Baby universes and energy (non)conservation in (1+1)-dimensional dilaton gravity

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We consider branching of baby universes off a parent one in (1+1)-dimensional dilaton gravity with 24 types of conformal matter fields. This theory is equivalent to string theory in a certain background in (D=26)-dimensional target space, so this process may be also viewed as the emission of a light string state by a heavy string. We find that bare energy is not conserved in 1+1 dimensions due to the emission of baby universes, and that the probability of this process is finite even for local distribution of matter in the parent universe. We present a scenario suggesting that the nonconservation of bare energy may be consistent with the locality of the baby universe emission process in 1+1 dimensions *and* the existence of the long-ranged dilaton field whose source is bare energy. This scenario involves the generation of longitudinal gravitational waves in the parent universe. [S0556-2821(97)01218-6]

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I. INTRODUCTION

Generally covariant (1+1)-dimensional theories provide convenient framework for considering various suspected properties of quantum gravity in 3+1 dimensions (for reviews see, e.g., Refs. [1,2]). In particular, the long-standing issue of the possible role of topology changing transitions and baby universes [3–8] may be naturally discussed in this framework. A special feature of 1+1 dimensions which is useful for the study of baby universes or wormholes is that some models admit their interpretation as string theories in higher-dimensional target space.

The simplest model of this sort is literally the theory of closed strings in the Minkowski target space of critical dimensions [9–12]. Indeed, macroscopic and microscopic string states may be interpreted as (1+1)-dimensional parent and baby universes, respectively. One feature inherent in that model is that the emission of a baby universe always requires nonconservation of bare energy in the parent universe[1] [11,16]. This nonconservation does not lead to any drastic consequences in the simplest stringy model; in particular, the rate of emission of baby strings is finite in the limit of infinite size of the parent string [12].

However, one important feature present in (3+1)-dimensional gravity is missing in the simplest stringy model. Namely, in 3+1 dimensions there exists a long-ranged gravitational field whose source is energy (Newton's gravity law), while there is no such field in that stringy model. Intuitively, one may suspect that the existence of the long-ranged field associated with energy and momentum may be an obstacle to energy nonconservation in local processes such as the emission of baby universes. To address this issue, more refined model than that of closed strings in critical dimensions, is needed.

A particularly simple (1+1)-dimensional model where the mass (energy of matter fields) produces long range effects, is the dilaton gravity with matter that has been widely discussed from the point of view of black hole physics [17] [for a careful analysis of the notion of Arnowitt-Deser-Misner (ADM) mass in that model see Refs. [18,19]]. Here we take a different attitude and consider the emission of baby universes, so we simplify the model as much as possible. In particular, we set the number of matter fields equal to 24 and the (1+1)-dimensional cosmological constant to zero. As stressed in Refs. [20,21], this model is equivalent to bosonic string theory (in critical dimension D=26 of target space) in the linear Dilaton background [to distinguish between dilaton fields in (1+1)-dimensional world and in D-dimensional target space, we call the former ''dilaton'' and the latter ''Dilaton,'' respectively]. Hence, the emission of a (1+1)-dimensional baby universe by a parent universe has an interpretation from the D-dimensional point of view as the emission of a light string state by a highly excited string state, in complete analogy to Ref. [11]. This process, in the leading order of string perturbation theory, is tractable both qualitatively and quantitatively; in particular, one is able to analyze whether it is accompanied by nonconservation of (bare) energy in 1+1 dimensions and whether its rate is large (unsuppressed) when baby universes are emitted locally in the (1+1)-dimensional parent universe. The discussion of these points is the main purpose of this paper.

The outline of the paper is as follows. In Sec. II we describe the model and some of its classical solutions in 1+1 dimensions. For technical reasons, the quantum version is conventiently constructed for the case of closed (1+1)-dimensional universe, so we present in Sec. II some classical solutions in the closed universe. This discussion will be useful to understand that the (bare) energy of localized distributions of matter fields still produces long-ranged effects, at least in some gauges, even though the total energy of the closed universe is always zero. In Sec. III we outline the quantum version of this model, which is known for some time (see, e.g., Refs. [22–25]), construct the states of parent

[1]It has been argued [11] that the emission of baby strings should lead to the loss of quantum coherence for one-dimensional observer at the parent string. Independently, it has been argued on general grounds [13] that the energy nonconservation is inevitable in modifications of quantum mechanics allowing for the loss of quantum coherence (see, however, Refs. [14,15]). So, energy nonconservation in the stringy model of Ref. [11] may not be too surprising.

universes [Di Vecchia–Del Giudice-Fubini (DDF) states], and vertex operators corresponding to the emission of baby universes. Section IV contains the main results of this paper. We consider the simplest DDF state of the parent universe, which can be interpreted as containing just two dressed matter ''particles,'' and analyze the emission of baby universes by this state in the lowest order of string perturbation theory. We find that this emission always occurs with the nonconservation of energy of matter in (1+1)-dimensional parent universe. We then proceed to the explicit calculation of the emission rate. Surprisingly, we find that the rate is unsuppressed even for localized distribution of matter in the parent universe, in spite of the long range field this matter produces. In Sec. V we conclude by presenting a scenario showing that the nonconservation of bare energy of matter may be consistent both with locality of the emission process and with the presence of long-ranged field; in this scenario, the ADM mass is conserved at the expense of the generation of ''longitudinal'' gravitational waves due to the emission of a baby universe.

II. MODEL AND CLASSICAL SOLUTIONS

A. The model

The action for the simplest version of (1+1)-dimensional dilaton gravity with conformal matter can be written in a form similar to Ref. [26]:

$$S=-\frac{1}{\pi}\int d^2\sigma\sqrt{-g}\left(-\frac{\gamma^2}{4}\phi R+g^{\alpha\beta}\partial_\alpha f^i\partial_\beta f^i\right), \qquad (1)$$

where ϕ is the dilaton field, f^i are matter fields, $i=1,\dots,24$, and γ is a positive coupling constant analogous to the Planck mass of (3+1)-dimensional gravity. The coupling constant γ may be absorbed into the dilaton field, but we will not do this for bookkeeping purposes. Both in infinite space and in the closed (1+1)-dimensional universe the field equations are simplified in the conformal gauge:

$$g_{\alpha\beta}=e^{2\rho}\eta_{\alpha\beta},$$

where η is the Minkowskian metrics in 1+1 dimensions. In this gauge, the fields ρ, ϕ, and f^i obey massless free field equations. There are also constraints

$$-\frac{1}{2}\gamma^2(\partial_\pm\phi\partial_\pm\rho-\tfrac{1}{2}\partial_\pm^2\phi)+\tfrac{1}{2}(\partial_\pm\mathbf{f})^2=0, \qquad (2)$$

ensuring that the total energy-momentum tensor vanishes.

B. Solutions in infinite space

Let us outline some classical solutions in this model. We begin with the case of infinite one-dimensional space, $\sigma^1\in(-\infty,+\infty)$, and consider localized distributions of matter. In this case one can further specify the gauge and choose

$$\rho=0 \qquad (3)$$

so that the space-time is flat. Equation (2) then determines the dilaton field ϕ for a given matter distribution. Indeed, the solution to Eq. (2) is, up to an arbitrary linear function of coordinates,

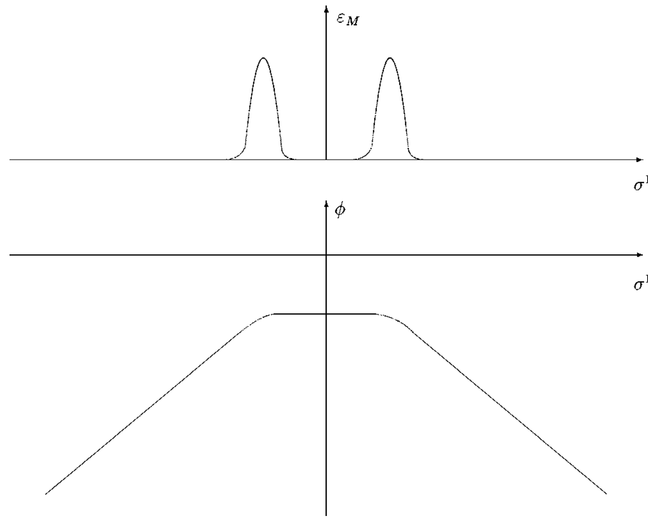

FIG. 1. Dilaton field produced by two lumps of matter in infinite space.

$$\phi(\sigma)=\phi_+(\sigma_+)+\phi_-(\sigma_-),$$

with

$$\phi_\pm=-\frac{1}{\gamma^2}\int d\sigma'_\pm|\sigma_\pm-\sigma'_\pm|(\partial_\pm\mathbf{f})^2(\sigma'_\pm). \qquad (4)$$

Hence, the energy momentum of matter fields produces long-ranged dilaton field which has a linear behavior at large $|\sigma^1|$. In particular, the ADM mass can be defined as

$$\mu_{\rm ADM}=-\frac{\gamma^2}{2\pi}\left[\frac{\partial\phi}{\partial\sigma^1}(\sigma^1\to+\infty)-\frac{\partial\phi}{\partial\sigma^1}(\sigma^1\to-\infty)\right]. \qquad (5)$$

In virtue of Eq. (4) it is equal to

$$\mu_{\rm ADM}=\int_{-\infty}^{+\infty}d\sigma^1\varepsilon_M(\sigma),$$

where

$$\varepsilon_M=\frac{1}{2\pi}[(\partial_0\mathbf{f})^2+(\partial_1\mathbf{f})^2]$$

is the energy density of matter which we will often call bare energy density.[2]

The dilaton field produced by two lumps of matter of equal energy and opposite momenta, moving towards each other (or from each other) with the speed of light, is shown in Fig. 1. Needless to say, the linear dependence of ϕ on σ^1 at large $|\sigma^1|$ is nothing but the Coulomb behavior of long-ranged field in one-dimensional space. In this respect the dilaton field in 1+1 dimensions is analogous to gravitational field of Newton's law in 3+1 dimensions.

[2]Note somewhat unconventional factor $1/\pi$ in the matter action in Eq. (1).

C. Solutions in closed space

For technical reasons, the quantum version of this model is conveniently formulated in a closed one-dimensional space. So, it is instructive to consider classical solutions in the closed universe. Let us study the classical theory (1) on a circle

$$\sigma^1 \in \left(-\frac{\pi}{2},+\frac{\pi}{2}\right).$$

The absolute length of the circle is irrelevant as we are dealing with scale-invariant action; what will matter is the relative size of matter distribution to the length of the universe. The gauge (3) is no longer possible in the closed universe; the closest analogue is the gauge in which the universe contracts (or expands) homogeneously,

$$\rho = -\frac{1}{\gamma} P^{(-)} \sigma^0, \tag{6}$$

where $P^{(-)}$ is some constant; our choice of normalization and notation will become clear later. In this gauge, the constraints (2) may again be used to determine the dilaton field for a given distribution of matter, provided the total spatial momentum of matter vanishes:

$$\int_{-\pi/2}^{+\pi/2} d\sigma^1 (\partial_+ \mathbf{f})^2 = \int_{-\pi/2}^{+\pi/2} d\sigma^1 (\partial_- \mathbf{f})^2. \tag{7}$$

The constraints (2) in the gauge (6) read

$$-\tfrac{1}{4}\gamma P^{(-)} \partial_\pm \phi - \tfrac{1}{4}\gamma^2 \partial_\pm^2 \phi = \tfrac{1}{2}(\partial_\pm \mathbf{f})^2. \tag{8}$$

Equation (7) is an immediate consequence of these constraints and the periodicity of the dilaton field in σ^1.

For a given matter distribution, the solution to Eq. (8), which is periodic in σ^1 with period π, is

$$\phi(\sigma^0,\sigma^1) = \phi_+(\sigma_+) + \phi_-(\sigma_-), \tag{9}$$

where

$$\phi_\pm(\sigma_\pm) = \int d\sigma_\pm G(\sigma_\pm, \sigma'_\pm) \tfrac{1}{2} (\partial_\pm \mathbf{f})^2 \tag{10}$$

and the Green function of Eq. (8) obeys the periodicity condition

$$\partial_u G(u+\pi, u') = \partial_u G(u,u').$$

At $u \in (-\pi/2, \pi/2)$, $u' \in (-\pi/2,\pi/2)$, we have explicitly

$$G(u,u') = \frac{2}{\gamma P^{(-)} \sinh(\pi P^{(-)}/2\gamma)} \exp\left[\frac{\pi P^{(-)}}{2\gamma} \epsilon(u-u')\right] \times \left(\exp\left[-\frac{P^{(-)}}{\gamma}(u-u')\right] - 1 \right), \tag{11}$$

where $\epsilon(u-u')$ is the usual step function.

It is instructive to consider the case $\gamma \gg 1$ (large ''Planck mass'') and study two narrow pulses of matter moving left and right and colliding at $\sigma^1 = 0$. These pulses may be approximated by the δ-function distribution

$$(\partial_\pm \mathbf{f})^2 = \frac{\pi}{2} \mu \delta(\sigma_\pm), \tag{12}$$

where the normalization is such that the constant μ would coincide with the ADM mass had the universe infinite size,

$$\mu = \int d\sigma^1 \frac{1}{2\pi} [(\partial_0 \mathbf{f})^2 + (\partial_1 \mathbf{f})^2]. \tag{13}$$

In this case Eq. (10) has particularly simple form (again at $|\sigma_\pm| < \pi/2$),

$$\phi_\pm = -\frac{P^{(+)}}{2\gamma}\sigma_\pm - \frac{\pi}{2\gamma^2}\mu|\sigma_\pm| + \frac{\mu}{2\gamma^2}\sigma_\pm^2 + O(\gamma^{-3}), \tag{14}$$

where

$$P^{(+)} = \frac{2\mu}{P^{(-)}}. \tag{15}$$

When the pulses are close to each other, one has for the total dilaton field at $|\sigma_\pm| \ll 1$,

$$\phi = -\frac{P^{(+)}}{\gamma}\sigma^0 - \frac{\pi}{2\gamma^2}\mu(|\sigma_+| + |\sigma_-|) + \cdots . \tag{16}$$

The first term in Eq. (16) describes spatially homogeneous component of the dilaton field, while the second term shows precisely the same Coulomb behavior as in the case of infinite space. In fact, the latter term coincides with the expression (4) [for narrow matter pulses as defined in Eq. (12)]. The terms omitted in Eq. (16) are of order of γ^{-2} and they become important at $|\sigma_\pm| \sim 1$; in particular, they ensure that $\phi(\sigma^0,\sigma^1)$ flattens out and has vanishing spatial derivatives at $\sigma^1 = \pm\pi/2$. The behavior of the dilaton field generated by narrow pulses of matter, which are close to each other, is schematically shown in Fig. 2.

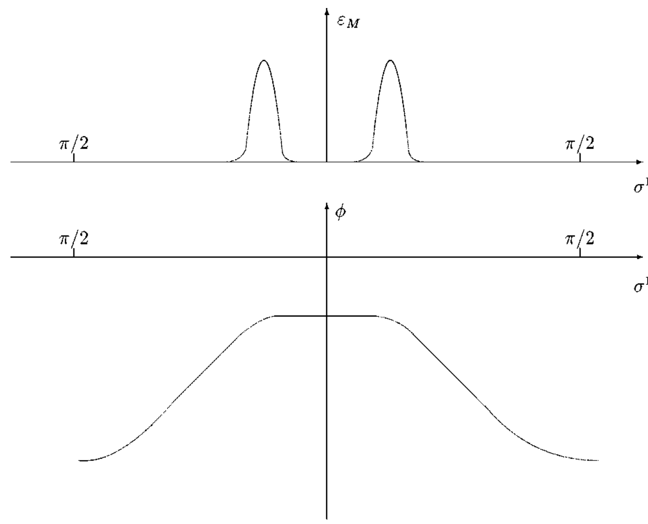

FIG. 2. Behavior of the dilaton field generated in closed space by narrow pulses of matter in close proximity.

Even though the ADM mass is, strictly speaking, zero in the closed universe, narrow pulses of matter produce the dilaton field that shows the Coulomb behavior not far away from the pulses. In this sense one can still use the notion of bare energy in the gauge (6). This bare energy (the energy of matter) can be observed by a one-dimensional observer by measuring the dilaton field outside the pulses but at distances small compared to the size of the universe. With these reservations, the formula for the ADM mass, Eq. (5), still makes sense (when all events occur and fields are studied in a small part of the universe). Clearly, all these observations apply only to those matter distributions whose conformal size, $\sigma^1 \sim r^{\text{pulse}}$, is small compared to π, the conformal size of the universe. In other words, we will be interested in considering large wave numbers:

$$n \sim \frac{1}{r^{\text{pulse}}} \gg 1. \tag{17}$$

The gauge (6) is not the only useful one in the closed universe. At large γ, one can choose the gauge

$$\phi' = -\frac{P^{(+)}}{\gamma}\sigma'^0, \tag{18}$$

where prime is used to denote the quantities in this gauge. In general, the coordinates in the two gauges are not too different:

$$\sigma'_\pm = \sigma_\pm + O(\gamma^{-1}). \tag{19}$$

The coordinate transformation has a particularly simple form in the case of two narrow pulses; it follows from Eqs. (14) and (15) that in that case

$$\sigma'_\pm = \sigma_\pm + \frac{\pi\mu}{\gamma P^{(+)}}|\sigma_\pm| - \frac{\mu}{\gamma P^{(+)}}\sigma_\pm^2 + O(\gamma^{-2}). \tag{20}$$

Furthermore, for two narrow pulses one obtains (at $|\sigma_\pm| < \pi/2$),

$$\rho'_\pm = -\frac{\pi\mu}{2\gamma P^{(+)}}\epsilon(\sigma'_\pm). \tag{21}$$

This expression, of course, solves the constraint (2) for matter distribution

$$(\partial'_\pm \mathbf{f})^2(\sigma'_\pm) = \frac{\pi}{2}\mu\,\delta(\sigma'_\pm). \tag{22}$$

The fact that the matter distributions (12) and (22) are essentially the same in the two gauges, is an immediate consequence of Eq. (19).

If the pulses of matter are not infinitely narrow, the above features remain valid qualitatively. The matter distributions in the two gauges (6) and (18) are the same up to corrections of order γ^{-1}; in particular, the spatial sizes of the lumps differ only by a factor $[1+O(\gamma^{-1})]$. The scale factor $\rho'(\sigma')$ changes rapidly in the regions of nonvanishing energy-momentum density of matter, and has the form shown in Fig. 3, the depth of the well being proportional to the total energy of mater. We will have to say more about this gauge in Sec. V.

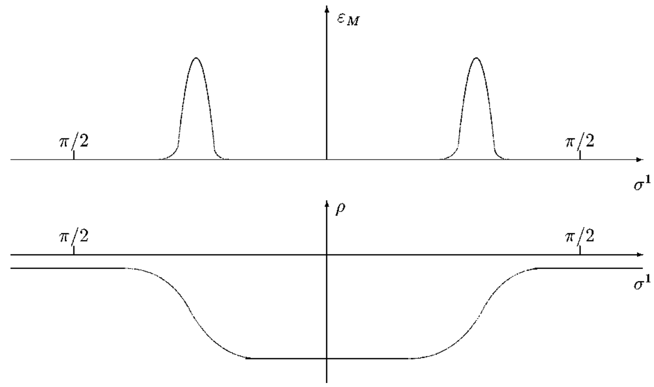

FIG. 3. Form of the scale factor $\rho'(\sigma')$.

III. QUANTUM STATES AND VERTEX OPERATORS

The model is quantized exactly in the same way as bosonic string theory in Minkowskian target space of $D=26$ dimensions. One introduces the notation

$$\phi = -\frac{1}{\gamma}(X^0 + X^1),$$

$$\rho = -\frac{1}{\gamma}(X^0 - X^1),$$

$$f^1, \ldots, f^{24} = X^2, \ldots, X^{25}. \tag{23}$$

Then, $X^\mu(\sigma)$, $\mu = 0,\ldots,25$ are canonically normalized free two-dimensional fields and the classical constraints (2) become

$$\tfrac{1}{2}\partial_\pm X_\mu \partial_\pm X^\mu - \tfrac{1}{4}\gamma \partial_\pm^2 X^{(+)} = 0, \tag{24}$$

where the summation is performed with Minkowskian D-dimensional metrics, $\eta_{\mu\nu} = (-1,+1,\ldots,+1)$ and

$$X^{(\pm)} = X^0 \pm X^1.$$

Upon quantization, the left- and right-moving components of these fields are decomposed in the usual way,

$$X_L^\mu(\sigma_+) = \tfrac{1}{2}x^\mu + \tfrac{1}{2}P^\mu \sigma_+ + \frac{i}{2}\sum_{k\neq 0}\frac{1}{k}\alpha_k^\mu e^{-2ik\sigma_+},$$

$$X_R^\mu(\sigma_-) = \tfrac{1}{2}x^\mu + \tfrac{1}{2}P^\mu \sigma_- + \frac{i}{2}\sum_{k\neq 0}\frac{1}{k}\tilde{\alpha}_k^\mu e^{-2ik\sigma_-},$$

where α_k^μ and $\tilde{\alpha}_k^\mu$ are the standard oscillator operators with string normalization. According to Eq. (24), the Virasoro operators are

$$L_0 = \frac{1}{8}P^2 + \sum_{k>0}\alpha_{-k}^\mu \alpha_k^\mu,$$

$$L_m = \frac{1}{2}P_\mu \alpha_m^\mu + \frac{1}{2}\sum_{k\neq 0,m} :\alpha_k^\mu \alpha_{m-k}^\mu: + \frac{i}{2}\gamma m \alpha_m^{(+)}, \tag{25}$$

and similarly for $\widetilde{L}_0$ and $\widetilde{L}_m$, where $\alpha_m^{(\pm)}=\alpha_m^0\pm\alpha_m^1$. Equation (25) shows that the model is equivalent to the string with background charge [22–25] or, in other words, to the bosonic string in linear Dilaton background [20,21],

$$\Phi(x)=\gamma(e_\mu x^\mu)=\gamma x^{(+)}, \tag{26}$$

with $e^\mu=(-1,1,0,\ldots,0)$ being a lightlike vector in D-dimensional target space.

It is clear from Eq. (25) that the spectrum of states in the target space is the same as the spectrum of the bosonic string in trivial background,

$$M^2=8(n-1), \quad n=0,1,\ldots\ .$$

The vertex operators are, however, slightly different. For example, the tachyon vertex operator of conformal dimension (1,1) is

$$V(Q)=:e^{iQ_\mu X^\mu+\gamma X^{(+)}}: \tag{27}$$

with $Q^2=8$. This modification can be understood as being due to the Dilaton background (26). Indeed, the effective action of tachyon field $\widetilde{T}$ in flat target space-time and in linear Dilaton background $\Phi(x)$ is

$$\int d^Dx\ \mathrm{e}^{-2\Phi}[-\tfrac{1}{2}(\partial_\mu\widetilde{T})^2+4\widetilde{T}^2+c\widetilde{T}^3+O(\widetilde{T}^4)].$$

By introducing the field $T=\mathrm{e}^{-\Phi}\widetilde{T}$, one rewrites this action in the form

$$\int d^Dx\left[-\frac{1}{2}(\partial_\mu T)^2-\frac{M_T}{2}T^2+ce^{\Phi}T^3+O(T^4)\right],$$

where $M_T^2=-8+(\partial_\mu\Phi)^2$, i.e., $M_T^2=-8$ for lightlike e_μ in Eq. (26). In this notation the kinetic and mass terms are conventional, while the trilinear vertex is proportional to e^{Φ}, precisely as required by Eq. (27). This argument can be generalized to interactions of the fields other than the tachyon [20,21,27].

Let us now consider highly excited string states (parent universes). They are conveniently constructed by making use of the DDF operators [28]. In the lightlike Dilaton background, the simplest choice of the DDF operators is

$$A_n^i=\int_{-\pi/2}^{+\pi/2}\frac{d\sigma_+}{\pi}\exp\left[4in\frac{e_\mu X_L^\mu(\sigma_+)}{e_\mu P^\mu}\right]\partial_+X_L^i(\sigma_+),$$

$$\widetilde{A}_{\tilde{n}}^i=\int_{-\pi/2}^{+\pi/2}\frac{d\sigma_-}{\pi}\exp\left[4i\tilde{n}\frac{e_\mu X_R^\mu(\sigma_-)}{e_\mu P^\mu}\right]\partial_-X_R^i(\sigma_-), \tag{28}$$

with the same lightlike vector e_μ. Here $i=2,\ldots,25$. These operators obey the usual oscillator commutational relations, and their commutational relations with the Virasoro operators [11] ensure that the state of the form

$$A_{-n_1}^{i_1}\cdots A_{-n_s}^{i_s}\cdots\widetilde{A}_{-\tilde{n}_1}^{j_1}\cdots\widetilde{A}_{-\tilde{n}_t}^{j_t}|\mathcal{P}\rangle \tag{29}$$

is a physical state provided that $|\mathcal{P}\rangle$ is the physical tachyon state and

$$n_1+\cdots+n_s=\tilde{n}_1+\cdots+\tilde{n}_t. \tag{30}$$

The DDF operators (28) are similar to those introduced in Ref. [25]. Another choice of DDF operators in this model has been considered in Refs. [23,25].

To make contact with the classical analysis of Sec. II, consider suitably modified coherent states

$$|\Psi_P\rangle=P_{L_0=\widetilde{L}_0}\exp\left(\sum_{n>0}\frac{1}{n}f_n^iA_{-n}^i+\sum_{\tilde{n}>0}\frac{1}{\tilde{n}}\tilde{f}_{\tilde{n}}^j\widetilde{A}_{-\tilde{n}}^j\right)|\mathcal{P}\rangle. \tag{31}$$

Here, f_n^i and $\tilde{f}_{\tilde{n}}^j$ are c-number amplitudes and $P_{L_0=\widetilde{L}_0}$ is a projector onto the subspace of vectors obeying

$$(L_0-\widetilde{L}_0)|\Psi\rangle=0.$$

This projector is needed to ensure the validity of Eq. (30) term by term in the expansion of $|\Psi_P\rangle$.

Let us impose the condition that

$$\sum_n \mathbf{f}_n\mathbf{f}_n^*=\sum_{\tilde{n}}\tilde{\mathbf{f}}_{\tilde{n}}\tilde{\mathbf{f}}_{\tilde{n}}^* \tag{32}$$

and take the amplitudes $\mathbf{f}_n$ and $\tilde{\mathbf{f}}_{\tilde{n}}$ to be large. Consider now matrix elements of the form

$$\langle\Psi_{P'}|O_M(\{\sigma_+\};\{\sigma_-\})|\Psi_P\rangle \tag{33}$$

and also

$$\langle\Psi_{P'}|\partial_-X_R^{(\pm)}(\sigma'_-)O_M(\{\sigma_+\};\{\sigma_-\})|\Psi_P\rangle, \tag{34}$$

$$\langle\Psi_{P'}|\partial_+X_L^{(\pm)}(\sigma'_+)O_M(\{\sigma_+\};\{\sigma_-\})|\Psi_P\rangle, \tag{35}$$

where the operators O_M are products of matter fields, in general at different points in (1+1)-dimensional space-time,

$$O_M=\partial_+X_L^{i_1}(\sigma_+^1)\cdots\partial_+X_L^{i_k}(\sigma_+^k)\cdot\partial_-X_R^{j_1}(\sigma_-^1)\cdots\partial_-X_R^{j_q}(\sigma_-^q) \tag{36}$$

and, as before, we take $i_1,\ldots,j_q=2,\ldots,25$, so that the operators O_M indeed contain matter fields only. The matrix elements (33) are then the correlators of matter fields in the coherent state (31), while the matrix elements (34) and (35) are the correlators of dilaton and metric fields with matter.

These matrix elements are calculated in the Appendix with the following result. Up to small corrections and trivial normalization factor, they coincide with classical correlators

$$\int_{-\pi/2}^{+\pi/2}\frac{d\xi^1}{\pi}O_M^{\rm cl}(\{\sigma_++\xi^1\};\{\sigma_--\xi^1\})$$
$$\equiv\int_{-\pi/2}^{+\pi/2}\frac{d\xi^1}{\pi}O_M^{\rm cl}(\{\sigma^0\};\{\sigma^1+\xi^1\}) \tag{37}$$

and

$$\int_{-\pi/2}^{+\pi/2} \frac{d\xi^1}{\pi} \partial_+ X_L^{\mathrm{cl},(\pm)}(\sigma_+ + \xi^1) O_M^{\mathrm{cl}}(\{\sigma_+ + \xi^1\};\{\sigma_- - \xi^1\}) \tag{38}$$

[and similarly for Eq. (34)], respectively, where O_M^{cl} is given by Eq. (36) with classical matter fields

$$\partial_+ X^{\mathrm{cl},i}(\sigma_+) = \frac{1}{2} P^i + \sum_{n>0} (f_n^i e^{-2in\sigma_+} + f_n^{*i} e^{2in\sigma_+}).$$

The classical field $X^{\mathrm{cl},(+)}$ is defined by

$$\partial_\pm X^{\mathrm{cl},(+)} = \mathcal{P}^{(+)},$$

while the field $\partial_\pm X^{\mathrm{cl},(-)}$ is to be found from the classical constraints (24). In short, the matrix elements such as Eq. (33) and Eq. (35) are equal to the corresponding classical expressions *in the gauge* (18), integrated over translations in one-dimensional space. In this way the classical picture is restored; the particular choice of the DDF operators, Eq. (28), corresponds to the gauge choice (18). Note that Eq. (32) is precisely the classical constraint $L_0 = \widetilde{L}_0$ written in this gauge.

Hence, the DDF operators (28) correspond to creation and annihilation of dressed matter excitations in (1+1)-dimensional space-time *in the gauge* $\phi = \mathrm{const} \times \sigma^0$. The corresponding wave numbers are n and $\widetilde{n}$, respectively. In what follows we will call (somewhat loosely) these excitations as ''dressed particles'' in the (1+1)-dimensional universe.

IV. NONCONSERVATION OF BARE ENERGY IN 1+1 DIMENSIONS AND EMISSION PROBABILITY

A. State of the parent universe

In this section we consider the simplest DDF state

$$|\Psi,n,i,j\rangle = \int d^{D-1}P \Psi(P) |P,n,i,j\rangle, \tag{39}$$

where

$$|P,n,i,j\rangle = \frac{1}{n} A_{-n}^i \widetilde{A}_{-n}^j |\mathcal{P}\rangle \tag{40}$$

and $\Psi(P)$ is the wave function of the center-of-mass motion in target space, in momentum representation. The normalization factor $1/n$ in Eq. (40) is chosen in such a way that the state $|P,n,i,j\rangle$ has the usual D-dimensional normalization (recall the string normalization of the oscillator operators, $[A_n^i, A_n^j] = n\delta^{ij}$). According to the discussion in Sec. III, we interpret this state as the state of a parent universe with two dressed matter particles (one left moving and one right moving) of equal wave numbers n, in the gauge (18). The normalization convention in Eq. (40) corresponds to ''two particles in entire one-dimensional space'' normalization of (1+1)-dimensional quantum field theory. These particles have equal bare energies and opposite bare momenta in 1+1 dimensions:

$$\epsilon_{\mathrm{left}} = \epsilon_{\mathrm{right}} = 2n, \tag{41}$$

$$p_{\mathrm{right}} = -p_{\mathrm{left}} = 2n.$$

We are interested in the limit [see Eq. (17)],

$$n \to \infty.$$

By making superpositions of the states (39) with different wave numbers n, one can construct states with localized distributions of matter in one-dimensional universe. This generalization is straightforward, so we stick to the state with fixed n. The necessity to consider wave packets (39) in a target space and not just plane waves (40) is due to the Dilaton background that increases indefinitely as $x^{(+)} \to \infty$: amplitudes of processes involving plane waves would be divergent in this background.

It is a matter of simple algebra to see that the total D-dimensional momentum of the state (40) is

$$P^k = \mathcal{P}^k, \quad k = 2, \ldots, 25,$$

$$P^{(+)} \equiv P^0 + P^1 = \mathcal{P}^{(+)},$$

$$P^{(-)} \equiv P^0 - P^1 = \mathcal{P}^{(-)} - \frac{8n}{\mathcal{P}^{(+)}}. \tag{42}$$

Hence, from D-dimensional point of view, the state (40) is interpreted as an excited string state at the nth level with the mass

$$M_n^2 = 8n - 8 \tag{43}$$

(recall that $|\mathcal{P}\rangle$ is the tachyon state). We consider for definiteness this state in the center-of-mass frame,

$$P^k = 0, \quad k = 2, \ldots, 25,$$

$$P^{(+)} = \mathcal{P}^{(+)} = M_n,$$

$$P^{(-)} = M_n, \tag{44}$$

although our discussion can be straightforwardly generalized to other frames. In terms of the wave packets (39), Eq. (44) means that the wave function of the center-of-mass motion, $|\Psi(P)\rangle$, is peaked near the values determined by Eq. (44).

B. Emission of baby universe: Energy nonconservation in 1+1 dimensions

The parent universe in the state (39) can emit a baby universe, a universe with no or small energy of matter particles. In D-dimensional language this process corresponds to the emission of a low-lying string state (tachyon, Dilaton, etc.) into D-dimensional target space.

Let us study whether this process always occurs with the nonconservation of bare energy in (1+1) dimensions. Since bare energy coincides with the level of the string, Eq. (41), we are interested in the change of the D-dimensional mass of the highly excited string due to the emission of the low-lying string state. At first sight, the presence of the time-dependent Dilaton background in target space might give rise to the nonconservation of D-dimensional energy and momentum in the emission process. In that case the emission of a low-lying state would not necessarily require the change of the level of the heavy string; in other words, the emission of a baby universe would not necessarily require nonconservation of bare energy in $1+1$ dimensions. Surprisingly, we will see in a moment that this is not the case: D-dimensional energy and momentum are in fact conserved in the presence of *linear* Dilaton background exactly as they do in flat target space with no background. Hence, the emission of a baby universe always occurs with the nonconservation of $(1+1)$-dimensional bare energy of matter in the parent universe.

The argument presented below is fairly general; the particular form of the initial state, Eq. (39), is unimportant. The nonconservation of energy in $(1+1)$-dimensional parent universe due to the emission of baby universes is a generic property of our model.

The argument for the *conservation* of D-dimensional energy momentum in the presence of the linear Dilaton background is conveniently presented by considering a theory of three scalar fields Φ_1, Φ_2, Φ_3 with cubic interaction and exponentially changing coupling. Let the cubic coupling be

$$g(x)=\exp(\Gamma_\mu x^\mu), \tag{45}$$

with real constant Γ_μ. Let us take Φ_1 to be the heaviest (mass M_i), and Φ_2 to have the mass M_f which for simplicity is close to M_i (but smaller than M_i). Let the field Φ_3 be light. We are interested in the process when a particle Φ_1 emits a particle Φ_3 and becomes a particle Φ_2.

Because of the unboundedness of the coupling (45), it does not make sense to consider plane waves of particles Φ_1 and Φ_2. Instead, one has to use wave packets. Namely, consider the amplitude of the decay of a wave packet $\Psi_i(\mathbf{x},t)$ describing the state of the particle Φ_1 into a wave packet Ψ_f of the particle Φ_2 plus a particle Φ_3 which has a fixed energy momentum Q_μ,

$$A=\int d\mathbf{x}dt g(\mathbf{x},t)\Psi_i(\mathbf{x},t)\Psi_f^*(\mathbf{x},t)\exp(-iQ_\mu x^\mu). \tag{46}$$

We will see in the next subsection that the amplitudes of string decays have similar form.

Let us specify the form of the wave packets Ψ_i and Ψ_f. Let us consider for definiteness the nonrelativistic regime and take wave packets narrow in momentum representation. Let us furthermore neglect the dispersion of the wave packets with time[3] (this can be achieved by confining the particles in moving potential wells). Thus, we take

$$\Psi_i=\exp\left(-iM_it-\frac{i}{2M_i}\mathbf{P}_i^2+i\mathbf{P}_i\mathbf{x}-\frac{\sigma^2}{2}(\mathbf{x}-\mathbf{v}_it)^2\right), \tag{47}$$

where σ, the width of the packet in momentum space, is small, and $\mathbf{v}_i=\mathbf{P}_i/M_i$ is the velocity of the particle, which is also assumed to be small for simplicity. Similarly, the wave packet Ψ_f has the form (47) with M_i, $\mathbf{P}_i$, $\mathbf{v}_i$ substituted by M_f, $\mathbf{P}_f$, $\mathbf{v}_f$.

The integral (46) is then Gaussian, and is straightforward to evaluate. The result, up to a preexponential factor, is

$$A=\exp\left[\frac{1}{2\sigma^2}[-F_1(\mathbf{\Delta P},\Delta E)+F_1(\mathbf{\Gamma},\Gamma_0)+iF_2(\mathbf{\Delta P},\mathbf{\Gamma},\Delta E,\Gamma_0)]\right], \tag{48}$$

where

$$\mathbf{\Delta P}=\mathbf{P}_i-\mathbf{P}_f-\mathbf{Q},$$

$$\Delta E=\left(M_i+\frac{\mathbf{P}_i^2}{2M_i}\right)-\left(M_f+\frac{\mathbf{P}_f^2}{2M_f}\right)-Q_0$$

are amounts of nonconservation of momentum and energy, and

$$F_1(\mathbf{\Delta P},\Delta E)=\frac{(\mathbf{\Delta P})^2}{4}+\frac{1}{(\mathbf{v}_i-\mathbf{v}_f)^2}[\Delta E-\tfrac{1}{2}(\mathbf{v}_i-\mathbf{v}_f)\mathbf{\Delta P}]^2,$$

$$F_1(\mathbf{\Gamma},\Gamma^0)=\frac{(\mathbf{\Gamma})^2}{4}+\frac{1}{(\mathbf{v}_i-\mathbf{v}_f)^2}[\Gamma^0-\tfrac{1}{2}(\mathbf{v}_i-\mathbf{v}_f)\mathbf{\Gamma}]^2.$$

The explicit form of F_2 is not important; it is sufficient to note that both F_1 and F_2 are real. The only important property of F_1 is that it is positive definite and vanishes iff $\mathbf{\Delta P}=\Delta E=0$.

The imaginary part of the exponent iF_2 in Eq. (48) is unimportant and cancels out in the probability. Then the probability factorizes into a term depending on $\mathbf{\Delta P}$ and ΔE and a term containing Γ_μ. The latter term is nothing but the overlap of the initial and final wave packets with the weight $g(x)$. More importantly, since $F_1(\mathbf{\Delta P},\Delta E)$ is multiplied by the large factor $1/\sigma^2$ in the exponent, and because $F_1(\mathbf{\Delta P},\Delta E)$ is non-negative and vanishes only at $\mathbf{\Delta P}=\Delta E=0$, the exponential factor containing $F_1(\mathbf{\Delta P},\Delta E)$ ensures conservation of energy and momentum in the limit of small σ exactly in the same manner as it does in the case of space-time-independent coupling.

The restriction to the nonrelativistic case and Gaussian wave packets is, in fact, not essential: the same argument goes through for relativistic and non-Gaussian wave packets (provided that they do not disperse with time).

The result that energy and momentum are conserved in spite of space-time dependence of the coupling is peculiar to the exponential coupling whose exponent linearly depends on x_μ: only in this case the dependences on ΔP_μ and Γ_μ factorize. For instance, if the coupling switches off at infinity in space-time (say, has finite support), then the same calcu-

[3]The dispersion of wave packets with time would complicate the analysis considerably. The coupling changes so rapidly that in the case of spreading wave packets the interaction often occurs in space-time very far away from the centers of the wave packets. In that region there are mostly modes with momenta quite different from the central values $\mathbf{P}_i$ or $\mathbf{P}_f$. Hence it is difficult to separate the effects of finite widths of the wave packets in momentum space from possible effects of momentum nonconservation.

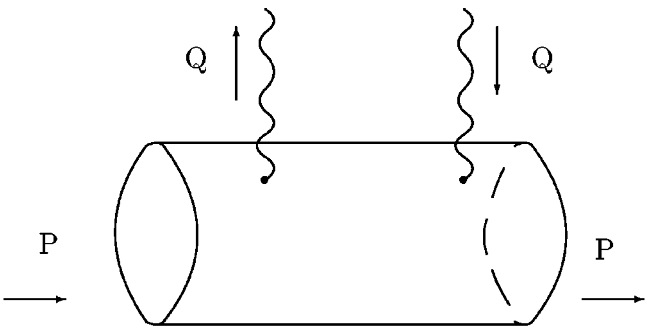

FIG. 4. Forward amplitude; curved lines denote the tachyon.

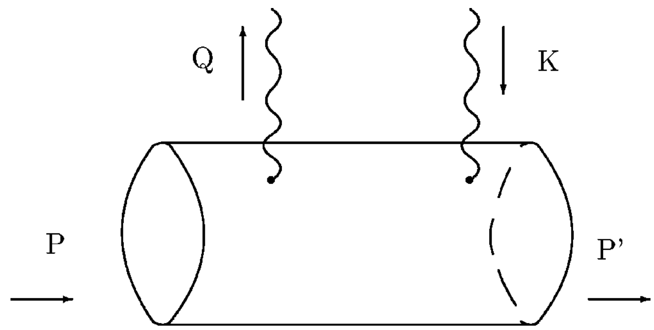

FIG. 5. Amplitude with different momenta.

lation leads to the usual result that energy and momentum are not conserved, and the amplitude is proportional to the Fourier component of the coupling, $\widetilde{g}(\mathbf{\Delta P},\Delta E)$.

C. Emission rate

Let us now turn to the actual calculation of the rate of the emission of a baby universe (low-lying string state) by a parent universe (excited string) in the state (39). We explicitly consider the emission of a tachyon, although the analysis, and results, are the same for the emission of a Dilaton or graviton as for the baby universe. Let the D-dimensional momentum of the outgoing tachyon be Q_μ. We first have to specify the range of Q_μ which is of interest for our purposes.

The emission of a baby universe due to the collision of (1+1)-dimensional particles with wave numbers equal to n has a chance to be local if the characteristic conformal time of the process of emission, $\Delta\sigma^0$, is of order

$$\Delta\sigma^0 \sim \frac{1}{n}.$$

This conformal time is related to the D-dimensional time via

$$\Delta x^0 = P^0 \Delta\sigma^0. \tag{49}$$

The D-dimensional time characteristic to the tachyon emission can be estimated in the center-of-mass frame of the decaying string as

$$\Delta x^0 \sim \frac{1}{Q^0}.$$

This is the time after which the tachyon is formed and splits off the initial string. Hence we are interested in the tachyon energies of order

$$Q^0 \sim \frac{n}{P^0} \sim \sqrt{n}, \tag{50}$$

where we made use of Eqs. (43) and (44). Since Q^0 is large, we neglect the tachyon mass where appropriate in what follows.

We are interested in the process in which the initial string at level n decays, by emitting a tachyon, into level n'. According to Eqs. (43) and (50), and because of energy conservation in D dimensions, we have

$$m \equiv n - n' \sim n. \tag{51}$$

We intend to sum up over all final states at given level n'. Usually, this summation is conveniently performed by evaluating the forward amplitude shown in Fig. 4 where curved lines denote the tachyon (cf. Ref. [12]). Because of the space-time dependent coupling, the procedure is somewhat tricky in our case. It will be convenient to consider first the amplitude with different momenta, as shown in Fig. 5, and set $P=P'$ and $K=Q$ in the end. The amplitude is given by

$$A^{ij}(P,P';K,Q) = \kappa^2 \langle P',n,i,j | V(K)\Delta V(Q) | P,n,i,j \rangle, \tag{52}$$

where κ is the string coupling constant, V is the vertex operator (27) and Δ is the usual string propagator (recall that the Virasoro operators L_0 and $\widetilde{L}_0$ coincide with conventional ones). Here we consider the plane wave (40) as the initial state; the fact that we actually have to deal with wave packets such as Eq. (39) has been already discussed in the previous subsection.[4]

The evaluation of the amplitude (52) is straightforward and parallels that of Ref. [12]. One finds

$$\begin{aligned} A^{ij}(P,P';K,Q) \\ &= \kappa^2 \int d^D x e^{i(P-Q)x} e^{-i(P'-K))x} e^{2\gamma x^{(+)}} \\ &\quad \times \int dz d\overline{z} (z\overline{z})^{1/4(P^\mu - i\gamma e^\mu)(-Q_\mu - i\gamma e_\mu)} \frac{1}{n^2} F^{ij}(z,\overline{z}), \end{aligned} \tag{53}$$

where we have kept the integration over the center-of-mass coordinate x and denoted

$$F^{ij}(z,\overline{z}) = |1-z|^{\frac{1}{2}(K^\mu - i\gamma e^\mu)(-Q_\mu - i\gamma e_\mu)} B^{ij}(z) B^{ij}(\overline{z}), \tag{54}$$

with

[4]The procedure below involves manipulations with formally divergent integrals, etc. This procedure can be checked by explicit analysis of amplitudes involving low-lying string states only.

$$B^{ij}(z)=\int \frac{du}{2\pi}\frac{du'}{2\pi}\frac{1}{u^{n+1}}\frac{1}{(u')^{n+1}}(1-u)^{-nK^{(+)}/P^{(+)}}\left(1-\frac{u}{z}\right)^{nQ^{(+)}/P^{(+)}}(1-u')^{nK^{(+)}/P'^{(+)}}(1-zu')^{-nQ^{(+)}/P'^{(+)}}$$
$$\times\left[\frac{1}{4}\left(P^i+Q^i\frac{u}{z-u}-K^i\frac{u}{1-u}\right)\left(P'^j+K^j\frac{u'}{1-u'}-Q^j\frac{zu'}{1-zu'}\right)+\delta^{ij}\frac{uu'}{(1-uu')^2}\right]. \quad (55)$$

The integrations here run around small circles in complex u and u' planes surrounding the origin. In Eqs. (53) and (54) we used the lightlike vector $e_\mu=(-1,1,0,\ldots,0)$ defined in Sec. III.

To extract the decay rate into final states at given level n', we expand $F(z,\overline{z})$ in a formal series in z, $\overline{z}$ (we omit superscripts i, j temporarily):

$$F(z,\overline{z})=\sum_{m,m'=-\infty}^{+\infty}F_{mm'}z^{-m}\overline{z}^{-m'}. \quad (56)$$

At $m=m'$ the corresponding integrals in Eq. (54) have poles:

$$F_{mm}\int dz d\overline{z}(z\overline{z})^{1/4(P^\mu i\gamma e^\mu)(-Q_\mu-i\gamma e_\mu)-m}$$
$$\sim\frac{\pi F_{mm}}{-\frac{1}{4}(P^\mu-i\gamma e^\mu)(-Q_\mu-i\gamma e_\mu)-m-1}$$
$$\sim\frac{8\pi F_{mm}}{(P_\mu-Q_\mu-i\gamma e_\mu)^2+(M_n^2-8m)}. \quad (57)$$

These pole terms lead to contributions to the amplitude (53) which can be written in the form

$$A^{ij}(P,P';K,Q)$$
$$=\sum_m 8\pi\kappa^2\frac{1}{n^2}F_{mm}\int d^Dxd^Dyd^DP_f e^{i(P-Q)x+\gamma x^{(+)}}$$
$$\times\frac{e^{-iP_f(x-y)}}{(2\pi)^D(-P_f^2-M_{n-m}^2)}e^{-i(P'-K)y+\gamma y^{(+)}}. \quad (58)$$

This expression is recognized as the sum of the amplitudes of processes going through states with masses M_{n-m} in a theory with trilinear coupling which exponentially depends on $x^{(+)}$. Hence, the probability of the decay into level $n'=n-m$ is determined by F_{mm}. The total probability involves also the overlap between the initial and final wave functions of the center-of-mass motion. This overlap has been considered in the previous subsection; it leads to the conservation of energy and momentum in D dimensions. Therefore, we can set $P'=P$ and $K=Q$ and obtain

$$\sum_f |A_f^{ij}(n\to n-m;Q)|^2=8\pi\kappa^2\frac{1}{n^2}F_{mm}^{ij}(P'=P;K=Q), \quad (59)$$

where A_f^{ij} denotes the amplitude of the decay into a final state f at level $(n-m)$ and a tachyon with momentum Q; the sum in the left-hand side runs over all final states at the level $(n-m)$. This expression should be integrated over the phase space of the two final string states.

To estimate the integral over the phase space, let us study first the behavior of F_{mm} at large n and m. We recall that we are interested in tachyons with energies $Q^0\sim\sqrt{n}$. Let us first consider the generic case,

$$Q^{(+)}\sim Q^{(-)}\sim Q^k\sim\sqrt{n}. \quad (60)$$

At $K=Q$ and $P'=P$, we have

$$F_{mm'}^{ij}=R_m^{ij}R_{m'}^{ij}, \quad (61)$$

where the form of R_m^{ij}‘ follows from Eqs. (54)–(56),

$$R_m^{ij}=\int\frac{du}{2\pi}\frac{du'}{2\pi}\frac{dz}{2\pi}\frac{1}{u^{n+1}(u')^{n+1}z^{-m+1}}$$
$$\times(1-z)^{-2}\left[\frac{\left(1-\frac{u}{z}\right)(1-u')}{(1-u)(1-zu')}\right]^{nQ^{(+)}/P^{(+)}}$$
$$\times\left[\frac{1}{4}\left(P^i+Q^i\frac{u(1-z)}{(z-u)(1-u)}\right)\right.$$
$$\left.\times\left(P^j+Q^j\frac{u'(1-z)}{(1-u')(1-zu')}\right)+\delta^{ij}\frac{uu'}{(1-uu')^2}\right]. \quad (62)$$

In the regime (60), R_m can be written as

$$R_m^{ij}=\int dudu'dzP^{ij}(u,u',z)e^{-nS(u,u',z)}, \quad (63)$$

where

$$S=\ln u+\ln u'-\frac{m}{n}\ln z-\frac{Q^{(+)}}{M_n}\left[-\ln(1-u)+\ln\left(1-\frac{u}{z}\right)\right.$$
$$\left.+\ln(1-u')-\ln(1-zu')\right]$$

and P^{ij} is a preexponential factor that depends on n only weakly. Here we made use of the fact that $P^{(+)}=M_n$ in the center-of-mass frame we consider.

In the regime (60) all terms in S are of order 1, so the integral in Eq. (63) can be calculated by saddle point technique. At the saddle point one finds

$$S=\chi\left(\frac{m}{n}\right)-\chi\left(\frac{m}{n}-\frac{2Q^{(+)}}{M_n}\right), \qquad (64)$$

where

$$\chi(\nu)=(1-\nu)\ln(1-\nu)+(1+\nu)\ln(1+\nu).$$

Making use of energy-momentum conservation, one obtains (again in the center-of-mass frame of the decaying string)

$$-\frac{m}{n}<\left(\frac{m}{n}-\frac{2Q^{(+)}}{M_n}\right)<\frac{m}{n},$$

so S is always positive. Therefore, we conclude that the decay probability is exponentially *suppressed* in the kinematical region (60),

$$P(n\rightarrow n-m;Q)\propto e^{-2nS}.$$

Equation (64) implies that the decay probability may be *unsuppressed* in the kinematical region different from Eq. (60), namely, at

$$Q^{(+)}\sim\frac{1}{\sqrt{n}}, \qquad (65)$$

i.e., at

$$\frac{nQ^{(+)}}{M_n}\sim 1. \qquad (66)$$

In this region,

$$nS=\chi'\left(\frac{m}{n}\right)\times 2\frac{nQ^{(+)}}{M_n}\sim 1.$$

Clearly, the saddle point calculation is not valid in this region, so we proceed in a different way. To estimate the integral in Eq. (62) in the regime (66), we make use of the following asymptotic formula [12]:

$$\int dz_1\cdots dz_k z_1^{-\lambda a_1}\cdots z_k^{-\lambda a_k}\prod(1-z_p)^{\alpha_p}$$
$$\times\prod(1-z_pz_q)^{\beta_{pq}}\prod(1-z_p/z_q)^{\gamma_{pq}}$$
$$\propto\left(\frac{1}{\lambda}\right)^{\Sigma\alpha_p+\sum\beta_{pq}+\sum\gamma_{pq}+k},$$

which is valid as $\lambda\rightarrow\infty$ with $a_p,\alpha_p,\beta_{pq},\gamma_{pq}$ fixed. We obtain

$$R_m^{ij}\propto n$$

at large n. We then recall Eqs. (59) and (61) and find that at large n,

$$\sum_f |A_f^{ij}(n\rightarrow n-m;Q)|^2=\text{ independent of } n$$

in the kinematical region (65).

To estimate the total emission probability, we note that in the region (65) one has

$$\mathbf{Q}_T^2\sim 1,$$

where $\mathbf{Q}_T=(0,0,Q^2,\ldots,Q^{25})$. Hence, the probability to decay into the level $(n-m)$ is of order

$$P(n\rightarrow n-m)=\int\frac{d^{D-2}Q_T}{(2\pi)^{D-2}}\,\frac{1}{Q^0M_nE_{n-m}}$$
$$\times\sum_f|A_f^{ij}(n\rightarrow n-m;Q)|^2\sim\frac{1}{n^{3/2}},$$

where $E_{n-m}\sim M_n\sim\sqrt{n}$ is the energy of the final excited string, and $Q^0\sim\sqrt{n}$ according to Eq. (50). The number of final states contributing to the decay Δm is of order n, so we have finally

$$\sum_m P(n\rightarrow n-m)\sim\frac{1}{\sqrt{n}}.$$

This is the decay probability per unit *D-dimensional* time x^0. To obtain its interpretation in (1+1)-dimensional terms, we recall Eq. (49) and find that the emission rate per unit conformal time σ^0 of (1+1)-dimensional universe is *independent* of n at large n.

This is our principal result: the rate of the emission of baby universes is unsuppressed at large n, when the emission process should occur locally. This emission rate is proportional to the collision rate of two narrow wave packets in one-dimensional universe of conformal size π, the proportionality constant being independent of the one-dimensional momenta of the ‘‘particles’’ or width of their wave packets and being determined by the string coupling constant only.

V. DISCUSSION AND CONCLUSION

Let us summarize our results for the simplest version of the dilaton gravity with conformal matter in 1+1 dimensions. We considered mostly the case of compact one-dimensional universe and studied pulses of matter whose size is small compared to the size of the universe (i.e., whose wave numbers n are large). At least at the classical level these pulses, in the gauge

$$\rho=\text{const}\times\sigma^0 \qquad (67)$$

produce long-ranged dilaton field which is approximately Coulomb at scales small compared to the size of the universe. The magnitude of this long-ranged field is proportional to the energy of matter, which we called bare energy. The notion of ADM mass makes sense at these scales and coincides with the ADM mass defined for infinite space.

To construct quantum states, it was convenient to work in a different gauge,

$$\phi=\text{const}\times\sigma^0. \qquad (68)$$

It is important that at large γ, the sizes of matter pulses in the two gauges are similar, again at the classical level. In other words, the pulses that are narrow in the gauge (67) are also

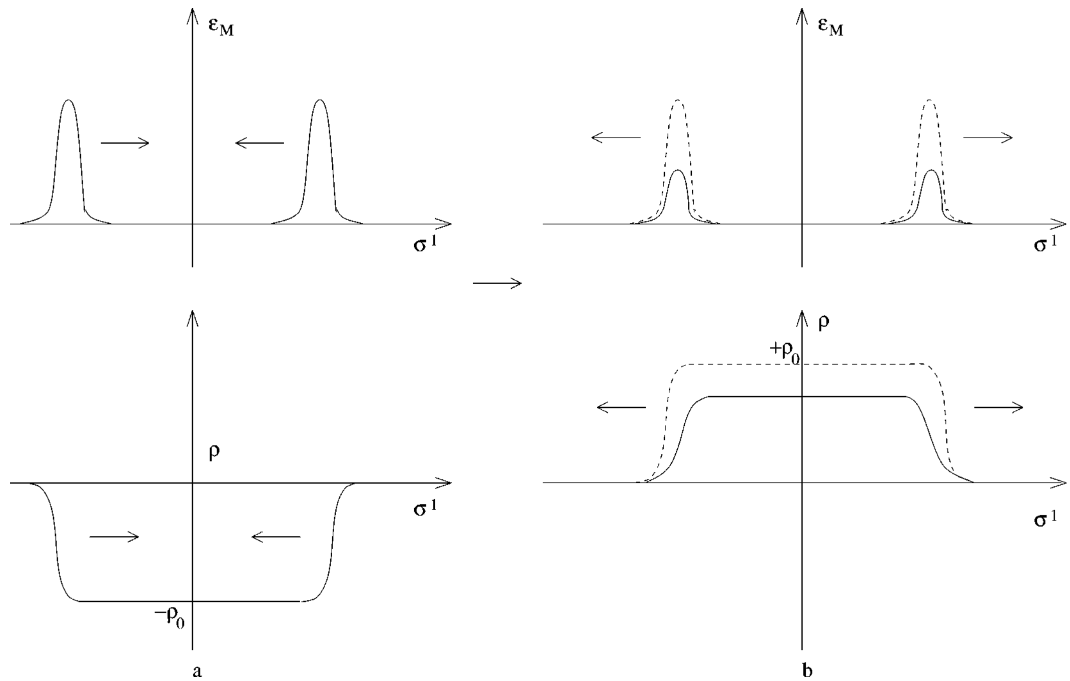

FIG. 6. (a) The field ρ and the matter energy density in the gauge (68). (b) Final state where matter energy is conserved (dashed lines) and where it is not conserved (solid lines).

narrow in the gauge (68), so the processes we were interested in are local in both gauges. Making use of the gauge (68), we considered, at the quantum level, the simplest state of the parent universe that contains one left-moving and one right-moving dressed matter ''particles'' with large wave number n. The collisions of these particles may eventually induce the emission of baby universes. If the relevant quantum numbers of the baby universe (D-dimensional momenta Q_μ of the microscopic string) are large enough, the emission process is local in the parent universe. We have found that the emission always occurs with the nonconservation of energy of matter, and that the probability of this process is finite at large n.

At first sight, there appears to be a conflict between the locality of the emission of a baby universe, and hence the locality of the nonconservation of matter energy, and the existence, in the gauge (67), of the long-ranged dilaton field whose strength is determined by the matter energy. To see that this conflict is only apparent, let us present a scenario consistent with both of the above properties. We stress that the following consideration is only a scenario, as its confirmation or rejection would require the analysis of the final state of the parent universe, which goes well beyond the scope of this paper. Also, the discussion below is essentially classical, while the actual analysis should necessarily be at the quantum level.

Let us again consider the collision of two narrow pulses of matter, and choose the gauge (68). In this gauge the field ρ and the matter energy density are those shown in Figs. 3 and 6(a). If the matter energy was conserved, the final state would be characterized by the configuration shown in Fig. 6(b) by dashed lines: the field ρ between the pulses would change from $-\rho_0$ to $+\rho_0$ where ρ_0 is determined by the total matter energy in the pulses [see Eqs. (21) and (12), (13)]. If the energy is not conserved at the moment of the collision (i.e., if the collision of the pulses induces splitting off of the baby universe), the height of the matter pulses and, correspondingly, the height of the plateau of ρ are smaller in the final state; this configuration is shown in Fig. 6(b) by solid lines.

Clearly, the process shown in Fig. 6 may be perfectly local in (1+1)-dimensional space-time. It shows that the nonconservation of matter energy does not require nonlocality. However, this process *cannot be transformed into the gauge* (67), as the field ρ does not obey the field equation $\partial_\alpha \partial^\alpha \rho = 0$ everywhere in space-time. To see what happens if the gauge (67) is chosen for the *initial* state, let us perform the gauge transformation that would transform the ''conventional'' configuraion (i.e., the configuration of the conventional process with energy conservation) into the gauge (67). In the case of infinitely narrow pulses this gauge transformation is the inverse of Eq. (20). Then the initial state is one shown in Fig. 2 (with $\rho = \mathrm{const}\cdot\sigma^0$ everywhere), while the final state is that shown in Fig. 7 by solid lines (only a small region of the universe is presented in Fig. 7; the final configuration of the conventional process with energy conservation is again shown by dashed lines for comparison). The final dilaton field ϕ in this gauge is the same as that of the conventional process; in particular, its long range behavior is not affected by the energy nonconservation. On the other hand, the field ρ in the final state is nontrivial and corresponds to longitudinal gravitational waves. It is the presence of these longitudinal waves that ensures the validity of the constraints after the collision, even though the energy of matter is not conserved and the dilaton field does not change asymptotically. Of course, the longitudinal ρ wave may be gauged away, but the corresponding gauge transformation would be nontrivial far away from the collision region, and would also induce longitudinal gravitational wave in the initial state.

In infinite space, the gauge (68) cannot be imposed, so we cannot use the arguments based on Fig. 6. However, the final states such as those shown in Fig. 7 are still possible in the

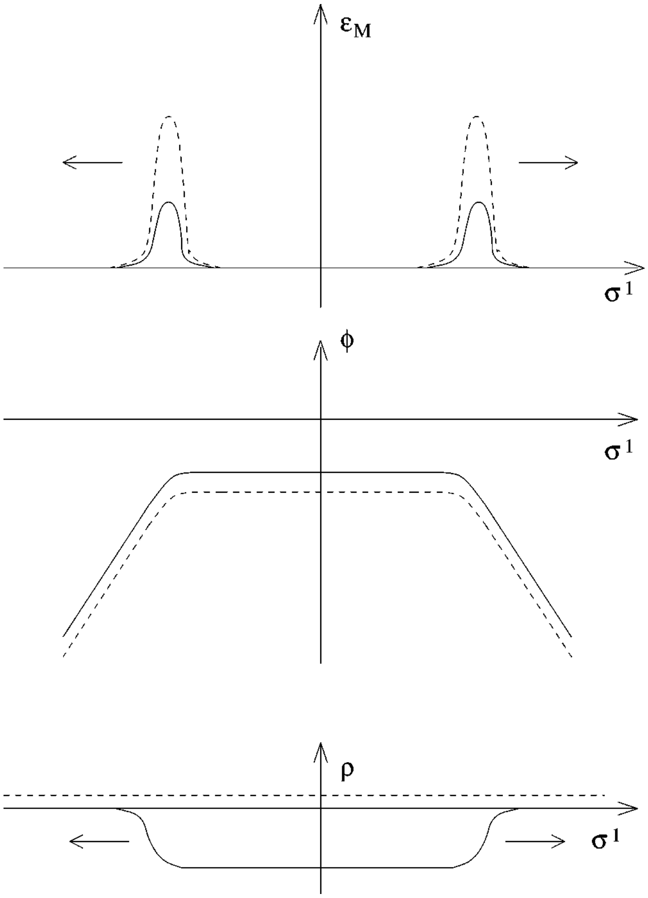

FIG. 7. Final state in the gauge (67) (solid lines), and with energy conservation (dashed lines).

gauge where $\rho=0$ initially. For these final states to appear, the field equations should be violated only in a small region of space-time (where the two pulses collide), and the entire process may occur locally. The ADM mass viewed from infinite distance is conserved, but this conservation is due to the appearance of the longitudinal gravitational waves that compensate for the nonconservation of matter energy.

It remains to be understood what part, if any, of the discussion of this paper may be relevant to (3+1)-dimensional theories. There exist semiclassical arguments, based on the study of fluctuations [29] about and analytical continuation [30] of the Euclidean wormhole solution of Ref. [5], favoring the interpretation of the wormhole as describing the process in which a baby universe branches off and then ''flies away'' in (mini)superspace. This process may be very similar to the one discussed in this paper in (1+1)-dimensional context. On the other hand, the possible nonconservation of bare energy was not explicit in the semiclassical treatment of (3+1)-dimensional Euclidean wormholes. An independent problem which can possibly be treated ''phenomenologically,'' as we did in this section, is whether the nonconservation of bare energy in 3+1 dimensions is consistent with locality, and, in particular, whether locality requires the generation of longitudinal gravitational waves. We hope our study of (1+1)-dimensional toy model will be helpful to understand these problems.

ACKNOWLEDGMENTS

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APPENDIX

Let us outline the calculation of the matrix elements (33) and (35) in the leading order in $\mathbf{f}_n$, $\tilde{\mathbf{f}}_{\tilde{n}}$. First, we make the projection onto the subspace $L_0=\tilde{L}_0$ explicit by writing the coherent state (31) in the form

$$|\Psi_P\rangle=\int_{-\pi/2}^{\pi/2}\frac{d\xi}{\pi}\exp\left(\sum_n \frac{1}{n}\mathbf{f}_n(\xi)\mathbf{A}_{-n}+\sum_{\tilde{n}}\frac{1}{\tilde{n}}\tilde{\mathbf{f}}_{\tilde{n}}(\xi)\tilde{\mathbf{A}}_{-\tilde{n}}\right)|\mathcal{P}\rangle, \tag{A1}$$

where

$$\mathbf{f}_n(\xi)=\mathbf{f}_n e^{2in\xi},$$

$$\tilde{\mathbf{f}}_{\tilde{n}}(\xi)=\tilde{\mathbf{f}}_{\tilde{n}} e^{-2i\tilde{n}\xi}. \tag{A2}$$

The representation (A1) coincides with Eq. (31) up to normalization. The fact that the state (A1) obeys the constraint $(L_0-\tilde{L}_0)|\Psi_P\rangle=0$ follows from the commutational relations of the DDF operators with L_0 and $\tilde{L}_0$,

$$[L_0,A_n^i]=-\frac{n}{2}A_n^i,$$

$$[\tilde{L}_0,A_n^i]=\frac{n}{2}A_n^i,$$

and similarly for $\tilde{A}_{\tilde{n}}^i$.

Consider now the norm of these states. One has

$$\langle\Psi_{P'}|\Psi_P\rangle=\left[\int\frac{d\xi_1 d\xi_2}{\pi^2}\exp\left(\sum \mathbf{f}_n\mathbf{f}_n^* e^{2in(\xi_1-\xi_2)}+\sum \tilde{\mathbf{f}}_{\tilde{n}}\tilde{\mathbf{f}}_{\tilde{n}}^* e^{-2i\tilde{n}(\xi_1-\xi_2)}\right)\right]\langle\mathcal{P}'|\mathcal{P}\rangle.$$

At large $\mathbf{f}_n$ and $\tilde{\mathbf{f}}_{\tilde{n}}$ this is a saddle point integral. Taking into account Eq. (32) we find that the integrand does not depend on $(\xi_1+\xi_2)$, while the saddle point in $(\xi_1-\xi_2)$ is at

$$\xi_1-\xi_2=0. \tag{A3}$$

Hence we obtain the usual result

$$\langle\Psi_{P'}|\Psi_P\rangle=\langle\mathcal{P}'|\mathcal{P}\rangle\exp\left(\sum \mathbf{f}_n\mathbf{f}_n^*+\sum \tilde{\mathbf{f}}_{\tilde{n}}\tilde{\mathbf{f}}_{\tilde{n}}^*\right)$$

up to a preexponential factor.

Let us turn to the matrix elements (33) involving matter fields only and consider explicitly the left-moving sector. The DDF operators can be written as

$$A_n^i=\int_{-\pi/2}^{\pi/2}\frac{d\sigma_+}{\pi}\exp\left(2in\frac{x^{(+)}}{P^{(+)}}+2in\sigma_+ -\frac{2n}{P^{(+)}}\sum\frac{1}{k}\alpha_k^{(+)}e^{-2ik\sigma_+}\right)\left(\frac{1}{2}P^i+\sum\alpha_q^i e^{-2iq\sigma_+}\right). \tag{A4}$$

Since P^i and $P^{(+)}$ commute with A_n^i, one can set

$$P^i=\mathcal{P}^i, P^{(+)}=\mathcal{P}^{(+)}$$

in the operator O_M for calculating the matrix element (33). Furthermore, $\alpha_k^{(+)}$ commute with O_M and with all factors in Eq. (A4), so one can set them equal to zero and write effectively

$$A_n^i=\exp\left(2in\frac{x^{(+)}}{P^{(+)}}\right)\alpha_n^i.$$

We have to calculate the matrix element

$$\langle\Psi_{P'}|\alpha_{-r_1}^{i_1}\cdots\alpha_{-r_s}^{i_s}\cdots\alpha_{p_1}^{j_1}\ldots\alpha_{p_t}^{j_t}|\Psi_P\rangle, \tag{A5}$$

with $r_1,\ldots,r_s,p_1,\ldots,p_t>0$, which is a building block of Eq. (33). Note that the operator ordering in Eq. (A5) is in fact not essential at large $\mathbf{f}_n$, as is usual in the classical limit. One finds for this matrix element

$$\int\frac{d\xi_1 d\xi_2}{\pi^2}\left\langle\mathcal{P}'\left|\exp\left[2i(r_1+\cdots+r_s-p_1-\cdots-p_t)\frac{x^{(+)}}{\mathcal{P}^{(+)}}\right](\mathrm{RM})\right|\mathcal{P}\right\rangle f_{r_1}^*(\xi_2)\cdots f_{r_s}^*(\xi_2)\cdot f_{p_1}(\xi_1)\cdots f_{p_t}(\xi_1)(\mathrm{RM}), \tag{A6}$$

where we omitted the superscripts $i_1,\ldots,j_t$; (RM) denotes the corresponding factors due to right-moving modes. All dependence on $(\xi_1+\xi_2)$ in this integral comes from the exponents in $f_r^*(\xi_2)$ and $f_p(\xi_1)$, see Eq. (A2), and similar exponents for right-moving components. This makes the matrix element in Eq. (A6) equal to $\langle\mathcal{P}'|\mathcal{P}\rangle$. The integral over $(\xi_1-\xi_2)$ is still of saddle point structure with the saddle point (A3). Hence the expression (A6) simplifies and becomes equal to

$$\langle\Psi_{P'}|\Psi_P\rangle\cdots\int\frac{d\xi}{\pi}f_{r_1}^*(\xi)\cdots f_{r_s}^*(\xi)\cdot f_{p_1}(\xi)\cdots f_{p_t}(\xi)(\mathrm{RM}).$$

We conclude that in the leading order in $\mathbf{f}_n$, $\tilde{\mathbf{f}}_{\tilde{n}}$, the calculation of the matrix elements of the matter operators is reduced to the substitution

$$\alpha_n\to e^{-2in\xi}f_n, \quad n>0,$$

$$\alpha_{-n}\to e^{2in\xi}f_n^*, \quad n>0,$$

with subsequent integration over ξ. This proves the relation between the matrix elements (33) and their classical counterparts (37).

Let us turn to the matrix elements (35). Since $P^{(+)}$ and $\alpha_k^{(+)}$ commute with O_M and with the DDF operators, the operator $\partial_+X_L^{(+)}$ reduces to

$$\partial_+X_L^{(+)}=\tfrac{1}{2}\mathcal{P}^{(+)}, \tag{A7}$$

when sandwiched as in Eq. (35). To find the matrix elements involving $\partial_+X_L^{(-)}$ one notices that in the leading order in $\mathbf{f}_n$, $\tilde{\mathbf{f}}_{\tilde{n}}$

$$\langle\Psi_{P'}|L_m O_M|\Psi_P\rangle=0, \tag{A8}$$

because the commutator of L_m and O_M does not contain $\mathbf{f}_n$, $\tilde{\mathbf{f}}_{\tilde{n}}$ (recall that $|\Psi_P\rangle$ and $|\Psi_{P'}\rangle$ are physical states). Equation (A7) and just established relation between the matrix elements (33) and classical correlators (37) immediately imply the desired relation between the matrix elements (35) and their classical versions (38). This relation can of course be obtained by an explicit calculation.

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