Doubly charged Higgsino contribution to the decays $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ and to the anomalous magnetic moment of the muon Δa_{μ} within the left-right supersymmetric model

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We present a detailed and complete calculation of the doubly charged Higgsino contribution to the decays $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ and to the anomalous magnetic moment of the muon Δa_{μ} within the left-right supersymmetric model. We include the mixing of the scalar partners of the left- and right-handed leptons, and show that it leads to a strong enhancement of the decay modes in certain scenarios. We find that the contribution of the doubly charged Higgsino can be close to the known experimental values and is reachable by future experiments. [S0556-2821(97)07119-1]

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I. INTRODUCTION

The quest for a supersymmetric grand unified theory is plagued by a lack of direct signals which would distinguish it from supersymmetry in general. Supersymmetry, in particular the minimal supersymmetric standard model (MSSM) [1] can be probed experimentally through the production of superpartners. However, the MSSM, while filling in some of the theoretical gaps of the standard model, fails to explain other phenomena such as the weak mixing angle, the small mass (or masslessness) of the known neutrinos, the origin of CP violation, to quote a few. Extended gauge structures such as grand unified theories, introduced to provide an elegant framework for the unification of forces [2], would connect the standard model with more fundamental structures such as superstrings, and also resolve the puzzles of the electroweak theory.

Phenomenologically, grand unified theories would either predict relationships between otherwise independent parameters of the standard model, or new interactions (i.e., interactions forbidden or highly suppressed in the standard model).

Among supersymmetric grand unified theories, SO(10) [3] and SU(5) [4] have received significant attention. In this article we shall study a model based on the left-right symmetric extension of the MSSM: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$. Its attraction is that on one hand the left-right supersymmetric (LRSUSY) model is an extension of the minimal supersymmetric standard model based on left-right symmetry and on the other hand it could be viewed as a low-energy realization of certain SUSY grand unified theories (GUT's) such as SO(10).

LRSUSY shares some of the attractive properties of the MSSM, such as providing a natural solution for the gauge hierarchy problem. LRSUSY also suppresses naturally rapid proton decay by disallowing terms in the Lagrangian that explicitly violate either baryon or lepton numbers [5]. It gauges the only quantum number left ungauged, $B-L$. The LRSUSY model also shares some of the attractive features of the original left-right symmetric model [6], such as providing a possible explanation for the smallness of the neutrino mass and for the origin of parity violation. Recently, this model has received a lot of attention, because it could offer a framework for solving both the strong and the weak CP problems [7], while automatically conserving R-parity.

So far, there is no experimental evidence for the right-handed interactions predicted by the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ theory, let alone by supersymmetry. Yet the foundation of LRSUSY has so many attractive features that the model deserves some experimental and theoretical investigation. The next generation of linear colliders will provide an excellent opportunity for such a study. The theoretical and experimental challenge lies in finding distinctive features for the left-right supersymmetric model, which allow it to be distinguished from both the SUSY version of the standard model and from the nonsupersymmetric version of the left-right theory [8]. Lepton-flavor violation decays are just the right type of phenomena. The LRSUSY model provides a natural framework for large lepton flavor-violating effects through two mechanisms. First, it can give rise to a leptonic decay width of the Z boson through both left-handed and right-handed scalar lepton mixing [9]. Second, it contains lepton-flavor-blind Higgsinos which couple to leptons only and enhance lepton-flavor violation.

In a recent paper [10] we analyzed the lepton flavor-violating decay $\mu \rightarrow e\gamma$, whose signature can be reliably calculated and, by the structure of the model, showed to be greatly enhanced over the MSSM, and already accessible at the current experimental accuracy.

The contribution of the doubly charged Higgs boson to lepton-flavor violation decays were considered in [11] and quite strong constraints on its mass and Yukawa couplings to leptons were presented. Here we will study a new source of enhancement coming from the existence, in the supersymmetric sector, of a doubly-charged Higgsino. Since the upper limits on the decays $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ are very tight [12] (4.9×10^{-11} and 1.0×10^{-12}, respectively) we are able to obtain values close to the experimental bounds and restrict some of the parameters in the theory.

A similar contribution from the doubly-charged Higgsino (with different mass parameters) appears in the calculation of another accurately measured quantity, the anomalous magnetic moment of the muon (AMMM) $a_\mu=(g-2)/2$. Its experimental value is $a_\mu^{\rm expt}=1165922(9)\times10^{-9}$ [12]. The measured deviation of the AMMM lies within a range of $-2\times10^{-8}\leqslant\Delta a_\mu^{\rm expt}\leqslant2.6\times10^{-8}$ [13].

Experiment E821, under way at Brookhaven National Laboratory (BNL), is designed to improve the current data on the AMMM by a factor of approximately 20. When completed, it would be possible to test deviations from the one-loop predictions of the minimal supersymmetric standard model, believed to arise most likely from supersymmetric contributions. For completeness, we also investigate the effect of the contribution of the doubly-charged Higgsino to the AMMM. We then restrict the parameter space of the left-right supersymmetric model by combining these effects.

Our paper is organized as follows: in Sec. II we give a brief description of the model, followed by the numerical analysis of the decay $\mu\to e\gamma$ in Sec. III and the AMMM in Sec. IV. We reach our conclusions in Sec. V. In addition, Appendix A and B will present the detailed analytical calculations.

II. THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The LRSUSY model, based on $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks) [8]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-},W^0)_L$, $(W^{+,-},W^0)_R$ and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bidoublets, $\Phi_u(\frac{1}{2},\frac{1}{2},0)$ and $\Phi_d(\frac{1}{2},\frac{1}{2},0)$, which are required to give masses to both the up and down quarks. In addition, the spontaneous symmetry breaking of the group $SU(2)_R\times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by introducing the Higgs triplet fields $\Delta_L(1,0,2)$ and $\Delta_R(0,1,2)$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated for the right-handed neutrino and a small one for the left-handed neutrino [6]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1,0,-2)$ and $\delta_R(0,1,-2)$, with quantum number $B-L=-2$ to insure cancellation of the anomalies that would otherwise occur in the fermionic sector. Given their strange quantum numbers, the δ_L and δ_R do not couple to any of the particles in the theory so their contribution is negligible for any phenomenological studies.

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire nonzero vacuum expectation values (VEV's). These values are

$$\langle\Delta_L\rangle=\begin{pmatrix}0&0\\ v_L&0\end{pmatrix},$$

$$\langle\Delta_R\rangle=\begin{pmatrix}0&0\\ v_R&0\end{pmatrix}\quad\text{and}\ \langle\Phi\rangle=\begin{pmatrix}\kappa&0\\ 0&\kappa' e^{i\omega}\end{pmatrix}.$$

$\langle\Phi\rangle$ causes the mixing of W_L and W_R bosons with CP-violating phase ω. In order to simplify, we will take the VEV's of the Higgs fields as $\langle\Delta_L\rangle=0$ and

$$\langle\Delta_R\rangle=\begin{pmatrix}0&0\\ v_R&0\end{pmatrix},$$

$$\langle\Phi_u\rangle=\begin{pmatrix}\kappa_u&0\\ 0&0\end{pmatrix}\quad\text{and}\ \langle\Phi_d\rangle=\begin{pmatrix}0&0\\ 0&\kappa_d\end{pmatrix}.$$

Choosing $v_L=\kappa'=0$ satisfies the more loosely required hierarchy $v_R\gg\max(\kappa,\kappa')\gg v_L$ and also the required cancellation of flavor-changing neutral currents. The Higgs fields acquire nonzero VEV's to break both parity and $SU(2)_R$. In the first stage of breaking, the right-handed gauge bosons, W_R and Z_R aquire masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L, which pick up masses proportional to κ_u and κ_d at the second stage of breaking.

The supersymmetric sector of the model, while preserving left-right symmetry, has four singly-charged charginos (corresponding to $\tilde\lambda_L$, $\tilde\lambda_R$, $\tilde\phi_u$, and $\tilde\phi_d$), in addition to $\tilde\Delta_L^-$, $\tilde\Delta_R^-$, $\tilde\delta_L^-$, and $\tilde\delta_R^-$. The model also has eleven neutralinos, corresponding to $\tilde\lambda_Z$, $\tilde\lambda_{Z'}$, $\tilde\lambda_V$, $\tilde\phi_{1u}^0$,$\tilde\phi_{2u}^0$, $\tilde\phi_{1d}^0$, $\tilde\phi_{2d}^0$, $\tilde\Delta_L^0$, $\tilde\Delta_R^0$, $\tilde\delta_L^0$, and $\tilde\delta_R^0$. It has been shown that in the scalar sector, the left-triplet Δ_L couplings can be neglected in phenomenological analyses of muon and tau decays [14]. Although Δ_L is not necessary for symmetry breaking [15] and is introduced only for preserving left-right symmetry, we will not neglect its couplings in the fermionic sector, since they have important consequences, as we will see later.

However, the doubly charged Δ_R^{--} is very important: it carries quantum number $B-L$ of 2 and couples only to leptons, therefore breaking lepton-quark universality. It and its supersymmetric partner could, as will be seen in the next section, play an important role in flavor-violating leptonic decays.

In the scalar matter sector, the LRSUSY contains two left-handed and two right-handed scalar fermions as partners of the ordinary leptons and quarks, which themselves come in left- and right-handed doublets. In general the left- and right-handed scalar leptons will mix together. Some of the effects of this mixing, such as enhancement of the AMMM, have been discussed elsewhere [5]. Only global lepton-family-number conservation would prevent $\tilde e$, $\tilde\mu$, and $\tilde\tau$ to mix arbitrarily. Permitting this mixing to occur, we could expect small effects to occur in the nonsupersymmetric sector, such as radiative muon or tau decays, in addition to other nonstandard effects such as massive neutrino oscillations and violation of lepton number itself. Allowing general mixings leads to six charged-scalar lepton states (involving 15 real angles and 10 complex phases) and six scalar neutrinos (also involving 15 real angles and 10 complex phases). In order to reduce the (large) number of parameters we shall assume in what follows that only two types of mixings dominate.

First, the scalar lepton (selectron, smuon, and stau) mix, but we shall assume for simplicity that only two generations

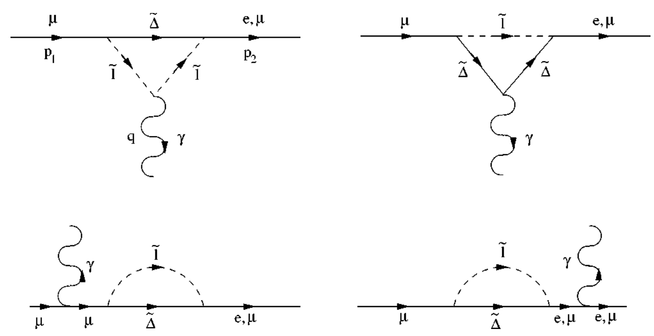

FIG. 1. The penguin diagrams with scalar leptons and doubly charged Higgsinos within the loop. In the left-right model without SUSY the same diagrams occur with $\tilde{\Delta}\leftrightarrow l$ and $\tilde{l}\leftrightarrow \Delta_{L,R}$.

of scalar leptons (the lightest) mix significantly.[1]

This mixing is described as follows: $\tilde{\mu}_{L,R}$ and $\tilde{e}_{L,R}$ with angle $\theta_{L,R}$; $\tilde{\nu}_{\mu_{L,R}}$ and $\tilde{\nu}_{e_{L,R}}$ with angle $\alpha_{L,R}$; so that, for example,

$$\tilde{l}_{L_1}=\tilde{\mu}_L\cos\theta_L+\tilde{e}_L\sin\theta_L\,, \tag{1}$$

$$\tilde{l}_{L_2}=-\tilde{\mu}_L\sin\theta_L+\tilde{e}_L\cos\theta_L\,, \tag{2}$$

and similarly for $\tilde{l}_{R_{1,2}}$ and $\tilde{\nu}_{L_{1,2}}$ and $\tilde{\nu}_{R_{1,2}}$.

Second, we have the mixing of the scalar partners to the left- and right-handed leptons, which is independent of the above mixing. We parametrize this second mixing as follows:

$$\tilde{e}_L=\cos\theta_{\tilde{e}}\tilde{e}_1-\sin\theta_{\tilde{e}}\tilde{e}_2\,, \tag{3}$$

$$\tilde{e}_R=\sin\theta_{\tilde{e}}\tilde{e}_1+\cos\theta_{\tilde{e}}\tilde{e}_2\,, \tag{4}$$

and similarly for the other generations. We express all these mixings by the matrix $\tilde{K}^a_{mn}$ in Appendix A.

The physical mass eigenstates are then given by Eqs. (1) and (2). Maximal slepton mixing would mean $\theta_{L,R}=\pi/4$ and maximal left-right slepton mixing would imply $\theta_{\tilde{e},\tilde{\mu}}=\pi/4$.

III. THE DECAY $\mu\to e\gamma$

We now consider the implications of these mixings in the lepton-flavor violating decay $\mu\to e\gamma$. The contribution of the doubly charged Higgsino to this process is represented by the Feynman diagrams of Fig. 1.

The detailed calculations are given in Appendix A. The matrix element given in Eq. (A6) leads to the following result:

$$iM=+\frac{e h_{LR}^2}{(4\pi)^2}\frac{m_\mu}{m_{\tilde{\Delta}}^2}\bar{u}_{p_2}i\sigma_{\mu\nu}q^\nu\{c_{\theta_L}s_{\theta_L}(B^L_{\tilde{e}}-B^L_{\tilde{\mu}})P_R + c_{\theta_R}s_{\theta_R}(B^R_{\tilde{e}}-B^R_{\tilde{\mu}})P_L\}u_{p_1}\epsilon_q^{*\mu}\,, \tag{5}$$

$$B^L_{\tilde{l}}=c^2_{\theta_{\tilde{l}}}B_{\tilde{l}_1}+s^2_{\theta_{\tilde{l}}}B_{\tilde{l}_2}+\frac{m_{\tilde{\Delta}}}{m_\mu}c_{\theta_{\tilde{l}}}s_{\theta_{\tilde{l}}}\left(\frac{c_{\theta_R}}{c_{\theta_L}}\tilde{B}_{\tilde{l}_1}-\frac{s_{\theta_R}}{s_{\theta_L}}\tilde{B}_{\tilde{l}_2}\right),$$

$$B^R_{\tilde{l}}=s^2_{\theta_{\tilde{l}}}B_{\tilde{l}_1}+c^2_{\theta_{\tilde{l}}}B_{\tilde{l}_2}+\frac{m_{\tilde{\Delta}}}{m_\mu}c_{\theta_{\tilde{l}}}s_{\theta_{\tilde{l}}}\left(\frac{c_{\theta_L}}{c_{\theta_R}}\tilde{B}_{\tilde{l}_1}-\frac{s_{\theta_L}}{s_{\theta_R}}\tilde{B}_{\tilde{l}_2}\right),$$

$$B_{\tilde{l}_m}=e_{\tilde{\Delta}}[-B_G(x^{\tilde{\Delta}}_{\tilde{l}_m})+B_F(x^{\tilde{\Delta}}_{\tilde{l}_m})]+e_{\tilde{l}}B_F(x^{\tilde{\Delta}}_{\tilde{l}_m}),$$

$$\tilde{B}_{\tilde{l}_m}=e_{\tilde{\Delta}}\tilde{B}_G(x^{\tilde{\Delta}}_{\tilde{l}_m})+e_\mu\tilde{B}_F(x^{\tilde{\Delta}}_{\tilde{l}_m}),$$

where $\tilde{l}$ is $\tilde{e}$ or $\tilde{\mu}$ and $m=1,2$ distinguishes the left-, right-handed scalar partners of the leptons.

In order not to have too many parameters, we assume that h_{LR} for different generations are all of the same order. We will however keep different the couplings within the first and second generation in the mixing angles $c_{\theta_{L,R}}$, Eqs. (1) and (2). Note that if we would include the mixing of all three generations, the expression would be far more complicated, in particular for the Higgsino mass term, since many more mixing combinations would occur. Knowing that $m_{\tilde{\Delta}}\gg m_l$ the mixing of the scalar leptons cannot be neglected. If we exclude the contribution of the left-handed Higgsino, then $B^L_{\tilde{l}}$ and the term proportional to the Higgsino mass are absent, as can be seen easily from Eq. (A6) (all $\tilde{K}^a_{m1}$ set to be 0).[2]

A matrix element of the form $iM=e\bar{u}_{p_2}i\sigma_{\mu\nu}q^\nu(a_LP_L+a_RP_R)u_{p_1}\epsilon_q^{*\mu}$ leads to a decay rate of $\Gamma(\mu\to e\gamma)=m_\mu^3(a_L^2+a_R^2)\alpha/4$ and $\Gamma(\mu\to 3e)=\frac{1}{3}[\alpha^2/(4\pi)](a_L^2+a_R^2)m_\mu^3\ln(m_\mu^2/m_e^2)-\frac{11}{4}]$. With $\Gamma(\mu\to\nu_\mu e^-\bar{\nu}_e)=(G_F^2/192\pi^3)m_\mu^5=(\alpha^2/384\pi s_W^4)(m_\mu/m_W)^4m_\mu$, where s_W is the Weinberg angle $\sin\theta_W$, we obtain the following branching ratio from Eq. (5):

$$B(\mu\to e\gamma)=\frac{3s_W^4}{8\pi^3}\frac{h_{LR}^4}{\alpha}\left(\frac{m_W}{m_{\tilde{\Delta}}}\right)^4[c^2_{\theta_L}s^2_{\theta_L}(B^L_{\tilde{e}}-B^L_{\tilde{\mu}})^2 + c^2_{\theta_R}s^2_{\theta_R}(B^R_{\tilde{e}}-B^R_{\tilde{\mu}})^2]. \tag{6}$$

For $B(\mu\to 3e)$, we use $\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)=(\alpha/3\pi)[\ln(m_\mu^2/m_e^2)-\frac{11}{4}]$.

[1]However, in Appendix A, we will present general analytical expressions obtained by including the mixing of all generations via a supersymmetric version of the Kobayashi-Maskawa matrix.

[2]The calculation of the diagrams with the doubly charged left- and right-Higgs bosons and leptons within the loop, which occur in the left-right model without SUSY are very similar ($e_{\tilde{\Delta}}\leftrightarrow e_{l^C}=+1$, $e_{\tilde{l}}\leftrightarrow e_{\Delta_{L,R}}$ and $x^{\tilde{\Delta}}_{\tilde{l}_m}\leftrightarrow x^l_{\Delta_{L,R}}=m^2_{\Delta_{L,R}}/m_l^2$). Limits of the couplings and the masses of the doubly charged Higgs boson have been given elsewhere and will not be presented here [11].

(I) If, in the first instance we neglect the mixing of the scalar partners of the left- and right-handed leptons, that is, setting $\sin\theta_{\tilde{l}}=0$, Eq. (6) leads to

$$B(\mu\to e\gamma)=\frac{3s_W^4}{8\pi^3}\frac{h_{LR}^4}{\alpha}\left(\frac{m_W}{m_{\tilde{\Delta}}}\right)^4[c_{\theta_L}^2 s_{\theta_L}^2(B_{\tilde{e}_1}-B_{\tilde{\mu}_1})^2 + c_{\theta_R}^2 s_{\theta_R}^2(B_{\tilde{e}_2}-B_{\tilde{\mu}_2})^2]. \tag{7}$$

The finite result depends strongly on the mass differences of the first and second generations. If we take, as required by left-right symmetry $\theta_L\equiv\theta_R\equiv\theta$ and $B_{\tilde{l}_1}=B_{\tilde{l}_2}$ with $\Delta m_{\tilde{l}}^2=\Delta m_l^2$ for different generations and $m_{\tilde{\Delta}}\sim m_{\tilde{l}}$, a first estimate is given by

$$B(\mu\to e\gamma)=\frac{3s_W^4}{4\pi^3}\frac{h_{LR}^4}{\alpha}\left(\frac{m_W}{m_{\tilde{\Delta}}}\right)^4 c_\theta^2 s_\theta^2\left(\frac{7}{120}\right)^2\left(\frac{m_\mu}{m_{\tilde{\Delta}}}\right)^4. \tag{8}$$

Equation (8) leads to $B(\mu\to e\gamma)\approx 2.97\times 10^{-16}h_{LR}^4 c_\theta^2 s_\theta^2$ for $m_{\tilde{\Delta}}=100$ GeV; far smaller than the experimental limit.

(II) However, the situation is changed when the mixing of the scalar partners of the right- and left-handed leptons is included. Since $m_l \ll m_{\tilde{l}}$ the mixing angle is given by $\sin\theta_{\tilde{l}}\sim m_l(A_l-\mu\tan\beta)/m_{\tilde{l}}^2\sim m_l/m_{\tilde{l}}$ with $(A_l-\mu\tan\beta)\sim m_{\tilde{l}}$, where A_l is the trilinear scalar interaction, μ the mixing mass terms of the Higgs bosons and $\tan\beta=v_2/v_1$ the ratio of their vacuum expectation values (VEV's). This leads to $B_{\tilde{l}}^L\approx B_{\tilde{l}_1}+(m_{\tilde{\Delta}}/m_{\tilde{l}})(m_l/m_\mu)(\tilde{B}_{\tilde{l}_1}-\tilde{B}_{\tilde{l}_2})$ where we assumed that after left-right symmetry breaking the relations $c_{\theta_L}/c_{\theta_R}\sim s_{\theta_L}/s_{\theta_R}\sim 1$ still hold.

If the left-right symmetry is conserved, we have $\tilde{B}_{\tilde{l}_1}=\tilde{B}_{\tilde{l}_2}$ and Eq. (7) is recovered. However, after breaking, the mass difference between the scalar partners of the left-handed and right-handed leptons is much larger than the mass difference between the generations. We expect $(m_{\tilde{l}_1}^2-m_{\tilde{l}_2}^2)/m_{\tilde{l}}^2 \sim 10^{-2}-10^{-1}$ [7,19]. That is $B_{\tilde{e}}^{L,R}-B_{\tilde{u}}^{L,R}\sim(m_{\tilde{\Delta}}/m_{\tilde{l}})(\tilde{B}_{\mu_2}-\tilde{B}_{\mu_1})\sim\frac{5}{12}(10^{-2}-10^{-1})$ with $m_{\tilde{\Delta}}\sim m_{\tilde{l}}$. With these values Eq. (6) leads to $B(\mu\to e\gamma)\approx 1.25\times(10^{-6}-10^{-4})h_{LR}^4 c_\theta^2 s_\theta^2$ for $m_{\tilde{\Delta}}=100$ GeV and thus far above the known experimental limit of 4.9×10^{-11}, leading to the constraint $h_{LR}^2<(10^{-7}-10^{-6})m_{\tilde{\Delta}}^2$ (GeV) when maximal electron-muon mixing is assumed.

IV. THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

The calculations to the AMMM are similar to the $\mu\to e\gamma$ decay. From Eq. (A7) we obtain the following result:

$$\Delta a_\mu=-\frac{2h_{LR}^2}{(4\pi)^2}\left(\frac{m_\mu}{m_{\tilde{\Delta}}}\right)^2\{B_{\tilde{\mu}}^L+B_{\tilde{\mu}}^R\}. \tag{9}$$

$B_{\tilde{\mu}}^L$ and $B_{\tilde{\mu}}^R$ as given in Eq. (5). Again $B_{\tilde{\mu}}^L$ is absent when we exclude the contribution of the left-handed Higgsino.

Equation (9) leads to

$$\Delta a_\mu=-\frac{2h_{LR}^2}{(4\pi)^2}\left(\frac{m_\mu}{m_{\tilde{\Delta}}}\right)^2(B_{\tilde{\mu}_1}+B_{\tilde{\mu}_2}). \tag{10}$$

Equation (10) leads to $\Delta a_\mu\approx -3.49\times 10^{-9}h_{LR}^2$ for $m_{\tilde{\Delta}}\sim m_{\tilde{\mu}}=100$ GeV, thus already very close to the experimental lower limit of $\Delta a_\mu^{\text{expt}}$, leading to the constraint $h_{LR}^2<5\times 10^{-4}m_{\tilde{\Delta}}^2$ (GeV).

V. CONCLUSIONS

We presented the results of the contribution of the doubly charged Higgsino to the decays $\mu\to e\gamma$ and $\mu\to 3e$ and to the anomalous magnetic moment of the muon Δa_μ. It was shown that when mixing of the scalar partners of the left- and right-handed leptons is neglected, the contribution to the muon decay modes is far below the experimental limits while the contribution to the anomalous magnetic moment of the muon is within reach of the BNL experiment E821. When mixing of the scalar leptons is included we obtain a constraint for the lepton-scalar lepton-doubly charged Higgsino coupling h_{LR} in the order of less than $10^{-3}m_{\tilde{\Delta}}$ (GeV) if maximal electron-muon mixing is assumed. The effect of this mixing on Δa_μ is negligible.

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APPENDIX A: THE MATRIX ELEMENTS

In this Appendix we present the final results of the calculations of the Feynman diagrams as shown in Fig. 1 with flavor nondiagonal couplings of the doubly charged Higgsino to the left- and right-handed leptons and their scalar partners. We define the following matrix:

$$iM=+\frac{eh_{LR}^2}{(4\pi)^2}(M_S+M_F+M_{\text{SE}}), \tag{A1}$$

where M_S is the matrix element for the two scalar leptons in the loop, M_F the one for the two doubly charged Higgsinos in the loop, and M_{SE} the one for the self-energy diagrams. The calculations of the diagrams are very similar to those presented in the Appendix A of [16], where we refer the interested reader to for more details. Here we only present the finite results for each diagram:

$$iM_F=\int_0^1 d\alpha_1 e_{\tilde\Delta}\bigg[\overline{u_{p_2}}(q^2\gamma_\mu-\not q q_\mu)(T_{11}^{aL}P_L+T_{22}^{aR}P_R)u_{p_1}\frac{1}{6}(2-3\alpha_1+\alpha_1^3)+\overline{u_{p_2}}i\sigma_{\mu\nu}q^\nu\bigg\{m_{\tilde\Delta}(1-\alpha_1)(T_{12}^{aL}P_L+T_{12}^{aR}P_R)$$
$$-\frac{1}{2}\alpha_1(1-\alpha_1)^2[m_{e,\mu}(T_{11}^{aL}P_L+T_{22}^{aR}P_R)+m_\mu(T_{11}^{aL}P_R+T_{22}^{aR}P_L)]\bigg\}u_{p_1}\bigg]\frac{\epsilon_q^{*\mu}}{D_{\rm SE}^{\tilde\Delta\tilde l}}, \tag{A2}$$

$$iM_S=\int_0^1 d\alpha_1 e_{\tilde l}\bigg\{\overline{u_{p_2}}(q^2\gamma_\mu-\not q q_\mu)(T_{11}^{aL}P_L+T_{22}^{aR}P_R)u_{p_1}\frac{1}{6}\alpha_1^3+\overline{u_{p_2}}i\sigma_{\mu\nu}q^\nu[m_{e,\mu}(T_{11}^{aL}P_L+T_{22}^{aR}P_R)$$
$$+m_\mu(T_{11}^{aL}P_R+T_{22}^{aR}P_L)]u_{p_1}\frac{1}{2}\alpha_1^2(1-\alpha_1)\bigg\}\frac{\epsilon_q^{*\mu}}{D_{\rm SE}^{\tilde\Delta\tilde l}}, \tag{A3}$$

$$iM_{\rm SE}=\int_0^1 d\alpha_1[-e_\mu\{\overline{u_{p_2}}i\sigma_{\mu\nu}q^\nu[-\alpha_1(1-\alpha_1)m_{\tilde\Delta}(T_{12}^{aL}P_L+T_{12}^{aR}P_R)]u_{p_1}\}\frac{\epsilon_q^{*\mu}}{D_{\rm SE}^{\tilde\Delta\tilde l}}, \tag{A4}$$

$$D_{\rm SE}^{\tilde\Delta\tilde l}=m_{\tilde\Delta}^2-(m_{\tilde\Delta}^2-m_{\tilde l_{a_m}}^2)\alpha_1,$$

$$T_{11}^{aL}:=K_{ae,\mu}^{*L}K_{a\mu}^L\widetilde K_{m1}^{a2},\quad T_{22}^{aR}:=K_{ae,\mu}^{*R}K_{a\mu}^R\widetilde K_{m2}^{a2},$$

$$T_{12}^{aL}:=K_{ae,\mu}^{*R}K_{a\mu}^L\widetilde K_{m1}^a\widetilde K_{m2}^a,\quad T_{12}^{aR}:=K_{ae,\mu}^{*L}K_{a\mu}^R\widetilde K_{m1}^a\widetilde K_{m2}^a,$$

where $e_{\tilde\Delta}=-2$, $e_{\tilde l}=+1$,and $e_\mu=-1$. We have to sum over the generation indices $a=1-3$ and over the eigenstates of the scalar partners of the left- and right-handed leptons $m=1,2$. The divergencies cancel after the summation of all diagrams $e_{\tilde\Delta}+e_{\tilde l}-e_\mu\equiv 0$. Because of this relation, many more terms like those proportional to $m_{\tilde\Delta}\alpha_1(1-\alpha_1)(p_1+p_2)^\mu$ drop out after summation and are not explicitly written down in Eqs. (A2)–(A4). Furthermore we made use of Eqs. (A.5–(A.8) in [16] and Eqs. (A.6)+(A.7) in [17]. After summation of Eqs. (A2)–(A4) the final matrix element is given by

$$iM=+\frac{eh_{LR}^2}{(4\pi)^2}\frac{1}{m_{\tilde\Delta}^2}\{\overline{u_{p_2}}(q^2\gamma_\mu-\not q q_\mu)(T_{11}^{aL}P_L+T_{22}^{aR}P_R)u_{p_1}\{e_{\tilde\Delta}[A_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+A_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]+e_{\tilde l}A_F(x_{\tilde l_{a_m}}^{\tilde\Delta})\}+\overline{u_{p_2}}i\sigma_{\mu\nu}q^\nu\{m_{\tilde\Delta}(T_{12}^{aL}P_L$$
$$+T_{12}^{aR}P_R)[e_{\tilde\Delta}\widetilde B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+e_\mu\widetilde B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})][m_{e,\mu}(T_{11}^{aL}P_L+T_{22}^{aR}P_R)+m_\mu(T_{11}^{aL}P_R+T_{22}^{aR}P_L)][e_{\tilde\Delta}[-B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]$$
$$+e_{\tilde l}B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]\}u_{p_1}\}\epsilon_q^{*\mu}. \tag{A5}$$

The final functions after Feynman integration are shown in Appendix B. Since we are not interested in CP violation we can skip the $*$ in $T_{kl}^{aL,R}$.[3] Furthermore for a photon on mass shell we have $q^2=0$ and due to gauge invariance $\epsilon_q^{*\mu}q_\mu=0$.

Neglecting the electron mass, the matrix element for the decay $\mu\to e\gamma$ is then given by

$$iM=+\frac{eh_{LR}^2}{(4\pi)^2}\frac{1}{m_{\tilde\Delta}^2}\overline{u_{p_2}}i\sigma_{\mu\nu}q^\nu\{m_\mu[K_{a1}^LK_{a2}^L\widetilde K_{m1}^{a2}P_R+K_{a1}^RK_{a2}^R\widetilde K_{m2}^{a2}P_L]\{e_{\tilde\Delta}[-B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]+e_{\tilde l}B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})\}$$
$$+m_{\tilde\Delta}\widetilde K_{m1}^a\widetilde K_{m2}^a(K_{a1}^RK_{a2}^LP_R+K_{a1}^LK_{a2}^RP_L)[e_{\tilde\Delta}\widetilde B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+e_\mu\widetilde B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]\}u_{p_1}\epsilon_q^{*\mu}. \tag{A6}$$

[3]We could also have skipped the L and R indices (that is $\theta_L=\theta_R$) since the difference between the left and right diagonalizing matrices lies in a relative minus sign of the SUSY phase; see Eq. (4) in [18]. This would simplify the results strongly, giving us an overall factor of $K_{ae,\mu}K_{a\mu}$ and $T_{12}^{aL}\equiv T_{12}^{aR}$. For the moment however we will keep θ_L different from θ_R.

After summation over the mass eigenstates of the scalar leptons and all the generations making use of the unitarity of the leptonic KM matrix Eq. (A6) leads to the branching ratio $B(\mu\rightarrow e\gamma)$ as given in Eq. (5).

The ANMM can be extracted from $V_\mu = (ie/2m_\mu)\,F(q^2)\,\overline{u_{p_2}}\,i\sigma_{\mu\nu}q^\nu u_{p_1}$ at $q^2=0$. We obtain the result

$$F(q^2)|_{q^2=0} = \Delta a_\mu = -\frac{2h_{LR}^2}{(4\pi)^2}\left(\frac{m_\mu}{m_{\tilde\Delta}}\right)^2\Bigg\{[|K_{a2}^L|^2\tilde K_{m1}^{a2}+|K_{a2}^R|^2\tilde K_{m2}^{a2}] \times[e_{\tilde\Delta}[-B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]+e_{\tilde l}B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})] + \frac{m_{\tilde\Delta}}{m_\mu}K_{a2}^R K_{a2}^L\tilde K_{m1}^a\tilde K_{m2}^a[e_{\tilde\Delta}\tilde B_G(x_{\tilde l_{a_m}}^{\tilde\Delta})+e_\mu\tilde B_F(x_{\tilde l_{a_m}}^{\tilde\Delta})]\Bigg\}. \tag{A7}$$

As above, after summation over all mass eigenstates and generations of the leptons Eq. (A7) leads to the result as given in Eq. (9).

APPENDIX B: THE FINAL FUNCTIONS

$$A_G(x_{\tilde l_{a_m}}^{\tilde\Delta}) = \frac{1}{6}\int_0^1 d\alpha_1[2-3\alpha_1^2-3\alpha_1(1-\alpha_1)]/D_{\tilde\Delta\tilde l_{a_m}} = \frac{1}{(1-x_{\tilde l_{a_m}}^{\tilde\Delta})^2}\frac{1}{2}\Bigg\{1-x_{\tilde l_{a_m}}^{\tilde\Delta}+\frac{1}{3}(1+2x_{\tilde l_{a_m}}^{\tilde\Delta}) \times\ln(x_{\tilde l_{a_m}}^{\tilde\Delta})\Bigg\}, \tag{B1}$$

$$A_F(x_{\tilde l_{a_m}}^{\tilde\Delta}) = \frac{1}{6}\int_0^1 d\alpha_1\alpha_1^3/D_{\tilde\Delta\tilde l_{a_m}} = \frac{1}{(1-x_{\tilde l_{a_m}}^{\tilde\Delta})^4}\frac{1}{36}\{-11+18x_{\tilde l_{a_m}}^{\tilde\Delta}-9x_{\tilde l_{a_m}}^{\tilde\Delta\,2} +2x_{\tilde l_{a_m}}^{\tilde\Delta\,3}-6\ln(x_{\tilde l_{a_m}}^{\tilde\Delta})\}, \tag{B2}$$

$$B_G(x_{\tilde l_{a_m}}^{\tilde\Delta}) = \frac{1}{2}\int_0^1 d\alpha_1\alpha_1(1-\alpha_1)/D_{\tilde\Delta\tilde l_{a_m}} = \frac{1}{(1-x_{\tilde l_{a_m}}^{\tilde\Delta})^3}\frac{1}{4}\{1-x_{\tilde l_{a_m}}^{\tilde\Delta\,2}+2x_{\tilde l_{a_m}}^{\tilde\Delta}\ln(x_{\tilde l_{a_m}}^{\tilde\Delta})\}, \tag{B3}$$

$$B_F(x_{\tilde l_{a_m}}^{\tilde\Delta}) = \frac{1}{2}\int_0^1 d\alpha_1\alpha_1^2(1-\alpha_1)/D_{\tilde\Delta\tilde l_{a_m}} = \frac{1}{(1-x_{\tilde l_{a_m}}^{\tilde\Delta})^4}\frac{1}{12}\{2+3x_{\tilde l_{a_m}}^{\tilde\Delta}-6x_{\tilde l_{a_m}}^{\tilde\Delta\,2}+x_{\tilde l_{a_m}}^{\tilde\Delta 3} +6x_{\tilde l_{a_m}}^{\tilde\Delta}\ln(x_{\tilde l_{a_m}}^{\tilde\Delta})\}, \tag{B4}$$

$$\tilde B_G(x_{\tilde l_{a_m}}^{\tilde\Delta}) = \int_0^1 d\alpha_1(1-\alpha_1)/D_{\tilde\Delta\tilde l_{a_m}} = \frac{1}{(1-x_{\tilde l_{a_m}}^{\tilde\Delta})^2}\{1-x_{\tilde l_{a_m}}^{\tilde\Delta}+x_{\tilde l_{a_m}}^{\tilde\Delta}\ln(x_{\tilde l_{a_m}}^{\tilde\Delta})\}, \tag{B5}$$

$$\tilde B_F(x_{\tilde l_{a_m}}^{\tilde\Delta}) = 2B_G(x_{\tilde l_{a_m}}^{\tilde\Delta}), \tag{B6}$$

$$D_{\tilde\Delta\tilde l_{a_m}} = 1-(1-x_{\tilde l_{a_m}}^{\tilde\Delta})\alpha_1,$$

$$x_{\tilde l_{a_m}}^{\tilde\Delta} = \frac{m_{\tilde l_{a_m}}^2}{m_{\tilde\Delta}^2}.$$

We have the following relations between the Eqs. (B.3)–(B.6) and Eqs. (4.10), (4.11), (4.17), and (4.21) of [5]. $(m_\mu^2/m_{\tilde\Delta}^2)B_F=G'/6$, $(m_\mu/m_{\tilde\Delta})\tilde B_F=G/2$, $m_\mu^2/m_{\tilde\Delta}^2(B_G-B_F)=F'/12$, and $(m_\mu/m_{\tilde\Delta})(\tilde B_G-\tilde B_F)=F/2$. Furthermore we have $(m_\mu/m_{\tilde\Delta})G=(2G'+F')/3$.

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