Radiative neutrino decay in hot media

José F. Nieves
Laboratory of Theoretical Physics, Department of Physics, University of Puerto Rico, P.O. Box 23343, Río Piedras, Puerto Rico 00931-3343

Palash B. Pal
Indian Institute of Astrophysics, Bangalore 560034, India
(Received 7 February 1997)

We calculate the rate for the radiative neutrino decay in a thermal background of electrons and photons, taking into account the effect of the stimulated emission of photons in the thermal bath. We show that the rate is enhanced by a large factor relative to the rate for the corresponding process in the vacuum. [S0556-2821(97)02013-4]

PACS number(s): 13.35.Hb, 11.10.Wx

The notion that the presence of an ambient medium modifies the properties of elementary particles is now well known. Sometimes the effects are dramatic, as is the case of the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism, which has been invoked to explain the solar neutrino puzzle [1]. There, under favorable conditions, the conversion of one neutrino flavor into another is greatly enhanced by the presence of the background. Some time ago it was observed [2] that the electromagnetic properties of neutrinos are also drastically modified within a medium when compared to the properties in vacuum. This fact can lead to such interesting phenomena as the Cherenkov radiation by massless neutrinos [3] and the possible explanation of pulsar velocities by neutrino oscillations biased by a magnetic field [4].

It has also been shown previously [5,6] that the rate for the radiative decay of a massive neutrino that propagates through a background of particles,

$$\nu(k)\rightarrow\nu'(k')+\gamma(q), \tag{1}$$

can be much larger than the rate for the corresponding process in the vacuum. In the calculation of Ref. [5] the background was taken to be a thermal bath of electrons, nucleons, and photons, as is the case of a plasma made of ordinary matter, as well as possibly their antiparticles. Because the medium contains electrons but not muons or tauons, the Glashow-Iliopoulos-Maiani (GIM) mechanism that inhibits the radiative neutrino decay in a vacuum is not operative anymore, and as a result the rate can increase by a considerable amount. It was also noted there that the rate increases even further due to the stimulated emission of photons in the thermal background. However, in those preliminary calculations the effect just mentioned was neglected.

We have calculated the radiative decay rate including the effect of the stimulated emission of photons. In this paper we present the results of that calculation, and we show that, under the appropriate conditions, the inclusion of this effect enhances the rate further by many orders of magnitude.

We follow the notation and conventions of Ref. [5]. The initial neutrino was assumed to have a mass m and to be traveling with a speed $\mathcal{V}$ in the medium, while the final neutrino ν' was assumed to be massless. For all the three particles involved, we disregarded the modification of the dispersion relations due to the presence of the matter background. As a consequence of this, only the transverse modes of the photon contribute. Then, retaining only the background-induced contribution to the decay process, the square of the transition matrix element was found to be

$$|\mathcal{M}|^2=m^2|U^*_{e\nu}U_{e\nu'}\mathcal{T}_T|^2\left[\frac{(k+k')\cdot v}{\Omega}-\frac{m^2}{2\Omega^2}\right], \tag{2}$$

where v^μ is the velocity four-vector of the background medium, $\Omega=q\cdot v$, the elements of the matrix U denote the mixing of the neutrinos with the electron, and

$$\mathcal{T}_T=-\sqrt{2}\ eG_F\int\frac{d^3p}{(2\pi)^3E}\{f_-(p)+f_+(p)\}. \tag{3}$$

Here $f_\pm$ stand for the positron and electron distribution functions which, in terms of the temperature $T=1/\beta$ and chemical potential μ, are given by

$$f_\pm(p)=\frac{1}{\exp[\beta(p\cdot v\mp\mu)]+1}, \tag{4}$$

with

$$p^\mu\equiv(E,\vec{p}),\quad E=\sqrt{\vec{p}^2+m_e^2}. \tag{5}$$

The differential decay rate can be written as

$$d\Gamma'=\frac{1}{2k_0}(2\pi)^4\delta^4(k-k'-q)|\mathcal{M}|^2\frac{d^3k'}{(2\pi)^32k_0'}\frac{d^3q}{(2\pi)^32q_0}\times[1+f(\Omega)], \tag{6}$$

where

$$f(\Omega)=\frac{1}{e^{\beta\Omega}-1} \tag{7}$$

is the Planck distribution. As already mentioned in Ref. [5], the effect of the stimulated emission is taken into account precisely by the factor $1+f(\Omega)$. In the earlier calculations it was set equal to unity, thus disregarding the effect of stimulated emission. In that case, integrating Eq. (6) in the rest frame of the medium, where $v^\mu=(1,\vec{0})$ and $\Omega=q_0$, we obtained

$$\Gamma'=\frac{m}{16\pi}|U^*_{e\nu}U_{e\nu'}\mathcal{T}_T|^2F(\mathcal{V}), \tag{8}$$

where

$$F(\mathcal{V})=\sqrt{1-\mathcal{V}^2}\left[\frac{2}{\mathcal{V}}\ln\left(\frac{1+\mathcal{V}}{1-\mathcal{V}}\right)-3\right]. \tag{9}$$

The result of including the effect of stimulated emission can be represented by replacing $F(\mathcal{V})$ in Eq. (8) by the new function

$$F_T(\mathcal{V},\beta m)\equiv F(\mathcal{V})+F_s(\mathcal{V},\beta m), \tag{10}$$

where F_s can be expressed as

$$F_s(\mathcal{V},\beta m)=\left(\frac{1-\mathcal{V}^2}{2\mathcal{V}}\right)\int_{x_1}^{x_2}dx f(\Omega)\left\{\frac{4}{x\sqrt{1-\mathcal{V}^2}}-\frac{2}{x^2}-1\right\}. \tag{11}$$

The variable x is related to the photon energy by

$$x\equiv\frac{2\Omega}{m}, \tag{12}$$

and the limits of integration are

$$x_1=\frac{1}{x_2}=\sqrt{\frac{1-\mathcal{V}}{1+\mathcal{V}}}. \tag{13}$$

The function $F(\mathcal{V})$ is given by a formula similar to Eq. (11), but with the factor $f(\Omega)$ replaced by 1, which yields the result quoted in Eq. (9). For arbitrary values of the temperature and incident neutrino energy F_s can be evaluated only numerically. Analytical expressions, which show the general features of the numerical results, can be obtained in some limiting situations as we now discuss.

For $\mathcal{V}=0$ the photons emitted in the decay have an energy $\Omega=m/2$, which is independent of the direction in which the photon is emitted. The stimulated emission factor then becomes

$$f(\Omega)=\frac{1}{e^{\beta m/2}-1} \tag{14}$$

and, therefore, it is simply a constant factor in the integration in Eq. (11). Thus, for slowly moving incident neutrinos, but any arbitrary value of the temperature,

$$F_s(\mathcal{V},\beta m)\approx F_s(0,\beta m)=\frac{1}{e^{\beta m/2}-1} \tag{15}$$

$$\approx\frac{2}{\beta m}\quad(\text{for }\beta m\ll 1), \tag{16}$$

where we have given, in Eq. (16), the limiting value for high temperaures, as indicated.

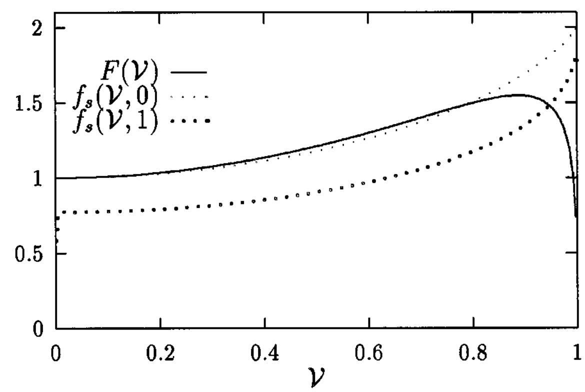

FIG. 1. Velocity dependence of the function $f_s(\mathcal{V},\beta m)$, defined in Eq. (19) of the text, for $\beta m=0$ and $\beta m=1$. Also shown is the velocity dependence of $F(\mathcal{V})$ defined in Eq. (9).

However, the effect of stimulated emission is important for temperatures that are much larger than the typical energy of the photons emitted in the decay process, which is of the order of the incident neutrino energy

$$k\cdot v=\frac{m}{\sqrt{1-\mathcal{V}^2}}. \tag{17}$$

In this regime $(\beta k\cdot v\ll 1)$, we can use the approximation

$$f(\Omega)\approx\frac{1}{\beta\Omega}, \tag{18}$$

in which case the expression for F_s can be evaluated analytically for arbitrary values of the incident neutrino energy. Introducing the function $f_s(\mathcal{V},\beta m)$ by writing

$$F_s(\mathcal{V},\beta m)\equiv\frac{2}{\beta m}f_s(\mathcal{V},\beta m), \tag{19}$$

the result of the integration for small βm is

$$f_s(\mathcal{V},0)=2-\frac{1-\mathcal{V}^2}{2\mathcal{V}}\ln\left(\frac{1+\mathcal{V}}{1-\mathcal{V}}\right). \tag{20}$$

Notice, in particular, that Eq. (20) reproduces the result of Eq. (16), $F_s(0,\beta m)=2/\beta m$, as it should.

In Fig. 1 we have plotted $f_s(\mathcal{V},0)$, from which we see that it is a very slowly increasing function that varies between 1 at $\mathcal{V}=0$ to 2 at $\mathcal{V}=1$. While $F(\mathcal{V})$ also is a slowly varying function for most values of $\mathcal{V}$, it drops sharply [6] near $\mathcal{V}=1$. However, $f_s(\mathcal{V},\beta m)$ does not drop below the value it has at $\mathcal{V}=0$, for any value of the temperature within the entire range of allowed values for $\mathcal{V}$. Moreover, it will have to be remembered that, in the decay rate, f_s is multiplied by $2/\beta m$, which is a large number for temperatures much higher than the mass of the decaying neutrino. It then follows that, for high temperatures, the factor $F+F_s$ is dominated by the leading term of F_s given by Eqs. (19) and (20), and the rate is given approximately by

$$\Gamma' \approx \Gamma_s \equiv \frac{1}{8\pi\beta} |U^*_{e\nu} U_{e\nu'} \mathcal{T}_T|^2 f_s(\mathcal{V},0) \quad \text{for } \beta m \ll 1. \tag{21}$$

Comparing this with Eq. (8), it follows that there can be a considerable enhancement of the rate when the temperature of the ambient photon gas is large compared to the mass of the decaying neutrino.

The formula in Eq. (21) has another peculiar character. Notice that, although it has a dependence on the mass matrix of the neutrinos through the elements of the mixing matrix, the decay rate does not directly depend on the mass of the decaying neutrino. Thus, irrespective of the mass, any neutrino will have the same decay rate apart from the difference coming from the mixing angles. To estimate this rate, we recall [5] that if the background medium consists of nonrelativistic electrons,

$$\mathcal{T}_T^{(\mathrm{NR})} = -\sqrt{2} e G_F n_e / m_e , \tag{22}$$

where n_e is the number density of the electrons. On the other hand, for a background of extremely relativistic electrons and positrons, assuming that the electron and photon temperatures are equal and the chemical potential is zero, one obtains

$$\mathcal{T}_T^{(\mathrm{ER})} = -\frac{e G_F}{3\sqrt{2}\beta^2}. \tag{23}$$

Inserting these expressions into Eq. (21), we obtain the decay rate

$$\Gamma'^{(\mathrm{NR})} \approx \alpha G_F^2 |U^*_{e\nu} U_{e\nu'}|^2 f_s(\mathcal{V},0) \frac{n_e^2}{m_e^2 \beta} = (4\times 10^{18}\ \mathrm{s})^{-1} |U^*_{e\nu} U_{e\nu'}|^2 f_s(\mathcal{V},0) \left(\frac{T}{1\ \mathrm{MeV}}\right) \times \left(\frac{n_e}{10^{24}\ \mathrm{cm}^{-3}}\right)^2 \tag{24}$$

for a background of nonrelativistic electrons, and

$$\Gamma'^{(\mathrm{ER})} \approx \alpha G_F^2 |U^*_{e\nu} U_{e\nu'}|^2 f_s(\mathcal{V},0) \frac{1}{36\beta^5} = (2.5\times 10^4\ \mathrm{s})^{-1} |U^*_{e\nu} U_{e\nu'}|^2 f_s(\mathcal{V},0) \left(\frac{T}{1\ \mathrm{MeV}}\right)^5 \tag{25}$$

for an extremely relativistic electron gas at zero chemical potential.

The quantitative study of the electromagnetic properties of neutrinos in a background of particles can lead to interesting physical phenomena and important applications. As already mentioned, the phenomenon of neutrino conversion in magnetized media has opened new possibilities for explaining the peculiar velocity of pulsars [4]. The results that have been presented in the present work can also have relevance in cosmological and astrophysical contexts such as the early Universe and the supernova, as well as in laboratory experiments related to rare processes and the coherent phenomena in crystals, where possible applications to the observation of induced radiative neutrino transitions have been mentioned [7].

It is important to stress that the results of the calculations presented here do not depend on any microscopic physics beyond the standard model, except for the assumption that neutrinos have a nonzero mass. Nevertheless, the physical implications of these results in any particular context depend not only on the macroscopic physics of the environment under consideration, but also on more specific and model-dependent assumptions such as, for example, those concerning the origin of the neutrino masses and the possibility of interactions beyond the standard model. These are typically necessary in order to assess the importance of possible competing processes in the particular situation at hand. A detailed analysis of such effects will be presented separately.

The research of J.F.N was supported in part by the U.S. National Science Foundation Grant No. PHY-9600924.

[1] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheyev and A. Y. Smirnov, Nuovo Cimento C **9**, 17 (1986). For a recent review see Palash B. Pal, Int. J. Mod. Phys. A **7**, 5387 (1992), and references therein.

[2] J. C. D'Olivo, José F. Nieves, and P. B. Pal, Phys. Rev. D **40**, 3679 (1989).

[3] J. C. D'Olivo, José F. Nieves, and P. B. Pal, Phys. Lett. B **365**, 178 (1996).

[4] A. Kusenko and G. Segrè, Phys. Rev. Lett. **77**, 4872 (1996).

[5] J. C. D'Olivo, José F. Nieves, and P. B. Pal, Phys. Rev. Lett. **64**, 1088 (1990).

[6] C. Giunti, C. W. Kim, and W. P. Lam, Phys. Rev. D **43**, 164 (1991).

[7] V. R. Zoller, Pis'ma Zh. Eksp. Teor. Fiz. **64**, 743 (1996) [JETP Lett. **64**, 788 (1996)].