Quark and antiquark polarization effects in $q\bar{q}$ and $q\bar{q}g$ final states in high energy collisions of polarized electrons and positrons

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The complete description of quark and antiquark spin polarization effects in high energy spin-polarized electron positron annihilation is to first order in α_s contained in nine hadron tensors. The hadron tensors are interrelated in such a way that only four of these tensors have to be calculated; the other five are obtained by simple transformations. The general basic cross section for description of longitudinal and transverse quark and antiquark polarization and polarization correlations is obtained. We find that the quark longitudinal polarization and longitudinal polarization correlations are in general considerable both for $q\bar{q}$ and $q\bar{q}g$ final states, and that the effect of a negative longitudinal electron beam polarization, which adds to the natural polarization, enhances the quark polarization effects. Transverse quark polarizations are in general small for relativistic quarks, being proportional to m_f/E. As for longitudinal polarization enhancement may be obtained by the use of longitudinal electron beam polarization. General analytic formulas for longitudinal and transverse quark polarization effects are given, including initial electron beam polarization. Specific analytic and numerical results for bottom and top quarks are presented. [S0556-2821(97)01213-7]

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I. INTRODUCTION

Spin polarization effects in high energy quark-antiquark-gluon production in electron positron annihilation seem to gain interest in current experiments. On the one hand, it is now possible to obtain substantial electron linear polarization at the SLAC Linear Collider [1] which improves measurements of cross sections and asymmetries. On the other hand, it is to be expected that observations of final state quark, antiquark, and gluon polarizations may improve our understanding of the physical production process itself, as well as the mechanism of jet production.

Previously, the gluon linear and circular polarization have been calculated for polarized electrons and positrons in the quark mass zero approximation [2], where it was shown that gluon bremsstrahlung has a remarkably high degree of linear polarization. The degree of polarization is strongly influenced by beam polarizations effects. It has been suggested [3] that measurements of gluon polarization effects through gluon jet oblateness could constitute sensitive tests on models of jet production.

The quark-antiquark polarization-independent cross section including quark mass effects has been studied by several groups [4,5] for unpolarized beam particles and [6] for polarized electrons and/or positrons, and final correct results for cross sections and asymmetries have been established.

Longitudinal polarization of quarks produced by unpolarized electrons and positrons obtained from the $\cos\theta$-even part of the cross section has been given by Körner *et al.* [7] and by Groote *et al.* [8] who in particular discuss bottom and top quarks. In a different, recent paper [9] the corresponding transverse quark polarization is discussed. Recently, Groote *et al.* [10] have given results for the $\cos\theta$-odd parts of the cross section related to the longitudinal quark polarization. In this work they discuss electron and positron polarizations.

The possibility of detection of polarization properties of the heavy quarks, c,b, and t have been studied recently [11]. In particular, it has been shown that as a result of the rapid top quark decay $t\rightarrow bW$, its polarization is little affected by hadronization. It is predicted that the polarization of the top quark is to a high degree transported to the W boson whose polarization can be studied from observations of its $W\rightarrow l\nu$ decay mode. Through this mechanism it might be possible to study the spin structure of the electroweak interaction of the top quark. Such knowledge is not obtainable from charm or beauty quarks since the strong hadronization effects effectively might depolarize the produced polarized quarks.

In the present paper we give a complete calculation of $q\bar{q}$ and $q\bar{q}g$ final state quark-antiquark polarization effects. The calculation is presented in such a way that the structure of the polarization-dependent hadronic tensors, which is basically complicated, is made apparent and may be checked in several ways throughout the calculation. It is shown in Secs. II and III how the different hadron tensors are related, and that the nine hadronic tensors are obtained by simple transformations from four tensors which need to be calculated. The general basic formula for the complete fermion polarization-dependent cross section is given in Sec. IV. This includes electron and positron transverse and longitudinal beam polarization effects and final state quark and antiquark longitudinal and transverse polarizations effects.

In Sec. V we obtain the cross section for longitudinally polarized quarks and antiquarks. The cross section contains quark and antiquark longitudinal polarizations and polarization correlations. We have restricted our calculation to longitudinally polarized electron and positron beams, since these are more interesting experimentally. The case of transverse polarized beams my be obtained from the formulas in Sec. IV when necessary. The explicit formulas for final state $q\bar{q}$ and $q\bar{q}g$ quark-antiquark polarization effects are given in Sec. VI. We also give the $q\bar{q}$ cross section at the Z_0 resonance to show how the Z_0 polarization for a longitudinally polarized electron beam couples to the quark and antiquark polarizations and polarization correlations. For $q\bar{q}g$ final states we define polarization-dependent form factors which are defined similarly to the form factors used in previous papers [6]. The longitudinal quark polarization $P_\parallel$ and polarization correlation $C_\parallel$ are defined in Sec. VII. Specific formulas for $P_\parallel$ and $C_\parallel$ are given for $q\bar{q}$ and $q\bar{q}g$ final states and numerical results are obtained for b and t quarks. In Sec. VIII transverse polarization of quarks is discussed and results for $q\bar{q}$ and $q\bar{q}g$ final states are given. In particular, we show that beam polarization may affect the magnitude of the transverse quark polarization considerably. Specifically, we show that a longitudinal polarization of the electron beam may under certain circumstances be transferred to a transverse polarization of the final state quark. Specific formulas are given and numerical results are obtained.

II. THE QUARK AND ANTIQUARK POLARIZATIONS

The cross section for flavor f, differential in angles and scaled quark and antiquark energies x and $\bar{x}$, respectively, is given by [12]

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega d\chi dx d\bar{x}} = \frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\{L_{\gamma\gamma}^{\mu\nu}H_{\gamma\gamma\mu\nu}^f + 2\mathrm{Re} f(s) L_{\gamma Z}^{\mu\nu}H_{\gamma Z\mu\nu}^f + |f(s)|^2 L_{ZZ}^{\mu\nu}H_{ZZ\mu\nu}^f\}, \tag{2.1}$$

where χ is the azimuthal angle of the electron momentum $\boldsymbol{p}_-$ in the coordinate system with the z axis along the quark momentum $\boldsymbol{q}$. $L_{\gamma\gamma}^{\mu\nu}, L_{\gamma Z}^{\mu\nu}$, and $L_{ZZ}^{\mu\nu}$ are the lepton (e_+, e_-) tensors and $H_{\gamma\gamma\mu\nu}^f, H_{\gamma Z\mu\nu}^f$, and $H_{ZZ\mu\nu}^f$ the corresponding hadron tensors for photon interaction, interference of photon and Z_0 interaction, and Z_0 interaction, respectively. We include the effects of electron and positron longitudinal and transverse polarization. The lepton tensors are given by

$$L_{\gamma\gamma}^{\mu\nu} = \Xi L_1^{\mu\nu} + \xi L_2^{\mu\nu} + L_3^{\mu\nu},$$

$$L_{\gamma Z}^{\mu\nu} = -(v\Xi - a\xi)L_1^{\mu\nu} - (v\xi - a\Xi)L_2^{\mu\nu} + vL_3^{\mu\nu} + aL_4^{\mu\nu}, \tag{2.2}$$

$$L_{ZZ}^{\mu\nu} = [(v^2+a^2)\Xi - 2va\xi]L_1^{\mu\nu} + [(v^2+a^2)\xi - 2va\Xi]L_2^{\mu\nu} - (v^2-a^2)L_3^{\mu\nu},$$

which include longitudinal polarization $P_\pm^\parallel$, effects $\Xi = 1 - P_+^\parallel P_-^\parallel$ and $\xi = P_-^\parallel - P_+^\parallel$. Transverse polarization effects are contained in $L_3^{\mu\nu}$ and $L_4^{\mu\nu}$. The lepton tensors are [12]

$$L_1^{\mu\nu} = 4(p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu}p_+p_-),$$

$$L_2^{\mu\nu} = -4i\varepsilon^{\mu\nu}_{\alpha\beta}p_+^\alpha p_-^\beta, \tag{2.3}$$

$$L_3^{\mu\nu} = 4(p_+p_-)(P_+^{\perp\mu}P_-^{\perp\nu} + P_-^{\perp\mu}P_+^{\perp\nu}) + (\boldsymbol{P}_+^\perp \boldsymbol{P}_-^\perp)L_1^{\mu\nu},$$

$$L_4^{\mu\nu} = 4i\varepsilon_{\alpha\beta\gamma\delta}[P_+^{\perp\alpha}p_+^\beta g^{\gamma\mu}(P_-^{\perp\delta}p_-^\nu - P_-^{\perp\nu}p_-^\delta) - (p_+, P_+^\perp \Leftrightarrow p_-, P_-^\perp)].$$

The hadronic tensors are

$$H_{\gamma\gamma\mu\nu}^f = \sum_{\mathrm{colors}, S, \bar{S}, e} H_{\mu\gamma}^f \overline{H_{\nu\gamma}^f} = (2s)^{-1}Q_f^2 H_{V\mu\nu}^f,$$

$$H_{\gamma Z\mu\nu}^f = \sum H_{\mu\gamma}^f\overline{H_{\nu Z}^f} = (2s)^{-1}Q_f[v_f H_{V\mu\nu}^f - a_f H_{A\mu\nu}^f], \tag{2.4}$$

$$H_{ZZ\mu\nu}^f = \sum H_{\mu Z}^f\overline{H_{\nu Z}^f} = (2s)^{-1}[(v_f^2 + a_f^2)H_{V\mu\nu}^f - 2a_f v_f H_{A\mu\nu}^f + 2a_f^2 m_f^2 H_{V\mu\nu}^{Zf}].$$

In order to simplify the presentation we define the term

$$\tilde{H}_{V\mu\nu}^f = H_{V\mu\nu}^f + H_{V\mu\nu}^{Zf}. \tag{2.5}$$

We extend the hadron tensors to include quark and antiquark polarizations

$$H_{V\mu\nu}^f = H_{V\mu\nu}^{0,f} + H_{V\mu\nu}^{S,f} + H_{V\mu\nu}^{\bar{S},f} + H_{V\mu\nu}^{S\bar{S},f}, \tag{2.6}$$

and similarly for $H_{A\mu\nu}^f$ and $H_{V\mu\nu}^{Zf}$. The hadron tensors are obtained in Appendix A:

$$\tilde{H}_{V\mu\nu}^{0,f} = -\tfrac{1}{4}\mathrm{Tr}\not{q}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha},$$

$$\tilde{H}_{V\mu\nu}^{S,f} = -\frac{m_f}{4}\mathrm{Tr}\gamma_5\not{S}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha},$$

$$\tilde{H}_{V\mu\nu}^{S\bar{S},f} = \frac{m_f^2}{4}\mathrm{Tr}\not{S}M_{\mu\alpha}\overline{\not{S}M_\nu^\alpha},$$

$$H_{A\mu\nu}^{0,f} = -\tfrac{1}{4}\mathrm{Tr}\gamma_5\not{q}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha},$$

$$H_{A\mu\nu}^{S,f} = -\frac{m_f}{4}\mathrm{Tr}\not{S}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha},$$

$$H_{A\mu\nu}^{S\bar{S},f} = \frac{m_f^2}{4}\mathrm{Tr}\gamma_5\not{S}M_{\mu\alpha}\overline{\not{S}M_\nu^\alpha}, \tag{2.7}$$

$$m_f^2 H_{V\mu\nu}^{0,Zf} = -\frac{m_f^2}{4}\mathrm{Tr}M_{\mu\alpha}\overline{M_\nu^\alpha},$$

$$m_f^2 H_{V\mu\nu}^{S,Zf} = -\frac{m_f}{4}\mathrm{Tr}\gamma_5\not{S}\not{q}M_{\mu\alpha}\overline{M_\nu^\alpha},$$

$$m_f^2 H_{V\mu\nu}^{S\bar{S},Zf} = -\tfrac{1}{4}\mathrm{Tr}\not{S}\not{q}M_{\mu\alpha}\overline{\not{S}\not{q}M_\nu^\alpha},$$

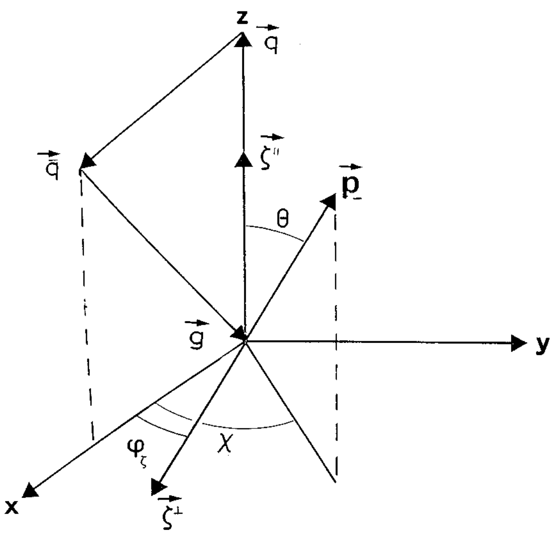

FIG. 1. Production of $q\overline{q}g$ final states (and $q\overline{q}$ final states for $g=0$). The production ($\boldsymbol{q}$-$\overline{\boldsymbol{q}}$-$\boldsymbol{g}$) plane is in the x-z plane, and the electron momentum $\boldsymbol{p}_-$ and linear polarization $\boldsymbol{P}_{el}$ are described by the polar angle θ and the azimuth angle χ. The unit longitudinal quark polarization vector $\boldsymbol{\zeta}^{\parallel}$ and transverse polarization vector $\boldsymbol{\zeta}^{\perp}$ give the directions of the respective quark polarizations.

and the corresponding $\widetilde{H}^{\overline{S},f}_{V\mu\nu}, H^{\overline{S},f}_{A\mu\nu}$, and $H^{\overline{S},Zf}_{V\mu\nu}$ tensors are obtained by the transformation $(q\leftrightarrow\overline{q}, S\leftrightarrow\overline{S}, \mu\leftrightarrow\nu, m_f\to -m_f)$. Here S_μ and $\overline{S}_\mu$ are the polarization four-vectors for the quark and antiquark, respectively, and the reduced matrix element $M_{\mu\alpha}$ is given by

$$M_{\mu\alpha}=W_\alpha\gamma_\mu+\frac{1}{2qg}\gamma_\alpha \not{g}\gamma_\mu-\frac{1}{2\overline{q}g}\gamma_\mu\not{g}\gamma_\alpha, \tag{2.8}$$

as explained in Appendix A. The quantity

$$W_\alpha=\frac{q_\alpha}{qg}-\frac{\overline{q}_\alpha}{\overline{q}g}, \tag{2.9}$$

is convenient for simplification of the calculations and for the presentation of the results. It satisfies $W\cdot g=0$, thereby demonstrating explicitly the gauge invariance of the matrix element, Eq. (2.8), $M_{\mu\alpha}g^\alpha=0$.

The quark polarization four-vector $S=(S_0,\boldsymbol{S})$ is given in terms of the polarization of the quark in its rest system $\boldsymbol{\zeta}$ by

$$S_0=\zeta^{\parallel}\frac{|\boldsymbol{q}|}{m_f},$$

$$\boldsymbol{S}=\boldsymbol{\zeta}^{\perp}+\zeta^{\parallel}\hat{\boldsymbol{q}}\frac{E_f}{m_f}. \tag{2.10}$$

Here $\boldsymbol{\zeta}^{\perp}$ is the transverse and $\zeta^{\parallel}=\boldsymbol{\zeta}\cdot\hat{\boldsymbol{q}}$ is the longitudinal polarization, Fig. 1. The covariant polarization satisfies

$$q\cdot S=0,\quad S^2=-\boldsymbol{\zeta}^2=-1, \tag{2.11}$$

for the unit polarization vector $\boldsymbol{\zeta}$.

III. THE HADRON TENSORS

The structure of the hadron tensors, Eq. (2.7), is such that only four trace calculations are necessary. The other hadron tensors are obtained by transformations indicated below. We choose the four tensors $\widetilde{H}^{S\overline{S},f}_{V\mu\nu}, H^{S\overline{S},f}_{A\mu\nu}, H^{S,Zf}_{V\mu\nu}$, and $H^{S\overline{S},Zf}_{V\mu\nu}$, which are explicitly given below.

The hadron tensors are

$$\widetilde{H}^{0,f}_{V\mu\nu}=-\frac{1}{m_f^2}\widetilde{H}^{S\overline{S},f}_{V\mu\nu}(S,\overline{S}\to q,\overline{q}), \tag{3.1}$$

$$\widetilde{H}^{S,f}_{V\mu\nu}=-\frac{1}{m_f}H^{S\overline{S},f}_{A\mu\nu}(\overline{S}\to\overline{q}), \tag{3.2}$$

$$\begin{aligned}\widetilde{H}^{S\overline{S},f}_{V\mu\nu}=m_f^2\bigg[&W^2S^\alpha\overline{S}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{S}^\beta\\&-\frac{1}{\overline{q}g}(\overline{S}Wg^\alpha-\overline{S}gW^\alpha)S^\beta-\frac{1}{(qg)^2}Sgg^\alpha\overline{S}^\beta\\&-\frac{1}{(\overline{q}g)^2}\overline{S}gg^\alpha S^\beta\bigg]t_{\alpha\mu\beta\nu},\end{aligned} \tag{3.3}$$

with

$$t_{\alpha\mu\beta\nu}=g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu}-g_{\alpha\beta}g_{\mu\nu}, \tag{3.4}$$

$$H^{0,f}_{A\mu\nu}=-\frac{1}{m_f^2}H^{S\overline{S},f}_{A\mu\nu}(S,\overline{S}\to q,\overline{q}), \tag{3.5}$$

$$H^{S,f}_{A\mu\nu}=-\frac{1}{m_f}\widetilde{H}^{S\overline{S},f}_{V\mu\nu}(\overline{S}\to\overline{q}), \tag{3.6}$$

$$\begin{aligned}H^{S\overline{S},f}_{A\mu\nu}=im_f^2\bigg[&W^2S^\alpha\overline{S}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{S}^\beta\\&+\frac{1}{\overline{q}g}(\overline{S}Wg^\alpha-\overline{S}gW^\alpha)S^\beta-\frac{1}{(qg)^2}Sgg^\alpha\overline{S}^\beta\\&+\frac{1}{(\overline{q}g)^2}\overline{S}gg^\alpha S^\beta\bigg]\varepsilon_{\alpha\mu\beta\nu}.\end{aligned} \tag{3.7}$$

Note that except for the change of sign on the third and fifth term and the replacement of $t_{\alpha\mu\beta\nu}$ by the antisymmetric tensor $i\varepsilon_{\alpha\mu\beta\nu}$, Eqs. (3.3) and (3.7) are identical:

$$H^{0,Zf}_{V\mu\nu}=\frac{1}{m_f^2}H^{S\overline{S},Zf}_{V\mu\nu}(S,\overline{S}\to q,\overline{q})=-W^2g_{\mu\nu}+\frac{2g_\mu g_\nu}{(qg)(\overline{q}g)}, \tag{3.8}$$

$$\begin{aligned}m_f^2H^{S,Zf}_{V\mu\nu}=im_f\bigg[&W^2S^\alpha q^\beta-\frac{1}{qg}(SWq^\alpha-qWS^\alpha)g^\beta\\&+\frac{1}{qg}(Sgq^\alpha-qgS^\alpha)\bigg(W^\beta-\frac{g^\beta}{\overline{q}g}\bigg)\bigg]\varepsilon_{\alpha\mu\beta\nu}.\end{aligned} \tag{3.9}$$

The hadron tensor $H^{S\bar{S},Zf}_{V\mu\nu}$ is very complicated when written out in full. By the use of Eq. (B3) in Appendix B we can write it in the form

$$m_f^2 H^{S\bar{S},Zf}_{V\mu\nu} = -\left[\frac{1}{2}W^2 S^\alpha q^\beta + \frac{1}{qg}\{(qWg^\beta - qgW^\beta)S^\alpha + (SWg^\alpha - SgW^\alpha)q^\beta\}\right]\bar{S}^\gamma \bar{q}^\delta(\varepsilon_{\alpha\beta\mu\varepsilon}\varepsilon^{\varepsilon}_{\gamma\delta\nu} + t_{\alpha\beta\mu\varepsilon}t^{\varepsilon}_{\gamma\delta\nu})$$
$$+ \frac{1}{(qg)(\bar{q}\bar{g})}[g_\mu q^\gamma S^\delta g^\sigma - g^\gamma_\mu(qgS^\delta - Sgq^\delta)g^\sigma]\bar{S}^\alpha \bar{q}^\beta(\varepsilon_{\alpha\beta\nu\varepsilon}\varepsilon^{\varepsilon}_{\sigma\gamma\delta} + t_{\alpha\beta\nu\varepsilon}t^{\varepsilon}_{\sigma\gamma\delta}) + (q,S \Leftrightarrow \overline{q,S}). \qquad (3.10)$$

The $\tilde{H}^{\bar{S},f}_{V\mu\nu}, H^{\bar{S},f}_{A\mu\nu}$, and $H^{\bar{S},Zf}_{V\mu\nu}$ tensors are obtained from corresponding S-dependent tensors by the transformation $(q\leftrightarrow\bar{q}, S\leftrightarrow\bar{S}, \mu\leftrightarrow\nu, m_f\rightarrow -m_f)$. For completeness we give in Appendix A the hadron tensors in terms of W_α written out in full. The calculations are checked by comparison with previously obtained results [12]:

$$H^{0,f}_{V\mu\nu} = \frac{1}{(qg)(\bar{q}\bar{g})}\left[\left(Qq - m_f^2\frac{Qg}{\bar{q}\bar{g}}\right)q^\alpha Q^\beta t_{\alpha\mu\beta\nu} - \left(Q^2 - 2m_f^2\frac{Qg}{\bar{q}\bar{g}}\right)q_\mu q_\nu + m_f^2[(Qg)g_{\mu\nu} - g_\mu g_\nu] + (q\Leftrightarrow\bar{q})\right],$$

$$H^{0,f}_{A\mu\nu} = \frac{-i}{(qg)(\bar{q}\bar{g})}\varepsilon_{\mu\nu\alpha\beta}\left[\left(Qq - m_f^2\frac{Qg}{\bar{q}\bar{g}}\right)q^\alpha Q^\beta - (q\Leftrightarrow\bar{q})\right], \qquad (3.11)$$

$$H^{0,Zf}_{V\mu\nu} = \frac{1}{(qg)(\bar{q}\bar{g})}\left[\left(q\bar{q} - m_f^2\frac{qg}{\bar{q}\bar{g}}\right)g_{\mu\nu} + g_\mu g_\nu + (q\Leftrightarrow\bar{q})\right].$$

In this way the calculations of $H^{S\bar{S},f}_{V\mu\nu}, H^{S\bar{S},f}_{A\mu\nu}, H^{S,f}_{V\mu\nu}$, and $H^{S,f}_{A\mu\nu}$ are checked by comparison with Eq. (3.11), by the use of the transformations indicated. Note also that the rather complicated tensor $H^{S\bar{S},Zf}_{V\mu\nu}$ equals $m_f^2 H^{0,Zf}_{V\mu\nu}$ for $S=q$ and $\bar{S}=\bar{q}$, which is a test on this calculation.

Expressed in terms of the momenta and polarizations the S- and $\bar{S}$-dependent tensors are

$$\tilde{H}^{S,f}_{V\mu\nu} = \frac{im_f}{(qg)(\bar{q}\bar{g})}\left[S^\alpha\left\{\left(Qq - m_f^2\frac{Qg}{\bar{q}\bar{g}}\right)(Q-q)^\beta + \left(q\bar{q} - m_f^2\frac{\bar{q}\bar{g}}{qg}\right)\bar{q}^\beta - \bar{q}\bar{g}q^\beta\right\} + S\bar{q}\bar{g}^\alpha\bar{q}^\beta + Sg\frac{\bar{q}\bar{g}}{qg}Q^\alpha\bar{q}^\beta\right]\varepsilon_{\alpha\mu\beta\nu}, \qquad (3.12)$$

$$\tilde{H}^{S\bar{S},f}_{V\mu\nu} = -\frac{m_f^2}{(qg)(\bar{q}\bar{g})}\left[\left(q\bar{q} - m_f^2\frac{qg}{\bar{q}\bar{g}}\right)\{S,\bar{S}\}_{\mu\nu} + \left(S\bar{q} - Sq\frac{\bar{q}\bar{g}}{qg}\right)\{\bar{S},g\}_{\mu\nu} + \frac{Sg}{qg}(\bar{q}\bar{g}\{\bar{S},Q\}_{\mu\nu} - Qg\{\bar{S},\bar{q}\}_{\mu\nu}) + (q,S\Leftrightarrow\overline{q,S})\right], \qquad (3.13)$$

where $\{a,b\}_{\mu\nu} = a^\alpha b^\beta t_{\alpha\mu\beta\nu} = a_\mu b_\nu + a_\nu b_\mu - ab\,g_{\mu\nu}$,

$$H^{S,f}_{A\mu\nu} = \frac{m_f}{(qg)(\bar{q}\bar{g})}\left[\left(Qq - m_f^2\frac{Qg}{\bar{q}\bar{g}} + Q\bar{q} - m_f^2\frac{Qg}{qg}\right)\{S,\bar{q}\}_{\mu\nu} + \left(\frac{Q^2}{2} - m_f^2\frac{Qg}{\bar{q}\bar{g}}\right)\{S,g\}_{\mu\nu} + \frac{Sg}{qg}(\bar{q}\bar{g}\{Q,\bar{q}\}_{\mu\nu} - Qg\{\overline{q,q}\}_{\mu\nu})\right.$$
$$\left. + S\bar{q}\{\bar{q},g\}_{\mu\nu} - \bar{q}\bar{g}\{S,Q\}_{\mu\nu}\right], \qquad (3.14)$$

$$H^{S\bar{S},f}_{A\mu\nu} = -\frac{im_f^2}{(qg)(\bar{q}\bar{g})}\left[\left\{\left(q\bar{q} - m_f^2\frac{qg}{\bar{q}\bar{g}}\right)S^\alpha + S\bar{q}\bar{g}^\alpha + \frac{Sg}{qg}(\bar{q}\bar{g}Q^\alpha - Qg\bar{q}^\alpha)\right\}\bar{S}^\beta\varepsilon_{\alpha\mu\beta\nu} - (q,S\Leftrightarrow\overline{q,S})\right], \qquad (3.15)$$

$$m_f^2 H^{S,Zf}_{V\mu\nu} = -\frac{im_f}{(qg)(\bar{q}\bar{g})}\left[q^\alpha(SgQ^\beta - S\bar{q}\bar{g}^\beta) + S^\alpha\left\{\left(q\bar{q} - m_f^2\frac{\bar{q}\bar{g}}{qg}\right)(Q-\bar{q})^\beta + \left(q\bar{q} - m_f^2\frac{qg}{\bar{q}\bar{g}}\right)q^\beta - qgQ^\beta + Qgq^\beta\right\}\right]\varepsilon_{\alpha\mu\beta\nu}, \qquad (3.16)$$

$$m_f^2 H_{V\mu\nu}^{S\bar{S},Zf} = -\left\{ \left[-\frac{q\bar{q}}{(qg)(\bar{q}g)} + \frac{1}{2}\left(\frac{m_f^2}{(qg)^2} + \frac{m_f^2}{(\bar{q}g)^2} \right) \right] S^\alpha q^\beta + \frac{1}{qg}\left[\left(-\frac{q\bar{q}}{\bar{q}g} + \frac{m_f^2}{qg} \right) g^\beta - \left(q^\beta - \frac{qg}{\bar{q}g}\bar{q}^\beta \right) \right] S^\alpha \right.$$

$$\left. + \left[\left(\frac{Sq}{qg} - \frac{S\bar{q}}{\bar{q}g} \right) g^\alpha - Sg\left(\frac{q^\alpha}{qg} - \frac{\bar{q}^\alpha}{\bar{q}g} \right) \right] q^\beta \right\} \bar{S}^\gamma \bar{q}^\delta (\varepsilon_{\alpha\beta\mu\epsilon}\varepsilon^\epsilon_{\gamma\delta\nu} + t_{\alpha\beta\mu\epsilon}t^\epsilon_{\gamma\delta\nu})$$

$$+ \frac{1}{(qg)(\bar{q}g)}[g_\mu q^\gamma S^\delta g^\sigma - g_\mu^\gamma (qgS^\delta - Sqq^\delta) g^\sigma] \bar{S}^\alpha \bar{q}^\beta (\varepsilon_{\alpha\beta\nu\epsilon}\varepsilon^\epsilon_{\sigma\gamma\delta} + t_{\alpha\beta\nu\epsilon}t^\epsilon_{\sigma\gamma\delta}) + (q,S \Leftrightarrow \bar{q},\bar{S}). \quad (3.17)$$

IV. THE POLARIZATION-DEPENDENT CROSS SECTION

The cross section for arbitrary (longitudinal and transversal) quark polarization and quark-antiquark polarization correlations may be written down using Eq. (2.1) with the hadron tensors Eqs. (3.12)–(3.17) and the lepton tensors Eqs. (2.2) and (2.3), where longitudinal as well as transverse electron and positron polarizations are included. In this way a complete description of all fermion polarizations effects to first order in the strong coupling constant α_s may be obtained.

The complete polarization-dependent cross section, Eq. (2.1), may be written in the form

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega\, d\chi\, dx\, d\bar{x}} = \frac{1}{4}\frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{2s}[h_f^{(1)}(s,\xi,\Xi)L_1^{\mu\nu}(H_{V\mu\nu}^{0,f} + H_{V\mu\nu}^{S\bar{S},f}) + h_f^{(2)}(s,\xi,\Xi)L_2^{\mu\nu}(H_{A\mu\nu}^{0,f} + H_{A\mu\nu}^{S\bar{S},f}) + \overline{m_f^2}h_f^{(5)}$$

$$\times(s,\xi,\Xi)L_1^{\mu\nu}(H_{V\mu\nu}^{0,Zf} + H_{V\mu\nu}^{S\bar{S},Zf}) + h_f^{(7)}(s,\xi,\Xi)L_2^{\mu\nu}(H_{V\mu\nu}^{S,f} + H_{V\mu\nu}^{\bar{S},f}) + h_f^{(8)}(s,\xi,\Xi)L_1^{\mu\nu}(H_{V\mu\nu}^{S,f} + H_{V\mu\nu}^{\bar{S},f})$$

$$+ \overline{m_f^2}h_f^{(9)}(s,\xi,\Xi)L_2^{\mu\nu}(H_{V\mu\nu}^{S,f} + H_{V\mu\nu}^{\bar{S},f}) + h_f^{(3)}(s,\xi,\Xi)L_3^{\mu\nu}(H_{V\mu\nu}^{0,f} + H_{V\mu\nu}^{S\bar{S},f}) + h_f^{(4)}(s,\xi,\Xi)L_4^{\mu\nu}(H_{V\mu\nu}^{0,f} + H_{V\mu\nu}^{S\bar{S},f})$$

$$+ h_f^{(6)}(s,\xi,\Xi)L_3^{\mu\nu}(H_{V\mu\nu}^{0,Zf} + H_{V\mu\nu}^{S\bar{S},Zf}) + h_f^{(10)}(s,\xi,\Xi)L_3^{\mu\nu}(H_{A\mu\nu}^{S,f} + H_{A\mu\nu}^{\bar{S},f}) + h_f^{(11)}(s,\xi,\Xi)L_4^{\mu\nu}(H_{A\mu\nu}^{S,f} + H_{A\mu\nu}^{\bar{S},f})], \quad (4.1)$$

where we have added a factor 1/4 to account for the specification of quark and antiquark polarization states. In Eq. (4.1) the coupling functions for longitudinally polarized electron and positron beam particles $h_f^{(i)}(s,\xi,\Xi)$ are given below in Eqs. (5.6) and (5.7) while the coupling functions $h^{(i)}(s)$ related to transversal beam polarization are given by [6,13]

$$h_f^{(3)} = Q_f^2 - 2Q_f \mathrm{Re} f(s) v v_f + |f(s)|^2 (v^2 - a^2)(v_f^2 + a_f^2),$$

$$h_f^{(4)} = -2Q_f \mathrm{Im} f(s) a v_f, \quad (4.2)$$

$$h_f^{(6)} = -|f(s)|^2 (v^2 - a^2) a_f^2,$$

and the new functions

$$h_f^{(10)} = -2Q_f \mathrm{Re} f(s) v a_f - 2|f(s)|^2 (v^2 - a^2) v_f a_f,$$

$$h_f^{(11)} = -2Q_f \mathrm{Re} f(s) a v_f. \quad (4.3)$$

The contribution of the cross section equation (4.1) is facilitated by the observation that $L_1^{\mu\nu}, L_3^{\mu\nu}$, and $L_4^{\mu\nu}$ are even in $\mu\nu$ while $L_2^{\mu\nu}$ is odd. Correspondingly, the hadron tensors $H_{V\mu\nu}^{0,f}$, $H_{V\mu\nu}^{S\bar{S},f}$, $H_{A\mu\nu}^{S,f}$, and $H_{A\mu\nu}^{\bar{S},f}$ are even in $\mu\nu$ while $H_{A\mu\nu}^{0,f}$, $H_{A\mu\nu}^{S\bar{S},f}$, $H_{V\mu\nu}^{S,f}$, and $H_{V\mu\nu}^{\bar{S},f}$ are odd.

V. THE CROSS SECTION FOR LONGITUDINALLY POLARIZED QUARKS AND ANTIQUARKS

In this section we shall discuss longitudinally polarized quarks and antiquarks. We shall restrict ourselves to longitudinally polarized beam electrons and positrons, since these are more interesting experimentally. The case of transverse lepton polarization may be included from Eq. (4.1) in the present formulation when needed.

Leaving out $L_3^{\mu\nu}$ and $L_4^{\mu\nu}$ terms, describing transverse beam polarization, we write the cross section equation (4.1) in the convenient form [12]

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega\, d\chi\, dx\, d\bar{x}} = \frac{1}{4}\frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})}[h_f^{(1)}(s,\xi,\Xi)(X_0 + X_0^{S\bar{S}}) + h_f^{(2)}(s,\xi,\Xi)(Y_0 + Y_0^{S\bar{S}}) + h_f^{(5)}(s,\xi,\Xi)(Z_0 + Z_0^{S\bar{S}})$$

$$+ h_f^{(7)}(s,\xi,\Xi)(X_0^S + X_0^{\bar{S}}) + h_f^{(8)}(s,\xi,\Xi)(Y_0^S + Y_0^{\bar{S}}) + h_f^{(9)}(s,\xi,\Xi)(Z_0^S + Z_0^{\bar{S}})], \quad (5.1)$$

where

$$X_0+X_0^{S\overline{S}}=(H_V^{0,f}+H_V^{S\overline{S},f})_{\alpha\beta}(p_+^\alpha p_-^\beta+p_+^\beta p_-^\alpha),$$

$$Y_0+Y_0^{S\overline{S}}=(H_A^{0,f}+H_A^{S\overline{S},f})_{\alpha\beta}(p_+^\alpha p_-^\beta-p_+^\beta p_-^\alpha),$$

$$Z_0+Z_0^{S\overline{S}}=(H_V^{0,Zf}+H_V^{S\overline{S},Zf})_{\alpha\beta}(p_+^\alpha p_-^\beta+p_+^\beta p_-^\alpha), \tag{5.2}$$

$$X_0^S=(H_V^{S,f})_{\alpha\beta}\overline{m_f^2}(p_+^\alpha p_-^\beta-p_+^\beta p_-^\alpha),$$

$$Y_0^S=(H_A^{S,f})_{\alpha\beta}(p_+^\alpha p_-^\beta+p_+^\beta p_-^\alpha),$$

$$Z_0^S=(H_V^{S,Zf})_{\alpha\beta}\overline{m_f^2}(p_+^\alpha p_-^\beta-p_+^\beta p_-^\alpha),$$

with the definitions for $(H_V^{0,f})_{\alpha\beta}$, etc.,

$$\frac{4}{s}(1-x)(1-\overline{x})H_{V\mu\nu}^{0,f}=(H_V^{0,f})^{\alpha\beta}t_{\alpha\mu\beta\nu},$$

$$\frac{4}{s}(1-x)(1-\overline{x})H_{V\mu\nu}^{S,f}=(H_V^{S,f})^{\alpha\beta}i\varepsilon_{\alpha\mu\beta\nu}, \tag{5.3}$$

$$\frac{4}{s}(1-x)(1-\overline{x})H_{A\mu\nu}^{0,f}=(H_A^{0,f})^{\alpha\beta}i\varepsilon_{\alpha\mu\beta\nu},$$

$$\frac{4}{s}(1-x)(1-\overline{x})H_{A\mu\nu}^{S,f}=(H_A^{S,f})^{\alpha\beta}t_{\alpha\mu\beta\nu}$$

for the symmetric and antisymmetric tensors, and

$$L_{\gamma\gamma}^{\mu\nu}=\Xi L_1^{\mu\nu}+\xi L_2^{\mu\nu},$$

$$L_{\gamma Z}^{\mu\nu}=-(v\Xi-a\xi)L_1^{\mu\nu}-(v\xi-a\Xi)L_2^{\mu\nu}, \tag{5.4}$$

$$L_{ZZ}^{\mu\nu}=[(v^2+a^2)\Xi-2va\xi]L_1^{\mu\nu}+[(v^2+a^2)\xi-2va\Xi]L_2^{\mu\nu},$$

with $\Xi=1-P_+^\parallel P_-^\parallel$ and $\xi=P_-^\parallel-P_+^\parallel$, and $L_1^{\mu\nu}$ and $L_2^{\mu\nu}$ given in Eq. (2.3). We have further used the multiplication rules [14]

$$t_{\mu\alpha\nu\beta}t^{\mu\gamma\nu\delta}=2(g_\alpha^\gamma g_\beta^\delta+g_\alpha^\delta g_\beta^\gamma),$$

$$\varepsilon_{\mu\alpha\nu\beta}\varepsilon^{\mu\gamma\nu\delta}=-2(g_\alpha^\gamma g_\beta^\delta-g_\alpha^\delta g_\beta^\gamma). \tag{5.5}$$

The coupling functions $h_f^{(1)},h_f^{(2)}$, and $h_f^{(5)}$ are given previously [12]

$$h_f^{(1)}(s,\xi,\Xi)=Q_f^2\Xi-2Q_f\mathrm{Re}f(s)(v\Xi-a\xi)v_f+|f(s)|^2[(v^2+a^2)\Xi-2va\xi](v_f^2+a_f^2),$$

$$h_f^{(2)}(s,\xi,\Xi)=2Q_f\mathrm{Re}f(s)(v\xi-a\Xi)a_f-2|f(s)|^2[(v^2+a^2)\xi-2va\Xi]v_fa_f, \tag{5.6}$$

$$h_f^{(5)}(s,\xi,\Xi)=2|f(s)|^2[(v^2+a^2)\Xi-2va\xi]a_f^2,$$

with

$$f(s)=\frac{1}{4\sin^2 2\theta_W}\frac{s}{s-M_Z^2+iM_Z\Gamma_Z^{\mathrm{tot}}}.$$

The new coupling functions related to the single polarizations S or $\overline{S}$ are

$$h_f^{(7)}(s,\xi,\Xi)=Q_f^2\xi-2Q_f\mathrm{Re}f(s)(v\xi-a\Xi)v_f+|f(s)|^2[(v^2+a^2)\xi-2va\Xi](v_f^2+a_f^2),$$

$$h_f^{(8)}(s,\xi,\Xi)=2Q_f\mathrm{Re}f(s)(v\Xi-a\xi)a_f-2|f(s)|^2[(v^2+a^2)\Xi-2va\xi]v_fa_f, \tag{5.7}$$

$$h_f^{(9)}(s,\xi,\Xi)=2|f(s)|^2[(v^2+a^2)\xi-2va\Xi]a_f^2.$$

Note that $h_f^{(7)}(\xi,\Xi)=h_f^{(1)}(\Xi,\xi),h_f^{(8)}(\xi,\Xi)=h_f^{(2)}(\Xi,\xi)$, and $h_f^{(9)}(\xi,\Xi)=h_f^{(5)}(\Xi,\xi)$, which reflect the transformation properties of $H_{V\mu\nu}^{0,f}$ to $H_{V\mu\nu}^{S,f}$, $H_{A\mu\nu}^{0,f}$ to $H_{A\mu\nu}^{S,f}$, and $H_{V\mu\nu}^{0,Zf}$ to $H_{V\mu\nu}^{S,Zf}$. It should be noted that the coupling functions are the same for interactions with no polarization dependence (e.g., $H_V^{0,f}$) as for interactions with $S,\overline{S}$ dependence (e.g., $H_V^{S\overline{S},f}$).

The X_0,Y_0, and Z_0 functions are given previously [12]:

$$X_0=\left(1-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-x}\right)[x^2(1+\beta_x^2\cos^2\theta)+\overline{m_f^2}]+\frac{\overline{m_f^2}}{16}[x_g^2(1+\cos^2\theta_g)-8x_g]+(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta}),$$

$$Y_0=2\left\{\left(x-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-x}\right)x\beta_x\cos\theta-(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta})\right\}, \tag{5.8}$$

$$Z_0=-\frac{\overline{m_f^2}}{4}\left\{4\left(1-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-x}\right)-x_g^2(1-\cos^2\theta_g)-4x_g+(x\Leftrightarrow\overline{x})\right\}.$$

The polarization functions are

$$\tilde{X}_0^{S}=\zeta^{\parallel}\Bigg\{x\bar{x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)(\cos\theta-\beta_x\beta_{\bar{x}}\cos\bar{\theta})+\left[\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)(2x-\bar{m}_f^2)-\bar{m}_f^2(1-x)\right]\cos\theta$$
$$-2\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\frac{\bar{x}(1-x)}{x\beta_x}\beta_{\bar{x}}\cos\bar{\theta}-\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)[x\beta_x\cos\theta+(2-x)\beta_{\bar{x}}\cos\bar{\theta}]\Bigg\}, \quad (5.9)$$

$$Y_0^{S}=\zeta^{\parallel}\Bigg\{x\bar{x}\left(x+\bar{x}-\frac{\bar{m}_f^2}{2}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)(\beta_x-\beta_{\bar{x}}\cos\bar{\theta}\cos\theta)-2(1-x)x\beta_x+x\left(1-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)$$
$$\times[x_g\beta_x+\cos\theta(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})]+\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)[x_g+\beta_{\bar{x}}\cos\bar{\theta}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})]$$
$$+\frac{2\bar{x}}{x\beta_x}\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left[1-x-\frac{\bar{x}x_g}{2}(1-\beta_{\bar{x}}^2\cos^2\bar{\theta})\right]\Bigg\}, \quad (5.10)$$

$$Z_0^{S}=-\bar{m}_f^2\zeta^{\parallel}\Bigg\{\left[1-x_g+\frac{x}{2}(2-\bar{x})\left(\frac{x+\bar{x}}{x}-\frac{x_g}{1-\bar{x}}\right)-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right]\cos\theta+\left[-\frac{1-x_g}{x^2}+\frac{1}{2}\left(\frac{x+\bar{x}}{x}-\frac{x_g}{1-\bar{x}}\beta_x^2\right)\right]\frac{x\bar{x}}{\beta_x}\beta_{\bar{x}}\cos\bar{\theta}\Bigg\}, \quad (5.11)$$

while the polarization correlation functions are given by

$$\tilde{X}_0^{S\bar{S}}=\zeta^{\parallel}\bar{\zeta}^{\parallel}\bar{x}\Bigg\{x\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)(\cos\theta\cos\bar{\theta}-\beta_x\beta_{\bar{x}})-\frac{1}{\beta_x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)[x_g\beta_{\bar{x}}+\cos\bar{\theta}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})]$$
$$-\frac{2\beta_{\bar{x}}}{x\beta_x}\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left(1-x-\frac{\bar{x}x_g}{2}\sin^2\bar{\theta}\right)\Bigg\}+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}), \quad (5.12)$$

$$Y_0^{S\bar{S}}=\zeta^{\parallel}\bar{\zeta}^{\parallel}\Bigg\{x\bar{x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)(\beta_x\cos\bar{\theta}-\beta_{\bar{x}}\cos\theta)+\bar{x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)[x_g\cos\bar{\theta}+\beta_{\bar{x}}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})]$$
$$+\frac{2}{x\beta_x}\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left[\bar{x}(1-x)-\frac{\bar{m}_f^2}{2}x_g\right]\cos\bar{\theta}\Bigg\}-(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}), \quad (5.13)$$

$$Z_0^{S\bar{S}}=\bar{m}_f^2\zeta^{\parallel}\bar{\zeta}^{\parallel}\Bigg\{\Bigg[-\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)\cos\theta+\frac{x_g}{2(1-\bar{x})}x\beta_x\bar{x}\beta_{\bar{x}}(\cos\bar{\theta}-\cos\vartheta\cos\theta)-\frac{1}{2}[x\beta_x+\bar{x}\beta_{\bar{x}}\cos\vartheta]$$
$$\times(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})\Bigg]\cos\bar{\theta}+\frac{1}{4}[x_g^2+(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})^2]\cos\vartheta\Bigg\}+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}). \quad (5.14)$$

Here $\cos\vartheta$ is given by

$$x^2\beta_x^2\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta=4(1-x)(1-\bar{x})(1-x_g)-\bar{m}_f^2x_g^2,$$

$$x\beta_x\bar{x}\beta_{\bar{x}}\cos\vartheta=x\bar{x}+2(1-x-\bar{x})+\bar{m}_f^2. \quad (5.15)$$

VI. QUARK-ANTIQUARK LINEAR POLARIZATION EFFECTS IN $q\bar{q}$ AND $q\bar{q}g$ FINAL STATES

It is of considerable use for the understanding of the polarization effects to write down the cross section for $e^+e^-\to q\bar{q}$ for longitudinally polarized electrons, positrons, and quarks and antiquarks. From the same procedure as above, replacing $M_{\mu\alpha}$ by γ_μ, one readily finds

$$\frac{d^2\sigma_f^{q\bar{q}}}{d\Omega}=\frac{3}{16}\frac{\alpha^2}{s}\beta\Big\{h_f^{(1)}(s,\xi,\Xi)[(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})(1+\cos^2\theta)+\overline{m_f^2}(1+\zeta^{\parallel}\overline{\zeta^{\parallel}})\sin^2\theta]+2h_f^{(2)}(s,\xi,\Xi)(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})\beta\cos\theta-\frac{\overline{m_f^2}}{2}h_f^{(5)}$$
$$\times(s,\xi,\Xi)[(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})(1+\cos^2\theta)+(1+\zeta^{\parallel}\overline{\zeta^{\parallel}})\sin^2\theta]+2h_f^{(7)}(s,\xi,\Xi)(\zeta^{\parallel}-\overline{\zeta^{\parallel}})\cos\theta$$
$$\times+h_f^{(8)}(s,\xi,\Xi)\beta(\zeta^{\parallel}-\overline{\zeta^{\parallel}})(1+\cos^2\theta)-h_f^{(9)}(s,\xi,\Xi)\overline{m_f^2}(\zeta^{\parallel}-\overline{\zeta^{\parallel}})\cos\theta\Big\},\qquad(6.1)$$

with $\beta=\sqrt{1-\overline{m_f^2}}$ and where the lepton longitudinal polarizations are contained in the coupling functions $h^{(i)}(s,\xi,\Xi)$, Eqs. (5.6) and (5.7).

It may be instructive to write down the $q\bar{q}$-cross section at the Z_0 resonance, which shows more clearly the correlations of electron and quark-antiquark polarizations:

$$\frac{d^2\sigma_f^{q\bar{q}}(2E=M_Z)}{d\Omega}=\frac{3}{16}\left(\frac{\alpha}{4\sin^2 2\theta_W}\right)^2\frac{1}{\Gamma^2}(v^2+a^2-2vaP_-)\beta\{(v_f^2+a_f^2)[(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})(1+\cos^2\theta)+\overline{m_f^2}(1+\zeta^{\parallel}\overline{\zeta^{\parallel}})\sin^2\theta]$$
$$-4P_{Z_0}v_fa_f(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})\beta\cos\theta-\overline{m_f^2}a_f^2[(1-\zeta^{\parallel}\overline{\zeta^{\parallel}})(1+\cos^2\theta)+(1+\zeta^{\parallel}\overline{\zeta^{\parallel}})\sin^2\theta]$$
$$+2P_{Z_0}(v_f^2+a_f^2)(\zeta^{\parallel}-\overline{\zeta^{\parallel}})\cos\theta-2v_fa_f\beta(\zeta^{\parallel}-\overline{\zeta^{\parallel}})(1+\cos^2\theta)-2P_{Z_0}\overline{m_f^2}a_f^2(\zeta^{\parallel}-\overline{\zeta^{\parallel}})\cos\theta\}.\qquad(6.2)$$

We have here for simplicity and also because it is experimentally relevant, considered polarized electrons only, with polarization P_-, while $P_+=0$. The polarization of the created and decaying Z_0 boson is [12]

$$P_{Z_0}=\frac{-2va+(v^2+a^2)P_-}{v^2+a^2-2vaP_-}.\qquad(6.3)$$

The cross section for $e^+e^-\to q\bar{q}g$ is obtained from Eq. (5.1), written out in the notation of Ref. [12] with $\mathcal{F}_i(x,\bar{x})$ the polarization-independent form factors, $\mathcal{F}_i^{\zeta}(x,\bar{x})$ and $\mathcal{F}_i^{\bar{\zeta}}(x,\bar{x})$ the polarization-dependent form factors, and $\mathcal{F}_i^{\zeta\bar{\zeta}}(x,\bar{x})$ the polarization correlation form factors. Integration over χ gives

$$\frac{d^4\sigma_f^{q\bar{q}g}}{d\Omega\,dx\,d\bar{x}}=\frac{\alpha^2}{8\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})}\Big\{h_f^{(1)}(s,\xi,\Xi)[\mathcal{F}_1(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_4(x,\bar{x})]+\frac{\overline{m_f^2}}{2}h_f^{(1)-}(s,\xi,\Xi)[\mathcal{F}_2(x,\bar{x})\cos^2\theta$$
$$+\tfrac{1}{2}\mathcal{F}_5(x,\bar{x})]+2h_f^{(2)}(s,\xi,\Xi)\mathcal{F}_3(x,\bar{x})\cos\theta+2h_f^{(7)}(s,\xi,\Xi)[\zeta^{\parallel}\mathcal{F}_1^{\zeta}(x,\bar{x})-\overline{\zeta^{\parallel}}\mathcal{F}_1^{\bar{\zeta}}(x,\bar{x})]\cos\theta+h_f^{(8)}(s,\xi,\Xi)$$
$$\times[\zeta^{\parallel}\{\mathcal{F}_2^{\zeta}(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_5^{\zeta}(x,\bar{x})\sin^2\theta\}-\overline{\zeta^{\parallel}}\{\mathcal{F}_2^{\bar{\zeta}}(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_5^{\bar{\zeta}}(x,\bar{x})\sin^2\theta\}]+h_f^{(9)}$$
$$\times(s,\xi,\Xi)\frac{\overline{m_f^2}}{2}[\zeta^{\parallel}\mathcal{F}_3^{\zeta}(x,\bar{x})-\overline{\zeta^{\parallel}}\mathcal{F}_3^{\bar{\zeta}}(x,\bar{x})]\cos\theta-\zeta^{\parallel}\overline{\zeta^{\parallel}}\Big(h_f^{(1)}(s,\xi,\Xi)[\mathcal{F}_1^{\zeta\bar{\zeta}}(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_4^{\zeta\bar{\zeta}}(x,\bar{x})]$$
$$-\frac{\overline{m_f^2}}{2}h_f^{(1)-}(s,\xi,\Xi)[\mathcal{F}_2^{\zeta\bar{\zeta}}(x,\bar{x})\cos^2\theta+\mathcal{F}_5^{\zeta\bar{\zeta}}(x,\bar{x})]+2h_f^{(2)}(s,\xi,\Xi)\mathcal{F}_3^{\zeta\bar{\zeta}}(x,\bar{x})\cos\theta\Big)\Big\},\qquad(6.4)$$

where $h_f^{(1)-}=h_f^{(1)}-h_f^{(5)}$ and $h_f^{(i)}=h_f^{(i)}(s,\xi,\Xi)$.

The form factors are given by the polarization-independent functions [12]

$$\mathcal{F}_1(x,\bar{x})=\left(1-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-x}\right)x^2\beta_x^2+\left(1-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}^2\beta_{\bar{x}}^2(1-\tfrac{3}{2}\sin^2\vartheta),$$

$$\mathcal{F}_2(x,\bar{x})=x_g^2-\tfrac{3}{2}\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta,$$

$$\mathcal{F}_3(x,\bar{x})=\left(x-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-x}\right)x\beta_x-\left(\bar{x}-\frac{\overline{m_f^2}}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}\beta_{\bar{x}}\cos\vartheta,\qquad(6.5)$$

$$\mathcal{F}_4(x,\bar{x})=2\left(1-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta+\bar{m}_f^2\left[2(1-x_g)+x_g^2\left(1-\frac{\bar{m}_f^2}{2(1-x)(1-\bar{x})}\right)\right],$$

$$\mathcal{F}_5(x,\bar{x})=\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta-2(x_g^2+4x_g-4)-2\bar{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})},$$

and the functions related to polarizations are

$$\mathcal{F}_1^{\zeta}(x,\bar{x})=x\left(x-\bar{x}\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}\cos\vartheta-\frac{\bar{m}_f^2}{2}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)+\frac{\bar{m}_f^2}{2}\left\{2(1-x_g)+\left(x+\bar{x}+\frac{1-x}{1-\bar{x}}x_g\right)\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}\cos\vartheta\right\}, \tag{6.6}$$

$$\mathcal{F}_2^{\zeta}(x,\bar{x})=x^2\beta_x\left(1-\frac{\bar{m}_f^2}{2x}\frac{x_g}{1-x}\right)+\frac{\bar{x}^2}{\beta_x}-\frac{\bar{m}_f^2}{2x\beta_x}\bar{x}\left(x+\bar{x}+x_g\frac{1-x}{1-\bar{x}}\right)-\frac{x_g}{2\beta_x}\left(1-\frac{\bar{m}_f^2}{2x}\frac{x_g}{1-\bar{x}}\right)(\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta+\bar{m}_f^2)$$
$$-\frac{x\bar{x}}{4}(\beta_x-\beta_{\bar{x}}\cos\vartheta)\left(\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta+\bar{m}_f^2\frac{x_g}{1-\bar{x}}\right), \tag{6.7}$$

$$\mathcal{F}_3^{\zeta}(x,\bar{x})=\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta-4(1-x_g)+\frac{\bar{m}_f^2x_g^2}{(1-x)(1-\bar{x})}, \tag{6.8}$$

$$\mathcal{F}_5^{\zeta}(x,\bar{x})=\frac{\bar{x}^2}{\beta_x}-\frac{\bar{m}_f^2}{2x\beta_x}\bar{x}\left(x+\bar{x}+x_g\frac{1-x}{1-\bar{x}}\right)-\frac{x_g}{2\beta_x}\left(1-\frac{\bar{m}_f^2}{2x}\frac{x_g}{1-\bar{x}}\right)(2\bar{x}^2\beta_{\bar{x}}^2\cos^2\vartheta+\bar{m}_f^2)$$
$$-\frac{x\bar{x}}{4}(\beta_x-\beta_{\bar{x}}\cos\vartheta)\left(2\bar{x}^2\beta_{\bar{x}}^2\cos^2\vartheta+\bar{m}_f^2\frac{x_g}{1-\bar{x}}\right)+\frac{\bar{m}_f^2}{2}x\left[\left(\frac{\bar{x}}{x}+1-\frac{x_g}{1-\bar{x}}\right)\bar{x}\beta_{\bar{x}}\cos\vartheta-\beta_x x_g\right]. \tag{6.9}$$

The $\mathcal{F}_i^{\bar{\zeta}}(x,\bar{x})$ functions are obtained by interchanging quark and antiquark quantities. For the case that we can compare with similar expressions in the works of Grooth *et al.*, we agree with Ref. [8] on our form factor $\mathcal{F}_5^{\zeta}(x,\bar{x})$ and with Ref. [10] on $\mathcal{F}_1^{\zeta}(x,\bar{x})$ and $\mathcal{F}_3^{\zeta}(x,\bar{x})$.

The polarization correlation functions are

$$\mathcal{F}_1^{\zeta\bar{\zeta}}(x,\bar{x})=\left[-x\bar{x}\left(1-x_g-\frac{\bar{m}_f^2}{4}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\cos\vartheta+\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}\bar{x}x_g\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)+\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)\right.$$
$$\left.\times(\bar{x}\beta_{\bar{x}}+x\beta_x\cos\vartheta)+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}})\right]-\frac{3}{2}\sin^2\vartheta\frac{\bar{x}^2\beta_{\bar{x}}}{\beta_x}\left[1-\frac{\bar{m}_f^2}{2}\left(1+\frac{\bar{x}}{x}+\frac{x_g^2}{x(1-\bar{x})}\right)\right], \tag{6.10}$$

$$\mathcal{F}_2^{\zeta\bar{\zeta}}(x,\bar{x})=\left((2-x_g)^2-\bar{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\cos\vartheta-x\beta_x\bar{x}\beta_{\bar{x}}\left(\frac{x_g}{1-x}-\frac{1}{2}\frac{x_g}{1-\bar{x}}+\frac{3}{2}\right)\sin^2\vartheta, \tag{6.11}$$

$$\mathcal{F}_3^{\zeta\bar{\zeta}}(x,\bar{x})=-\frac{x\bar{x}}{2}\left(2(1-x_g)-\frac{\bar{m}_f^2}{2}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)(\beta_x\cos\vartheta-\beta_{\bar{x}})-\frac{x_g}{2}\left(1-x_g-\frac{\bar{m}_f^2}{2}\right)(\bar{x}\cos\vartheta-x)+\frac{\bar{m}_f^2}{4}x_g\left(\frac{\bar{x}^2}{x}\cos\vartheta-\frac{x^2}{\bar{x}}\right)$$
$$-\frac{1}{2}(x\beta_x+\bar{x}\beta_{\bar{x}}\cos\vartheta)\left[\left(1-x_g-\frac{\bar{m}_f^2}{2}\right)(\bar{x}\beta_{\bar{x}}-x\beta_x)-\frac{\bar{m}_f^2}{2}\left(\frac{\bar{x}^2}{x}\beta_{\bar{x}}-\frac{x^2}{\bar{x}}\beta_x\right)\right]-\left(x-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left(\bar{x}-\frac{\bar{m}_f^2}{2}\frac{x_g}{1-x}\right)$$
$$\times\left(\frac{1-x}{x\beta_x}\cos\vartheta-\frac{1-\bar{x}}{\bar{x}\beta_{\bar{x}}}\right), \tag{6.12}$$

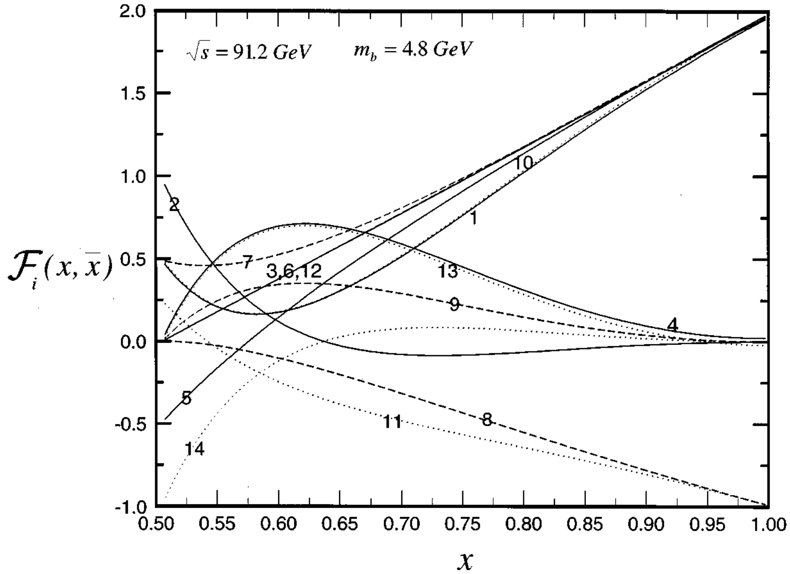

FIG. 2. The form factors $\mathcal{F}_i(x,\bar{x})$ for the b quark for $x=\bar{x}$ at the Z_0 resonance, $2E=91.2$ GeV. The numbers attached to the curves refer to 1–5: $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$, and $(1/4)\mathcal{F}_5$; 6–9: $\mathcal{F}_1^{\zeta}, \mathcal{F}_2^{\zeta}, (1/4)\mathcal{F}_3^{\zeta}$, and $\mathcal{F}_5^{\zeta}$; 10–14: $\mathcal{F}_1^{\zeta\bar{\zeta}}, (1/4)\mathcal{F}_2^{\zeta\bar{\zeta}}, \mathcal{F}_3^{\zeta\bar{\zeta}}, \mathcal{F}_4^{\zeta\bar{\zeta}}$, and $\mathcal{F}_5^{\zeta\bar{\zeta}}$, respectively.

$$\mathcal{F}_4^{\zeta\bar{\zeta}}(x,\bar{x}) = \left[x\bar{x}\left(1 - x_g - \frac{\bar{m}_f^2}{4}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)(\cos\vartheta + \beta_x\beta_{\bar{x}}) + \frac{\bar{x}}{\beta_x}\left(1 - x_g - \frac{\bar{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)(x_g\beta_{\bar{x}} - \bar{x}\beta_{\bar{x}} - x\beta_x\cos\vartheta) \right.$$
$$\left. - 2\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}(1-\bar{x})(1-x_g)\left(x - \frac{\bar{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right) + (x \Leftrightarrow \bar{x}, \beta_x \Leftrightarrow \beta_{\bar{x}}) \right] + 2\sin^2\vartheta\,\frac{\bar{x}^2\beta_{\bar{x}}}{\beta_x}\left[1 - \frac{\bar{m}_f^2}{2}\left(1 + \frac{\bar{x}}{x} + \frac{x_g^2}{x(1-\bar{x})}\right)\right], \tag{6.13}$$

$$\mathcal{F}_5^{\zeta\bar{\zeta}}(x,\bar{x}) = -x_g^2\cos\vartheta - \frac{1}{2}\frac{1-x}{1-\bar{x}}x\beta_x\bar{x}\beta_{\bar{x}}\sin^2\vartheta. \tag{6.14}$$

We give in Figs. 2, 3, and 4 the form factors as functions of x for $x=\bar{x}$ for bottom and top quarks.

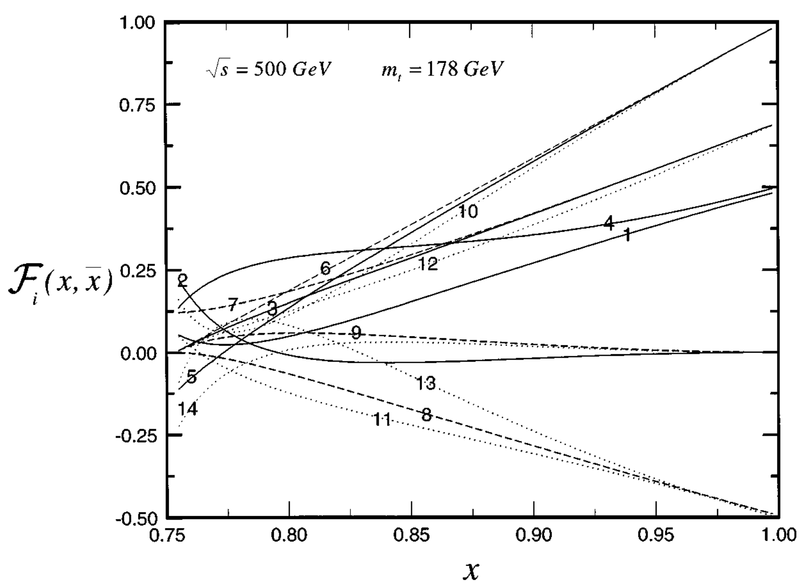

FIG. 3. Same as Fig. 2 for the top quark for the energy $2E=500$ GeV.

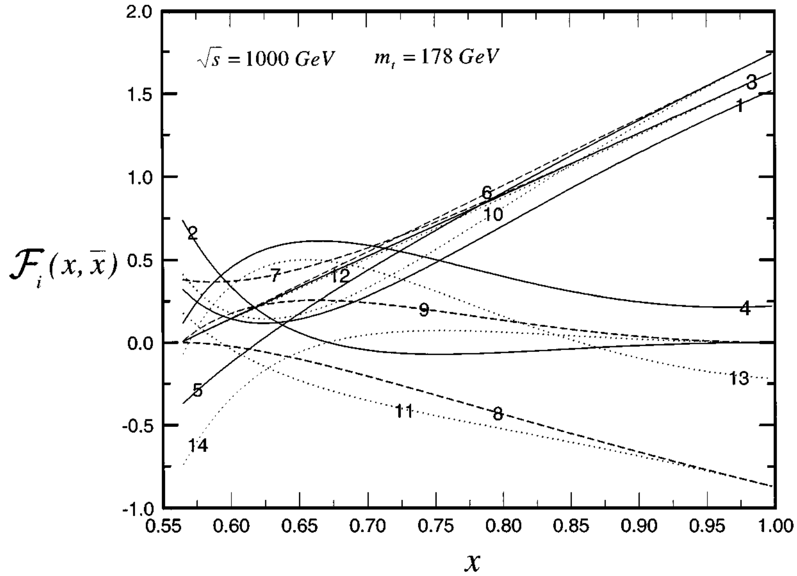

FIG. 4. Same as Fig. 2 for the top quark for the energy $2E = 1000$ GeV.

VII. LONGITUDINAL QUARK POLARIZATION AND POLARIZATION CORRELATIONS

From the cross sections, Eqs. (6.1), (6.2), and (6.4), we obtain the longitudinal polarization of the emitted quark in the final states $q\bar{q}$ or $q\bar{q}g$,

$$P_{\parallel} = \frac{d\sigma(\zeta^{\parallel}=1) - d\sigma(\zeta^{\parallel}=-1)}{d\sigma(\zeta^{\parallel}=1) + d\sigma(\zeta^{\parallel}=-1)}. \qquad (7.1)$$

We define the longitudinal polarization correlation as a measure of how often quarks and antiquarks are emitted with the same helicities, $\zeta^{\parallel}\overline{\zeta^{\parallel}} = +1$, compared to opposite helicities, $\zeta^{\parallel}\overline{\zeta^{\parallel}} = -1$,

$$C_{\parallel} = \frac{d\sigma(\zeta^{\parallel}\overline{\zeta^{\parallel}}=1) - d\sigma(\zeta^{\parallel}\overline{\zeta^{\parallel}}=-1)}{d\sigma(\zeta^{\parallel}\overline{\zeta^{\parallel}}=1) + d\sigma(\zeta^{\parallel}\overline{\zeta^{\parallel}}=-1)}. \qquad (7.2)$$

Note that in the first case we sum over the states $\zeta^{\parallel} = \overline{\zeta^{\parallel}} = \pm 1$ while in the second case the sum is taken over the states $\zeta^{\parallel} = -\overline{\zeta^{\parallel}} = \pm 1$.

For the presentation of the results it is convenient to consider polarization effects odd and even in $\cos\theta$ separately. This separates the forward-backward antisymmetric cross section from the symmetric cross section. We use the forward and backward cross sections σ_F and σ_B, respectively, and define the forward-minus-backward polarization

$$P_{\parallel,FB} = \frac{\sigma_F(\zeta^{\parallel}=1) - \sigma_F(\zeta^{\parallel}=-1) - [\sigma_B(\zeta^{\parallel}=1) - \sigma_B(\zeta^{\parallel}=-1)]}{\sigma(\zeta^{\parallel}=1) - \sigma(\zeta^{\parallel}=-1)}. \qquad (7.3)$$

For the $q\bar{q}$ final states we find from Eq. (6.1) the longitudinal quark polarization, when the antiquark polarization is not recorded:

$$P_{\parallel}^{q\bar{q}}(\theta) = \frac{[2h_f^{(7)}(s) - h_f^{(9)}(s)\overline{m_f^2}]\cos\theta + h_f^{(8)}(s)\beta(1+\cos^2\theta)}{h_f^{(1)}(s)(1+\cos^2\theta + \overline{m_f^2}\sin^2\theta) - h_f^{(5)}(s)\overline{m_f^2} + 2h_f^{(2)}(s)\beta\cos\theta}. \qquad (7.4)$$

For the integrated cross section the polarization is

$$P_{\parallel}^{q\bar{q}} = \frac{2h_f^{(8)}(s)\beta}{h_f^{(1)}(s)(2+\overline{m_f^2}) - \dfrac{3}{2}\overline{m_f^2}h_f^{(5)}(s)}, \qquad (7.5)$$

which becomes, for energies at the Z_0 resonance, from Eq. (6.2),

$$P_{\parallel}^{q\bar{q}}(2E = M_Z) = -\frac{2v_f a_f \beta}{v_f^2\left(1 + \dfrac{\overline{m_f^2}}{2}\right) + a_f^2\beta^2}, \qquad (7.6)$$

which is independent of the beam polarization. All quarks which may be produced at this energy, i.e., all except the top quark, have sizable longitudinal polarizations, -93% for d,s, and b quarks and -60% for u and c quarks at the Z_0 resonance. We give in Figs. 5, 6, and 7 the angular distribution of the longitudinal polarization, Eq. (7.4), for the b

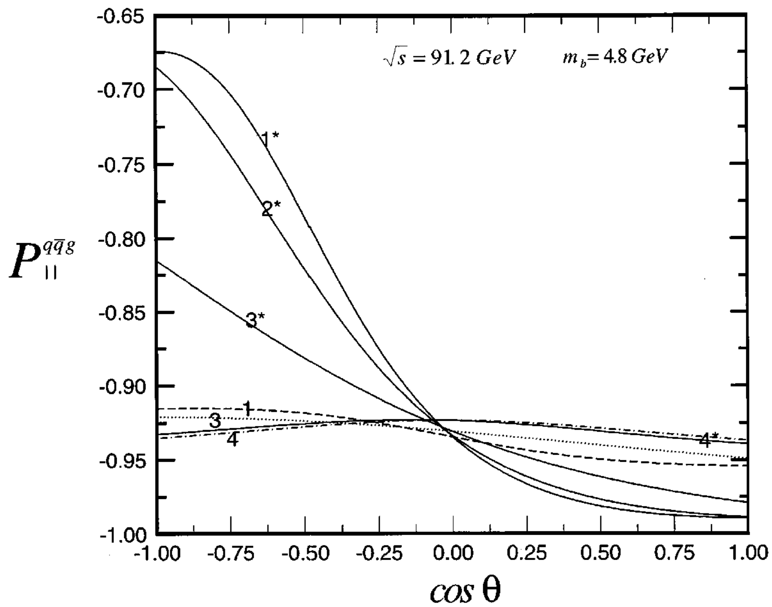

FIG. 5. Longitudinal polarization of the b quark at the Z_0 resonance as a function of $\cos\theta$ for $q\bar{q}g$ final states. Curves 1, 3, and 4 are for $x=\bar{x}=1$, 0.60, and 0.51, respectively, for an unpolarized electron beam, while curves $1^*,2^*,3^*$, and 4^* are for $x=\bar{x}=1$, 0.75, 0.60, and 0.51, respectively, for a longitudinally polarized electron beam, $P_{\rm el}^{\parallel}=-0.63$. The curves for $x=\bar{x}=1$ give at the same time the quark polarizations for $q\bar{q}$ final states.

quark at the Z_0 resonance and for the top quark at the energies $E=250$ GeV and $E=500$ GeV. We also include here the effects of electron beam polarization.

The forward-minus-backward polarization defined in Eq. (7.3) is

$$P_{\parallel,FB}^{q\bar{q}}=\frac{3}{4}\,\frac{2h_f^{(7)}(s)-h_f^{(9)}(s)\overline{m}_f^2}{h_f^{(1)}(s)(2+\overline{m}_f^2)-\frac{3}{2}\overline{m}_f^2h_f^{(5)}(s)},\qquad(7.7)$$

which at the Z_0 resonance becomes

$$P_{\parallel,FB}^{q\bar{q}}(2E=M_Z)=\frac{3}{4}P_{Z_0}\frac{v_f^2+a_f^2\beta^2}{v_f^2\left(1+\frac{\overline{m}_f^2}{2}\right)+a_f^2\beta^2}.\qquad(7.8)$$

Here the polarization is proportional to the Z_0 polarization and, therefore, depends strongly on the beam polarizations. For no beam polarization the forward-minus-backward polarizations of all quarks u,d,s,c, and b are moderate, while for a beam polarization of $\approx$ 60% the quark polarizations are of the same order of magnitude as $P_{\parallel}^{q\bar{q}}$, Eq. (7.6).

For the integrated cross section the longitudinal polarization correlation defined in Eq. (7.2) is

$$C_{\parallel}^{q\bar{q}}=-\frac{h_f^{(1)}(s)(2-\overline{m}_f^2)-\frac{1}{2}\overline{m}_f^2h_f^{(5)}(s)}{h_f^{(1)}(s)(2+\overline{m}_f^2)-\frac{3}{2}\overline{m}_f^2h_f^{(5)}(s)}.\qquad(7.9)$$

For the $q\bar{q}g$ final states we find from Eq. (6.4) the longitudinal quark polarization corresponding to Eq. (7.4) for $q\bar{q}$ states

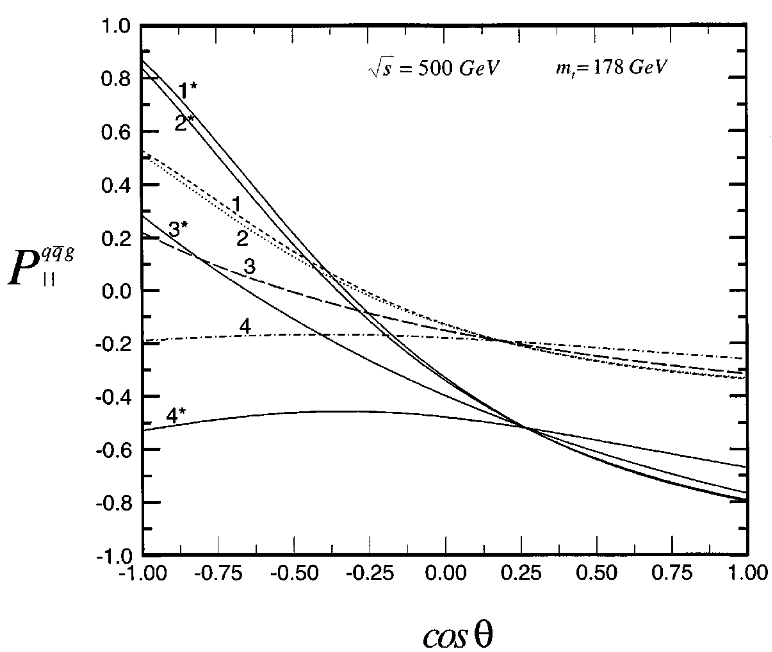

FIG. 6. Same as Fig. 5 for the top quark for the energy $2E=500$ GeV. Here the curves 1–4 correspond to $x=\bar{x}=1$, 0.90, 0.80, and 0.76, respectively, for no beam polarization, while the curves marked with asterisks are for a beam polarization, $P_{\rm el}^{\parallel}=-0.63$.

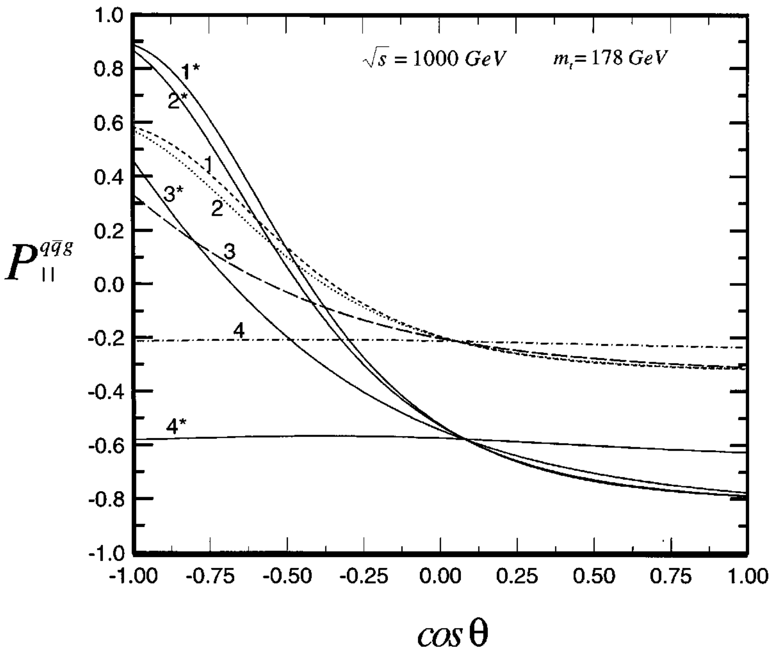

FIG. 7. Same as Fig. 5 for the top quark for the energy $2E=1000$ GeV. Here the curves 1–4 correspond to $x=\bar{x}=1$, 0.85, 0.70, and 0.57, respectively, for no beam polarization, while the curves marked with asterisks are for a beam polarization, $P_{\rm el}^{\parallel}=-0.63$.

$$P_{\parallel}^{q\bar{q}g}(\theta)=\left\{\left[2h_f^{(7)}(s)\mathcal{F}_1^{\zeta}(x,\bar{x})+h_f^{(9)}(s)\frac{\overline{m_f^2}}{2}\mathcal{F}_3^{\zeta}(x,\bar{x})\right]\cos\theta+h_f^{(8)}(s)[\mathcal{F}_2^{\zeta}(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_5^{\zeta}(x,\bar{x})\sin^2\theta]\right\}\left\{h_f^{(1)}(s)\times[\mathcal{F}_1(x,\bar{x})(1+\cos^2\theta)+\mathcal{F}_4(x,\bar{x})]+\frac{\overline{m_f^2}}{2}h_f^{(1)-}(s)[\mathcal{F}_2(x,\bar{x})\cos^2\theta+\tfrac{1}{2}\mathcal{F}_5(x,\bar{x})]+2h_f^{(2)}(s)\mathcal{F}_3(x,\bar{x})\cos\theta\right\}^{-1}. \quad (7.10)$$

Note that the longitudinal polarization is sizable for large regions of angles θ, as shown in Fig. 5 for b quarks and Figs. 6 and 7 for top quarks, and that the electron beam polarization is to a substantial degree transferred to the quark. Our results have overlapping features with results of Refs. [8] and [10] which represent $q\bar{q}$ final states including order α_s radiative corrections, initiated by unpolarized beams.

The polarization for the θ-integrated cross section becomes

$$P_{\parallel}^{q\bar{q}g}(x,\bar{x})=\frac{h_f^{(8)}(s)[2\mathcal{F}_2^{\zeta}(x,\bar{x})+\mathcal{F}_5^{\zeta}(x,\bar{x})]}{h_f^{(1)}(s)[2\mathcal{F}_1(x,\bar{x})+\frac{3}{2}\mathcal{F}_4(x,\bar{x})]+\frac{\overline{m_f^2}}{4}h_f^{(1)-}(s)[\mathcal{F}_2(x,\bar{x})+\frac{3}{2}\mathcal{F}_5(x,\bar{x})]}. \quad (7.11)$$

We give in Fig. 8, $P_{\parallel}^{q\bar{q}g}(x,\bar{x})$ for $x=\bar{x}$, for b quark at the resonance and for top quark for energies $E=250$ GeV and

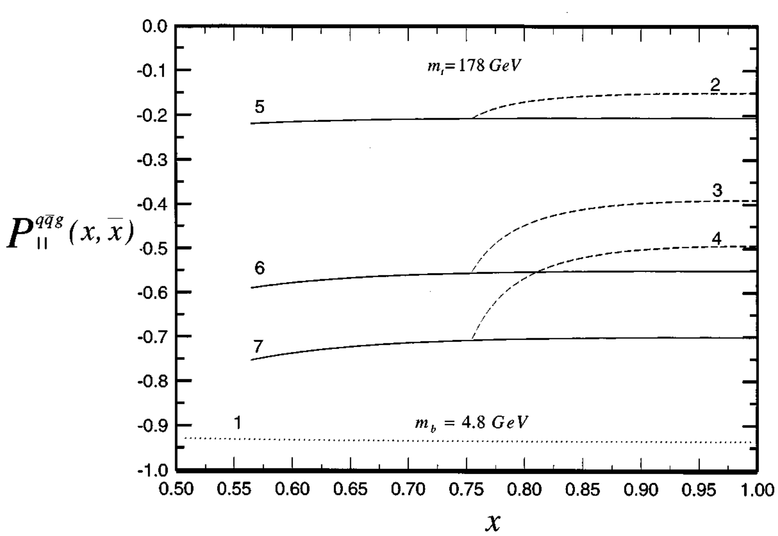

FIG. 8. Longitudinal quark polarization for the cross section integrated over θ as a function of x for $x=\bar{x}$. Curve 1 is for the b quark at the Z_0 resonance irrespective of the polarization of the electron-positron beams. Curves 2–4 are for the top quark for the energy $2E=500$ GeV. Curves 2, 3, and 4 are for an unpolarized beam, for $P_{\rm el}^{\parallel}=-0.63$, and for $P_{\rm el}^{\parallel}=-1.00$, respectively. Curves 5–7 are for the top quark for the energy $2E=1000$ GeV. Curves 5, 6, and 7 are for an unpolarized beam, for $P_{\rm el}^{\parallel}=-0.63$, and for $P_{\rm el}^{\parallel}=-1.00$, respectively.

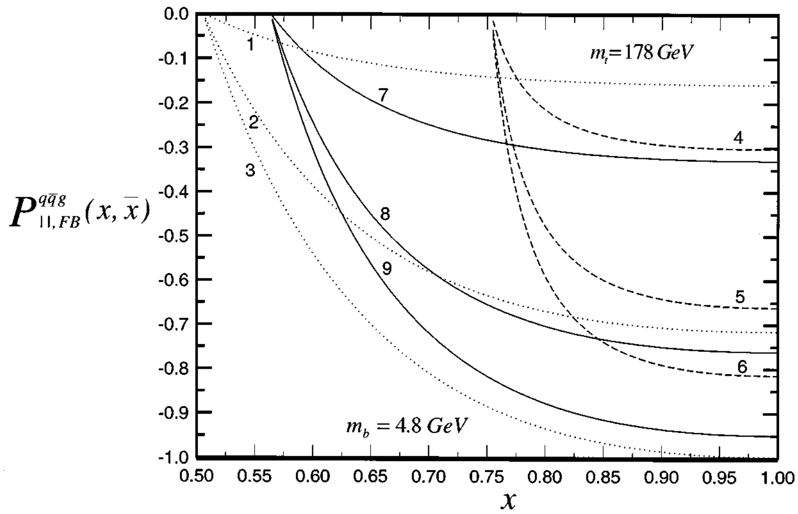

FIG. 9. The forward-minus-backward polarization as a function of x for $x=\overline{x}$. Curves 1–3 are for the b quark at the Z_0 resonance. Curves 1, 2, and 3 are for an unpolarized electron-positron beam, for $P^{\|}_{\rm el}=-0.63$, and for $P^{\|}_{\rm el}=-1.00$, respectively. Curves 4–6 are for the top quark for the energy $2E=500$ GeV. Curves 4, 5, and 6 are for an unpolarized beam, for $P^{\|}_{\rm el}=-0.63$, and for $P^{\|}_{\rm el}=-1.00$, respectively. Curves 7–9 are for the top quark for the energy $2E=1000$ GeV. Curves 7, 8, and 9 are for an unpolarized beam, for $P^{\|}_{\rm el}=-0.63$, and for $P^{\|}_{\rm el}=-1.00$, respectively.

$E=500$ GeV.

The forward-minus-backward polarization defined in Eq. (7.3) becomes for $q\overline{q}g$ final states

$$P^{q\overline{q}g}_{\|,FB}(x,\overline{x})=\frac{3}{4}\,\frac{2h_f^{(7)}(s)\mathcal{F}_1^{\zeta}(x,\overline{x})+h_f^{(9)}(s)\dfrac{\overline{m}_f^2}{2}\mathcal{F}_3^{\zeta}(x,\overline{x})}{h_f^{(1)}(s)\left[2\mathcal{F}_1(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_4(x,\overline{x})\right]+\dfrac{\overline{m}_f^2}{4}h_f^{(1)-}(s)\left[\mathcal{F}_2(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_5(x,\overline{x})\right]}. \tag{7.12}$$

Inspection of Eq. (5.2) for $h_f^{(7)}(s)$ and $h_f^{(9)}(s)$ shows that the polarization is proportional to P_{Z_0} at the Z_0 resonance as for the case of $q\overline{q}$ final states. $P^{q\overline{q}g}_{\|,FB}(x,\overline{x})$ is given in Fig. 9 for b and top quarks. It should be noted that while $P_{\|}^{q\overline{q}g(x,\overline{x})}$ is sizable for unpolarized beams, the forward-minus-backward polarization $P_{\|,FB}^{q\overline{q}g(x,\overline{x})}$ is dependent on a high beam polarization to be of any importance.

The linear polarization correlation for $q\overline{q}g$ final states for the θ-integrated cross section is

$$C_{\|}^{q\overline{q}g}(x,\overline{x})=-\frac{h_f^{(1)}(s)\left[2\mathcal{F}_1^{\zeta\overline{\zeta}}(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_4^{\zeta\overline{\zeta}}(x,\overline{x})\right]-\dfrac{\overline{m}_f^2}{4}h_f^{(1)-}(s)\left[\mathcal{F}_2^{\zeta\overline{\zeta}}(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_5^{\zeta\overline{\zeta}}(x,\overline{x})\right]}{h_f^{(1)}(s)\left[2\mathcal{F}_1(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_4(x,\overline{x})\right]+\dfrac{\overline{m}_f^2}{4}h_f^{(1)-}(s)\left[\mathcal{F}_2(x,\overline{x})+\dfrac{3}{2}\mathcal{F}_5(x,\overline{x})\right]}. \tag{7.13}$$

We give $C_{\|}^{q\overline{q}g}(x,\overline{x})$ in Fig. 10 for b and top quarks.

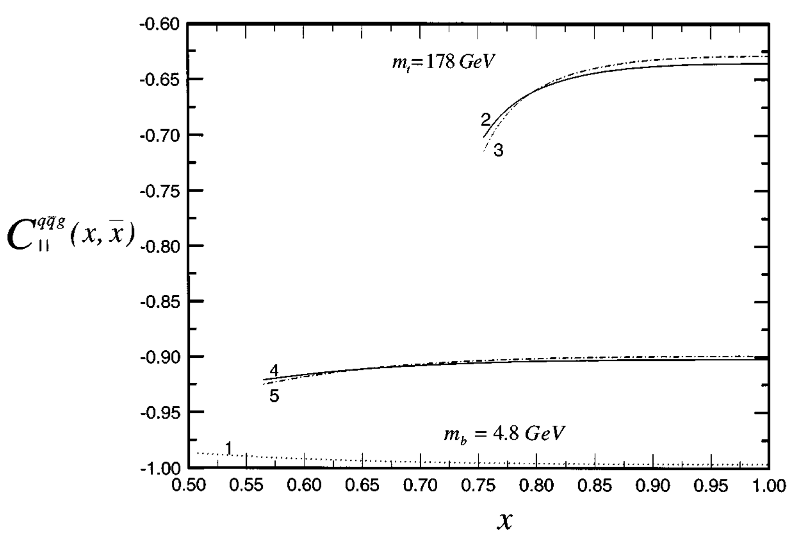

FIG. 10. The longitudinal polarization correlation as a function of x for $x=\overline{x}$. Curve 1 is for the b quark at the Z_0 resonance irrespective of the polarization of the electron-positron beams. Curves 2 and 3 are for the top quark for the energy $2E=500$ GeV, for unpolarized beams and for $P^{\|}_{\rm el}=-1.00$, respectively. Curves 4 and 5 are the same as 2 and 3 for the energy $2E=1000$ GeV.

VIII. TRANSVERSE QUARK POLARIZATION

We obtain in this section the cross sections for transverse quark and antiquark polarization in $q\bar{q}$ and $q\bar{q}g$ final states. We do not discuss here the transverse polarization correlations. We note that the transverse polarization terms in the cross section are proportional to the reduced mass $\overline{m}_f$ and are, therefore, small for light quarks in contrast with the longitudinal polarization-dependent terms discussed above. The only exception is the transverse polarization correlation term described by $h_f^{(5)}(s)Z_0^{S\bar{S}}$ in Eq. (5.1). This can be seen most easily from the relevant hadron tensors, Eqs. (3.12), (3.14), and (3.16) for terms linear in S and Eq. (3.17) for $S\bar{S}$ correlation. From Eq. (2.10) the transverse quark polarization four-vector is given by

$$S^{\perp}=(0,\boldsymbol{\zeta}^{\perp}). \tag{8.1}$$

For the $q\bar{q}$ production process only one scalar product $\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-=-\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_+$ contains $\boldsymbol{\zeta}^{\perp}$, therefore the (maximum) transverse polarization lies *in* the production ($\boldsymbol{p}_-$-$\boldsymbol{q}$) plane. The polarization perpendicular to the production plane vanishes. The cross section for quark polarization is easily obtained, corresponding to Eq. (6.1):

$$\frac{d^2\sigma_f^{q\bar{q}}}{d\Omega}=\frac{3}{16}\frac{\alpha^2}{s}\beta\{h_f^{(1)}(s,\xi,\Xi)(1+\cos^2\theta+\overline{m}_f^2\sin^2\theta)+2h_f^{(2)}(s,\xi,\Xi)\beta\cos\theta-h_f^{(5)}(s,\xi,\Xi)\overline{m}_f^2+\overline{m}_f\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-[2h_f^{(7)}(s,\xi,\Xi)+h_f^{(8)}$$
$$\times(s,\xi,\Xi)\beta\cos\theta-h_f^{(9)}(s,\xi,\Xi)]\}, \tag{8.2}$$

where $\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-=\zeta^{\perp}\sin\theta\cos\varphi_{\zeta}$ with the azimuth angle φ_{ζ} measured in the positive sense from the ($\boldsymbol{p}_-$-$\boldsymbol{q}$) plane, Fig. 1. At the Z_0 resonance the cross section becomes

$$\frac{d^2\sigma_f^{q\bar{q}}}{d\Omega}(2E=M_Z)=\frac{3}{16}\left(\frac{\alpha}{4\sin^2 2\theta_W}\right)^2\frac{1}{\Gamma^2}(v^2+a^2-2vaP_-)\beta\{(v_f^2+a_f^2)(1+\cos^2\theta+\overline{m}_f^2\sin^2\theta)-4P_{Z_0}v_fa_f\beta\cos\theta-2\overline{m}_f^2a_f^2$$
$$+2\overline{m}_f\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-[P_{Z_0}v_f^2-v_fa_f\beta\cos\theta]\}. \tag{8.3}$$

For the $q\bar{q}g$ production cross section the transverse polarization is contained in the scalar products $\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-$ and $\boldsymbol{\zeta}^{\perp}\bar{\boldsymbol{q}}=-\boldsymbol{\zeta}^{\perp}\boldsymbol{g}$, and the transverse polarization does not any more, in general, lie in the ($\boldsymbol{p}_-$-$\boldsymbol{q}$) plane. The cross section corresponding to Eq. (6.4) is found to be given by

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega\, d\chi\, dx\, d\bar{x}}=\frac{\alpha^2}{4(2\pi)^2}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})}\Bigg\{h_f^{(1)}(s,\xi,\Xi)X_0+h_f^{(2)}(s,\xi,\Xi)Y_0+h_f^{(5)}(s,\xi,\Xi)Z_0+\frac{\overline{m}_f}{2}[\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-\{h_f^{(7)}(s,\xi,\Xi)f_X$$
$$+h_f^{(8)}(s,\xi,\Xi)f_Y+h_f^{(9)}(s,\xi,\Xi)f_Z\}+\boldsymbol{\zeta}^{\perp}\hat{\bar{\boldsymbol{q}}}\{h_f^{(7)}(s,\xi,\Xi)g_X+h_f^{(8)}(s,\xi,\Xi)g_Y+h_f^{(9)}(s,\xi,\Xi)g_Z\}]\Bigg\}, \tag{8.4}$$

where the f and g form factors, related to transverse polarizations, are functions of x and $\bar{x}$, and of the angles θ,φ, and χ. It should be noted that the magnitude and direction of the transverse polarization depends on the azimuth angle χ between the production ($\boldsymbol{q}$-$\bar{\boldsymbol{q}}$-$\boldsymbol{g}$) plane and $(\hat{\boldsymbol{p}}_-)^{\perp}$, with $\boldsymbol{q}$ along the z axis,

$$\boldsymbol{\zeta}^{\perp}\hat{\boldsymbol{p}}_-=\zeta^{\perp}\sin\theta\cos(\chi-\varphi_{\zeta}), \tag{8.5}$$

where φ_{ζ} now is measured in the positive sense from the ($\boldsymbol{q}$-$\bar{\boldsymbol{q}}$-$\boldsymbol{g}$) plane.

We demonstrate this effect in Eq. (8.4), where we did not integrate the cross section over azimuth angle χ, as was done for the longitudinal polarization cross section.

The scalar product $\boldsymbol{\zeta}^{\perp}\hat{\bar{\boldsymbol{q}}}$ is

$$\boldsymbol{\zeta}^{\perp}\hat{\bar{\boldsymbol{q}}}=-\zeta^{\perp}\sin\vartheta\cos\varphi_{\zeta}. \tag{8.6}$$

The f and g form factors are given by

$$f_X=8(x+\bar{x}-1)-2\overline{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})},$$

$$f_Y=2x\beta_x\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\cos\theta$$
$$-2\bar{x}\beta_{\bar{x}}\left(x+\bar{x}-1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\cos\bar{\theta},$$

$$f_Z=-2[x+\bar{x}-x_g(2-\bar{x})]+\overline{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})}, \tag{8.7}$$

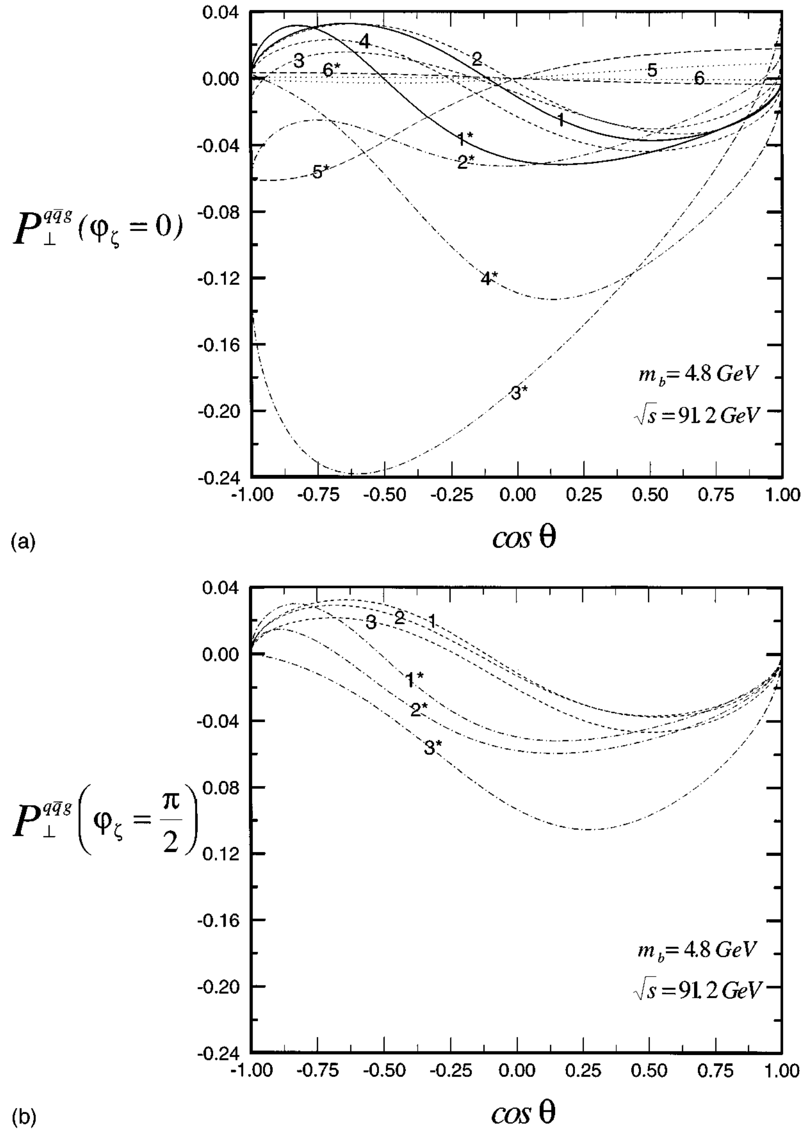

FIG. 11. (a) Transverse polarization in the production plane ($\varphi_\zeta = 0$) for the b quark at the Z_0 resonance for $q\bar{q}g$ final states. Curves 1–4 are for $x = \bar{x} = 1$, 0.90, 0.60, and 0.51, respectively, and $\chi = 0$ for unpolarized electron-positron beams, while the corresponding curves marked with asterisks are for $P_{\rm el}^{\parallel} = -0.63$. The curves 5 and 6 are for $x = \bar{x} = 0.90$ and $x = \bar{x} = 0.51$ for $\chi = \pi/2$ for unpolarized beams, while the corresponding curves marked with asterisks are for $P_{\rm el}^{\parallel} = -0.63$. The curves for $x = \bar{x} = 1$ give at the same time the quark polarization for $q\bar{q}$ final states. (b) Transverse polarization perpendicular to the production plane ($\varphi_\zeta = \pi/2$) for the b quark at the Z_0 resonance for $q\bar{q}g$ final states for $\chi = \pi/2$. Curves 1–3 are for $x = \bar{x} = 0.90$, 0.70, and 0.51, respectively, for unpolarized electron-positron beams, while the corresponding curves marked with asterisks are for $P_{\rm el}^{\parallel} = -0.63$.

$$g_X = 4x\beta_x \bar{x}\beta_{\bar{x}}\cos\theta + 2\bar{x}^2\beta_{\bar{x}}^2\frac{x-\bar{x}}{1-\bar{x}}\cos\bar{\theta},$$

$$g_Y = \frac{\bar{x}^2\beta_{\bar{x}}}{1-\bar{x}}[\bar{x} - x + (1-x)\bar{x}\beta_{\bar{x}}^2\cos^2\bar{\theta}] - x\beta_x\bar{x}^2\beta_{\bar{x}}^2\cos\theta\cos\bar{\theta},$$

$$g_Z = x\beta_x\bar{x}\beta_{\bar{x}}(\bar{x}-4)\cos\theta - x\bar{x}^2\beta_{\bar{x}}^2\cos\bar{\theta}.$$

The transverse polarization is given by

$$P_\perp = \frac{d\sigma(\varphi_\zeta) - d\sigma(\varphi_\zeta + \pi)}{d\sigma(\varphi_\zeta) + d\sigma(\varphi_\zeta + \pi)}. \tag{8.8}$$

For the $q\bar{q}$ final state we find, from Eq. (8.2),

$$P_\perp^{q\bar{q}} = \bar{m}_f\cos\varphi_\zeta\sin\theta\frac{2h_f^{(7)}(s) + h_f^{(8)}(s)\beta\cos\theta - h_f^{(9)}(s)}{h_f^{(1)}(s)(1+\cos^2\theta + \bar{m}_f^2\sin^2\theta) - h_f^{(5)}(s)\bar{m}_f^2 + 2h_f^{(2)}(s)\beta\cos\theta}. \tag{8.9}$$

The corresponding formula at the Z_0 resonance is, from Eq. (8.3),

$$P_\perp^{q\bar{q}}(2E = M_Z) = 2\bar{m}_f\cos\varphi_\zeta\sin\theta\frac{P_{Z_0}v_f^2 - v_f a_f\beta\cos\theta}{(v_f^2 + a_f^2)(1+\cos^2\theta + \bar{m}_f^2\sin^2\theta) - 2\bar{m}_f^2 a_f^2 - 4P_{Z_0}v_f a_f\beta\cos\theta}, \tag{8.10}$$

which shows that electron beam polarization equation (6.3) and Fig. 1 in Ref. [6], may affect the transverse polarization considerably. In fact, for the integrated cross section the polarization is proportional to P_{Z_0}:

$$P_\perp^{q\bar{q}}(2E=M_Z)=\tfrac{3}{8}\pi P_{Z_0}\frac{\bar{m}_f}{1+\frac{\bar{m}_f^2}{2}+\frac{a_f^2}{v_f^2}\beta^2}\cos\varphi_\zeta, \tag{8.11}$$

which gives for a b quark $P_\perp^{q\bar{q}}=0.040P_{Z_0}$.

Note that at $\theta=\pi/2$ the Z_0 longitudinal polarization is to a larger degree transferred to the quark as a transverse polarization in the ($\boldsymbol{q}$-$\boldsymbol{p}_-$) plane [from Eq. (8.10)]:

$$P_\perp^{q\bar{q}}(2E=M_Z)=2P_{Z_0}\frac{\bar{m}_f}{1+\bar{m}_f^2+\frac{a_f^2}{v_f^2}\beta^2}, \tag{8.12}$$

which gives for a b quark $P_\perp^{q\bar{q}}=0.068P_{Z_0}$. This gives a -4.9% transverse polarization of a b quark for an electron beam polarization of -63%. With no beam polarization the b quark transverse polarization is as low as -1.1%. Large effects of electron beam polarization are demonstrated in Figs. 11 and 12 for b and top quarks for $q\bar{q}$ final states.

The transverse quark polarization for $q\bar{q}g$ final states from Eq. (8.4) is given by

$$\begin{aligned}P_\perp^{q\bar{q}g}(x,\bar{x})=&\frac{\bar{m}_f}{2}\{\sin\theta\cos(\chi-\varphi_\zeta)[h_f^{(7)}(s)f_X+h_f^{(8)}(s)f_Y\\&+h_f^{(9)}(s)f_Z]-\sin\vartheta\cos\varphi_\zeta[h_f^{(7)}(s)g_X+h_f^{(8)}\\&\times(s)g_Y+h_f^{(9)}(s)g_Z]\}\{h_f^{(1)}(s)X_0+h_f^{(2)}(s)Y_0\\&+h_f^{(5)}(s)Z_0\}^{-1}.\end{aligned} \tag{8.13}$$

We give in Figs. 11 and 12 $P_\perp^{q\bar{q}g}(x,\bar{x})$ for b and top quarks for $\varphi_\zeta=0$, transverse polarization in the production plane, and for $\varphi_\zeta=\pi/2$, transverse polarization perpendicular to the production plane. It should be noted that also for $q\bar{q}g$ final states the electron beam polarization has a dramatic effect on the quark polarization, in particular in the vicinity of $\theta=\pi/2$. This may be understood as a transfer of longitudinal electron polarization to transverse polarization of the quark, when the quark is emitted close to perpendicular to the electron beam direction. It should be noted that this gives a check on the sign of the quark polarization: the negative natural and electron beam polarizations give a final state negative quark polarization. To the extent that our findings can be compared with the results of Ref. [9], we are in general agreement with their results except that they define their transverse quark polarization with opposite sign. It should, however, be noted that our results are differential in $\bar{x}$ and also χ as mentioned above, and that we include the effect of electron beam polarization.

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APPENDIX A

It is convenient for the calculation to rewrite the Z-coupling matrix element

$$\bar{u}_f\left[\not{e}\frac{\not{q}+\not{g}+m_f}{2qg}\gamma_\mu(v_f-a_f\gamma_5)+\gamma_\mu(v_f-a_f\gamma_5)\frac{\bar{\not{q}}+\not{g}-m_f}{2\bar{q}g}\not{e}\right]v_f,$$

in the form $\bar{u}_f M_{\mu\alpha}(v_f-a_f\gamma_5)e^\alpha v_f$ with

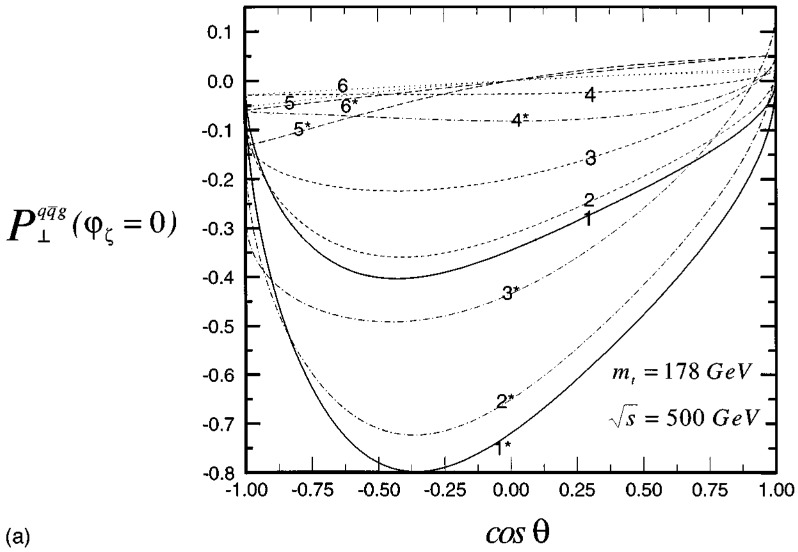
$$M_{\mu\alpha}=W_\alpha\gamma_\mu+\frac{\gamma_\alpha\not{g}\gamma_\mu}{2qg}-\frac{\gamma_\mu\not{g}\gamma_\alpha}{2\bar{q}g},\quad W_\alpha=\frac{q_\alpha}{qg}-\frac{\bar{q}_\alpha}{\bar{q}g}. \tag{A1}$$

By removal of the explicit appearance of the mass, $M_{\mu\alpha}$ has become an odd function in γ, which appears to simplify trace calculations. When we apply the projection operators $\frac{1}{4}(1+\gamma_5\not{s})(\not{q}+m_f)$ and $\frac{1}{4}(1+\gamma_5\bar{\not{s}})(\bar{\not{q}}-m_f)$ for quarks and antiquarks, respectively, we find for the ZZ hadron tensor $H^f_{ZZ\mu\nu}$, Eq. (2.4), which is summed over gluon polarizations,

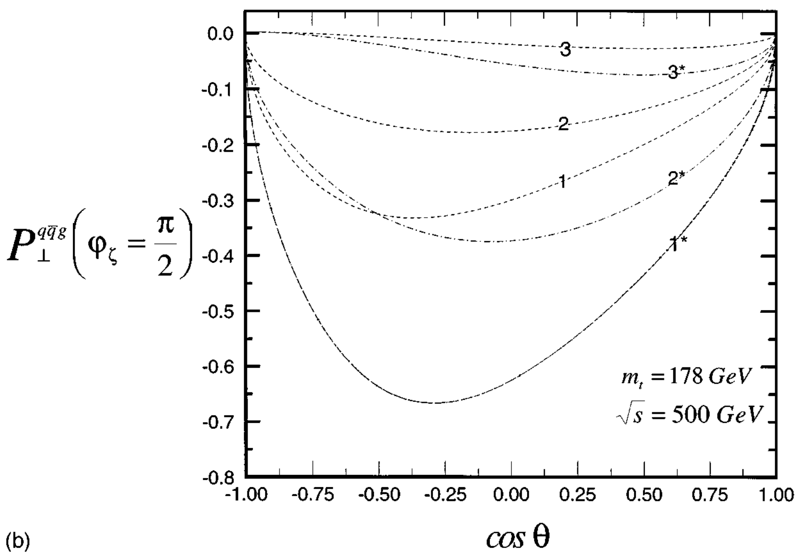
$$\begin{aligned}\frac{1}{8s}H^f_{ZZ\mu\nu}=&-\tfrac{1}{4}\mathrm{Tr}\,(1+\gamma_5\not{s})(\not{q}+m_f)M_{\mu\alpha}(v_f-a_f\gamma_5)(1+\gamma_5\bar{\not{s}})(\bar{\not{q}}-m_f)(v_f+a_f\gamma_5)\bar{M}_\nu^\alpha\\=&-\tfrac{1}{4}\mathrm{Tr}\,\{(v_f^2+a_f^2)[\not{q}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha}+m_f\gamma_5\not{s}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha}-m_f\not{q}M_{\mu\alpha}\gamma_5\overline{\not{s}M_\nu^\alpha}-m_f^2\not{s}M_{\mu\alpha}\overline{\not{s}M_\nu^\alpha}]\\&-2v_fa_f[\gamma_5\not{q}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha}+m_f\not{s}M_{\mu\alpha}\overline{\not{q}M_\nu^\alpha}-m_f\not{q}M_{\mu\alpha}\overline{\not{s}M_\nu^\alpha}-m_f^2\gamma_5\not{s}M_{\mu\alpha}\overline{\not{s}M_\nu^\alpha}]\\&-(v_f^2-a_f^2)[m_f^2M_{\mu\alpha}\bar{M}_\nu^\alpha+m_f\gamma_5\not{s}\not{q}M_{\mu\alpha}\bar{M}_\nu^\alpha-m_fM_{\mu\alpha}\gamma_5\overline{\not{s}\not{q}M_\nu^\alpha}+\not{s}\not{q}M_{\mu\alpha}\overline{\not{s}\not{q}M_\nu^\alpha}]\}.\end{aligned} \tag{A2}$$

From this equation follow the hadron tensors listed in Eq. (2.7) by comparison with Eqs. (2.4) and (2.5):

$$\tilde{H}^{0,f}_{V\mu\nu}=-\left[\frac{1}{2}W^2q^\alpha\bar{q}^\beta+(qW-1)\frac{g^\alpha\bar{q}^\beta}{qg}-W^\alpha\bar{q}^\beta+(q\Leftrightarrow\bar{q})\right]t_{\alpha\mu\beta\nu},$$



(a)



(b)

FIG. 12. (a) Same as Fig. 11(a) for the top quark for $2E=500$ GeV. Curves 1–4 are for $x=\overline{x}=1$, 0.95, 0.85, and 0.76, respectively, and $\varphi_\zeta=\chi=0$ for no beam polarization. Curves 5 and 6 are for $x=\overline{x}=0.95$ and $x=\overline{x}=0.76$ for $\varphi_\zeta=0, \chi=\pi/2$ for no beam polarization. The corresponding curves marked with asterisks give the transverse quark polarization for $P_{\rm el}^{\parallel}=-0.63$. (b) Same as Fig. 11(b) for the top quark for $2E=500$ GeV. Curves 1–3 are for $x=\overline{x}=0.95$, 0.85, and 0.76, respectively, and $\varphi_\zeta=\chi=\pi/2$ for no beam polarization. The corresponding curves marked with asterisks give the transverse quark polarization for $P_{\rm el}^{\parallel}=-0.63$.

$$\widetilde{H}^{S,f}_{V\mu\nu}=im_f\left[W^2S^\alpha\overline{q}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{q}^\beta+\frac{\overline{q}W}{\overline{q}g}g^\alpha S^\beta-W^\alpha S^\beta-\frac{Sg}{(qg)^2}g^\alpha\overline{q}^\beta+\frac{1}{\overline{q}g}g^\alpha S^\beta\right]\varepsilon_{\alpha\mu\beta\nu},$$

$$\widetilde{H}^{S\overline{S},f}_{V\mu\nu}=m_f^2\left[\frac{1}{2}W^2S^\alpha\overline{S}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{S}^\beta-\frac{Sg}{(qg)^2}g^\alpha\overline{S}^\beta+(q,S\Leftrightarrow\overline{q,S})\right]t_{\alpha\mu\beta\nu},$$

$$H^{0,f}_{A\mu\nu}=-i\left[\frac{1}{2}W^2q^\alpha\overline{q}^\beta+(qW-1)\frac{g^\alpha\overline{q}^\beta}{qg}-W^\alpha\overline{q}^\beta-(q\Leftrightarrow\overline{q})\right]\varepsilon_{\alpha\mu\beta\nu},$$

$$H^{S,f}_{A\mu\nu}=-m_f\left[W^2S^\alpha\overline{q}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{q}^\beta-\frac{\overline{q}W}{\overline{q}g}g^\alpha S^\beta+W^\alpha S^\beta-\frac{Sg}{(qg)^2}g^\alpha\overline{q}^\beta-\frac{1}{\overline{q}g}g^\alpha S^\beta\right]t_{\alpha\mu\beta\nu}, \tag{A3}$$

$$H^{S\overline{S},f}_{A\mu\nu}=im_f^2\left[\frac{1}{2}W^2S^\alpha\overline{S}^\beta+\frac{1}{qg}(SWg^\alpha-SgW^\alpha)\overline{S}^\beta-\frac{Sg}{(qg)^2}g^\alpha\overline{S}^\beta-(q,S\Leftrightarrow\overline{q,S})\right]\varepsilon_{\alpha\mu\beta\nu},$$

$$m_f^2H^{0,Zf}_{V\mu\nu}=m_f^2\left[-\frac{1}{2}W^2g_{\mu\nu}+\frac{g_\mu g_\nu}{(qg)(\overline{q}g)}+(q\Leftrightarrow\overline{q})\right],$$

$$m_f^2H^{S,Zf}_{V\mu\nu}=im_f\left[W^2S^\alpha q^\beta-\frac{1}{qg}(SWq^\alpha-qWS^\alpha)g^\beta+\frac{1}{qg}(Sgq^\alpha-qgS^\alpha)\left(W^\beta-\frac{g^\beta}{\overline{q}g}\right)\right]\varepsilon_{\alpha\mu\beta\nu},$$

$$m_f^2 H_{V\mu\nu}^{S\overline{S},Zf} = -\left[\frac{1}{2}W^2 S^\alpha q^\beta + \frac{1}{qg}\{(qWg^\beta - qgW^\beta)S^\alpha + (SWg^\alpha - SgW^\alpha)q^\beta\}\right]\overline{S}^\gamma\overline{q}^\delta(\varepsilon_{\alpha\beta\mu\varepsilon}\varepsilon^\varepsilon_{\gamma\delta\nu} + t_{\alpha\beta\mu\varepsilon}t^\varepsilon_{\gamma\delta\nu})$$

$$+ \frac{1}{(qg)(\overline{qg})}[g_\mu q^\gamma S^\delta g^\sigma - g_\mu^\gamma(qgS^\delta - Sgq^\delta)g^\sigma]\overline{S}^\alpha\overline{q}^\beta(\varepsilon_{\alpha\beta\nu\varepsilon}\varepsilon^\varepsilon_{\sigma\gamma\delta} + t_{\alpha\beta\nu\varepsilon}t^\varepsilon_{\sigma\gamma\delta}) + (q,S \Leftrightarrow \overline{q,S}).$$

APPENDIX B

It is sometimes useful to note the relation [14]

$$\gamma_\mu\gamma_\nu\gamma_\alpha = i\gamma_5\varepsilon_{\mu\nu\alpha\beta}\gamma^\beta + t_{\mu\nu\alpha\beta}\gamma^\beta, \tag{B1}$$

with $\varepsilon_{\mu\nu\alpha\beta}$ the completely antisymmetric tensor with $\varepsilon_{0123}=1$ and

$$t_{\mu\nu\alpha\beta} = g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}.$$

With the help of relation (B1) one can write down in closed form the trace of any number of γ's, the well-known traces

$$\mathrm{Tr}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\sigma = 4t_{\mu\nu\alpha\sigma},$$
$$\mathrm{Tr}\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\sigma = 4i\varepsilon_{\mu\nu\alpha\sigma}, \tag{B2}$$

and the not so well-known traces

$$\mathrm{Tr}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\omega\gamma_\gamma\gamma_\sigma = 4(\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\beta_{\omega\gamma\sigma} + t_{\mu\nu\alpha\beta}t^\beta_{\omega\gamma\sigma}),$$

$$\mathrm{Tr}\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\omega\gamma_\gamma\gamma_\sigma = 4i(\varepsilon_{\mu\nu\alpha\beta}t^\beta_{\omega\gamma\sigma} - t_{\mu\nu\alpha\beta}\varepsilon^\beta_{\omega\gamma\sigma}). \tag{B3}$$

The similarity between $\varepsilon_{\mu\nu\alpha\beta}$ and $t_{\mu\nu\alpha\beta}$ is shown in the relations

$$\varepsilon_{\mu\alpha\nu\beta}\varepsilon^{\alpha\beta}_{\omega\sigma} = -2(g_{\mu\omega}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\omega}),$$
$$t_{\mu\alpha\nu\beta}t^{\alpha\beta}_{\omega\sigma} = 2(g_{\mu\omega}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\omega}), \tag{B4}$$

and

$$t_{\mu\nu\alpha\beta}t^\beta_{\omega\gamma\sigma} = g_{\mu\nu}t_{\omega\gamma\sigma\alpha} - g_{\mu\alpha}t_{\omega\gamma\sigma\nu} + g_{\nu\alpha}t_{\omega\gamma\sigma\mu},$$
$$t_{\mu\nu\alpha\beta}\varepsilon^\beta_{\omega\gamma\sigma} = g_{\mu\nu}\varepsilon_{\omega\gamma\sigma\alpha} - g_{\mu\alpha}\varepsilon_{\omega\gamma\sigma\nu} + g_{\nu\alpha}\varepsilon_{\omega\gamma\sigma\mu}, \tag{B5}$$

and

$$\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\beta_{\omega\gamma\sigma} = g_{\mu\omega}t_{\nu\alpha\gamma\sigma} - g_{\mu\gamma}t_{\nu\alpha\omega\sigma} + g_{\mu\sigma}t_{\nu\alpha\omega\gamma} - g_{\nu\alpha}t_{\mu\omega\gamma\sigma}. \tag{B6}$$

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