda_b(v_1)\rangle = \bar{u}(v_2)(f_1^A \gamma_\mu \gamma_5 + f_2^A v_{1\mu} \gamma_5 + f_3^A v_{2\mu} \gamma_5)u(v_1). \quad (1)$$

In the following we switch to a more generic notation and identify the labels b and c with 1 and 2, respectively. In Eq. (1) we have used velocity covariants to define our form factors as is appropriate when discussing the ramifications of heavy quark symmetry. We define the velocity transfer variable ω by $\omega = v_1 \cdot v_2$, as usual. We use a conventional state normalization and normalize our spinors by $\bar{u}u = 2M$. The order 1 HQET predictions for the form factors read as follows [7–9]:

$$\text{order } 1: \quad f_1^V(\omega) = f_1^A(\omega) := F(\omega)$$

$$f_2^V(\omega) = f_3^V(\omega) = f_2^A(\omega) = f_3^A(\omega) = 0. \quad (2)$$

The order 1 reduced form factor $F(\omega)$ satisfies the zero recoil normalization condition $F(\omega = 1) = 1$ [8,9].

Before writing down the $O(1/m_Q)$ corrections we note that there are two different sources for the $O(1/m_Q)$ corrections that come into play. First one has a local contribution from the $1/m_Q$ corrected HQET current which is proportional to the product of the binding energy of the Λ_Q baryon denoted by $\bar{\Lambda}$ ($\bar{\Lambda} \approx M_Q - m_Q \approx 600$ MeV) and the order 1 reduced form factor $F(\omega)$. Second there is a nonlocal contribution coming from the kinetic energy term of the $1/m_Q$ corrected HQET Lagrangian. The evaluation of this contri-

bution brings in a new reduced form factor which will be denoted[1] by $\eta(\omega)$. Accordingly we have (see, e.g., [10])

$$f_1^V(\omega,m_1,m_2)=F(\omega)+\frac{1}{2}\left[\frac{1}{m_c}+\frac{1}{m_b}\right](\eta(\omega)+\bar{\Lambda}F(\omega)),$$

$$f_1^A(\omega,m_1,m_2)=F(\omega)+\frac{1}{2}\left[\frac{1}{m_c}+\frac{1}{m_b}\right]\left(\eta(\omega)\bar{\Lambda}F(\omega)\frac{\omega-1}{\omega+1}\right),$$

$$f_2^V(\omega,m_1,m_2)=f_2^A(\omega,m_1,m_2)=-\frac{1}{m_c}\frac{\bar{\Lambda}F(\omega)}{1+\omega},$$

$$f_3^V(\omega,m_1,m_2)=-f_3^A(\omega,m_1,m_2)=-\frac{1}{m_b}\frac{\bar{\Lambda}F(\omega)}{1+\omega}, \quad (3)$$

where $\eta(\omega)$ satisfies the zero recoil normalization condition $\eta(\omega=1)=0$. We have written out the m_1,m_2 dependence in the arguments of the form factors in order to clearly exhibit the scaling structure of the various contributions in Eq. (3). Equation (3) shows that, up to $O(1/m_Q)$, the six form factors are given in terms of the order 1 function $F(\omega)$, the $O(1/m_Q)$ function $\eta(\omega)$, and the constant $\bar{\Lambda}$. One of the entreating features of HQET is that, up to $O(1/m_Q)$, one retains a zero recoil normalization condition for the form factors which reads [6]

$$f_1^V(1)+f_2^V(1)+f_3^V(1)=1,$$

$$f_1^A(1)=1. \quad (4)$$

Note that the linear combinations of amplitudes written down in Eq. (4) are nothing but the vector and axial vector current s-wave amplitudes, respectively. They give the dominant contributions at pseudothreshold (or zero recoil) as $\omega\to1$. Put in a different language, the vector combination and axial vector term in Eq. (4) make up the so-called allowed Fermi and Gamow-Teller transitions, respectively, which are induced by the time component V_0 of the vector current and space components A_i ($i=1,2,3$) of the axial vector current. It is important to keep in mind that only the s-wave amplitudes Eq. (4) are constrained by HQET to $O(1/m_Q)$ at zero recoil.

A different but equivalent representation of the order $1+O(1/m_Q)$ HQET result Eqs. (2), (3) may be written down in the form [6]

order $1+O(1/m_Q)$:

$$f_1^V(\omega,m_1,m_2)=f_1^A(\omega,m_1,m_2)\left(1+\left[\frac{1}{m_1}+\frac{1}{m_2}\right]\frac{\bar{\Lambda}}{\omega+1}\right),$$

$$f_2^V(\omega,m_1,m_2)=f_2^A(\omega,m_1,m_2)=-f_1^A(\omega,m_1,m_2)\frac{1}{m_2}\frac{\bar{\Lambda}}{\omega+1},$$

$$f_3^V(\omega,m_1,m_2)=-f_3^A(\omega,m_1,m_2)$$

$$=-f_1^A(\omega,m_1,m_2)\frac{1}{m_1}\frac{\bar{\Lambda}}{\omega+1}, \quad (5)$$

where the $O(1/m_Q)$ zero recoil normalization condition now reads

$$f_1^A(\omega,m_1,m_2)=1. \quad (6)$$

It is not difficult to see that the two representations (3) and (5) are equivalent at $O(1/m_Q)$ but start to be different at $O(1/m_Q^2)$. The representation (5) is somewhat simpler in that there is only the one ω-dependent function $f_1^A(\omega,m_1,m_2)$. The representation (3) has the advantage that the various order 1 and $O(1/m_Q)$ contributions remain clearly identified. It is for this reason that we shall work with the representation (3) in the following.

Let us now turn to our model calculation to determine the ω dependence of the order 1 reduced form factor $F(\omega)$ and the [order $1+O(1/m_Q)$] form factor $f_1^A(\omega)$, or, in the representation (3), of the $O(1/m_Q)$ reduced form factor $\eta(\omega)$. We employ the method introduced by Neubert and Rieckert [5] which allows one to determine the hadronic form factors as functions of the scaling variable ω once they are known at $q^2=0$. The idea of Neubert and Rieckert is as follows: For general momentum transfer squared q^2 the relation between q^2 and the scaling variable ω is given by

$$\omega=v_1\cdot v_2=\frac{M_1^2+M_2^2-q^2}{2M_1M_2}. \quad (7)$$

Neubert and Rieckert propose to compute quark model form factors at $q^2=0$ where one has[2]

$$\omega(q^2=0)=\frac{1}{2}\left(\frac{M_1}{M_2}+\frac{M_2}{M_1}\right). \quad (8)$$

Then by varying the ratio M_1/M_2 at the point $q^2=0$ in the quark model calculation one can extract the form factor values for all values of the scaling variable $\omega\geq1$ provided one is using appropriate scaling form factors. The appropriate

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[1]It is important to realize that there is no contribution from the chromomagnetic $1/m_Q$ term in the HQET Lagrangian in the case of the $\Lambda_b\to\Lambda_c$ transitions since the light-side transition is a spin-0 to spin-0 transition in this case.

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[2]The maximum recoil point $q^2=0$ is privileged in the IMF quark model approach of BSW [11]. At $q^2=0$ the IMF overlap integrals allow for a specific interpretation in terms of space integrals of the ''good'' current components that are the charges of a broken collinear symmetry at infinite momentum. This means that, in the limit of a strict collinear symmetry combining spin and flavor [i.e., an $SU(4)_W$ symmetry acting on two spin states and two appropriate quark flavors], the normalized overlap integrals of the ''good'' current components are generators of this collinear spin-flavor group. A drawback of the IMF at $q^2=0$ is that it cannot be related to a frame of finite momentum by any Lorentz transformation.

scaling form factors of the quark model are identified by comparison with the HQET structure Eq. (2) or (3). Explicitly, we shall relate the form factors at $q^2 = 0$ to overlap integrals as is done in the BSW scheme. Using a diquark model for the infinite momentum frame (IMF) wave function [11], the overlap integrals can be evaluated. It follows trivially that the order 1 form factors $f_1^V = f_1^A = F(\omega)$ are normalized for $M_1 = M_2$ or $\omega = 1$ since they correspond to an overlap integral which is normalized to one, i.e., $\langle M_Q | M_Q \rangle = 1$ for identical hadron states. The overlap integrals and thus the quark model form factors can be expanded with respect to the inverse heavy quark masses. One can identify the zeroth and first order terms in this expansion with the same expansion in HQET and thereby compute the ω dependence of the order 1 and $O(1/m_Q)$ reduced form factors that appear in Eqs. (3) or (5) by varying the mass ratio M_1/M_2.

II. INFINITE MOMENTUM FRAME WAVE FUNCTIONS

As explained before we shall employ the approach of BSW [11] to calculate form factors at $q^2 = 0$ in terms of relativistic bound state wave functions in the infinite momentum frame. In the relativistic BSW approach the hadrons are described as relativistic bound states of a heavy active quark Q_1 and a heavy or light spectator state, which, in our case is a spin-isospin zero light diquark state. A relativistic bound state of a quark-diquark pair in the IMF is written as

$$
\begin{aligned}
|\mathbf{P}, M_i ; J, J_z\rangle = \sqrt{2}(2\pi)^{3/2} \sum_{s_1 s_2} \int d^3 p_1 d^3 p_2 \delta^3 \\
\times (\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2) \Phi_i^{J,J_z}(\mathbf{p}_{1\perp}, x_1 ; s_1, s_2) \\
\times a_1^\dagger(\mathbf{p}_1, s_1) a_2^\dagger(\mathbf{p}_2, s_2) |0\rangle,
\end{aligned}
\tag{9}
$$

where $a_1^\dagger(\mathbf{p}_1, s_1)$ denotes the creation operator for the heavy quark and $a_2^\dagger(\mathbf{p}_2, s_2)$ the creation operator for the light diquark and where $\mathbf{p}_1(\mathbf{p}_2)$, $s_1(s_2)$ represent the momentum and spin of the heavy quark (light diquark), respectively. The fraction of the longitudinal momentum carried by the active heavy quark Q_1, is denoted by $x_1 = p_{1z}/P$, $\mathbf{p}_{1\perp} = (p_{1x}, p_{1y})$ is the relative transverse momentum of the active heavy quark. We use a conventional state normalization $\langle \mathbf{P}' | \mathbf{P} \rangle = 2P^0 (2\pi)^3 \delta^3(\mathbf{P} - \mathbf{P}')$ so that

$$
\sum_{s_1 s_2} \int d^2 p_\perp dx |\Phi_i^{JJ_z}(\mathbf{p}_\perp, x; s_1, s_2)|^2 = 1.
\tag{10}
$$

In the following we suppress spin labels. The IMF heavy ground state baryon wave function is constructed in complete analogy to the heavy ground state meson case. The light antiquark in the meson case is substituted by a light diquark in the baryon case. In our definition the wave function describes a quark-diquark Fock state for the Λ-type baryon in which the spin degrees of freedom decouple from the momentum. The two light degrees of freedom are treated as a single quasielementary constituent and are represented by a spin-isospin zero diquark with $[ud]$ quantum numbers.

Because the diquark always shows up as a spectator in the overlap integrals it is of no significance whether or not it is a true bound state. Color indices are omitted for convenience. $\Phi_i(x_1, p_\perp)$ denotes the hadronic IMF (null plane) wave function normalized to one where $i = b, c$. This momentum space wave function is assumed to factorize into its longitudinal and transverse momentum dependence, i.e., into its x and $p_\perp$ dependence as usual. Large transverse momenta are assumed to be strongly suppressed by introducing an exponential cutoff. Such a picture is corroborated by many observations in inclusive processes. One thus has

$$
\Phi_i(x_1, p_\perp) = N_i \phi_i(x_1) \exp\{-b_i^2 p_\perp^2\}.
\tag{11}
$$

N_i is a normalization factor whose value is fixed once the x_1 dependence of $\phi_i(x_1)$ is specified. The oscillator parameter b_i characterizes a soft process scale below which there is no suppression by the wave function. It may be subject to mass corrections. Therefore we make the ansatz $b_i = b + \bar{b}/M_i$. Interpreting the transverse momenta as Fermi motion of the baryon constituents, the oscillator parameter b_i is adjusted such that realistic mean $p_\perp$'s are obtained, i.e., a mean $p_\perp$ is of the order of a few hundred MeV. From this consideration we expect a value of order $1-2 \text{ GeV}^{-1}$ for b. Guided by results for mesonic decays [12] we expect $\bar{b}$ to be about 0.1.

The x_1 dependence of the hadronic wave function $\phi_i(x_1)$ is controlled by the long distance behavior of QCD. The calculation of the x_1 dependence would require nonperturbative methods as, e.g., lattice gauge theories. As there are no nonperturbative results yet we have to rely on educated guesses. Most appropriate for our purposes is the wave function

$$
\phi_i(x_1) = \bar{N}_i x_1^n (1 - x_1)^m \exp\{-b_i^2 M_i^2 (x_1 - x_{i0})^2\}.
\tag{12}
$$

This wave function is a generalization of the meson wave function proposed by Bauer, Stech, and Wirbel [11]. It has already been used for the description of heavy baryons in the large recoil region [13] as well as for light baryons in a quark-diquark model [14].

The wave function (12) exhibits a pronounced maximum at $x_{i0} = 1 - \alpha_i / M_i$ where α_i is the difference between the masses of the heavy hadron and the heavy quark, i.e., $\alpha_i = M_i - m_i$ which can be expanded in terms of inverse powers of M_i [15]. We take into account only the first two terms of this series $\alpha_i = \alpha + \bar{\alpha}/M_i$. In the zero binding approximation α is approximately equal to the diquark mass and lies in the range of $0.5 - 1.0$ GeV. The size of the correction term $\bar{\alpha}$ has been estimated from baryonic QCD sum rules to amount to $0.1 - 0.3 \text{ GeV}^2$ [16]. Finally, $\bar{N}_i$ is a further normalization constant fixed by the requirement

$$
\int_0^1 dx_1 \phi_i^*(x_1) \phi_i(x_1) = 1,
\tag{13}
$$

which depends on the values chosen for the endpoint powers n and m.

The behavior of the wave function (12) in the endpoint regions $x_1 \to 0$ and $x_1 \to 1$ is controlled by the power dependent terms x_1^n and $(1-x_1)^m$. As the endpoint behavior of the light diquark system can be expected to be the same as that of the heavy quark the choice $n=m$ seems to be a natural one. We note that for light baryons this choice is not optimal [14]. Finally, in our numerical work we take $n=m=1/2$. This is suggested by the similarity of the baryonic heavy quark-light diquark system and the mesonic heavy quark-light antiquark system for which this choice has been found to be appropriate [11]. For the sake of comparison we also compute numerical results for the powers $n=m=1$. We mention that the polarization predictions of our model are only weakly dependent on the choice of n and m, whereas the rate prediction does depend on the choice of n and m.

III. MODEL FORM FACTORS

To leading order in the IMF momentum P one finds two relations for the heavy baryon decay form factors. These two relations correspond to the 0- and 3-component of the transition current which represent leading order contributions in the IMF momentum P and are thus termed ''good.'' The 1 and 2 components are ''bad,'' because in the latter, particles moving with $x<0$ or $x>1$ cannot be excluded. For these the extra powers of P in the denominator can be compensated by similar factors in the numerator from the matrix elements of J_1 and J_2, thus mimicking a constant behavior though they may hide terms proportional to P as $P \to \infty$. A more phenomenological argument why we have decided to keep only first order expressions in the matrix elements is that many approximations were made in our parton model approach. First of all, though we set $x=p_{1z}/P$, there may be components of the heavy quark momentum perpendicular to the z direction of $\mathbf{P}$. Such transverse momenta, as well as off-shell effects, the light cone factorization in x and $p_\perp$, which is not rotationally invariant, and the decoupling of spin and orbital momenta lead to modifications of order $1/P$. Despite of this the bad current relations are in agreement with HQET as we show in an Appendix, there is only a little difficulty with the reduced form factor $\eta(\omega)$. Our subsequent analysis is based on the good current components only.

The good relations between the form factors corresponding to the 0 and 3 components of the current transitions at $q^2=0$ read

$$f_1^V(\omega,M_1,M_2) + \frac{1}{2}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)(M_2 f_2^V(\omega,M_1,M_2)$$
$$+ M_1 f_3^V(\omega,M_1,M_2)) = I(\omega,M_1,M_2), \quad (14)$$

$$f_1^A(\omega,M_1,M_2) + \frac{1}{2}\left(\frac{1}{M_1}-\frac{1}{M_2}\right)(M_2 f_2^A(\omega,M_1,M_2)$$
$$+ M_1 f_3^A(\omega,M_1,M_2)) = I(\omega,M_1,M_2), \quad (15)$$

where $I(\omega,M_1,M_2)$ is an overlap integral between the initial and final state baryons and is given by

$$I(\omega,M_1,M_2) = \int_0^1 dx_1 \phi_2^*(x_1)\phi_1(x_1). \quad (16)$$

Note that in the elastic case $M_1=M_2$ (which implies $\omega=1$ at $q^2=0$) one reads off the normalization conditions $f_1^V(1)+f_2^V(1)+f_3^V(1)=1$ and $f_1^A(1)=1$ from Eqs. (11)–(15). We emphasize, though, that Eqs. (11)–(15) imply no normalization condition for $M_1 \neq M_2$ at $\omega=1$.

In order to extract further information from Eqs. (11)–(15) we have to expand the form factors $f_i^V(\omega,M_1,M_2)$ and $f_i^A(\omega,M_1,M_2)$, and the overlap function $I(\omega,M_1,M_2)$ into appropriate scaling functions that depend on the scaling variable ω only.

Let us first expand the overlap function $I(\omega,M_1,M_2)$ into inverse powers of the heavy baryon mass $1/M_i$, i.e.,

$$I(\omega,M_1,M_2) = I^{(0)}(\omega) + \left[\frac{1}{M_1}+\frac{1}{M_2}\right]I^{(1)}(\omega) + \cdots$$
$$:= I^{(0)}(\omega)\left(1 + \left[\frac{1}{M_1}+\frac{1}{M_2}\right]\tilde{I}^{(1)}(\omega) + \cdots\right), \quad (17)$$

where

$$I^{(0)}(\omega) = \sqrt{1/\omega}^{2m+1}\exp\left\{-\kappa^2\frac{\omega-1}{2\omega}\right\}\frac{H_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}{H_{2m}(\kappa)}, \quad (18)$$

$$\tilde{I}^{(1)}(\omega) = \frac{\omega-1}{\omega}\left[\frac{2m+1}{2}\frac{\bar{b}}{b} - \sqrt{2}\kappa(\bar{\alpha}b+\alpha\bar{b}) + \kappa\alpha\bar{b}/(\sqrt{2}\omega)\right] + \frac{n}{\sqrt{2}b}\left[\frac{H_{2m+1}(\kappa)}{H_{2m}(\kappa)} - \sqrt{2/\omega(\omega+1)}\frac{H_{2m+1}(\kappa\sqrt{(\omega+1)/2\omega})}{H_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}\right]$$
$$- \frac{1}{\sqrt{2}}(\bar{\alpha}b+\alpha\bar{b})\left[\frac{H'_{2m}(\kappa)}{H_{2m}(\kappa)} - \sqrt{2/\omega(\omega+1)}\frac{H'_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}{H_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}\right] + \frac{1}{2}\alpha\bar{b}\frac{\omega-1}{\sqrt{\omega^3(\omega+1)}}\frac{H'_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}{H_{2m}(\kappa\sqrt{(\omega+1)/2\omega})}. $$
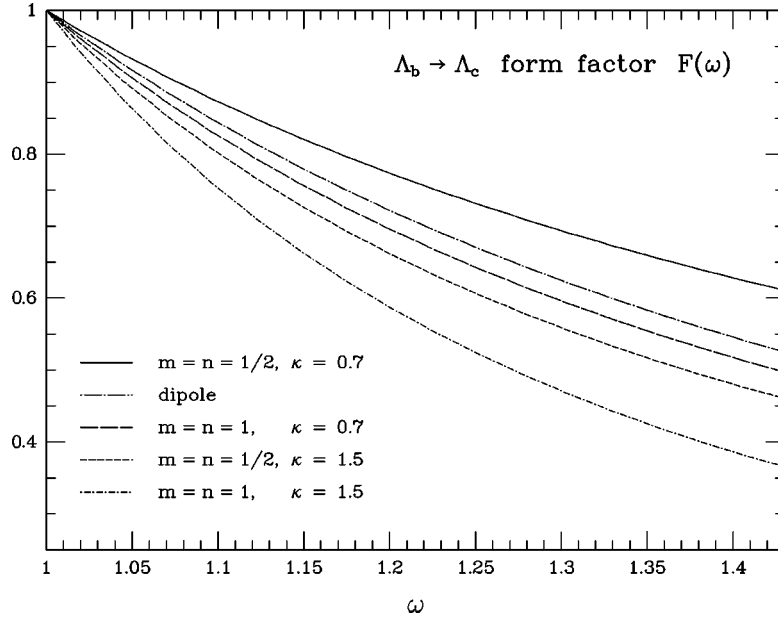$$(19)$$

FIG. 1. Dipole form factor and IMF quark model predictions for the order 1 reduced form factor $F(\omega)$ as functions of $\omega = v_1 \cdot v_2$ ($n = m = 1/2, 1$; $\kappa = 0.7, 1.5$).

Furthermore we have introduced the following abbreviations:

$$H_l(x) = \int_{-x}^{\infty} dz (z+x)^l e^{-z^2},$$

$$H_l'(x) = dH_l(x)/dx$$

$$\kappa = \sqrt{2} \alpha b. \tag{20}$$

Note that the scaling functions $I^{(0)}(\omega)$ and $\widetilde{I}^{(1)}(\omega)$ obey the normalization conditions $I^{(0)}(1) = 1$ and $\widetilde{I}^{(1)}(1) = 0$.

It is clear that the two constraint relations (14), (15) do not suffice to determine the six unknown form factors. However, when one inserts the HQET relations (3) or (5) into the determining relations one can then solve for the reduced HQET form factors. Using, e.g., the representation (3) for the HQET expansion of the form factors one can then solve the two $q^2 = 0$ relations Eqs. (14), (15) to obtain

$$\text{order 1:} \quad F(\omega) = I^{(0)}(\omega),$$

$$O(1/m_Q): \quad \eta(\omega) = I^{(0)}(\omega) \left[2\widetilde{I}^{(1)}(\omega) - \bar{\Lambda} \frac{\omega - 1}{\omega + 1} \right]. \tag{21}$$

It is quite evident that up to order $O(1/m_Q)$ the solution (21) satisfies the zero recoil normalization condition (4) for the form factors since $F(1) = 1$ and $\eta(1) = 0$. Note that F depends on κ only, i.e., on the product of the two model parameters α and b whereas η depends on all the parameters, $\alpha, \bar{\alpha}, b, \bar{b}$.

It is noteworthy that the $q^2 = 0$ constraint relations (14), (15) provide no constraint on the HQET parameter $\bar{\Lambda}$. For consistency reasons we fix

$$\bar{\Lambda} = \alpha. \tag{22}$$

Of interest is also the slope of the order 1 reduced form factor $F(\omega)$ at $\omega = 1$ which can in fact be obtained in closed form. In terms of the usual $\omega \simeq 1$ parametrization

$$F(\omega) = 1 - \rho^2 (\omega - 1) + \cdots , \tag{23}$$

one finds

$$\rho^2 = \frac{2m+1}{2} + \frac{1}{2} \kappa^2 + \frac{1}{4} \kappa \frac{H_{2m}'(\kappa)}{H_{2m}(\kappa)}. \tag{24}$$

IV. NUMERICAL RESULTS

We are now in a position to discuss the numerical implications of the IMF quark model in terms of decay spectra, decay rates, and asymmetry parameters.

In Fig. 1 we plot our model predictions for the ω dependence of the order 1 reduced form factor $F(\omega)$ for the two values $n = m = 1/2$ and $n = m = 1$. In both cases $F(\omega)$ is evaluated for either value of κ, 0.7 and 1.5. For $\alpha \simeq 0.5 - 0.6$ GeV the latter value of κ corresponds to $b = 1.7 - 2.1$ GeV^{-1} which is characteristic of light baryons [14,17]. $\kappa = 0.7$, on the other hand, corresponds to $b = 0.8 - 1.0$ GeV^{-1} which implies a Λ_b radius about half as large as that of light baryons. A value of about 1 GeV^{-1} for the parameter b seems realistic to us. For the sake of comparison we also plot the ω dependence of a dipole-behaved form factor $F^{\text{dipole}}(\omega)$ which is appropriately normalized to one at zero recoil. The normalized dipole form factor is given by[3]

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[3]By rewriting Eq. (25) in terms of the momentum transfer variable q^2 one recovers the familiar dipole representation $F^{\text{dipole}}(q^2) = N(q^2)(1 - q^2/m_{\text{FF}}^2)^{-2}$, where $N(q^2)$ normalizes the dipole form factor to one at the zero recoil point $q^2 = (M_1 - M_2)^2$.
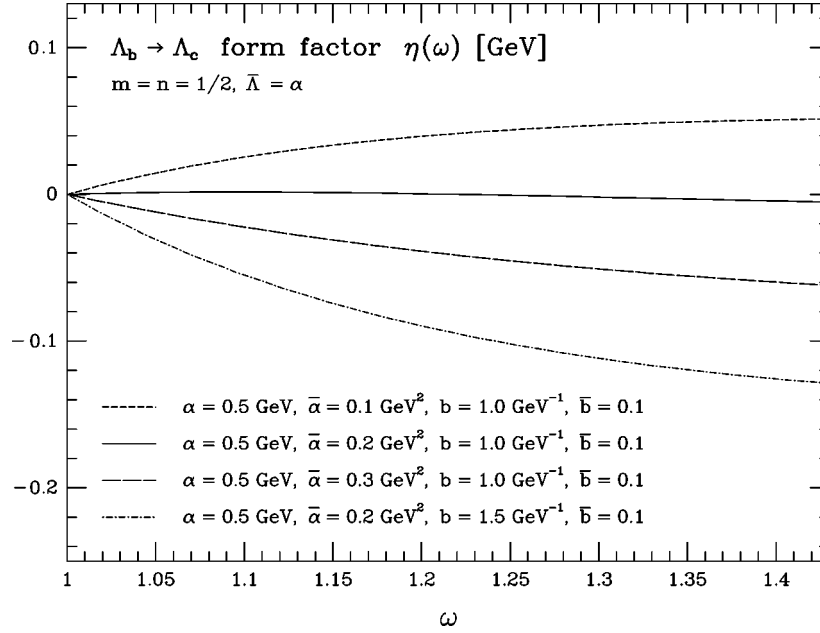
FIG. 2. IMF quark model predictions for the $O(1/m_Q)$ reduced form factor $\eta(\omega)$ as functions of $\omega = v_1 \cdot v_2$ ($m = n = 1/2$).

$$F^{\text{dipole}}(\omega) = \left(1 + \frac{2M_1 M_2 (\omega - 1)}{m_{\text{FF}}^2 - (M_1 - M_2)^2} \right)^{-2}. \quad (25)$$

As the form factor mass m_{FF} in the dipole form factor we take the expected mass value of the $J^P = 1^- (b\bar{c})$ vector meson, i.e., $m_{\text{FF}} = 6.34$ GeV. For the Λ_b and Λ_c masses we take $M_1 = M_{\Lambda_b} = 5.621$ GeV [4] and $M_2 = M_{\Lambda_c} = 2.285$ GeV.

The IMF quark model form factors $F(\omega)$ fall more quickly than the dipole form factor except for our preferred choice $n = m = 1/2$ and $\kappa = 0.7$. We mention that the QCD sum rule analysis of [18] results in a form factor behavior which is well approximated by

$$F(\omega) = \frac{2}{\omega + 1} \exp\left[-(2\rho^2 - 1) \frac{\omega - 1}{\omega + 1} \right] \quad (26)$$

with $\rho \simeq 1$. Thus, the form factor of [18] is even slightly flatter than our preferred form factor. The falloff behavior of the various form factors $F(\omega)$ in Fig. 1 can be conveniently characterized by comparing the charge radii ρ^2 of the form factors defined in Eq. (23). We obtain [see Eq. (24)]

$$\rho^2 = \begin{cases} 1.44 & \text{IMF model } (n = m = 1/2, \ \kappa = 0.7) \\ 3.04 & \text{IMF model } (n = m = 1, \ \kappa = 1.5) \\ 1.77 & \text{dipole model.} \end{cases} \quad (27)$$

To set the scale of the slope we remind the reader that the normalized dipole form factor in the infinite mass limit is given by $[1 + \frac{1}{2}(\omega - 1)]^{-2}$ and thus has a slope $\rho^2 = 1$. We mention that in the heavy meson case slope values between $\rho^2 = 1$ and $\rho^2 = 2$ are being discussed in the literature. As mentioned before the slope of the sum rule form factor of [18] is also $\rho^2 \simeq 1$.

A first measurement of the slope parameter in the transition $\Lambda_b \rightarrow \Lambda_c$ has been reported in [19]. The value quoted is

$$\rho^2 = 1.81^{+0.70}_{-0.67}(\text{stat}) \pm 0.32(\text{syst}). \quad (28)$$

The IMF model value ($n = m = 1/2$, $\kappa = 0.7$) and the dipole model value both lie within the error bars of this measurement.

As explained in Sec. II our preferred choice for the endpoint behavior of the heavy quark-light diquark IMF wave function is $n = m = 1/2$. In the following we shall no longer discuss the choice $n = m = 1$, in particular as the slope of the corresponding order 1 form factor at $\omega = 1$ seems unrealistically big for any reasonable value of κ.

In Fig. 2 we show our results for the $O(1/m_Q)$ reduced form factor $\eta(\omega)$ obtained with various sets of the model parameters. It turns out that $\eta(\omega)$ is very small in the Λ_b decay region and, depending on the parameter values, it can be positive or negative. Judging from the smallness of the nonleading reduced form factor $\eta(\omega)$ it will not be an easy task to measure it. Our results for the form factor $\eta(\omega)$ for $b = 1.0$ GeV^{-1} and $\bar{\alpha} = 0.2$ GeV^2 are close to those obtained from QCD sum rules [20].

Inserting the reduced form factors $F(\omega)$ and $\eta(\omega)$ into Eq. (3), we evaluate the form factors $f_i^{V,A}$ to $O(1/m_Q)$. We use particle masses throughout in Eq. (3). This is perfectly legitimate since the difference between particle and quark masses is an $O(1/m_Q^2)$ effect. In Fig. 3 we exhibit the ω dependence of the form factors $f_i^{V,A}$ for our preferred set of parameters $m = n = 1/2$, $\alpha = 0.5$ GeV, and $b = 1.0$ GeV^{-1} as well as, according to the discussion in Sec. II, $\bar{\alpha} = 0.2$ GeV^2 and $\bar{b} = 0.1$. Looking at Fig. 2 this choice of parameters gives a nonleading reduced form factor $\eta(\omega)$ which is practically zero. The $1/m_Q$ corrections to the form factors can be seen to be quite moderate as the comparison with the order 1 reduced form factor $F(\omega)$ shows (remember that $f_2^V = f_3^V = f_2^A = f_3^A = 0$ at order 1). The axial form factor $f_1^A(\omega)$ is predicted to be rather similar to its order 1 counterpart $F(\omega)$: the difference amounts to maximally $\approx 3\%$ at $\omega_{\max}$.

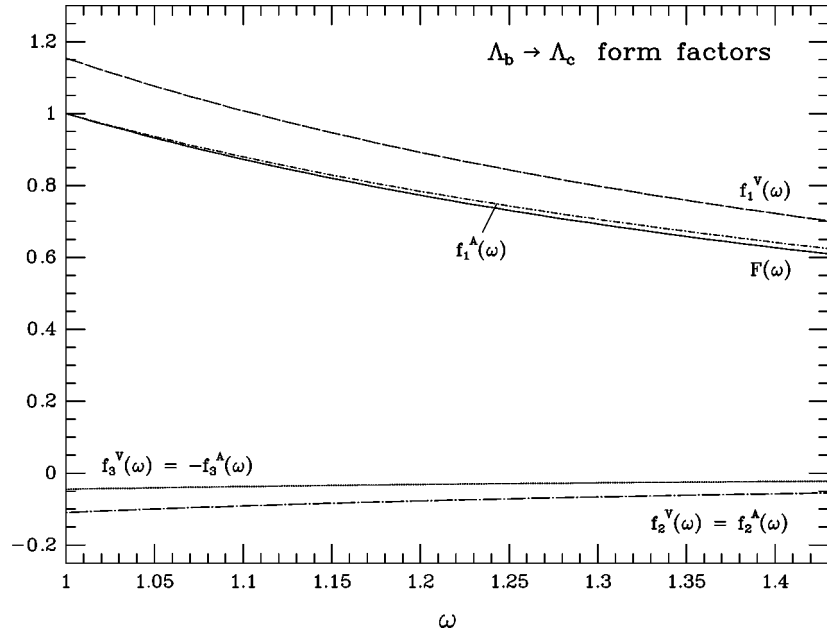FIG. 3. Order $1+O(1/m_Q)$ IMF quark model form factors as functions of $\omega = v_1 \cdot v_2$ ($n=m=1/2$). Order 1 form factor $F(\omega)$ is also shown.

The form factors f_2^V, f_3^V, f_2^A, and f_3^A acquire slight nonzero values at $O(1/m_Q)$ which are largest at zero recoil. They never amount to more than $\approx 10\%$ of f_1^A though.

To proceed further let us first state the linear relations between the "velocity" form factors defined in Sec. I and the helicity amplitudes that enter into the formulas for physical observables. One has [21]

$$\sqrt{q^2} H_{1/2\ 0}^{V,A} = \sqrt{2M_1 M_2(\omega \mp 1)}[(M_1 \pm M_2)f_1^{V,A}$$
$$\pm M_2(\omega \pm 1)f_2^{V,A} \pm M_1(\omega \pm 1)f_3^{V,A}],$$

$$H_{1/2\ 1}^{V,A} = -2\sqrt{M_1 M_2(\omega \mp 1)}f_1^{V,A}, \qquad (29)$$

where $H_{\lambda_2,\lambda_W}^{V,A}$ are the helicity amplitudes for the vector (V) and axial vector (A) current induced $1/2^+ \rightarrow 1/2^+ + W_{\text{off-shell}}^-$ transitions (λ_2 and λ_W are the helicities of the final state baryon and $W_{\text{off-shell}}^-$, respectively). The upper and lower signs in Eq. (29) stand for the vector (V) current and axial vector (A) current contributions, respectively, where the total helicity amplitude is given by

$$H_{\lambda_2\lambda_W} = H_{\lambda_2\lambda_W}^V - H_{\lambda_2\lambda_W}^A \qquad (30)$$

for a left-chiral $\gamma_\mu(1-\gamma_5)$ transition. The remaining helicity amplitudes are related to the above two helicity amplitudes by parity. One has
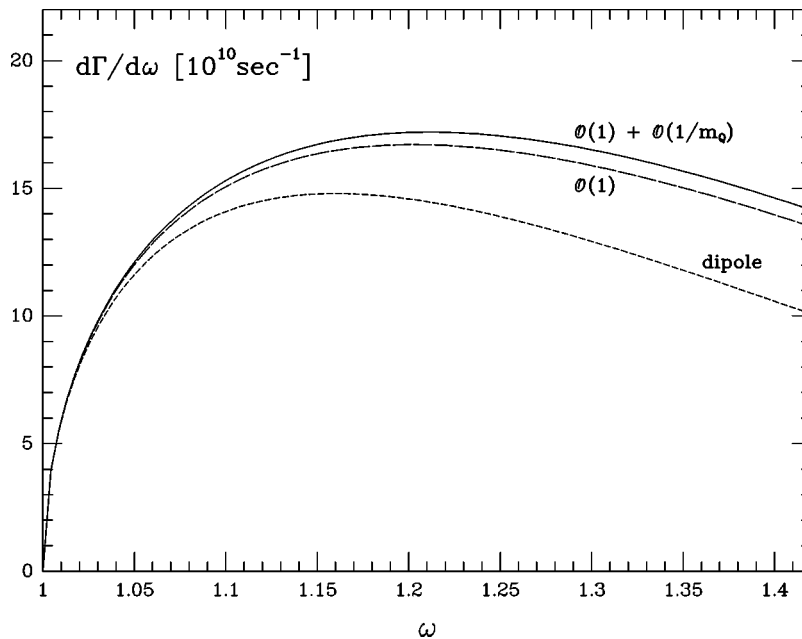

FIG. 4. ω spectrum of decay rate in the dipole model, the order $1+O(1/m_q)$ IMF quark model, and in the order 1 IMF quark model.
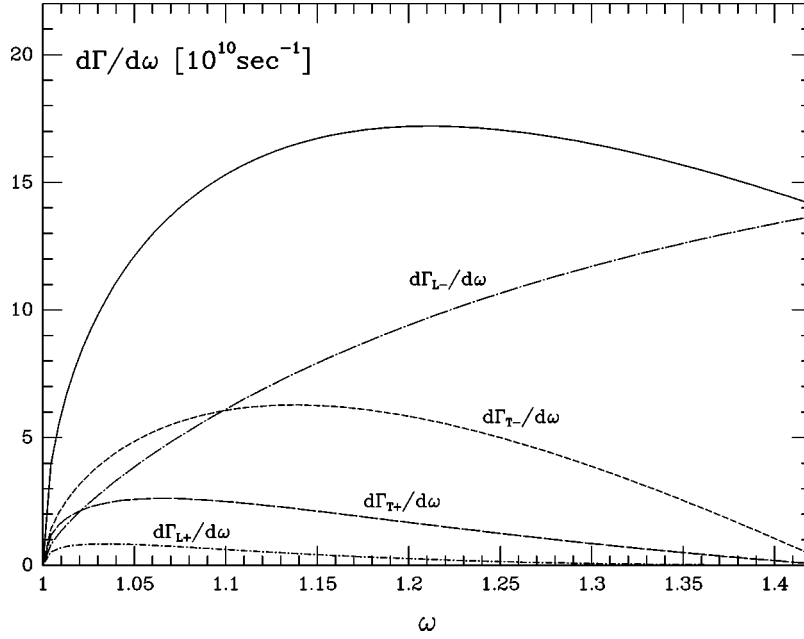
FIG. 5. ω spectrum of decay rate and partial rates into given helicity components for order $1 + O(1/m_Q)$ IMF quark model.

$$H^{V,A}_{-\lambda_2 - \lambda_W} = \pm H^{V,A}_{\lambda_2 \lambda_W}. \tag{31}$$

For the differential decay rate one then obtains [22]

$$\frac{d\Gamma}{d\omega} = \frac{G^2}{(2\pi)^3} |V_{bc}|^2 \frac{q^2 p M_2}{12 M_1} (|H_{1/2\ 1}|^2 + |H_{-1/2\ -1}|^2$$

$$+ |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2)$$

$$:= \frac{d\Gamma_{T_+}}{d\omega} + \frac{d\Gamma_{T_-}}{d\omega} + \frac{d\Gamma_{L_+}}{d\omega} + \frac{d\Gamma_{L_-}}{d\omega}, \tag{32}$$

where p is the c.m. momentum of the Λ_c $[p = M_2\sqrt{(\omega+1)(\omega-1)}]$. In the second line of Eq. (32) we have defined rates into particular helicity components through $\Gamma_{T_+}(\propto |H_{1/2\ 1}|^2)$, $\Gamma_{T_-}(\propto |H_{-1/2\ -1}|^2)$, $\Gamma_{L_+}(\propto |H_{1/2\ 0}|^2)$, and $\Gamma_{L_-}(\propto |H_{-1/2\ 0}|^2)$, where T and L denote the transverse and longitudinal components of the current transition.

In Fig. 4 we plot the velocity transfer dependence of the differential decay rate for the IMF quark model with and without $1/m_Q$ corrections and again compare them to the predictions of the dipole model. To be definite we have chosen $V_{bc} = 0.044$. For other values of V_{bc} the decay rate is to be scaled by $(V_{bc}/0.044)^2$. As would be anticipated from the comparison of the form factors in Fig. 1 the differential rates of the IMF quark model are larger than the dipole model rates. The IMF quark model spectrum peaks at larger values of ω than the dipole model spectrum. In the case of the IMF quark model, the $1/m_Q$ corrections tend to slightly increase the order 1 rates.

In Fig. 5 we show the longitudinal/transverse composition of the differential decay rate for the IMF quark model calculated up to $O(1/m_Q)$. The longitudinal rate Γ_{L_-} (propor-

tional to $(|H_{-1/2\ 0}|^2)$ dominates except at low ω. The left-chiral nature of the underlying $b \to c$ transition is reflected in the dominance of the transverse negative rate Γ_{T_-} over the transverse positive rate Γ_{T_+} and the longitudinal negative rate Γ_{L_-} over the longitudinal positive rate Γ_{L_+}. This has interesting experimental implications as will be discussed later on. As Fig. 6 shows, the difference between the transverse and longitudinal negative and positive rates is quite marked for high lepton momenta which are best suited for experimental detection.

The total decay rate Γ_{tot} and the partial rates into given helicity states of the W^- (or the current) are listed in Table I. For the sake of comparison we also show results for the dipole model, the free quark decay model and from an alternative IMF model [17] which has many features in common with the model presented in the present paper. We will comment on the model proposed in [17] below. In all cases discussed in this paper the longitudinal rate Γ_{L_-} dominates over the transverse rates Γ_{T_+} and Γ_{T_-} while the longitudinal positive rate Γ_{L_+} is small. As expected from the left-handed current coupling the transverse negative rate Γ_{T_-} dominates over the transverse positive rate Γ_{T_+} and the longitudinal negative rate Γ_{L_-} dominates over the longitudinal positive rate Γ_{L_+}.

As already apparent from the differential rates Fig. 4 the IMF quark model has the largest total rate Γ_{tot}; the $O(1/m_Q)$ corrections are small and increase the order 1 rate by 3%. It is quite instructive to compare the computed rates with the free quark decay (FQD) rates. For $m_b = 4.73$ GeV and $m_c = 1.55$ GeV one obtains $\Gamma_{tot} = 7.52 \times 10^{10}$ s^{-1}. If one takes $m_b = 5.621$ GeV and $m_c = 2.285$ GeV (a choice which incorporates phases space effects correctly) one finds $\Gamma_{tot} = 11.42 \times 10^{10}$ s^{-1}. The latter case corresponds to taking
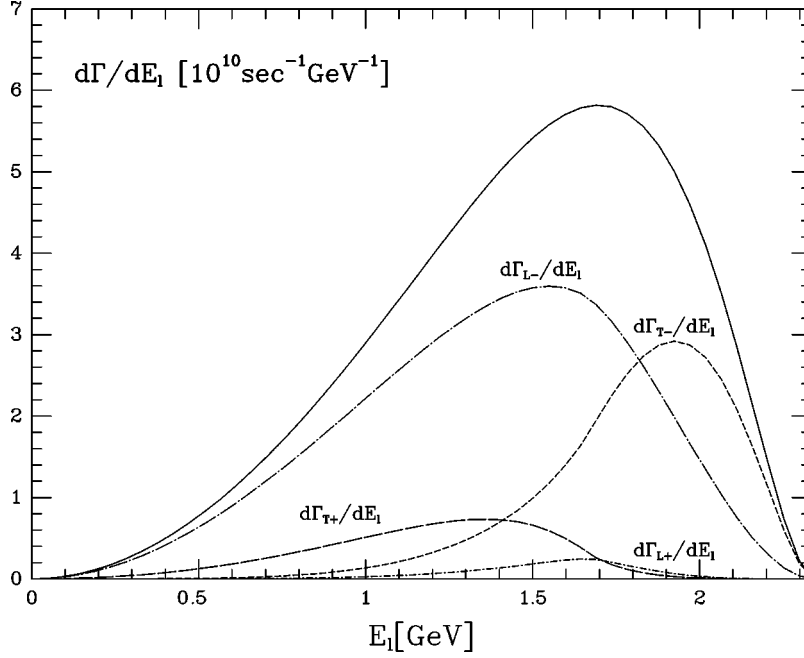
FIG. 6. Lepton energy spectrum of decay rate and partial rates into given helicity components for order $1 + O(1/m_Q)$ IMF quark model.

structureless form factors in the dipole model or in the order 1 HQET calculation.[4] The difference in rate between the form factor models and the ''structureless'' rate $\Gamma_{tot} \approx 7.52 - 11.42 \times 10^{10}$ s^{-1} would have to be filled out by the contribution of higher Λ_c resonances and continuum states.

The decay products in the quasi-two-body decay $\Lambda_b \rightarrow \Lambda_c^+ + W^-$ are highly polarized. The polarization of the decay products can be analyzed by monitoring the angular decay distributions of their subsequent decays. The structure of the lepton-side decay $W^- \rightarrow l^- + \bar{\nu}_l$ is determined by the standard model ($V-A$) coupling and has 100% analyzing power. For the hadron side the two-body decay $\Lambda_c^+ \rightarrow \Lambda + \pi^+$ is best suited for this analysis since the decay structure has recently been determined in two experiments [23,24] which obtained

$$\alpha_{\Lambda_c} = \begin{cases} -1.0^{-0.0}_{+0.4} & [23] \\ -0.96 \pm 0.42 & [24] \end{cases} \quad (33)$$

for the asymmetry parameter that characterizes the decay $\Lambda_c^+ \rightarrow \Lambda + \pi^+$.

The respective polar angle distributions are given by the following expressions [22]:

lepton side: $W(\Theta) = 1 + 2\alpha' \cos\Theta + \alpha'' \cos^2\Theta$,

hadron side: $W(\Theta_\Lambda) = 1 + \alpha\alpha_{\Lambda_c} \cos\Theta_\Lambda$,   (34)

where Θ and Θ_Λ are the polar angles of the lepton and the Λ in the ($l^- \bar{\nu}_l$) c.m. system and the Λ_c rest system, respec-

tively (see [22]). The asymmetry parameters in Eq. (34) can be expressed in terms of the helicity amplitudes and read

$$\alpha' = \frac{|H_{1/2\ 1}|^2 - |H_{-1/2\ -1}|^2}{|H_{1/2\ 1}|^2 + |H_{-1/2\ -1}|^2 + 2(|H_{-1/2\ 0}|^2 + |H_{1/2\ 0}|^2)}, \quad (35)$$

$$\alpha'' = \frac{|H_{1/2\ 1}|^2 + |H_{-1/2\ -1}|^2 - 2(|H_{-1/2\ 0}|^2 + |H_{1/2\ 0}|^2)}{|H_{1/2\ 1}|^2 + |H_{-1/2\ -1}|^2 + 2(|H_{-1/2\ 0}|^2 + |H_{1/2\ 0}|^2)}, \quad (36)$$

$$\alpha = \frac{|H_{1/2\ 1}|^2 - |H_{-1/2\ -1}|^2 + |H_{1/2\ 0}|^2 - |H_{-1/2\ 0}|^2}{|H_{1/2\ 1}|^2 + |H_{-1/2\ -1}|^2 + |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}. \quad (37)$$

The asymmetry parameters α' and α'' are specific components of the polarization density matrix of the off-shell W, whereas the asymmetry parameter α is the longitudinal po-

TABLE I. Total rates and partial rates into given helicity states. Row 1: order 1 IMF quark model; row 2: order $1 + O(1/m_Q)$ IMF quark model; row 3: dipole model; row 4: free quark decay model (FQD) (or flat form factor model) with $m_b = M_{\Lambda_b} = 5.621$ GeV and $m_c = M_{\Lambda_c} = 2.285$ GeV; row 5: order $1 + O(1/m_Q)$ IMF quark model of [17]. Parameters for the IMF quark model are $m = n = 1/2$, $\alpha = 0.5$, $\bar{\alpha} = 0.2$ GeV^2, $b = 1.0$ GeV^{-1}, and $\bar{b} = 0.1$ Rates are given in units of 10^{10} s^{-1}.

	Γ_{tot}	Γ_{T+}	Γ_{T-}	Γ_{L+}	Γ_{L-}
Order 1	6.32	0.61	1.78	0.13	3.79
Order $1 + O\left(\dfrac{1}{m_Q}\right)$	6.50	0.62	1.82	0.14	3.92
Dipole	5.43	0.55	1.58	0.12	3.17
FQD	11.4	0.92	2.90	0.18	7.43
[17]	4.89	0.44	1.53	0.10	2.82

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[4]Judging from the numbers in Table I the exclusive semileptonic decay rate $\Lambda_b \rightarrow \Lambda_c^+ + l^- + \bar{\nu}_l$ would be predicted to amount to 57–87% (IMF quark model) and 48–72% (dipole model) of the total inclusive semileptonic rate $\Lambda_b \rightarrow \Lambda_c^+ + X + l^- + \bar{\nu}_l$.

TABLE II. Mean values of various asymmetry parameters. Model parameters as in Table I.

	α	α'	α''	γ	α_P	γ_P
Order 1	-0.76	-0.11	-0.53	0.55	0.39	-0.16
Order $1+O\left(\dfrac{1}{m_Q}\right)$	-0.77	-0.11	-0.54	0.55	0.40	-0.16
Dipole	-0.75	-0.12	-0.51	0.57	0.37	-0.17
FQD	-0.81	-0.10	-0.60	0.50	0.46	-0.14
[17]	-0.78	-0.14	-0.49	0.53	0.33	-0.15

larization P_z of the daughter baryon Λ_c. Mean values of the above three asymmetry parameters are listed in Table II. In calculating the mean asymmetries one has to integrate numerator and denominator separately. We also show results for asymmetry parameters in Table II obtained from the dipole model, the free quark decay model and from the alternative IMF model [17].

Alternatively one can define forward-backward asymmetries by averaging over events in the respective forward (F) and backward (B) hemispheres of the two decays and then by taking the ratio $A_{FB}=(F-B)/(F+B)$. One then has

$$\text{lepton side:} \quad A_{FB}=-\frac{\alpha'}{1+\frac{1}{3}\alpha''},$$

$$\text{hadron side:} \quad A_{FB}=\frac{1}{2}\alpha\alpha_{\Lambda_c}, \tag{38}$$

where the forward hemispheres are defined with respect to the momentum direction of the W^- and Λ_b, i.e., $\pi/2\leq\Theta<\pi$ and $0\leq\Theta_\Lambda<\pi/2$, respectively. Judging from the large value of the measured asymmetry parameter α_{Λ_c} in Eq. (33) and the numbers in Table II, the FB asymmetry on the hadron side can be expected to be comfortably large in comparison to the small FB asymmetry on the lepton side.

The FB asymmetry measures are quite interesting when one wants to determine the chirality of the $b\to c$ transition. In the left-chiral case, as predicted by the standard model, the c quark, and thereby the Λ_c baryon, is predominantly in the negative helicity state. As the asymmetry parameter α_{Λ_c} is also negative, and thereby the helicity of the Λ predominantly negative, the helicities want to align, and one has an altogether positive FB asymmetry. A right-chiral $b\to c$ transition would, on the contrary, yield a negative FB asymmetry. The size of the predicted FB asymmetry is large enough to accommodate even large errors in this measurement to exclude or confirm the SM prediction for the chirality of the $b\to c$ transition.

The FB asymmetry on the lepton side is again predicted to be positive if the $b\to c$ transition is left-chiral. Again this can be understood from simple helicity arguments. There are, however, two reasons that the lepton side FB asymmetry measure is not optimal. First, it is predicted to be quite small ($A_{FB}=0.167$ in our model), and second, one uses the left handedness of the lepton current as input to analyze the chirality of the $b\to c$ coupling. If the weak interaction were mediated by a non-SM right-handed gauge boson W_R with

right-handed couplings at the lepton *and* hadron side one would have the same lepton-side FB asymmetry even though the $b\to c$ transition is right chiral.[5]

The hadron-side FB asymmetry measure involves P-odd spin-momentum correlations and thus is a direct measure of the $b\to c$ chirality whereas the lepton-side FB asymmetry involves P-even momentum-momentum correlations only and is therefore not optimally suited for the determination of the $b\to c$ chirality, unless, of course, one presumes the handedness of $W^-\to l^-\bar{\nu}_l$ is known.

In Table II we also list the value of the azimuthal asymmetry parameter γ which describes the azimuthal correlation of the lepton-side and the hadron-side decay planes. The corresponding azimuthal distribution is given by

$$\frac{d\Gamma}{dq^2d\chi}\propto 1-\frac{3\pi^2}{32\sqrt{2}}\gamma\alpha_\Lambda\cos\chi, \tag{39}$$

where

$$\gamma=\frac{2\,\text{Re}(H_{-1/2\,0}H^*_{1/2\,1}+H_{1/2\,0}H^*_{-1/2\,-1})}{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2+|H_{-1/2\,0}|^2+|H_{1/2\,0}|^2} \tag{40}$$

and where χ is the relative azimuth of the two decay planes (see [22]).

The asymmetry parameter γ can be seen to be the transverse component P_x (in the lepton plane) of the polarization vector of the daughter baryon Λ_c. Since we have taken the decay amplitudes to be relatively real there is no P_y component (out of the lepton plane) and correspondingly no azimuthal term proportional to $\sin\chi$ in the angular decay distribution (39). The presence of a P_y polarization component would signal the presence of CP-violating effects and/or final state interaction effects which we shall not discuss in this paper.

For completeness we list in Table II also the values for the asymmetry parameters α_p and γ_p relevant for polarized Λ_b decays. They are related to polar and azimuthal correlations between the polarization vector of the Λ_b and the momentum of the Λ_c and the decay products of the Λ_c as described in [22]. An analysis of these decay correlations is of relevance for Λ_b's originating from Z decays which are expected to be produced with a substantial amount of polarization [26]. We must mention, though, that a first analysis of the polarization of Λ_b's from the Z did not confirm theoretical expectations of a large Λ_b polarization [27].

Table II shows that the asymmetry parameters are not very dependent on whether the IMF quark model or the dipole model is used as input despite the fact that there are rate differences between the two. When taking the asymmetry ratios differences in the ω dependence of the form factors tend to drop out and one remains with the underlying spin dynamics which is approximately the same in both models. Even when going to the extreme case of choosing flat form factors for f_1^V and f_1^A the asymmetry values do not change

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[5]A viable model involving a right-handed W_R that is consistent with all present data has been recently proposed in Ref. [25].

much (see Table II). We have checked that the same statement holds true when one chooses $n=m=1$ for the endpoint power behavior in the IMF quark model.

Finally we remark that the form factors and the numerical rate values presented in this paper are derived from unrenormalized current vertices. The renormalization effects can easily be incorporated as discussed in detail in [28] and in [17]. The renormalization effects are very small close to the zero recoil point $\omega=1$ and become largest at maximum recoil $q^2=0$. Numerically they tend to increase the rates by approximately 10% but leave the asymmetry values practically unchanged.

V. SUMMARY AND CONCLUSIONS

We have used a infinite momentum frame quark model that was improved by using results from HQET to calculate the $\Lambda_b\to\Lambda_c$ transition form factors and to give detailed predictions for rates, spectra and polarization dependent observables in the semileptonic decay $\Lambda_b\to\Lambda_c^+ + l^- + \overline{\nu}_l$ ($l=e,\mu$). We have employed heavy quark–light diquark IMF wave functions in which the x dependence of the wave function resembles that of a mesonic heavy quark–light antiquark system. In our analysis we have only made use of the so-called good components of the quark transition currents.

It is important to realize that any BSW-type infinite momentum frame quark model calculation does not take into account possible spin-spin interactions between the heavy side and the light side as they occur in general in a $O(1/m_Q)$ HQET treatment. It is for this reason that a calculation such as the one presented in [5] will lead to inconsistencies when comparing the infinite momentum quark model results with the general $O(1/m_Q)$ HQET structure. In the case of the calculation of [5] this inconsistency can be exhibited by taking also the $B^*\to D^*$ channel into account, in addition to the $B\to D,D^*$ treated in [5]. As mentioned before there are no spin-spin interactions between the heavy side and the light side in the case of the $\Lambda_b\to\Lambda_c$ transitions and thus the infinite momentum frame structure is fully consistent with the $O(1/m_Q)$ HQET structure in this special case.

At this point it is appropiate to summarize the uncertainties in our model predictions. The most important parameters are n and m controlling the powers of x_1 and $1-x_1$ in the wave function. Of less importance are the oscillator parameter b and the mass difference α which determines the position of the maximum of the wave function. The order 1 reduced form factor $F(\omega)$ depends only on m and on the product of α and b while the $O(1/m_Q)$ form factor $\eta(\omega)$ depends on all parameters. While our analytical results are given for all values of n and m we have discussed two specific choices of the parameters n and m in our numerical analysis, namely $n=m=1/2$ and $n=m=1$. The choice $n=m=1/2$ is characterized by a relatively small value of the charge radius ρ^2 and, hence, a large exclusive semileptonic decay rate of the Λ_b which will amount to a substantial fraction of the total inclusive semileptonic Λ_b decay rate. Larger values of m lead to larger values of the charge radius and thus to smaller exclusive semileptonic decay rates. Although we favor the choice $m=1/2$ the question of which value one finally has to choose for m has to settled by experiment.

There is first experimental evidence that $n=m=1/2$ is the preferred choice [19]. For a given value of m the magnitude of the $O(1/m_Q)$ reduced form factor and thereby the rate increases with n. Whereas the rates are strongly dependent on the choice of n and m our polarization predictions show only a weak dependence on the choice of n and m.

Before concluding this paper we would like to comment on an alternative IMF quark model calculation of the $\Lambda_b\to\Lambda_c$ transition form factors [17]. The IMF approach of [17] has many features in common with the present model calculation: The large form factors f_1^V and f_1^A are calculated from overlap integrals of the initial and final state hadronic wave functions for which the same quark-diquark light-cone wave function is used as here. The other form factors f_2^V, f_3^V, f_2^A, and f_3^A are estimated in the same way as we do here, namely from the HQET relations.

The difference between the two approaches is that in [17] the overlap integrals are evaluated for all values of the momentum transfer q^2 (or ω) for given masses M_c,M_b while in this paper we use overlap integrals only at $q^2=0$ varying ω through the mass ratio M_c/M_b. As was mentioned above the overlap integrals at $q^2=0$ are on rather safe theoretical grounds, whereas those at $q^2\neq0$ are somewhat less reliable. On the other hand, the IMF that is being used here cannot be reached from any frame of finite momentum by a Lorentz transformation.

The results of the two models are, on the other hand, rather similar mathematically as well as numerically (cf. Tables I, II). This gives us additional confidence in our predictions. The main difference between the predictions is that the form factors used in this paper exhibit a singularity at $\omega=0$ [see Eq. (18)] which has no physical interpretation while the form factor singularity in [17] appears at $\omega=-1$ at physical threshold. However, due to the normalization condition at zero recoil this difference does not matter much numerically in the decay region.

ACKNOWLEDGMENT

APPENDIX: FORM FACTOR RELATIONS INCLUDING THE BAD CURRENT COMPONENTS

In this appendix we list the two additional equations relating form factors and quark model overlap integrals when also the bad quark current components are used. Using similar techniques as in the main part of the paper one finds

$$f_1^V(\omega,M_1,M_2)=\frac{m_1-m_2}{M_1-M_2}J(\omega,M_1,M_2),$$

$$f_1^A(\omega,M_1,M_2)=\frac{m_1+m_2}{M_1+M_2}J(\omega,M_1,M_2), \qquad (A1)$$

where one now has a different overlap integral

$$J(\omega,M_1,M_2)=\int_0^1 dx_1 \phi_2^*(x_1)\,\frac{1}{x_1}\,\phi_1(x_1). \qquad \text{(A2)}$$

It is clear that only the form factors f_1^V and f_1^A associated with the covariants γ_μ and $\gamma_\mu\gamma_5$ enter into the bad current relations since the covariants $v_{1\mu}$, $v_{2\mu}$, $v_{1\mu}\gamma_5$, and $v_{2\mu}\gamma_5$ have no transverse components.

Proceeding as in the main text we expand the overlap integral $J(\omega,M_1,M_2)$ according to

$$J(\omega,M_1,M_2)=I^{(0)}(\omega)\left(1+\left[\frac{1}{M_1}+\frac{1}{M_2}\right]\widetilde{J}^{(1)}(\omega)+\cdots\right), \qquad \text{(A3)}$$

where

$$\widetilde{J}^{(1)}(\omega)=\widetilde{I}^{(1)}(\omega)+\frac{1}{2b}\,\frac{1}{\sqrt{\omega(\omega+1)}}\,\frac{H_{2m+1}[\kappa\sqrt{(\omega+1)/2\omega}]}{H_{2m}[\kappa\sqrt{(\omega+1)/2\omega}]}. \qquad \text{(A4)}$$

Note that the zeroth order coefficient of $J(\omega)$ is identical to the zeroth order coefficient $I^{(0)}(\omega)$ [see Eq. (17)]. Similarly we expand the mass factors in Eq. (A1) up to $O(1/M_Q)$ using $m_i+\alpha_i=M_i$. One obtains

$$\frac{m_1+m_2}{M_1+M_2}=1-\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\frac{\alpha}{\omega+1};\quad \frac{m_1-m_2}{M_1-M_2}=1. \qquad \text{(A5)}$$

In Sec. III we have argued that the relations (A1) obtained from the bad current components are not so reliable. One can nevertheless ponder the question which kind of relations one

obtains for the form factors if the combined set of four equations (14), (15) and (A1) is used as a starting point. In particular one can ask oneself whether one can now derive the HQET relations Eqs. (2), (3) or (5).

At order 1 one derives from the bad current relations Eq. (A1) $f_1^{V(0)}(\omega)=f_1^{A(0)}(\omega)=I^{(0)}(\omega)=F(\omega)$ where the zero recoil condition $f_1^{A(0)}(\omega=1)=1$ is satisfied. Substituting this result into the good current relations Eqs. (14), (15) one finds $f_2^{V(0)}(\omega)M_2+f_3^{V(0)}(\omega)M_1=0$ and $f_2^{A(0)}(\omega)M_2+f_3^{A(0)}(\omega)M_1=0$. Further one has the charge conjugation relations $f_2^{V(0)}(\omega)=f_3^{V(0)}(\omega)$ and $f_2^{A(0)}(\omega)=-f_3^{A(0)}(\omega)$ which then imply that $f_2^{V(0)}=f_3^{V(0)}=f_2^{A(0)}=f_3^{A(0)}=0$. This then establishes that the order 1 HQET result Eq. (2) can in fact be derived when both the good and bad current quark model relations are used.

Turning now to the $O(1/m_Q)$ results, the bad current relation Eq. (A1) immediately leads to the first of the HQET results in Eq. (5) upon using the expansion (A5). As for the good current relations, one has to identify the HQET parameter $\bar{\Lambda}$ with the quark model parameter α, i.e., $\bar{\Lambda}=\alpha$.

The reduced form factor $\eta(\omega)$ [see (3)] can also be calculated from the bad current relations (A1) upon using the expansion (A5) again. In fact one finds

$$\eta_{\text{b.c.}}(\omega)=I^{(0)}(\omega)[2\widetilde{J}^{(1)}(\omega)-\bar{\Lambda}], \qquad \text{(A6)}$$

which differs from the solution (21) obtained from the good current relations. While the reduced form factor $\eta_{\text{b.c.}}$ is fairly small in the $\Lambda_b\to\Lambda_c$ decay region as compared to $F(\omega)$ it is not zero at $\omega=1$. With regard to the approximations involved in the bad current components (see the discussion in Sec. III) this little inconsistency in the $O(1/m_Q)$ results can be tolerated allowing us to conclude that even the bad current relations are in reasonable agreement with HQET.

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