# Virtualities of quarks and gluons in a QCD vacuum and nonlocal condensates within the single instanton approximation

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We calculate the lowest-dimensional nonlocal quark and gluon condensates within the single instanton approximation of the instanton liquid model. As a result, we determine the values of average virtualities of quarks $\lambda_q^2 \equiv \langle :\bar{q} D^2 q: \rangle / \langle :\bar{q} q: \rangle$ and gluons $\lambda_g^2 \equiv \langle :G \widetilde{D}^2 G: \rangle / \langle :GG: \rangle$ in the QCD vacuum and obtain parameterless predictions for the ratio $\lambda_g^2 / \lambda_q^2 = 12/5$, and for some ratios of different vacuum condensates of higher dimensions. The nonlocal properties of quark and gluon condensates are analyzed, and the insufficiency of the single instanton approximation is discussed. [S0556-2821(97)05117-5]

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## I. INTRODUCTION

The nonperturbative vacuum of QCD is densely populated by long-wave fluctuations of gluon and quark fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, condensates: $\langle :\bar{q} q: \rangle$, $\langle :G_{\mu\nu}^a G_{\mu\nu}^a: \rangle$, $\langle :\bar{q}(\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a/2) q: \rangle$, etc. The nonzero quark condensate $\langle :\bar{q} q: \rangle$ is responsible for the spontaneous breakdown of chiral symmetry, and its value was estimated a long time ago within the current algebra approach. The importance of the QCD vacuum properties for hadron phenomenology have been established by Shifman, Vainshtein, and Zakharov [1]. They used the operator product expansion (OPE) to relate the behavior of hadron current correlation functions at short distances to a small set of condensates. The values of low-dimensional condensates were obtained phenomenologically from the QCD sum rule (QCD SR) analysis of various hadron channels.

Values of higher-dimensional condensates are known with less accuracy since usually in the range of applicability of QCD SR the static hadron properties: lepton widths, masses, etc., are less sensitive to respective corrections. The whole series of power corrections characterizes the nonlocal structure of vacuum condensates.

Nonlocality of the quark condensate is characterized by the parameter [2]

$$\lambda_q^2 = \frac{\langle :\bar{q} D^2 q: \rangle}{\langle \bar{q} q: \rangle}, \tag{1}$$

where $D_\mu = \partial_\mu - ig A_\mu^a \lambda^a/2$ is a covariant derivative. This

quantity is treated as average virtuality of quarks in the QCD vacuum and characterizes the space width of quark distribution. By the equation of motion in the chiral limit the parameter $\lambda_q^2$ is also related to the mixed quark-gluon condensate

$$m_0^2 = \frac{\left\langle :\bar{q}\left( ig \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} \right) q: \right\rangle}{\langle :\bar{q} q: \rangle}, \quad \lambda_q^2 = \frac{m_0^2}{2}. \tag{2}$$

This quantity has been estimated by QCD SR for baryons to $m_0^2 = 0.8 \pm 0.2$ GeV$^2$ [3], and the lattice QCD (LQCD) calculations yield $m_0^2 = 1.1 \pm 0.1$ GeV$^2$ [4]. Within the instanton model the mixed condensate has first been obtained in the single instanton approximation in [5].[1] Recently, similar calculations have been performed in a more advanced instanton vacuum model [6] with the result $m_0^2 \approx 4/\rho_c^2$, where $\rho_c$ is the characteristic size of the instanton fluctuation in the QCD vacuum. Below, we reproduce this result in another way. As for the nonperturbative properties of gluons in the QCD vacuum, new precise LQCD measurement of the gauge-invariant bilocal correlator of the gluon field strengths has become available down to a distance of 0.1 fm [7].

As it has been proposed in [2], the nonlocal properties of vacuum condensates are of principal importance in the study of the distribution functions of quarks and gluons in hadrons. There, it has been shown that this problem can be correctly considered only if a certain nonlocal form of the vacuum condensates is suggested. Physically, it means that vacuum quarks and gluons can flow through the vacuum with nonzero momentum. To construct the simplest ansatzes for the shape of the nonlocal condensates, in [2,8] some general

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[1]The result given in [5] differs from a correct one by factor 1/2.

properties of these functions and the restricted information about their first derivatives have been used.

On the other hand, in QCD there is an instanton [9,10], a well-known nontrivial nonlocal vacuum solution of the classical Euclidean QCD field equations with the finite action and size $\rho$. The importance of instantons for QCD is that it is believed that an interacting instanton ensemble provides a realistic microscopic picture of the QCD vacuum in the form of ''instanton liquid'' [5,11,12] (see, e.g., a recent review [13]). It has been argued on phenomenological grounds that the distribution of instantons over sizes is peaked at a typical value $\rho_c \approx 1.7$ GeV$^{-1}$ and the ''liquid'' is dilute in the sense that the mean separation between instantons is much larger than the average instanton size. Moreover, the quark Green functions are dominated by zero energy modes localized around the instanton. The effects of condensate nonlocality within the instanton liquid model have implicitly been used in QCD SR for the pion [14] and nucleon [15], where they appear as exponential corrections to the sum rules along with power corrections typical of the OPE approach.

In this paper, we start a systematic discussion of nonlocal condensates within the instanton liquid model of the QCD vacuum. As a first step, we calculate average virtualities of quarks and gluons in the QCD vacuum in the single instanton (SI) approximation to the instanton liquid model. Next, we attempt to obtain the correlation functions $f(\nu)$ which describe distributions over virtuality $\nu$ of quarks and gluons in the nonperturbative vacuum. The approximation used works well for large virtualities, but fails in the description of physically argued distributions at small virtualities (or long distances). The reason is that in order to have a realistic model of vacuum distributions, the important effects of long-wave vacuum configurations have to be included in the vacuum model [16].

The paper is organized as follows. In the second section, the general properties of nonlocal condensates are briefly discussed. The quark and gluon average virtualities $\lambda^2$ are estimated within the single instanton approximation in the third section. To guarantee the gauge invariance, we have introduced the Schwinger $\hat{E}$ exponent as an operator element of the nonlocal vacuum averages. In the fourth section, we analyze the space coordinate behavior of nonlocal condensates. The main asymptotics of the correlation functions $f(\nu)$ at large virtualities $\nu$ are derived. We also demonstrate insufficiency of the SI approximation to obtain the realistic behavior at large distances. There, we point out the physical reason for the failure of the approach used in the large distance region and suggest a way to solve this problem.

## II. THE QUARK AND GLUON DISTRIBUTION FUNCTIONS IN THE QCD VACUUM

To begin, we outline some basic elements of the approach with the nonlocal vacuum condensates. The simplest bilocal scalar condensate $M(x)$ or, in other words, the nonperturbative part of the gauge-invariant quark propagator has the form (in the below definitions we shall follow works [2,8])

$$M(x) \equiv \langle:\bar{q}(0)\hat{E}(0,x)q(x):\rangle \equiv \langle:\bar{q}(0)q(0):\rangle Q(x^2). \quad (3)$$

Here, $\hat{E}(x,y) = P\,\exp[i\int_x^y A_\mu(z)dz^\mu]$ is the path-ordered Schwinger phase factor (the integration is performed along

the straight line) required for gauge invariance and $A_\mu(z) = gA_\mu^a(z)\lambda^a/2$. In the same manner, we will consider the correlator $D^{\mu\nu,\rho\sigma}(x)$ of gluonic strengths $G_{\mu\nu}(x) = gG_{\mu\nu}^a(x)\lambda^a/2$:

$$D^{\mu\nu,\rho\sigma}(x-y) \equiv \langle:TrG^{\mu\nu}(x)\hat{E}(x,y)G^{\rho\sigma}(y)\hat{E}(y,x):\rangle. \quad (4)$$

The correlator may be parameterized in the form consistent with general requirements of gauge and Lorenz symmetries as [17,8,19]:

$$D^{\mu\nu,\rho\sigma}(x) \equiv \frac{1}{24}\langle:g^2G^2:\rangle\Bigg\{(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})[D(x^2)$$
$$+ D_1(x^2)] + (x_\mu x_\rho g_{\nu\sigma} - x_\mu x_\sigma g_{\nu\rho} + x_\nu x_\sigma g_{\mu\rho}$$
$$- x_\nu x_\rho g_{\mu\sigma})\frac{\partial D_1(x^2)}{\partial x^2}\Bigg\}, \quad (5)$$

where $\langle:G^2:\rangle = \langle:G_{\mu\nu}^a(0)G_{\mu\nu}^a(0):\rangle$ is a gluon condensate, and $Q(x^2)$, $D(x^2)$, and $D_1(x^2)$ are invariant functions that characterize nonlocal properties of condensates.

The vacuum expectation values (VEVs) such as $\langle:\bar{q}q:\rangle, \langle:g^2G^2:\rangle, \langle:\bar{q}D^2q:\rangle, \ldots$ appear as expansion coefficients of the correlators $M(x)$ and $D^{\mu\nu,\rho\sigma}(x)$ in a Taylor series in the variable $x^2/4$. The coordinate dependence of the scalar condensates $Q(x^2)$ and $D(x^2)$, normalized at zero by $Q(0) = 1$ and $D(0) + D_1(0) = 1$, can conventionally be parametrized similarly to the well-known $\alpha$ representation for the propagator:[2]

$$Q(x^2) = \int_0^\infty \exp\left(\frac{x^2}{4\alpha}\right)f_q\left(\frac{1}{\alpha}\right)\frac{d\alpha}{\alpha^2}, \quad (6)$$

$$D(x^2) + D_1(x^2) = \int_0^\infty \exp\left(\frac{x^2}{4\alpha}\right)f_g\left(\frac{1}{\alpha}\right)\frac{d\alpha}{\alpha^2}. \quad (7)$$

The properties and the role of the correlation functions $f(\nu)$ have been discussed in detail in [2,8]. The explicit form of $f(\nu)$ completely fixes the coordinate dependence of the condensates and can be determined in the future QCD vacuum theory. Evidently, $f(\nu) \sim \delta(\nu), \delta^{[1]}(\nu), \ldots$, would correspond to the standard VEVs $\langle:\bar{q}q:\rangle, m_0^2, \ldots$, while the behavior $f(\nu) \sim$ const would simulate free propagation. We expect that the realistic $f(\nu)$ occurs somewhere in between these two extremes. Thus, it is a continuous function concentrated around a certain finite value $\lambda^2$ and rapidly decaying to zero as $\nu$ goes to 0 or $\infty$.

The correlation function $f_q(\nu)$ describes the virtuality distribution of quarks in the nonperturbative vacuum [2]. Its $n$ moment is proportional to the VEV of the local operator with the covariant derivative squared $D^2$ to the $n$th power:

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[2]One has to remember that in this work we make use of the Euclidean space and $x^2 < 0$.

$$\int_0^\infty \nu^n f_q(\nu) d\nu = \frac{1}{\Gamma(n+2)} \frac{\langle :\overline{q}(D^2)^n q: \rangle}{\langle :\overline{q}q: \rangle}. \tag{8}$$

It is natural to suggest that VEVs in the right-hand side (RHS) of Eq. (8) should exist for any $n$. It means that the decrease of $f(\nu)$ for large arguments has to be faster than any inverse power of $\nu$, e.g., like some exponential

$$f_q(\nu) \sim \exp(-\text{const} \times \nu) \quad \text{as} \quad \nu \to \infty. \tag{9}$$

The two lowest moments give the normalization conditions and the average vacuum virtualities of quarks $\lambda_q^2$ and gluons $\lambda_g^2$:

$$\int_0^\infty f_q(\nu) d\nu = 1, \quad \int_0^\infty \nu f_q(\nu) d\nu = \frac{1}{2} \frac{\langle :\overline{q}D^2 q: \rangle}{\langle :\overline{q}q: \rangle} \equiv \frac{\lambda_q^2}{2},$$

$$(\lambda_q^2 \simeq 0.4 \text{ GeV}^2, \text{QCD SR [3]}), \tag{10}$$

$$\int_0^\infty f_g(\nu) d\nu = 1,$$

$$\int_0^\infty \nu f_g(\nu) d\nu = \frac{1}{2} \frac{\langle :G_{\mu\nu}^a (\widetilde{D}^2) G_{\mu\nu}^a: \rangle}{\langle :G^2: \rangle} \equiv \frac{\lambda_g^2}{2}. \tag{11}$$

Note that the quark correlator (3) has a direct physical interpretation in the heavy quark effective theory (HQET) of heavy-light mesons as it describes the propagation of a light quark in the color field of an infinitely heavy quark [5,13]. This behavior has been analyzed in detail in [20]. There, it was demonstrated that for large distances $|x|$ the correlator is dominated by the contribution of the lowest state of a heavy-light meson with energy $\Lambda_q$: $Q(x^2) \sim \exp(-\Lambda_q |x|)$. This law provides the behavior of $f(\nu)$ at small $\nu$:

$$f_q(\nu) \sim \exp(-\Lambda_q^2/\nu) \quad \text{as} \quad \nu \to 0. \tag{12}$$

In the case of gluon correlator (5) the correlation length $L_g$ has recently been estimated in the LQCD calculations [7]. The quantity $\Lambda_g = 1/L_g$ plays a similar role as $\Lambda_q$ for the quark distribution, i.e., $D(x^2) \sim \exp(-\Lambda_g |x|)$ for large $|x|$. It is formed at typical distances of an order of 0.5 fm and describes long range vacuum fluctuations of gluon field.

In works [21,22], the arguments in favor of a definite continuous dependence of $f(\nu)$ have been analyzed and different *Ansatze* for these functions were suggested which are consistent with the requirements (9), (12). In particular, one ansatz has been constructed by the simplest combination of both these asymptotics

$$f_q(\nu) \sim \exp\left(-\frac{\Lambda_q^2}{\nu} - \sigma_q^2 \nu\right) \tag{13}$$

with the parameters $\Lambda_q \simeq 0.45$ GeV and $\sigma_q^2 \simeq 10$ GeV$^{-2}$. This ansatz has been successfully applied in QCD SR for a pion and its radial excitations [22], and the main features of the pion have been described: the mass spectrum of pion radial excitations $\pi'$ and $\pi''$ which is in agreement with the experiment and the shapes of the wave functions of $\pi$ and $\pi'$ which have been confirmed by an independent analysis in

[2,21]. Thus, we will regard the form (13) as following directly from the pion phenomenology. Below, we will make some conclusions about the form of the correlation function $f_q(\nu)$ using concrete solutions for the instanton field and quark zero mode around it.

## III. VACUUM AVERAGE VIRTUALITIES IN THE SINGLE INSTANTON APPROXIMATION

Let us consider an instanton solution of the classical Yang-Mills equations in the Euclidean space [9]. It is well known that in the vicinity of the instanton the quark amplitudes are dominated by the localized mode with zero energy [10]. We will consider the expressions for the instanton field and quark zero mode in the axial gauge $A_\mu(z)n^\mu = 0$ since in this gauge with the vector $n_\mu = x_\mu - y_\mu$ the Schwinger factor $\hat{E}(x,y) = 1$. The expressions in the axial gauge for the instanton $(+)$ [anti-instanton $(-)$] field

$$A_{\mu(ax)}(x) = R(x)A_{\mu(\text{reg})}R(x)^+ + iR(x)\partial_\mu R(x)^+,$$

$$G_{\mu\nu(ax)}(x) = R(x)G_{\mu\nu(\text{reg})}R(x)^+, \tag{14}$$

and the quark zero mode

$$\Psi_{ax}^\pm(x) = R_\pm(x)\Psi_{\text{reg}}^\pm(x), \tag{15}$$

where

$$R_\pm(x) = \exp[\pm i(\vec{x}\vec{\tau})\alpha(x)],$$

$$\alpha(x) = \frac{|\vec{x}|}{\sqrt{x^2 + \rho^2}} \arctan \frac{x_4}{\sqrt{x^2 + \rho^2}}$$

have been introduced in [23]. In Eqs. (14) and (15) the expressions for the instanton and quark fields in the regular gauge are given by

$$A_{\mu,\text{reg}}^{\pm a}(x) = \eta_{\mu\nu}^{\mp a} \frac{2x_\nu}{x^2 + \rho^2}, \quad G_{\mu\nu,\text{reg}}^{\pm a}(x) = -\eta_{\mu\nu}^{\mp a} \frac{4\rho^2}{(x^2 + \rho^2)^2}, \tag{16}$$

$$\Psi_{\text{reg}}^\pm(x) = \varphi_{\text{reg}}(x)\xi^\pm, \quad \varphi_{\text{reg}}(x) = \frac{\rho}{\pi(x^2 + \rho^2)^{3/2}}. \tag{17}$$

In Eqs. (14)–(17), $x = (x_4, \vec{x})$ is a relative coordinate with respect to the position of the instanton center $z$. The solutions (14) and (15) are given within the SU(2) subgroup of the SU$_c$(3) theory [$\tau_a$ are the corresponding generators normalized according to Tr$(\tau_a \tau_b) = 1/2 \delta^{ab}$] and the following notation is introduced: $\eta_{\mu\nu}^{a\pm} = \epsilon_{4a\mu\nu} \pm \frac{1}{2}\epsilon_{abc}\epsilon_{bc\mu\nu}$ are the 't Hooft symbols, $\xi^\pm \overline{\xi}^\pm = 1/8\gamma_\mu \gamma_\nu (1 \pm \gamma_5)/2 U\tau_\mu^\mp \tau_\nu^\pm U^+$ with $\tau^\pm = (\pm i, \vec{\tau})$, and $U$ is the matrix of color space rotations.

In the SI background in the zero mode approximation the bilocal quark and gluon condensates acquire the form

$$M_q(x) = \langle :\overline{q}_{(ax)}(0)q_{(ax)}(x): \rangle$$

$$= -\sum_\pm n_c^\pm \int d^4z \int d\Omega \frac{\text{Tr}[\Psi_{ax}^\pm(x-z)\overline{\Psi}_{ax}^\pm(-z)]}{m_q^*}, \tag{18}$$

$$D^{\mu\nu,\rho\sigma}(x) = \langle :G^a_{(ax)\mu\nu}(0) G^a_{(ax)\rho\sigma}(x): \rangle$$

$$= \frac{1}{12} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \sum_{\pm} n_c^{\pm} \int d^4z$$

$$\times \int d\Omega\, G^{\pm a}_{\delta\delta'(ax)}(x-z) G^{\pm a}_{\delta\delta'(ax)}(-z). \quad (19)$$

Here, $n_c^{\pm}$ is the effective instanton / anti-instanton density. The collective coordinate $z$ of the instanton center and its color space orientation are integrated over. In the SI approximation the term in Eq. (5) with the second Lorenz structure does not appear. This fact is due to the specific topological structure of the instanton solution. Both the Lorenz structures

will appear in the RHS of Eq. (19) if one takes into account the long-wave background fields [16].

The averaging over the instanton orientations in the color space is carried out by using the relation $\int d\Omega\, U^a_b U^{+c}_d = 1/N_c \, \delta^a_d \delta^c_b$, where $N_c$ is the number of colors. Using the definitions (3)–(5), (18), and (19) we obtain

$$Q_{ax}(x^2) = \frac{8\rho^2}{\pi} \int_0^\infty dr\, r^2$$

$$\times \int_{-\infty}^\infty dt\, \frac{\cos\left\{ \frac{r}{R} \left[ \arctan\left( \frac{t+|x|}{R} \right) - \arctan\left( \frac{t}{R} \right) \right] \right\}}{[R^2+t^2]^{3/2}[R^2+(t+|x|)^2]^{3/2}},$$

$$(20)$$

$$D_{ax}(x^2) \equiv D(x^2) + D_1(x^2) = \frac{24\rho^4}{\pi} \int_0^\infty dr\, r^2 \int_{-\infty}^\infty dt\, \frac{1 - \frac{4}{3}\sin^2\left\{ \frac{r}{R} \left[ \arctan\left( \frac{t+|x|}{R} \right) - \arctan\left( \frac{t}{R} \right) \right] \right\}}{(R^2+t^2)^2[R^2+(t+|x|)^2]^2}, \quad (21)$$

where $R^2 = \rho^2 + r^2$, $r = |\vec{z}|$, $t = z_4$. In the derivation of these equations we have used a reference frame where the instanton sits at the origin and $x^\mu$ is parallel to one of the coordinate axes, say $\mu = 4$, serving as a ''time'' direction (i.e., $\vec{x} = 0$, $x_4 = |x|$). Expression (20) corresponds to that derived in [5,23] and expression (21) was derived in [24].[3]

In the derivation of (20) and (21) the following relations between the quark and gluon condensates, on the one hand, and the effective density $n_c = n^+ + n^-$, $(n^+ = n^-)$ and the effective quark mass $m_q^*$, on the other hand, have been used

$$\langle :\bar{q}(0) q(0): \rangle = -\frac{n_c}{m_q^*}, \quad \langle :g^2 G^2: \rangle = 32\pi^2 n_c. \quad (22)$$

These relations are valid in the mean field approximation of the instanton liquid model [13] and provide the normalization conditions in Eqs. (18) and (19). Let us emphasize two features of expressions (20) and (21).

First, it is important that the factors $\cos(\dots)$ or $\sin^2(\dots)$ in the numerator of integrands reflect the presence of the $\hat{E}$ factor in the definition of the bilocal condensates.

Second, the correlators $Q(x^2) = Q_{ax}(x^2)$ and $D_{ax}(x^2)$ are gauge-invariant objects by construction. Therefore, the same expressions for the correlators can be derived using any other gauge. But the axial gauge used seems to be the most adequate in this case.

From Eqs. (20) and (21) one may derive the average virtualities of vacuum quarks and gluons in the SI approximation which characterize the behavior of nonperturbative propagators at short distances in the instanton field:

$$\lambda_q^2 = -8 \left. \frac{dQ_{ax}(x)}{dx^2} \right|_{x=0} = 2\frac{1}{\rho_c^2},$$

$$\lambda_g^2 = -8 \left. \frac{dD_{ax}(x)}{dx^2} \right|_{x=0} = \frac{24}{5}\frac{1}{\rho_c^2}, \quad \lambda_g^2 = \frac{12}{5}\lambda_q^2. \quad (23)$$

In expressions (23) for $\lambda^2$, factor 8 arises from the expansion of correlators in the variable $x^2/4$ and also due to the definition of $\lambda_{q(g)}^2$, (10) and (11). The result for $\lambda_q^2$ in Eq. (23) agrees with the value for the mixed condensate derived in [6] if the relation (2) is used.

We see that our result coincides numerically with that derived from the QCD SR, (10), if the effective size of the instanton is approximately chosen as $\rho_c \approx 2$ GeV$^{-1}$:

$$\lambda_q^2 \approx 0.5 \text{ GeV}^2, \quad \lambda_g^2 \approx 1.2 \text{ GeV}^2. \quad (24)$$

This value is quite close to the commonly accepted typical instanton radii 1.7 GeV$^{-1}$ chosen to reproduce the phenomenological properties of the instanton vacuum (see review [13]). The recent analysis of the instanton liquid parameters on the lattice [25] leads to an estimate $\rho_c \approx 1.7$ GeV$^{-1}$. It is interesting to note that gluons are distributed more compact than quarks in the QCD vacuum as it follows from (23). To demonstrate this it is instructive to compare the short-distance correlation lenghts for quark $l_q = 1/\lambda_q \approx 0.28$ fm and gluon $l_g = 1/\lambda_g \approx 0.18$ fm distributions in the QCD vacuum ($\rho_c \approx 2$ GeV$^{-1}$).

We ignore the effects of radiative corrections to the condensates connected with a possible change of normalization point $\mu$ where the condensates are defined. These effects as well as the effects due to nonzero modes contributions are not very important. Thus, the SI approximation works fairly well in describing virtuality of vacuum quarks (gluons) and

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[3]We are grateful to A. Radyushkin who communicated us this reference.

nonlocal properties of condensates at short distances. In the next section, we are going to study the shape of nonlocal condensates in more detail.

The relation of the quantity $\lambda_g^2$ to the combination of VEVs of dimension six has been obtained in [18] (see also [19]):

$$\frac{\lambda_g^2}{2} = \frac{\langle :g^3 f G^3: \rangle}{\langle :g^2 G^2: \rangle} - \frac{\langle :g^4 J^2: \rangle}{\langle :g^2 G^2: \rangle}, \tag{25}$$

where $\langle :g^3 f G^3: \rangle = \langle :g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c: \rangle$, $J^2 = J_\mu^a J_\mu^a$, and $J_\mu^a = \bar{q}(x) \lambda^a/2 \, \gamma_\mu q(x)$. This formula is analogous to Eq. (2) for quarks and relates short distance characteristic (23) of nonlocal gluon condensate $D(x)$ to the standard VEVs of higher dimensions. The estimation $\langle :g^3 f G^3: \rangle \approx 12/5\rho_c^2 \langle :g^2 G^2: \rangle$ following from Eqs. (23) and (25) (without the second numerically small term) coincides with that obtained in [1,5] in a different way. The latter relation in Eq. (23) and the expressions for $\lambda_q^2$, (1), and $\lambda_g^2$, (25), allow us to obtain a new parameterless relation

$$\frac{\langle :g^3 f G^3: \rangle}{\langle :g^2 G^2: \rangle} = \frac{3}{5} \frac{\langle :\bar{q}(i g \sigma_{\mu\nu} G_{\mu\nu}) q: \rangle}{\langle :\bar{q} q: \rangle}$$
$$+ \frac{\langle :g^4 J^2: \rangle}{\langle :g^2 G^2: \rangle} \quad \text{(SI approximation)} \tag{26}$$

and then to estimate a poorly known value of $\langle :g^3 f G^3: \rangle$:

$$\frac{\langle :g^3 f G^3: \rangle}{\langle :g^2 G^2: \rangle} \approx (0.45 \pm 0.12) \text{ GeV}^2.$$

To obtain this value, we have used the approximation $\langle :g^4 J^2: \rangle \approx -\frac{4}{3} g^2 \langle :g \bar{u} u: \rangle^2$ [1] and the estimation for $m_0^2$ [3].

The expressions for $Q_{ax}(x)$ and $D_{ax}(x)$ may be considered as generation functions to obtain the condensates of higher dimensions in the SI approximation. From a technical point of view this procedure is more convenient than the direct calculations of them. In Appendix A we present some new relations for quark VEVs of dimension seven and gluon VEVs of dimension eight in the SI approximation.

## IV. NONLOCAL CONDENSATES WITHIN THE SINGLE INSTANTON APPROXIMATION

The aim of this section is to study the form of the distributions over virtuality of quarks and gluons in the SI approximation. To understand the main asymptotical behavior of correlators at short and long distances it is enough to inspect the expressions (3) and (4) dropping the Schwinger $\hat{E}$ factor. We will also consider numerical effects connected with the neglect of this factor.

To this goal, let us first calculate the correlators using the regular gauge and neglecting the $\hat{E}$ factor. The corresponding expressions are given by Eqs. (20) and (21) with the changes $\cos(\dots) \to 1$ and $\sin(\dots) \to 0$ in the integrands and are reduced to

$$Q^{\text{reg}}(x^2) = \frac{2}{y^2} \left( 1 - \frac{1}{\sqrt{1+y^2}} \right), \tag{27}$$

$$D^{\text{reg}}(x^2) = \frac{3}{4y^2(1+y^2)} \left( \frac{1+2y^2}{y\sqrt{1+y^2}} \ln|\sqrt{1+y^2}+y| - 1 \right), \tag{28}$$

where the dimensionless parameter $y = x/2\rho$ is introduced.

From Eqs. (27) and (28) we easily find for the average virtualities

$$\lambda_{q,\text{reg}}^2 = \frac{3}{2} \frac{1}{\rho^2}, \quad \lambda_{g,\text{reg}}^2 = \frac{16}{5} \frac{1}{\rho^2},$$

which are about 30% less than the corresponding gauge-invariant ''physical'' values in Eq. (23). The same quantities (without $\hat{E}$ factor) calculated in the singular gauge look like

$$\lambda_{q,\text{sing}}^2 = \frac{9}{2} \frac{1}{\rho^2}, \quad \lambda_{g,\text{sing}}^2 = \frac{96}{5} \frac{1}{\rho^2}. \tag{29}$$

Thus, we see that the gauge dependence is very strong and the results derived without $\hat{E}$ factors may be far from being correct numerically.[4]

Now, let us consider the correlation functions $f(\nu)$ in the regular gauge. To this end, we make the inverse Laplace transform of the correlators (27) and (28) and obtain

$$f_q^{\text{reg}}(\nu) = 2\rho^2 \text{erfc}(\rho\sqrt{\nu}), \tag{30}$$

$$f_g^{\text{reg}}(\nu) = \frac{3}{2} \rho^2 \left( \frac{\rho^2 \nu}{2} \right) \exp\left( -\frac{\rho^2 \nu}{2} \right) K_0\left( \frac{\rho^2 \nu}{2} \right), \tag{31}$$

where $\text{erfc}(t) = 1 - \text{erf}(t)$ is the error function and $K_0(t)$ is the MacDonald function. Then, it is easy to obtain large $\nu$ asymptotics of these functions

$$f_q^{\text{reg}}(\nu) = 2\rho^2 \frac{e^{-\rho^2 \nu}}{\sqrt{\rho^2 \nu \pi}} \left[ 1 + O\left( \frac{1}{\nu} \right) \right], \tag{32}$$

$$f_g^{\text{reg}}(\nu) = \frac{3}{4} \rho^2 e^{-\rho^2 \nu} \sqrt{\rho^2 \nu \pi} \left[ 1 + O\left( \frac{1}{\nu} \right) \right], \tag{33}$$

which reflect the behavior of the corresponding correlators in the region of small $x$. The same exponential asymptotics have physical correlation functions $f_{q(g)}(\nu)$ resulting from the gauge-invariant correlators (20) and (21). Thus, we can conclude that the model of nonlocal condensates in the SI approximation can reproduce the main exponential asymptotical behavior $\sim \exp(-\sigma\nu)$ of the physical correlation functions at large virtualities (short distances), and the phenomenological parameter $\sigma$ in Exp. (13) may be identified as $\sigma \simeq \rho_c$.

As to the description of the small virtuality (long distance) region, this approximation fails since in that regime $f(\nu \to 0)$ decays too slowly in contradiction with the physically argued ''color screening'' exponential asymptotics given in Eq. (12). In other words, the correlators in Eqs. (27) and (28) decrease too slowly at large $x$. These conclusions

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[4] Note, that an estimate for $\lambda_q^2$ calculated by nongauge-invariant manner [which is close to $\lambda_{q,\text{sing}}^2$ in Eq. (29)] is presented in [26].

remain valid for the physical case of gauge-invariant correlators (20) and (21), that is easily seen from the behavior of corresponding numerators of the integrands at large $x$. As it is explained in [16], the SI approximation considered in the present paper does not correspond to the real physical vacuum picture. We should take into account the important long-wave background fields too. These fields modify the long-distance behavior of the correlators and lead to appearance of the ''second scale'' parameters $\Lambda_q$ and $\Lambda_g = 1/L_g$ in quark and gluon distributions, respectively [see Exp. (12) and discussion there]. This effect allows us to reproduce the long- and short-distance behavior of the physical correlators (13) in a complete form. It is also shown that the effect of long-wave vacuum fluctuations is not very essential for the values of $\lambda_{q(g)}$ related to short distances.

## V. CONCLUSION

The instanton model provides a way for constructing of the nonlocal vacuum condensates. We have obtained the expressions for the nonlocal gluon $\langle :\mathrm{Tr} G^{\mu\nu}(x)\hat{E}(x,y)G^{\rho\sigma}(y)\hat{E}(y,x): \rangle$ and quark $\langle :\bar{q}(0)\hat{E}(0,x)q(x): \rangle$ condensates within the single instanton approximation to the instanton liquid model. The average virtualities of quarks $\lambda_q^2$ and gluons $\lambda_g^2$ in the QCD vacuum are derived. The results are $\lambda_q^2 = 2/\rho_c^2$ for vacuum quarks, and $\lambda_g^2 = \frac{24}{5}(1/\rho_c^2)$ for vacuum gluons. The value of $\lambda_q^2$ estimated in the QCD SR analysis [3] is reproduced at $\rho_c \approx 2 \text{ GeV}^{-1}$. This number is close to the estimate from the phenomenology of the QCD vacuum in the instanton liquid model [13,25]. The model provides parameterless predictions for the ratio $\lambda_g^2/\lambda_q^2 = 12/5$ and the relation (26) for the vacuum averages of dimension six.

The calculations have been performed in a gauge-invariant manner by using the expressions for the instanton field and quark zero mode in the axial gauge [23]. It is shown that the usage of the singular gauge [in neglecting the Schwinger gauge factor $\hat{E}(0,x)$] in the calculations of nongauge-invariant quantities leads to a strong numerical deviation from correct values.

The behavior of the correlation functions demonstrates that in the single instanton approximation the model of nonlocal condensates can well reproduce the asymptotic behavior of the functions (13) at large virtualities (short distances). This conclusion agrees with the analysis of the meson correlators presented in [13]. There, the coordinate dependencies of the correlators calculated in the SI approximation and in more refined instanton liquid models are close each to other up to distances $\sim 0.5$ fm. The results for nonlocal quark (20) and gluon (21) condensates are also supported by the independent test of their local characteristics $\lambda_q^2$ and $\lambda_g^2$. The latter may be obtained from standard VEVs calculated within the instanton model in [1], [5], and [6].

Nevertheless, the approximation used fails in the description of the small virtuality (long distance) regime. The reason is the neglect of long-wave vacuum fluctuations in the single instanton approximation. In the forthcoming paper [16] we will prove that inclusion of the effects of these fluctuations cures this disease.

## APPENDIX

Here, we present the relations between the derivatives $d^2Q(x^2)/2!d^2x^2$ and $d^2D(x^2)/2!d^2x^2$ calculated in the SI approximation and the same quantities expressed via the quark and gluon VEVs obtained in [19] (below we put the current quark mass $m_q = 0$),

$$\frac{d^2Q_{ax}(x^2)}{2!d^2x^2} \equiv \frac{7}{120}\frac{1}{\rho_c^4}$$

$$= \frac{3Q_1^7 - \frac{3}{2}Q_2^7 - 3Q_3^7 + Q_4^7}{4!24\langle\bar{q}q\rangle} \quad \text{(SI approximation)},$$

$$\tag{A1}$$

$$\frac{d^2D_{ax}(x^2)}{2!d^2x^2} \equiv \frac{5}{21}\frac{1}{\rho_c^4}$$

$$= \frac{\frac{7}{2}G_{1-2}^8 + 23G_{3-4}^8 + 30G_5^8 + 8G_6^8 - 3G_7^8}{4!6G^4}$$

$$\text{(SI approximation)}, \tag{A2}$$

where the quark condensate basis was chosen in the form

$$Q_1^7 = \langle\bar{q}G_{\mu\nu}G_{\mu\nu}q\rangle, \quad Q_2^7 = i\langle\bar{q}G_{\mu\nu}\widetilde{G}_{\mu\nu}\gamma_5 q\rangle,$$

$$Q_3^7 = \langle\bar{q}G_{\mu\lambda}G_{\lambda\nu}\sigma_{\mu\nu}q\rangle, \quad Q_4^7 = i\langle\bar{q}D_\mu J_\nu\sigma_{\mu\nu}q\rangle,$$

and the gluon condensate basis was chosen as

$$G^4 = \langle\mathrm{Tr}G_{\mu\nu}G_{\mu\nu}\rangle,$$

$$G_1^8 = \langle\mathrm{Tr}G_{\mu\nu}G_{\mu\nu}G_{\alpha\beta}G_{\alpha\beta}\rangle, \quad G_2^8 = \langle\mathrm{Tr}G_{\mu\nu}G_{\alpha\beta}G_{\mu\nu}G_{\alpha\beta}\rangle,$$

$$G_3^8 = \langle\mathrm{Tr}G_{\mu\alpha}G_{\alpha\nu}G_{\nu\beta}G_{\beta\mu}\rangle, \quad G_4^8 = \langle\mathrm{Tr}G_{\mu\alpha}G_{\alpha\nu}G_{\mu\beta}G_{\beta\nu}\rangle,$$

$$G_5^8 = i\langle\mathrm{Tr}J_\mu G_{\mu\nu}J_\nu\rangle, \quad G_6^8 = i\langle\mathrm{Tr}J_\lambda[D_\lambda G_{\mu\nu}, G_{\mu\nu}]\rangle,$$

$$G_7^8 = \langle\mathrm{Tr}J_\mu D^2 J_\mu\rangle,$$

and the notation $G_{i-j}^8 = G_i^8 - G_j^8$ is used.

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