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Exclusive semileptonic decays of charmed and *b*-flavored mesons

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We investigate the exclusive semileptonic decays of $(B,B_s;D,D_s)$ mesons into less heavy as well as light mesons in a field-theoretic framework based on the independent quark model with a confining potential in scalar-vector-harmonic form. With the recoil effect properly taken into account, the present model describes consistently the semileptonic decays of charmed and b-flavored mesons, agreeing well with the experimental data. The transition form factors in the heavy to heavy decays, in particular, comply with the heavy quark symmetry relations expected from HQET. The CKM parameters extracted in this formalism are close to the existing data. The model prediction also satisfies the Isgur-Wise relation connecting the form factors of the semileptonic $(B\rightarrow\rho e\nu)$ and that of rare radiative decay $(B\rightarrow\rho\gamma)$. [S0556-2821(97)03719-3]

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I. INTRODUCTION

The semileptonic decays of charmed and b-flavored mesons are important sources of information on the fundamental parameters of the weak interaction, namely, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements which are required to realize the CP-violating effects within the minimal standard model picture. This is because the hadronic matrix element in such decays involves only a single quark current and hence can be calculated with greater reliability than those of the nonleptonic decays. The semileptonic decay amplitudes are given by the product of leptonic and hadronic $V-A$ current. The corresponding matrix elements for the D- and B-meson decays are determined by considering the confinement effects that describe the initial and final meson bound states. This can provide valuable direct information about the internal structure of the system containing a heavy and a light quark. Semileptonic decays can also provide important clues for the estimation of prominent nonleptonic decays of heavy mesons.

Some exclusive channels of the semileptonic decays of D and B mesons have been identified and measured [1–6]. With the ongoing efforts in the charm and bottom sectors, more and more precise experimental data on these and other unidentified channels are expected in the near future. For theoretical analysis of these decays, one requires knowledge of the relevant transition form factors in terms of which the weak current matrix elements are represented in a Lorentz-covariant way. Although the form factors are manifestation of nonperturbative QCD processes, they cannot be reliably calculated from the QCD Lagrangian and therefore one is usually forced to rely on different phenomenological models [7–15]. Some of the nonrelativistic phenomenological models which are frequently cited in the literature include the constituent quark model of Isgur, Scora, Grinstein, and Wise (ISGW) [8] based on the nonrelativistic meson wave function [9], that of Altomari and Wolfenstein (AW) [10], and the modified quark model of Gilman and Singleton (GS) [11]. Among the relativistic constituent quark models which are often referred to are the Bauer-Stech-Wirbel model (BSW) [12] based on the light cone wave function, its extension by Korner and Schuler (KS) [13] and W. Jaus (WJ) [14], and the constituent quark model of Faustov, Galkin, and Mishurov [15] based on the quasipotential approach in quantum field theory. All the constituent quark models including those referred to above essentially attempt to determine the invariant transition form factors and their q^2 (four-momentum transfer squared) dependence in the monopole or dipole ansatz with an end-point normalization either at $q^2=0$ or $q^2=q^2_{\rm max}$. While most of these models provide more or less a consistent picture of all the aspects of B-meson semileptonic decays, they fail to describe the charmed meson decays. These models predict almost equal decay rates for the transitions, $D\rightarrow Ke\nu$ and $D\rightarrow K^*e\nu$. They also find a comparable population of the transverse and longitudinal polarization states of the final K^* in $D\rightarrow K^*e\nu$ transition. On the other hand, the experiments [2,3] show that the rate for $D\rightarrow Ke\nu$ is about twice that for $D\rightarrow K^*e\nu$ with the K^* produced dominantly in a longitudinally polarized state. The predictions of all the quark models cited above except the [WJ] model have not been so consistent with the experimental data in all aspects of D- and B-meson semileptonic decays taken together. It therefore appears that a completely consistent analysis of the weak decay form factors within the framework of the relativistic constituent quark model has not so far been accomplished. This may be mainly due to the fact that, in all these models, a truly relativistic bound-state character of the participating mesons has not been adequately reflected while calculating the hadronic matrix element.

Of course, in recent years, a new theoretical approach known as heavy quark effective theory (HQET) has emerged for analyzing so-called heavy-light hadrons with one of the constituent quarks belonging to the heavy flavor sector. A number of separate ideas underlying HQET have been published in several papers including some basic ones [16,17]. The two pioneering works by Isgur and Wise [18], which are most frequently cited in particle physics in recent years, have in fact, played a major role in synthesizing and extending the development of HQET. An extensive review of the history of HQET development is given by Neubert [19]. Georgi [16] and Grinstein [16] first applied HQET and the operator product expansion to the problem of semileptonic B decays and found that the lowest order term in a $(1/m_b)$ expansion corresponds to the result from a free-quark-decay model, assuming that m_b is suitably defined. Furthermore, they showed that there are no nonperturbative QCD corrections of order $\Lambda_{\rm QCD}/m_b$. Thus one can write

$$\Gamma(B\to X)=\Gamma(b\to x)+O(1/m_b^2).$$

Bigi *et al.* [20–22] argued that $O(1/{m_b}^2)$ corrections are likely to be small with a natural scale set by $(1\ {\rm GeV}^2)/m_b^2$ and a significant enhancement of the nonleptonic rate would, therefore, have to come from perturbative corrections. Their estimation of the perturbative corrections using heavy quark expansion yielded a B-semileptonic branching fraction close to the range of the experimental data [6]. Recently much attention have been devoted to applying heavy quark expansion [23–25] to analyze the lepton-energy spectrum, in particular, to the so-called problematic end-point region. Using heavy quark expansion Shifman *et al.* [26] predicted the semileptonic decay rate from which the theoretical uncertainty in the value of the CKM parameter $|\mathcal{V}_{cb}|$ was found to be 5% or less. The literature on semileptonic decays is enormous. A very extensive review on these decays has been given by Richman and Burchet [27], which briefly describes the analysis of semileptonic decays by all the available phenomenological models as well as that of various model-independent approaches including HQET.

Although HQET in the limit of $\Lambda_{\rm QCD}/m_Q\to 0$ appears aesthetically neat and sound, simplifying the problem to a considerable extent by relating each of the relevant form factors to a single one called the Isgur-Wise function ξ, it is not possible to predict theoretically the q^2 dependence of such a unique function in a straightforward way except through an appeal to nonperturbative lattice QCD [28]. In the absence of any theoretically well-motivated and straightforward approach, one is inclined to resort to taking a suitable phenomenological approach which adequately reflects the bound-state character of the participating hadrons with the relativistic constituent quarks confined within. Scora [29] improved the (ISGW) model [8] by taking into account the constraints imposed by HQET, relativistic correction factors, hyperfine distortions of the wave functions, and form factors with more realistic high-recoil behavior. Such a phenomenological model, to be known as the (ISGW2) model [30], yielded more reliable predictions in various sectors than the (ISGW) model [8] consistent with the experimental data. As an alternative suitable scheme, we had employed the relativistic independent quark model based on an average confining potential in the scalar-vector harmonic form [31–38] $U(r)=\frac{1}{2}(1+\gamma^0)(ar^2+V_0)$ (where a and V_0 are potential parameters of this model) to investigate the exclusive semileptonic decays of D and B mesons in our earlier work [32]. The predictive power of such a model has been tested earlier in describing the static hadronic properties in the mesonic and baryonic sectors [32] as well as in the radiative [33,34], weak radiative [35], leptonic [36], weak leptonic [37], and rare radiative [38] decays of light and heavy mesons.

In our earlier analysis of the exclusive semileptonic decays of D and B mesons [32], we had derived the transition form factors and their q^2 dependence in their kinematic range from underlying constituent-quark dynamics without resorting to any kind of pole ansatz as considered in other quark models. Contrary to the failure of the quark models [8–13], we predicted that the decay widths of $(D\to K,K^*)$ as well as $R=\Gamma(D\to K^*e\nu)/\Gamma(D\to Ke\nu)$ are in good agreement with the experimental data. The polarization of K^* was found to be dominantly in a transverse state contrary to the experimental data [6]. Nevertheless, our prediction for K^* polarization was certainly comparable to the measurement of the Mark Collaboration [3]. Thus the model analysis [32] projected a wholesome picture of the D-meson semileptonic decay into the strange mesons $(D\to K,K^*)$. However the story was different in B-meson semileptonic transitions to charmed mesons $(B\to D,D^*)$. Though the polarization ratio $\Gamma_L(B\to D^*)/\Gamma_T(B\to D^*)$ was found to lie within the range having large experimental uncertainty [6], the decay widths for $B\to D,D^*$ transitions were largely overestimated. This apparent failure in the B sector has been closely investigated and has been found to be entirely due to the simplifying assumption adopted in the numerical evaluation of the integral for the hadronic matrix element in the model. The relevant hadronic matrix element expressed as an integral at the constituent-quark level involves in its integrand the momentum squared $(\vec{p}_1+\vec{k})^2$ of the nonspectator quark of the daughter meson which is recoiling with momentum $\vec{k}$ in the parent meson rest frame. Since all possible directions of the quark momentum $\vec{p}_1$ and the recoil momentum $\vec{k}$ are ultimately being taken into account through the integrations leading to the decay widths, we considered $(\vec{p}_1+\vec{k})^2\simeq(|\vec{p}_1|^2+|\vec{k}|^2)$ to be a good simplifying approximation. But on closer scrutiny we find that such an approximation is justified only in the case of transitions involving not too large recoil momentum as in case of radiative [34] as well as semileptonic transitions such as $(D\to K,K^*)$ [32].

Therefore we consider it worthwhile to reinvestigate the exclusive semileptonic transitions of charmed and b-flavored mesons without resorting to any such simplifying assumption in evaluating the relevant hadronic matrix elements. We would also extend our calculation to study the semileptonic transitions of $(D,D_s;B,B_s)$ mesons into other light meson channels involving (π,η,η') and (ρ,ω,ϕ), which are important for estimating the CKM parameters such as $\mathcal{V}_{cd}, \mathcal{V}_{ub}$, etc.

The paper is organized in the following manner. The general formalism for the semileptonic decay is briefly described in Sec. II. Section III provides the model framework and calculation of the transition form factors and their q^2 dependence in the kinematic range. Our numerical results for the form factors, decay widths, ratio of decay widths, ratio of polarization states, etc., are presented in Sec. IV. There we discuss the q^2 dependence of the form factors in relation to the expectation of HQET, extraction of the CKM parameters, and the relation between the form factors of the semileptonic and rare radiative B decays. Section V contains our conclusion.

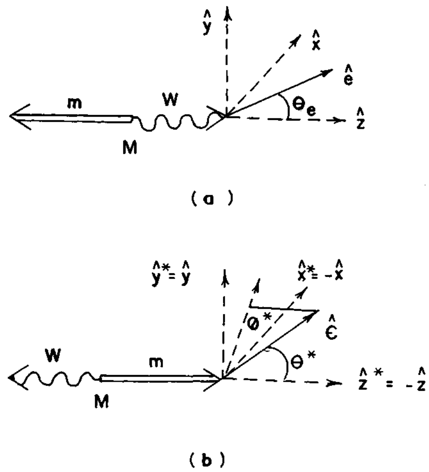

FIG. 1. Coordinate system for the semileptonic decay of a heavy meson: (a) the decaying virtual W and (b) the decaying final vector meson.

II. GENERAL FORMALISM AND KINEMATICS

For the description of the exclusive semileptonic transitions of mesons such as $M\to me\nu$, the general formalism with the appropriate kinematics has been derived and reported elsewhere [11,32]. Nevertheless, for completeness only we present here a brief outline of the same.

The S-matrix element describing the process $M\to me\nu$ is given in the familiar form

$$S_{fi}=(2\pi)^4\delta^{(4)}(P-k-p-p')\times(i\mathcal{M}_{fi})\frac{1}{\sqrt{V^4 2E_e 2E_\nu 2E_M 2E_m}}, \tag{1}$$

where the transition matrix element is

$$\mathcal{M}_{fi}=\frac{G_F}{\sqrt{2}}\mathcal{V}_{Qq}L^\mu H_\mu. \tag{2}$$

Here the leptonic and hadronic amplitudes are defined, respectively, as

$$L^\mu=\overline{u}_e(\vec{p},\delta_1)\gamma^\mu(1-\gamma^5)v_\nu(\vec{p}\,',\delta_2), \tag{3}$$

$$H_\mu=\langle m(k)|J^h_\mu(0)|M(P)\rangle. \tag{4}$$

The kinematics is conveniently described by introducing the dimensionless variable $y=q^2/M^2$, where the four-momentum transfer $q=P-k=p+p'$. In the limit of vanishing lepton mass, the kinematically allowed limit of y becomes

$$0\leqslant y\leqslant\left(1-\frac{m}{M}\right)^2. \tag{5}$$

The coordinate system chosen here is such that the daughter meson momentum $\vec{k}$ is along the negative Z axis with the charged lepton momentum $\vec{p}$ at an angle θ_e to the Z axis [Fig. 1(a)] in the $e\nu$ center-of-mass frame. The Y axis is oriented perpendicular to the plane containing the final momenta. The kinematic quantities such as the energy momentum of the daughter meson and lepton in the parent meson rest frame are given, respectively, by

$$\widetilde{E}_m=\frac{1}{2}M\left[1+\frac{m^2}{M^2}-y\right],$$

$$|\widetilde{k}|=K=\frac{1}{2}M\left[\left(1-\frac{m^2}{M^2}-y\right)^2-4\frac{m^2}{M^2}y\right]^{1/2},$$

$$\widetilde{E}_e=\frac{1}{2}K\cos\theta_e+\frac{1}{4}M\left(1-\frac{m^2}{M^2}+y\right). \tag{6}$$

Such quantities in the $e\nu$ center-of-mass frame are obtained as

$$E_m=\frac{M}{2\sqrt{y}}\left[1+\frac{m^2}{M^2}-y\right],$$

$$|\vec{k}|=K/\sqrt{y},$$

$$E_e=E_\nu=\frac{M}{2}\sqrt{y}. \tag{7}$$

With the hadronic weak current $J^h_\mu=(V_\mu-A_\mu)$, the hadronic amplitude in Eq. (4) is conventionally expressed in terms of the Lorentz-invariant form factors. For the semileptonic transitions of the type $(0^-\to0^-)$, where a pseudoscalar meson is in the final state, only the hadronic vector current contributes, which is expressed as

$$\langle m(k)|V_\mu(0)|M(P)\rangle=f_+(q^2)(P+k)_\mu+f_-(q^2)(P-k)_\mu. \tag{8}$$

On the other hand, for transitions of the type $(0^-\to1^-)$ with a vector meson in the final state, the corresponding expressions are

$$\langle m(k,\epsilon^*)|V_\mu(0)|M(P)\rangle=ig(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(P+k)^\rho\times(P-k)^\sigma, \tag{9}$$

$$\langle m(k,\epsilon^*)|A_\mu(0)|M(P)\rangle=f(q^2)\epsilon^*_\mu+a_+(q^2)(\epsilon^*\cdot P)\times(P+k)_\mu-a_-(q^2)\times(\epsilon^*\cdot P)(P-k)_\mu. \tag{10}$$

Here $\epsilon^*\equiv(\epsilon^*_0,\vec{\epsilon}^*)$, with $\epsilon^*\cdot k=0$, represents the vector meson polarization. It can be shown as in Refs. [11,32] that in the $e\nu$ frame, the leptonic tensor $L^{\mu\nu}=L^\mu L^\nu$ of the invariant transition amplitude squared $|\mathcal{M}|^2$ provides a nonvanishing spatial contribution in the limit of vanishing lepton mass. Therefore the effective hadronic amplitude turns out to be spacelike and can be expressed in the $(e\nu)$ center-of-mass frame as follows. For $(0^-\to0^-)$ transitions one obtains, from Eq. (8),

$$\vec{H}=(\vec{P}+\vec{k})f_+(q^2), \tag{11}$$

and for $(0^-\to1^-)$ transitions, one finds, from Eqs. (9) and (10),

$$\vec{H}=2i\sqrt{y}Mg(q^2)(\vec{\epsilon}^*\times\vec{k})-f(q^2)\vec{\epsilon}^*-2a_+(q^2)(\epsilon^*\cdot P)\vec{k}. \tag{12}$$

One can note here that in the $e\nu$ center-of-mass frame the form factors $f_-(q^2)$ and $a_-(q^2)$ do not contribute to $\vec{H}$ for the $(0^-\to 0^-)$ and $(0^-\to 1^-)$ transitions, respectively. It is useful to expand $\vec{H}$ in terms of the helicity basis (effectively of the virtual W) in the form

$$\vec{H}=H_+\hat{e}_+ + H_-\hat{e}_- + H_0\hat{e}_0, \tag{13}$$

where

$$\hat{e}_\pm=\frac{1}{\sqrt{2}}(\mp\hat{x}-i\hat{y}), \quad \hat{e}_0=\hat{z}. \tag{14}$$

The polarization vector $\hat{\epsilon}^*$ with the polar and azimuthal angle (θ^*,ϕ^*) in the vector meson helicity frame [Fig. 1(b)], can be Lorentz transformed to the $(e\nu)$ center-of-mass frame so as to be expressed as

$$\hat{\epsilon}^*=\frac{1}{\sqrt{2}}\sin\theta^* e^{i\phi^*}\hat{e}_+ - \frac{1}{\sqrt{2}}\sin\theta^* e^{-i\phi^*}\hat{e}_- - \frac{E_m}{m}\cos\theta^*\hat{e}_0. \tag{15}$$

Now using the expansion as per Eqs. (13), (14), and (15), integrating over polar and azimuthal angles, and finally summing over the daughter meson polarization, the differential decay rate in the parent meson rest frame can be obtained from the general expression

$$d\Gamma(M\to me\nu)=\frac{1}{2M}|\mathcal{M}|^2(2\pi)^4\delta^{(4)}(P-k-p-p')\times\prod_f\frac{d^3k_f}{(2\pi)^3 2E_f} \tag{16}$$

in the form

$$\frac{d\Gamma}{dy}=\frac{G_F{}^2|\mathcal{V}_{Qq}|^2KM^2y}{96\pi^3}[|\overline{H_+}|^2+|\overline{H_-}|^2+|\overline{H_0}|^2], \tag{17}$$

where the reduced helicity amplitudes $(\overline{H_+},\overline{H_-},\overline{H_0})$ are obtained in terms of the invariant form factors in the following manner: for $(0^-\to 0^-)$ transitions

$$\overline{H_\pm}=0, \ \overline{H_0}=-2\frac{K}{\sqrt{y}}f_+(q^2) \tag{18}$$

and for $(0^-\to 1^-)$ transitions

$$\overline{H_\pm}=[f(q^2)\mp 2MKg(q^2)], \tag{19}$$

$$\overline{H_0}=\frac{M}{2m\sqrt{y}}\left[\left(1-\frac{m^2}{M^2}-y\right)f(q^2)+4K^2a_+(q^2)\right]. \tag{20}$$

The contribution of $|\overline{H_0}|^2$ and that of $(|\overline{H_+}|^2+|\overline{H_-}|^2)$ in Eq. (17) provides, respectively, the longitudinal and transverse polarization modes.

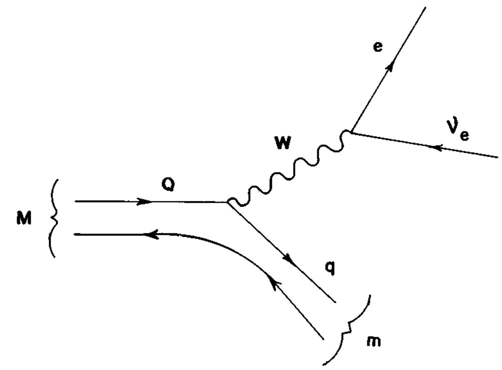

FIG. 2. The semileptonic decay of a heavy quark Q into a lighter quark q and a virtual W which become a lepton and neutrino.

Thus realizing the transition form factors $[f_+(q^2), f(q^2), g(q^2)$, and $a_+(q^2)]$ from within the dynamical scheme of the suitable phenomenological model, it is possible to predict the longitudinal and transverse widths and hence the decay widths as well as the ratio of the decay widths and polarization, etc., using the expressions in Eqs. (17)–(20).

III. MODEL FRAMEWORK AND WEAK DECAY FORM FACTORS

As discussed in Sec. II, exclusive semileptonic transitions are usually described by the invariant transition matrix expressed at the mesonic level in its familiar form of Eq. (1). However, at the constituent level, transitions of this type are basically pictured as the weak decay of the heavy quark Q of the parent meson to a less heavy or light quark q belonging to the daughter meson via the emission of a virtual W boson which subsequently disintegrates into a charged lepton and its neutrino (Fig. 2). The antiquark here remains as mere spectator. In fact, the decay of the meson physically occurs between the momentum eigenstates of the participating mesons. Therefore, in a field-theoretic calculation, one should take into account the meson states by the appropriate momentum wave packets, which, in the present model, is taken in the general form [32,34,36–38]

$$|M(\vec{P},S_M)\rangle=\frac{1}{\sqrt{N(\vec{P})}}\sum_{\lambda_1\lambda_2\in S_M}\zeta^M_{q_1q_2}(\lambda_1,\lambda_2)\int d\vec{p}_1 d\vec{p}_2\times\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{P})\mathcal{G}_M(\vec{p}_1,\vec{p}_2)\times\hat{b}^\dagger_{q_1}(\vec{p}_1,\lambda_1)\hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_2,\lambda_2)|0\rangle. \tag{21}$$

With the normalization defined as

$$\langle M(\vec{P})|M(\vec{P}')\rangle=\delta^{(3)}(\vec{P}-\vec{P}'), \tag{22}$$

the overall normalization factor is obtained in an integral form as

$$N(\vec{P})=\int d\vec{p}_1|\mathcal{G}_M(\vec{p}_1,\vec{P}-\vec{p}_1)|^2. \tag{23}$$

Here $\hat{b}^{\dagger}_{q_1}$ and $\hat{\tilde{b}}^{\dagger}_{q_2}$ stand for the quark and antiquark creation operators. $\mathcal{G}_M(\vec{p}_1,\vec{p}_2)$ represents the effective momentum distribution amplitude for the quark-antiquark pair inside the meson bound state, which is taken as [32,34,36–39]

$$\mathcal{G}_M(\vec{p}_1,\vec{P}-\vec{p}_1)=\sqrt{G_{q_1}(\vec{p}_1)\widetilde{G}_{q_2}(\vec{P}-\vec{p}_1)}. \tag{24}$$

Here $G_q(\vec{p})$ refers to the momentum probability amplitude for the constituent quark q to have momentum $\vec{p}$ inside the meson in its lowest eigenmode. Although the bound quark-antiquark pairs inside the meson are in definite energy states without having a definite momentum of their own, it is possible to find out their momentum probability amplitudes by taking a suitable momentum-space projection of the corresponding orbitals $\phi^{(+)}_{q\lambda}(\vec{r})$ or $\phi^{(-)}_{q\lambda}(\vec{r})$ derivable from the model [31–38] as

$$G_q(\vec{p})=\frac{i\pi\mathcal{N}_q}{2\alpha_q\lambda_q}\sqrt{(E_p+m_q)/E_p}(E_p+E_q)\exp(-\vec{p}^2/4\alpha_q). \tag{25}$$

Finally $\zeta^M_{q_1q_2}(\lambda_1,\lambda_2)$ stands for the spin-flavor coefficients which may include appropriate mixing angles when necessary, corresponding to the parent and the daughter meson. It is worthwhile to mention the mixing angle convention followed here while describing the transition involving pseudoscalar mesons (η,η') and the vector mesons (ϕ,ω). If we define pure strange and pure nonstrange components of the vector (ϕ,ω) and pseudoscalar (η,η') mesons as

$$(\phi_{\rm s},\eta_{\rm s})\equiv-(s\overline{s})$$

and

$$(\omega_{\rm ns},\eta_{\rm ns})\equiv\frac{1}{\sqrt{2}}(u\overline{u}+d\overline{d}),$$

then the flavor contents of physical ϕ and ω can be expressed in terms of $\delta_V=[\arcsin(1/\sqrt{3})-\theta_V]$ as

$$\begin{pmatrix}\phi\\ \omega\end{pmatrix}=\begin{pmatrix}\cos\delta_V & \sin\delta_V\\ -\sin\delta_V & \cos\delta_V\end{pmatrix}\begin{pmatrix}\phi_{\rm s}\\ \omega_{\rm ns}\end{pmatrix}. \tag{26}$$

Similarly the flavor contents of physical (η,η') can be expressed in terms of $\delta_P=[\arcsin(1/\sqrt{3})-\theta_P]$ as

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\delta_P & \sin\delta_P\\ -\sin\delta_P & \cos\delta_P\end{pmatrix}\begin{pmatrix}\eta_{\rm s}\\ \eta'_{\rm ns}\end{pmatrix}, \tag{27}$$

where $\theta_V=39°$ and $\theta_P=-(10.1)°$ stand for the mixing angle as required by the quadratic mass formula [40] in the vector and pseudoscalar meson sector, respectively, yielding the mixing angle deviation from the ideal one to be used here as [33,34]

$$\delta_V=-3.7°,$$

$$\delta_P=45°. \tag{28}$$

Thus starting from the constituent level dynamics represented by $\mathcal{G}_M(\vec{p}_1,\vec{p}_2)$ through the relevant momentum probability amplitudes $G_{q_1}(\vec{p}_1)$ and $\widetilde{G}_{q_2}(\vec{p}_2)$ and specifying the appropriate spin-flavor coefficients that include the relevant mixing angles when necessary, one can define the momentum wave packet through Eq. (21) corresponding to the parent as well as daughter mesons. Then it is straightforward to calculate the S-matrix element corresponding to the diagram in Fig. 2 for the semileptonic transition $M\rightarrow me\nu$ as in Refs. [32,38].

It may be mentioned here that starting from constituent level dynamics, the energy-momentum conservation as depicted through the δ function of Eq. (1) at the composite level is not realizable in a straightforward manner. Although three-momentum conservation is automatically guaranteed here at the mesonic level in terms of the constituent quark-antiquark momenta, it is not so transparent in the case of energy conservation. This is due to the fact that the constituent level dynamics considered here in zeroth order only cannot ensure the complete bound-state character effecting the total mass energy of the participating mesons. In order to realize appropriate energy conservation at the mesonic level, we extract out the energy delta function $\delta(E_{p_1}-E_{k+p_1}-E_e-E_\nu)$ from within the quark level integral of the hadronic amplitude in the form $\delta(M-\widetilde{E}_m-E_e-E_\nu)$ with an ansatz that $(E_{p_1}+E_{p_2})$ and $(E_{k+p_1}+E_{p_2})$ in the δ function argument can effectively be equated in an integrated sense to the parent meson mass M and the daughter meson energy $\widetilde{E}_m$, respectively. To compensate any possible mismatch arising out of such an ansatz, we incorporate an *ad hoc* mismatch factor in the form

$$\sqrt{\frac{M}{E_{p_1}+E_{p_2}}}\sqrt{\frac{\widetilde{E}_m}{E_{k+p_1}+E_{p_2}}}$$

into the quark level integral defining the hadronic amplitude. Finally we ensure an appropriate phase-space factor at the mesonic level, incorporating the covariant normalization of meson states to realize the S matrix in the standard form of Eq. (1), where the hadronic amplitude H_μ is obtained in the parent meson rest frame with $P\equiv(M,0,0,0)$ as [32,38]

$$H_\mu=\frac{M\widetilde{E}_m}{\sqrt{N_M(0)N_m(\vec{k})}}\times\int\frac{d\vec{p}_1\mathcal{G}_M(\vec{p}_1,-\vec{p}_1)\mathcal{G}_m(\vec{k}+\vec{p}_1,-\vec{p}_1)}{\sqrt{E_{p_1+k}E_{p_1}(E_{p_1+k}+E_{p_2})(E_{p_1}+E_{p_2})}}\langle S_m|\Gamma_\mu|S_M\rangle. \tag{29}$$

Here $\langle S_m|\Gamma_\mu|S_M\rangle$ with $\Gamma_\mu=\gamma_\mu(1-\gamma^5)$ represents symbolically the appropriate spin-matrix element for the process $M\rightarrow me\nu$ in the form

$$\langle S_m|\Gamma_\mu|S_M\rangle=\sum_{\lambda_1,\lambda_2\in S_M;\lambda_1',\lambda_2'\in S_m}\zeta^M_{Q\overline{q'}}(\lambda_1,\lambda_2)\times\zeta^m_{q\overline{q'}}(\lambda_1',\lambda_2')\overline{u_q}(\vec{k}+\vec{p}_1,\lambda_1')\Gamma_\mu u_Q(\vec{p}_1,\lambda_1). \tag{30}$$

As in Ref. [32] we calculate the spin-matrix elements for the $(0^-\to0^-)$ and $(0^-\to1^-)$-transitions corresponding to the vector and axial vector currents separately and obtain the expressions for respective hadronic matrix elements using Eqs. (29) and (30). A term-by-term comparison of the expressions for hadronic matrix elements so obtained with those from the form factor expansions in Eqs. (8), (9), and (10) provides transition form factors in the form [32]

$$f_+=\frac{1}{2M}\int d\vec{p}_1C(p_1)[(E_{p_1}+m_Q)(E_{p_1+k}+m_q+M-\tilde{E}_m)+\vec{p}_1^{\,2}],$$

$$g=-\frac{1}{2M}\int d\vec{p}_1C(p_1)(E_{p_1}+m_Q),$$

$$f=-\int d\vec{p}_1C(p_1)[(E_{p_1+k}+m_q)(E_{p_1}+m_Q)-\vec{p}_1^{\,2}/3],$$

$$a_+=-\frac{1}{2M^2}(f+2Mmg), \tag{31}$$

with

$$C(p_1)=\frac{M\tilde{E}_m}{\sqrt{N_M(0)N_m(\vec{k})}}\frac{\mathcal{G}_M(\vec{p}_1,-\vec{p}_1)\mathcal{G}_m(\vec{k}+\vec{p}_1,-\vec{p}_1)}{\sqrt{E_{p_1+k}E_{p_1}(E_{p_1+k}+m_q)(E_{p_1}+m_Q)(E_{p_1+k}+E_{p_2})(E_{p_1}+E_{p_2})}}. \tag{32}$$

The transition form factors in Eq. (31), in fact, embody the appropriate q^2 dependence. They can also be written in the dimensionless forms as often cited in the literature to treat all of them in the same footing as

$$F_1(q^2)=f_+(q^2),$$

$$V(q^2)=(M+m)g(q^2),$$

$$A_1(q^2)=(M+m)^{-1}f(q^2),$$

$$A_2(q^2)=-(M+m)a_+(q^2). \tag{33}$$

It is to be noted here that E_{p_1} and E_{k+p_1} stand for the energy of the nonspectator quarks belonging to the parent and daughter mesons, respectively, such that $E_{p_1}=\sqrt{\vec{p}_1^{\,2}+m_Q^2}$ and $E_{k+p_1}=\sqrt{(\vec{k}+\vec{p}_1)^2+m_q^2}$. In Ref. [32] we had taken $(\vec{k}+\vec{p}_1)^2\simeq\vec{k}^2+\vec{p}_1^{\,2}$, considering it to be a good approximation since all possible directions of the quark momentum $\vec{p}_1$ and the recoil momentum $\vec{k}$ are ultimately considered through the integrations leading to the decay widths. In doing so we had ignored the angular-dependent factor of $2\vec{k}\cdot\vec{p}_1$ not only from the algebraic terms in the integrand involving E_{k+p_1} but also from the exponential function $\exp[-(\vec{k}+\vec{p}_1)^2/4\alpha_q]$ in $G_q(\vec{k}+\vec{p}_1)$ pertaining to the nonspectator quark of the daughter meson. Eventually the quark level integrals of the form factors in Eqs. (31) and (33) turn out to be Gaussian, which could be calculated [32] using the familiar Gaussian quadrature technique. In fact, such a simplifying assumption seems to be all the more inappropriate in the transitions involving large momentum transfer yielding to the large recoil momentum of the daughter meson. Therefore in the present calculation we do not resort to such a simplifying assumption. Instead we take the squared momentum of the quark of the daughter meson as such with $(\vec{k}+\vec{p}_1)^2=\vec{k}^2+\vec{p}_1^{\,2}+2\vec{k}\cdot\vec{p}_1$ and perform the quark level integration appropriately for the transition form factors in Eq. (31) or (33). Then we can determine the reduced helicity amplitudes of Eqs. (18), (19), and (20) and hence estimate the decay widths and polarization modes as well as the corresponding ratios for specific channels of the semileptonic transitions using Eq. (17). The q^2 dependence of the form factors can also be studied and compared with the expectation of HQET and predictions of other models.

IV. RESULTS AND DISCUSSION

We estimate the semileptonic decays of heavy flavored mesons $(D,D_s;B,B_s)$ in specific exclusive channels such as

$$B\to(D,D^*;\pi,\rho),$$

$$B_s\to(D_s,D_s^*;K,K^*),$$

$$D\to(K,K^*;\pi,\rho;\eta,\eta';\phi,\omega),$$

$$D_s\to(K,K^*;\eta,\eta';\phi,\omega).$$

The parameters primarily required for the estimation are the flavor-independent potential parameters (a,V_0) and the quark masses (m_Q,m_q). We take the values of the parameters as those obtained for the present model in its earlier applications to several hadronic phenomena in the mesonic and baryonic sectors [31,32,34–38]. Accordingly we take the potential parameters as

$$(a,V_0)\equiv(0.017166\ \text{GeV}^3,\ -0.1375\ \text{GeV}) \tag{34}$$

and the quark masses and the corresponding quark binding energies in GeV as

$$m_u=0.07875,\quad E_u=0.47125,$$

TABLE I. Decay width, polarization ratio, and decay width ratio in $(D^0\to K^-, K^{*-})$ and $(\overline{B^0}\to D^+, D^{*+})$ transitions.

Physical quantity	(ISGW) [a] [8]	(GS) [11]	(BSW) [12]	(WJ) [14]	Previous prediction [32]	Present prediction	Experiment [6]
$\Gamma(D^0\to K^-)$ $\times(10^{10}\ \mathrm{s}^{-1})$	8.50	7.10	7.89	6.72	7.68	7.97	8.2 ± 0.4
$\Gamma(D^0\to K^{*-})$ $\times(10^{10}\ \mathrm{s}^{-1})$	9.13	9.50	9.03	4.28	5.20	5.23	4.6 ± 0.4
$\frac{\Gamma_L(D^0\to K^{*-})}{\Gamma_T(D^0\to K^{*-})}$	1.09	1.21	0.90	1.44	0.52	0.44	1.23 ± 0.13
$\frac{\Gamma(D^0\to K^{*-})}{\Gamma(D^0\to K^-)}$	1.07	1.34	1.14	0.64	0.68	0.66	$0.60\pm0.09\pm0.07$
$\Gamma(\overline{B^0}\to D^+)$ $\times(10^{10}\ \mathrm{s}^{-1})$	2.05	2.60	1.50	1.63	2.73	0.98	1.27 ± 0.33
$\Gamma(\overline{B^0}\to D^{*+})$ $\times(10^{10}\ \mathrm{s}^{-1})$	4.66	4.90	4.10	4.10	5.10	2.52	2.96 ± 0.27
$\frac{\Gamma_L(\overline{B^0}\to D^{*+})}{\Gamma_T(\overline{B^0}\to D^{*+})}$	0.97	0.88	1.07	1.17	0.77	0.64	$1.105\pm0.74\pm0.6$
$\frac{\Gamma(\overline{B^0}\to D^{*+})}{\Gamma(\overline{B^0}\to D^+)}$	2.27	1.88	2.67	2.46	1.87	2.56	$2.6^{+1.1+1.0}_{-0.8-0.8}$

[a]The ISGW2 model [30] predicts $\Gamma(D^0\to K^{*-})/\Gamma(D^0\to K^-)=0.54$.

$$m_d=0.07875,\ E_d=0.47125,$$

$$m_s=0.31575,\ E_s=0.59100,$$

$$m_c=1.49276,\ E_c=1.57951,$$

$$m_b=4.77659,\ E_b=4.76633. \tag{35}$$

The relevant CKM parameters are taken to be the central values of Ref. [6] as

$$\mathcal{V}_{cb}=0.041,\ \mathcal{V}_{cs}=1.01,$$

$$\mathcal{V}_{ub}=0.0032,\ \mathcal{V}_{cd}=0.224. \tag{36}$$

The meson masses used here are the experimentally observed ones. With all these model parameters having been fixed earlier in the present model, we perform, in a way, a parameter-free calculation. We first evaluate numerically the transition form factors in Eq. (31) from which we predict the values of the decay widths $\Gamma(M\to me\nu)$, the polarization ratios $\Gamma_L(0^-\to1^-)/\Gamma_T(0^-\to1^-)$, and the ratios of the decay widths for the $D\to K,K^*$ and $B\to D,D^*$ transitions using Eq. (17). The results are summarized in Table I in comparison with the predictions of our earlier calculation [32] and those of other models along with the experimental data. We find that the present predictions on $D\to K,K^*$ transitions remain almost unaffected whereas those on the $B\to D,D^*$ transitions get a significant improvement over what we obtained in Ref. [32], providing, in both cases, very good agreement with the experimental data. This is because of the fact that a relatively small recoil momentum involved in $D\to K,K^*$ transitions is found to have a marginal effect only on the evaluation of the integrals defining the transition form factors in Eq. (31). On the other hand, the recoil momentum involved in $B\to D,D^*$ transitions is, in fact, very large which provides a significant damping effect in the evaluation of the integrals of Eq. (31), yielding to the predictions considerably suppressed compared to the overestimated values of Ref. [32] to be in close agreement with the experiment. On closer scrutiny we find that the suppression in the prediction for the transition form factors and hence for the decay widths is mainly due to the exponential factor $\exp[-2\vec{k}\cdot\vec{p}_1/4\alpha_q]$ that appears in the momentum probability amplitude $G_q(\vec{k}+\vec{p}_1)$ of the nonspectator quark of the daughter meson. This vindicates our earlier contention that the simplifying assumption taken in Ref. [32] that ignores the angular dependence arising out of $\hat{k}\cdot\vec{p}_1$ throughout the calculation is more or less justified in the study of the transitions of the type $D\to K,K^*$ involving not too large recoil momentum. But it is certainly not justified in studying the $B\to D,D^*$ type transitions with large recoil momentum. It is not strange to find here the ratios of decay widths, $\Gamma(0^-\to1^-)/\Gamma(0^-\to0^-)$, in both the sectors to be close to the central values of the corresponding experimental data. Our prediction of $\Gamma(B\to D^*)/\Gamma(B\to D)=2.56$ remains well within the asymptotic QCD prediction (2-3) from HQET. It may be mentioned here that most of the quark models including [8,11,12] fail to analyze the transition $D^0\to K^*e\nu$ and reproduce the experimental data for $\Gamma(D^0\to K^*e\nu)$ as well as $\Gamma(D^0\to K^*e\nu)/\Gamma(D^0\to Ke\nu)$. By incorporating various relativistic corrections originally ignored in the ISGW model [8], an ISGW2 model [30] calculation of the transition form factors yielded $\Gamma(D^0\to K^{*-}e\nu)/\Gamma(D^0\to K^-e\nu)=0.54$ which is comparable to the presently predicted value in agreement with the experimental data.

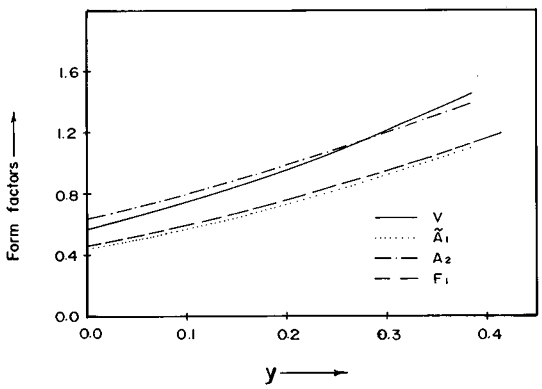

FIG. 3. Variation of the form factors relevant for the decays $\overline{B^0}\to D^{*+},D^+$ in the entire kinematic range of y.

The daughter mesons K^* in $D\to K^*$ and D^* in $B\to D^*$ transitions are found to have their spin polarization predominantly transverse in nature contrary to predictions of most other quark models. Our prediction on the polarization ratio $\Gamma_L(D\to K^*)/\Gamma_T(D\to K^*)$ is close to the measurement of the Mark III Collaboration [3]. However, the more precise measurements [6] of the CLEO Collaboration correspond to the predominance of longitudinally polarized states for K^* as well as D^*. We can see no mechanism within the present model that would give such a strong enhancement of the longitudinal component as the CLEO results imply. Altomari and Wolfenstein (AW) [10] have pointed out that the polarization is sensitive to the value of the a_+ (and hence of A_2) form factor which contributes to the longitudinal component Γ_L only. By including the recoil effect appropriately without taking any simplifying assumption, we have a fairly reliable estimation of the transition form factors including A_2. A relatively higher value of A_2 so obtained, in fact, contributes destructively towards the longitudinal decay mode, yielding to the low value of the polarization ratio consistently in both the sectors. For a consistency check, we have also reproduced the results listed in Table I using a different but straightforward formalism developed in Ref. [10].

We then study the q^2 dependence of the form factors in Eq. (31) for $B\to D,D^*$ and $D\to K,K^*$ transitions which are shown in Figs. 3 and 4, respectively. According to HQET, the form factors are expected to satisfy the heavy quark symmetry relations [19] in the entire kinematic range following

$$F_1(q^2)\simeq V(q^2)\simeq A_2(q^2)\simeq \tilde{A}_1(q^2), \tag{37}$$

where

$$\tilde{A}_1(q^2)=\left[1-\frac{q^2}{(M+m)^2}\right]^{-1}A_1(q^2). \tag{38}$$

Heavy quark symmetry also leads to model-independent normalization at zero recoil, yielding the form factors at $q^2\to q^2_{\max}$ [19] as

$$F_1(q^2_{\max})\simeq\frac{(M+m)}{2\sqrt{Mm}},$$

$$V(q^2_{\max})\simeq A_2(q^2_{\max})\simeq\frac{(M+m^*)}{2\sqrt{Mm^*}},$$

$$A_1(q^2_{\max})\simeq\frac{2\sqrt{Mm^*}}{(M+m^*)}. \tag{39}$$

The symmetry relations in Eqs. (37)–(39) are, in fact, model-independent consequences of QCD in the limit of heavy quark mass $m_Q\gg\Lambda_{\rm QCD}$, which can be checked from the model calculation so as to test the applicability of the model in the study of the semileptonic decay of heavy mesons. In the $B\to D,D^*$ transitions involving both the participating quarks to be heavy (m_c and $m_b\gg\Lambda_{\rm QCD}$), the form factors are expected to obey the asymptotic QCD predictions. But the same is not expected from the form factors in the $D\to K,K^*$ transitions (as evident in Fig. 4), since the underlying assumption in HQET requiring the s quark to be very heavy is certainly not a good approximation. In Fig. 3 we observe that the q^2 dependence of the $B\to D,D^*$ transition form factors shows only a marginal deviation from the heavy quark symmetry relation in Eqs. (37)–(39) comparing well with the expectation of HQET as well as the result of the HQET-based calculation [19]. Here we obtain a closer agreement with the heavy quark symmetry relation than what was achieved in the previous calculation [32]. It may be pointed out here that the relation for q^2 dependence of the transition $(B\to D,D^*)$ form factors assumed in the many other quark model calculations [8,12,13] is not generally in accord with the heavy quark symmetry relation [19]. We also obtained the values of the form factors at $q^2=q^2_{\max}$ as

$$F_1(q^2_{\max})\simeq 1.20\ (1.13),$$

$$V(q^2_{\max})\simeq 1.45\ (1.13),$$

$$A_2(q^2_{\max})\simeq 1.39\ (1.13),$$

$$A_1(q^2_{\max})\simeq 0.88\ (0.89), \tag{40}$$

in reasonable agreement with the model-independent normalization at zero recoil (in parentheses) expected from heavy quark symmetry.

We also evaluate the form factors at $q^2\to 0$. The results are listed in Tables II and III for $B\to D,D^*$ and $D\to K,K^*$

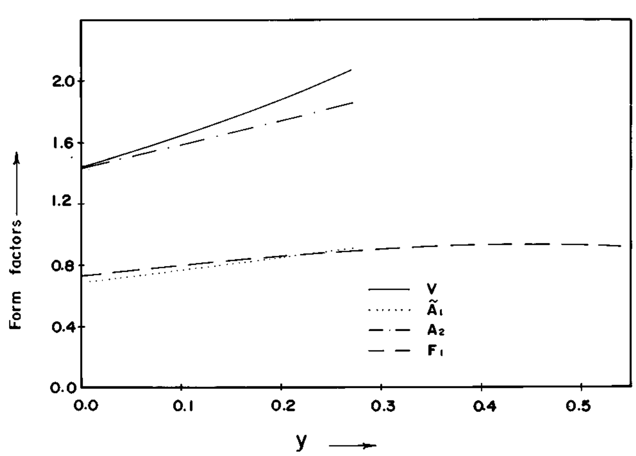

FIG. 4. Variation of the form factors relevant for the decays $D^0\to K^{*-},K^-$ in the entire kinematic range of y.

TABLE II. Transition form factors at $q^2=0$ in the decay $\overline{B^0}\to D^+,D^{*+}$ along with other model predictions.

Form factor	(ISGW) [8]	(GS) [11]	(BSW) [12]	(WJ) [14]	Previous prediction [32]	Present prediction	Experiment [6]
$V(0)$		0.95	0.71	0.69	1.11	0.57	
$A_1(0)$		0.69	0.65	0.35	0.93	0.44	
$A_2(0)$		0.80	0.69	0.56	1.31	0.63	
$F_1(0)$			0.69	0.67	0.97	0.46	

transitions, respectively. We observe that the values obtained in the present calculation are closer to the predictions of other standard models than those in our previous calculation [32], where they were found to be mostly overestimated.

We have extended the calculation to study the semileptonic decays of $(B,B_s;D,D_s)$ mesons into other channels involving (π,η,η',K,D_s) and $(\rho,\phi,\omega,K^*,D_s^*)$ mesons. This is achieved by a suitable replacement of the quarks, mesons, and other relevant parameters including the CKM parameters. The predictions on the decay widths, polarization ratios, and the branching ratios of the above kinematically allowed channels are provided in Table IV in comparison with the available experimental data. We find that the present predictions in most of the transitions involving not too high momentum transfer more or less agree with the available data. The results for the decays of D^+ and D_s^+ to (η,η') are compatible with the η-η' mixing angle $\theta_p\simeq-10°$ as required by the quadratic mass formula. However, if one takes $\theta_p\simeq-20°$ as obtained from the measurement of $\Gamma_n(2\gamma)$, which is also close to the mixing angle predicted by linear mass formula, results for the partial decay widths and corresponding branching ratios change to

$$\Gamma(D^+\to\eta e\nu)=1.75\times10^9\ \mathrm{s}^{-1},$$

$$B(D^+\to\eta e\nu)=1.85\times10^{-3},$$

$$\Gamma(D^+\to\eta' e\nu)=0.41\times10^9\ \mathrm{s}^{-1},$$

$$B(D^+\to\eta' e\nu)=0.43\times10^{-3},$$

$$\Gamma(D_s^+\to\eta e\nu)=2.95\times10^{10}\ \mathrm{s}^{-1},$$

$$B(D_s^+\to\eta e\nu)=1.38\times10^{-2},$$

$$\Gamma(D_s^+\to\eta' e\nu)=3.10\times10^{10}\ \mathrm{s}^{-1},$$

$$B(D_s^+\to\eta' e\nu)=1.44\times10^{-2}.$$

It is observed that the predictions for $D_s^+\to(\eta,\eta')$ with $\theta_p\simeq-20°$ are no longer within the range of presently available imprecise data. However, the above cited results together with those in Table IV with $\theta_p=-10°$ provide a range of variation of predictions for these decay modes. The q^2 involved in the transition $B\to\pi e\nu$ and $B\to\rho e\nu$ are, in fact, very large since the initial meson is heavy and daughter meson is light. In general, the larger the value of q^2, the larger will be the variation in the form factors and less reliable is the prediction over the full range. The model predictions for these decay modes are not considered very reliable. Therefore it is not surprising to find various model predictions on $B(B\to\pi e\nu)$ to vary by an order of magnitude. Present predictions for $(B\to\pi e\nu)$ and $(B\to\rho e\nu)$ are found to be within the limits of large experimental uncertainty.

One can also extract the CKM parameters from within the dynamical scheme of the model, taking the experimental data on the branching ratio and the lifetime as inputs. The appropriate expression for $\mathcal{V}_{Qq}$ is

$$|\mathcal{V}_{Qq}|^2\simeq\left(\frac{B_{\mathrm{expt}}(M\to me\nu)}{\tau_M}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{\widetilde{\Gamma}(M\to me\nu)}\right), \tag{41}$$

where the reduced partial decay width is given by

$$\widetilde{\Gamma}(M\to me\nu)=\frac{\Gamma(M\to me\nu)}{|\mathcal{V}_{Qq}|^2}. \tag{42}$$

We calculate $\widetilde{\Gamma}(M\to me\nu)$ from the model and evaluate the CKM parameters using Eq. (41). The results are listed in Table V and are found to be in agreement with the estimated values as per Ref. [6].

Finally we use the presently predicted value of the form factor V and A_1 to test the HQET relations [41] between the form factors of the semileptonic and rare radiative B decays. Isgur and Wise [41] have shown that in the limit of infinitely heavy b-quark mass, an exact relation connects the form fac-

TABLE III. Transition form factors at $q^2=0$ in the decay $D^0\to K^-,K^{*-}$ along with the predictions of other models and the experiment.

Form factor	(ISGW) [8]	(GS) [11]	(BSW) [12]	(WJ) [14]	Previous prediction [32]	Present prediction	Experiment [6]
$V(0)$	1.10	1.46	1.27	0.79	1.32	1.43	1.0 ± 0.2
$A_1(0)$	0.80	0.74	0.88	0.59	0.77	0.69	0.55 ± 0.03
$A_2(0)$	0.80	0.55	1.15	0.36	1.48	1.47	0.40 ± 0.08
$F_1(0)$	0.80	0.70	0.75	0.70	0.80	0.73	0.75 ± 0.03

TABLE IV. Decay width, polarization ratio, and branching ratio of $M\to me\nu$ transitions along with the experimental data.

Physical process $M\to me\nu$	Decay width $\Gamma(M\to me\nu)$ (s^{-1})	Polarization ratio $\frac{\Gamma_L(0^-\to1^-)}{\Gamma_T(0^-\to1^-)}$	Branching ratio $B(M\to me\nu)$	Experiment $B(M\to me\nu)$ [6]
$\overline{B^0}\to\pi^-e^+\nu$	4.53×10^7		0.71×10^{-4}	$(1.34\pm0.35\pm0.28)\times10^{-4}$
$\overline{B^0}\to\rho^-e^+\nu$	1.35×10^8	0.42	2.10×10^{-4}	$(2.28\pm0.36\pm0.59^{+.00}_{-0.46})\times10^{-4}$
$\overline{B_s}\to D_s^-e^+\nu$	8.61×10^9		1.39×10^{-2}	
$\overline{B_s}\to D_s^{-*}e^+\nu$	2.26×10^{10}	0.61	3.64×10^{-2}	
$\overline{B_s}\to K^-e^+\nu$	4.36×10^7		7.01×10^{-5}	
$\overline{B_s}\to K^{-*}e^+\nu$	1.15×10^8	0.30	1.85×10^{-4}	
$D^0\to\pi^-e^+\nu$	0.62×10^{10}		2.57×10^{-3}	$(3.8^{+1.2}_{-1.0})\times10^{-3}$
$D^0\to\rho^-e^+\nu$	3.37×10^9	0.18	1.40×10^{-2}	
$D^+\to\eta e^+\nu$	1.30×10^9		1.38×10^{-3}	
$D^+\to\eta' e^+\nu$	0.62×10^9		0.66×10^{-3}	
$D^+\to\phi e^+\nu$	0.33×10^7	0.15	0.035×10^{-2}	$<2.09\times10^{-2}$
$D^+\to\omega e^+\nu$	1.66×10^9	0.17	0.18×10^{-2}	
$D^+\to\overline{K^0}e^+\nu$	8.09×10^{10}		8.55×10^{-2}	$(6.7\pm0.8)\times10^{-2}$
$D^+\to\overline{K^0}{}^*e^+\nu$	5.29×10^{10}	0.44	5.94×10^{-2}	$(4.8\pm0.4)\times10^{-2}$
$D_s^+\to\eta e^+\nu$	4.49×10^{10}		2.10×10^{-2}	$(2.5\pm0.7)\times10^{-2}$
$D_s^+\to\eta' e^+\nu$	2.31×10^{10}		1.08×10^{-2}	$(0.87\pm0.34)\times10^{-2}$
$D_s^+\to\phi e^+\nu$	4.62×10^{10}	0.41	2.15×10^{-2}	$(1.9\pm0.5)\times10^{-2}$
$D_s^+\to\omega e^+\nu$	0.37×10^9	0.47	0.017×10^{-2}	
$D_s^+\to\overline{K^0}e^+\nu$	6.14×10^9		2.87×10^{-3}	
$D_s^+\to\overline{K^0}{}^*e^+\nu$	3.01×10^9	0.15	1.41×10^{-3}	

tors V and A_1 of the $(B\to\rho e\nu)$ transition with the rare radiative decay form factor F of $B\to\rho\gamma$ defined by

$$\langle\rho(p_\rho,e^*)|\overline{u}_e\sigma_{\mu\nu}q^\nu P_R b|B(0)\rangle = i\epsilon_{\mu\nu\tau\sigma}e^{*\nu}p_B^\tau p_\rho^\sigma F(q^2) + [e_\mu^*(M_B^2-M_\rho^2)(e^*\cdot q)(p_B+p_\rho)]G(q^2). \quad (43)$$

This relation is valid for q^2 values sufficiently close to $q^2_{\max}=(M_B-M_\rho)^2$ and reads

$$F(q^2)=\frac{(q^2+M_B^2-M_\rho^2)}{2M_B}\frac{V(q^2)}{(M_B+M_\rho)}+\frac{(M_B+M_\rho)}{2M_B}A_1(q^2). \quad (44)$$

It has been argued in [42] that in these processes the soft contributions dominate over the hard perturbative ones and thus the Isgur-Wise relation in Eq. (44) could be extended to whole kinematic range of q^2. From Eq. (43) we estimate the rare radiative decay form factor $F(q^2\to0)$ for $B\to\rho\gamma$ in the present model and find

$$F(0)^{B\to\rho\gamma}=0.049. \quad (45)$$

Using Eq. (44) as well as the values of the form factors $V(0)$ and $A_1(0)$ we again find that

$$F(0)^{B\to\rho\gamma}=0.048, \quad (46)$$

TABLE V. CKM parameters along with the corresponding values estimated as per [6].

CKM parameters $\mathcal{V}_{Qq}$	Process from which extracted	Present prediction $\mathcal{V}_{Qq}$	Estimated $\mathcal{V}_{Qq}$ [6]
$\mathcal{V}_{cb}$	$\overline{B^0}\to D^{+*}e^-\nu$	0.044 ± 0.002	0.041 ± 0.003
$\mathcal{V}_{cb}$	$\overline{B^0}\to D^+e^-\nu$	0.046 ± 0.006	0.041 ± 0.003
$\mathcal{V}_{ub}$	$\overline{B^0}\to\rho^-e^+\nu$	$(0.334\pm0.069)\times10^{-2}$	$(0.328\pm0.112)\times10^{-2}$ [a]
$\mathcal{V}_{ub}$	$\overline{B^0}\to\pi^-e^+\nu$	$(0.441\pm0.101)\times10^{-2}$	$(0.328\pm0.112)\times10^{-2}$ [a]
$\mathcal{V}_{cs}$	$D^0\to K^{-*}e^+\nu$	0.972 ± 0.081	1.01 ± 0.18
$\mathcal{V}_{cs}$	$D^0\to K^-e^+\nu$	1.06 ± 0.029	1.01 ± 0.18
$\mathcal{V}_{cd}$	$D^0\to\pi^-e^+\nu$	$0.273^{+0.042}_{-0.041}$	0.224 ± 0.016

[a] $\mathcal{V}_{ub}$ is evaluated from the estimated values of $\mathcal{V}_{ub}/\mathcal{V}_{cb}$ and $\mathcal{V}_{cb}$ as per Ref. [6].

which is in accord with Eq. (45).

V. CONCLUSION

We reinvestigate $B \to D, D^*$ and $D \to K, K^*$ semileptonic transitions in the relativistic independent quark model based on a confining potential in the scalar-vector-harmonic form. We find that with the recoil effect taken appropriately into consideration in the numerical integration involved without resorting to any simplifying assumption, the model calculation provides a satisfactory description of the semileptonic transitions discussed here. We extend a similar calculation to the semileptonic transitions of $(D, D_s; B, B_s)$ mesons into various other kinematically allowed channels and our predictions are found to agree reasonably with the available experimental data. The model predictions in those sectors where there exist no experimental data stand to guide future experiments in the corresponding sectors. Our results in the heavy to heavy transitions compare well with the expectation of the heavy quark symmetry relations of HQET. The CKM parameters extracted from the model calculation are close to the estimated values as per Ref. [6]. Finally we consider the Isgur-Wise relation between the form factors of the semileptonic and rare radiative B-meson decays in the limit of infinitely heavy b-quark mass. It is found that our prediction is quite in conformity with this relation.

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[1] Mark II Collaboration, J. Alder *et al.*, Phys. Rev. Lett. **60**, 89 (1988); **62**, 1821 (1989); Phys. Lett. B **196**, 107 (1987).

[2] E691 Collaboration, J. C. Anjos *et al.*, Phys. Rev. Lett. **62**, 722 (1989); **62**, 1587 (1989).

[3] Mark III Collaboration, Z. Bai *et al.*, Phys. Rev. Lett. **66**, 1011 (1991). The value of the polarization ratio $\Gamma_L(D^0 \to K^{*-})/\Gamma_T(D^0 \to K^{*-})$ is found here to be $0.5^{+1.0+0.1}_{-0.1-0.2}$.

[4] ARGUS Collaboration, H. Albrecht *et al.* Phys. Lett. B **255**, 634 (1991); **219**, 121 (1989); **229**, 175 (1989); **197**, 452 (1987).

[5] E653 Collaboration, K. Kodama *et al.*, Phys. Rev. Lett. **66**, 1819 (1991).

[6] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[7] For early work, see S. Adler and R. Dashen, *Current Algebra* (Benjamin, New York, 1968); A. Ali and I. C. Yang, Phys. Lett. **65B**, 275 (1976); D. Fakirov and B. Stech, Nucl. Phys. **B133**, 315 (1978); M. B. Gavela, Phys. Lett. **83B**, 367 (1979); C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982); F. E. Close, G. J. Gounaris, and J. E. Paschalis, Phys. Lett. **149B**, 209 (1984); H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984); S. Godfrey, University of Toronto report, 1984; A. Ali, Z. Phys. C **1**, 25 (1979); A. Ali, J. G. Korner, G. Kramer, and J. Willrodt, *ibid.* **1**, 269 (1979).

[8] B. Grinstein, N. Isgur, and M. B. Wise, Phys. Rev. Lett. **56**, 298 (1986); Caltech Report No. CALT-68-1311, 1986 (unpublished); N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); N. Isgur and D. Scora, *ibid.* **40**, 1491 (1989).

[9] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); R. Kokoski and N. Isgur, *ibid.* **35**, 907 (1987).

[10] T. Altomari and L. Wolfenstein, Phys. Rev. Lett. **58**, 1583 (1987); Phys. Rev. D **37**, 681 (1988).

[11] F. J. Gilman and R. L. Singleton, Jr., Phys. Rev. D **41**, 142 (1990); J. G. Korner and G. A. Schuler, Mainz Report No. MZ-TH/88-14, 1988 (unpublished); Phys. Lett. B **226**, 185 (1989); H. Hagiwara, A. D. Martin, and M. F. Wade, *ibid.* **228**, 144 (1989).

[12] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); M. Bauer and M. Wirbel, *ibid.* **42**, 671 (1989).

[13] J. G. Korner and G. A. Schuler, Z. Phys. C **38**, 511 (1988); J. G. Korner, K. Schilcher, M. Wirbel, and Y. L. Wu, *ibid.* **48**, 663 (1990); G. Kramer, T. Mannel, and G. A. Schuler, *ibid.* **51**, 649 (1991).

[14] Wolfgang Jaus, Z. Phys. C **54**, 611 (1992); Phys. Rev. D **41**, 3394 (1990).

[15] R. N. Faustov, V. O. Galkin, and A. Yu. Mishurov, Phys. Rev. D **53**, 6302 (1996); R. N. Faustov and V. O. Galkin, *ibid.* **52**, 5131 (1995).

[16] E. V. Shuryak, Nucl. Phys. **B93**, 134 (1980); S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987); E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990); H. Georgi, *ibid.* **240**, 447 (1990); B. Grinstein, Nucl. Phys. **B339**, 253 (1990).

[17] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1987) [**47**, 511 (1988)]; A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).

[18] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); M. B. Wise, in *Particle Physics—The Factory Era*, Proceedings of the Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell *et al.* (World Scientific, Singapore, 1991), and references therein.

[19] M. Neubert, Phys. Lett. B **264**, 455 (1991); Phys. Rep. **245**, 259 (1994), and references therein.

[20] I. I. Bigi and N. G. Uraltsev, Phys. Lett. B **280**, 271 (1992).

[21] I. I. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B **293**, 430 (1992).

[22] I. I. Bigi, B. Blok, M. Shifman, and A. I. Vainshtein, Phys. Lett. B **323**, 408 (1994); I. I. Bigi, M. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994).

[23] I. I. Bigi, M. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).

[24] A. F. Falk, E. Jenkins, A. V. Manohar, and M. B. Wise, Phys. Rev. D **49**, 4553 (1994); A. F. Falk, Z. Ligeti, M. Neubert, and Y. Nir, Phys. Lett. B **326**, 145 (1994).

[25] A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994); M. Neubert, *ibid.* **49**, 3392 (1994).

[26] M. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. D **51**, 2217 (1995).

[27] J. D. Richman and P. R. Burchat, Rev. Mod. Phys. **67**, 893 (1995).

[28] M. Cristafulli, G. Martinelli, and C. T. Sachrajda, Phys. Lett. B **223**, 90 (1989); C. Bernard, A. El-Khandra, and A. Soni, in *Lattice '88*, Proceedings of the International Symposium, Batavia, Illinois, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 186 (1989)]; V. Lubiez, G. Martinelli, and C. T. Sachrajda, Nucl. Phys. **B356**, 301 (1991).

[29] D. Scora, Ph.D. thesis, University of Toronto, 1993.

[30] D. Scora and N. Isgur, CEBAF Report No. CEBAF-TH-94-14, 1994.

[31] N. Barik, B. K. Dash, and P. C. Dash, Pramana **29**, 543 (1987); N. Barik and B. K. Dash, Phys. Rev. D **33**, 1925 (1986); N. Barik, B. K. Dash, and M. Das, *ibid.* **32**, 1725 (1985); N. Barik and B. K. Dash, *ibid.* **34**, 2092 (1986); **34**, 2803 (1986); B. E. Palladino and P. Leal Ferriera, IFT Sao Paulo Report No. IFT/P-35/88 (unpublished).

[32] N. Barik and P. C. Dash, Phys. Rev. D **53**, 1366 (1996).

[33] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **46**, 3856 (1992).

[34] N. Barik and P. C. Dash, Phys. Rev. D **49**, 299 (1994).

[35] N. Barik and P. C. Dash, Mod. Phys. Lett. A **10**, 103 (1995).

[36] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **47**, 1001 (1993).

[37] N. Barik and P. C. Dash, Phys. Rev. D **47**, 2788 (1993).

[38] N. Barik, S. Kar, and P. C. Dash (unpublished).

[39] B. Margolis and R. R. Mendel, Phys. Rev. D **28**, 468 (1983).

[40] Particle Data Group, C. G. Wohl *et al.*, Rev. Mod. Phys. **56**, S1 (1984).

[41] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).

[42] S. Narison, Phys. Lett. B **345**, 166 (1995); **327**, 354 (1994); G. Burdman and J. F. Donoghue, *ibid.* **270**, 55 (1991).