Pseudoscalar heavy quarkonium decays with both relativistic and QCD radiative corrections

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We estimate the decay rates of $\eta_c \to 2\gamma$, $\eta_c' \to 2\gamma$, and $J/\psi \to e^+e^-$, $\psi' \to e^+e^-$, by taking into account both relativistic and QCD radiative corrections. The decay amplitudes are derived in the Bethe-Salpeter formalism. The Bethe-Salpeter equation with a QCD-inspired interquark potential is used to calculate the wave functions and decay widths for these $c\bar{c}$ states. We find that the relativistic correction to the ratio $R \equiv \Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-)$ is negative and tends to compensate the positive contribution from the QCD radiative correction. Our estimates give $\Gamma(\eta_c \to 2\gamma) = (6-7)$ keV and $\Gamma(\eta_c' \to 2\gamma) = 2$ keV, which are smaller than their nonrelativistic values. The hadronic widths $\Gamma(\eta_c \to 2g) = (17-23)$ MeV and $\Gamma(\eta_c' \to 2g) = (5-7)$ MeV are then indicated accordingly to the first-order QCD radiative correction, if $\alpha_s(m_c) = 0.26-0.29$. The decay widths for the $b\bar{b}$ states are also estimated. We show that when making the assumption that the quarks are on their mass shells our expressions for the decay widths will become identical to that in the nonrelativistic QCD theory to the next to leading order of v^2 and α_s. [S0556-2821(97)02613-1]

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I. INTRODUCTION

Heavy quarkonium decays may provide useful information on understanding the nature of interquark forces and decay mechanisms. Both QCD radiative corrections and relativistic corrections are important for charmonium decays, because for charmonium the strong-coupling constant $\alpha_s(m_c) \approx 0.3$ [defined in the modified minimal subtraction $\overline{\text{MS}}$ scheme] and the velocity squared of the quark in the meson rest frame $v^2 \approx 0.3$, both are not small. These corrections for bottomonium decays are also appreciable. Decay rates of heavy quarkonia in the nonrelativistic limit with QCD radiative corrections have been studied (see, e.g., Refs. [1–3]). However, the decay rates of many processes are subject to substantial relativistic corrections. In the present paper, we will investigate relativistic corrections to the pseudoscalar quarkonium decays such as $\eta_c \to 2\gamma$ and $\eta_c' \to 2\gamma$, and give an estimate of their widths by taking into account both relativistic and QCD radiative corrections. (For a brief report on this result, see Ref. [4].) For comparison we will also study the leptonic decays of the vector charmonium such as $J/\psi \to e^+e^-$ and $\psi' \to e^+e^-$. The corresponding results for the P-wave and D-wave $c\bar{c}$ states have been given in Ref. [5] by using the same method.

These pseudoscalar charmonium decays are interesting. Experimentally, the branching ratio of $\eta_c \to 2\gamma$ may provide an independent determination of α_s at the charm quark mass, but the measured $\Gamma(\eta_c \to 2\gamma)$ ranges from 6 to 28 keV [6], and the measured η_c total width is also uncertain. As for the η_c', its existence needs to be confirmed, and its two γ decay mode is being searched for by the E835 experiment at the Fermi Lab $p\bar{p}$ collider, and its hadronic decay modes are being studied by the BES Collaboration at the Beijing Electron-Positron Collider (BEPC).

A lot of theoretical work have been done on charmonium and, in particular, on these pseudoscalar charmonium decays [4,5,7–11]. The nonrelativistic quark model gives $\Gamma(\eta_c \to 2\gamma) = 8.5$ keV (using the observed J/ψ leptonic width as input), while the QCD sum rule approach predicts a value of 4.6 ± 0.4 keV [9]. References [12–15] have given some results on the two-photon width of quarkonia in the relativistic quark models. In these papers, two-photon decay widths of S-wave, P-wave, and D-wave quarkonium states were calculated. But detail discussions about the source of relativistic corrections were not given, and they have not elaborated the relationship between their model and the newly developed factorization formalism for production and decay of heavy quarkonium.

Recently, there have been significant progresses in the study of heavy quarkonium decays based on a more fundamental approach of the nonrelativistic QCD (NRQCD) effective theory [16,17]. The factorization theorem was further discussed, and some important issues (e.g., the infrared divergences in the P-wave state decay rates) were clarified in this study. The NRQCD theory combined with nonperturbative lattice simulations have achieved many interesting results on heavy quarkonium spectrum and decays [18–20].

In this paper, we will use the Bethe-Salpeter (BS) formalism [21] to derive the decay amplitudes and to calculate the decay widths of S-wave heavy quarkonium. The meson will be treated as a bound state consist of a pair of constituent quark and antiquark (i.e., higher Fock states such as $|Q\overline{Q}g\rangle$ and $|Q\overline{Q}gg\rangle$ are neglected, which may be justified to the first-order relativistic corrections of S-wave heavy quarkonium decays) and described by the BS wave function which satisfies the BS equation. A phenomenological QCD-inspired interquark potential will be used to solve for the wave functions and to calculate the decay widths. Both relativistic and QCD radiative corrections to next-to-leading order will be considered based on the factorization assumption for the long-distance and short-distance effects. The remainder of this paper is organized as follows. In Sec. II we derive the Decay amplitudes in the BS formalism. In Sec. III we give the spectrum and decay widths of S-wave heavy quarkonia. A summary and discussion will be given in the last section.

II. DECAY AMPLITUDES IN THE BS FORMALISM

We first consider the $\eta_c \to 2\gamma$ decay. This process proceeds via the $c\bar{c}$ annihilation. In the Bethe-Salpeter (BS) formalism the annihilation matrix element can be written as

$$\langle 0|\overline{QI}Q|P\rangle = \int d^4q \mathrm{Tr}[I(q,P)\chi_P(q)], \tag{1}$$

where $|P\rangle$ represents the heavy quarkonium state, $P(q)$ is the total (relative) momentum of the $Q\bar{Q}$, $\chi_P(q)$ is its four-dimensional BS wave function, and $I(q,P)$ is the interaction vertex of the $Q\bar{Q}$ with other fields (e.g., the photons or gluons) which, in general, may also depend on the variable q^0 (the time-component of the relative momentum). If $I(q,P)$ is independent of q^0 (e.g., if the quarks are on their mass shells in the annihilation), this equation can be written as

$$\langle 0|\overline{QI}Q|P\rangle = \int d^3q \mathrm{Tr}[I(\vec{q},P)\Phi_P(\vec{q})], \tag{2}$$

where

$$\Phi_P(\vec{q}) = \int dq^0 \chi_P(q) \tag{3}$$

is the three-dimensional BS wave function of the $Q\bar{Q}$ meson. Note that in this approximation the decay amplitude is greatly simplified and only the three-dimensional BS wave function is needed (but this does not necessarily require the interquark interaction to be instantaneous). In the BS formalism in the meson rest frame, where $\vec{p}_1 = -\vec{p}_2 = \vec{q}$, $P=(M,0)$, and $p_1(p_2)$ is the quark(antiquark) momentum, M is the meson mass, for the S-wave state, we have

$$\Phi_P^{0^-}(\vec{q}) = \Lambda_+^1(\vec{q})\gamma^0(1+\gamma^0)\gamma_5\gamma^0\Lambda_-^2(-\vec{q})\varphi(\vec{q}),$$

$$\Phi_P^{1^-}(\vec{q}) = \Lambda_+^1(\vec{q})\gamma^0(1+\gamma^0)\not{e}\gamma^0\Lambda_-^2(-\vec{q})f(\vec{q}), \tag{4}$$

where $\Phi_P^{0^-}(\vec{q})$, and $\Phi_P^{1^-}(\vec{q})$ represent the three-dimensional wave functions of the 0^- and 1^- mesons, respectively, $\not{e} = e_\mu\gamma^\mu$, e_μ is the polarization vector of the 1^- meson, φ and f are scalar functions which can be obtained by solving the BS equation for the 0^- and 1^- mesons, and $\Lambda_+(\Lambda_-)$ are the positive (negative) energy projector operators:

$$\Lambda_+^1(\vec{q}) = \Lambda_+(\vec{p}_1) = \frac{1}{2E}(E+\gamma^0\vec{\gamma}\cdot\vec{p}_1 + m\gamma^0),$$

$$\Lambda_-^2(-\vec{q}) = \Lambda_-(\vec{p}_2) = \frac{1}{2E}(E-\gamma^0\vec{\gamma}\cdot\vec{p}_2 - m\gamma^0),$$

$$E = \sqrt{\vec{q}^2+m^2}. \tag{5}$$

The general formalism of the BS wave function for any state $^{2S+1}L_J$ and the corresponding reduced BS equation were derived in Ref. [5].

For process $\eta_c \to 2\gamma$ with the photon momenta and polarizations q_1, ϵ_1 and q_2, ϵ_2, the decay amplitude can be written as

$$T = \langle 0|\overline{c}\Gamma_{\mu\nu}(q)c|\eta_c\rangle \epsilon_1^\mu(\lambda_1)\epsilon_2^\nu(\lambda_2) + \langle 0|\overline{c}\Gamma'_{\mu\nu}(q)c|\eta_c\rangle \epsilon_2^\mu(\lambda_2)\epsilon_1^\nu(\lambda_1), \tag{6}$$

where $p_1(p_2)$ is the charm quark(antiquark) momentum, $p=p_1-q_1$, $p'=p_1-q_2$, m and M represent the masses of the c quark and the η_c meson, respectively, and

$$\Gamma_{\mu\nu}(q) = \gamma_\mu \frac{e^2e_Q^2}{\hat{p}-m}\gamma_\nu, \quad \Gamma'_{\mu\nu}(q) = \gamma_\mu \frac{e^2e_Q^2}{\hat{p}'-m}\gamma_\nu, \tag{7}$$

$e_Q = \frac{2}{3}$ for $Q=c$. Since $p_1^0 + p_2^0 = M$, as usual, we take [1,10]

$$p_1^0 = p_2^0 = \frac{M}{2}. \tag{8}$$

Thus, $p^0 = \frac{1}{2}M - q_1^0 = 0$, $p'^0 = \frac{1}{2}M - q_2^0 = 0$, and the amplitude T becomes independent of q^0. Employing Eqs. (2) and (4), we get

$$T = B\epsilon^{\rho\sigma\mu\nu}q_{1\rho}q_{2\sigma}\epsilon_{1^\mu}(\lambda_1)\epsilon_{2^\nu}(\lambda_2)e^2e_Q^2 - B'\epsilon^{\rho\sigma\mu\nu}q_{1\rho}q_{2\sigma}\epsilon_{1^\nu}(\lambda_1)\epsilon_{2^\mu}(\lambda_2)e^2e_Q^2, \tag{9}$$

where

$$B = B',$$

$$B = i\frac{2m}{M}\int d\vec{q}\frac{\sqrt{\vec{q}^2+m^2}+m}{(\vec{q}^2+m^2)(\vec{q}^2+\vec{q}_1^2+m^2-2\vec{q}\cdot\vec{q}_1)}\varphi(\vec{q}). \tag{10}$$

Using $q_1\cdot\epsilon_1 = 0$ and $q_2\cdot\epsilon_2 = 0$, it is easy to get the decay width

$$\Gamma(\eta_c \to 2\gamma) = 3M^3\pi\alpha^2e_Q^4|B|^2. \tag{11}$$

In the nonrelativistic (NR) limit ($M \simeq 2m$, $\vec{q}^2 \to 0$)

$$|B|^2 = \frac{1}{2m^5}|\psi(0)|^2, \tag{12}$$

where we have used the relation

$$\int \varphi(\vec{q})d\vec{q} = \frac{\sqrt{M}}{2}\psi(0), \tag{13}$$

where $\psi(0)$ is the Schrödinger wave function at origin in coordinate space. Substituting Eq. (12) into Eq. (11), we get

$$\Gamma^{\mathrm{NR}}(\eta_c \to 2\gamma) = 12\pi\alpha^2e_Q^4|\psi(0)|^2/m^2, \tag{14}$$

where $\Gamma^{\mathrm{NR}}(\eta_c \to 2\gamma)$ represents the decay width of $\eta_c \to 2\gamma$ in the nonrelativistic limit, which is consistent with that given in Ref. [1]. The QCD radiative correction to this process has been given in Ref. [1]. Recently, in the framework of NRQCD the factorization formulas for the long-distance and short-distance effects were found to involve a double expansion in the quark relative velocity v and in the QCD coupling constant α_s [16]. To next to leading order in both v^2 and α_s, as an approximation, we may write

$$\Gamma(\eta_c \to 2\gamma) = 3M^3 \pi \alpha^2 e_Q^4 |B|^2 \left(1 - \frac{3.4\alpha_s(m_c)}{\pi}\right), \quad (15)$$

where the strong-coupling constant $\alpha_s(m_c)$ is defined in the $\overline{\mathrm{MS}}$ scheme. By expanding B in Eq. (10) in terms of $\vec{q}^2/m^2$, to the next to leading order of v^2 we have

$$B = \frac{16i}{M(M^2+4m^2)} \int d\vec{q}\, \varphi(\vec{q}) \left(1 - \frac{11}{12}\frac{\vec{q}^2}{m^2}\right). \quad (16)$$

We see that the relativistic kinematic effect is to suppress the $\eta_c \to 2\gamma$ decay width.

For comparison with the process $J/\psi \to e^+e^-$, we also give the decay amplitude for the $Q\bar{Q}$ annihilation into an electron with momentum k_1 and helicity r_1 and a positron with momentum k_2 and helicity r_2. Here the interaction vertex $I(P,q) = -ie\gamma_\mu$, which is independent of q^0, and the amplitude can be written as

$$T = e^2 e_Q \langle 0|\bar{c}\gamma_\mu c|J/\psi\rangle \bar{u}_{r_1}(k_1)\gamma^\mu v_{r_2}(k_2)\frac{1}{M^2}. \quad (17)$$

Define the decay constant f_V by

$$f_V M e_\mu \equiv \langle 0|\bar{c}\gamma_\mu c|J/\psi\rangle = \int d\vec{q}\, \mathrm{Tr}[\gamma_\mu \Phi_{\vec{P}}(\vec{q})], \quad (18)$$

where e_μ is the polarization vector of J/ψ meson. Then with Eq. (4) we find

$$f_V = \frac{2\sqrt{3}}{M} \int d\vec{q} \left(\frac{m+E}{E} - \frac{\vec{q}^2}{3E^2}\right) f(\vec{q}), \quad (19)$$

where $E = \sqrt{\vec{q}^2 + m^2}$. Summing over the polarizations of the final states and averaging over that of the initial states, it is easy to get the decay width

$$\Gamma(J/\psi \to e^+e^-) = \frac{4}{3}\pi\alpha^2 e_Q^2 f_V^2/M. \quad (20)$$

In the nonrelativistic limit it is reduced to the well known result

$$\Gamma^{\mathrm{NR}}(J/\psi \to e^+e^-) = 16\pi\alpha^2 e_Q^2 |\psi(0)|^2/M^2. \quad (21)$$

Including also the QCD radiative correction [1], we will get

$$\Gamma(J/\psi \to e^+e^-) = \frac{4}{3}\pi\alpha^2 e_Q^2 \frac{f_V^2}{M}\left(1 - \frac{5.3\alpha_s(m_c)}{\pi}\right). \quad (22)$$

To the next to leading order of v^2, f_V is expressed as

$$f_V = -\frac{4\sqrt{3}}{M} \int d\vec{q}\, f(\vec{q}) \left(1 - \frac{5}{12}\frac{\vec{q}^2}{m^2}\right). \quad (23)$$

Again, the relativistic kinematic correction is to reduce the leptonic decay width. Comparing the two photon width with the leptonic width, we get

$$R \equiv \frac{\Gamma(\eta_c \to 2\gamma)}{\Gamma(J/\psi \to e^+e^-)} = \frac{9}{4} M_{\eta_c}^3 M_{J/\psi} e_Q^2 \frac{|B|^2}{f_V^2}\left(1 + 1.96\frac{\alpha_s(m_c)}{\pi}\right), \quad (24)$$

and in the nonrelativistic limit it becomes

$$R^{\mathrm{NR}} \equiv \frac{\Gamma^{\mathrm{NR}}(\eta_c \to 2\gamma)}{\Gamma^{\mathrm{NR}}(J/\psi \to e^+e^-)} = \frac{4}{3}\left(1 + 1.96\frac{\alpha_s(m_c)}{\pi}\right). \quad (25)$$

In fact, there are two sources of relativistic corrections: (i) the correction of relativistic kinematics which appears explicitly in the decay amplitudes; (ii) the correction due to interquark dynamics (e.g., the well-known Breit-Fermi interactions), which mainly causes the correction to the bound state wave functions. In general, due to the attractive spin-spin force induced by one gluon exchange for the 0^- meson, the η_c wave function at origin becomes larger than its nonrelativistic value and one might expect the width of $\eta_c \to 2\gamma$ to be enhanced after taking relativistic corrections into account. However, because the kinematic relativistic correction to the decay rates is in the opposite direction and can be even larger, the overall relativistic correction to the decay width of η_c is found to be negative.

III. SPECTRA AND DECAY WIDTHS OF S-WAVE HEAVY QUARKONIA

To calculate the decay widths, we need to know the wave functions $\varphi(\vec{q})$ for the 0^- meson and $f(\vec{q})$ for the 1^- meson, which are determined mainly by the long-distance interquark dynamics. In the absence of a deep understanding for quark confinement at present, we will follow a phenomenological approach by using QCD inspired interquark potentials including both spin-independent and spin-dependent potentials, which are supported by both lattice QCD calculations and heavy quark phenomenology, as the interaction kernel in the BS equation. We begin with the bound state BS equation [21] in momentum space,

$$(\not{q}_1 - m_1)\chi_P(q)(\not{q}_2 + m_2) = \frac{i}{2\pi}\int d^4k\, G(P,q-k)\chi_P(k), \quad (26)$$

where q_1 and q_2 represent the momenta of quark and antiquark, respectively, and $G(P,q-k)$ is the interaction kernel which dominates the interquark dynamics. In solving Eq. (26), we will employ the instantaneous approximation since for heavy quarks the interaction is dominated by instantaneous potentials [15,22]. Meanwhile, we will neglect negative energy projectors in the quark propagators which are of even higher orders [23]. We then get the reduced Salpeter equation [21] for the three-dimensional BS wave function $\Phi_P(\vec{q})$ defined in Eq. (3),

$$\Phi_P(\vec{q})=\frac{1}{P^0-E_1-E_2}\Lambda_+^1\gamma^0\int d^3kG(P,\vec{q}-\vec{k})\Phi_P(\vec{k})\gamma^0\Lambda_-^2, \tag{27}$$

where $G(P,\vec{q}-\vec{k})$ represents the instantaneous potential.

We employ the following interquark potentials including a screened long-ranged confinement potential (Lorentz scalar) and a short-ranged one-gluon exchange (OGE) potential (Lorentz vector) [24]:

$$V(r)=V_S(r)+\gamma_\mu\otimes\gamma^\mu V_V(r),$$
$$V_S(r)=\lambda r\frac{(1-e^{-\alpha r})}{\alpha r},$$
$$V_V(r)=-\frac{4}{3}\frac{\alpha_s(r)}{r}e^{-\alpha r}, \tag{28}$$

where the introduction of the factor $e^{-\alpha r}$ is to regulate the infrared divergence and also to incorporate the color screening effects of the dynamical light quark pairs on the $Q\overline{Q}$ linear confinement potential. This is consistent with the result derived from lattice calculations [25]. In fact, when $\alpha\rightarrow 0$, the screened long-range potential will become the standard linear confinement potential and for small values of α, say $\alpha<0.06$ GeV, the mass sprectrum for the low-lying states are quite stable and insensitive to the value of α. We will also find later that the dependence of the decay widths on the parameter α is very weak. In momentum space the potentials become [24]

$$G(\vec{p})=G_S(\vec{p})+\gamma_\mu\otimes\gamma^\mu G_V(\vec{p}),$$
$$G_S(\vec{p})=-\frac{\lambda}{\alpha}\delta^3(\vec{p})+\frac{\lambda}{\pi^2}\frac{1}{(\vec{p}^2+\alpha^2)^2},$$
$$G_V(\vec{p})=-\frac{2}{3\pi^2}\frac{\alpha_s(\vec{p})}{\vec{p}^2+\alpha^2}, \tag{29}$$

where $\alpha_s(\vec{p})$ is the quark-gluon running coupling constant and is assumed to become a constant of order 1 as $\vec{p}^2\rightarrow 0$:

$$\alpha_s(\vec{p})=\frac{12\pi}{27}\frac{1}{\ln(a+\vec{p}^2/\Lambda_{\mathrm{QCD}}^2)}. \tag{30}$$

The constants λ, α, a, and Λ_{QCD} are the parameters that characterize the potential.

Substituting Eqs. (4) and (29) into Eq. (27), one derives the equation for the 0^- meson wave function $\varphi(\vec{q})$ in the meson rest frame:

$$\begin{aligned}M\varphi_1(\vec{q})=&(E_{q1}+E_{q2})\varphi_1(\vec{q})+\frac{1}{4E_{q1}E_{q2}}\bigg(-(E_{q1}E_{q2}+m_1m_2+\vec{q}^2)\int d^3k[G_S(\vec{q}-\vec{k})-4G_V(\vec{q}-\vec{k})]\varphi_1(\vec{k})\\&-(E_{q1}m_2+E_{q2}m_1)\int d^3k[G_S(\vec{q}-\vec{k})+2G_V(\vec{q}-\vec{k})]\frac{m_1+m_2}{E_{k1}+E_{k2}}\varphi_1(\vec{k})+(E_{q1}+E_{q2})\int d^3kG_S(\vec{q}-\vec{k})\\&\times(\vec{q}\cdot\vec{k})\frac{m_1+m_2}{E_{k1}m_2+E_{k2}m_1}\varphi_1(\vec{k})+(m_1-m_2)\int d^3k[G_S(\vec{q}-\vec{k})+2G_V(\vec{q}-\vec{k})](\vec{q}\cdot\vec{k})\frac{E_{k1}-E_{k2}}{E_{k1}m_2+E_{k2}m_1}\varphi_1(\vec{k})\bigg),\end{aligned} \tag{31}$$

where $E_{qi}=\sqrt{\vec{q}^2+m_i^2}$, $E_{ki}=\sqrt{\vec{k}^2+m_i^2}$, $(i=1,2)$, and

$$\varphi_1(\vec{q})=\frac{(m_1+m_2+E_{q1}+E_{q2})(E_{q1}m_2+E_{q2}m_1)}{4E_{q1}E_{q2}(m_1+m_2)}\varphi(\vec{q}). \tag{32}$$

The normalization condition $\int d^3q\mathrm{Tr}\{\Phi^\dagger(\vec{q})\Phi(\vec{q})\}=(2\pi)^{-3}2M$ for the BS wave function leads to [24]

$$\int d^3q\frac{(m_1+E_{q1})(m_2+E_{q2})}{8E_{q1}E_{q2}}|\varphi(\vec{q})|^2=\frac{M}{(4\pi)^3}. \tag{33}$$

For the 1^- meson we have

$$\begin{aligned}Mf_1(\vec{q})=&(E_{q1}+E_{q2})f_1(\vec{q})-\frac{E_{q1}+m_1+E_{q2}+m_2}{4E_{q1}E_{q2}[3(E_{q1}+m_1)(E_{q2}+m_2)+\vec{q}^2]}\bigg\{(E_{q1}E_{q2}+m_1m_2+\vec{q}^2)\int d^3k[G_S(\vec{q}-\vec{k})\\&-2G_V(\vec{q}-\vec{k})]\frac{3(E_{k1}+m_1)(E_{k2}+m_2)+\vec{k}^2}{E_{k1}+E_{k2}+m_1+m_2}f_1(\vec{k})-2\vec{q}^2\int d^3k[G_S(\vec{q}-\vec{k})-2G_V(\vec{q}-k)]\frac{E_{k2}m_1+E_{k1}m_2}{m_1+m_2}f_1(\vec{k})\\&+(E_{q1}m_2+E_{q2}m_1)\int d^3kG_S(\vec{q}-\vec{k})\frac{3(E_{k1}+m_1)(E_{k2}+m_2)-\vec{k}^2}{E_{k1}+E_{k2}+m_1+m_2}f_1(\vec{k})-(m_1+m_2)\int d^3k[G_S(\vec{q}-\vec{k})\end{aligned}$$

TABLE I. Mass spectrum (in MeV) and electromagnetic decay widths (in keV) of the $c\bar{c}$ system (decay widths for $\alpha=0.04$ are given in parentheses).

State	Expt. mass	Theo. mass	Expt. width	Theo. width	Resag S [15]	Resag V [15]
1^1S_0	2980	2979		5.5(6.6)	4.2	3.8
1^3S_1	3097	3097	5.26 ± 0.37	4.5(6.2)	8.05	9.21
2^1S_0		3618		2.1(1.8)		
2^3S_1	3685	3677	2.14 ± 0.21	2.5(2.9)	4.30	5.87
3^3S_1	4040	4062	0.75 ± 0.15	1.7(2.0)	3.05	4.81
4^3S_1	4415	4353	0.47 ± 0.10	1.3(1.2)	2.16	3.95

$$+4G_V(\vec{q}-\vec{k})](\vec{q}\cdot\vec{k})f_1(\vec{k})-2(E_{q1}-E_{q2})\int d^3k[G_S(\vec{q}-\vec{k})-2G_V(\vec{q}-\vec{k})](\vec{q}\cdot\vec{k})\frac{E_{k1}+m_1}{E_{k1}+m_1+E_{k2}+m_2}$$
$$\times f_1(\vec{k})+\int d^3k[4G_S(\vec{q}-\vec{k})-8G_V(\vec{q}-\vec{k})](\vec{q}\cdot\vec{k})^2\frac{f_1(\vec{k})}{E_{k1}+m_1+E_{k2}+m_2}-2(m_1-m_2)\int d^3kG_S(\vec{q}-\vec{k})$$
$$\times\frac{E_{k1}-E_{k2}}{m_1+m_2}(\vec{q}\cdot\vec{k})f_1(\vec{k})+(E_{q1}+3E_{q2})\int d^3kG_S(\vec{q}-\vec{k})(\vec{q}\cdot\vec{k})f_1(\vec{k})-(6E_{q1}+2E_{q2})$$
$$\times\int d^3kG_V(\vec{q}-\vec{k})\vec{q}\cdot\vec{k}f_1(\vec{k}), \tag{34}$$

where

$$f_1(\vec{q})=\frac{E_{q1}+m_1+E_{q2}+m_2}{4E_{q1}E_{q2}}f(\vec{q}). \tag{35}$$

The normalization condition

$$\int d^3q\mathrm{Tr}\{\Phi^\dagger(\vec{q})\Phi(\vec{q})\}=(2\pi)^{-3}2M$$

for the BS wave function leads to [24]

$$\int d^3q\frac{(m_1+E_{q1})(m_2+E_{q2})}{8E_{q1}E_{q2}}|f(\vec{q})|^2=\frac{M}{(4\pi)^3}. \tag{36}$$

To leading order in the nonrelativistic limit, Eqs. (31) and (34) are just ordinary nonrelativistic Schrödinger equations with simply a spin-independent linear plus a Coulomb potential. To first order of v^2, Eqs. (31) and (34) become the well-known Breit equations for the 0^- and 1^- mesons with both spin-independent and spin-dependent potentials from vector (one-gluon) exchange and scalar (confinement) exchange. For the P wave and D wave, wave functions and corresponding reduced BS equations have been derived in other papers [26].

For the heavy quarkonium $c\bar{c}$ and $b\bar{b}$ systems, $m_1=m_2=m$, Eqs. (31) and (34) become much simpler. By solving these equations we can find the wave functions for the 0^- and 1^- mesons. Here not only the ground-state wave functions but also the radial excitation wave functions are obtained.

Substituting the obtained BS wave functions into Eqs. (10), (15), (19), and (22), respectively, we then get the decay widths for both the 0^- and 1^- quarkonium states. In the calculation, the following parameters have been chosen:

$$m_c=1.5\ \text{GeV},\quad m_b=4.88\ \text{GeV},$$
$$\lambda=0.23\ \text{GeV}^2,\quad \Lambda_{\mathrm{QCD}}=0.22\ \text{GeV},$$
$$\alpha=0.06\ \text{GeV},\quad a=e=2.7183. \tag{37}$$

With these parameters, we then get the mass spectrum and decay widths for $c\bar{c}$ (Table I) and $b\bar{b}$ (Table II) systems. Here the decay widths for the $^1S_0$ states mean $\Gamma(^1S_0\to2\gamma)$ while those for the $^3S_1$ states mean $\Gamma(^3S_1\to e^+e^-)$. In these results we have included both relativistic and QCD radiative corrections, for which $\alpha_s(m_c)=0.29$ and $\alpha_s(m_b)=0.20$ are taken [3,26]. In Tables I and II, we have listed the results of [15], where the interaction kernel was taken as

$$V(r)=V_S(r)+\gamma_\mu\otimes\gamma^\mu V_V(r), \tag{38}$$

corresponding to the S column in Tables I and II or

$$V(r)=-\gamma_0\otimes\gamma_0V_S(r)+\gamma_\mu\otimes\gamma^\mu V_V(r), \tag{39}$$

corresponding to the V column in Tables I and II. In Eqs. (38) and (39), $V_S(r)$ and $V_V(r)$ read

$$V_S(r)=a_c+b_cr$$

and

$$V_V(r)=-\frac{4\alpha_s(r)}{3r}\quad \text{for}\ r>r_0,$$

TABLE II. Mass spectrum (in MeV) and electromagnetic decay widths (in keV) of the $b\bar{b}$ system.

State	Expt. mass	Theo. mass	Exp. width	Theo. width	Resag S [15]	Resag V [15]
1^1S_0		9412		0.45		
1^3S_1	9460	9460	1.32 ± 0.03	1.22	0.80	0.84
2^1S_0		9999		0.21		
2^3S_1	10 023	10 019	0.58 ± 0.10	0.72	0.54	0.57
3^3S_1	10 355	10 376	0.48 ± 0.05	0.53	0.44	0.47
4^3S_1	10 580	10 652	0.24 ± 0.05	0.43	0.40	0.49
5^3S_1	10 860	10 881	0.31 ± 0.07	0.36		
6^3S_1	11 020	11 078	0.130 ± 0.030	0.30		

$$V_V(r)=a_G r^2+b_G \quad \text{for} \quad r\leqslant r_0,$$

where potential parameters include a_c, b_c, a_G, and b_G.

We see that the mass spectra are in agreement with data. In particular, we predict the hyperfine splittings

$$m(J/\psi)-m(\eta_c)=118 \text{ MeV},$$

$$m(\Upsilon)-m(\eta_b)=48 \text{ MeV}.$$

The leptonic widths for vector $c\bar{c}$ and $b\bar{b}$ states are also in resonably good agreement with data. The calculated $\Gamma(J/\psi\to e^+e^-)$ is slightly smaller than its experimental value while the results of [15] are smaller for $^1S_0\to\gamma\gamma$ and larger for $^3S_1\to e^+e^-$ compared with the $c\bar{c}$ experimental data. It is also obvious that our results for the decay widths are insensitive to the choice of α.

In order to reduce the uncertainty in our calculation for the absolute decay widths for $\eta_c\to 2\gamma$ and $\eta_c'\to 2\gamma$, we use the calculated ratios $\Gamma(\eta_c\to 2\gamma)/\Gamma(J/\psi\to e^+e^-)=1.23$ and $\Gamma(\eta_c'\to 2\gamma)/\Gamma(\psi'\to e^+e^-)=0.85$, and the observed $\Gamma(J/\psi\to e^+e^-)=5.26\pm0.37$ keV and $\Gamma(\psi'\to e^+e^-)=2.14\pm0.21$ keV, and then get

$$\Gamma(\eta_c\to 2\gamma)\approx 6.5 \text{ keV},$$

$$\Gamma(\eta_c'\to 2\gamma)\approx 1.8 \text{ keV}.$$

These values should be viewed as our prediction for the pseudoscalar decay widths.

In order to examine the sensitivity of the calculated decay widths to the potential parameters, we choose another set of parameters:

$$m_c=1.5 \text{ GeV}, \quad \lambda=0.23 \text{ GeV}^2, \quad \Lambda_{\text{QCD}}=0.18 \text{ GeV},$$

$$\alpha=0.06 \text{ GeV}, \quad a=e=2.7183.$$

We then get

$$\Gamma(J/\psi\to e^+e^-)=5.7 \text{ keV}, \quad \Gamma(\psi'\to \mathrm{e}^+\mathrm{e}^-)=2.7 \text{ keV},$$

$$\Gamma(\eta_c\to 2\gamma)=6.3 \text{ keV}, \quad \Gamma(\eta_c'\to 2\gamma)=1.8 \text{ keV}.$$

In order to see further the sensitivity of the decay widths to the parameters (especially the charm quark mass) we have also used another two sets of parameters:

$$m_c=1.4 \text{ GeV}, \quad \lambda=0.24 \text{ GeV}^2,$$

$$m_c=1.6 \text{ GeV}, \quad \lambda=0.22 \text{ GeV}^2, \tag{40}$$

with the other potential parameters (Λ_{QCD}, α, a) unchanged from Eq. (37), and found

$$\Gamma(\eta_c\to 2\gamma)=7.0(5.5) \text{ keV}, \quad \Gamma(\eta_c'\to 2\gamma)=1.7(1.5) \text{ keV} \tag{41}$$

for $m_c=1.4$ (1.6) GeV, where the experimental value of $\Gamma(J/\psi\to e^+e^-)=5.26$ keV (as input) and the calculated ratio R in Eq. (24) are used to give predictions for the pseudoscalar decay widths.

We see that for smaller charm quark masses $\Gamma(\eta_c\to 2\gamma)$ gets enhanced. This tendency is in line with the QCD sum rule result [9]. We estimate that $\Gamma(\eta_c\to 2\gamma)=(6-7)$ keV is consistent with the CLEO data [27] $\Gamma(\eta_c\to 2\gamma)=(5.9^{+2.1}_{-1.8}\pm1.9)$ keV, and the E760 data [28] 7 ± 3 keV, and slightly smaller than the L3 data [29] $(8.0\pm2.3\pm2.4)$ keV. Our results for η_c and η_c' distinguish them from the nonrelativistic values, which can be obtained by using the ratio R^{NR} [Eq. (25)] and the experimental values of $\Gamma[J/\psi(\psi')\to e^+e^-]$:

$$\Gamma^{\text{NR}}(\eta_c\to 2\gamma)=8.5 \text{ keV}, \quad \Gamma^{\text{NR}}(\eta_c'\to 2\gamma)=3.4 \text{ keV}. \tag{42}$$

In particular, our prediction $\Gamma(\eta_c'\to 2\gamma)=2$ keV is significantly smaller than its nonrelativistic value.

We may further use these results to give an estimate for the total widths of η_c and η_c'. Note the branching ratio

$$B(P\to 2\gamma)\approx\frac{\Gamma(P\to 2\gamma)}{\Gamma(P\to 2g)}=\frac{9\alpha^2 e_Q^4}{2\alpha_s^2}\left(\frac{1-3.4\alpha_s/\pi}{1+4.8\alpha_s/\pi}\right) \tag{43}$$

is free of the relativistic correction. Using the calculated two γ widths $\Gamma(P\to 2\gamma)=6.2(1.8)$ keV for $P=\eta_c(\eta_c')$ and the strong-coupling constant at the mass of the charm quark $\alpha_s(m_c)=0.26-0.29$ [3,5,11], we will get

$$\Gamma_{\text{tot}}(\eta_c)=17-23 \text{ MeV},$$

$$\Gamma_{\text{tot}}(\eta_c')=5.0-6.7 \text{ MeV}. \tag{44}$$

This is the prediction for the total widths up to the next to leading order of QCD radiative corrections, but higher order corrections may further modify this result. With the present Particle Data Group values [6] $\Gamma_{\text{tot}}(\eta_c)=10.3^{+3.8}_{-3.4}$ MeV and $\Gamma(\eta_c\to 2\gamma)=6.0^{+2.0}_{-1.7}$ keV, however, a value of

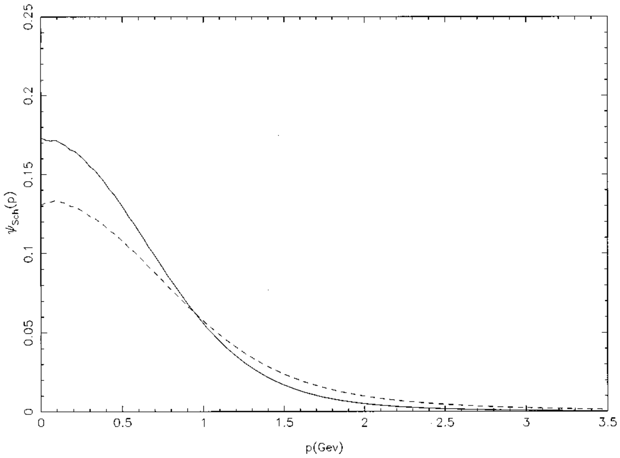

FIG. 1. Wave functions $\psi_{\rm Sch}(p)$ (normalized in momentum space) of J/ψ (solid line) and η_c (dashed line) by solving the BS equation with $m_c = 1.5$ GeV.

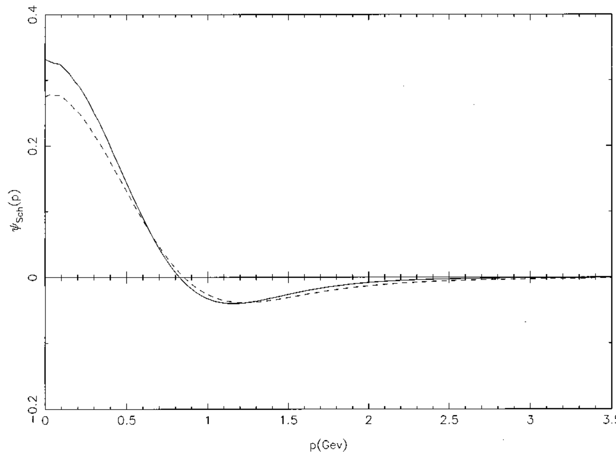

FIG. 2. Wave functions $\psi_{\rm Sch}(p)$ (normalized in momentum space) of first excited state ψ' (solid line) and η_c' (dashed line) by solving the BS equation with $m_c = 1.5$ GeV.

$\alpha_s(m_c) \approx 0.20$ will be indicated, which is significantly lower than expected from other experiments and theoretical studies on the QCD scale parameter. Therefore, it will be very interesting to see the accuracy of the experiment or to take the higher order QCD radiative correction more seriously.

IV. DISCUSSIONS

We finally discuss the relation between our approach and the NRQCD theory. In fact, our decay widths can be written in terms of the standard Schrödinger wave function (with relativistic corrections) $\psi_{\rm Sch}(\vec{q})$, which is related to $\varphi(\vec{q})$ [or $f(\vec{q})$] through the normalization condition (33) [or (36)] which leads to

$$\psi_{\rm Sch}(\vec{q}) = \frac{1}{\sqrt{M}}\left(\frac{m+E}{E}\right)\varphi(\vec{q}),$$

$$(2\pi)^3 \int d^3q \quad \psi^*_{\rm Sch}(\vec{q})\psi_{\rm Sch}(\vec{q}) = 1. \tag{45}$$

In Figs. 1 and 2 the solved wave function $\psi_{\rm Sch}(\vec{q})$ for the $1S$ and $2S$ charmonium states using parameters (37) are shown. We see explicit differences between cases of $J=0$ and $J=1$, while they are the same in the nonrelativistic limit. The spin-dependent forces (induced mainly by one-gluon exchange) not only cause the fine splittings of masses of $^3S_1$ and $^1S_0$, but also make the wave functions of $^1S_0$ and $^3S_1$ different from each other. Mainly due to the attractive spin-spin force induced by one-gluon exchange for the 0^{-+} meson, the $^1S_0$ wave function in momentum space becomes fatter than the $^3S_1$ wave function in which the spin-spin force is repulsive.

For the above discussed pseudoscalar (P) and vector (V) heavy quarkonium decays [see Eqs. (10), (15), (19), and (22)] to the next-to-leading order in v^2 and α_s we then have

$$\Gamma(P\to 2\gamma) = \frac{192\pi\alpha^2 e_Q^4 M^2}{(M^2+4m^2)^2}\left(1-\frac{3.4\alpha_s(m_Q)}{\pi}\right)$$

$$\times\left|\int d^3q\left(1-\frac{2\vec{q}^2}{3m^2}\right)\psi_{\rm Sch}(\vec{q})\right|^2, \tag{46}$$

$$\Gamma(V\to e^+e^-) = \frac{16\pi\alpha^2 e_Q^2}{M^2}\left(1-\frac{16\alpha_s(m_Q)}{3\pi}\right)$$

$$\times\left|\int d^3q\left(1-\frac{\vec{q}^2}{6m^2}\right)\psi_{\rm Sch}(\vec{q})\right|^2. \tag{47}$$

In above expressions M is the mass of the meson. In previous calculations we have taken M as their observed values for η_c and J/ψ. However, we may take the on-shell condition, which assumes the quark and antiquark to be on the mass shell [see Eq. (8)],

$$q_1^0 = q_2^0 = M/2 = E = \sqrt{m^2+\vec{q}^2}, \tag{48}$$

to replace the observed value of the meson mass M then Eqs. (46) and (47) will become

$$\Gamma(P\to 2\gamma) = \frac{12\pi\alpha^2 e_Q^4}{m^2}\left(1-\frac{3.4\alpha_s(m_Q)}{\pi}\right)$$

$$\times\left|\int d^3q\left(1-\frac{2\vec{q}^2}{3m^2}\right)\psi_{\rm Sch}(\vec{q})\right|^2, \tag{49}$$

$$\Gamma(V\to e^+e^-) = \frac{4\pi\alpha^2 e_Q^2}{m^2}\left(1-\frac{16\alpha_s(m_Q)}{3\pi}\right)$$

$$\times\left|\int d^3q\left(1-\frac{2\vec{q}^2}{3m^2}\right)\psi_{\rm Sch}(\vec{q})\right|^2. \tag{50}$$

It is easy to see that to the first order of v^2, in coordinate space, Eqs. (49) and (50) can be expressed as

$$\Gamma(P\to 2\gamma) = \frac{3\alpha^2 e_Q^4}{m^2}\left(1-\frac{3.4\alpha_s(m_Q)}{\pi}\right)\left[|R(0)|^2 + \frac{4}{3m^2}{\rm Re}[R^*(0)\nabla^2 R(0)]\right], \tag{51}$$

$$\Gamma(V\to e^+e^-)=\frac{\alpha^2 e_Q^2}{m^2}\left(1-\frac{16\alpha_s(m_Q)}{3\pi}\right)\left[|R(0)|^2+\frac{4}{3m^2}\mathrm{Re}[R^*(0)\nabla^2 R(0)]\right], \quad (52)$$

where $R(0)$ is the Schrödinger radial wave function at the origin of the $P(P=\eta_c,\eta_c')$ or V $(V=J/\psi,\psi')$ meson. These expressions are exactly the same as those given in Ref. [16] with the NRQCD effective theory, if we identify our bound state wave functions with their regularized operator matrix elements: i.e.,

$$R(0)=\sqrt{\frac{2\pi}{3}}\epsilon\langle 0|\chi^\dagger\sigma\psi|V\rangle, \quad (53)$$

$$\nabla^2 R(0)=-\sqrt{\frac{2\pi}{3}}\epsilon\left\langle 0|\chi^\dagger\sigma\left(\frac{-i}{2}\overleftrightarrow{D}\right)^2\psi|V\right\rangle \times[1+O(v^2/c^2)]. \quad (54)$$

In the NRQCD theory, the expectation values of the quark operators are well defined [16] and can be calculated with lattice simulations, which is a more fundamental method for describing nonperturbative dynamics than the quark potential model. In our approach the wave functions (and their derivatives) are estimated on the basis of the QCD-inspired potential model by solving the BS equation. Although this is not a first principle theory and it is difficult to control the systematic accuracy within the potential model, it may provide a rather useful estimate of the decay rates, since not only the zeroth order spin-independent potential but also the first-order spin-dependent potential, i.e., the Breit-Fermi Hamiltonian, which stems from one gluon exchange and has a good theoretical and phenomenological basis, are considered in the calculation, and different quark masses are also chosen to estimate the uncertainties in the calculation. In fact, the potentials are required to reproduce the observed mass difference between η_c and J/ψ and the J/ψ leptonic decay width, and then give predictions for the pseudoscalar mesons. This may reduce the uncertainty in the calculation of pseudoscalar decay widths. Nevertheless, for more reliable estimates we hope that these decay widths of heavy quarkonium can be eventually calculated from more fundamental theoretical methods, e.g., the lattice QCD simulations. It will be interesting to see the numerical results in the NRQCD approach and compare them with our results.

In summary, we have estimated the photonic widths and hadronic widths for pseudoscalar heavy quarkonium states, and the leptonic widths for vector heavy quarkonium states as well, by taking into account both relativistic and QCD radiative corrections. The photonic widths of η_c and η_c' tend to take lower values than the nonrelativistic result. We hope that experiments in the future, especially the E835 and BES experiments or experiments at the τ-charm factory, will be able to make more accurate measurements on the branching ratios and the total widths for the η_c and η_c' particles. This will provide the basis for testing theoretical predictions.

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