Contribution of $W^{\pm}H^{\mp}Z_i$ vertices to anomalous magnetic dipole and electric quadrupole moments of the W boson in the extra U(1) superstring-inspired model

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$W^{\pm}H^{\mp}Z_i$ ($i=1,2$) vertices available exclusively in the extra U(1) superstring-inspired model have been utilized to evaluate the W-W-γ vertex contribution at the one loop level. The expressions so obtained have been used to estimate the anomalous magnetic dipole moment (Δk_{WZ_iH}) and electric quadrupole moment (ΔQ_{WZ_iH}) of the W^+ boson. The contribution of the Z_2WW vertex is also added to these values. The resulting values in the unit of $(-\alpha/\pi)$ when the Z_2 mass varies from 555 to 620 GeV have the following ranges: for $m_{t(\mathrm{CDF})}=175.6$ GeV, $\Delta k_{[\mathrm{U}(1)]}$, from 25.402 to 41.559, $\Delta Q_{[\mathrm{U}(1)]}$, from 6.886 to 10.858; for $m_{t(\mathrm{D0})}=169$ GeV, $\Delta k_{[\mathrm{U}(1)]}$, from 20.821 to 34.121, $\Delta Q_{[\mathrm{U}(1)]}$, from 5.738 to 9.033. These are larger than the standard model radiative correction contributions but an order of magnitude smaller than those predicted by the composite model of Abbott and Farhi. [S0556-2821(97)03619-9]

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In recent years precision electroweak measurements have started acquiring a sensitivity at which they are capable of testing the standard model (SM) having the gauge symmetry $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ at the level of its electroweak radiative corrections and exploring the small ''new physics'' effects [1–3]. Already the precision measurements of the weak interaction parameters have reached the level of 1% accuracy of the determination of the W mass and the parameters of the Z^0 resonance. As such, any deviation from standard model expectations could be a hint for additional tree level interactions involving physics beyond the SM, or a signal for further loop corrections. The prevailing framework for physics beyond the SM includes technicolor theories including composite models [4–7], supersymmetry, and superstring theories [8].

An interesting outcome of the precision measurement of various electroweak parameters has been the prediction that the parameters [9] S (and possibly T) should have negative values [1,9,10]. In fact the result has been used to constrain [1] the technicolor models which by and large give positive S values, with the exception of a recent observation by Luty and Sundrum [11]. On the other hand, Marciano and Rosner [1] and Holdom [12] have shown that models having an extra U(1) with an additional Z' do give a negative contribution to S. Recently we have been exploring the phenomenology of the extra U(1) models emanating from the E_6 group in superstring theory [13]. The fact that the extra U(1) model may provide a viable alternative as a model for physics beyond the SM has further motivated us to explore some new aspects related to this model, in particular, our recent calculations for the S, T, U parameters in the extra U(1) model do give reasonable negative values [14].

The static properties of the W boson, such as the anomalous magnetic dipole moment (Δk) and electric quadrupole moment (ΔQ) are two important quantities and may provide a crucial test for the SM. Bardeen *et al.* and others [15], have calculated these quantities in the SM in the massless fermion limits. These have been modified recently for the massive t quark by Couture and Ng [16] who also examined the contribution to (Δk) and (ΔQ) in the E_6 vector lepton model, where extra leptons—both neutral and charged types—form doublets with the corresponding known leptons but there is no extra Z'. In the extra U(1) model coming from E_6 group in superstring theory there is an extra Z' boson and extended Higgs structure involving at least two doublets and one singlet for each family [13,17]. This provides additional vertices $W^{\pm}H^{\mp}Z_i$ ($i=1,2$) in the model. It is, therefore, of some interest to explore the contribution of these vertices to the static quantities, e.g., the anomalous magnetic moment Δk and the electric quadrupole moment ΔQ of the W boson. We report on the contribution of this work at one loop level. Since a copious supply of the W^+W^- beam is expected from the CERN e^+e^- collider LEP II, we hope that this work could be used to measure these parameters with high precision and thereby new predictions may possibly be tested.

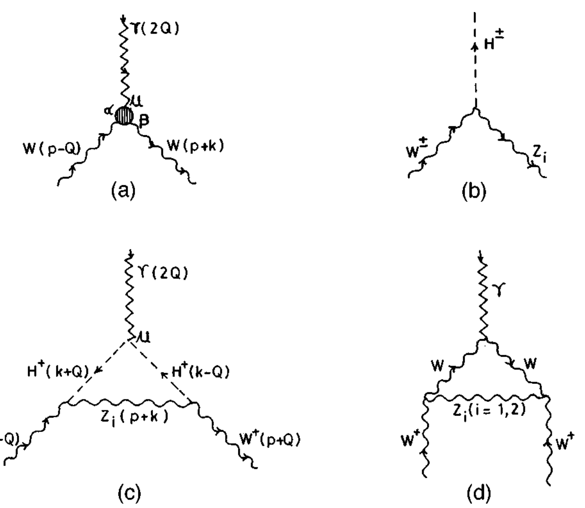

FIG. 1. (a) The W-W-γ vertex in general. (b) The $W^{\pm}$ $H^{\mp}$ Z_i vertex of the extra U(1) model. (c) Two one loop diagrams involving the $W^+H^+Z_i$ ($i=1,2$) vertices contribution to the W-W-γ vertex. (d) The one loop diagram involving the Z_2WW vertex.

When the particles involved are on the mass shell, the most general CP-invariant $W^+W^-\gamma$ vertex may be written as [15–16]

$$\tau^{\mu\alpha\beta}=ie\Big\{A[2p^\mu g^{\alpha\beta}+4(Q^\beta g^{\alpha\mu}-Q^\alpha g^{\beta\mu})]+2(k-1)\times(Q^\beta g^{\alpha\mu}-Q^\alpha g^{\beta\mu})+4\frac{\Delta Q}{M_W^2}p^\mu Q^\alpha Q^\beta\Big\}. \tag{1}$$

This is shown schematically in Fig. 1(a). In addition, the $W^{\pm}H^{\mp}Z_i$ ($i=1,2$) vertices of the extra U(1) model [13] are shown in Fig. 1(b), the one loop diagram for the W-W-γ vertex in Fig. 1(c), and for the Z_2WW vertex in Fig. 1(d). The $W^{\pm}H^{\mp}Z_i$ couplings (c_i) are given by [13,17]

$$c_1=\frac{5}{6}\,e\,\sin 2\beta\,\sin\theta_E M_{Z_1}, \tag{2a}$$

$$c_2=\frac{5}{6}\,e\,\sin 2\beta\,\cos\theta_E M_{Z_1}, \tag{2b}$$

where β and θ_E are the mixing angles, respectively, in the charged Higgs sector and the Z-Z' boson sector. In the latter case Z_1,Z_2 define the mass eigenstates corresponding to the flavor states Z-Z', respectively.

Using the couplings given in Eqs. (2) for the $W^{\pm}H^{\mp}Z_i$ vertices and the usual coupling for γH^+H^-, we get for the triple W-W-γ vertex of Fig. 1(c), the following expression in the Landau gauge:

$$-i\tau^{\alpha\beta\mu}=2e\sum_{i=1,2}\int\frac{d^4k}{(2\pi)^4}\,c_i^2\,\frac{\{g^{\alpha\beta}-(p^\alpha p^\beta+k^\alpha k^\beta+k^\alpha p^\beta+p^\alpha k^\beta)/M_{Z_i}^2\}k^\mu}{[(p+k)^2-M_{Z_i}^2][(k-Q)^2-M_H^2][(k+Q)^2-M_H^2]}. \tag{3}$$

In order to simplify right-hand side (RHS), we follow the usual procedure [18]: namely, (i) change the variable to $k'=k+p(1-x-y)-Q(x-y)$, (ii) introduce an arbitrary mass μ for regularizing the couplings and make use of dimensional regularization.

After lengthy but straightforward algebra, we get

$$-i\tau^{\alpha\beta\mu}=4e\sum_{i=1,2}\frac{d^nk'c_i^2}{M_{Z_i}^2(2\pi)^n}\int_0^1dx\int_0^{1-x}dy[k'^\alpha k'^\beta k'^\mu+A_1^\beta(x,y;p,Q)k'^\alpha k'^\mu+A_2^\alpha(x,y;p,Q)k'^\mu k'^\beta+A_3^\mu(x,y;p,Q)k'^\alpha k'^\beta$$
$$+A_4^{\alpha\beta}(x,y;p,Q)k'^\mu+A_5^{\beta\mu}(x,y;p,Q)k'^\alpha+A_6^{\mu\alpha}(x,y;p,Q)k'^\beta+A_7^{\alpha\beta\mu}(x,y;p,Q)]\,\frac{1}{[k'^2-M_i'^2]^3}. \tag{4}$$

Since in the denominator $Q=0$, the contribution from $A_4^{\alpha\beta}k'^\mu$, $A_5^{\mu\beta}k'^\alpha$, $A_6^{\alpha\mu}k'^\beta$ will vanish [18]. Also

$$M_i'^2=M_{Z_i}^2-p^2(1-x-y)(x+y)+Q^2(x-y)^2-2pQ(1-x-y)(x-y)-(M_{Z_i}^2-M_{H^+}^2)(x+y). \tag{5}$$

After very lengthy and tedious algebra [18] and with the use of the Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) [19] regularization scheme, we get an expression of the type

$$\tau^{\alpha\beta\mu}=\sum_i ieF_i\Big[2p^\mu g^{\alpha\beta}+4(Q^\beta g^{\alpha\mu}-Q^\alpha g^{\beta\mu})I_1(Z_i)+(Q^\beta g^{\alpha\mu}Q^\alpha g^{\beta\mu})I_2(Z_i)+\frac{(Q^\alpha Q^\beta p^\mu)}{M_W^2}I_3(Z_i)+\frac{(Q^\alpha Q^\beta Q^\mu)}{M_W^2}I_4(Z_i)\Big], \tag{6}$$

where

$$F_1=\left(\frac{100\pi^2}{9}\right)\sin^2 2\beta M_{Z_1}^2\,\sin^2\theta_E\left(\frac{\alpha}{\pi}\right),$$

$$F_2=\left(\frac{100\pi^2}{9}\right)\sin^2 2\beta M_{Z_1}^2\,\cos^2\theta_E\left(\frac{\alpha}{\pi}\right), \tag{7}$$

and

$$I_1(Z_i)=\sum_{i=1}^{2}\frac{1}{(64\pi^2M_{Z_i})}\int_0^1dx\int_0^{(1-x)}dy\Big[\left(\frac{1-3x-3y}{2}\right)\{\ln(C_iy^2+b_iy+a_i)-(C_iy^2+b_iy+a_i-1)\}$$
$$\times\frac{-(1/4)\{y^3+(3x-1)y^2+(3x^2-2x-4/C_i)y+x^3-x^2+(1-x)(1/C_i)\}}{(y^2+p_iy+q_i)}\Big], \tag{8}$$

$$I_2(Z_i)=-\sum_{i=1}^{2}\frac{1}{(32\pi^2M_{Z_i}^2)}\int_0^1 dx\int_0^{(1-x)}dy\Bigg[(1-3x-3y)\{\ln(C_iy^2+b_iy+a_i)-(C_iy^2+b_iy+a_i)\}$$
$$\times\frac{-(1/2)\{y^3+(3x-1)y^2+(3x^2-2x-4/C_i)y+x^3-x^2+(1-x)(1/C_i)\}}{(y^2+p_iy+q_i)}\Bigg], \tag{9}$$

$$I_3(Z_i)=-\sum_{i=1}^{2}\frac{1}{(32\pi^2M_{Z_i}^2)}\int_0^1 dx\int_0^{(1-x)}dy\ \frac{\{3y^3+(3x-1)y^2+(x^2-2x)y+(3x^3-x^2)\}}{(y^2+p_iy+q_i)}, \tag{10}$$

$$I_4(Z_i)=-\sum_{i=1}^{2}\frac{1}{(64\pi^2M_{Z_i}^2)}\frac{(1-x-y)^3}{(y^2+p_iy+q_i)}, \tag{11a}$$

with

$$C_i=\frac{M_W^2}{M_{Z_i}^2},$$
$$b_i=\left(\frac{2xM_W^2-d_i}{M_{Z_i}^2}\right),$$
$$a_i=\left(\frac{1+(x^2M_W^2-xd_i)}{M_{Z_i}^2}\right),$$
$$d_i=(M_W^2+M_{Z_i}^2-M_{H^+}^2),$$
$$p_i=\left(2x-\frac{d_i}{M_W^2}\right),$$
$$q_i=\left(x^2-x\,\frac{d_i}{M_W^2}+\frac{1}{C_i}\right). \tag{11b}$$

A comparison of Eqs. (1) and (6) gives

$$\Delta k_{(WZ_iH)}=\sum_{i=1}^{2}\frac{1}{2}F_iI_2(Z_i), \tag{12}$$

$$\Delta Q_{(WZ_iH)}=\sum_{i=1}^{2}\frac{1}{4}F_iI_3(Z_i), \tag{13}$$

where $\Delta k=(k-1)$.

The integrals $I_2(Z_i)$ and $I_3(Z_i)$ have been evaluated analytically by the standard procedure. We obtain finally the expressions

$$\Delta k_{(WZ_iH)}=\sum_{i=1}^{2}\frac{25}{144}\sin^2 2\beta\,\frac{M_{Z_1}^2}{M_{Z_i}^2}\left(\frac{\sin\theta_E}{\cos\theta_E}\right)^2\left(\frac{\alpha}{\pi}\right)\Bigg[\left(\frac{15}{4}+\frac{7}{2}\frac{1}{C_i}-\frac{d_i}{12M_{Z_i}^2}+\frac{9}{20}C_i+\frac{1}{4}\frac{d_i}{M_W^2}+\frac{d_i^2}{M_W^4}\right)$$
$$+\sqrt{\Delta_i}\left(\frac{3}{2}\frac{d_i}{M_W^2}-\frac{7}{2C_i}+\frac{5}{2}\frac{d_i^2}{M_W^4}\right)\left\{\arctan\left(\frac{f_i}{2\sqrt{\Delta_i}}\right)+\arctan\left(\frac{d_i}{2M_W^2\sqrt{\Delta_i}}\right)\right\}+\left(\frac{1}{C_i}-\frac{7}{4}\frac{d_i}{C_iM_W^2}-\frac{1}{4}\frac{d_i^2}{M_W^4}+\frac{1}{4}\frac{d_i^3}{M_W^6}\right)$$
$$\times\left\{\frac{d_i}{M_W^2}\left\{\arctan\left(\frac{f_i}{\sqrt{\Delta_i}}\right)+\arctan\left(\frac{d_i}{M_W^2\sqrt{\Delta_i}}\right)\right\}-\frac{1}{2}\ln(\Delta_i+f_i^2)-\frac{1}{2}\ln\left(\frac{d_i^2}{M_W^4}+\Delta_i\right)\right.$$
$$+\left(\frac{3}{2}M_{H^+}^2-\frac{3}{C_i}+\frac{19}{8}\frac{d_i}{C_iM_W^2}+\frac{5}{2}\frac{d_i}{M_W^2}-\frac{23}{8}\frac{d_i^2}{M_W^4}+\frac{1}{2}\frac{d_i^3}{M_W^6}\right)\ln\left(\frac{M_{H^+}^2}{M_W^2}\right)$$
$$+\left(-\frac{1}{2}+\frac{3}{2C_i}-\frac{19}{8}\frac{d_i}{C_iM_W^2}-\frac{1}{8}\frac{d_i^2}{M_W^4}+\frac{1}{4}\frac{d_i^3}{M_W^6}\right)\ln\left(\frac{M_{Z_i}^2}{M_W^2}\right)\Bigg], \tag{14}$$

$$\Delta Q_{(WZ_iH)}=-\sum_{i=1}^{2}\frac{25}{288}\sin^2 2\beta\,\frac{M_{Z_1}^2}{M_{Z_i}^2}\left(\begin{matrix}\sin\theta_E\\ \cos\theta_E\end{matrix}\right)^2\left(\frac{\alpha}{\pi}\right)\Bigg[\left(-\frac{1}{6}-\frac{5}{4}\frac{1}{C_i}+\frac{d_i}{6M_W^2}+\frac{23}{24}\frac{d_i^2}{M_W^4}\right)+\left(\frac{1}{C_i}-\frac{d_i^2}{M_W^4}+\frac{13}{12}\frac{d_i^3}{M_W^6}-\frac{13}{2}\frac{d_i}{C_iM_W^2}\right)\frac{1}{2}\ln\left(\frac{M_{H^+}^2}{M_W^2}\right)+\left(\frac{d_i^2}{M_W^2}-\frac{d_i^3}{M_W^6}+\frac{3d_i}{C_iM_W^2}\right)\frac{1}{4}\ln\left(\frac{M_{Z_i}^2}{M_W^2}\right)+\left(-\frac{1}{C_i}-\frac{5d_i}{C_iM_W^2}+\frac{d_i^2}{2M_W^4}-\frac{7}{12}\frac{d_i^3}{M_W^6}\right)\times\frac{1}{2}\ln\left(\frac{3}{4}\frac{d_i^2}{M_W^4}+\frac{1}{C_i}\right)+\sqrt{\Delta_i}\left(\frac{3}{C_i}+\frac{d_i}{M_W^2}-\frac{d_i^2}{M_W^4}\right)\left\{\arctan\left(\frac{f_i}{2\sqrt{\Delta_i}}\right)+\arctan\left(\frac{d_i}{2M_W^2\sqrt{\Delta_i}}\right)\right\}+\frac{1}{\sqrt{\Delta_i}}\left(-\frac{16}{3}+\frac{16}{3}\frac{d_i}{M_W^2}+\frac{13}{2}\frac{d_i^3}{M_W^6}+\frac{67}{192}\frac{d_i^4}{M_W^8}-\frac{2}{C_i^2}-\frac{d_i^2}{M_W^4C_i}\right)\arctan\left(\frac{f_i}{\sqrt{\Delta_i}}\right)+\frac{1}{\sqrt{\Delta_i}}\left(-\frac{7}{4C_i^2}+\frac{d_i}{C_iM_W^2}-\frac{17}{8}\frac{d_i^2}{C_iM_W^4}-\frac{d_i^3}{2M_W^6}+\frac{115}{192}\frac{d_i^4}{M_W^8}\right)\arctan\left(\frac{d_i}{M_W^2\sqrt{\Delta_i}}\right\}\Bigg], \tag{15a}$$

with

$$f_i=\left(2-\frac{d_i}{M_W^2}\right),$$

$$\Delta_i=\left(\frac{1}{C_i}-\frac{d_i^2}{4M_W^4}\right). \tag{15b}$$

Further, in order to evaluate the contribution of the Z_2WW vertex of Fig. 1(d) we make use of the paper by Bardeen *et al.* [15] (Figs. 1 and 2) and write directly the following expressions by incorporating appropriate modifications coming from the extra U(1) model under discussion:

$$\Delta k_{(Z_2WW)}=\left(\frac{\alpha}{\pi}\right)\frac{\sin^2\theta_E}{2\sin^2\theta_W R'}\left[\left(\frac{1}{2}R'^3+\frac{19}{4}R'^2-\frac{97}{3}R'+\frac{20}{3}\right)-\left(\frac{R'^4}{4}+2R'^3-\frac{23}{2}R'^2+8R'\right)\ln R'+\frac{\left(\frac{3}{5}R'^5-5R'^4-18R'^3+52R'^2-16R'\right)}{(R'^2-4R')^{1/2}}\left\{\operatorname{arctanh}\frac{R'}{(R'^2-4R')^{1/2}}-\operatorname{arctanh}\frac{(R'-2)}{(R'^2-4R')^{1/2}}\right\}\right] \tag{16a}$$

and

$$\Delta Q_{(Z_2WW)}=\left(\frac{\alpha}{\pi}\right)\frac{\sin^2\theta_E}{12\sin^2\theta_W}(8+R')\left[\left(\frac{1}{6}+\frac{3}{2}R'-R'^2\right)+\frac{(R'^3-3R'^2+R')}{2}\ln R'+\frac{(R'^4-5R'^3+5R'^2)}{(R'^2-4R')^{1/2}}\left\{\operatorname{arctanh}\frac{R'}{(R'^2-4R')^{1/2}}-\operatorname{arctanh}\frac{(R'-2)}{(R'^2-4R')^{1/2}}\right\}\right], \tag{16b}$$

with

$$R'=\left(\frac{M_{Z_2}^2}{M_W^2}\right).$$

The total values of Δk and ΔQ coming exclusively from the extra U(1) model, over and above the SM values [15,16] are then given by

$$\Delta k_{[\mathrm{U}(1)]}=\Delta k_{(WZ_iH)}+\Delta k_{(Z_2WW)}, \tag{17a}$$

$$\Delta Q_{[\mathrm{U}(1)]}=\Delta Q_{(WZ_iH)}+\Delta Q_{(Z_2WW)}. \tag{17b}$$

In order to find the values of M_{H^+}, β, and θ_E, we solve the relevant renormalization group equations (RGE's) [20,21] at the one loop level and ascertain the Yukawa couplings λ_t, λ_D, λ_H, and A_H at the weak interaction scale ($\sqrt{s}=M_W$). The final values of $\lambda_t,\lambda_D,\lambda_H$ are then chosen to be consistent with the unitarity bound [22], and A_H is ascertained by making use of the no scale scenario in the relevant RGE's. The values of $\nu=\langle 0|H|0\rangle$, $\overline{\nu}=\langle 0|\overline{H}|0\rangle$, are then adjusted in accordance with the presently known values of the top quark mass [23,24] by adjusting β. The values of $x=\langle 0|N|0\rangle$ are estimated by making use of Collider Detector at Fermilab (CDF) limits [25] on M_{Z_2} which is varied from 555 to 620

TABLE I. The calculated values of anomalous magnetic moments $\Delta k_{(WZ_iH)}$, $\Delta k_{(Z_2WW)}$, $\Delta k_{[\mathrm{U}(1)]}$, and electric quadrupole moments $\Delta Q_{(WZ_iH)}$, $\Delta Q_{(Z_2WW)}$, $\Delta Q_{[\mathrm{U}(1)]}$ of the W^+ boson as contributions of the $W^+H^+Z_i$ and Z_2WW vertices and their sums as a function of Z_2 mass for the top quark CDF value $m_t = 175.6$ GeV (central value).

M_{Z_2} (GeV)	x (GeV)	θ_E (rad)	M_{H^+} (GeV)	Δk in units of $(-\alpha/\pi)$			ΔQ in units of $(-\alpha/\pi)$		
				$\Delta k_{(WZ_iH)}$	$\Delta k_{(Z_2WW)}$	$\Delta k_{[\mathrm{U}(1)]}$	$\Delta Q_{(WZ_iH)}$	$\Delta Q_{(Z_2WW)}$	$\Delta Q_{[\mathrm{U}(1)]}$
555	1315.714	−0.0101	991.886	23.952	1.450	25.402	6.810	0.076	6.886
565	1339.603	−0.0097	1010.172	25.993	1.516	27.509	7.330	0.079	7.409
575	1363.488	−0.0094	1028.468	28.162	1.583	29.745	7.880	0.083	7.963
585	1387.370	−0.0090	1046.758	30.465	1.651	32.116	8.462	0.086	8.548
595	1411.247	−0.0087	1065.039	32.906	1.721	34.627	9.075	0.089	9.164
605	1435.125	−0.0084	1083.316	35.493	1.793	37.286	9.723	0.093	9.816
615	1458.998	−0.0081	1101.586	38.229	1.866	40.095	10.405	0.096	10.501
620	1470.934	−0.0080	1110.718	39.656	1.903	41.559	10.760	0.098	10.858

GeV. The set of values of ν, $\overline{\nu}$, x, A_H, and λ_H so obtained are then used to evaluate [21] M_{H^+} and θ_E. The procedure is described in the Appendix.

Calculated values of $\Delta k_{(WZ_iH)}$, $\Delta Q_{(WZ_iH)}$, $\Delta k_{(Z_2WW)}$, $\Delta Q_{(Z_2WW)}$ and also the total values $\Delta k_{[U(1)]}$, $\Delta Q_{[U(1)]}$ are shown in Tables I and II for the top quark mass values [23] $m_{t(\mathrm{CDF})} = 175.6$ GeV (central value) [24] and $m_{t(\mathrm{D0})} = 169$ GeV (central value), respectively, as a function of M_{Z_2} CDF limits [25]. Therefore, the values reported are consistent with the presently known mass of the top quark as predicted by the CDF [23] and D0 groups [24] and with the stringent CDF mass limits on the mass of Z'. But are the values of the Z-Z' mixing angle θ_E as reported in Tables I and II consistent with the presently known experimental limits on its value provided by, e.g., the CERN e^+e^- collider LEP or SLAC Large Detector (SLD) etc.? According to Rizzo [26] there is no simple answer to this question for two reasons: (i) The value of θ_E depends upon the precise values taken for the top quark and Higgs boson masses used for doing radiative correction calculations in the SM and (ii) the $E(6)$ model one picks up, as Z' has a coupling which depends on the Z-Z' mixing angle which has different versions in different $E(6)$ models [27]. As such, just comparing our values with those predicted by some experimental group without considering the aforesaid factors appropriately will not be correct. However [26], there appears to be a general consensus that independently of the factors (i) and (ii), one may take the value of θ_E to be around <0.005 radians.

In fact, Riemann [28] has recently informed us that for the $E_\eta(6)$ model that we are considering, the ALEPH, CERN PPE/97-10 limits on θ_E are [29]

$$-0.21 < Z\text{-}Z' \text{ mix angle } < 0.12,$$

TABLE II. The values of anomalous magnetic moments $\Delta k_{(WZ_iH)}$, $\Delta k_{(Z_2WW)}$, $\Delta k_{[\mathrm{U}(1)]}$ and electric quadrupole moments $\Delta Q_{(WZ_iH)}$, $\Delta Q_{(Z_2WW)}$, $\Delta Q_{[\mathrm{U}(1)]}$ of the W^+ boson as contributions of the $W^+H^+Z_i$ and Z_2WW vertices and their sums as a function of Z_2 mass for the top quark D0 value of $m_t = 169$ GeV (central value).

M_{Z_2} (GeV)	x (GeV)	θ_E (rad)	M_{H^+} (GeV)	Δk in units of $(-\alpha/\pi)$			ΔQ in units of $(-\alpha/\pi)$		
				$\Delta k_{(WZ_iH)}$	$\Delta k_{(Z_2WW)}$	$\Delta k_{[\mathrm{U}(1)]}$	$\Delta Q_{(WZ_iH)}$	$\Delta Q_{(Z_2WW)}$	$\Delta Q_{[\mathrm{U}(1)]}$
555	1316.110	−0.0090	965.727	19.664	1.157	20.821	5.677	0.061	5.738
565	1339.993	−0.0087	983.556	21.345	1.209	22.554	6.109	0.063	6.172
575	1363.873	−0.0084	1001.377	23.132	1.262	24.394	6.565	0.065	6.630
585	1387.749	−0.0081	1019.190	25.029	1.317	26.346	7.048	0.069	7.117
595	1411.623	−0.0078	1036.997	27.040	1.373	28.413	7.557	0.071	7.628
605	1435.495	−0.0075	1054.797	29.172	1.430	30.602	8.094	0.074	8.168
615	1459.364	−0.0073	1072.592	31.427	1.489	32.916	8.660	0.077	8.737
620	1470.297	−0.0072	1081.486	32.603	1.518	34.121	8.955	0.078	9.033

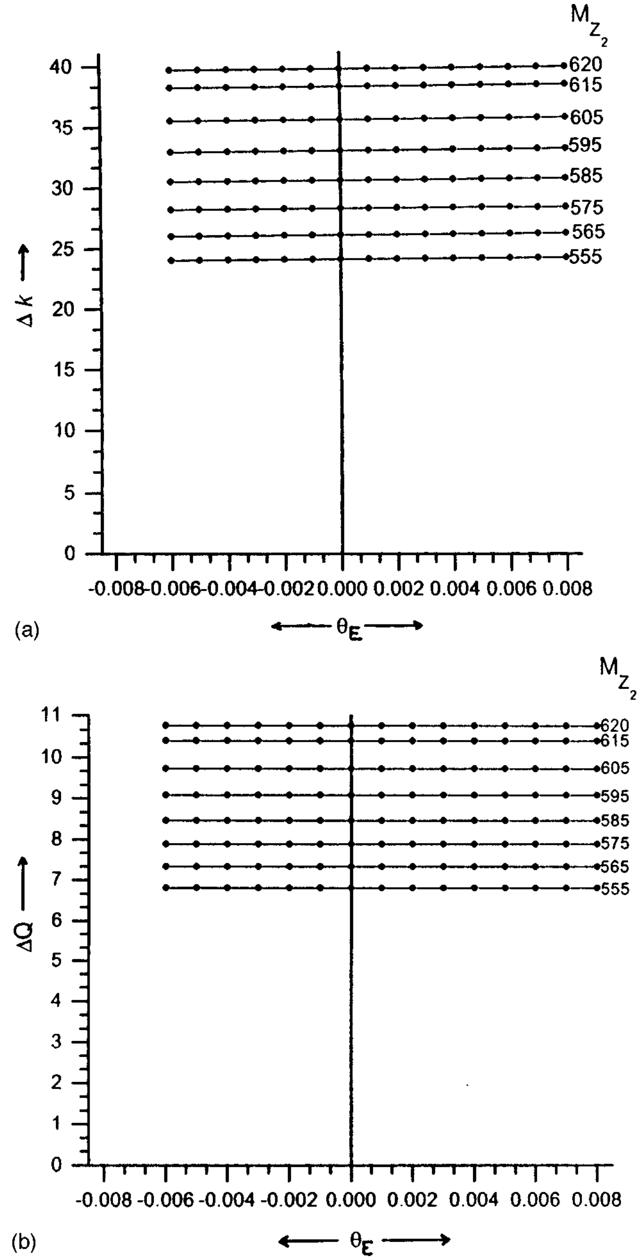

FIG. 2. (a) The variation of $\Delta k^{(\mathrm{CDF})}_{WZ_iH}$ against the Z-Z' mixing angle θ_E. (b) The variation of $\Delta Q^{(\mathrm{CDF})}_{WZ_iH}$ against θ_E.

$$-0.007 < Z\text{-}Z' \text{ mix angle } < 0.009.$$

We may, therefore, remark that *prima facie* our values of θ_E as given in Tables I and II are not very much different from these limits. However, a precise comparison is possible only when the experimental limits on θ_E are obtained by making use of the top quark and Higgs masses that we have used. To our knowledge, no such experimental limits are available. Under the circumstances, a precise statement in this regard is not possible.

It may, however, be pointed out that a slight variation in the value of θ_E in this range of values hardly has any noticeable contribution to $\Delta k_{(WZ_iH)}$ and $\Delta Q_{(WZ_iH)}$. As an illustration, we demonstrate this fact in Figs. 2(a) and 2(b) by showing $\Delta k_{(WZ_iH)}$ vs θ_E and $\Delta Q_{(WZ_iH)}$ vs θ_E, respectively, by considering θ_E values in the range -0.006 to $+0.008$ rad pertaining to the second of the aforesaid ALEPH, etc., limits. The graphs are for the CDF top quark mass value. Identical conclusions follow for the D0 top quark mass value.

We notice that the values of $\Delta k_{(WZ_iH)}$ and $\Delta Q_{(WZ_iH)}$ given in Tables I and II are larger than the corresponding $\Delta k_{(Z_2WW)}$ and $\Delta Q_{(Z_2WW)}$ values as they are also larger than the SM radiative correction contributions which are usually of the order of (α/π) only. The latter aspect may raise the possibility of questioning the validity of a perturbative calculation for this problem. We, however, notice that $\Delta k_{(WZ_iH)}$ and $\Delta Q_{(WZ_iH)}$ do not directly appear as coupling parameters in the matrix element [Eqs. (2) and (3)] and, as such, their values cannot be used to ascertain the validity of the perturbative calculations. The coupling parameter used is e which is a weak parameter, and therefore a perturbative calculation for the problem is perfectly valid. In a loop diagram calculation the physical requirement is the finiteness of the amplitude, irrespective of the value, which it is.

The problem of strong coupling comes in the case of composite models. In fact, in a recent calculation [30] of this problem using the composite model of Abbott and Farhi [31], the values of Δk and ΔQ for the W boson are shown to be two to three orders of magnitude greater than the SM radiative corrections, and they are therefore larger than our values.

Further, a comparison of our values with those from the $E(6)$ vector lepton model [32] reported in Ref. [16] shows that the latter are some what smaller than our values. But these are constrained by limits on the masses of heavy lepton doublet

$$\begin{pmatrix} E \\ N_E \end{pmatrix}_R$$

and also by a much smaller value of the top quark mass. Here again a precise comparison is not possible.

Finally, it may be emphasized that the increase in the magnitude of $\Delta k_{(WZ_iH)}$ and $\Delta Q_{(WZ_iH)}$ with an increase in the M_{Z_2} value appears to be in contravention of the decoupling theorem [33]. But a closer examination of the numerical values of various terms comprising Eqs. (14) and (15) for $\Delta k_{(WZ_iH)}$ and $\Delta Q_{(WZ_iH)}$, respectively, reveals that whereas the decoupling term $(\cos^2\theta_E/M^2_{Z_2})$ reduces the amplitude by a factor of about 0.8 for Table I, the other terms arising out of the renormalization procedure— both BPHZ and dimensional regularization,—enhance the amplitude by a factor $\cong 2$, as one varies M_{Z_2} from 555 to 620 GeV. As a consequence, the overall increase of the amplitude becomes of the order of 1.6. Identical conclusions are applicable for Table II. The enhancement in the values of $\Delta k_{(Z_2WW)}$ and $\Delta Q_{(Z_2WW)}$ come from dimensional regularization.

In any case, it does not appear to be very remote from the experimental reach to discern the extra U(1) model from the SM on the basis of $W^{\pm}H^{\mp}Z_i$ vertex contributions in terms of $\Delta k_{(WZ_iH)}$, $\Delta Q_{(WZ_iH)}$. The reported results decidedly encourage a search for the static properties of the W boson in experiments probing three boson vertices—in particular, the W-W-γ vertex at LEP II and 500 GeV Next-Linear Collider (NLC) [34].

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APPENDIX

In order to ascertain the evolution of the superpotential couplings λ_i's ($i=t,D,H$) of the extra U(1) model, we make use of the procedure outlined by Durand and Lopez [35] in their paper on the ''flipped SU(5)×U(1) model.''

Beginning with the superpotential [20,21]

$$W=\lambda_t t^c qH+\lambda_D ND^cD+\lambda_b b^c q\overline{H}+\lambda_H N\overline{H}H+\lambda_\tau \tau^c l\overline{H}, \tag{A1}$$

we proceed by solving the relevant one loop renormalization-group equations (RGE) of the extra U(1) model [35,36]. The RGE's for the gauge couplings are [36,37]

$$\frac{dg_a}{dt}=\frac{b_a g_a^3}{8\pi}, \tag{A2}$$

with $a=3,2,1,E$, $b_a=(0,3,9,9)$, $t=(1/2\pi)\ln(\sqrt{s}/M_w)$. These have the solutions

$$g_a^2(t)=\left(\frac{g_a^2(0)}{1-\frac{b_a}{4\pi}g_a^2(0)t}\right). \tag{A3}$$

Further, for the Yukawa couplings (λ_i's), the RGE's are [20,21]

$$\frac{d\lambda_t}{dt}=\frac{\lambda_t}{4\pi}\left[3\lambda_t^2+\frac{1}{2}\lambda_H^2-\frac{8}{3}g_3^2-\frac{3}{2}g_2^2-\frac{5}{6}g_1^2\right], \tag{A4}$$

$$\frac{d\lambda_D}{dt}=\frac{\lambda_D}{4\pi}\left[\frac{5}{2}\lambda_D^2+\lambda_H^2-\frac{8}{3}g_3^2-\frac{5}{6}g_1^2\right], \tag{A5}$$

$$\frac{d\lambda_H}{dt}=\frac{\lambda_H}{4\pi}\left[\frac{3}{2}\lambda_t^2+2\lambda_H^2+\frac{3}{2}\lambda_D^2-\frac{3}{2}g_2^2-g_1^2\right]. \tag{A6}$$

We do not consider RGE's for λ_b and λ_τ, treating them as negligible [20]. Further, the contributions of $a=1,E$, are combined ($\alpha_a=g_a^2/4\pi$) as $\alpha_1=\alpha_E$ at the one loop level [20]. It is interesting to note that if any of the Eqs. (A4)–(A6) are solved for a specific λ with all others λ's being put to zero, one obtains a singular point in the solution beyond which the corresponding coupling diverges at $\sqrt{s}=M_G$. For this case equations (A4)–(A6) are then reduced to the form [35]

$$\frac{d\lambda_i}{dt}=\frac{\lambda_i}{4\pi}\left(A_i\lambda_i^2-\sum_{j=1}^{3}C_{ij}g_j^2\right), \tag{A7}$$

where $i=(t,D,H)$, and $j=1,2,3$. These will have solutions of the form

$$\lambda_i^2(t)=\frac{\lambda_i^2(0)}{[1-(A_i/2\pi)\lambda_i^2(0)G_i(t)]}\left[1-\frac{b_a}{4\pi}g_a^2(0)t\right]^{2C_{ia}/b_a}, \tag{A8}$$

with

$$G_i(t)=\int_0^t\prod_{a=1}^{3}\left[1-\frac{b_a}{4\pi}g_a^2(0)t\right]^{2C_{ia}/b_a}dt', \tag{A9}$$

$$A_i=\begin{pmatrix}3\\ \frac{5}{2}\\ 2\end{pmatrix},$$

and

$$C_{ia}=\begin{pmatrix}\frac{8}{3} & \frac{3}{2} & \frac{5}{6}\\ \frac{8}{3} & 0 & \frac{5}{6}\\ 0 & \frac{3}{2} & 1\end{pmatrix}. \tag{A10}$$

Here $\lambda_i(0)$ denotes the values of λ_i at $t=0$, i.e., at $\sqrt{s}=M_W$. As pointed out earlier $\lambda_i(t)$ will diverge at the grand unification scale ($\sqrt{s}=M_G$) if

$$\lambda_i(0)=\lambda_{i,c}=\left(\frac{2\pi}{A_iG_i(t_G)}\right)^{1/2}, \tag{A11}$$

where the $\lambda_{i,c}$'s are called critical couplings. The divergence of the couplings at a high-mass scale is closely related to the triviality of the scalar field theories [38]. The condition that the couplings diverge at $\sqrt{s}=M_G$, defines a critical surface in the space of the λ_i's at $\sqrt{s}=M_W$, and the extra U(1) model is inconsistent for couplings outside this surface.

Using [27,37] $\sin^2\theta_W=0.2314$, $\alpha_3(0)=\alpha_3(M_W)=0.11$, and making use of the fact that $g_2(t_G)=g_3(t_G)$, we obtain for the grand unifications scale $M_G=5.36\times10^{15}M_W$ GeV and $t_G=5.77$. The values of the critical couplings $\lambda_{t,c}(0)=1.265$, $\lambda_{D,c}(0)=1.266$, and $\lambda_{H,c}(0)=0.953$ are then used with the corresponding $T_i(0)$ values $T_{t,c}(0)=0.127$, $T_{D,c}(0)=0.128$, and $T_{H,c}=0.072$, where

$$T_i=\frac{\lambda_i^2}{4\pi}. \tag{A12}$$

It is interesting to point out that these values are very close to the upper bound on the λ_i's at ($\sqrt{s}=M_W$), obtained by applying unitarity requirement [22]. These latter values are [22]

$$\lambda_t(M_W)=1.263,\quad \lambda_D(M_W)=1.264,\quad \lambda_H(M_W)=0.948. \tag{A13}$$

This closeness of unitarity limits on the λ_i's and $\lambda_{i,c}$'s is easily understood [35] by rewriting Eq. (A8) as an equation for $\lambda_{i,0}$ as

$$\lambda_{i,0}^2=\lambda_{i,c}^2\left[1+\frac{\lambda_{i,c}^2}{\lambda_i'^2}\right]^{-1}\approx\lambda_{i,c}^2\left[1-\frac{\lambda_{i,c}^2}{\lambda_i'^2}\right], \tag{A14}$$

where

$$\lambda_i'^2=\lambda_i^2(t)\prod_{j=1}^{3}\left[1-\frac{b_a}{4\pi}\,g_a^2(0)t_G\right]; \tag{A15}$$

obviously $\lambda_{i,c}^2/\lambda_i'^2\ll 1$.

The unitarity limits on $\lambda_{i,0}$ at $\sqrt{s}=M_w$ are therefore slightly smaller than the critical couplings $\lambda_{i,c}$. It is obvious that $\lambda_{i,0}$'s correspond to the values of λ_i at $\sqrt{s}=M_w$, i.e., at the weak interaction scale.

In order to be on the safe side we choose these couplings to be slightly smaller than the unitarity limiting values, i.e.,

$$\lambda_t(M_W)=1.250,\quad \lambda_D(M_W)=1.250,\quad \lambda_H(M_W)=0.940, \tag{A16}$$

with $T_t(M_w)=0.124$, $T_D(M_w)=0.124$, $T_H(M_w)=0.70$. For the top quark mass (m_t) we use the relation [20,21]

$$m_t=\lambda_t\nu=\lambda_t\nu_0\cos\beta, \tag{A17}$$

which gives $m_{t(\max)}=\lambda_t\nu_0=217.5$ GeV, with $\nu_0=174$ GeV, and $\tan\beta=\overline{\nu}/\nu$. This value is in agreement with the upper limits arrived at by some authors [20,35] and is somewhat higher than those recently quoted by the CDF [23] and D0 [24] Collaborations:

(i) CDF [23]:

$$m_t=175.6\pm 5.7(\text{stat})\pm 7.1(\text{syst})\ \text{GeV}, \tag{A18}$$

(ii) D0 [24]:

$$m_t=169\pm 8(\text{stat})\pm 8(\text{syst})\ \text{GeV}.$$

Making use of the central values given in Eq. (A18) and the relation. (A17), we get, corresponding to $m_{t(\mathrm{CDF})}$, $\nu=140.5$ GeV and $\overline{\nu}=102.6$ GeV, and corresponding to $m_{t(\mathrm{D0})}$, $\nu=135.2$ GeV and $\overline{\nu}=109.5$ GeV.

The corresponding values of x are calculated by using the mass formula of M_{Z_2} [13,21]

$$M_{Z_2}^2=\frac{1}{2}\,(1+b)+((1-b)^2+4a^2)^{1/2}M_{Z_1}^2, \tag{A19}$$

with

$$a=\frac{1}{3}\sin\theta_W\left(\frac{4\nu^2-\overline{\nu^2}}{\nu^2+\overline{\nu^2}}\right),$$

with

$$b=\frac{1}{9}\sin^2\theta_W\left(\frac{25x^2+16\nu^2+\overline{\nu^2}}{\nu^2+\overline{\nu^2}}\right).$$

We use the recent M_{Z_2} mass limits quoted by the CDF group [25] for evaluating x corresponding to the aforesaid values of ν and $\overline{\nu}$. It may be pointed out that Z_1 and Z_2 correspond to the mass eigenstate of Z and Z', respectively, through the relation [13]

$$Z=Z_1\cos\theta_E-Z_2\sin\theta_E,$$

$$Z'=Z_1\sin\theta_E+Z_2\cos\theta_E,$$

with $\tan 2\theta_E=(2a/1-b)$. From a knowledge of the values of ν, $\overline{\nu}$, and x, the corresponding values of θ_E are evaluated for the aforesaid ranges of Z_2 mass limits [25].

In order to evaluated M_{H^+}, we make use of the following formula of Ellis *et al.* [21]:

$$M_{H^+}^2=M_W^2-\lambda_H A_H x\left(\frac{\overline{\nu}}{\nu}+\frac{\nu}{\overline{\nu}}\right)-\lambda_H(\nu^2+\overline{\nu^2}). \tag{A20}$$

For ascertaining $A_H(t)$, we solve the relevant one loop renormalization group equation [20,21]

$$\frac{dA_i(t)}{dt}=C_{ia}\alpha_a M_a-K_{ij}T_jA_j, \tag{A21}$$

with $M_a=m_{1/2}\alpha_a(t)/\alpha(0)$, and where $m_{1/2}$ is the gaugino mass at the grand unified theory (GUT) scale. A solution for $A_i(t)$ for the no scale scenario is [21]

$$A_i(t)=\sum_a C_{ia}m_{1/2}\left(\frac{\alpha_a b_a t}{1+\alpha_a b_a t}\right). \tag{A22}$$

This gives for $i=H$, the expression

$$A_H(t)=m_{1/2}t\left[\frac{3\alpha_2(0)}{(1+3\alpha_2(0)t)}+\frac{2}{9}\,\frac{\alpha_1(0)}{(1+9\alpha_1(0)t)}\right]$$
$$\approx 0.677m_{1/2}(t). \tag{A23}$$

This value of $A_H(t)$ is in agreement with the limits given in Ref. [39]. In order to evaluate $m_{1/2}$, we make use of the no scale scenario elaborated in Refs. [21,39–41] and write

$$m_{1/2}=-\frac{1}{4}\,bg^2\left(\frac{1}{16S_RT_R^3}\right)^{1/2}\lambda_H\left(\frac{\nu}{\overline{\nu}}x+\frac{\overline{\nu}}{\nu}x+\frac{\nu\overline{\nu}}{x}\right), \tag{A24}$$

with [21] $b=27/16\pi^2$, $S_R=1/g^2$, and $T_R=O(g^2)$, where S_R and T_R are established dynamically [21,40,41]. For a numerical evaluation of $m_{1/2}$, we take the multiplying numerical factor in Eq. (24) as the proportionality number of [21] T_R and use the simplified equation

$$m_{1/2}=b\lambda_H\left(\frac{\nu}{\overline{\nu}}x+\frac{\overline{\nu}}{\nu}x+\frac{\nu\overline{\nu}}{x}\right). \tag{A25}$$

Further, in order to ascertain $m_{1/2}(t)$ we use the no scale prescription of Refs. [21,40] as follows. We choose the renormalization point Q and take $Q=O(m_{1/2})$. The effective potential near $Q=Q_0$ defined by $V_{\mathrm{eff}}(Q_0)=0$, then has the form

$$V_{\mathrm{eff}}=e^{-4t}P(t), \tag{A26}$$

with $t=\ln Q$, where the function $P(t)$ depends upon radiative corrections. We take conjecturally

$$P=(t-t_0)^n+\cdots, \tag{A27}$$

with $t_0 = \ln Q_0$, being the scale where symmetry breaking starts to appear. The minimum of Eq. (A26) is given by $P = \frac{1}{4} P' = \frac{1}{4} dP/dt$. This gives, on retaining only the first term in the expansion, the expression

$$t = t_0 + \frac{1}{4} n. \tag{A28}$$

One then gets for $n=1$, the expression $Q/Q_0 = e^{1/4}$, which gives [41,42]

$$m_{1/2}(Q) = m_{1/2}(Q_0) e^{1/4}$$

or

$$m_{1/2}(t) = m_{1/2}(t_0)(1.28) = 1.28 m_{1/2}. \tag{A29}$$

Thus $m_{1/2}(t)$ is obtained by multiplying the expression of Eq. (A25) by 1.28. The values of $m_{1/2}(t)$ so obtained are then used in $A_H(t)$ by using Eq. (A23). Finally M_{H^+} is evaluated by using Eq. (A20).

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