# Electromagnetic mass splittings of $\pi$, $a_1$, $K$, $K_1(1400)$, and $K^*(892)$

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To one-loop order and $O(\alpha_{\rm EM})$, the electromagnetic mass splittings of $\pi$, $a_1$, $K$, $K_1(1400)$, and $K^*(892)$ are calculated in the framework of $U(3)_L\times U(3)_R$ chiral field theory. The logarithmic divergences emerging in the Feynman integrations of the mesonic loops are factorized by using an intrinsic parameter $g$ of this theory. No other additional parameters or counterterms are introduced to absorb the mesonic loop divergences. When $f_\pi$, $m_\rho$, and $m_a$ are taken as inputs, the parameter $g$ will be determined and all the physical results are finite and fixed. Dashen's theorem is satisfied in the chiral SU(3) limit of this theory and a rather large violation of the theorem is revealed at the order of $m_s$ or $m_K^2$. Mass ratios of light quarks have been determined. A relation for electromagnetic corrections to masses of axial-vector mesons is obtained. It could be regarded as a generalization of Dashen's theorem. Comparing with data, it is found that the nonelectromagnetic mass difference of $K^*$ is in agreement with the estimation of Schechter, Subbaraman, and Weigel. [S0556-2821(97)06319-4]

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## I. INTRODUCTION

Calculating the electromagnetic mass splittings of the low-lying mesons is an important issue in nonperturbative (NP) quantum chromodynamics (QCD). This topic has intrigued particle physicists for many years [1–9]. Recently, a chiral field theory of pseudoscalar, axial-vector and vector mesons [called the $U(3)_L\times U(3)_R$ chiral field theory of mesons] has been proposed [10,11]. This theory can be regarded as a realization of chiral symmetry, current algebra, and vector meson dominance (VMD). In this paper, we try to present systematical calculations of electromagnetic masses of $\pi$, $a_1$, $K$, $K_1(1400)$, and $K^*(892)$ in the framework of this theory.

It is well known that chiral perturbation theory ($\chi$PT) is rigorous and phenomenologically successful in describing the physics of the pseudoscalar mesons at low energies [12]. The effective Lagrangian of $\chi$PT depends on ten chiral coefficients that are determined by a comparison with the experimental low-energy information. Models attempting to extend the $\chi$PT to include more low-lying mesons should predict these ten coefficients by fitting data in $\chi$PT. $U(3)_L\times U(3)_R$ chiral field theory has been studied at the tree level [11], and the theoretical results agree well with data. This theory has also been successfully applied to study $\tau$ mesonic decays systematically [13]. In Ref. [14] the ten coefficients of $\chi$PT have been predicted at about $\Lambda\sim 2$ GeV in this theory. The coefficients of $\chi$PT are expressed by a universal coupling constant $g$ and the ratio $f_\pi^2/m_\rho^2$, which have been fixed in [10,11]. The authors of Refs. [15,16] have found that the vector meson dominates the structure of the phenomenological chiral Lagrangian. Two of the coefficients obtained in Ref. [10] are the same as the ones in Ref. [15]. The relations $2(L_1+L_2)+L_3=0$ and $L_4^V=L_6^V=L_7^V=0$ found in Ref. [14] have already been obtained in Ref. [16]. A very small $L_8$ predicted in Ref. [14] is not in contradiction with the $L_8^V=0$ found in Ref. [16]. The expression of $L_9$ presented in Ref. [14] is similar to the one obtained in Ref. [16]. When taking $g=1$, the $L_2^V=G_V^2/16M_V^2$ is the same as the expression presented in Ref. [14].

In Ref. [17], starting from the $U(3)_L\times U(3)_R$ chiral field theory of mesons, the authors use the path integration method to derive $L_1$, $L_2$, $L_3$, $L_9$, and $L_{10}$. The results are in agreement with the experimental values of the $L_i$ at $\mu=m_\rho$ in $\chi$PT. Therefore, the low-energy limit of this theory is indeed equivalent to $\chi$PT and the QCD constraints discussed in Ref. [16] are met by this theory.

$U(3)_L\times U(3)_R$ chiral field theory of mesons provides a unified description of meson physics at low energies. VMD in the meson physics is a natural consequence of this theory instead of an input. Therefore, the dynamics of the electromagnetic interactions of mesons has been introduced and established naturally. On the other hand, this theory starts with a chiral Lagrangian of quantum quark fields within mesonic background fields and the chiral dynamics for mesons comes from the path integration over quark fields. A cutoff $\Lambda$ (or $g$ in Ref. [10]) has to be introduced to absorb the logarithmic divergences due to quark loops. Thus $g$ (or $\Lambda$) will serve as an intrinsic parameter in this truncated fields theory. Therefore, it is legitimate to use the $g$ to factorize the loga-

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rithmic divergences of loop diagrams in calculating the electromagnetic mass splittings of the low-lying mesons [18].

The basic Lagrangian of this chiral fields theory is (hereafter we use the notations in Refs. [10,11])

$$\mathcal{L}=\overline{\psi(x)}[i\gamma\cdot\partial+\gamma\cdot v+\gamma\cdot a\gamma_5-mu(x)]\psi(x)+\frac{1}{2}m_1^2(\rho_i^\mu\rho_{\mu i}+\omega^\mu\omega_\mu+a_i^\mu a_{\mu i}+f^\mu f_\mu)+\frac{1}{2}m_2^2(K_\mu^{*a}K^{*a\mu}+K_1^\mu K_{1\mu})+\frac{1}{2}m_3^2(\phi_\mu\phi^\mu+f_s^\mu f_{s\mu}), \tag{1}$$

with

$$u(x)=\exp[i\gamma_5(\tau_i\pi_i+\lambda_a K^a+\eta+\eta')],$$

$$a_\mu=\tau_i a_\mu^i+\lambda_a K_{1\mu}^a+\left(\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8\right)f_\mu+\left(\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8\right)f_{s\mu},$$

$$v_\mu=\tau_i\rho_\mu^i+\lambda_a K_\mu^{*a}+\left(\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8\right)\omega_\mu+\left(\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8\right)\phi_\mu, \tag{2}$$

where $i=1,2,3$ and $a=4,5,6,7$. The $\psi$ in Eq. (1) are $u,d,s$ quark fields. $m$ is a parameter related to the quark condensate. Here the mesons are bound states in QCD and they are not fundamental fields. Therefore, in Eq. (1) there are no kinetic terms for these fields and the kinetic terms will be generated from quark loops.

According to Refs. [10,11], the effective Lagrangian $\mathcal{L}_{\rm Re}$ and $\mathcal{L}_{\rm Im}$ can be evaluated by performing the path integrations over quark fields. In order to absorb the logarithmic divergences in the effective Lagrangian, as mentioned above, it is necessary to introduce a universal coupling constant $g$ as

$$g^2=\frac{8}{3}\frac{N_c}{(4\pi)^{D/2}}\frac{D}{4}\left(\frac{\mu^2}{m^2}\right)^{\epsilon/2}\Gamma\left(2-\frac{D}{2}\right)=\frac{1}{6}\frac{F^2}{m^2}. \tag{3}$$

Also, following Refs. [10,11], after defining the physical meson fields, we have

$$m_a^2=\left(\frac{1}{1-\frac{1}{2\pi^2g^2}}\right)\left(m_\rho^2+\frac{F^2}{g^2}\right), \tag{4}$$

$$m_{K_1}^2=\left(\frac{1}{1-\frac{1}{2\pi^2g^2}}\right)\left(m_{K^*}^2+\frac{F^2}{g^2}\right), \tag{5}$$

with

$$F^2=\frac{f_\pi^2}{1-\frac{2c}{g}},\quad c=\frac{f_\pi^2}{2gm_\rho^2}, \tag{6}$$

$$F^2=\frac{f_k^2}{1-\frac{2c'}{g}},\quad c'=\frac{f_k^2}{2gm_{K^*}^2}, \tag{7}$$

$$m^2=\frac{F^2}{6g^2}. \tag{8}$$

Combining Eq. (4) with Eq. (6) and taking $f_\pi, m_\rho, m_a$ as inputs, the parameter $g$ will be fixed.

VMD has been well established in studying electromagnetic interactions of hadrons [19]. To the present theory, the interactions between photon and the vector meson fields of $\rho_0$, $\omega$, and $\phi$ can be found through the substitutions [11]:

$$\rho_\mu^3\to\rho_\mu^3+\frac{1}{2}egA_\mu, \tag{9}$$

$$\omega_\mu\to\omega_\mu+\frac{1}{6}egA_\mu, \tag{10}$$

$$\phi_\mu\to\phi_\mu-\frac{1}{3\sqrt{2}}egA_\mu. \tag{11}$$

The $\rho^3$- (or $\rho^0$-) photon, $\omega$-photon, and $\phi$-photon interaction Lagrangians are

$$\mathcal{L}_{\rho\gamma}=-\frac{1}{2}eg\partial_\mu\rho_\nu^3(\partial^\mu A^\nu-\partial^\nu A^\mu), \tag{12}$$

$$\mathcal{L}_{\omega\gamma}=-\frac{1}{6}eg\partial_\mu\omega_\nu(\partial^\mu A^\nu-\partial^\nu A^\mu), \tag{13}$$

$$\mathcal{L}_{\phi\gamma}=\frac{1}{3\sqrt{2}}eg\partial_\mu\phi_\nu(\partial^\mu A^\nu-\partial^\nu A^\mu). \tag{14}$$

Using $\mathcal{L}_i(\phi,\gamma,\dots)|_{\phi=\pi,a,v}$, we can calculate the $S$ matrix:

$$S_\phi=\left\langle\phi\left|T\exp\left(i\int dx^4\mathcal{L}_i(\phi,\gamma,\dots)\right)-1\right|\phi\right\rangle\Big|_{\phi=\pi,a,v}. \tag{15}$$

On the other hand, $S_\phi$ can also be expressed in terms of the effective Lagrangian of $\phi$ as

$$S_\phi=\left\langle\phi\left|i\int d^4x\mathcal{L}_{\rm eff}(\phi)\right|\phi\right\rangle.$$

Noting $\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}m_\phi^2\phi^2$, then the electromagnetic interaction correction to the mass of $\phi$ reads

$$\delta m_\phi^2=\frac{2iS_\phi}{\langle\phi|\phi^2|\phi\rangle}, \tag{16}$$

where $\langle\phi|\phi^2|\phi\rangle=\langle\phi|\int d^4x\ \phi^2(x)|\phi\rangle$. We adopt dimensional regularization to do loop calculations and use Eq. (3) to factorize the divergences. Thus all of virtual-photon contributions to the masses of the low-lying mesons can be computed systematically and analytically.

The purposes of our investigations in this paper are threefold, which are stated as follows.

(i) We try to present a systematic method to derive the electromagnetic masses of the mesons by employing $U(3)_L \times U(3)_R$ chiral field theory. Nearly 30 years ago, Das *et al.* [1] obtained a finite result of the $\pi^+ - \pi^0$ mass difference by using current algebra techniques and especially relying on Weinberg's second sum rule to cancel the divergences in it (for further investigations on it, see [4,5]). However, Weinberg's second sum rule is not satisfied experimentally [20–22]. Actually, many people have found the existence of the divergence in calculating the electromagnetic mass of $\pi$ mesons in the effective chiral Lagrangian theories [6–9]. In particular, in Ref. [6], when the corrections of perturbative QCD to $m_{\pi^+} - m_{\pi^0}$ were investigated in a chiral model, a dependence of $m_{\pi^+} - m_{\pi^0}$ on an ultraviolet cutoff has been revealed. In Refs. [8,9], in order to remove this divergence, the counterterms have been introduced. These facts mean that we could not expect such a cancellation between the divergent terms to work without any additional assumptions, in particular, when the strange-flavor mesons are involved. The method in this paper is systematic and the logarithmic divergences from mesonic loops can be factorized by using the intrinsic parameter ($g$ or $\Lambda$) of the theory. There is no need to introduce other parameters or counterterms to absorb the mesonic loop divergences. The spirit of this method will be shown in Sec. II by reexamining the calculations of the electromagnetic mass difference of charge and neutral $\pi$ mesons in the present theory.

(ii) It is straightforward to extend our method to the studies of the electromagnetic masses of strange-flavor mesons. The smallness of $u,d$ quark masses allows the calculations in the chiral limit for nonstrange mesons. However, the large strange quark mass will bring a significant contribution to the electromagnetic self-energies of the strange-flavor mesons. Dashen's theorem [3] states that the square electromagnetic mass differences between the charged pseudoscalar mesons and their corresponding neutral partners are equal in the chiral SU(3) limit, i.e.,

$$(m_{K^+}^2 - m_{K^0}^2)_{\rm EM} = (m_{\pi^+}^2 - m_{\pi^0}^2)_{\rm EM}.$$

The subscript EM denoted the electromagnetic mass. The significant SU(3) symmetry breaking will lead to the violations of this theorem. Furthermore, it has been known that the $\pi^+ - \pi^0$ mass difference is almost entirely electromagnetic in origin; however, the contributions of the $K^+ - K^0$ mass difference are from both electromagnetic interactions and the $u$-$d$ quark mass difference. Thus it is of interest to calculate the electromagnetic mass difference between $K^+$ and $K^0$ to leading order in the quark mass expansion both to increase the understanding of the low-energy dynamics and to aid in the extraction of current mass ratios of light quarks. The latter reflects the breaking effect of isospin symmetry [5,23,24]. Therefore, the quark mass term $-\bar{\psi} M \psi$ [$M = {\rm diag}(m_u, m_d, m_s)$ is the quark mass matrix], which represents the explicit chiral symmetry breaking in the present theory, should be added into Eq. (1) when the electromagnetic masses of the strange-flavor mesons are calculated. The nonzero quark masses will yield the mass terms of pseudoscalar mesons in addition to $\mathcal{L}_{\rm Re}$ (explicit quark mass parameter do not occur in the abnormal part effective Lagrangian). To leading order in the quark mass expansion, the masses of the octet pseudoscalar mesons have been derived in Ref. [25] (Gell-Mann–Oakes–Renner formulas), which read

$$m_{\pi^+}^2 = m_{\pi^0}^2 = -\frac{2}{f_\pi^2}(m_u + m_d)\langle 0|\bar{\psi}\psi|0\rangle,$$

$$m_{K^+}^2 = -\frac{2}{f_k^2}(m_u + m_s)\langle 0|\bar{\psi}\psi|0\rangle,$$

$$m_{K^0}^2 = -\frac{2}{f_k^2}(m_d + m_s)\langle 0|\bar{\psi}\psi|0\rangle,$$

$$m_\eta^2 = -\frac{2}{3f_\eta^2}(m_u + m_d + 4m_s)\langle 0|\bar{\psi}\psi|0\rangle, \qquad (17)$$

where $\langle 0|\bar{\psi}\psi|0\rangle$ is the quark condensate of the light flavors [10,26].

(iii) All of the low-lying mesons including pseudoscalar, vector, and axial-vector mesons are involved in this theory. This makes it possible to evaluate the electromagnetic masses of vector and axial-vector mesons in addition to the pseudoscalar $\pi$ and $K$. The electromagnetic mass splittings of $a_1$ and $K_1(1400)$ are calculated, and in the chiral SU(3) limit we obtain the relation

$$(m_{a^+}^2 - m_{a^0}^2)_{\rm EM} = (m_{K_1^+}^2 - m_{K_1^0}^2)_{\rm EM},$$

which could be regarded as a generalization of Dashen's theorem. The electromagnetic masses of $K^*(892)$ are also derived. Using the experimental value of $m_{K^{*+}} - m_{K^{*0}}$, the nonelectromagnetic mass difference of $K^{*+}$ and $K^{*0}$ is estimated. The result is close to the one given in Ref. [27].

The contents of this paper are organized as follows. In Sec. II we discuss the electromagnetic mass splitting of $\pi$ mesons and in Sec. III the electromagnetic mass splitting of $a_1$ mesons. In Sec. IV we will extend this method to the case of $K$ mesons and give the violations of Dashen's theorem at leading order in the quark mass expansion. In Sec. V we discuss the electromagnetic mass splitting of $K_1(1400)$ and in Sec. VI the electromagnetic mass splitting of $K^*(892)$. In Sec. VII we discuss and give a summary of the results.

## II. $\pi^+ - \pi^0$ ELECTROMAGNETIC MASS DIFFERENCE

In this section and in Sec. III we will restrict our calculations to the two-flavor case because the strange quark has no effect on the electromagnetic self-energies of pions and $a_1$ mesons, and the smallness of $u,d$ quark masses allows the calculations in the chiral limit. Note that the contributions from $\mathcal{L}_{\rm Im}$ are proportional to $m_\pi^2$, which can be neglected in the chiral limit. Thus, from $\mathcal{L}_{\rm Re}$ [Eq. (13) in Ref. [10]], the interaction Lagrangians contributing to $\pi^+ - \pi^0$ electromagnetic mass difference for massless pions read

$$\mathcal{L}_{\rho\rho\pi\pi} = \frac{2F^2}{g^2 f_\pi^2}\rho_\mu^i \rho^{j\mu}(\pi^2\delta_{ij} - \pi_i\pi_j) + \frac{1}{\pi^2 g^2 f_\pi^2}\partial_\nu \rho_\mu^i \partial^\nu \rho^{j\mu}(\pi^2\delta_{ij} - \pi_i\pi_j), \qquad (18)$$

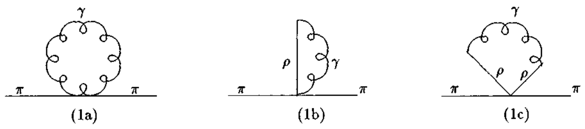


FIG. 1. One-loop Feynman diagrams contributing to the electromagnetic mass difference between $\pi^+$ and $\pi^0$. The curly line is the photon line.

$$\mathcal{L}_{\rho\pi a}=-\frac{2F^2\gamma}{f_\pi g^2}\,\rho^i_\mu\epsilon_{ijk}\pi_k a^{j\mu}+\frac{\gamma}{f_\pi g^2\pi^2}\,\rho^i_\mu\epsilon_{ijk}\pi_k\partial^2 a^{j\mu}, \tag{19}$$

$$\mathcal{L}_{\rho\pi\pi}=\frac{2}{g}\,\rho^i_\mu\epsilon_{ijk}\pi_k\left(-\partial^\mu\pi_j+\frac{1}{2\pi^2F^2}\partial^2\partial^\mu\pi_j\right), \tag{20}$$

where $\gamma=(1-1/2\pi^2g^2)^{-1/2}$.

Using VMD, i.e., the substitution (9), and Eqs. (18)–(20) we get all of the corresponding photon-$\pi$ interaction Lagrangians $\mathcal{L}_{\gamma\gamma\pi\pi}$, $\mathcal{L}_{\gamma\rho\pi\pi}$, $\mathcal{L}_{\gamma\pi a}$, and $\mathcal{L}_{\gamma\pi\pi}$. Combining them with $\mathcal{L}_{\rho\gamma}$ [Eq. (12)], we can calculate $S_\pi$ [Eq. (15)] and obtain the $\pi^+$-$\pi^0$ mass difference due to electromagnetic interactions. The corresponding Feynman diagrams are shown in Figs. 1–3. Denoting the corresponding $S$ matrices as $S_\pi(1)$, $S_\pi(2)$, and $S_\pi(3)$, respectively, we have

$$S_\pi=S_\pi(1)+S_\pi(2)+S_\pi(3).$$

We will compute them separately up to $O(e^2)$ below. In order to show the gauge independence of the final results explicitly, we take the most general linear gauge condition for electromagnetic fields to all diagram calculations in this paper. Namely, the $A_\mu$ propagator with an arbitrary gauge parameter $a$ is taken to be

$$\Delta_{F^{(\gamma)}_{\mu\nu}}(x-y)=\int\frac{d^4k}{(2\pi)^4}\Delta_{F_{\mu\nu}}^{(\gamma)}(k)e^{-ik(x-y)},$$

$$\Delta_{F^{(\gamma)}_{\mu\nu}}(k)=-\frac{i}{k^2}\left[g_{\mu\nu}-(1-a)\frac{k_\mu k_\nu}{k^2}\right]. \tag{21}$$

First, we compute $S_\pi(1)$ (Fig. 1). From Eqs. (15), (12), (18), and (9) we have

$$S_\pi(1)=\left\langle\pi\middle|T\left[i\int d^4x_1\mathcal{L}_{\gamma\gamma\pi\pi}(x_1)+\frac{i^2}{2!}2\times\int d^4x_1d^4x_2\mathcal{L}_{\gamma\rho\pi\pi}(x_1)\mathcal{L}_{\rho\gamma}(x_2)+\frac{i^3}{3!}3\times\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{\rho\rho\pi\pi}(x_1)\mathcal{L}_{\rho\gamma}(x_2)\right.\right.$$

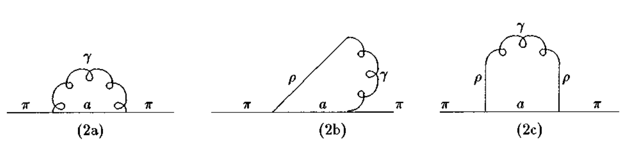


FIG. 2. Same as Fig. 1.

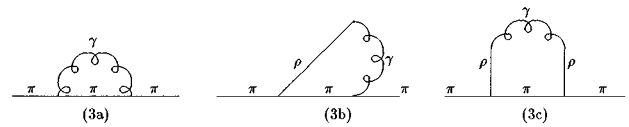


FIG. 3. Same as Fig. 1.

$$\left.\left.\mathcal{L}_{\rho\gamma}(x_3)\right]\middle|\pi\right\rangle$$

$$=\frac{e^2g^2}{4}\left\langle\pi\middle|i\int d^4x[\pi_1^2(x)+\pi_2^2(x)]\frac{1}{g^2f_\pi^2}\left\{2F^2g^{\mu\nu}\Delta_{F_{\mu\nu}}^{(\gamma\rho)}(x-y)|_{x=y}+\frac{1}{\pi^2}g^{\mu\nu}g^{\lambda\rho}\partial^x_\lambda\partial^y_\rho\Delta_{F_{\mu\nu}}^{(\gamma\rho)}(x-y)|_{x=y}\right\}\middle|\pi\right\rangle, \tag{22}$$

where

$$\Delta_{F_{\mu\nu}}^{(\gamma\rho)}(x-y)=\int\frac{d^4k}{(2\pi)^4}\Delta_{F_{\mu\nu}}^{(\gamma\rho)}(k)e^{-ik(x-y)},$$

$$\Delta_{F_{\mu\nu}}^{(\gamma\rho)}(k)=-\frac{i}{k^2}\left[\frac{m_\rho^4}{(k^2-m_\rho^2)^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^2}\right]. \tag{23}$$

We call $\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)$ the photon propagator within $\rho$ (see Appendix A for details).

It is easy to check that Eq. (22) can be reobtained by the following steps: First compute Fig. 1(a) by using $\mathcal{L}_{\gamma\gamma\pi\pi}$ and second substitute $\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)$ for $\Delta_{F^{(\gamma)}_{\mu\nu}}(x-y)$ in it and then arrive at Eq. (22) again. It is constructive that the substitution of $\Delta_{F^{(\gamma)}_{\mu\nu}}\to\Delta_{F^{(\gamma\rho)}_{\mu\nu}}$ in the above is the consequence of VMD. This rule is generally valid for all VMD processes in the two-flavor case and it is useful for practical calculations.

Using Eq. (16) and substituting Eq. (23) into Eq. (22), we get the total contributions of Figs. 1(a), 1(b), and 1(c) to $m^2_{\pi^+}-m^2_{\pi^0}$:

$$(m^2_{\pi^+}-m^2_{\pi^0})_1=\frac{2iS_\pi(1)}{\langle\pi|\int d^4x(\pi_1^2+\pi_2^2)|\pi\rangle}$$

$$=i\frac{e^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\left(F^2+\frac{k^2}{2\pi^2}\right)\times\left[\frac{m_\rho^4}{k^2(k^2-m_\rho^2)^2}(D-1)+\frac{a}{k^2}\right], \tag{24}$$

where $D=4-\epsilon$. According to the rule of dimensional regularization, i.e., 't Hooft–Veltman conjecture [28], the last term in Eq. (24) will vanish. Therefore, $(m^2_{\pi^+}-m^2_{\pi^0})_1$ is gauge independent.

Second, from Eqs. (19), (12), and (15) and using the substitution (9), we have

$$S_\pi(2)=\left\langle \pi\right|T\left\{\frac{i^2}{2!}\int d^4x_1d^4x_2\mathcal{L}_{\pi a\gamma}(x_1)\mathcal{L}_{\pi a\gamma}(x_2)+\frac{i^3}{3!}6\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{\pi a\rho}(x_1)\mathcal{L}_{\pi a\gamma}(x_2)\mathcal{L}_{\rho\gamma}(x_3)+\frac{i^4}{4!}6\int d^4x_1d^4x_2d^4x_3d^4x_4\mathcal{L}_{\pi a\rho}(x_1)\mathcal{L}_{\pi a\rho}(x_2)\times\mathcal{L}_{\rho\gamma}(x_3)\mathcal{L}_{\rho\gamma}(x_4)\right\}\left|\pi\right\rangle. \quad (25)$$

Straightforward calculation shows

$$S_\pi(2)=-\frac{e^2\gamma^2}{2g^2f_\pi^2}\left\langle\pi\right|\int d^4p\ \pi_a(p)\pi_a(-p)\times(2\pi)^4\Gamma_2(p^2)\left|\pi\right\rangle, \quad (26)$$

where $a=1,2$, and

$$\pi_a(p)=\frac{1}{(2\pi)^4}\int d^4x\ \pi_a(x)e^{-ipx}, \quad (27)$$

$$\Gamma_2(p^2)=\int\frac{d^4k}{(2\pi)^4}\left(F^2+\frac{k^2}{2\pi^2}\right)^2\frac{g^{\mu\nu}-\frac{k^\mu k^\nu}{m_a^2}}{-k^2+m_a^2}\times\left[\frac{m_\rho^4}{q^2(q^2-m_\rho^2)^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+a\frac{q_\mu q_\nu}{q^4}\right]. \quad (28)$$

Here $q=p-k$. On the $\pi$-mass shell, $p^2=m_\pi^2=0$ (chiral limit), so we have

$$S_\pi(2)=-\frac{e^2\gamma^2}{2g^2f_\pi^2}\left\langle\pi\right|\int d^4x\ \pi_a(x)\pi_a(x)\left|\pi\right\rangle\Gamma_2(p^2=0), \quad (29)$$

where $\int d^4p\,\pi_a(p)\pi_a(-p)(2\pi)^4=\int d^4x\ \pi_a(x)\pi_a(x)$ [see Eq. (27)] has been used. From Eq. (28) the gauge-dependent term of $\Gamma_2(p^2=0)$ is

$$a\int\frac{d^4k}{(2\pi)^4}\left(F^2+\frac{k^2}{2\pi^2}\right)^2\frac{1}{m_a^2k^2}.$$

This term is equal to zero according to 't Hooft–Veltman conjecture in dimensional regularization. Therefore, $S_\pi(2)$ is gauge independent. Thus, using Eq. (16), we get

$$(m_{\pi^+}^2-m_{\pi^0}^2)_2=-\frac{ie^2\gamma^2}{g^2f_\pi^2}\Gamma_2(p^2=0)=i\frac{e^2\gamma^2}{g^2f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\times\left(F^2+\frac{k^2}{2\pi^2}\right)^2\frac{m_\rho^4(D-1)}{k^2(k^2-m_\rho^2)^2(k^2-m_a^2)}. \quad (30)$$

The $S_\pi(3)$ corresponding to Fig. 3 reads

$$S_\pi(3)=\left\langle \pi\right|T\left\{\frac{i^2}{2!}\int d^4x_1d^4x_2\mathcal{L}_{\pi\pi\gamma}(x_1)\mathcal{L}_{\pi\pi\gamma}(x_2)+\frac{i^3}{3!}6\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{\pi\pi\rho}(x_1)\mathcal{L}_{\pi\pi\gamma}(x_2)\mathcal{L}_{\rho\gamma}(x_3)+\frac{i^4}{4!}6\int d^4x_1d^4x_2d^4x_3d^4x_4\mathcal{L}_{\pi\pi\rho}(x_1)\mathcal{L}_{\pi\pi\rho}(x_2)\times\mathcal{L}_{\rho\gamma}(x_3)\mathcal{L}_{\rho\gamma}(x_4)\right\}\left|\pi\right\rangle=-\frac{e^2}{2F^4}\left\langle\pi\right|\int d^4x\ \pi_a(x)\pi_a(x)\left|\pi\right\rangle\Gamma_3(p^2=0), \quad (31)$$

where

$$\Gamma_3(p^2=0)=\int\frac{d^4k}{(2\pi)^4}\left(F^2+\frac{k^2}{2\pi^2}\right)^2\frac{k^\mu k^\nu}{k^2}\times\left[\frac{m_\rho^4}{k^2(k^2-m_\rho^2)^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^4}\right]=\int\frac{d^4k}{(2\pi)^4}a\frac{1}{k^2}\left(F^2+\frac{k^2}{2\pi^2}\right)^2. \quad (32)$$

Using dimensional regularization, we have $\Gamma_3(p^2=0)=0$; then $S_\pi(3)=0$ and

$$(m_{\pi^+}^2-m_{\pi^0}^2)_3=0. \quad (33)$$

The total $\pi^+-\pi^0$ mass difference is the sum of Eqs. (21), (30), and (33), which is

$$m_{\pi^+}^2-m_{\pi^0}^2=i\frac{e^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}(D-1)m_\rho^4\times\frac{F^2+\frac{k^2}{2\pi^2}}{k^2(k^2-m_\rho^2)^2}\left[1+\frac{\gamma^2}{g^2}\frac{F^2+\frac{k^2}{2\pi^2}}{k^2-m_a^2}\right]. \quad (34)$$

The integration calculation for Eq. (34) is standard. We get the result of

$$m_{\pi^+}^2-m_{\pi^0}^2=\frac{3\alpha_{\rm EM}m_\rho^4}{8\pi f_\pi^2}\left\{\frac{\gamma^2}{g^2\pi^2m_\rho^2}\left(F^2+\frac{m_a^2}{2\pi^2}\right)+\left(2+\frac{\gamma^2}{g^2\pi^2}\right)\times\left(\frac{F^2}{m_\rho^2}+\frac{1}{3\pi^2}-8\chi_\rho\right)-\frac{2\gamma^2}{g^2(m_a^2-m_\rho^2)}\times\left(F^2+\frac{m_a^2}{2\pi^2}\right)^2\left(\frac{1}{m_\rho^2}+\frac{1}{m_a^2-m_\rho^2}\ln\frac{m_\rho^2}{m_a^2}\right)\right\}, \quad (35)$$

where $\alpha_{\rm EM}=e^2/4\pi=1/137$ and

$$\chi_\rho=\left(\frac{\mu^2}{m_\rho^2}\right)^{\epsilon/2}\frac{1}{(4\pi)^{D/2}}\Gamma\left(2-\frac{D}{2}\right). \quad (36)$$

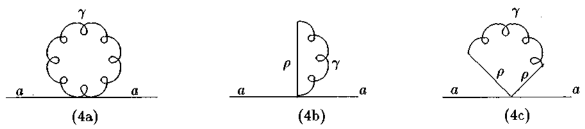


FIG. 4. One-loop Feynman diagrams contributing to the electromagnetic mass difference between $a_1^+$ and $a_1^0$. The curly line is the photon line.

It is essential that the logarithmic divergence in Eq. (35) [or Eq. (36)] can be factorized by using the intrinsic parameter $g$ in this theory. Comparing Eq. (3) with Eq. (36), we have

$$\chi_\rho=\frac{1}{8}g^2+\frac{1}{32\pi^2}+\frac{1}{16\pi^2}\ln\frac{f_\pi^2}{6(g^2m_\rho^2-f_\pi^2)}, \quad (37)$$

where Eq. (6) has been used. When $g$ is determined, $\chi_\rho$ will be fixed and the final result of Eq. (35) is finite.

The determination of $g$ can be done by taking $f_\pi$, $m_\rho$, and $m_a$ as inputs. Substituting $f_\pi=0.186$ GeV, $m_\rho=0.768$ GeV, and $m_a=1.20$ GeV into Eqs. (4) and (6), we obtain

$$g=0.39. \quad (38)$$

Then

$$m_{\pi^+}^2-m_{\pi^0}^2=0.001\,465\ \text{GeV}^2=2m_\pi\times 5.3\ \text{MeV}, \quad (39)$$

which is in reasonable agreement with the experimental value of $2m_\pi\times 4.6$ MeV [22].

## III. $a_1^+-a_1^0$ ELECTROMAGNETIC MASS DIFFERENCE

The interaction Lagrangians contributing to the $a_1^+-a_1^0$ electromagnetic mass difference read

$$\mathcal{L}_{\rho\rho aa}=-\frac{2\gamma^2}{g^2}\rho_\mu^j\rho_\nu^k(a^{j\mu}a^{k\nu}-g^{\mu\nu}a^{j\lambda}a_\lambda^k)+\frac{\gamma^2}{\pi^2g^4}\rho_\mu^j\rho^{k\mu}(\delta_{jk}a_\nu^i a^{i\nu}-a_\nu^j a^{k\nu}), \quad (40)$$

$$\mathcal{L}_{\rho aa}=\frac{2}{g}\left(1-\frac{\gamma^2}{g^2\pi^2}\right)\epsilon_{ijk}a_\mu^j a_\nu^k\partial^\nu\rho^{i\mu}-\frac{2}{g}\epsilon_{ijk}\rho_\mu^i a_\nu^j(\partial^\mu a^{k\nu}-\gamma^2\partial^\nu a^{k\mu}), \quad (41)$$

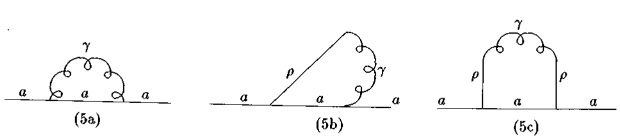


FIG. 5. Same as Fig. 4.

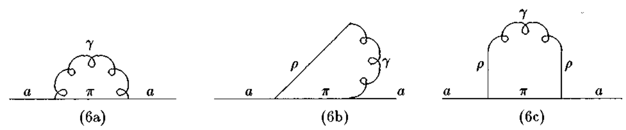


FIG. 6. Same as Fig. 4.

$$\mathcal{L}_{\rho a\pi}=\frac{2}{g}\epsilon_{ijk}\rho_\mu^i[c_1\pi_j a^{k\mu}+c_2(\partial_\nu\pi_j\partial^\mu a^{k\nu}-a_\nu^k\partial^\mu\partial^\nu\pi_j)]+\frac{2}{g}\epsilon_{ijk}(\partial_\mu\rho_\nu^i-\partial_\nu\rho_\mu^i)[c_3\partial^\mu(a^{k\nu}\pi_j)+c_4\partial^\mu\pi_j a^{k\nu}], \quad (42)$$

where

$$c_1=\frac{\gamma}{f_\pi g}\left[F^2+\left(\frac{1}{2\pi^2}-2cg\right)m_a^2\right], \quad (43)$$

$$c_2=\frac{\gamma}{2f_\pi\pi^2g}\left(1-\frac{2c}{g}\right), \quad (44)$$

$$c_3=\frac{3\gamma}{2f_\pi\pi^2g}\left(1-\frac{2c}{g}\right)+\frac{2\gamma c}{f_\pi}, \quad (45)$$

$$c_4=\frac{2\gamma c}{f_\pi}. \quad (46)$$

The corresponding photon-$a_1$ interaction Lagrangians $\mathcal{L}_{\gamma\gamma aa}$, $\mathcal{L}_{\gamma aa}$, and $\mathcal{L}_{\gamma a\pi}$ can be constructed by the substitution (9) and Eqs. (40)–(42). It is similar to the preceding section that these Lagrangians and $\mathcal{L}_{\rho\gamma}$ [Eq. (12)] provide the dynamics for the mass splitting of $a_1$ due to electromagnetic interactions. The Feynman diagrams are shown in Figs. 4–6. The corresponding $S$ matrices are denoted as $S_a(1)$, $S_a(2)$, and $S_a(3)$ and

$$S_a=S_a(1)+S_a(2)+S_a(3). \quad (47)$$

We calculate $S_a(1)$, $S_a(2)$, and $S_a(3)$ separately in the following.

For Fig. 4, from Eqs. (40), (9), and (12) we have

$$S_a(1)=\left\langle a\left|T\left[i\int d^4x_1\mathcal{L}_{\gamma\gamma aa}(x_1)+\frac{i^2}{2!}2\int d^4x_1d^4x_2\mathcal{L}_{\gamma\rho aa}(x_1)\mathcal{L}_{\rho\gamma}(x_2)+\frac{i^3}{3!}3\times\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{\rho\rho aa}(x_1)\mathcal{L}_{\rho\gamma}(x_2)\mathcal{L}_{\rho\gamma}(x_3)\right]\right|a\right\rangle. \quad (48)$$

Using Eq. (15), we get

$$(m^2_{a^+}-m^2_{a^0})_1$$

$$=ie^2\frac{\gamma^2\langle a|\int d^4x\ a^{i\mu}a^{i\nu}|a\rangle-\langle a|\int d^4x\ a^{i\lambda}a^i_\lambda|a\rangle g^{\mu\nu}}{\langle a|\int d^4x\ a^i_\mu a^{i\mu}|a\rangle}\times\int\frac{d^4k}{(2\pi)^4}\frac{m^4_\rho}{k^2(k^2-m^2_\rho)^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right), \tag{49}$$

where $\underline{i}=1,2$.

For Fig. 5, from Eqs. (41), (9), and (12) we have

$$S_a(2)=\left\langle a\left|T\left\{\frac{i^2}{2!}\int d^4x_1d^4x_2\mathcal{L}_{aa\gamma}(x_1)\mathcal{L}_{aa\gamma}(x_2)+\frac{i^3}{3!}6\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{aa\rho}(x_1)\mathcal{L}_{aa\gamma}(x_2)\mathcal{L}_{\rho\gamma}(x_3)+\frac{i^4}{4!}6\int d^4x_1d^4x_2d^4x_3d^4x_4\mathcal{L}_{aa\rho}(x_1)\mathcal{L}_{aa\rho}(x_2)\times\mathcal{L}_{\rho\gamma}(x_3)\mathcal{L}_{\rho\gamma}(x_4)\right\}\right|a\right\rangle. \tag{50}$$

Using Eq. (15), we obtain

$$(m^2_{a^+}-m^2_{a^0})_2=\frac{ie^2}{\langle a|\int d^4x\ a^{i\mu}a^i_\mu|a\rangle}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-2p\cdot k}\frac{m^4_\rho}{k^2(k^2-m^2_\rho)^2}\left\{\left\langle a\left|\int d^4x\ a^{i\mu}a^i_\mu\right|a\right\rangle\times\left[4m^2_a+(b^2+2b\gamma^2)k^2+2\gamma^4p\cdot k-\frac{4(pk)^2}{k^2}-\frac{1}{m^2_a}[bk^2-(b-\gamma^2)p\cdot k]^2\right]+\left\langle a\left|\int d^4x\ a^i_\mu a^i_\nu\right|a\right\rangle k^\mu k^\nu\left[-(3b^2-4b+4)+D(b+\gamma^2)^2+4\gamma^2-6b\gamma^2-2\gamma^4-\frac{2\gamma^4p\cdot k}{k^2}+\frac{1}{m^2_ak^2}[bk^2-2(1-\gamma^2)p\cdot k]^2\right]\right\}, \tag{51}$$

where $b=1-\gamma^2/\pi^2g^2$ and $p$ is the external momentum of $a_1$ fields. The Fourier transformation for mass-shell $a_1$ fields is

$$a^i_\mu(p)=\frac{1}{(2\pi)^4}\int d^4x\ a^i_\mu(x)e^{-ipx},$$

with

$$p^2=m^2_a,\quad p^\mu a^i_\mu(p)=0. \tag{52}$$

For Fig. 6, from Eqs. (42), (9), and (12), we have

$$S_a(3)=\left\langle a\left|T\left\{\frac{i^2}{2!}\int d^4x_1d^4x_2\mathcal{L}_{a\pi\gamma}(x_1)\mathcal{L}_{a\pi\gamma}(x_2)+\frac{i^3}{3!}6\int d^4x_1d^4x_2d^4x_3\mathcal{L}_{a\pi\rho}(x_1)\mathcal{L}_{a\pi\gamma}(x_2)\mathcal{L}_{\rho\gamma}(x_3)+\frac{i^4}{4!}6\int d^4x_1d^4x_2d^4x_3d^4x_4\mathcal{L}_{a\pi\rho}(x_1)\mathcal{L}_{a\pi\rho}(x_2)\mathcal{L}_{\rho\gamma}\times(x_3)\mathcal{L}_{\rho\gamma}(x_4)\right\}\right|a\right\rangle \tag{53}$$

and

$$(m^2_{a^+}-m^2_{a^0})_3=\frac{-ie^2}{\langle a|\int d^4xa^i_\mu a^{i\mu}|a\rangle}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\times\frac{m^4_\rho}{k^2(k^2-m^2_\rho)^2}\left\{\left\langle a\left|\int d^4x\ a^i_\mu a^{i\mu}\right|a\right\rangle\times(c_1-3c_2p\cdot k+c_3k^2)^2+\left\langle a\left|\int d^4x\ a^i_\mu a^i_\nu\right|a\right\rangle k^\mu k^\nu\times\left[c_2m^2_a-\frac{(c_1-2c_2p\cdot k+c_3k^2)^2}{k^2}\right]\right\}. \tag{54}$$

It needs to be checked that $(m^2_{a^+}-m^2_{a^0})_{1,2,3}$ are gauge independent. The gauge-dependent terms of $(m^2_{a^+}-m^2_{a^0})_1$, which come from Fig. 4(a), will vanish according to the rule of dimensional regularization.

The gauge-dependent terms in $S_a(2)$ [to be denoted as $S_a(2)_G$] come from Fig. 5(a). Using VMD, the correspondent photon-meson interaction Lagrangian is

$$\mathcal{L}_{\gamma aa}=eb\epsilon_{3jk}a^j_\mu a^k_\nu\partial^\nu A^\mu-e\epsilon_{3jk}A_\mu a^j_\nu(\partial^\mu a^{k\nu}-\gamma^2\partial^\nu a^{k\mu}). \tag{55}$$

Then

$$S_a(2)_G = a' \frac{e^2}{2}\left\langle a \left| \int d^4x\, a^i_\mu a^{i\mu} \right| a \right\rangle \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\left\{1-\frac{2pk}{k^2} + \frac{k^2}{D}\left[-\frac{\gamma^4}{k^2}-\frac{(1-\gamma^2)^2}{m_a^2k^2}(k^2-2pk)\right]\right\}, \tag{56}$$

where $a'$ is gauge parameter. 't Hooft–Veltman conjecture will make sure that $S_a(2)$ is gauge independent.

The photon-meson interaction Lagrangian contributing to the gauge-dependent term $S_a(3)_G$ [Fig. 6(a)] is

$$\mathcal{L}_{\gamma a\pi} = e\epsilon_{3jk}A_\mu[c_1\pi_j a^{k\mu} + c_2(\partial^\nu\pi_j\partial^\mu a^k_\nu - a^{k\nu}\partial^\mu\partial_\nu\pi_j)]. \tag{57}$$

We will have

$$S_a(3)_G = -a'\frac{e^2}{2}\left\langle a \left| \int d^4x\, a^i_\mu a^{i\mu} \right| a \right\rangle \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \times\left[\frac{(c_1-c_2m_a^2)^2}{k^2(p-k)^2}+\frac{c_2(c_1-c_2m_a^2)}{k^2}+\frac{c_2^2(p-k)^2}{k^2}\right]. \tag{58}$$

The third term will vanish because of dimensional regularization. By Eq. (5) and the definition of $c_1$ and $c_2$, we obtain that $c_1-c_2m_a^2=0$. Thus $S_a(3)_G=0$.

The $g$ has been determined in Eq. (38) and the logarithmic divergences in the above Feynman integrations [Eqs. (49), (51), and (54)] can also be factorized by using Eq. (37), so there are no further unknown parameters in the expressions of $(m^2_{a^+}-m^2_{a^0})_{1,2,3}$. After a long but straightforward calculation, we can get the final results for $(m^2_{a^+}-m^2_{a^0})_{1,2,3}$, whose form is very tedious. The numerical results for them are

$$(m^2_{a^+}-m^2_{a^0})_1 = -0.000\,648\ \mathrm{GeV}^2,$$

$$(m^2_{a^+}-m^2_{a^0})_2 = -0.002\,688\ \mathrm{GeV}^2,$$

$$(m^2_{a^+}-m^2_{a^0})_3 = 0.001\,896\ \mathrm{GeV}^2. \tag{59}$$

Totally,

$$(m^2_{a^+}-m^2_{a^0})_{\mathrm{EM}} = -0.001\,440\ \mathrm{GeV}^2 = -2m_a\times 0.57\ \mathrm{MeV}. \tag{60}$$

## IV. $K^+-K^0$ ELECTROMAGNETIC MASS DIFFERENCE AND THE VIOLATION OF DASHEN'S THEOREM

In this section and Secs. V and VI our method is extended to the studies of the electromagnetic self-energies of the strange-flavor mesons. As mentioned above, the large strange quark mass will result in the SU(3) symmetry breaking playing an important role in these calculations. Dashen's theorem, which states that the electromagnetic contributions to the difference between the mass square of kaons and pions are equal, is valid only in the chiral SU(3) limit. Corrections to the electromagnetic self-energies to the leading order in quark mass expansion are sure to lead to the violation of Dashen's theorem. Therefore, it is necessary to evaluate the electromagnetic self-energies of the strange-flavor mesons and the corrections to Dashen's theorem to the order of $m_s$ or $m_K^2$.

From Eq. (3) in Ref. [11] ($\mathcal{L}_{\mathrm{Re}}$), the interaction Lagrangians that can contribute to the electromagnetic mass difference between $K^+$ and $K^0$ are

$$\mathcal{L}_{KKvv} = \frac{1}{f_k^2g^2}\Bigg\{2F^2\rho^{3\mu}v^8_\mu(K^+K^- - K^0\overline{K^0}) + \frac{1}{\pi^2}\partial_\nu\rho^{3\mu}\partial^\nu v^8_\mu(K^+K^- - K^0\overline{K^0}) + \left[\frac{1+\left(1-\frac{2c'}{g}\right)^2}{\pi^2} - 8c'^2\right]\rho^{3\mu}v^8_\mu(\partial_\nu K^+\partial^\nu K^- - \partial_\nu K^0\partial^\nu\overline{K^0}) - \frac{2\left(1-\frac{2c'}{g}\right)}{\pi^2}\rho^3_\nu v^8_\mu(K^+\partial^{\mu\nu}K^- - K^0\partial^{\mu\nu}\overline{K^0} + \mathrm{H.c.}) + 4c'^2\rho^3_\mu v^8_\nu(\partial^\mu K^+\partial^\nu K^- - \partial^\mu K^0\partial^\nu\overline{K^0} + \mathrm{H.c.})\Bigg\}, \tag{61}$$

$$\mathcal{L}_{KKv} = \frac{i}{g}\alpha_1[\rho^3_\mu(K^+\partial^\mu K^- - K^0\partial^\mu\overline{K^0}) + v^8_\mu(K^+\partial^\mu K^- + K^0\partial^\mu\overline{K^0})] - \frac{i}{g}\alpha_2[\rho^3_\mu(K^+\partial^2\partial^\mu K^- - K^0\partial^2\partial^\mu\overline{K^0}) + v^8_\mu(K^+\partial^2\partial^\mu K^- + K^0\partial^2\partial^\mu\overline{K^0})] + \frac{i}{g}\alpha_3[\rho^3_\mu(\partial^{\mu\nu}K^+\partial_\nu K^- - \partial^{\mu\nu}K^0\partial_\nu\overline{K^0}) + v^8_\mu(\partial^{\mu\nu}K^+\partial_\nu K^- + \partial^{\mu\nu}K^0\partial_\nu\overline{K^0})] + \mathrm{H.c.}, \tag{62}$$

$$\mathcal{L}_{KK_1v}=\frac{i}{g}\beta_1[\rho^3_\mu(K^+K_1^{-\mu}-K^0\overline{K_1^0}^{\mu})+v^8_\mu(K^+K_1^{-\mu}+K^0\overline{K_1^0}^{\mu})]+\frac{i}{g}\beta_2[\rho^3_\nu(\partial^{\mu\nu}K^+K_{1\mu}-\partial^{\mu\nu}K^0\overline{K^0_{1\mu}})+v^8_\nu(\partial^{\mu\nu}K^+K_{1\mu}$$

$$+\partial^{\nu\theta}K^0\overline{K^0_{1\mu}})]+\frac{i}{g}\beta_3[\rho^3_\mu(K_1^{+\mu}\partial^2K^- -K_1^{0\mu}\partial^2\overline{K^0})+v^8_\mu(K_1^{+\mu}\partial^2K^- +K_1^{0\mu}\partial^2\overline{K^0})]-\frac{i}{g}\beta_4[\rho^3_\mu(K^+\partial^2K_1^{-\mu}$$

$$-K^0\partial^2\overline{K_1^0}^{\mu})+v^8_\mu(K^+\partial^2K_1^{-\mu}+K^0\partial^2\overline{K_1^0}^{\mu})]-\frac{i}{g}\beta_3[\rho^3_\nu(K^+\partial^{\mu\nu}K^-_{1\mu}-K^0\partial^{\mu\nu}\overline{K^0_{1\mu}})+v^8_\nu(K^+\partial^{\mu\nu}K^-_{1\mu}+K^0\partial^{\mu\nu}\overline{K^0_{1\mu}})]$$

$$-\frac{i}{g}\beta_5[\rho^3_\mu(\partial_\nu K^+_{1\mu}\partial^\nu K^- -\partial_\nu K^0_{1\mu}\partial^\nu\overline{K^0})+v^8_\mu(\partial_\nu K^+_{1\mu}\partial^\nu K^- +\partial_\nu K^0_{1\mu}\partial^\nu\overline{K^0})]+\text{H.c.}, \tag{63}$$

with

$$\alpha_1=\frac{\left(1-\frac{2c'}{g}\right)F^2}{f_k^2},$$

$$\alpha_2=\frac{\left(1-\frac{2c'}{g}\right)}{2\pi^2f_k^2}+\frac{3\left(1-\frac{2c'}{g}\right)^2}{2\pi^2f_k^2}-\frac{4c'^2}{f_k^2},$$

$$\alpha_3=\frac{\left(1-\frac{2c'}{g}\right)^2}{\pi^2f_k^2}-\frac{4c'^2}{f_k^2} \tag{64}$$

and

$$\beta_1=\frac{\gamma F^2}{gf_k},\quad \beta_2=\frac{\gamma}{2\pi^2gf_k}\left(1-\frac{2c'}{g}\right),$$

$$\beta_3=\frac{3\gamma}{2\pi^2gf_k}\left(1-\frac{2c'}{g}\right)+\frac{2\gamma c'}{f_k},\quad \beta_4=\frac{\gamma}{2\pi^2gf_k},$$

$$\beta_5=\frac{3\gamma}{2\pi^2gf_k}\left(1-\frac{2c'}{g}\right)+\frac{4\gamma c'}{f_k}. \tag{65}$$

Here $v$ denotes the vector mesons including $\rho$, $\omega$, and $\phi$. $v^8_\mu=\omega_\mu-\sqrt{2}\phi_\mu$ and $\partial^{\mu\nu}=\partial^\mu\partial^\nu$. Distinguishing from the case of massless pions system, the nonzero strange quark mass, i.e., $m_k^2\neq 0$, will bring about the contributions to $m^2_{K^+}-m^2_{K^0}$ from the abnormal part of the effective Lagrangian. These vertices have been found by the evaluation of $(1/g)K^*_{a\mu}\langle\overline{\psi}\lambda_a\gamma^\mu\psi\rangle$ in Ref. [11]:

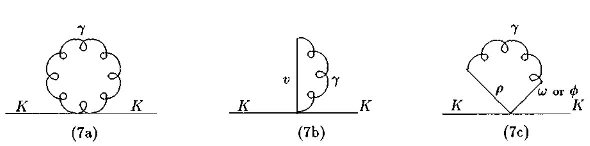


FIG. 7. One-loop Feynman diagrams contributing to the electromagnetic mass difference between $K^+$ and $K^0$. The curly line is the photon line and $v$ denotes neutral vector mesons $\rho$, $\omega$, and $\phi$.

$$\mathcal{L}_{K^*Kv}=-\frac{3}{2\pi^2g^2}\frac{2}{f_k}\epsilon^{\mu\nu\alpha\beta}K^+_\mu\partial_\beta K^-\left(\frac{1}{2}\partial_\nu\rho^3_\alpha+\frac{1}{2}\partial_\nu\omega_\alpha+\frac{\sqrt{2}}{2}\partial_\nu\phi_\alpha\right)-\frac{3}{2\pi^2g^2}\frac{2}{f_k}\epsilon^{\mu\nu\alpha\beta}K^0_\mu\partial_\beta\overline{K^0}\left(-\frac{1}{2}\partial_\nu\rho^3_\alpha+\frac{1}{2}\partial_\nu\omega_\alpha+\frac{\sqrt{2}}{2}\partial_\nu\phi_\alpha\right)+\text{H.c.} \tag{66}$$

Here we adopt the following definitions for the strange-flavor mesons:

$$K^\pm=\frac{1}{\sqrt{2}}(K^4\pm iK^5),\quad K^0(\overline{K^0})=\frac{1}{\sqrt{2}}(K^6\pm iK^7),$$

$$K^\pm_{1\mu}=\frac{1}{\sqrt{2}}(K^4_{1\mu}\pm iK^5_{1\mu}),\quad K^0_{1\mu}(\overline{K^0_{1\mu}})=\frac{1}{\sqrt{2}}(K^6_{1\mu}\pm iK^7_{1\mu}),$$

$$K^\pm_\mu=\frac{1}{\sqrt{2}}(K^4_\mu\pm iK^5_\mu),\quad K^0_\mu(\overline{K^0_\mu})=\frac{1}{\sqrt{2}}(K^6_\mu\pm iK^7_\mu). \tag{67}$$

The interaction Lagrangians between the photon and $K$-meson $\mathcal{L}_{\gamma\gamma KK}$, $\mathcal{L}_{\gamma KK}$, $\mathcal{L}_{\gamma K_1K}$, and $\mathcal{L}_{\gamma K^*K}$ can be obtained by the substitutions (9)–(11) and Eqs. (61)–(63) and (66). The Feynman diagrams contributing to the electromagnetic mass difference between $K^+$ and $K^0$ are shown in Figs. 7–10. The corresponding $S$ matrices are denoted as $S_K(1)$, $S_K(2)$, $S_K(3)$, and $S_K(4)$, respectively.

In Sec. II we obtained $S_\pi$ by substituting $\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)$ for $\Delta_{F^{(\gamma)}_{\mu\nu}}(x-y)$ after computing Figs. 1(a), 2(a), and 3(a). Here the involved vector mesons are not only $\rho$ mesons but also $\omega$ and $\phi$ mesons. So it is not as simple as in the case of pions. Practical calculations will show that we can get $S_K$ by changing the form of this substitution (see Appendix B). Specifically, for $S_K(1)$, $S_K(2)$, and $S_K(3)$ coming from the $\mathcal{L}_{\text{Re}}$, the corresponding propagator of the substitution should be $\Delta_{F^{(\gamma v)}_{1\mu\nu}}$ instead of $\Delta_{F^{(\gamma\rho)}_{\mu\nu}}$:

$$\Delta_{F^{(\gamma v)}_{1\mu\nu}}(x-y)=\int\frac{d^4k}{(2\pi)^4}\Delta_{F^{(\gamma v)}_{1\mu\nu}}(k)e^{-ik(x-y)},$$

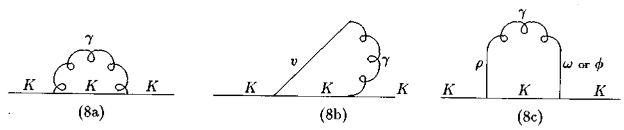


FIG. 8. Same as Fig. 7.

$$\Delta_{F^{(\gamma v)}_{1\mu\nu}}(k)=-\frac{i}{k^2}\left\{\left[\frac{1}{3}\frac{m_\rho^2 m_\omega^2}{(k^2-m_\rho^2)(k^2-m_\omega^2)}+\frac{2}{3}\frac{m_\rho^2 m_\phi^2}{(k^2-m_\rho^2)(k^2-m_\phi^2)}\right]\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^2}\right\}. \tag{68}$$

Obviously, under the SU(3) limit $m_\rho=m_\omega=m_\phi$, $\Delta_{F^{(\gamma v)}_{1\mu\nu}}$ will go back $\Delta_{F^{(\gamma\rho)}_{\mu\nu}}$. However, for $S_K(4)$, which receives the contributions from the abnormal part Lagrangian $\mathcal{L}_{\rm Im}$, the substituting propagator should be $\Delta_{F^{(\gamma v)}_{2\mu\nu}}$:

$$\Delta_{F^{(\gamma v)}_{2\mu\nu}}(x-y)=\int\frac{d^4k}{(2\pi)^4}\Delta_{F^{(\gamma v)}_{2\mu\nu}}(k)e^{-ik(x-y)},$$

$$\Delta_{F^{(\gamma v)}_{2\mu\nu}}(k)=-\frac{i}{k^2}\left\{\left[\frac{1}{3}\frac{m_\rho^2 m_\omega^2}{(k^2-m_\rho^2)(k^2-m_\omega^2)}-\frac{2}{3}\frac{m_\rho^2 m_\phi^2}{(k^2-m_\rho^2)(k^2-m_\phi^2)}\right]\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^2}\right\}. \tag{69}$$

Note that $\Delta_{F^{(\gamma v)}_{2\mu\nu}}$ is different from $\Delta_{F^{(\gamma v)}_{1\mu\nu}}$ (see Appendix B).

Thus it is easy to obtain the contributions of Figs. 7–10 to $m^2_{K^+}-m^2_{K^0}$, respectively. The contribution of Fig. 7 is

$$\begin{aligned}(\Delta m_K^2)_1=(m^2_{K^+}-m^2_{K^0})_1&=\frac{iS_K(1)}{\langle K|\int d^4x\, K^+K^-|K\rangle}\\&=i\frac{e^2}{f_k^2}\int\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)(D-1)\\&\quad\times\left[\frac{1}{3}\frac{m_\rho^2 m_\omega^2}{k^2(k^2-m_\rho^2)(k^2-m_\omega^2)}+\frac{2}{3}\frac{m_\rho^2 m_\phi^2}{k^2(k^2-m_\rho^2)(k^2-m_\phi^2)}\right],\end{aligned} \tag{70}$$

with

$$F_K^2=F^2+\left[\frac{1+\left(1-\frac{2c'}{g}\right)+\left(1-\frac{2c'}{g}\right)^2}{2\pi^2}-3c'^2\right]p^2,$$

where $p$ is the external momentum of kaons and $p^2=m_K^2$ on the $K$-mass shell. The contribution of Fig. 8 is

$$\begin{aligned}(\Delta m_K^2)_2=(m^2_{K^+}-m^2_{K^0})_2&=\frac{iS_K(2)}{\langle K|\int d^4x K^+K^-|K\rangle}\\&=-ie^2\int\frac{d^4k}{(2\pi)^4}\frac{X^\mu X^\nu\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)}{(p-k)^2-m_K^2}\\&\quad\times\left[\frac{1}{3}\frac{m_\rho^2 m_\omega}{(k^2-m_\rho^2)(k^2-m_\omega^2)}+\frac{2}{3}\frac{m_\rho^2 m_\phi^2}{(k^2-m_\rho^2)(k^2-m_\phi^2)}\right],\end{aligned} \tag{71}$$

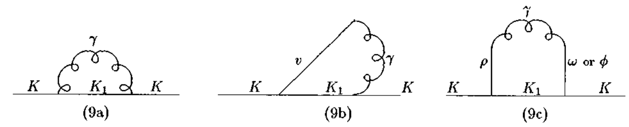


FIG. 9. Same as Fig. 7.

with

$$X_\mu=\alpha_1(q_\mu+p_\mu)+\alpha_2(q^2q_\mu+p^2p_\mu)-\alpha_3(p\cdot q)(q_\mu+p_\mu), \tag{72}$$

$$q=p-k.$$

The contribution of Fig. 9 is

$$\begin{aligned}(\Delta m_K^2)_3=(m^2_{K^+}-m^2_{K^0})_3&=\frac{iS_K(3)}{\langle K|\int d^4x K^+K^-|K\rangle}\\&=ie^2\int\frac{d^4k}{(2\pi)^4}Y_{\mu\nu}\frac{g^{\mu\nu}-\frac{q^\mu q^\nu}{m^2_{K_1}}}{q^2-m^2_{K_1}}\\&\quad\times\left[\frac{1}{3}\frac{m_\rho^2 m_\omega^2}{k^2(k^2-m_\rho^2)(k^2-m_\omega^2)}+\frac{2}{3}\frac{m_\rho^2 m_\phi^2}{k^2(k^2-m_\rho^2)(k^2-m_\phi^2)}\right],\end{aligned} \tag{73}$$

where

$$\begin{aligned}Y_{\mu\nu}=&(\beta_1+\beta_3p^2+\beta_4q^2-\beta_5p\cdot q)^2\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)-2(\beta_1\\&+\beta_4q^2)\beta_2p_\mu p_\nu+\beta_3^2q_\mu q_\nu\left(p^2-\frac{(p\cdot k)^2}{k^2}\right)+2(\beta_1\\&+\beta_4q^2-\beta_5p\cdot q)\beta_3p_\mu q_\nu-2(\beta_1+\beta_4q^2-\beta_5p\cdot q)\end{aligned}$$

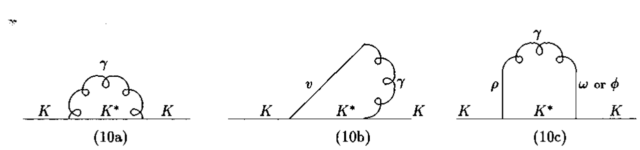


FIG. 10. Same as Fig. 7.

$$\beta_2\frac{p\cdot k}{k^2}p_\nu q_\mu+\beta_3\frac{p\cdot k}{k^2}k_\mu q_\nu\Bigg) \tag{74}$$

and $q=p-k$.

The contribution of Fig. 10 is

$$\begin{aligned}(\Delta m_K^2)_4=(m_{K^+}^2-m_{K^0}^2)_4&=\frac{iS_K(4)}{\langle K|\int d^4xK^+K^-|K\rangle}\\&=-\frac{9ie^2}{2\pi^4g^2f_k^2}\int\frac{d^4k}{(2\pi)^4}\frac{p^2k^2-(p\cdot k)^2}{(p-k)^2-m_{K*}^2}\\&\quad\times\left[\frac{1}{3}\frac{m_\rho^2m_\omega^2}{k^2(k^2-m_\rho^2)(k^2-m_\omega^2)}\right.\\&\quad\left.-\frac{2}{3}\frac{m_\rho^2m_\phi^2}{k^2(k^2-m_\rho^2)(k^2-m_\phi^2)}\right].\end{aligned} \tag{75}$$

The gauge independence of $(m_{K^+}^2-m_{K^0}^2)_{1,2,3,4}$ should be examined. The gauge-dependent terms in $(m_{K^+}^2-m_{K^0}^2)_1$ will vanish according to 't Hooft–Veltman conjecture, which is similar to the cases of $S_\pi(1)$ and $S_a(1)$.

The gauge-independent terms in $S_K(2)$ [to be denoted as $S_K(2)_G$] come from Fig. 8(a). Using VMD, $\mathcal{L}_{\gamma KK}$ can be constructed from $\mathcal{L}_{KKv}$. Thus we have

$$\begin{aligned}S_K(2)_G=&-ae^2\left\langle K\middle|\int d^4x\ K^+K^-\middle|K\right\rangle\\&\times\int\frac{d^4k}{(2\pi)^4}\frac{X_\mu X_\nu k^\mu k^\nu}{(q^2-m_K^2)(k^2)^2}.\end{aligned} \tag{76}$$

From Eq. (72) we have

$$\begin{aligned}X_\mu k^\mu=&-\alpha_1(q^2-p^2)-\alpha_2(p^2-p\cdot k+k^2)(q^2-p^2)\\&+\alpha_3(p^2-p\cdot k)(q^2-p^2).\end{aligned}$$

The mass shell condition leads to $p^2=m_K^2$, so the term $q^2-m_K^2$ in the denominator of $S_K(2)_G$ will be reduced. This means that the contribution of $S_K(2)_G$ is zero in the framework of the dimensional regularization.

Likewise, we will obtain $S_K(3)_G$ [Fig. 9(a)], which is

$$\begin{aligned}S_K(3)_G=&ae^2\left\langle K\middle|\int d^4x\ K^+K^-\middle|K\right\rangle\int\frac{d^4k}{(2\pi)^4}\\&\times\frac{1}{q^4}\frac{g^{\mu\nu}-\dfrac{k^\mu k^\nu}{m_{K_1}^2}}{k^2-m_{K_1}^2}(W_1p_\mu-W_2k_\mu)(W_1p_\nu-W_2k_\nu)\\=&ae^2\left\langle K\middle|\int d^4x\ K^+K^-\middle|K\right\rangle\int\frac{d^4k}{(2\pi)^4}\frac{1}{q^4}\\&\times\left\{W_1^2\frac{p^2-\dfrac{(p\cdot k)^2}{m_{K_1}^2}}{k^2-m_{K_1}^2}+2W_1W_2\frac{p\cdot k}{m_{K_1}^2}-W_2^2\frac{k^2}{m_{K_1}^2}\right\},\end{aligned} \tag{77}$$

where

$$W_1=\beta_1+(\beta_4-\beta_2-\beta_6)k^2+\beta_2m_K^2,$$

$$W_2=\beta_1+\beta_4k^2-\beta_6p\cdot k,$$

$$\beta_6=2\gamma c'f_k.$$

The contributions of the second and third terms in Eq. (77) are zero because of 't Hooft–Veltman conjecture. Since our calculations are only to the order of $m_K^2$, the denominator of the first term in Eq. (77), $k^2-m_{K_1}^2$, can also be reduced. Here the relation $\beta_1+(\beta_4-\beta_2-\beta_6)m_{K_1}^2=0$, which can be easily obtained by Eq. (7), has been used. Thus $S_K(3)$ is gauge independent.

The gauge-dependent terms $S_K(4)_G$ [Fig. 10(a)], which receive contributions from the abnormal part of the effective Lagrangian $\mathcal{L}_{KK*\gamma}$, are

$$\begin{aligned}S_K(4)_G=&-a\frac{3e^2}{4\pi^4g^2f_k^2}\left\langle K\middle|\int d^4x\ K^+K^-\middle|K\right\rangle\\&\times\int\frac{d^4q}{(2\pi)^4}p_\beta p_{\beta'}\epsilon^{\mu\nu\alpha\beta}\epsilon^{\mu'\nu'\alpha'\beta'}\\&\times\frac{g_{\mu\mu'}-\dfrac{(p-q)_\mu(p-q)_{\mu'}}{m_{K*}^2}}{(p-q)^2-m_{K*}^2}\frac{q_\nu q_{\nu'}q_\alpha q_{\alpha'}}{q^4}.\end{aligned} \tag{78}$$

It is obvious that $S_K(4)_G$ will vanish because of the totally antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$.

From Eqs. (70), (71), (73), and (75), it is not difficult to conclude that the contributions of $S_K(2)$ and $S_K(4)$ are proportional to $p^2$. So in the chiral limit $p^2=m_K^2=0$, only $S_K(1)$ and $S_K(3)$ contribute to $m_{K^+}^2-m_{K^0}^2$. Then we have

$$\begin{aligned}\Delta m_{Km_s=0}^2=&i\frac{e^2}{f_k^2}\int\frac{d^4k}{(2\pi)^4}(D-1)\left(F^2+\frac{k^2}{2\pi^2}\right)\\&\times\left(1+\frac{\gamma^2}{g^2}\frac{F^2+\dfrac{k^2}{2\pi^2}}{k^2-m_{K_1}^2}\right)\\&\times\left[\frac{1}{3}\frac{m_\rho^2m_\omega^2}{k^2(k^2-m_\rho^2)(k^2-m_\omega^2)}\right.\\&\left.+\frac{2}{3}\frac{m_\rho^2m_\phi^2}{k^2(k^2-m_\rho^2)(k^2-m_\phi^2)}\right];\end{aligned} \tag{79}$$

Taking $f_k=f_\pi$, $m_\rho=m_\omega=m_\phi$, and $m_{K_1}=m_a$, the above equation reduces to Eq. (34). This indicates that Dashen's theorem is automatically obeyed in the chiral SU(3) limit of the present theory. However, SU(3) symmetry-breaking effects will lead to the violation of Dashen's theorem. The total $\Delta m_K^2$ [the sum of $(m_{K^+}^2-m_{K^0}^2)_{1,2,3,4}$], which is evaluated to the order of $m_K^2$, can be read off from Eqs. (70), (71), (73), and (75). It is straightforward to perform these Feynman integrations, although the calculating processes and the results

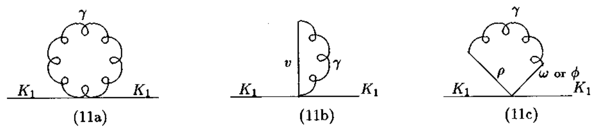


FIG. 11. One-loop Feynman diagrams contributing to the electromagnetic mass difference between $K_1^+$ and $K_1^0$. The curly line is the photon line and $v$ denotes neutral vector mesons $\rho$, $\omega$, and $\phi$.

are not as simple as that in the case of pions. We do not present the final expressions of $(m^2_{K^+}-m^2_{K^0})_{1,2,3,4}$ here. Note that only the logarithmic divergences are involved in the above Feynman integrations, which can be factorized by using Eq. (37). $f_k$ is determined from Eqs. (7) and (8), not as an input, and $g=0.39$ still holds. Numerically, the results of Eqs. (70), (71), (73), and (75) are

$$(m^2_{K^+}-m^2_{K^0})_1=0.002\,193\ \text{GeV}^2,$$

$$(m^2_{K^+}-m^2_{K^0})_2=-0.000\,430\ \text{GeV}^2,$$

$$(m^2_{K^+}-m^2_{K^0})_3=0.000\,571\ \text{GeV}^2,$$

$$(m^2_{K^+}-m^2_{K^0})_4=0.000\,139\ \text{GeV}^2.$$

Totally, we have

$$(\Delta m^2_K)_{\text{EM}}=(m^2_{K^+}-m^2_{K^0})_{\text{EM}}=0.002\,473\ \text{GeV}^2 =2m_K\times 2.5\ \text{MeV}. \tag{80}$$

Then the correction to Dashen's theorem beyond the chiral limit is

$$\rho_{\text{EM}}=\frac{(m^2_{K^+}-m^2_{K^0})_{\text{EM}}}{(m^2_{\pi^+}-m^2_{\pi^0})_{\text{EM}}}=1.68,$$

$$(\Delta m^2_K)_{\text{EM}}-(\Delta m^2_\pi)_{\text{EM}}=1.08\times 10^{-3}\ \ \text{GeV}^2. \tag{81}$$

The results show a rather large violation of Dashen's theorem, which is in correspondence with the one by Donoghue *et al.* [5] and Bijnens *et al.* [6,29].

It has been known that the mass difference between $K^+$ and $K^0$ receives the contributions from both electromagnetic self-energy and mass difference of $m_u$ and $m_d$, i.e.,

$$(m^2_{K^+}-m^2_{K^0})_{\text{expt}}=(m^2_{K^+}-m^2_{K^0})_{\text{EM}}+(m^2_{K^+}-m^2_{K^0})_{\text{QM}}. \tag{82}$$

Employing the value of $(m^2_{K^+}-m^2_{K^0})_{\text{EM}}$ and experimental data of the mass difference between $K^+$ and $K^0$ [22], we obtain

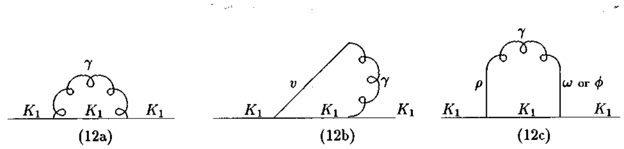


FIG. 12. Same as Fig. 11.

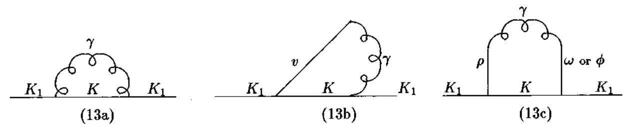


FIG. 13. Same as Fig. 11.

$$(m^2_{K^+}-m^2_{K^0})_{\text{QM}}=-0.006\,346\ \ \text{GeV}^2=-2m_K\times 6.4\ \ \text{MeV}. \tag{83}$$

The use of the result of $(m^2_{K^+}-m^2_{K^0})_{\text{QM}}$ together with Eq. (17) will yield mass ratios of light quarks

$$\frac{m_u+m_d}{m_s+\hat{m}}=\frac{f_\pi^2 m_\pi^2}{f_k^2 m_K^2}=0.070,$$

$$\frac{m_d-m_u}{m_s-\hat{m}}=\frac{f_k^2(m^2_{K^0}-m^2_{K^+})_{\text{QM}}}{f_k^2 m_K^2-f_\pi^2 m_\pi^2}=0.028,$$

where $\hat{m}=(m_u+m_d)/2$. These above results can be translated into

$$\frac{m_d}{m_s}=0.050,\quad \frac{m_d-m_u}{m_s}=0.027,\quad \frac{m_u}{m_d}=0.44.$$

The results are in agreement with the data of light quark mass ratios [19]. Similar results have been given recently by Bijnens *et al.* [30], Leutwyler [31], and Duncan *et al.* [32]. The value of $m_u/m_d=0.44$ reflects the breaking of isospin symmetry in the present theory.

Finally, using the value of $m_s=175\pm 16$ MeV, which is obtained with QCD sum rules [33] in the modified minimal subtraction ($\overline{\text{MS}}$) scheme at scale $\mu=1$ GeV, we can calculate $m_u$ and $m_d$ with the above mass ratios. The result reads

$$m_u(1\ \ \text{GeV}^2)=3.8\pm 0.3\ \ \text{MeV},$$

$$m_d(1\ \ \text{GeV}^2)=8.7\pm 0.8\ \ \text{MeV}.$$

## V. $K_1^+ - K_1^0$ ELECTROMAGNETIC MASS DIFFERENCE

The Lagrangians $\mathcal{L}_{K_1K_1vv}$, $\mathcal{L}_{K_1K_1v}$, and $\mathcal{L}_{K_1Kv}$, which contribute to the electromagnetic self-energies of the $K_1$ meson, are

$$\mathcal{L}_{K_1K_1vv}=-\frac{2}{g^2}\Big[\rho^3_\mu v^{8\mu}(K^+_{1\nu}K_1^{-\nu}-K^0_{1\nu}\overline{K_1^{0\nu}}) -\frac{\gamma^2}{2}\rho^3_\mu v^8_\nu(K_1^{+\mu}K_1^{-\nu}-K_1^{0\mu}\overline{K_1^{0\nu}}+\text{H.c.})\Big], \tag{84}$$

$$\mathcal{L}_{K_1K_1v}=\frac{i}{g}\left(1-\frac{\gamma^2}{\pi^2g^2}\right)[\partial^\nu\rho^3_\mu(K_1^{+\mu}K^-_{1\nu}-K_1^{0\mu}\overline{K^0_{1\nu}}) +\partial^\nu v^8_\mu(K_1^{+\mu}K^-_{1\nu}+K_1^{0\mu}\overline{K^0_{1\nu}})]-\frac{i}{g}\rho^3_\nu[K_1^{+\mu}(\partial^\nu K^-_{1\mu} -\gamma^2\partial_\mu K_1^{-\nu})-K_1^{0\mu}(\partial^\nu\overline{K^0_{1\mu}}-\gamma^2\partial_\mu\overline{K_1^{0\nu}})] -\frac{i}{g}v^8_\nu[K_1^{+\mu}(\partial^\nu K^-_{1\mu}-\gamma^2\partial_\mu K_1^{-\nu})+K_1^{0\mu}(\partial^\nu\overline{K^0_{1\mu}} -\gamma^2\partial_\mu\overline{K_1^{0\nu}})]+\text{H.c.}, \tag{85}$$

$$\mathcal{L}_{KK_1v}=\frac{i}{g}\beta_1[\rho^3_\mu(K^+K_1^{-\mu}-K^0\overline{K_1^{0\mu}})+v^8_\mu(K^+K_1^{-\mu} +K^0\overline{K_1^{0\mu}})]+\frac{i}{g}\beta_2[\rho^3_\nu(\partial^{\mu\nu}K^+K^-_{1\mu}-\partial^{\mu\nu}K^0\overline{K^0_{1\mu}}) +v^8_\nu(\partial^{\mu\nu}K^+K^-_{1\mu}+\partial^{\mu\nu}K^0\overline{K^0_{1\mu}})] +\frac{i}{g}\beta_3[\rho^3_\mu(K_1^{+\mu}\partial^2K^- -K_1^{0\mu}\partial^2\overline{K^0})+v^8_\mu(K_1^{+\mu}\partial^2K^- +K_1^{0\mu}\partial^2\overline{K^0})]-\frac{i}{g}\beta_4[\rho^3_\mu(K^+\partial^2K_1^{-\mu}-K^0\partial^2\overline{K_1^{0\mu}}) +v^8_\mu(K^+\partial^2K_1^{-\mu}+K^0\partial^2\overline{K_1^{0\mu}})] -\frac{i}{g}\beta_5[\rho^3_\mu(\partial_\nu K^+_{1\mu}\partial^\nu K^- -\partial_\nu K^0_{1\mu}\partial^\nu\overline{K^0}) +v^8_\mu(\partial_\nu K^+_{1\mu}\partial^\nu K^- +\partial_\nu K^0_{1\mu}\partial^\nu\overline{K^0})]+\text{H.c.} \tag{86}$$

The photon-meson interaction Lagrangians can be obtained by combining the above Lagrangians with the substitutions (9)–(11) and the corresponding Feynman diagrams have been shown in Figs. 11–13. The examination of gauge independence can be done in the same way as in the previous sections.

From Fig. 11 we have

$$(m^2_{K_1^+}-m^2_{K_1^0})_1=ie^2\,\frac{\gamma^2\langle K_1|\int d^4x\ K_1^{\mu+}K_1^{\nu-}|K_1\rangle-\langle K_1|\int d^4x\ K_1^{\lambda+}K^-_{1\lambda}|K_1\rangle g^{\mu\nu}}{\langle K_1|\int d^4xK^+_{1\mu}K_1^{\mu-}|K_1\rangle} \times\int\frac{d^4k}{(2\pi)^4}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\left[\frac{1}{3}\,\frac{m^2_\rho m^2_\omega}{k^2(k^2-m^2_\rho)(k^2-m^2_\omega)}+\frac{2}{3}\,\frac{m^2_\rho m^2_\phi}{k^2(k^2-m^2_\rho)(k^2-m^2_\phi)}\right]. \tag{87}$$

From Fig. 12 we obtain

$$(m^2_{K_1^+}-m^2_{K_1^0})_2=\frac{ie^2}{\langle K_1|\int d^4xK_1^{\mu+}K^-_{1\mu}|K_1\rangle}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-2p\cdot k}\left\{\left\langle K_1\left|\int d^4xK_1^{\mu+}K^-_{1\mu}\right|K_1\right\rangle \times\left[4m^2_{K_1}+(b^2+2b\gamma^2)k^2+2\gamma^4p\cdot k-\frac{4(p\cdot k)^2}{k^2}-\frac{1}{m^2_{K_1}}[bk^2-(b-\gamma^2)p\cdot k]^2\right] +\left\langle K_1\left|\int d^4xK^+_{1\mu}K^-_{1\nu}\right|K_1\right\rangle k^\mu k^\nu\left[-(3b^2-4b+4)+D(b+\gamma^2)^2+4\gamma^2-6b\gamma^2-2\gamma^4-\frac{2\gamma^4p\cdot k}{k^2} +\frac{1}{m^2_{K_1}k^2}[bk^2-2(1-\gamma^2)p\cdot k]^2\right]\right\}\left[\frac{1}{3}\,\frac{m^2_\rho m^2_\omega}{k^2(k^2-m^2_\rho)(k^2-m^2_\omega)}+\frac{2}{3}\,\frac{m^2_\rho m^2_\phi}{k^2(k^2-m^2_\rho)(k^2-m^2_\phi)}\right]. \tag{88}$$

From Fig. 13 we get

$$(m^2_{K_1^+}-m^2_{K_1^0})_3=\frac{-ie^2}{\langle K_1|\int d^4x\ K^+_{1\mu}K_1^{\mu-}|K_1\rangle}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(p-k)^2-m^2_K}\left\{\left\langle K_1\left|\int d^4x\ K^+_{1\mu}K_1^{\mu-}\right|K_1\right\rangle \times(\beta_1'-3\beta_2p\cdot k+\beta_3k^2)^2+\left\langle K_1\left|\int d^4x\ K^+_{1\mu}K^-_{1\nu}\right|K_1\right\rangle k^\mu k^\nu\left[\beta_2m^2_{K_1}-\frac{(\beta_1'-2\beta_2p\cdot k+\beta_3k^2)^2}{k^2}\right]\right\} \times\left[\frac{1}{3}\,\frac{m^2_\rho m^2_\omega}{k^2(k^2-m^2_\rho)(k^2-m^2_\omega)}+\frac{2}{3}\,\frac{m^2_\rho m^2_\phi}{k^2(k^2-m^2_\rho)(k^2-m^2_\phi)}\right], \tag{89}$$

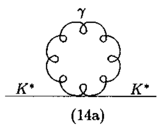

FIG. 14. One-loop Feynman diagrams contributing to the electromagnetic mass difference between $K^{*+}$ and $K^{*0}$. The curly line is the photon line and $v$ denotes neutral vector mesons $\rho$, $\omega$, and $\phi$.

with

$$\beta_1'=\beta_1+(\beta_3+\beta_4-\beta_5)m_{K_1}^2.$$

Comparing Eqs. (87)–(89) and Eqs. (49), (51), and (54) and taking $f_k=f_\pi$, $m_K^2=m_\pi^2=0$, $m_\rho=m_\omega=m_\phi$, and $m_a=m_{K_1}$, we can conclude that

$$(m_{a^+}^2-m_{a^0}^2)_i=(m_{K_1^+}^2-m_{K_1^0}^2)_i, \quad i=1,2,3.$$

This means that the square mass difference coming from the electromagnetic interaction between the charged axial-vector mesons and their corresponding neural partners are equal in the chiral SU(3) limit, i.e.,

$$(m_{a^+}^2-m_{a^0}^2)_{\rm EM}=(m_{K_1^+}^2-m_{K_1^0}^2)_{\rm EM}, \tag{90}$$

which is similar to Dashen's theorem for the pseudoscalar $\pi$ and $K$ mesons. Certainly, the SU(3) symmetry breaking will bring about the violation of the above equation.

After carrying out the Feynman integrations of Eqs. (87)–(89), the numerical results for $m_{K_1^+}^2-m_{K_1^0}^2$ are

$$(m_{K_1^+}^2-m_{K_1^0}^2)_1=-0.000\,781 \ \ {\rm GeV}^2,$$

$$(m_{K_1^+}^2-m_{K_1^0}^2)_2=-0.003\,474 \ \ {\rm GeV}^2,$$

$$(m_{K_1^+}^2-m_{K_1^0}^2)_3=0.001\,252 \ \ {\rm GeV}^2.$$

Thus the correction of the electromagnetic mass to $K_1(1400)$ mesons is

$$(m_{K_1^+}^2-m_{K_1^0}^2)_{\rm EM}=-0.003\,003 \ \ {\rm GeV}^2=-2m_{K_1}\times 1.1 \ \ {\rm MeV}. \tag{91}$$

This result gives a very large violation of Eq. (90):

$$\frac{(m_{K_1^+}^2-m_{K_1^0}^2)_{\rm EM}}{(m_{a^+}^2-m_{a^0}^2)_{\rm EM}}=2.08. \tag{92}$$

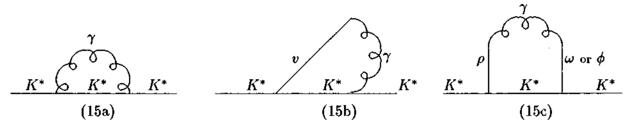

FIG. 15. Same as Fig. 14.

### VI. $K^{*+}-K^{*0}$ ELECTROMAGNETIC MASS DIFFERENCE

The Lagrangians contributing to $m_{K^{*+}}-m_{K^{*0}}$ come from both the normal part of the effective Lagrangian $\mathcal{L}_{\rm Re}$ and the abnormal part $\mathcal{L}_{\rm Im}$. $\mathcal{L}_{KK^*v}$ deriving from $\mathcal{L}_{\rm Im}$ is exactly Eq. (66):

$$\mathcal{L}_{K^*K^*vv}=-\frac{2}{g^2}\rho_\mu^3 v^{8\mu}(K_\nu^+K^{-\nu}-K_\nu^0\overline{K^{0\nu}}) +\frac{1}{g^2}\rho_\mu^3 v_\nu^8(K^{+\mu}K^{-\nu}-K^{0\mu}\overline{K^{0\nu}}+{\rm H.c.}), \tag{93}$$

$$\mathcal{L}_{K^*K^*v}=\frac{i}{g}[\partial_\nu\rho_\mu^3(K^{+\mu}K^{-\nu}-K^{0\mu}\overline{K^{0\nu}})+\partial_\nu v_\mu^8(K^{+\mu}K^{-\nu} +K^{0\mu}\overline{K^{0\nu}})]-\frac{i}{g}\rho_\nu^3[K_\mu^+(\partial^\nu K^{-\mu}-\partial^\mu K^{-\nu}) -K_\mu^0(\partial^\nu\overline{K^{0\mu}}-\partial^\mu\overline{K^{0\nu}})]-\frac{i}{g}v_\nu^8[K_\mu^+(\partial^\nu K^{-\mu} -\partial^\mu K^{-\nu})+K_\mu^0(\partial^\nu\overline{K^{0\mu}}-\partial^\mu\overline{K^{0\nu}})]+{\rm H.c.} \tag{94}$$

Substitutions (9)–(11) together with Eqs. (66), (93), and (94) will produce the photon–$K^*$-meson interaction Lagrangians $\mathcal{L}_{K^*K^*\gamma\gamma}$, $\mathcal{L}_{K^*K^*\gamma}$, and $\mathcal{L}_{K^*K\gamma}$. The one-loop Feynman diagrams contributing to electromagnetic mass splitting of $K^{*+}$ and $K^{*0}$ are shown in Figs. 14–16. The gauge-dependent terms from Figs. 14(a) and 15(a) will vanish in the framework of dimensional regularization and one from Fig. 16 will also vanish due to the totally antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$ in Eq. (66). It is straightforward to evaluate the contributions to $m_{K^{*+}}^2-m_{K^{*0}}^2$ from Figs. 14–16 one by one.

The contribution of Fig. 14 is

$$(m_{K^{*+}}^2-m_{K^{*0}}^2)_1=-\frac{i9e^2}{4}\int\frac{d^4q}{(2\pi)^4}\times\left[\frac{1}{3}\frac{m_\rho^2m_\omega^2}{q^2(q^2-m_\rho^2)(q^2-m_\omega^2)}+\frac{2}{3}\frac{2m_\rho^2m_\phi^2}{q^2(q^2-m_\rho^2)(q^2-m_\phi^2)}\right]. \tag{95}$$

The contribution of Fig. 15 is

$$\begin{aligned}(m^2_{K^{*+}}-m^2_{K^{*0}})_2 ={}& \frac{ie^2}{\langle K^*|\int d^4x\, K^+_\mu K^{-\mu}|K^*\rangle}\int \frac{d^4q}{(2\pi)^4}\Bigg\{\Big\langle K^*\Big|\int d^4x\, K^+_\mu K^{\mu-}\Big|K^*\Big\rangle\Bigg[k^2-\frac{(k^2)^2}{m^2_{K^*}}+4p^2+4q^2+\frac{4(p\cdot q)^2}{m^2_{K^*}}\\ &-\frac{4q^2 p\cdot q}{m^2_{K^*}}-\frac{4(p\cdot q)^2}{q^2}\Bigg]+\Big\langle K\Big|\int d^4x\, K^+_\mu K^-_\nu\Big|K^*\Big\rangle q^\mu q^\nu\Bigg[8+\frac{4p\cdot q}{m^2_{K^*}}-\frac{4(p\cdot q)^2}{q^2 m^2_{K^*}}-\frac{k^2}{q^2}\\ &+\frac{(k^2)^2}{q^2m^2_{K^*}}\Bigg]\Bigg\}\frac{1}{k^2-m^2_{K^*}}\Bigg[\frac{1}{3}\frac{m^2_\rho m^2_\omega}{q^2(q^2-m^2_\rho)(q^2-m^2_\omega)}+\frac{2}{3}\frac{m^2_\rho m^2_\phi}{q^2(q^2-m^2_\rho)(q^2-m^2_\phi)}\Bigg],\end{aligned} \tag{96}$$

where $p$ is the external momentum of $K^*$ mesons and $k=p-q$. For mass-shell $K^*$ mesons, $p^2=m^2_{K^*}$ and $p^\mu K_\mu(p)=0$. Here $K_\mu(p)$ is the Fourier transformation of the $K^*$-mesons field

$$K_\mu(p)=\frac{1}{(2\pi)^4}\int d^4x\, K_\mu(x)e^{-ipx}.$$

The contribution of Fig. 16 is

$$\begin{aligned}(m^2_{K^{*+}}-m^2_{K^{*0}})_3 ={}& \frac{ie^2}{\langle K^*|\int d^4x\, K^+_\mu K^{-\mu}|K^*\rangle}\frac{9}{4\pi^4 g^2 f_k^2}\int\frac{d^4q}{(2\pi)^4}\Bigg\{\Big\langle K^*\Big|\int d^4x\, K^+_\mu K^{-\mu}\Big|K^*\Big\rangle[p^2q^2-(p\cdot q)^2]\\ &-\Big\langle K^*\Big|\int d^4x\, K^+_\mu K^-_\nu\Big|K^*\Big\rangle q^\mu q^\nu p^2\Bigg\}\frac{1}{k^2-m^2_K}\Bigg[\frac{1}{3}\frac{m^2_\rho m^2_\omega}{q^2(q^2-m^2_\rho)(q^2-m^2_\omega)}-\frac{2}{3}\frac{m^2_\rho m^2_\phi}{q^2(q^2-m^2_\rho)(q^2-m^2_\phi)}\Bigg].\end{aligned} \tag{97}$$

The Feynman integrations of $(m^2_{K^{*+}}-m^2_{K^{*0}})_{1,3}$ are finite; only the logarithmic divergence emerges in $(m^2_{K^{*+}}-m^2_{K^{*0}})_2$, which can be factorized by using Eq. (37). Performing these Feynman integrations is standard. The numerical results are

$$(m^2_{K^{*+}}-m^2_{K^{*0}})_1=-0.000\,938\ \text{GeV}^2,$$

$$(m^2_{K^{*+}}-m^2_{K^{*0}})_2=-0.001\,547\ \text{GeV}^2,$$

$$(m^2_{K^{*+}}-m^2_{K^{*0}})_3=-0.000\,662\ \text{GeV}^2.$$

The electromagnetic mass correction to a total of $K^*(892)$ mesons is

$$\begin{aligned}(m^2_{K^{*+}}-m^2_{K^{*0}})_{\text{EM}}&=-0.003\,147\ \text{GeV}^2\\ &=-2m_{K^*}\times 1.76\ \text{MeV}.\end{aligned} \tag{98}$$

However, the mass difference between $K^{*+}$ and $K^{*0}$ receives contributions not only from the virtual photon exchange, but also from the other nonelectromagnetic interactions, such as isospin symmetry breaking, which is similar to the case of pseudoscalar $K$ mesons. So we have

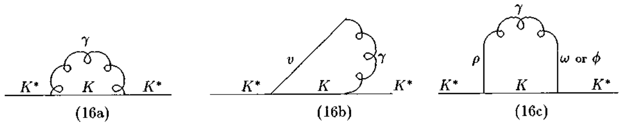


FIG. 16. Same as Fig. 14.

$$\begin{aligned}(m_{K^{*+}}-m_{K^{*0}})_{\text{expt}}={}&(m_{K^{*+}}-m_{K^{*0}})_{\text{EM}}\\ &+(m_{K^{*+}}-m_{K^{*0}})_{\text{nonEM}}.\end{aligned} \tag{99}$$

Using the experimental value of $(m_{K^{*+}}-m_{K^{*0}})_{\text{expt}}=-6.7\pm 1.2$ MeV [22], we obtain

$$(m_{K^{*+}}-m_{K^{*0}})_{\text{nonEM}}=-4.94\pm 1.2\ \text{MeV}. \tag{100}$$

The nonelectromagnetic mass difference of $(m_{K^{*+}}-m_{K^{*0}})_{\text{nonEM}}$, which comes from isospin breaking effects, has been evaluated [27,34]. In Ref. [27], Schechter *et al.* predicted that $(m_{K^{*+}}-m_{K^{*0}})_{\text{nonEM}}$ would be from $-2.04$ MeV to $-6.78$ MeV. By choosing the best-fit parameter, they concluded that $(m_{K^{*+}}-m_{K^{*0}})_{\text{nonEM}}=-4.47$ MeV, which is close to our result of Eq. (100).

## VII. SUMMARY AND DISCUSSION

In the framework of the present theory, the dynamics of meson fields comes from the quark-loop integrations within mesonic background fields. The logarithmic divergence due to the quark-loop integrations is absorbed into the parameter $g$ [Eq. (3)] in this truncated field theory. Thus both meson's effective Lagrangians with VMD and criteria to factorize the logarithmic divergences in the loop calculations are well established. In this paper, by using this theory, we have computed all one-loop diagrams contributing to the electromagnetic mass splitting of the low-lying mesons including pseudoscalar mesons $\pi$ and $K$, axial-vector mesons $a_1$ and $K_1(1400)$, and vector meson $K^*(892)$. Fortunately, no other higher-order divergences but the logarithmic divergences emerge in the Feynman integrations of the above loop diagrams. Therefore, it is reasonable to factorize these logarithmic divergences by using the intrinsic parameter $g$ in this theory, which is determined by the experimental values of $f_\pi$, $m_\rho$, and $m_{a_1}$. Then it is unnecessary to introduce other additional parameters or counterterms into this theory to absorb the mesonic loop divergences. The dimensional regularization has been employed and the gauge independence of the calculations is examined.

The electromagnetic mass splittings of $\pi$ and $a_1$ are calculated in the chiral limit because of the smallness of $u$ and $d$ quark masses, and the result of $m_{\pi^+}-m_{\pi^0}$ is close to the experimental data. However, the electromagnetic mass splittings of the strange-flavor mesons $K$, $K_1$, and $K^*$ have been evaluated to the order of $m_s$ or $m_K^2$ because of the large strange quark mass. Thus a rather large violation of Dashen's theorem [which holds in the chiral SU(3) limit of the present theory] has been revealed at leading order in the quark mass expansion. The mass ratios of light quarks have been calculated and masses of $u,d$ quarks have been estimated by employing the value of $m_s$ obtained with QCD sum rules. It has been found that there exists a different relation for axial-vector mesons, i.e., $(m^2_{a^+}-m^2_{a^0})_{\rm EM}=(m^2_{K_1^+}-m^2_{K_1^0})_{\rm EM}$ is obeyed in the chiral SU(3) limit. Moreover, the nonelectromagnetic mass difference between $K^{*+}$ and $K^{*0}$ is estimated by using the experimental value of $m_{K^{*+}}-m_{K^{*0}}$ with $(m_{K^{*+}}-m_{K^{*0}})_{\rm EM}$ calculated in this paper.

The electromagnetic self-energies of the other low-lying mesons, such as vector mesons $\rho$,$\omega$,$\phi$(1020) and pseudoscalar mesons $\eta,\eta'(960)$, also need to be evaluated. However, the quadratic or higher-order divergences will emerge in the Feynman integrations of the loop calculations of $\rho$, $\omega$, and $\phi$. It is unsuitable to factorize these higher-order divergences by the parameter $g$ in which only the logarithmic divergence is involved. As for $\eta$ and $\eta'$, the U(1) anomaly problem and the mixing of $\eta$ and $\eta'$ should be taken into account. The investigation on these problems is beyond the scope of the present work.

## ACKNOWLEDGMENTS

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## APPENDIX A: FEYNMAN RULES AND THE PHOTON PROPAGATOR WITHIN $\rho$

The propagators taken in this paper are as follows: the pseudoscalar-meson fields

$$\langle 0|T\phi(x)\phi(y)|0\rangle=\int\frac{d^4k}{(2\pi)^4}\,\Delta_F(k)e^{-ik(x-y)},$$

$$\Delta_F(k)=\frac{i}{k^2-m^2+i\epsilon} \tag{A1}$$

and the vector-meson fields

$$\langle 0|T(V^i_\mu(x)V^j_\nu(y)|0\rangle=\int\frac{d^4k}{(2\pi)^4}\,\delta_{ij}\Delta_{F\mu\nu}(k)e^{-ik(x-y)},$$

$$\Delta_{F\mu\nu}(k)=\frac{-i}{k^2-m_V^2+i\epsilon}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{m_V^2}\right), \tag{A2}$$

where $V^i_\mu(x)=a^i_\mu(x)$, $\rho^i_\mu(x)$, $\omega_\mu(x)$, $\phi_\mu(x)$, $K_{1\mu}(x)$, and $K_\mu(x)$.

For the photon propagator within $\rho$, from Eqs. (22) and (12) we have

$$\begin{aligned}\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)=&\bigg\langle 0\bigg|T\bigg\{A_\mu(x)A_\nu(y)-2i\int d^4x_1A_\mu(x)\rho^3_\nu(y)\\&\times\{\partial_\lambda\rho^3_\sigma(x_1)[\partial^\lambda A^\sigma(x_1)-\partial^\sigma A^\lambda(x_1)]\}\\&-\frac{1}{2}\int d^4x_1d^4x_2\rho^3_\mu(x)\rho^3_\nu(y)\{\partial_\lambda\rho^3_\sigma(x_1)\\&\times[\partial^\lambda A^\sigma(x_1)-\partial^\sigma A^\lambda(x_1)]\}\{\partial_\alpha\rho^3_\beta(x_2)\\&\times[\partial^\alpha A^\beta(x_2)-\partial^\alpha A^\beta(x_2)]\}\bigg\}\bigg|0\bigg\rangle.\end{aligned} \tag{A3}$$

Using Eqs. (22) and (A2), we get

$$\begin{aligned}&\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)\\&=\int\frac{d^4k}{(2\pi)^4}(-i)\frac{1}{k^2}e^{-ik(x-y)}\bigg\{a\frac{k_\mu k_\nu}{k^2}+\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\\&\quad\times\left[1-\frac{2k^2}{k^2-m_\rho^2}+\frac{k^4}{(k^2-m_\rho^2)^2}\right]\bigg\}.\end{aligned}$$

Then

$$\begin{aligned}\Delta_{F^{(\gamma\rho)}_{\mu\nu}}(x-y)=&\int\frac{d^4k}{(2\pi)^4}(-i)\frac{1}{k^2}\bigg[\frac{m_\rho^4}{(k^2-m_\rho^2)^2}\\&\times\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^2}\bigg]e^{-ik(x-y)}.\end{aligned} \tag{A4}$$

This is Eq. (23).

## APPENDIX B: $\Delta_{F^{(\gamma v)}_{1\mu\nu}}$ AND $\Delta_{F^{(\gamma v)}_{2\mu\nu}}$

The photon propagator within $\rho$ can be generalized to the photon propagator within $v$ including $\rho$, $\omega$, and $\phi$ to simplify the corresponding calculations of the strange-flavor mesons. In this appendix, as an example, we display the whole calculating process of Fig. 7 to deduce $\Delta_{F^{(\gamma v)}_{1\mu\nu}}$.

$\mathcal{L}_{KKvv}$ has been shown in Eq. (61). The corresponding photon-meson couplings $\mathcal{L}_{KK\gamma\gamma}$, $\mathcal{L}_{KK\rho\gamma}$, $\mathcal{L}_{KK\omega\gamma}$, and $\mathcal{L}_{KK\phi\gamma}$, which contribute to electromagnetic mass differences between $K^+$ and $K^0$, can be obtained by the substitutions (9)–(11). All the one-loop Feynman diagrams contributing to $(m^2_{K^+}-m^2_{K^0})_1$ are shown in Figs. 7(a)–7(c) and the corresponding $S$ matrices are denoted as $S_K(1)_i$, $i=a,b,c$. Thus we have

$$S_K(1)_a=i\left\langle K\left|T\int d^4x\ \mathcal{L}_{KK\gamma\gamma}(x)\right|K\right\rangle$$
$$=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle \frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{-i}{k^2}g^{\mu\nu}\left[\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+a\frac{k_\mu k_\nu}{k^2}\right] \tag{B1}$$

and

$$S_K(1)_b=S_K(1)_\rho+S_K(1)_\omega+S_K(1)_\phi,$$

with

$$S_K(1)_\rho=\frac{i^2}{2!}2\left\langle K\left|T\int d^4x\ d^4y\ \mathcal{L}_{KK\rho\gamma}(x)\mathcal{L}_{\rho\gamma}(y)\right|K\right\rangle$$
$$=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle \frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{i}{(-k^2+m_\rho^2)(-k^2)}k^2g^{\mu\nu}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right), \tag{B2}$$

$$S_K(1)_\omega=\frac{i^2}{2!}2\left\langle K\left|T\int d^4x\ d^4y\ \mathcal{L}_{KK\omega\gamma}(x)\mathcal{L}_{\omega\gamma}(y)\right|K\right\rangle$$
$$=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle \frac{1}{3}\frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{i}{(-k^2+m_\omega^2)(-k^2)}k^2g^{\mu\nu}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right), \tag{B3}$$

$$S_K(1)_\phi=\frac{i^2}{2!}2\left\langle K\left|T\int d^4x\ d^4y\mathcal{L}_{KK\phi\gamma}(x)\mathcal{L}_{\phi\gamma}(y)\right|K\right\rangle$$
$$=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle \frac{2}{3}\frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{i}{(-k^2+m_\phi^2)(-k^2)}k^2g^{\mu\nu}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right). \tag{B4}$$

and

$$S_K(1)_c=S_K(1)_{\rho\omega}+S_K(1)_{\rho\phi},$$

with

$$S_K(1)_{\rho\omega}=\frac{i^3}{3!}6\left\langle K\left|T\int d^4x\ d^4y\ d^4z\ \mathcal{L}_{KK\rho\omega}(x)\mathcal{L}_{\rho\gamma}(y)\mathcal{L}_{\omega\gamma}(z)\right|K\right\rangle=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle\frac{1}{3}\frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{i}{(-k^2+m_\rho^2)(-k^2+m_\omega^2)(-k^2)}(k^2)^2g^{\mu\nu}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right), \tag{B5}$$

$$S_K(1)_{\rho\phi}=\frac{i^3}{3!}6\left\langle K\left|T\int d^4x\ d^4y\ d^4z\ \mathcal{L}_{KK\rho\phi}(x)\mathcal{L}_{\rho\gamma}(y)\mathcal{L}_{\phi\gamma}(z)\right|K\right\rangle=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle\frac{2}{3}\frac{ie^2}{f_k^2}\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)$$
$$\times\frac{i}{(-k^2+m_\rho^2)(-k^2+m_\phi^2)(-k^2)}(k^2)^2g^{\mu\nu}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right). \tag{B6}$$

Thus the total contribution of Fig. 7 is

$$S_K(1)=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle\frac{ie^2}{f_k^2}\int\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)\frac{i}{-k^2}g^{\mu\nu}\left\{\left[\frac{1}{3}\frac{m_\rho^2m_\omega^2}{(k^2-m_\rho^2)(k^2-m_\omega^2)}\right.\right.$$
$$\left.\left.+\frac{2}{3}\frac{m_\rho^2m_\phi^2}{(k^2-m_\rho^2)(k^2-m_\phi^2)}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\right]+a\frac{k_\mu k_\nu}{k^2}\right\}$$
$$=\left\langle K\left|\int d^4x\ K^+K^-\right|K\right\rangle\frac{ie^2}{f_k^2}\int\frac{d^4k}{(2\pi)^4}\left(F_K^2+\frac{k^2}{2\pi^2}\right)g^{\mu\nu}\Delta_{F_{1\mu\nu}^{(\gamma\nu)}}(k). \tag{B7}$$

Here $\Delta_{F1\mu\nu}^{(\gamma\nu)}(k)$ is exactly Eq. (68).

A similar procedure can be easily applied to Figs. 8–10. We can conclude that Figs. 7–9, which receive contributions from the normal part of the effective Lagrangian $\mathcal{L}_{\mathrm{Re}}$, yield the same expression of $\Delta_{F_{1\mu\nu}^{(\gamma\nu)}}(k)$; however, Fig. 10, which is from the abnormal part of the effective Lagrangian $\mathcal{L}_{\mathrm{Im}}$, gives the form of $\Delta_{F_{2\mu\nu}^{(\gamma\nu)}}(k)$ [Eq. (69)]. The difference between $\Delta_{F_{1\mu\nu}^{(\gamma\nu)}}(k)$ and $\Delta_{F_{2\mu\nu}^{(\gamma\nu)}}(k)$ comes from $\omega$ and $\phi$ mesons fields always appearing as the combination $\omega_\mu-\sqrt{2}\phi_\mu$ in $\mathcal{L}_{\mathrm{Re}}$ and as the combination $\omega_\mu+\sqrt{2}\phi_\mu$ in $\mathcal{L}_{\mathrm{Im}}$ [see Eq. (66)].

[1] T. Das, G. Guralnik, V. Mathur, F. Low, and J. Young, Phys. Rev. Lett. **18**, 759 (1967).
[2] R. H. Socolow, Phys. Rev. **137**, 1221 (1965); I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, Phys. Rev. Lett. **19**, 1064 (1967); B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968); A. Zee, Phys. Rep. **3**, 127 (1972); P. Langacker and H. Pagels, Phys. Rev. D **8**, 4620 (1973); N. Deshpande, D. Dicus, K. Johnson, and V. Teplitz, *ibid.* **15**, 1885 (1977); K. Maltman and D. Kotchan, Mod. Phys. Lett. A **5**, 2457 (1990); K. Maltman, T. Goldman, and G. L. Stephenson, Jr., Phys. Lett. B **234**, 158 (1990); V. Dmitrasinovic, H.-J. Schulze, R. Tegen, and R. H. Lemmer, Phys. Rev. D **52**, 2855 (1995); J. F. Donoghue and A. F. Perez, *ibid.* **55**, 7075 (1997).
[3] R. Dashen, Phys. Rev. **183**, 1245 (1969).
[4] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982); R. D. Peccei and J. Sola, Nucl. Phys. **B281**, 1 (1987); T. N. Pham, Phys. Lett. B **374**, 205 (1996).
[5] J. F. Donoghue, B. R. Holstein, and D. Wyler, Phys. Rev. Lett. **69**, 3444 (1992); Phys. Rev. D **47**, 2089 (1993).
[6] J. Bijnens and E. de Rafael, Phys. Lett. B **273**, 483 (1991); W. A. Bardeen, J. Bijnens, and J.-M. Gerard, Phys. Rev. Lett. **62**, 1343 (1989).
[7] J. Bijnens and J. Prades, Nucl. Phys. **B490**, 239 (1997).
[8] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1995).
[9] R. Urech, Nucl. Phys. **B433**, 234 (1995); R. Baur and R. Urech, Phys. Rev. D **53**, 6552 (1996).
[10] B. A. Li, Phys. Rev. D **52**, 5165 (1995).
[11] B. A. Li, Phys. Rev. D **52**, 5184 (1995).
[12] S. Weinberg, Physica A **96**, 327 (1979); J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985); H. Leutwyler, Ann. Phys. (N.Y.) **235**, 165 (1994).
[13] B. A. Li, Phys. Rev. D **55**, 1425 (1997); **55**, 1436 (1997).
[14] B. A. Li (unpublished).
[15] J. F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev. D **39**, 1947 (1989); J. F. Donoghue and B. R. Holstein, *ibid.* **40**, 238 (1989).
[16] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).
[17] X. J. Wang and M. L. Yan (unpublished).
[18] B. A. Li and M. L. Yan (unpublished).
[19] J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969).
[20] J. J. Sakurai, Phys. Rev. Lett. **19**, 803 (1967).
[21] D. Ebert and H. Reinhardt, Nucl. Phys. **B271**, 188 (1986).
[22] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[23] D. J. Gross, S. B. Treiman, and F. Wilczek, Phys. Rev. D **19**, 2188 (1979).
[24] D. Kaplan and A. Manohar, Phys. Rev. Lett. **56**, 2004 (1986).
[25] M. Gell-mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[26] B. A. Li (unpublished).
[27] J. Schechter, A. Subbaraman, and H. Weigel, Phys. Rev. D **48**, 339 (1993).
[28] G. Leibbrandt, Rev. Mod. Phys. **47**, 849 (1975); G. 't Hooft and M. Veltman (private communication).
[29] J. Bijnens, Phys. Lett. B **306**, 343 (1993).
[30] J. Bijnens, J. Prades, and E. de Rafael, Phys. Lett. B **348**, 226 (1995).
[31] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).
[32] A. Duncan, E. Eichten, and H. Thacker, Phys. Rev. Lett. **76**, 3894 (1996).
[33] M. Jamin and M. Münz, Z. Phys. C **66**, 633 (1995); K. G. Chetyrkin, C. A. Dominguez, D. Pirjol, and K. Schicher, Phys. Rev. D **51**, 5090 (1995).
[34] J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 1992), pp. 368–370.