Wave function corrections and off-forward gluon distributions in diffractive J/ψ electroproduction

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Diffractive production of J/ψ particles by virtual photons on a proton target is studied with a view towards understanding two important corrections to the leading order result. First, the effect of Fermi motion of the heavy quarks is studied by performing a systematic expansion in the relative velocity, and a simple correction factor is derived. This is considerably less than estimated previously. Second, since the kinematics necessarily requires that nonzero momentum be transferred to the proton, off-forward gluon distributions are probed by the scattering process. To estimate the importance of the off-forwardness, we compute, in leading order perturbation theory, the extent of deviation from the usual forward gluon distribution in a quark.

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I. INTRODUCTION

There is considerable interest in diffractive electroproduction of J/ψ mesons off protons at high energies because this process is important for studying the gluon density in a proton at low values of the Bjorken x. This interest stems from the simplicity of the leading order QCD expression for longitudinally polarized photons which was first derived by Ryskin [1,2]:

$$\frac{d\sigma}{dt}(\gamma^* + P \to J/\psi + P) = \frac{16\pi^3 M\Gamma_{ee}}{3Q^6}\alpha_s(\overline{Q^2})[\xi g(\xi,\overline{Q^2})]^2, \tag{1}$$

where $\overline{Q^2} = 1/4(Q^2+M^2)$, M is the J/ψ mass, and Γ_{ee} is the decay width into leptons. The above equation was derived under the assumption of $s \gg Q^2 \gg M^2 \gg t$, and that Fermi motion of the quarks in the meson can be entirely neglected. It was further supposed that the gluon density appearing in Eq. (1) is that which would be measured in some inclusive hard process, i.e., that it corresponds to the matrix element of gluon operators between states of equal momentum.

In this paper we shall examine the effect of relaxing two assumptions which go into Eq. (1). The first is to take into account the correction arising from the Fermi motion of the $c\bar{c}$ pair. In the work of Brodsky *et al.* [4] this motion is contained in the vector-meson light-cone wave function $\psi^V(k_\perp, x)$, a quantity which is in principle calculable from lattice QCD but whose presently unknown form is an important source of ambiguity. For example, Frankfurt *et al.* [3] conclude that wave function effects suppress J/ψ production by a factor of 3 or more. However, Ryskin *et al.* [2] estimate a suppression factor of $0.4 \leqslant F^2 \leqslant 0.6$. The detailed shape of the wave function appears to be an important source of the difference.

The method of treating the diffractive process, as well as Fermi motion corrections, used in this paper differs from previously used methods in an essential way. Rather than work in the infinite momentum frame and in the $A^+=0$ gauge, we shall choose the rest frame of the J/ψ and the Coulomb gauge for the soft gluons in the meson wave function. This is the natural choice for heavy-quark systems because one can then use systematic procedures, such as nonrelativistic quantum chromodynamics [5] (NRQCD) or the method developed in Refs. [6–8], in order to evaluate quarkonium observables of interest to any desired level of accuracy. However, for the gluons in the fast-moving proton we shall continue to use the $A^+=0$ gauge because this is the natural gauge to use for parton distributions. The two kinds of gluons have very different momenta and hence are effectively distinguishable, justifying the use of two different gauges in two different parts of the same Feynman diagram. It turns out that a gauge-invariant correction factor, derived in this paper, multiplies Eq. (1):

$$\left(1 + \frac{8}{9}\frac{\nabla^2\phi}{M^2\phi}\right).$$

The second derivative of the wave function is understood to be evaluated at the origin. It is a nonperturbative quantity whose value has to be inferred from some other quarkonium processes, such as decays or production, involving large momentum transfer. In previous work [6,7] its value was estimated:

$$\frac{\nabla^2\phi}{M^2\phi} \approx -0.07.$$

The correction factor due to Fermi motion is therefore around 0.96, a value considerably below the other estimates [2,3]. Hence, the ambiguity in extracting the normalization of the gluon distribution may be under better control than anticipated so far.

The second issue to be considered in this paper is the gluon distribution which appears in Eq. (1). Recently Ji [10] identified certain twist-two ''off-forward'' quark distributions inside the proton which, when measured, will reveal the orbital angular momentum content of the proton. Subsequently Radyushkin [11] extended the discussion to the off-forward or ''asymmetric'' gluon distribution in the proton and pointed out that diffractive vector meson electroproduc-

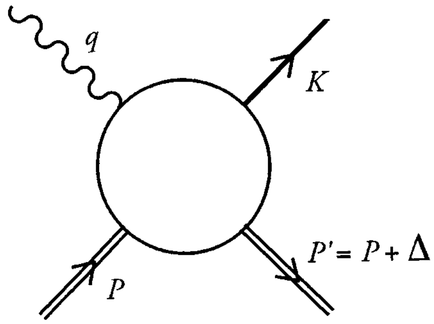

FIG. 1. Definition of kinematic variables for J/ψ diffractive production off a proton target by a virtual photon.

tion necessarily measures this quantity. Here we have examined this issue further by considering gluon radiation from a quark and explicitly computed the off-forward gluon distribution in a quark to leading order in α_s. This enables an estimate to be made of the extent to which the gluon distribution measured in J/ψ diffractive production differs from that which would be measured in some inclusive process like $\gamma+P\rightarrow J/\psi+X$.

II. FERMI MOTION

A. Kinematics

We consider a massless proton target and the $t=0$ limit. Define two null vectors p^μ and n^μ with $p^2=n^2=0$ and $p\cdot n=1$:

$$p^\mu=\frac{\Lambda}{\sqrt{2}}(1,0,0,1),\quad n^\mu=\frac{1}{\sqrt{2}\Lambda}(1,0,0,-1). \tag{2}$$

p^μ is also the proton's momentum. Although we shall not need to do so explicitly, Λ can be adjusted to bring the produced J/ψ to rest. The kinematic region of interest is considered to be $s\gg Q^2\gg M^2$. With the definitions

$$\xi=-\frac{q^2}{2p\cdot q},\quad q^2=-Q^2, \tag{3}$$

the other momenta in Fig. 1 are

$$q^\mu=-\xi p^\mu+\frac{Q^2}{2\xi}n^\mu,$$

$$P'^\mu=p^\mu+\Delta^\mu,$$

$$\Delta^\mu=-\xi p^\mu,$$

$$K^\mu=\frac{\xi M^2}{Q^2}p^\mu+\frac{Q^2}{2\xi}n^\mu. \tag{4}$$

The polarization vectors of the longitudinally polarized photon and J/ψ are, respectively,

$$\varepsilon_L^\mu=\frac{\xi}{Q}p^\mu+\frac{Q}{2\xi}n^\mu,$$

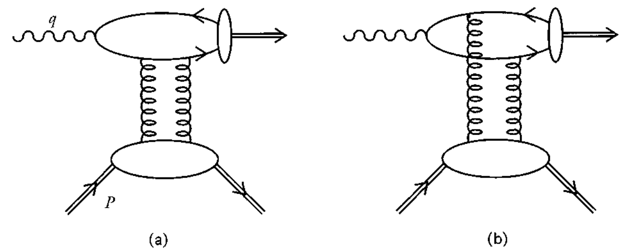

FIG. 2. Diagrams which give a nonzero contribution at order Q^{-1} and v^0. The relative weight at this order $a{:}b$ is as 1:2. Two other diagrams, which are numerically equal by time-reveral invariance, are not shown. The complete expression is given in Eq. (16) .

$$E_L=-\xi\frac{M}{Q^2}p^\mu+\frac{Q^2}{2M\xi}n^\mu. \tag{5}$$

These obey $\varepsilon_L\cdot\varepsilon_L=1$, $\varepsilon_L\cdot q_L=0$, and $E_L\cdot E_L=-1$, $E_L\cdot K_L=0$, with $K^2=M^2$. We have kept only leading terms and set $\Delta_\perp\approx 0$.

B. Diagrams

The leading order contribution to J/ψ diffractive production is given by the sum of the diagrams shown in Fig. 2, to which must be added the contribution of two other diagrams that give the same numerical values because of time-reversal symmetry. Consider, by way of example, the first of these which has the expression

$$A_1=\int\frac{d^4k}{(2\pi)^4}\frac{d^4\ell}{(2\pi)^4}\mathrm{Tr}[S_{\mu\nu}(k,\Delta)H_1^{\mu\nu}(k,\ell)M(\ell)],$$

$$H_1^{\mu\nu}(k,\ell)=e_Qg^2\gamma^\mu S_F(k+q-K/2+\ell)\gamma^\nu \times S_F(q-K/2+\ell)\not\epsilon(q). \tag{6}$$

The perturbative part $H^{\mu\nu}(k,\ell)$ is different for the other diagram but the other factors in Eq. (6) remain unchanged. We have not indicated color explicitly in the above; its inclusion will amount to a simple factor which will be inserted at the end of the calculation. The nonperturbative information of the vector meson is contained in the Bethe-Salpeter wave function $M(\ell)$:

$$M(\ell)=\int d^4x e^{i\ell\cdot x}\langle K,E|T[\psi(x/2)\overline{\psi(}-x/2)]|0\rangle. \tag{7}$$

In the above, ℓ^μ and x^μ are, respectively, the relative momentum and relative distance of the $c\bar{c}$ pair. The nonperturbative information of the gluons in the proton is contained in $S^{\mu\nu}$:

$$S^{\mu\nu}(k,\Delta)=\int d^4x e^{i(k+\Delta/2)\cdot x}\langle P'|T[A^\mu(-x/2)A^\nu(x/2)]|P\rangle. \tag{8}$$

While the diagrams in Fig. 2 contain the leading order contribution to the cross section, they also contain parts which are next to leading order (NLO). The sense in which these are to be understood as ''higher order'' will be made precise

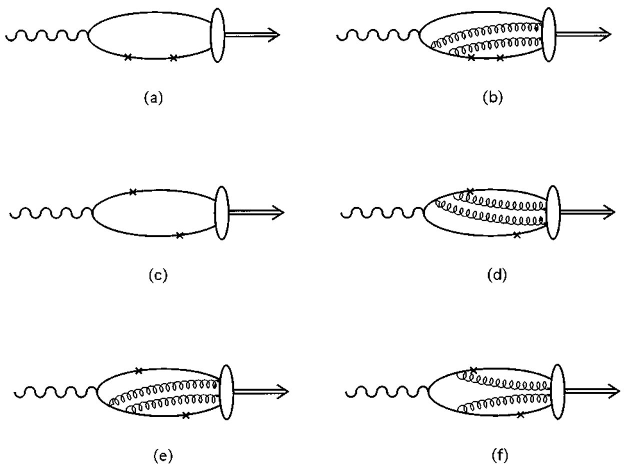

FIG. 3. Diagrams which give a nonzero contribution at order Q^{-1} and v^2. The crosses denote connection to external gluons originating from the proton. The relative weight at this order a:b:c:d:e:f is as -1:1:-2:2:2:-4. Note that each internal gluon zero-momentum gluon line, in the Coulomb gauge, is actually just a differentiation of the quark propagator. Six other diagrams, which are numerically equal by time-reversal invariance, are not shown. The complete expression is given in Eq. (16).

later. Other diagrams will have to be included (see Fig. 3) for a complete calculation at the (NLO) level.

C. Expansion

The diffractive process considered here has two large scales, $Q^2 \gg M^2 \gg \Lambda^2_{\rm QCD}$. Since a $c\bar{c}$ system is close to being a nonrelativistic Coulombic bound state, it allows for an expansion in powers of the heavy quark relative velocity. Hence it is useful to expand the inner integral in Eq. (6):

$$\Omega(k) = \int \frac{d^4\ell}{(2\pi)^4} H^{\mu\nu}(k,\ell) M(\ell) = \sum_{n=0}^{\infty} \Omega_n^{\mu\nu}$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial}{\partial \ell^{\alpha_1}} \cdots \frac{\partial}{\partial \ell^{\alpha_n}} H^{\mu\nu}|_{\ell=0} M^{\alpha_1 \cdots \alpha_n}, \qquad (9)$$

where

$$M^{\alpha_1 \cdots \alpha_n} = \int \frac{d^4\ell}{(2\pi)^4} \ell^{\alpha_1} \cdots \ell^{\alpha_n} M(\ell)$$
$$= i\partial^{\alpha_1} \cdots i\partial^{\alpha_n} \langle K,E | T[\psi(x/2)\overline{\psi}(-x/2)]|0\rangle|_{x=0}. \qquad (10)$$

The set of constants $M^{\alpha_1 \cdots \alpha_n}$ provides a description equivalent to that of the original Bethe-Salpeter (BS) wave function in Eq. (7) . The expansion, Eq. (9), is useful because the quarks are nearly on mass shell: $(1/2K+\ell)^2 \approx m^2$ implies that all components of ℓ^μ are small relative to the quark mass m in the meson's rest frame and, in particular, $\ell \cdot n \sim (m/Q)^2$. In the large Q^2 limit this implies considerable simplification, giving a limit approximately independent of ℓ:

$$\frac{1}{(k+q-K/2+\ell)^2 - m^2 + i\varepsilon} \approx \frac{2\xi}{Q^2} \frac{1}{k\cdot n - \xi + i\varepsilon}, \qquad (11)$$

$$\frac{1}{(q-K/2+\ell)^2 - m^2 + i\varepsilon} \approx -\frac{2}{Q^2}. \qquad (12)$$

The inclusion of Fermi motion requires that we keep a sufficient number of derivatives with respect to ℓ in Eq. (9). These may be computed using the simple Ward identity

$$\frac{\partial}{\partial \ell_\alpha} S_F = -S_F \gamma^\alpha S_F, \qquad (13)$$

and the $Q^2 \to \infty$ limit should be taken after performing the trace algebra. Stated in words, a differentiation of either propagator in Eq. (6) with respect to ℓ splits that propagator into two. Since we shall work upto $O(v^2)$, only two derivatives of $H^{\mu\nu}(k,\ell)$ are needed.

D. Gauge invariance

It is obvious from the occurrence of the ordinary derivatives in Eq. (10), or the form of the BS wave function Eq. (7), that gauge invariance has been violated. In earlier work on quarkonium processes [6–8] we have encountered an identical situation — the diagrams in Fig. 2 yield expressions which are not gauge invariant to $O(v^2)$ and one needs to consider additional diagrams, which are higher order in α_s. These are shown in Fig. 3. The gluon fields indicated in these diagrams combine with the ordinary derivatives to yield covariant derivatives, $\partial^\alpha \to D^\alpha$, thereby restoring gauge invariance. In the Coulomb gauge, the contribution of explicit gluons is $O(v^3)$ and so the reduction of the Bethe-Salpeter equation performed in Ref. [9] without explicit gluons is adequate up to $O(v^2)$. We therefore arrive at the following gauge-invariant matrix elements:

$$\langle K,E|\psi\overline{\psi}|0\rangle = \frac{1}{2} M^{1/2}\left(\phi + \frac{\nabla^2\phi}{M^2}\right) \not{E}^* \left(1 + \frac{\not{K}}{M}\right)$$
$$- \frac{1}{6} M^{1/2} \frac{\nabla^2\phi}{M^2} \not{E}^* \left(1 - \frac{\not{K}}{M}\right),$$

$$\langle K,E|\psi i\overleftrightarrow{D}_\alpha \overline{\psi}|0\rangle = \frac{1}{3} M^{3/2} \frac{\nabla^2\phi}{M^2} E^{*\beta}$$
$$\times \left(g_{\alpha\beta} + i\epsilon_{\alpha\beta\mu\nu}\gamma^\mu\gamma_5 \frac{K^\nu}{M}\right),$$

$$\langle K,E|\psi i\overleftrightarrow{D}_\alpha i\overleftrightarrow{D}_\beta \overline{\psi}|0\rangle = \frac{1}{6} M^{5/2} \frac{\nabla^2\phi}{M^2}$$
$$\times \left(g_{\alpha\beta} - \frac{K_\alpha K_\beta}{M^2}\right) \not{E}^* \left(1 + \frac{\not{K}}{M}\right). \qquad (14)$$

In the above, ϕ and $\nabla^2\phi$ are the nonrelativistic wave function and its second derivative evaluated at zero separation. Inclusion of $\nabla^2\phi$ amounts to taking the first step towards inclusion of Fermi motion.

E. Traces

All the ingredients are now in place for calculating the trace of the quark loops. Because we shall need only the leading twist piece, symmetric in μ and ν, it will be sufficient to calculate

$$\Omega_n=\mathrm{Tr}\sum_{i=1,2}\Omega_n^{ii}=(-g_{\mu\nu}+p_\mu n_\nu+p_\nu n_\mu)\mathrm{Tr}[\Omega_n^{\mu\nu}] \tag{15}$$

for $n=0,1,2$ (n is the order of differentiation with respect to ℓ) and then keep only the leading order term in $O(1/Q)$. We record below the results of the calculation listing, for clarity, the relative contribution of only those diagrams which give a nonzero contribution:

$$\Omega_0=-\frac{4e_Q g^2\phi(0)}{M^{1/2}Q}2(1+2)\left(1+\frac{2}{3}\frac{\nabla^2\phi}{M^2\phi}\right)+O(1/Q^3),$$

$$\Omega_1=O(1/Q^3),$$

$$\Omega_2=\frac{4e_Q g^2\phi(0)}{M^{1/2}Q}2\left(\frac{2}{3}+\frac{4}{3}+\frac{4}{3}-\frac{8}{3}\right)\frac{\nabla^2\phi}{M^2\phi}+O(1/Q^3). \tag{16}$$

The factor of 2 multiplying the brackets in the above equations comes from the diagrams which are permutations of the ones shown. The sum over all diagrams is

$$\Omega=-\frac{24e_Q g^2}{M^{1/2}Q}\phi(0)\left(1+\frac{4}{9}\frac{\nabla^2\phi}{M^2\phi}\right)+O(1/Q^3). \tag{17}$$

Note that this leading order contribution is in fact independent of the gluon momentum k in the $Q^2\to\infty$ limit. The term in the brackets represents the correction due to the Fermi motion of the heavy quarks and its square is precisely the factor which modifies Eq. (1).

III. GLUON DISTRIBUTION

A. Asymmetric distribution

Let us now return to the amplitude for diffractive scattering, a typical contribution to which is given by Eq. (6). The photon and proton both move along the $\hat{z}$ direction, and the gluons in the proton have limited $k_\perp^2$ and k^2. This means that one can perform a systematic collinear expansion in these quantities just as in the treatment of deep-inelastic scattering [12]:

$$H^{\mu\nu}(k,\ell)=H^{\mu\nu}(k^+,\ell)+(k-k^+)_\alpha\partial^\alpha \times H^{\mu\nu}(k,\ell)|_{k=k^+}+\cdots. \tag{18}$$

Keeping only the first, leading twist, term gives, in the $A^+=0$ gauge,

$$\int\frac{d^4k}{(2\pi)^4}S_{\mu\nu}(k,\Delta)H^{\mu\nu}(k,\ell) =\int dy\int\frac{d\lambda}{2\pi}e^{i\lambda(y-\xi/2)} \times\left\langle P'\left|\left[A_\mu\left(-\frac{\lambda}{2}n\right)A_\nu\left(\frac{\lambda}{2}n\right)\right]\right|P\right\rangle H^{\mu\nu}(y,\ell). \tag{19}$$

In the above we have set $x^-=\lambda n^-$ and $k^+=yp^+$. The time-ordering operation becomes irrelevant on the light cone.

The inner integral will now be analyzed following the discussion given by Radyushkin [11]. Define the "asymmetric distribution function" $F_\xi(X)$ as below:

$$\left\langle P'\left|n^-G^{+i}\left(-\frac{\lambda}{2}n\right)n^-G^{+i}\left(\frac{\lambda}{2}n\right)\right|P\right\rangle =\frac{1}{2}\overline{u(p')}\not{n}u(p)\int_0^1 dX\{e^{i\lambda(X-\xi/2)}+e^{-i\lambda(X-\xi/2)}\}F_\xi(X). \tag{20}$$

A sum over transverse components ($i=1,2$) is implied. The proton spinor product is $\overline{u(p')}\not{n}u(p)=2\sqrt{1-\xi}$, with the initial and final protons having the same helicity and $p'=(1-\xi)p$. Making a Fourier transformation yields

$$\theta(y)F_\xi(y)+\theta(\xi-y)F_\xi(\xi-y) =\frac{1}{\sqrt{1-\xi}}\int\frac{d\lambda}{2\pi}e^{i\lambda(y-\xi/2)} \times\left\langle P'\left|n^-G^{+i}\left(-\frac{\lambda}{2}n\right)n^-G^{+i}\left(\frac{\lambda}{2}n\right)\right|P\right\rangle. \tag{21}$$

It is instructive to insert a complete set of states for $y>\xi>0$:

$$F_\xi(y)=\frac{y(y-\xi)}{\sqrt{1-\xi}}\sum_k\delta(y-1+x)\langle P'|A^i|k\rangle\langle k|A^i|P\rangle. \tag{22}$$

Here $x=k\cdot n$ with $0<x<1$ is the momentum fraction carried by the intermediate state. Comparing with the usual (diagonal) gluon distribution function for $\xi=0$ it immediately follows that

$$F_{\xi=0}(y)=yg(y),$$

$$g(y)=y\sum_k\delta(y-1+x)\langle P|A^i|k\rangle\langle k|A^i|P\rangle. \tag{23}$$

We shall now relate the matrix element in Eq. (19) to $F_\xi(y)$. Inverting the relation $G^{+i}=\partial^+A^i$ gives

$$A^i(\lambda n)=n^-\int_0^\infty d\sigma G^{+i}(\lambda n+\sigma n). \tag{24}$$

Inserting the above into Eq. (19) and using the definition of $F_\xi(y)$ in Eq. (20),

$$\int \frac{d\lambda}{2\pi} e^{i\lambda(y-\xi/2)} \left\langle P' \left| A^i\left(-\frac{\lambda}{2}n\right) A^i\left(\frac{\lambda}{2}n\right) \right| P \right\rangle$$

$$= -\sqrt{1-\xi}\left\{ \frac{F_\xi(y)}{y(\xi-y-i\varepsilon)} + \frac{F_\xi(\xi-y)}{(\xi-y-i\varepsilon)(y-i\varepsilon)} \right\}. \tag{25}$$

The imaginary part of the above for $y>0$ is

$$-\pi\sqrt{1-\xi}\frac{F_\xi(\xi)}{\xi}\,\delta(\xi-y)$$

and, hence,

$$\mathrm{Im}\int \frac{d^4k}{(2\pi)^4}\frac{d^4\ell}{(2\pi)^4}\mathrm{Tr}[S_{\mu\nu}(k,\Delta)H^{\mu\nu}(k,\ell)M(\ell)]$$

$$= -\frac{1}{2}\pi\sqrt{1-\xi}\frac{F_\xi(\xi)}{\xi}\Omega. \tag{26}$$

B. Cross section

All the ingredients are now in place for calculating the cross section for the diffractive process under consideration:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A|^2$$

$$= \frac{1}{16\pi(Q^2/\xi)^2}\left(\frac{2}{3\sqrt{3}}\right)^2\left(\frac{1}{2}\pi\sqrt{1-\xi}\frac{F_\xi(\xi)}{\xi}\Omega\right)^2. \tag{27}$$

The factor $2/3\sqrt{3}$ comes from summing over colors, and Ω is the quantity calculated in the previous section, Eq. (17), from the expansion of the heavy quark loop integral. Defining Γ to be the leading order decay width into lepton pairs,

$$\Gamma = \frac{16\pi e_Q^2\alpha_e^2}{M^2}, \tag{28}$$

yields the important result

$$\frac{d\sigma}{dt} = \frac{16\pi^3 M\Gamma}{3Q^6}\alpha_s(\overline{Q^2})[\sqrt{1-\xi}F_\xi(\xi)]^2\left(1+\frac{8\nabla^2\phi}{9M^2\phi}\right). \tag{29}$$

Making the approximate identification

$$\sqrt{1-\xi}F_\xi(\xi) \approx \xi g(\xi), \tag{30}$$

and setting the last factor to unity reproduces Eq. (1) once again. This identification was motivated by Eq. (23) but the exact relation between $F_\xi(\xi)$ and $g(\xi)$ is far from clear.

C. Perturbative gluon distribution

$F_\xi(\xi)$ and $g(\xi)$ can be known only if the nonperturbative structure of the proton state is known. However, it would be highly desirable to have at least some partial knowledge of their structure. To this end, consider the following simple solvable problem: Imagine that the target proton is replaced by a single quark which can radiate a gluon. Its light-cone wave function can be computed order by order in perturbation theory, and the leading order matrix element is

$$\langle P's'|A^i|ks\rangle = g\frac{k^+}{p'^+}\frac{1}{k_\perp^2}\sum_\lambda \overline{u}(p's')\not\varepsilon(l\lambda)u(ps)\varepsilon^{*i}(l\lambda), \tag{31}$$

where l and λ are the momenta and transverse polarizations of the emitted gluon. Using

$$\sum_\lambda \varepsilon^\mu(l\lambda)\varepsilon^{\nu*}(l\lambda) = -g^{\mu\nu} + \frac{l^\mu n^\nu + l^\nu n^\mu}{l\cdot n}, \tag{32}$$

and summing over $i=1,2$ gives

$$\langle P'|A^i|k\rangle\langle k|A^i|P\rangle = g^2\frac{2x}{\sqrt{1-\xi}}\frac{1}{k_\perp^2}\frac{1+x^2-\xi}{(1-x)(1-x-\xi)}. \tag{33}$$

Inserting this into Eq. (22) yields

$$F_\xi(y) = g^2\frac{2y(y-\xi)}{(1-\xi)}\int\frac{d^2k}{16\pi^3k_\perp^2}\int dx$$

$$\times\,\delta(y-1+x)\frac{1+x^2-\xi}{(1-x)(1-x-\xi)}. \tag{34}$$

The last integral is both infrared and ultraviolet divergent. It is regulated by inserting a low momentum scale cutoff $\mu=O(\Lambda_{\mathrm{QCD}})$ and a high momentum cutoff $k_\perp=O(Q)$. Multiplying by the color factor $C_F=4/3$, we arrive at the perturbative *asymmetric* gluon distribution inside a quark:

$$F_\xi(y) = \frac{2\alpha_s}{3\pi}\left\{1+\frac{(1-y)^2}{(1-\xi)}\right\}\ln\frac{Q^2}{\mu^2}. \tag{35}$$

Note that

$$\sqrt{1-\xi}F_\xi(\xi) = \frac{2\alpha_s}{3\pi}\sqrt{1-\xi}\{1+(1-\xi)\}\ln\frac{Q^2}{\mu^2}$$

$$= \frac{4\alpha_s}{3\pi}\left(1-\xi+\frac{1}{8}\xi^2+\cdots\right), \tag{36}$$

but that the usual perturbative *symmetric* distribution, which can also be obtained by first putting $\xi=0$ in Eq. (35) and then setting $y=\xi$, is

$$\xi g(\xi) = \frac{4\alpha_s}{3\pi}\left(1-\xi+\frac{1}{2}\xi^2\right). \tag{37}$$

Comparison of the last two formulas gives an estimate of the extent to which the asymmetric distribution departs from the symmetric one as ξ becomes larger.

Finally, we remark that there exists some confusion in the literature about various factors of 2 and 4. First, it is claimed in the work of Brodsky *et al.* [4] that the cross section displayed in Eq. (1) must be multiplied by 1/4. We do not find this to be the case; the result of Ryskin [1,2] appears to be correct. This point has been corroborated in Ref. [3]. A second possible point of confusion concerning the relation between $F_\xi(y)$ and the usual gluon distribution $g(\xi)$ has also now been resolved following the corrected definition (which I have used in the final version of this paper) of $F_\xi(y)$ in Ref. [11].

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