Heavy baryon transitions in a relativistic three-quark model

M. A. Ivanov and V. E. Lyubovitskij
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia

J. G. Körner
Johannes Gutenberg-Universität, Institut für Physik, D-55099 Mainz, Germany

P. Kroll
Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal 1, Germany

(Received 26 December 1996)

Exclusive semileptonic decays of bottom and charm baryons are considered within a relativistic three-quark model with a Gaussian shape for the baryon-three-quark vertex and standard quark propagators. We calculate the baryonic Isgur-Wise functions, decay rates, and asymmetry parameters. [S0556-2821(97)05111-4]

PACS number(s): 13.30.Ce, 12.39.Ki, 14.20.Lq, 14.20.Mr

I. INTRODUCTION

The investigation of semileptonic (SL) decays of heavy hadrons allows one to determine the unknown Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, i.e., V_{bc} and V_{bu} in bottom meson and baryon decays. These play a fundamental role in the physics of weak interactions. The CKM matrix elements can be extracted from the inclusive SL width of heavy hadron [1] or decay spectra [2] and from the exclusive differential rates of $B\to D^\star l\nu$, $\Lambda_b\to\Lambda_c l\nu$, $\ldots$, extrapolated to the point of zero recoil [1,3,4]. Other characteristics of semileptonic decays (momentum dependence of transition form factors, exclusive decay rates, asymmetry parameters, etc.) are also important for our understanding of the heavy hadron structure.

From a modern point of view the appropriate theoretical framework for the analysis of hadrons containing a single heavy quark is the heavy quark effective theory (HQET) [5–11] based on a systematic $1/m_Q$ expansion of the QCD Lagrangian. The leading order of the HQET expansion, when the heavy quark mass goes to infinity, corresponds to the case of heavy quark symmetry (or Isgur-Wise symmetry) [6]. Because of the Isgur-Wise (IW) symmetry the structure of weak currents of low-lying baryons is simplified. The form factors of these transitions are expressed through a few universal functions. Unfortunately, HQET can give predictions only for the normalization of the form factors at zero recoil. Once one moves away from the zero recoil point one has to take recourse to full nonperturbative calculations.

This paper focuses on exclusive SL decays of the ground state bottom and charm baryons. Recently, the activity in this field has started to make contact with experiment due to the observation of the CLEO Collaboration [12] of the heavy-to-light SL decay mode $\Lambda_c^+\to\Lambda e^+\nu_e$. Also the ALEPH [13] and OPAL [14] Collaborations expect to observe the exclusive mode $\Lambda_b\to\Lambda_c l\nu$ in the near future. Therefore, a theoretical study of the SL decays of heavy baryons seems to be very important.

In [15,16] a model for QCD bound states composed of light and heavy quarks was proposed. The model is the Lagrangian formulation of the NJL model with separable interaction [17,18] but its advantage consists in the possibility of studying baryons as relativistic systems of three quarks. The general framework was developed for light mesons [15,16] and baryons [16,19], and also for heavy-light hadrons [20]. Particularly, in Refs. [15,16] the pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0\to\gamma$ transition, the pion charge form factor, and the strong πNN form factor have been calculated and good agreement with the data has been achieved with three parameters. Two of the parameters are range parameters characterizing the size of mesons and baryons. The remaining parameter is the constituent quark mass. In Ref. [19] the approach developed in [15,16] was applied to a calculation of the electromagnetic form factors of nucleons. Some preliminary results on SL decays of heavy-light baryons were already presented in [20].

The purpose of the present work is to give a description of the properties of baryons containing a single heavy quark within the framework proposed in [15,16] and developed in [19,20]. Namely, we report the calculation of observables in semileptonic decays of bottom and charm baryons: Isgur-Wise functions, asymmetry parameters, decay rates, and distributions.

II. MODEL

We start with a brief review of our approach [15,16] based on interaction Lagragians coupling hadrons with constituent quarks and *vice versa*. It was found [15,16,19,20] that this approach successfully describes low-energy hadronic properties like decay constants, form factors, etc. Here we are going to apply this approach to the calculation of baryonic observables when the baryons contain a heavy (b or c) quark.

Let y_i $(i=1,2,3)$ be the spatial four-coordinates of quarks with masses m_i, respectively. They are expressed through the center of mass coordinate (x) and relative Jacobi coordinates ($\xi_1,\ldots$) as

$$y_1=x-3\xi_1\frac{m_2+m_3}{\Sigma_i m_i},$$

$$y_2 = x + 3\xi_1 \frac{m_1}{\Sigma_i m_i} - 2\xi_2\sqrt{3}\frac{m_3}{m_2+m_3}, \qquad (1)$$

$$y_3 = x + 3\xi_1 \frac{m_1}{\Sigma_i m_i} + 2\xi_2\sqrt{3}\frac{m_2}{m_2+m_3},$$

where

$$x = \frac{\Sigma_i m_i y_i}{\Sigma_i m_i}, \quad \xi_1 = \frac{1}{3}\left(\frac{m_2 y_2 + m_3 y_3}{m_2+m_3} - y_1\right), \quad \xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}.$$

We assume that the momentum distribution of the constituents inside a baryon is modeled by an effective relativistic vertex function

$$F\left(\frac{\Lambda_B^2}{18}\sum_{i<j}(y_i - y_j)^2\right)$$

which depends on the sum of relative coordinates only in the configuration space and a cutoff parameter Λ_B. Generally speaking, the shape of this function should be defined from an equation on the bound states and will depend on the flavors of quarks. To reduce the number of free parameters, we will use the universal function (Gaussian) for all flavors with the different values of parameter Λ_B. The Gaussian shape guarantees ultraviolet convergence of matrix elements. At the same time the vertex function is a phenomenological description of the long distance QCD interactions between quarks and gluons. In this paper we will consider the semileptonic decays of baryons with one heavy quark (b or c). Thus there are at least three different values for Λ_B corresponding to (s,d,u), (c,d,u), and (b,d,u) sectors. But, as we show below, the exhibition of the Isgur-Wise symmetry in the heavy quark limit ($m_Q \to \infty$) gives that Λ_B parameter should be the same for charm and bottom baryons:

$$\mathcal{L}_B^{\rm int}(x) = g_B \overline{B}(x)\int dy_1 \int dy_2 \int dy_3 \delta\left(x - \frac{\Sigma_i m_i y_i}{\Sigma_i m_i}\right) \times F\left(\frac{\Lambda_B^2}{18}\sum_{i<j}(y_i - y_j)^2\right) J_B(y_1,y_2,y_3) + \text{H.c.} \qquad (2)$$

with $J_B(y_1,y_2,y_3)$ being the three-quark current with quantum numbers of a baryon B:

$$J_B(y_1,y_2,y_3) = \Gamma_1 q^{a_1}(y_1) q^{a_2}(y_2) C\Gamma_2 q^{a_3}(y_3)\varepsilon^{a_1 a_2 a_3}. \qquad (3)$$

Here $\Gamma_{1(2)}$ are strings of Dirac matrices, $C=\gamma^0\gamma^2$ is the charge conjugation matrix, and a_i are the color indices.

The choice of baryonic currents depends on two different cases: (a) light baryons composed from u,d,s quarks; (b) heavy-light baryons with a single heavy quark b or c.

In the case of light baryons we shall work in the limit of isospin invariance by assuming that the masses of u and d quarks are equal to each other, i.e., $m_u = m_d = m$. The breaking of SU(3) symmetry is taken into account via a difference of strange and nonstrange quark masses $m_s - m \neq 0$. Thus, for baryons composed either of u or d quarks (nucleons, Δ-isobar) or of s quarks (Ω-hyperon) the coordinates of quarks may be written as

$$y_1 = x - 2\xi_1, \quad y_2 = x + \xi_1 - \xi_2\sqrt{3}, \quad y_3 = x + \xi_1 + \xi_2\sqrt{3}.$$

If a light baryon contains a single strange quark with mass m_s and two nonstrange quarks (u or d) with a mass m each as in Λ and Σ hyperons one gets

$$y_1 = x - 6\xi_1\frac{m}{2m+m_s},$$

$$y_2 = x + 3\xi_1\frac{m_s}{2m+m_s} - \xi_2\sqrt{3},$$

$$y_3 = x + 3\xi_1\frac{m_s}{2m+m_s} + \xi_2\sqrt{3},$$

where y_1 is the coordinate of the strange quark and y_2 and y_3 are the coordinates of nonstrange quarks.

For a baryon with two strange quarks and a single nonstrange quark (as, e.g., in the Ξ hyperons) one obtains

$$y_1 = x - 6\xi_1\frac{m_s}{2m_s+m},$$

$$y_2 = x + 3\xi_1\frac{m}{2m_s+m} - \xi_2\sqrt{3},$$

$$y_3 = x + 3\xi_1\frac{m}{2m_s+m} + \xi_2\sqrt{3},$$

where y_1 now is the coordinate of the nonstrange quark and y_2 and y_3 are the coordinates of the strange quarks.

The spin-flavor structure of light baryonic currents with quantum numbers $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ has been studied in detail in the papers [21–26]. It was shown that there are two possibilities to choose the baryonic currents with $J^P = \frac{1}{2}^+$:

TABLE I. Three-quark currents of light baryons.

Baryon	Three-quark current
Proton	$J_p^T(y_1,y_2,y_3) = [u^a(y_1)u^b(y_2)C\gamma^5 d^c(y_3) + \gamma^5 u^a(y_1)u^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$
Neutron	$J_n^T(y_1,y_2,y_3) = [d^a(y_1)d^b(y_2)C\gamma^5 u^c(y_3) + \gamma^5 d^a(y_1)d^b(y_2)Cu^c(y_3)]\varepsilon^{abc}$
Ξ^- hyperon	$J_{\Xi^-}^T(y_1,y_2,y_3) = [s^a(y_1)s^b(y_2)C\gamma^5 d^c(y_3) + \gamma^5 s^a(y_1)s^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$
Λ^0 hyperon	$J_{\Lambda^0}^T(y_1,y_2,y_3) = [s^a(y_1)u^b(y_2)C\gamma^5 d^c(y_3) + \gamma^5 s^a(y_1)u^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$

vector variant $J_B^V(y_1,y_2,y_3)$

$$=\gamma^\mu\gamma^5 q^{a_1}(y_1)q^{a_2}(y_2)C\gamma_\mu q^{a_3}(y_3)\varepsilon^{a_1a_2a_3}, \qquad (4)$$

tensor variant $J_B^T(y_1,y_2,y_3)$

$$=\sigma^{\mu\nu}\gamma^5 q^{a_1}(y_1)q^{a_2}(y_2)C\sigma_{\mu\nu} q^{a_3}(y_3)\varepsilon^{a_1a_2a_3}. \qquad (5)$$

Both of these forms have been used in [26,27] for studying the electromagnetic and strong properties of light baryons. It was shown that the *tensor variant* is more suitable for the description of the data. For this reason we will use the tensor current in the approach developed in this paper. For convenience the tensor current can be transformed into a sum of *pseudoscalar* [$\Gamma_1=I,\Gamma_2=\gamma_5$ in Eq. (3)] and *scalar currents* [$\Gamma_1=\gamma_5,\Gamma_2=I$ in Eq. (3)] using the Fierz transformations:

$$(\sigma^{\mu\nu}\gamma^5)_{i_1i_2}(C\sigma_{\mu\nu})_{i_3i_4}=-2[I_{i_1i_4}(C\gamma_5)_{i_3i_2}+\gamma^5_{i_1i_4}C_{i_3i_2}]+4[I_{i_1i_4}(C\gamma_5)_{i_3i_2}+\gamma^5_{i_1i_4}C_{i_3i_2}].$$

For example, a *tensor current* for the proton

$$J_p^T(y_1,y_2,y_3)=\sigma^{\mu\nu}\gamma^5 d^{a_1}(y_1)u^{a_2}(y_2)C\sigma_{\mu\nu}u^{a_3}(y_3)\varepsilon^{a_1a_2a_3}$$

written in $S+P$ form becomes

$$J_p^T(y_1,y_2,y_3)=4[u^{a_1}(y_3)u^{a_2}(y_2)C\gamma_5 d^{a_3}(y_1)+\gamma_5 u^{a_1}(y_3)u^{a_2}(y_2)Cd^{a_3}(y_1)]\varepsilon^{a_1a_2a_3}$$

in the Fierz transformed form. After exchanging the variables $y_1\leftrightarrow y_3$ in the interaction Lagrangian of the proton with quarks we have

$$\begin{aligned}\mathcal{L}_P^{\mathrm{int},T}(x)&=4g_p^T\overline{p(x)}\int dy_1\int dy_2\int dy_3\,\delta\left(x-\frac{\Sigma_i m_i y_i}{\Sigma_i m_i}\right)F\left(\frac{\Lambda_B^2}{18}\sum_{i<j}(y_i-y_j)^2\right)[u^{a_1}(y_1)u^{a_2}(y_2)C\gamma_5 d^{a_3}(y_3)\\&\quad+\gamma_5 u^{a_1}(y_1)u^{a_2}(y_2)Cd^{a_3}(y_3)]\varepsilon^{a_1a_2a_3}+\mathrm{H.c.}\\&=4g_p^T\overline{p(x)}\int d\xi_1\int d\xi_2 F(\Lambda_B^2\cdot[\xi_1^2+\xi_2^2])[u^{a_1}(x-2\xi_1)u^{a_2}(x+\xi_1-\xi_2\sqrt{3})C\gamma_5 d^{a_3}(x+\xi_1+\xi_2\sqrt{3})\\&\quad+\gamma_5 u^{a_1}(x-2\xi_1)u^{a_2}(x+\xi_1-\xi_2\sqrt{3})Cd^{a_3}(x+\xi_1+\xi_2\sqrt{3})]\varepsilon^{a_1a_2a_3}+\mathrm{H.c.}\end{aligned} \qquad (6)$$

The Fourier transform of the vertex form factor is defined as

$$F(\Lambda_B^2\cdot[\xi_1^2+\xi_2^2])=\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\times\exp(-ik_1\xi_1-ik_2\xi_2)F\left(\frac{k_1^2+k_2^2}{\Lambda_B^2}\right). \qquad (7)$$

Table I contains a set of *tensor currents* for nucleons, Λ^0, and Ξ^- hyperons in the $S+P$ form which will be used in our calculations.

Next we turn to the discussion of heavy-light baryonic currents. Suppose that the heavy-quark mass is much larger than the light-quark masses ($m_Q\gg m_{q_1},m_{q_2}$). From Eq. (1) one then obtains

$$y_1=y_Q=x,$$

$$y_2=y_{q_1}=x+3\xi_1-2\xi_2\sqrt{3}\frac{m_{q_2}}{m_{q_1}+m_{q_2}},$$

$$y_3=y_{q_2}=x+3\xi_1+2\xi_2\sqrt{3}\frac{m_{q_1}}{m_{q_1}+m_{q_2}},$$

where y_1 is the coordinate of heavy quark, and y_2 and y_3 are the coordinates of light quarks q_1 and q_2.

It is convenient to transform the Jacobi coordinates of Eq. (1) to remove the light-quark mass dependence

$$\xi_1\to\xi_1-\frac{\xi_2}{\sqrt{3}}\frac{m_{q_1}-m_{q_2}}{m_{q_1}+m_{q_2}},$$

$$\xi_2\to\xi_2.$$

Then we have

$$y_1=y_Q=x,\quad y_2=y_{q_1}=x+3\xi_1-\xi_2\sqrt{3},$$

$$y_3=y_{q_2}=x+3\xi_1+\xi_2\sqrt{3}.$$

The problem of the spin-flavor structure of heavy-light baryonic currents was analyzed in Refs. [23–25]. It was shown that, in the static limit $\vec{p}_Q\to 0$ (this is equivalent to the heavy quark limit $m_Q\to\infty$), Λ-type baryons (Λ_Q, Ξ_Q) containing a light diquark system with zero spin may be described by either of the following nonderivative three-quark currents:

$$J^P_{\Lambda_{h_Q}}=\varepsilon^{abc}h_Q^a u^b C\gamma^5 d^c,$$

$$J^A_{\Lambda_{h_Q}}=\varepsilon^{abc}h_Q^a u^b C\gamma^0\gamma^5 d^c,$$

where h_Q denotes the effective static field of the heavy quark.

In the same vein there are two currents for Ω-type baryons (Ω_Q, Σ_Q and $\Omega_Q^\star$, $\Sigma_Q^\star$) containing a light diquark system with spin 1

$$J^{V}_{\Omega_{h_Q}}=\varepsilon^{abc}\vec{\gamma}\gamma_5 h^a_Q s^b C\vec{\gamma}s^c,$$

$$J^{V;k}_{\Omega^\star_{h_Q}}=\varepsilon^{abc}\left[h^a_Q s^b C\gamma^k s^c+\frac{1}{3}\gamma^k\vec{\gamma}h^a_Q s^b C\vec{\gamma}s^c\right],$$

$$J^{T}_{\Omega_{h_Q}}=\varepsilon^{abc}\vec{\gamma}\gamma_5 h^a_Q s^b C\gamma^0\vec{\gamma}s^c,$$

$$J^{T;k}_{\Omega^\star_{h_Q}}=\varepsilon^{abc}\left[h^a_Q s^b C\gamma^0\gamma^k s^c+\frac{1}{3}\gamma^k\vec{\gamma}h^a_Q s^b C\gamma^0\vec{\gamma}s^c\right],$$

where $k=1,2,3$, $(\gamma^k)^2=-3$. The currents $J^{I;k}_{\Omega^\star_{h_Q}}$ $(I=V,T)$ satisfy the spin-3/2 Rarita-Schwinger condition $\gamma^k J^{I;k}_{\Omega^\star_{h_Q}}=0$. In this paper we work with Lorentz-covariant representations of the HQET heavy-light currents mentioned above [23–25].

Our currents are listed below:

pseudoscalar variant $\quad J^P_{\Lambda_{h_Q}}\to J^P_{\Lambda_Q}=\varepsilon^{abc}Q^a u^b C\gamma^5 d^c,$ (8)

axial variant $\quad J^A_{\Lambda_{h_Q}}\to J^A_{\Lambda_Q}=\varepsilon^{abc}\gamma_\mu Q^a u^b C\gamma^\mu\gamma^5 d^c,$ (9)

vector variant $\quad J^V_{\Omega_{h_Q}}\to J^V_{\Omega_Q}=\varepsilon^{abc}\gamma_\mu\gamma^5 Q^a s^b C\gamma^\mu s^c,$ (10)

$$J^{V;k}_{\Omega^\star_{h_Q}}\to J^{V;\mu}_{\Omega^\star_Q}+J^{(\perp)V;\mu}_{\Omega^\star_Q},$$

$$J^{V;\mu}_{\Omega^\star_Q}=\varepsilon^{abc}Q^a s^b C\gamma^\mu s^c, \tag{11}$$

$$J^{(\perp)V;\mu}_{\Omega^\star_Q}=-\frac{1}{4}\varepsilon^{abc}\gamma^\mu\gamma_\nu Q^a s^b C\gamma^\nu s^c,$$

tensor variant $\quad J^T_{\Omega_{h_Q}}\to J^T_{\Omega_Q}=\varepsilon^{abc}\sigma_{\mu\nu}\gamma_5 Q^a s^b C\sigma^{\mu\nu}s^c,$ (12)

$$J^{T;k}_{\Omega^\star_{h_Q}}\to J^{T;\mu}_{\Omega^\star_Q}+J^{(\perp)T;\mu}_{\Omega^\star_Q},$$

$$J^{T;\mu}_{\Omega^\star_Q}=-i\varepsilon^{abc}\gamma_\nu Q^a s^b C\sigma^{\mu\nu}s^c, \tag{13}$$

$$J^{(\perp)T;\mu}_{\Omega^\star_Q}=\frac{i}{4}\varepsilon^{abc}\gamma^\mu\gamma_\alpha\gamma_\nu Q^a s^b C\sigma^{\alpha\nu}s^c.$$

The currents $J^{(\perp)I;\mu}_{\Omega^\star_Q}(I=V,T)$ are orthogonal to the corresponding baryon field with spin 3/2: $\overline{\Omega^\star_Q}^{\mu}\cdot J^{(\perp)I;\mu}_{\Omega^\star_Q}=0$ and can, therefore, be omitted in the interaction Lagrangian (2). Thus, for heavy-light baryons with spin 3/2 we use the currents $J^V_{\Omega^\star_Q}$ (11) and $J^T_{\Omega^\star_Q}$ (13).

In Table II we give the quark content, the quantum numbers (spin-parity J^P, spin S_{qq}, and isospin I_{qq} of light diquark) and the experimental (when available) and theoretical mass spectrum of heavy baryons [28,29] which will be analyzed in this paper. Square brackets [ ] and round brackets { } denote antisymmetric and symmetric flavor and spin combinations of the light degrees of freedom.

TABLE II. Quantum numbers of heavy-light baryons.

Baryon	Quark content	J^P	(S_{qq},I_{qq})	Mass (GeV)
Λ_c^+	$c[ud]$	$\frac{1}{2}^+$	(0,0)	2.285
Ξ_c^+	$c[us]$	$\frac{1}{2}^+$	(0,1/2)	2.466
Σ_c^{++}	$c\{uu\}$	$\frac{1}{2}^+$	(1,1)	2.453
Ω_c^0	$c\{ss\}$	$\frac{1}{2}^+$	(1,0)	2.719
$\Sigma_c^{\star++}$	$c\{uu\}$	$\frac{3}{2}^+$	(1,1)	2.510
$\Omega_c^{\star 0}$	$c\{ss\}$	$\frac{3}{2}^+$	(1,0)	2.740
Λ_b^0	$b[ud]$	$\frac{1}{2}^+$	(0,0)	5.640
Ξ_b^+	$b[us]$	$\frac{1}{2}^+$	(0,1/2)	5.800
Σ_b^+	$b\{uu\}$	$\frac{1}{2}^+$	(1,1)	5.820
Ω_b^-	$b\{ss\}$	$\frac{1}{2}^+$	(1,0)	6.040

We write down the Lagrangian which describes the interaction of Λ_Q baryon with quarks in the heavy quark limit:

$$\mathcal{L}^{\rm int}_{\Lambda_Q}(x)=g_{\Lambda_Q}\overline{\Lambda_Q}(x)\Gamma_1 Q^a(x)\int d\xi_1$$
$$\times\int d\xi_2 F(\Lambda^2_{B_Q}\cdot[\xi_1^2+\xi_2^2])u^b(x+3\xi_1-\xi_2\sqrt{3})$$
$$\times C\Gamma_2 d^c(x+3\xi_1+\xi_2\sqrt{3})\varepsilon^{abc}+\text{H.c.}, \tag{14}$$

where

$$\Gamma_1\otimes C\Gamma_2=\begin{cases} I\otimes C\gamma^5 & \textit{pseudoscalar current},\\ \gamma_\mu\otimes C\gamma^\mu\gamma^5 & \textit{axial vector current}.\end{cases}$$

One can see that the heavy quark is factorized out from light degrees of freedom in this limit. The vertex form factor F characterizes the distribution of u and d quarks inside the Λ_Q baryon. It is readily seen that the Lagrangian (14) exhibits the flavor symmetry (symmetry under exchange b with c) if the parameter Λ_{B_Q} will be the same for charm and bottom baryons.

Next we discuss the model parameters. First, there are the baryon-quark coupling constants and the vertex function in the Lagrangian (2). The coupling constants are calculated from *the compositeness condition* (see Ref. [30]), i.e., the renormalization constant of the baryon wave function is set equal to zero, $Z_B=1-g_B^2\Sigma'_B(M_B)=0$, with Σ_B being the baryon mass operator [see Fig. 1(a) for light baryons and Fig. 1(b) for heavy baryons]. The expressions for the mass operators of light baryons $\Sigma_B(p)$ and heavy-light ones $\Sigma_{B_Q}(p)$ are written as

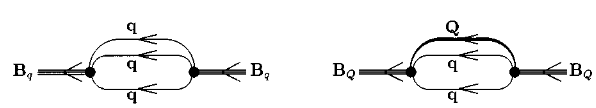

FIG. 1. (a) Light baryon mass operator. (b) Heavy-light baryon mass operator.

$$\Sigma_B(p)=g_B^2\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}F^2\left(\frac{(3k+2p)^2+3k'^2}{\Lambda_B^2}\right)\times\Gamma_1 S_q(k+p)\Gamma_2 \mathrm{Tr}\left[\Gamma_1' S_q\left(\frac{k'-k}{2}\right)\times\Gamma_2' S_q\left(\frac{k'+k}{2}\right)\right], \tag{15}$$

$$\Sigma_{B_Q}(p)=g_{B_Q}^2\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}F^2\left(\frac{9k^2+3k'^2}{\Lambda_{B_Q}^2}\right)\times\Gamma_1 S_Q(k+p)\Gamma_2 \mathrm{Tr}\left[\Gamma_1' S_q\left(\frac{k'-k}{2}\right)\times\Gamma_2' S_q\left(\frac{k'+k}{2}\right)\right], \tag{16}$$

with $S_q(k)$ and $S_Q(k)$ being the light and heavy quark propagators, respectively. All color, flavor, and combinatorial coefficients are omitted.

For light quark propagator with a mass m_q we shall use the standard form of the free fermion propagator

$$\langle 0|T[q(x)\overline{q(y)}]|0\rangle=\int\frac{d^4k}{(2\pi)^4 i}e^{-ik(x-y)}S_q(k),$$

$$S_q(k)=\frac{1}{m_q-\not k}. \tag{17}$$

For the heavy quark propagator we will use the leading term in the inverse mass expansion. Suppose $p=M_{B_Q}v$ is the heavy baryon momentum. We introduce the parameter $\overline{\Lambda}_{\{q_1q_2\}}=M_{\{Qq_1q_2\}}-m_Q$ which is the difference between the heavy baryon mass $M_{\{Qq_1q_2\}}\equiv M_{B_Q}$ and the heavy quark mass. Keeping in mind that the vertex function falls off sufficiently fast such that the condition $|k|\ll m_Q$ holds where k is the virtual momentum of light quarks, one has

$$S_Q(p+k)=\frac{1}{m_Q-(\not p+\not k)}=\frac{m_Q+M_{B_Q}\not v+\not k}{m_Q^2-M_{B_Q}^2-2M_{B_Q}vk-k^2}=S_v(k,\overline{\Lambda}_{\{q_1q_2\}})+O\left(\frac{1}{m_Q}\right),$$

$$S_v(k,\overline{\Lambda}_{\{q_1q_2\}})=-\frac{(1+\not v)}{2(v\cdot k+\overline{\Lambda}_{\{q_1q_2\}})}. \tag{18}$$

In what follows we will assume that $\overline{\Lambda}\equiv\overline{\Lambda}_{uu}=\overline{\Lambda}_{dd}=\overline{\Lambda}_{du}$, $\overline{\Lambda}_s\equiv\overline{\Lambda}_{us}=\overline{\Lambda}_{ds}$. Thus there are three independent parameters: $\overline{\Lambda}$, $\overline{\Lambda}_s$, and $\overline{\Lambda}_{ss}$.

A drawback of our approach is the lack of confinement. This can in principle be corrected by changing the analytic properties of the light-quark propagator. We leave the investigation of this possibility for future studies. For the time being we shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by postulating the following condition: the baryon mass must be less than the sum of constituent quark masses $M_B<\Sigma_i m_{q_i}$.

In the case of heavy-light baryons the restriction $M_B<\Sigma_i m_{q_i}$ implies that the parameter $\overline{\Lambda}_{\{q_1q_2\}}$ must be less than the sum of light quark masses $\overline{\Lambda}_{\{q_1q_2\}}<m_{q_1}+m_{q_2}$. The last constraint serves as the upper limit for our choices of the parameter $\overline{\Lambda}_{\{q_1q_2\}}$.

Actually, the compositeness condition is equivalent to the normalization of the elastic form factors to one at zero momentum transfer. This may be readily seen from the Ward indentity which relates the vertex function with the mass operator on mass shell. We have

$$\Lambda^\mu_{B\to B\gamma}(p,p)|_{\not p=M_B}=g_B^2\frac{\partial\Sigma_B(p)}{\partial p^\mu}\bigg|_{\not p=M_B}=\gamma^\mu g_B^2\Sigma_B'(p)|_{\not p=M_B}, \tag{19}$$

where the vertex function is related to the baryon elastic form factor by

$$\Lambda^\mu_{B\to B\gamma}(p,p)=\gamma^\mu F_B(0). \tag{20}$$

From this the normalization of the form factor mentioned above immediately follows.

The vertex function F is an arbitrary function except that it should make the Feynman diagrams ultraviolet finite, as we have mentioned above. In the papers [15,16] we have found that the basic physical observables of pion and nucleon low-energy physics depend only weakly on the choice of the vertex functions. As was mentioned above, in this paper we choose a Gaussian vertex function for simplicity. In Minkowski space we write

$$F\left(\frac{k_1^2+k_2^2}{\Lambda_B^2}\right)=\exp\left(\frac{k_1^2+k_2^2}{\Lambda_B^2}\right),$$

where Λ_B is the Gaussian range parameter which may be related to the size of a baryon. Note that all calculations are done in the Euclidean region ($k_i^2=-k_{iE}^2$) where the above vertex function decreases very rapidly. It was found in [19]

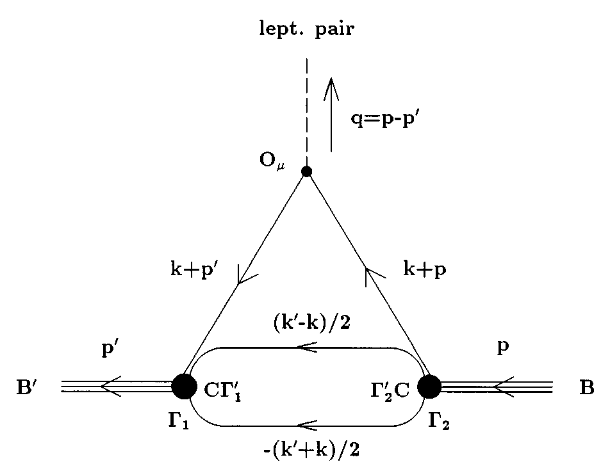

FIG. 2. Semileptonic decay of heavy-light baryon.

that for nucleons ($B=N$) the value $\Lambda_N=1.25$ GeV gives a good description of the nucleon's static characteristics (magnetic moments, charge radii) and its form factors in the spacelike region for Q^2 up to 1 GeV^2. Here, we will use the value $\Lambda_{B_q}\equiv\Lambda_N=1.25$ GeV for all light baryons. Now we will demonstrate that the requirement of the normalization of the baryonic IW functions $\zeta(\omega)$ and $\xi_1(\omega)$ at zero recoil $\omega=1$ to one $[\zeta(1)=1,\ \xi_1(1)=1]$ imposes the restriction on the choice of Λ_{B_Q}: $\Lambda_{B_b}=\Lambda_{B_c}$. The expressions for the baryonic IW functions for arbitrary values of Λ_{B_Q} are written as

$$\zeta(\omega)=\frac{\Phi(\sqrt{2\Lambda^2_{B_b}\Lambda^2_{B_c}/(\Lambda^2_{B_b}+\Lambda^2_{B_c})},\omega)}{\sqrt{\Phi(\Lambda_{B_b},1)}\sqrt{\Phi(\Lambda_{B_c},1)}}, \tag{21}$$

$$\Phi(\Lambda_{B_Q},w)=\Lambda^4_{B_Q}\int_0^\infty dxx\int_0^\infty\frac{dyy}{(y+1)^2}\int_0^1 d\phi\int_0^1 d\theta\left[\frac{m_q^2}{\Lambda_{B_Q}}+\frac{1}{6[3+4y+4y^2\theta(1-\theta)]}+\frac{x^2[1+2y+4y^2\theta(1-\theta)]}{4(1+y)^2}\right.$$
$$\left.\times[1+2\phi(1-\phi)(\omega-1)]\right]\exp\left[-24y\frac{m_q^2}{\Lambda^2_{B_Q}}-\frac{3+4y+4y^2\theta(1-\theta)}{1+y}\left(6x^2[1+2\phi(1-\phi)(\omega-1)]-12x\frac{\bar\Lambda}{\Lambda_Q}\right)\right].$$

It is easily seen that $\zeta(1)=1$ only when $\Lambda_{B_b}=\Lambda_{B_c}$. The parameter $\Lambda_{B_Q}=\Lambda_{B_b}=\Lambda_{B_c}$ is an adjustable parameter in our calculations.

Thus there are three sets of adjustable parameters in our model: the constituent light quark masses m_q ($m=m_u=m_d$ and m_s), the range cutoff parameters Λ_B (Λ_{B_q} and Λ_{B_Q}) and a set of $\bar\Lambda_{\{q_1q_2\}}$ subsidiary parameters: $\bar\Lambda$, $\bar\Lambda_s$, and $\bar\Lambda_{\{ss\}}$. The parameters $m=420$ MeV and $\Lambda_{B_q}=1.25$ GeV were fixed in Ref. [19] from a best fit to the data on electromagnetic properties of nucleons. The parameters Λ_{B_Q}, m_s, $\bar\Lambda$ are determined in this paper from the analysis of the $\Lambda_c^+\to\Lambda^0+e^++\nu_e$ decay data. The following values are obtained: $\Lambda_Q=2.5$ GeV, $m_s=570$ MeV, and $\bar\Lambda=710$ MeV.

The parameters $\bar\Lambda_s$ and $\bar\Lambda_{\{ss\}}$ cannot be adjusted at present since at present there are no experimental data on the decays of heavy-light baryons containing one or two strange quarks.

III. MATRIX ELEMENTS OF SEMILEPTONIC DECAYS OF BOTTOM AND CHARM BARYONS

In our model the semileptonic decays of bottom and charm baryons are described by the standard triangle quark diagram (Fig. 2). The matrix elements describing heavy-to-heavy ($b\to c$) and heavy-to-light ($c\to s$) transitions can be written as follows.

$b\to c$ transition

$$\overline{u}(v')M_\Gamma(v,v')u(v)$$
$$=g_{B_b}g_{B_c}\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}\mathrm{Tr}\left[\Gamma_1'S_q\left(\frac{k'-k}{2}\right)\Gamma_2'S_q\left(\frac{k'+k}{2}\right)\right]\exp\left(\frac{18k^2+6k'^2}{\Lambda^2_{B_Q}}\right)\overline{u}(v')\Gamma_1S_{v'}(k,\bar\Lambda)\Gamma S_v(k,\bar\Lambda)\Gamma_2u(v). \tag{22}$$

$c\to s$ transition

$$\overline{u}(p')M_\Gamma(p,v')u(v)=g_{B_s}g_{B_c}\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}\mathrm{Tr}\left[\Gamma_1'S_q\left(\frac{k'-k}{2}\right)\Gamma_2'S_q\left(\frac{k'+k}{2}\right)\right]\exp\left(\frac{9k^2+3k'^2}{\Lambda^2_{B_Q}}\right)\exp\left(\frac{9(k+\alpha p')^2+3k'^2}{\Lambda^2_{B_q}}\right)$$
$$\times\overline{u}(p')\Gamma_1S_s(k+p')\Gamma S_v(k,\bar\Lambda)\Gamma_2u(v), \tag{23}$$

$$\alpha=\frac{2m}{2m+m_s}.$$

Here Tr[ ] corresponds to the light quark loop obtained after a standard transformation which involves the charge conjugation matrix C

$$(C\Gamma_1')^{\alpha\mu}S_q^{\mu\nu}\left(\frac{k'-k}{2}\right)(\Gamma_2'C)^{\nu\beta}S_q^{\alpha\beta}\left(-\frac{k'+k}{2}\right)=\mathrm{Tr}\left[\Gamma_1'S_q\left(\frac{k'-k}{2}\right)\Gamma_2'S_q\left(\frac{k'+k}{2}\right)\right].$$

Calculational details of the matrix elements (22) and (23) are given in the Appendix.

We now turn to the discussion of matrix elements of heavy-to-heavy baryonic decays. In this paper we consider decays of bottom baryons (Λ_b^0, Ξ_b^0, Σ_b^+, and Ω_b^-) into pseudoscalar charmed baryons (Λ_c^+, Σ_c^{++}, and Ω_c^0) and pseudovector states ($\Sigma_c^{\star++}$ and $\Omega_c^{\star+}$). The matrix elements describing weak transitions between heavy baryons can be decomposed into a set of relativistic form factors. In the HQL these form factors are proportional to three universal functions ζ,ξ_1,ξ_2 of the variable $\omega=v\cdot v'$, the so-called Isgur-Wise functions [31,32]. The function $\zeta(\omega)$ describes the $b-c$ transitions of Λ-type baryons. The functions $\xi_1(\omega)$ and $\xi_2(\omega)$ describe transitions of Ω-type baryons.

Weak hadronic currents describing the transition of a heavy baryon $B_b(v)$ with four-velocity v to a heavy baryon $B_c^{(\star)}(v)$ with v' are written as [31–34] follows.

$\Lambda_b\to\Lambda_c$ *transition*

$$\langle\Lambda_c(v')|\overline{b}\Gamma c|\Lambda_b(v)\rangle=\zeta(\omega)\overline{u}(v')\Gamma u(v).$$

$\Omega_b\to\Omega_c(\Omega_c^\star)$ *transition*

$$\langle\Omega_c(v') \text{ or } \Omega_c^\star(v')|\overline{b}\Gamma c|\Lambda_b(v)\rangle = \overline{B}_c^{\mu}(v')\Gamma B_b^{\nu}(v)[-\xi_1(\omega)g_{\mu\nu}+\xi_2(\omega)v_\mu v'_\nu],$$

where the spinor tensor $B_b^\nu(v)$ satisfies the Rarita-Schwinger conditions $v_\nu B_b^\nu(v)=0$ and $\gamma_\nu B_b^\nu(v)=0$. The spin wave functions are written as

$$B_Q^\mu(v)=\frac{\gamma^\mu+v^\mu}{\sqrt{3}}\gamma^5 u_{\Omega_Q}(v) \text{ for } \Omega_Q \text{ states and}$$

$$B_Q^\mu(v)=u^\mu_{\Omega_Q^\star}(v) \text{ for } \Omega_Q^\star \text{ states,}$$

where $u_{\Omega_Q}(v)$ is the usual spin-1/2 spinor and the spinor $u^\mu_{\Omega_Q^\star}(v)$ is the usual Rarita-Schwinger spinor. Note that the Ward indentity between the derivative of the mass operator of heavy-light baryons and the vertex function (22) with $\Gamma=\gamma_\mu$ and $v=v'$ ensures the correct normalization of the functions $\zeta(\omega)$ and $\xi_1(\omega)$ at $\omega=1$.

In the heavy quark limit the matrix element of the transition of heavy baryon containing a scalar light diquark into light baryons is described by two relativistic form factors f_1 and f_2. For example, the typical hadronic current for $\Lambda_c\to\Lambda^0$ transition is written as

$$\langle\Lambda(p')|\overline{s}O_\mu c|\Lambda_c(v)\rangle = \overline{u}_\Lambda(p')[f_1(p'\cdot v)+\not{v}f_2(p'\cdot v)]O_\mu u_{\Lambda_c}(v).$$

IV. RESULTS

In this section we give numerical results on the observables of semileptonic decays of bottom and charm baryons: the baryonic Isgur-Wise functions, decay rates, and asymmetry parameters in the two-cascade decays $\Lambda_b\to\Lambda_c[\to\Lambda_s\pi]+W[\to l\nu_l]$ and $\Lambda_c\to\Lambda_s[\to p\pi]+W[\to l\nu_l]$. Our model contains a number of parameters. The cutoff parameter Λ_{B_q} and the light quark mass m_q are taken from a fit to proton and neutron data [19]. The cutoff parameter Λ_{B_Q} relevant for heavy-light baryons, the binding energy $\overline{\Lambda}=M_{B_Q}-m_Q$, and the strange quark mass m_s are fixed by comparison with the experimentally measured decay $\Lambda_c^+\to\Lambda^0+e^++\nu_e$. We have checked that the Isgur-Wise functions ξ_1 and ξ_2 satisfy the model-independent Bjorken-Xu inequalities [35]. We give a detailed description of the $\Lambda_c^+\to\Lambda^0+e^++\nu_e$ decay, which was recently measured by CLEO Collaboration [12]. In what follows we will use the following values for the CKM matrix elements: $|V_{bc}|=0.04$, $|V_{cs}|=0.975$.

A. Baryonic Isgur-Wise functions

In Sec. II we have introduced heavy-light baryonic currents. We present a full list of possible currents (without derivatives) with the quantum numbers of baryons $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$. For simplicity we restrict ourselves to only one variant of the three-quark currents for each kind of heavy-light baryon: *pseudoscalar current* (8) for Λ_Q-type baryons and *vector currents* (10),(11) for Ω_Q-type baryons. A justification of this procedure may be taken from the QCD sum rule analysis of [25] where it was found that, using the *axial current* for Λ_Q baryons and *tensor currents* for $\Omega_Q^{(\star)}$ baryons, one obtains results which are not very different from the ones with the *pseudoscalar current* and the *vector currents*. The direct calculation of the IW functions with currents (8) and (10),(11) gives the following results:

$$\zeta(\omega)=\frac{F_0(\omega)}{F_0(1)},\quad \xi_1(\omega)=\frac{F_1(\omega)}{F_1(1)},\quad \xi_2(\omega)=\frac{F_2(\omega)}{F_1(1)}, \tag{24}$$

$$F_I(\omega)=\int_0^\infty dxx\int_0^\infty\frac{dyy}{(y+1)^2}\int_0^1 d\phi\int_0^1 d\theta R_I(\omega)\exp[-6S(\beta)(4\mu_q^2-\overline{\lambda}^2)]$$
$$\times\exp\left[-12x^2S(\beta)\phi(1-\phi)(\omega-1)-6S(\beta)(x-\overline{\lambda})^2-24\mu_q^2(1-2\theta)^2\frac{y^2}{1+y}\right],$$

where

$$R_0(\omega)=\mu_q^2+\frac{1}{6S(\beta)(1+y)}+\frac{x^2\beta}{4(1+y)^2}[1+2\phi(1-\phi)(\omega-1)],$$

$$R_1(\omega)=\mu_q^2+\frac{1}{12S(\beta)(1+y)}+\frac{x^2\beta}{4(1+y)^2}[1+2\phi(1-\phi)(\omega-1)],$$

$$R_2(\omega)=\frac{x^2\beta}{2(1+y)^2}\phi(1-\phi)=\frac{R_1(\omega)-R_1(1)}{\omega-1},$$

$$\beta=1+2y+4y^2\theta(1-\theta),\quad S(\beta)=\frac{2}{3}+\frac{\beta}{3(1+y)},\quad \mu_q=\frac{m_q}{\Lambda_Q},\quad \overline{\lambda}=\frac{\overline{\Lambda}}{\Lambda_Q}.$$

All mass-dimension variables are scaled by the parameter Λ_Q. Hence the IW functions depend only on two parameters μ_q and $\overline{\lambda}$. We reiterate that the functions ζ and ξ_1 are normalized to one at zero recoil due to the existence of a Ward identity relating the vertex function with the derivative of the heavy-light baryon mass operator as discussed after Eq. (7). Contrary to this the normalization of the ξ_2 function is model-dependent. In our model the value $\xi_2(1)$ satisfies the inequality $0<\xi_2(1)<1/2$ and depends on the choice of the parameters μ_q and $\overline{\lambda}$.

It is easy to show that the baryonic IW functions can be rewritten in the form

$$\zeta(\omega)=\frac{\sum_{N=0}^{\infty} C_N^{\zeta}\overline{\lambda}^N\Phi_N(\omega)}{\sum_{N=0}^{\infty} C_N^{\zeta}\overline{\lambda}^N}\leqslant\Phi_0(\omega)=\frac{\ln(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}},$$

$$\xi_1(\omega)=\frac{\sum_{N=0}^{\infty} C_N^{\xi_1}\overline{\lambda}^N\Phi_N(\omega)}{\sum_{N=0}^{\infty} C_N^{\xi_1}\overline{\lambda}^N}\leqslant\Phi_0(\omega)=\frac{\ln(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}},$$

$$\xi_2(\omega)=\frac{\sum_{N=0}^{\infty} C_N^{\xi_2}\overline{\lambda}^N[\Phi_N(\omega)-\Phi_{N+1}(\omega)]}{(\omega-1)\sum_{N=0}^{\infty} C_N^{\xi_1}\overline{\lambda}^N}<\frac{\Phi_0(\omega)-\Phi_1(\omega)}{\omega-1}=\frac{1}{\omega^2-1}\left(\frac{\omega\ln(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}}-1\right).$$

Here

$$\Phi_N(\omega)=\int_0^1\frac{d\phi}{[1+2(w-1)\phi(1-\phi)]^{N/2+1}}\leqslant\Phi_0(\omega)\quad\text{for}\quad\forall N\geqslant 0,$$

$$C_N^F=\frac{(2\sqrt{6})^N\Gamma(N/2+1)}{12\Gamma(N)}\int_0^1 d\theta\int_0^\infty dy\,y\,\frac{S^{N/2-1}(\beta)}{(1+y)^2}\exp(-24\mu_q^2 y)\Delta_F>0,\quad F=\zeta,\xi_1,\xi_2,$$

$$\Delta_\zeta=\mu_q^2+\frac{1}{6S(\beta)(1+y)}+\left(\frac{N}{2}+1\right)\frac{\beta}{24S(\beta)(1+y)^2},$$

$$\Delta_{\xi_1}=\mu_q^2+\frac{1}{12S(\beta)(1+y)}+\left(\frac{N}{2}+1\right)\frac{\beta}{24S(\beta)(1+y)^2},$$

$$\Delta_{\xi_2}=\left(\frac{N}{2}+1\right)\frac{\beta}{24S(\beta)(1+y)^2},$$

$$\Gamma(N)=\int_0^\infty dt\,t^{N-1}\exp(-t)\quad\text{is the }\gamma\text{ function.}$$

Note that $\zeta(\omega)$ and $\xi_1(\omega)$ become largest when $\overline{\lambda}=0$:

$$\zeta(\omega) \equiv \xi_1(\omega) \equiv \Phi_0(\omega). \tag{25}$$

An increase of $\overline{\lambda}$ leads to a suppression of the IW functions in the physical kinematical region of the variable ω, i.e., in the region

$$1 \leqslant \omega \leqslant \omega_{\max} = \frac{M^2_{B_Q} + M^2_{B'_Q}}{2 M_{B_Q} M_{B'_Q}}. \tag{26}$$

The radii of the form factors ζ and ξ_1 are defined as

$$F(\omega) = 1 - \rho^2_F(\omega - 1) + \ldots, \quad F = \zeta, \xi_1. \tag{27}$$

It is easy to show that ρ^2_ζ and $\rho^2_{\xi_1}$ have the lower bound

$$\rho^2_\zeta = \frac{1}{3} + 2\overline{\lambda}\frac{I(2,2)}{I(1,2)} \geqslant \frac{1}{3}, \quad \rho^2_{\xi_1} = \frac{1}{3} + 2\overline{\lambda}\frac{I(2,1)}{I(1,1)} \geqslant \frac{1}{3} \tag{28}$$

since the integral

$$I(M,N) = \int_0^1 d\theta \int_0^\infty dx \int_0^\infty \frac{dy\, y}{(1+y)^2} S(\beta) x^M \left[\mu_q^2 + \frac{N}{12 S(\beta)(1+y)} + \frac{x^2 \beta}{4(1+y)^2}\right] \exp[-6S(\beta)x(x - 2\overline{\lambda}) - 24\mu_q^2 y]$$

is always positive.

As was shown in [35], the IW functions ξ_1 and ξ_2 must respect the two model-independent Bjorken-Xu inequalities. The first inequality

$$1 \geqslant B(\omega) = \frac{2+\omega^2}{3}\xi_1^2(\omega) + \frac{(\omega^2-1)^2}{3}\xi_2^2(\omega) + \frac{2}{3}(\omega - \omega^3)\xi_1(\omega)\xi_2(\omega) \tag{29}$$

is derived from the Bjorken sum rule for semileptonic Ω_b decays to the ground state and to low-lying negative-parity excited charmed baryon states in the HQL. The inequality (29) implies a second inequality, namely a model-independent restriction of the slope (radius) of the form factor $\xi_1(\omega)$

$$\rho^2_{\xi_1} \geqslant \frac{1}{3} - \frac{2}{3}\xi_2(1). \tag{30}$$

Let us check whether our IW functions ξ_1 and ξ_2 respect these inequalities. First, the inequality (30) for the slope of the ξ_1 function can be seen to be satisfied because from Eq. (28) one has $\rho^2_{\xi_1} \geqslant 1/3$ and further $\xi_2(1) > 0$ from Eq. (24).

To check the inequality (29) we rewrite it in the form

$$1 \geqslant B(\omega) = \frac{2}{3}\xi_1^2(\omega) + \frac{1}{3}[\omega\xi_1(\omega) - \xi_2(\omega)(\omega^2-1)]^2. \tag{31}$$

One can show that the combination $\omega\xi_1(\omega) - \xi_2(\omega)(\omega^2-1)$ satisfies the following condition: $\xi_1(\omega) \leqslant \omega\xi_1(\omega) - \xi_2(\omega)(\omega^2-1) \leqslant \omega\xi_1(\omega)$. Hence

$$\xi_1^2(\omega) \leqslant B(\omega) \leqslant \frac{2+\omega^2}{3}\xi_1^2(\omega). \tag{32}$$

From the inequalities (31) and (32) one finds an upper limit for the function $\xi_1(\omega)$:

$$\xi_1(\omega) \leqslant \sqrt{3/(2+\omega^2)}. \tag{33}$$

The results for the IW functions $\zeta(\omega)$ and $\xi_1(\omega)$ are plotted in Figs. 3–7 in the physical region $1 \leqslant \omega \leqslant \omega_{\max}$. The function $\xi_1(\omega)$ is shown for the two cases: (a) decay of Σ_b baryon and (b) decay of Ω_b baryon. In Figs. 3 and 4 we demonstrate the sensitivity of the ζ function on the choice of the parameters $\overline{\Lambda}$ and Λ_Q when one is varied and the other one is fixed. Below (in Sec. IV B) we will show that the best

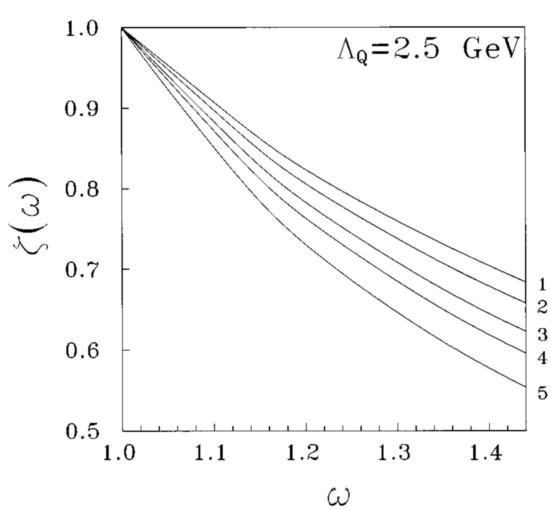

FIG. 3. $\zeta(\omega)$ function. (1) $\overline{\Lambda}$=600 MeV. (2) $\overline{\Lambda}$=650 MeV. (3) $\overline{\Lambda}$=710 MeV. (4) $\overline{\Lambda}$=750 MeV. (5) $\overline{\Lambda}$=800 MeV.

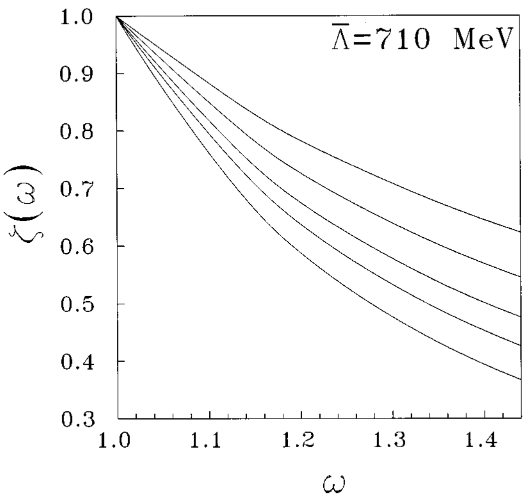

FIG. 4. $\zeta(\omega)$ function. (1) Λ_Q=2.5 GeV. (2) Λ_Q=2.0 GeV. (3) Λ_Q=1.7 GeV. (4) Λ_Q=1.5 GeV. (5) $\Lambda_Q \equiv \Lambda_q$=1.25 GeV.

description of the experimental data for $\Lambda_c^+ \to \Lambda^0 + e^+ + \nu_e$ decay is obtained with the choice of parameters Λ_Q=2.5 GeV, $\bar{\Lambda}$=710 MeV, and m_s=570 MeV. In Fig. 3 the $\zeta(\omega)$ function is shown for $\bar{\Lambda}$ values between 600 MeV and 800 MeV, where the parameter Λ_Q is assumed to be 2.5 GeV. It is seen that an increase of $\bar{\Lambda}$ leads to a suppression of the baryonic IW function ζ. In Fig. 4 the dependence of ζ on the value Λ_Q is plotted for $\bar{\Lambda}$=710 MeV. One can see that a decrease of Λ_Q leads to a suppression of $\zeta(\omega)$. In Fig. 5 we give the *best fit* for the IW function ζ (Λ_Q=2.5 GeV, $\bar{\Lambda}$=710 MeV). For comparison the results of other phenomenological approaches are shown too where we compare with results obtained from QCD sum rules [24], IMF models [38,39], MIT bag model [40], a simple quark model (SQM) [42], and the dipole formula [39]. Our result is close to the QCD sum rule result [24]. For quick reference we want to remark that in the physical region our function ζ can be well approximated by the formula

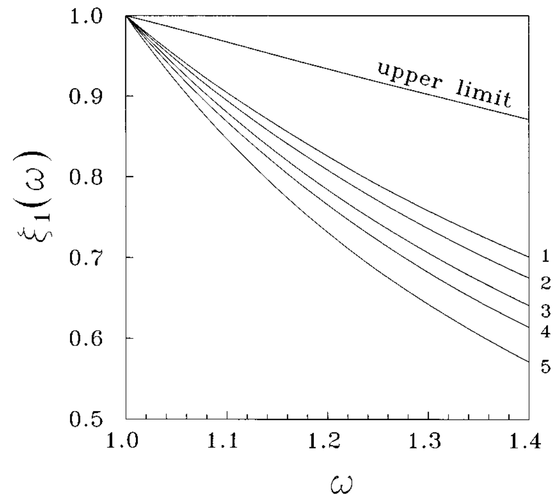

FIG. 6. $\xi_1(\omega)$ function (Σ_b decay). (1) $\bar{\Lambda}$=600 MeV. (2) $\bar{\Lambda}$=650 MeV. (3) $\bar{\Lambda}$=710 MeV. (4) $\bar{\Lambda}$=750 MeV. (5) $\bar{\Lambda}$=800 MeV.

$$\zeta(\omega) \approx \left[\frac{2}{1+\omega} \right]^{1.7+1/\omega}. \tag{34}$$

In Figs. 6 and 7 we analyze the ω dependence of the ξ_1 form factor. We exhibit the dependence of $\xi_1(\omega)$ on the choice of $\bar{\Lambda}$ for Σ_b baryon decays (Fig. 6) and for Ω_b baryon decays (Fig. 7). For both cases Λ_Q is put equal to 2.5 GeV. In the analysis of the Ω_b form factor we use m_s=570 MeV. We also present results on the upper limit (22) for the function $\xi_1(\omega)$. In Fig. 7 we also compare to a simple quark model calculation of [43]. We want to emphasize that for both cases, Σ_b and Ω_b baryon decays, our ζ_1 does not exceed the upper limit (33) except in a narrow region of very small (unphysical) values of $\bar{\Lambda}$: $\bar{\Lambda} \leqslant$ 60 MeV. Thus we conclude that the Bjorken-Xu inequality is respected by our model.

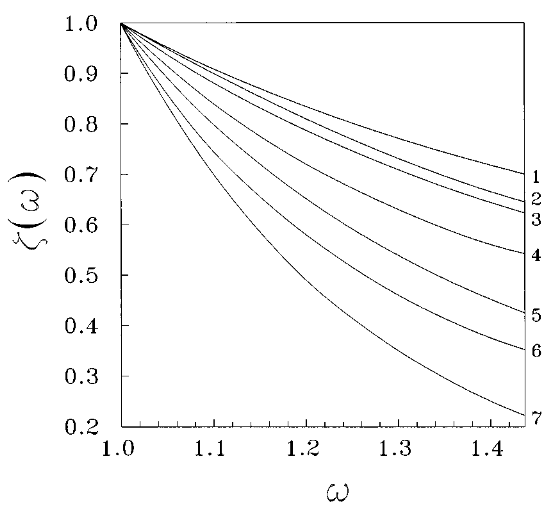

FIG. 5. $\zeta(\omega)$ function. (1) SQM (Ref. [42]). (2) QCD SR (Ref. [24]). (3) Our result ($\bar{\Lambda}$=710 MeV; Λ_Q=2.5 GeV). (4) Dipole (Ref. [39]). (5) MIT bag (Ref. [40]). (6) IMF (Ref. [39]). (7) IMF (Ref. [38]).

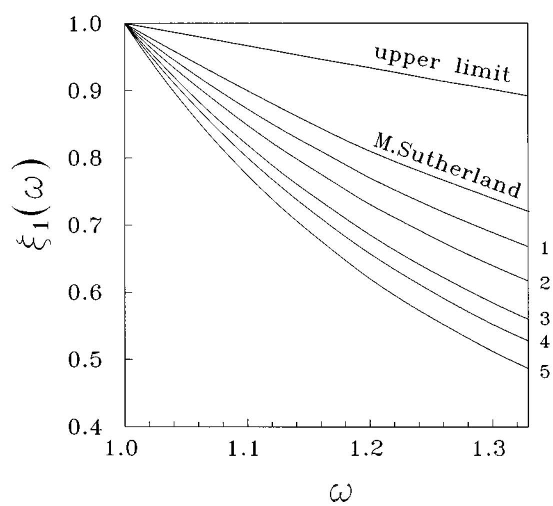

FIG. 7. $\xi_1(\omega)$ function (Ω_b decay). (1) $\bar{\Lambda}_{ss}$=800 MeV. (2) $\bar{\Lambda}_{\{ss\}}$=900 MeV. (3) $\bar{\Lambda}_{\{ss\}}$=1000 MeV. (4) $\bar{\Lambda}_{\{ss\}}$=1050 MeV. (5) $\bar{\Lambda}_{\{ss\}}$=1100 MeV.

TABLE III. The charge radius ρ_ζ^2 of Λ_b baryon at Λ_Q=2.5 GeV.

$\bar\Lambda$ (MeV)	600	625	650	675	700	710	725	750	675	690	800
ρ_ζ^2	1.04	1.09	1.12	1.22	1.30	1.33	1.38	1.47	1.59	1.68	1.76

The results for the charge radii are listed in Tables III–VI for various sets of the adjustable parameters. For comparison we quote the results for the charge radii predicted by other phenomenological approaches: $\rho_\zeta^2=3.04$ (IMF model) [39], $\rho_\zeta^2=1.78$ (dipole formula) [39], $\rho_\zeta^2=2.28$ (MIT bag model) [40], $\rho_\zeta^2=1$ and $\rho_{\xi_1}^2=1.02-1.18$ (simple quark model) [42,43], and $\rho_\zeta^2=0.55\pm0.15$ (QCD sum rules) [44].

B. Rates, distributions, and asymmetry parameters in $b\to c$ baryonic decays

In this section we present on numerical results for rates, distributions, and asymmetry parameters in the $b\to c$ flavor changing baryon decays. The standard expressions for observables of semileptonic decays of bottom baryons (decay rates, differential distributions, leptonic spectra, and asymmetry parameters) have simple forms when expressed in terms of helicity amplitudes $H_{\lambda_f\lambda_W}$ [36,28], where λ_f is helicity of the final state baryon and λ_W is the helicity of the off mass-shell W boson. The HQL helicity amplitudes describing transitions of bottom baryon into charm ones are expressed through IW functions in the following way:

$$H_{\pm\frac{1}{2}\pm1}=-2\sqrt{M_iM_f}(\sqrt{\omega-1}\mp\sqrt{\omega+1})\times\begin{cases}\zeta(\omega), & \Lambda_b\to\Lambda_c \text{ decay}\\ \frac{1}{3}\xi_T(\omega), & \Omega_b\to\Omega_c \text{ decay}\\ \pm\frac{\sqrt{2}}{3}\xi_T(\omega), & \Omega_b\to\Omega_c^\star \text{ decay,}\end{cases}$$

$$H_{\pm\frac{1}{2}0}=\frac{1}{\sqrt{\omega_{\max}-\omega}}\times\begin{cases}\zeta(\omega)[M_+\sqrt{\omega-1}\mp M_-\sqrt{\omega+1}], & \Lambda_b\to\Lambda_c \text{ decay}\\ \frac{1}{3}[M_+\sqrt{\omega-1}\xi_{L_+}(\omega)\mp M_-\xi_{L_-}(\omega)\sqrt{\omega+1}], & \Omega_b\to\Omega_c \text{ decay}\\ \frac{\sqrt{2}}{3}[M_+\sqrt{\omega-1}\xi_{L_+^\star}(\omega)\mp M_-\xi_{L_-^\star}(\omega)\sqrt{\omega+1}], & \Omega_b\to\Omega_c^\star \text{ decay,}\end{cases}$$

$$H_{\pm\frac{3}{2}\pm1}=\mp2\xi_1(\omega)\sqrt{\tfrac{2}{3}M_iM_f}[\sqrt{\omega-1}\mp\sqrt{\omega+1}],\quad \Omega_b\to\Omega_c^\star \text{ decay,}$$

where

$$M_\pm=M_i\pm M_f,\quad \xi_T=\xi_1\omega-\xi_2(\omega^2-1),$$

$$\xi_{L_\pm}=\xi_1(\omega\pm2)-\xi_2(\omega^2-1),\quad \xi_{L_\pm^\star}=\xi_1(\omega\mp1)-\xi_2(\omega^2-1),$$

$$\omega_{\max}=\frac{M_i^2+M_f^2}{2M_iM_f}.$$

The decay rates of semileptonic decays are then given by

$$\Gamma=\int_1^{\omega_{\max}}d\omega\frac{d\Gamma}{d\omega},\quad \frac{d\Gamma}{d\omega}=\frac{d\Gamma_{T_+}}{d\omega}+\frac{d\Gamma_{T_-}}{d\omega}+\frac{d\Gamma_{L_+}}{d\omega}+\frac{d\Gamma_{L_-}}{d\omega}, \tag{35}$$

TABLE IV. The charge radius ρ_ζ^2 of Λ_b baryon at $\bar\Lambda$=710 MeV.

Λ_Q (GeV)	1.25	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.3	2.5
ρ_ζ^2	2.93	2.82	2.63	2.47	2.31	2.15	2.0	1.87	1.75	1.64	1.46	1.33

TABLE V. The charge radius $\rho^2_{\xi_1}$ of Σ_b baryon at $\Lambda_Q=2.5$ GeV.

$\bar{\Lambda}$ (MeV)	600	625	650	675	700	710	725	750	675	690	800
$\rho^2_{\xi_1}$	1.05	1.09	1.12	1.22	1.32	1.35	1.38	1.50	1.59	1.68	1.80

where the indices T and L denote partial contributions of transverse ($\lambda_W=\pm 1$) and longitudinal ($\lambda_W=0$) components of the current transitions. Partial differential distributions are given by

$$\frac{d\Gamma_{T_\pm}}{d\omega}=\kappa_\omega\times\begin{cases}|H_{\pm\frac{1}{2}\pm 1}|^2 & \text{for } \frac{1}{2}^+\to\frac{1}{2}^+ \text{ transition}\\ |H_{\pm\frac{1}{2}\pm 1}|^2+|H_{\pm\frac{3}{2}\pm 1}|^2 & \text{for } \frac{1}{2}^+\to\frac{3}{2}^+ \text{ transition,}\end{cases}$$

$$\frac{d\Gamma_{L_\pm}}{d\omega}=\kappa_\omega|H_{\pm\frac{1}{2}0}|^2,\quad \kappa_\omega=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\frac{M_f^3}{6}(\omega_{\max}-\omega)\sqrt{\omega^2-1}.$$

Tables VII–XI list our predictions for the semileptonic rates of beauty baryons. In Table VII we present the results for total and partial rates for various $b\to c$ decay modes. The adjustable parameters are chosen as $m_s=570$ MeV, $\Lambda_Q=2.5$ GeV, $\bar{\Lambda}=710$ MeV, $\bar{\Lambda}_s=850$ MeV, and $\bar{\Lambda}_{\{ss\}}=1000$ MeV. In Table VIII we compare our results for total rates with the predictions of other phenomenological approaches: constituent quark model [28], spectator quark model [37], and nonrelativistic quark model [41]. The dependence of the total rates on the parameters $\bar{\Lambda}$, $\bar{\Lambda}_s$, and $\bar{\Lambda}_{\{ss\}}$ are shown in Tables IX–XI.

The differential distributions for $\Lambda_b^0\to\Lambda_c^+e^-\bar{\nu}$ decay are plotted in Fig. 8.

Leptonic spectra $d\Gamma/dE_l$ are calculated according to the sum

$$\frac{d\Gamma}{dE_l}=\frac{d\Gamma_{T_+}}{dE_l}+\frac{d\Gamma_{T_-}}{dE_l}+\frac{d\Gamma_{L_+}}{dE_l}+\frac{d\Gamma_{L_-}}{dE_l}. \tag{36}$$

Expressions for partial leptonic spectra are given by

$$\frac{d\Gamma_{T_\pm}}{dE_l}=\int_{\omega_{\min}(E_l)}^{\omega_{\max}}d\omega\,\kappa_E(1\pm\cos\Theta)^2|H_{\pm\frac{1}{2}\pm 1}|^2,$$

$$\frac{d\Gamma_{L_\pm}}{dE_l}=\int_{\omega_{\min}(E_l)}^{\omega_{\max}}d\omega\,\kappa_E(1-\cos^2\Theta)^2|H_{\pm\frac{1}{2}0}|^2,$$

$$\kappa_E=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\frac{M_{\Lambda_c}^2}{8}(\omega_{\max}-\omega),$$

$$\cos\Theta=\frac{E_l^{\max}-2E_l+M_{\Lambda_c}(\omega_{\max}-\omega)}{M_{\Lambda_c}\sqrt{\omega^2-1}},$$

$$E_l^{\max}=\frac{M_{\Lambda_b}^2-M_{\Lambda_c}^2}{2M_{\Lambda_b}},\quad \omega_{\min}(E_l)=\omega_{\max}-2\frac{E_l}{M_{\Lambda_c}}\frac{E_l^{\max}-E_l}{M_{\Lambda_b}-2E_l}.$$

Our results on leptonic spectra in semileptonic $\Lambda_b\to\Lambda_c$ transitions are shown in Fig. 9.

Finally, we consider the cascade decay $\Lambda_b\to\Lambda_c[\to\Lambda_s\pi]+W[\to l\nu_l]$ which is characterized by a set of asymmetry parameters. The formalism and a detailed analysis of the asymmetry parameters is presented in [36,28]. In terms of helicity amplitudes the asymmetry parameters of nonpolarized Λ_b decays ($\alpha,\alpha',\alpha'',\gamma$) and polarized Λ_b decays (α_P,γ_P) are given by the following expressions:

$$\alpha=\frac{H_T^-+H_L^-}{H_T^++H_L^+},\quad \alpha'=\frac{H_T^-}{H_T^++2H_L^+},\quad \alpha''=\frac{H_T^+-2H_L^+}{H_T^++2H_L^+},$$

$$\gamma=\frac{2H_\gamma}{H_T^++H_L^+},\quad \alpha_P=\frac{H_T^--H_L^-}{H_T^++H_L^+},\quad \gamma_P=\frac{2H_{\gamma_P}}{H_T^++H_L^+}, \tag{37}$$

$$H_T^\pm=|H_{1/2\,1}|^2\pm|H_{-1/2\,-1}|^2,\quad H_L^\pm=|H_{1/2\,0}|^2\pm|H_{-1/2\,0}|^2,$$

$$H_\gamma=\mathrm{Re}(H_{-1/2\,0}H^*_{1/21}+H_{1/2\,0}H^*_{-1/2\,-1}),$$

$$H_{\gamma_P}=\mathrm{Re}(H_{1/2\,0}H^*_{-1/2\,0}).$$

TABLE VI. The charge radius $\rho^2_{\xi_1}$ of Ω_b baryon at $\Lambda_Q=2.5$ GeV.

$\bar{\Lambda}_{\{ss\}}$ (MeV)	800	850	875	900	925	950	975	1000	1025	1050	1075	1100
$\rho^2_{\xi_1}$	1.44	1.58	1.66	1.74	1.82	1.92	2.02	2.12	2.25	2.39	2.56	2.79

TABLE VII. Decay rates of bottom baryons (in 10^{10} sec^{-1}) for $|V_{bc}|=0.04$.

Process	$\Gamma_{\rm total}$	Γ_T	Γ_{T_+}	Γ_{T_-}	Γ_L	Γ_{L_+}	Γ_{L_-}
$\Lambda_b^0\to\Lambda_c^+e^-\overline{\nu_e}$	5.39	2.07	0.53	1.54	3.32	0.11	3.21
$\Xi_b^0\to\Xi_c^+e^-\overline{\nu_e}$	5.27	2.02	0.54	1.48	3.25	0.11	3.14
$\Sigma_b^+\to\Sigma_c^{++}e^-\overline{\nu_e}$	2.23	0.33	0.08	0.25	1.90	1.49	0.41
$\Omega_b^-\to\Omega_c^0e^-\overline{\nu_e}$	1.87	0.29	0.08	0.21	1.58	1.26	0.32
$\Sigma_b^+\to\Sigma_c^{*++}e^-\overline{\nu_e}$	4.56	2.07	0.54	1.53	2.49	1.09	1.40
$\Omega_b^-\to\Omega_c^{*0}e^-\overline{\nu_e}$	4.01	1.89	0.53	1.36	2.12	0.95	1.17

TABLE VIII. Model results for rates of bottom baryons (in 10^{10} sec^{-1}) for $|V_{bc}|=0.04$.

Process	Ref. [37]	Ref. [41]	Ref. [28]	Our results
$\Lambda_b^0\to\Lambda_c^+e^-\overline{\nu_e}$	5.9	5.1	5.14	5.39
$\Xi_b^0\to\Xi_c^+e^-\overline{\nu_e}$	7.2	5.3	5.21	5.27
$\Sigma_b^+\to\Sigma_c^{++}e^-\overline{\nu_e}$	4.3			2.23
$\Sigma_b^+\to\Sigma_c^{\star++}e^-\overline{\nu_e}$				4.56
$\Omega_b^-\to\Omega_c^0e^-\overline{\nu_e}$	5.4	2.3	1.52	1.87
$\Omega_b^-\to\Omega_c^{\star0}e^-\overline{\nu_e}$			3.41	4.01

TABLE IX. Dependence of rates on $\overline{\Lambda}$ for $|V_{bc}|=0.04$.

	$\overline{\Lambda}$ (MeV)				
Process	600	650	710	750	800
$\Lambda_b^0\to\Lambda_c^+e^-\overline{\nu_e}$	6.10	5.83	5.39	5.19	4.74
$\Sigma_b^+\to\Sigma_c^{++}e^-\overline{\nu_e}$	2.51	2.39	2.23	2.11	1.92
$\Sigma_b^+\to\Sigma_c^{*++}e^-\overline{\nu_e}$	4.99	4.81	4.56	4.35	4.03

TABLE X. Dependence of rates on $\overline{\Lambda_s}$ for $|V_{bc}|=0.04$.

	$\overline{\Lambda_s}$ (MeV)			
Process	760	800	850	900
$\Xi_b^0\to\Xi_c^+e^-\overline{\nu_e}$	5.81	5.58	5.27	4.93

TABLE XI. Dependence of rates on $\overline{\Lambda_{\{ss\}}}$ for $|V_{bc}|=0.04$.

	$\overline{\Lambda_{\{ss\}}}$ (MeV)				
Process	900	950	1000	1050	1100
$\Omega_b^-\to\Omega_c^0e^-\overline{\nu_e}$	2.09	1.98	1.87	1.72	1.54
$\Omega_b^-\to\Omega_c^{*0}e^-\overline{\nu_e}$	4.44	4.23	4.01	3.75	3.43

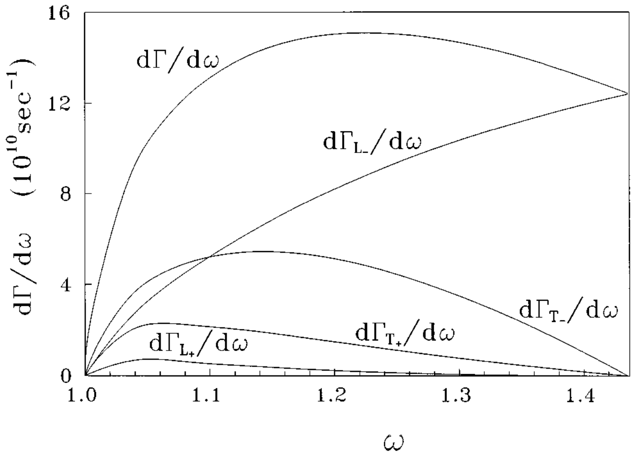

FIG. 8. Differential distribution.

We evaluate the average magnitudes (see Table XII) of the asymmetry parameters ($\langle\alpha\rangle,\langle\alpha'\rangle$, etc.) as results of separate ω integrations of numerators and denominators. We found that the results weakly depend on the behavior of the Λ_b baryonic IW function.

For example, it may be seen from the analytical expression for the asymmetry parameter α',

$$\alpha' = -\frac{\displaystyle\int_1^{\omega_{\rm max}} d\omega\,\zeta^2(\omega)(\omega^2-1)(\omega_{\rm max}-\omega)}{\displaystyle\int_1^{\omega_{\rm max}} d\omega\,\zeta^2(\omega)\sqrt{\omega^2-1}[\omega^2-1+2\omega(\omega_{\rm max}-\omega)]}. \tag{38}$$

We give also in Table XII the results of the IMF approach [39] and dipole model [39]. One can see that our results are closer to the results of dipole model. It follows that the behavior of the IW function for $\Lambda_b\to\Lambda_c$ transition (see Fig. 5) is found to be similar to the dipole function used in the dipole model.

C. Heavy-to-light baryon decays

In this subsection we consider the heavy-to-light semileptonic modes. In particular the process $\Lambda_c^+\to\Lambda^0+e^++\nu_e$ which was recently investigated by the CLEO Collaboration [12] is studied in detail. In the heavy mass limit ($m_C\to\infty$) its transition matrix element is defined by two form factors f_1 and f_2 (see Sec. III). Assuming identical dipole forms for the form factors (as in the model of Körner and Krämer [36]), CLEO found that $R=f_2/f_1=-0.25\pm0.14\pm0.08$. Our form factors have different q^2 dependences. In other words, the quantity $R=f_2/f_1$ has a q^2 dependence in our approach. In

TABLE XII. Asymmetry parameters of Λ_b decay.

Model	α	α'	α''	γ	α_P	γ_P
Our	-0.76	-0.12	-0.53	0.56	0.39	-0.16
Dipole [39]	-0.75	-0.12	-0.51	0.57	0.37	-0.17
IMF [39]	-0.71	-0.12	-0.46	0.61	0.33	-0.19

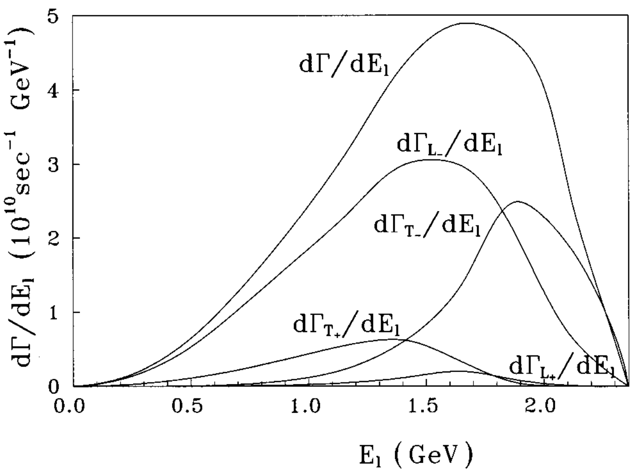

FIG. 9. Leptonic spectrum.

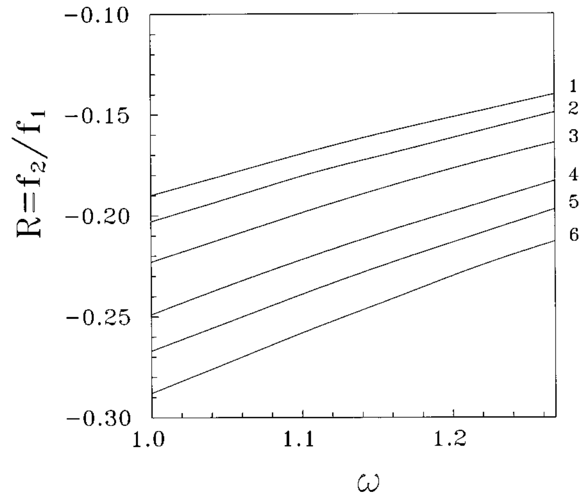

FIG. 10. Form factor ratios $R=f_2/f_1$ for $\Lambda_c^+\to\Lambda^0+e^+\nu$ decay. (1) $\bar{\Lambda}=650$ MeV. (2) $\bar{\Lambda}=710$ MeV. (3) $\bar{\Lambda}=725$ MeV. (4) $\bar{\Lambda}=750$ MeV. (5) $\bar{\Lambda}=775$ MeV. (6) $\bar{\Lambda}=800$ MeV.

Fig. 10 we plot the results for R in the kinematical region $1\leqslant\omega\leqslant\omega_{\max}$ for different magnitudes of the $\bar{\Lambda}$ parameter.

It is seen that larger values of $\bar{\Lambda}$ lead to an increase of the ratio R. The best fit to the experimental data is achieved for the following set of parameters: $m_s=570$ MeV, $\Lambda_Q=2.5$ GeV, and $\bar{\Lambda}=710$ MeV. In this case the ω dependence of the form factors f_1, f_2 and their ratio R are shown in Fig. 11. Particularly, we get $f_1(q^2_{\max})=0.8$, $f_2(q^2_{\max})=-0.18$, $R=-0.22$ at zero recoil ($\omega=1$ or $q^2=q^2_{\max}$) and $f_1(0)=0.38$, $f_2(0)=-0.06$, $R=-0.16$ at maximum recoil ($\omega=\omega_{\max}$ or $q^2=0$). Note that our results for $q^2_{\max}$ are close to those of the nonrelativistic quark model [41]: $f_1(q^2_{\max})=0.75$, $f_2(q^2_{\max})=-0.17$, $R=-0.23$.

Our result for R agrees well with the experimental data [12] $R=-0.25\pm0.14\pm0.08$. The predictions for the decay rate $\Gamma(\Lambda_c^+\to\Lambda^0e^+\nu_e)=7.22\times10^{10}$ sec^{-1} and for the asymmetry parameter $\alpha_{\Lambda_c}=-0.812$ also coincide with the experiment: $\Gamma_{\rm expt}=7.0\pm2.5\times10^{10}$ sec^{-1} and $\alpha^{\rm expt}_{\Lambda_c}=-0.82^{+0.09+0.06}_{-0.06-0.03}$, respectively, as well as with the result of [41] $\Gamma=7.1\times10^{10}$ sec^{-1}. Note that the agreement with the experimental rate measurement crucially depends on the use of the Λ^0 three-quark current in its SU(3)-flavor symmetric form (see Table I) which leads to the presence of the flavor-suppression factor $N_{\Lambda_c\Lambda}=1/\sqrt{3}$ for $\Lambda_c^+\to\Lambda^0e^+\nu_e$. If the SU(3) symmetric structure of Λ^0 hyperon is not taken into account the predicted rate for $\Lambda_c^+\to\Lambda^0e^+\nu_e$ becomes too large (see discussion in Refs. [28,41]).

In Table XIII we present our predictions for some modes of semileptonic heavy-to-light transitions (for $\bar{\Lambda}_s=850$ MeV, $\bar{\Lambda}_{\{ss\}}=1000$ MeV). Also the results obtained in other approaches are tabulated. Note that the flavor-suppression factor for the modes $\Xi_c^0\to\Xi^-e^+\nu_e$, $\Lambda_b^0\to pe^-\overline{\nu_e}$, and $\Lambda_c^+\to ne^+\nu_e$ is equal to $1/\sqrt{2}$.

Finally, in Table XIV we give the predictions for the average magnitudes of the asymmetry parameters for the cascade decay $\Lambda_c\to\Lambda_s[\to p\pi]+W[\to l\nu_l]$ which are expected to be measured in the near future by the COMPASS Collaboration [47]. For comparison, the results of paper [36] for $R=f_2/f_1=-0.25$ are also given. The results of both approaches are in a good agreement that again may be explained by similar (dipolelike) behavior of the weak form factors f_1 and f_2 in our model and in the paper [36].

V. CONCLUSION

We have developed a relativistic model [15,16,19,20] for QCD bound states composed of light quarks and a heavy quark. In fact, this model is the Lagrangian formulation of the NJL model with separable interaction [17,18] and its advantage consists in the possibility of studying baryons as three-quark states as multiquark and exotic objects. We have used our approach to study the properties of baryons containing a single heavy quark. We have calculated the observables of semileptonic decays of bottom and charm baryons: Isgur-Wise functions, asymmetry parameters, decay rates, and distributions. We obtained analytical expressions for the baryon IW functions: ζ ($\Lambda_b\to\Lambda_c$ transition), ξ_1 and ξ_2 ($\Omega_b\to\Omega_c^{(\star)}$ transition). We checked the model-independent Bjorken-Xu inequalities for the ξ_1 and ξ_2 functions and their

TABLE XIII. Heavy-to-light decay rates (in 10^{10} sec^{-1}) for $|V_{bc}|=0.04$, $|V_{cs}|=0.975$.

Process	Quantity	Ref. [37]	Ref. [41]	Ref. [45]	Ref. [46]	Our	Expt. [29]
$\Lambda_c^+\to\Lambda^0e^+\nu_e$	Γ	9.8	7.1	5.36	7	7.22	7.0 ± 2.5
$\Xi_c^0\to\Xi^-e^+\nu_e$	Γ	8.5	7.4		9.7	8.16	
$\Lambda_b^0\to pe^-\overline{\nu_e}$	$\Gamma/\lvert V_{bu}\rvert^2$			6.48×10^2		7.47×10^2	
$\Lambda_c^+\to ne^+\nu_e$	$\Gamma/\lvert V_{cd}\rvert^2$				0.17×10^2	0.26×10^2	

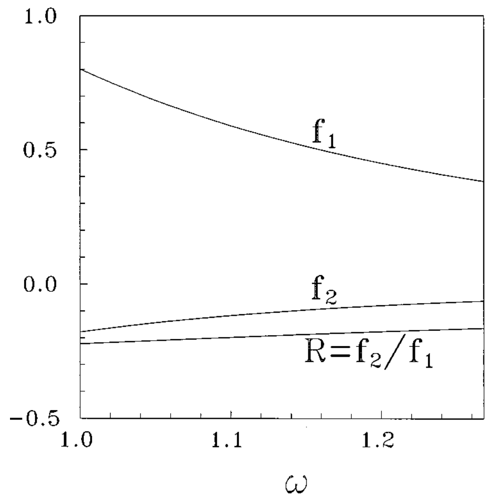

FIG. 11. Form factors and their ratio for $\Lambda_c^+ \to \Lambda^0 + e^+ \nu$ decay.

derivatives at the zero recoil point. It is shown that inequality for the charge radius of ξ_1 [see Eq. (30)] is automatically respected in our model. The inequality (29) for the ω dependence of ξ_1 and ξ_2 leads to an upper limit for ξ_1 [see Eq. (33)] which is respected in our model for reasonable values of the parameter $\bar{\Lambda}$. We have also applied our model to the calculation of heavy-to-light semileptonic decay processes motivated by the recent experimental observation of the $\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$ decay by the CLEO Collaboration [12]. Note that this decay was used for adjusting the model parameters: the strange quark mass m_s, the range cutoff parameter Λ_{B_Q}, and the mass shift parameter $\bar{\Lambda}$. The success in reproducing the correct experimental rate $\Gamma(\Lambda_c^+ \to \Lambda^0 e^+ \nu_e)$ requires the use of the Λ^0 three-quark current in the SU(3)-flavor symmetric form (see Table I). Predictions for other semileptonic heavy-to-light rates are also given. Finally, we have given predictions for the asymmetry parameters of the cascade decay $\Lambda_c \to \Lambda_s[\to p\pi] + W[\to l\nu_l]$.

ACKNOWLEDGMENTS

M.A.I. and V.E.L thank Mainz and Wuppertal Universities for the hospitality where a part of this work was completed. This work was supported in part by the INTAS Grant No. 94-739, the Heisenberg-Landau Program, by the Russian Fund of Fundamental Research (RFFR) under Contract No. 96-02-17435-a, and the State Committee of the Russian Federation for Education (Project No. 95-0-6.3-67, Grant Center at S.-Petersburg State University).

APPENDIX: THE CALCULATION TECHNIQUE

To elucidate the calculation of the matrix elements (22) and (23) we consider the two generic integrals in a Euclidean space

$$I_1(p_E'^2, w_E) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k_E'}{\pi^2} \exp\left(-\frac{9k_E^2+3k_E'^2}{\Lambda_{B_Q}^2}\right) \exp\left(-\frac{9(k_E+\alpha p')^2+3k_E'^2}{\Lambda_{B_q}^2}\right) \frac{1}{m^2+(k_E+k_E')^2/4} \frac{1}{m^2+(k_E-k_E')^2/4}$$
$$\times \frac{1}{m_s^2+(k_E+p_E')^2} \frac{1}{k_E v_E - \bar{\Lambda}}, \quad \alpha = \frac{2m}{2m+m_s}, \tag{A1}$$

$$I_2(w_E) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k_E'}{\pi^2} \exp\left(-\frac{18k_E^2+6k_E'^2}{\Lambda_Q^2}\right) \frac{1}{m^2+(k_E+k_E')^2/4} \times \frac{1}{m^2+(k_E-k_E')^2/4} \frac{1}{k_E v_E - \bar{\Lambda}} \frac{1}{k_E v_E' - \bar{\Lambda}}, \tag{A2}$$

where α is defined in Eq. (23). The final light baryon state carrying the Euclidean momenta p_E' is on mass-shell: $p_E'^2 = -M'^2$. The dimensionless variable w_E is defined as $w_E = v_E \cdot p_E'/M' = -w$.

The first integral appears in the calculation of heavy-to-light form factors, the second one in the calculation of the heavy-to-heavy case.

Scaling all momentum variables in Eq. (A1) by Λ_{B_q} and Eq. (A2) by Λ_{B_Q} and using the Feynman parametrization

$$\frac{1}{A} = \int_0^\infty d\alpha \, \exp(-\alpha A),$$

we have

TABLE XIV. Asymmetry parameters of Λ_c decay.

Model	α	α'	α''	γ	α_P	γ_P
Our	-0.81	-0.13	-0.56	0.50	0.40	-0.15
Körner and Krämer [36]	-0.82	-0.13	-0.56	0.47	0.39	-0.14

$$I_1(-M'^2,w_E)=2\Lambda_{B_q}[6(1+R)]^4\ \exp[-36m^2(1+R)-9M'^2\alpha^2]\int_0^\infty\cdots\int_0^\infty d\beta_1\cdots d\beta_4 t^2(\beta)\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}$$
$$\times\exp\left[-3(1+R)(1+\beta_3+\beta_4)\left(k'_E+k_E\frac{\beta_3-\beta_4}{1+\beta_3+\beta_4}\right)^2\right]$$
$$\times\exp\left[-3(1+R)t(\beta)(1+\beta_2)\left(k_E+\frac{v_E\beta_1+p'_Er_1}{1+\beta_2}\right)^2\right]\exp\left[-\frac{3(1+R)t(\beta)}{1+\beta_2}[\beta_1+M'\beta_2-\overline{\Lambda}(1+\beta_2)]^2\right.$$
$$\left.+6M'(1+R)t(\beta)(w_E+1)\frac{\beta_1\beta_2}{1+\beta_2}\right]\exp\left[\frac{18M'\alpha}{1+\beta_2}(w_E\beta_1-M'r_2)-12m^2(1+R)\frac{(\beta_3-\beta_4)^2}{1+\beta_3+\beta_4}\right.$$
$$\left.-3(1+R)t(\beta)(4m^2-\overline{\Lambda^2})\right]\exp[-3(1+R)t(\beta)\beta_2(m_s^2-(M'-\overline{\Lambda})^2)],\tag{A3}$$

$$I_2(w_E)=4\Lambda_{B_Q}^2 12^4\ \exp[-72m^2]\int_0^\infty\cdots\int_0^\infty d\beta_1\cdots d\beta_4 t^2(\beta)\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\exp\left[-6(1+\beta_3+\beta_4)\left(k'_E+k_E\frac{\beta_3-\beta_4}{1+\beta_3+\beta_4}\right)^2\right]$$
$$\times\exp[-6t(\beta)(k_E+v_E\beta_1+v'_E\beta_2)^2]\exp[-6t(\beta)(\beta_1+\beta_2-\overline{\Lambda})^2+12t(\beta)(w_E+1)\beta_1\beta_2]$$
$$\times\exp\left[-24m^2\frac{(\beta_3-\beta_4)^2}{1+\beta_3+\beta_4}-6t(\beta)(4m^2-\overline{\Lambda^2})\right].\tag{A4}$$

The notation is as follows:

$$R=\frac{\Lambda_{B_q}^2}{\Lambda_{B_Q}^2},\quad t(\beta)=\frac{3+4(\beta_3+\beta_4)+4\beta_3\beta_4}{1+\beta_3+\beta_4},$$

$$r_1=\beta_2+\frac{3\alpha}{(1+R)t(\beta)},\quad r_2=\beta_2+\frac{3\alpha}{2(1+R)t(\beta)}.$$

After a change of variables for k'_E, k_E and integrations we arrive at

$$I_1(-M'^2,-w)=32\Lambda_{B_q}\ \exp[-36m_q^2(1+R)-9m'^2\alpha^2]\int_0^\infty\cdots\int_0^\infty\frac{d\beta_1\cdots d\beta_4}{(1+\beta_3+\beta_4)^2(1+\beta_2)^2}\ \exp[-3(1+R)t(\beta)$$
$$\times\beta_2(m_s^2-(M'-\overline{\Lambda})^2)]\exp\left[-\frac{3(1+R)t(\beta)}{1+\beta_2}[\beta_1+M'\beta_2-\overline{\Lambda}(1+\beta_2)]^2-6M'(1+R)t(\beta)(w-1)\frac{\beta_1\beta_2}{1+\beta_2}\right]$$
$$\times\exp\left[-\frac{18M'\alpha}{1+\beta_2}(w\beta_1+M'r_2)-12m^2(1+R)\frac{(\beta_3-\beta_4)^2}{1+\beta_3+\beta_4}-3(1+R)t(\beta)(4m^2-\overline{\Lambda^2})\right],\tag{A5}$$

$$I_2(-w)=64\Lambda_{B_Q}^2\ \exp[-72m^2]\int_0^\infty\cdots\int_0^\infty\frac{d\beta_1\cdots d\beta_4}{(1+\beta_3+\beta_4)^2}\ \exp[-6t(\beta)(\beta_1+\beta_2-\overline{\Lambda})^2-12t(\beta)(w-1)\beta_1\beta_2]$$
$$\times\exp\left[-24m^2\frac{(\beta_3-\beta_4)^2}{1+\beta_3+\beta_4}-6t(\beta)(4m^2-\overline{\Lambda^2})\right].\tag{A6}$$

---

[1] M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. D **51**, 2217 (1995).
[2] B. Blok and T. Mannel, Phys. Rev. D **51**, 2208 (1995).
[3] M. Neubert, Phys. Rep. **245**, 259 (1994); A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993).
[4] J.G. Körner and D. Pirjol, Phys. Lett. B **334**, 399 (1994); J.G. Körner, K. Melnikov, and O. Yakovlev, Z. Phys. C **68**, 439 (1995).
[5] D. Politzer and M.B. Wise, Phys. Lett. B **206**, 681 (1988).
[6] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[7] G. Lepage and B.A. Thacker, in *Field Theory on the Lattice*, Proceedings of the International Symposium, Seillac, France, 1987, edited by A. Billiore *et al.* [Nucl. Phys. B (Proc. Suppl.) **4**, 199 (1990)].
[8] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990).

[9] B. Grinstein, Nucl. Phys. **B339**, 253 (1990).
[10] H. Georgi, Phys. Lett. B **240**, 447 (1990).
[11] F. Hussain, Phys. Lett. B **249**, 295 (1990).
[12] CLEO Collaboration, G. Crawford *et al.*, Phys. Rev. Lett. **75**, 624 (1995); CLEO Collaboration, T. Bergfeld *et al.*, Report No. CLEO 94-4, 1994 (unpublished).
[13] ALEPH Collaboration, D. Decamp *et al.*, Phys. Lett. B **278**, 209 (1992).
[14] OPAL Collaboration, P.D. Acton *et al.*, Phys. Lett. B **281**, 394 (1992).
[15] I.V. Anikin, M.A. Ivanov, N.B. Kulimanova, and V.E. Lyubovitskij, Phys. At. Nuclei **57**, 1082 (1994).
[16] I.V. Anikin, M.A. Ivanov, N.B. Kulimanova, and V.E. Lyubovitskij, Z. Phys. C **65**, 681 (1995).
[17] T. Goldman and R.W. Haymaker, Phys. Rev. D **24**, 724 (1981).
[18] H. Ito, W.W. Buck, and F. Gross, Phys. Rev. C **45**, 1918 (1992); R.D. Bowler and M.C. Birse, Nucl. Phys. **A582**, 655 (1995).
[19] M.A. Ivanov, M.P. Locher, and V.E. Lyubovitskij, Few-Body Syst. **21**, 131 (1996).
[20] M.A. Ivanov and V.E. Lyubovitskij, *Proceedings of HADRON-95* (World Scientific, Singapore, 1996), p. 396; V.E. Lyubovitskij and M.A. Ivanov, *Proceedings of ''BARYON'95''* (World Scientific, Singapore, 1996), p. 547.
[21] B.L. Ioffe, Nucl. Phys. **B188**, 317 (1981).
[22] Y. Chung, Nucl. Phys. **B197**, 55 (1982).
[23] E.V. Shuryak, Nucl. Phys. **B198**, 83 (1982).
[24] A.G. Grozin and O.I. Yakovlev, Phys. Lett. B **285**, 254 (1992); **291**, 441 (1992).
[25] S. Groote, J.G. Körner, and O.I. Yakovlev, Phys. Rev. D **54**, 3447 (1996).
[26] G.V. Efimov, M.A. Ivanov, and V.E. Lyubovitskij, Few-Body Syst. **6**, 17 (1989).
[27] G.V. Efimov, M.A. Ivanov, and V.E. Lyubovitskij, Z. Phys. C **47**, 583 (1990).
[28] J.G. Körner, D. Pirjol, and M. Krämer, Prog. Part. Nucl. Phys. **33**, 787 (1994).
[29] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[30] G.V. Efimov and M.A. Ivanov, *The Quark Confinement Model of Hadrons* (IOP, Philadelphia, 1993).
[31] N. Isgur and M. Wise, Nucl. Phys. **B348**, 276 (1991).
[32] H. Georgi, Nucl. Phys. **B348**, 293 (1991).
[33] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B355**, 38 (1991).
[34] F. Hussain *et al.*, Nucl. Phys. **B370**, 259 (1992).
[35] Q.P. Xu, Phys. Rev. D **48**, 5429 (1993).
[36] J.G. Körner and M. Krämer, Phys. Lett. B **275**, 495 (1992); P. Białas *et al.*, Z. Phys. C **57**, 115 (1993).
[37] R. Singleton, Jr., Phys. Rev. D **43**, 2939 (1991).
[38] X.-H. Guo and P. Kroll, Z. Phys. C **59**, 567 (1993).
[39] B. König, J.G. Körner, M. Krämer, and P. Kroll, Report No. DESY 93-011, 1993 (unpublished).
[40] M. Sadzikowski and K. Zalewski, Z. Phys. C **59**, 677 (1993).
[41] H.-Y. Cheng and B. Tseng, Phys. Rev. D **53**, 1457 (1996).
[42] B. Holdom, M. Sutherland, and J. Mureika, Phys. Rev. D **49**, 2359 (1994).
[43] M. Sutherland, Z. Phys. C **63**, 111 (1994).
[44] Y.-B. Dai *et al.*, Phys. Lett. B **387**, 379 (1996).
[45] A. Datta, Report No. UH-511-825-95 1995, hep-ph/9504429 (unpublished).
[46] C.W. Luo, Report No. Alberta-Thy-32-96, 1996 (unpublished).
[47] COMPASS Collaboration, G. Baum *et al.*, Report No. CERN-SPSLC-96-14, 1996 (unpublished).