# Black holes in the Brans-Dicke-Maxwell theory

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The black hole solutions in the higher dimensional Brans-Dicke-Maxwell theory are investigated. We find that the presence of the nontrivial scalar field depends on the spacetime dimensions $(D)$. When $D=4$, the solution corresponds to the Reissner-Nordström black hole with a constant scalar field. In higher dimensions $(D>4)$, one finds the charged black hole solutions with the nontrivial scalar field. The thermal properties of the charged black holes are discussed and the reason why the nontrivial scalar field exists are explained. Also the solutions for higher dimensional Brans-Dicke theory are given for comparison. [S0556-2821(97)03818-6]

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As is well known, differing from general relativity with the metric, the Brans-Dicke (BD) theory [1] describes gravitation in terms of the metric as well as a scalar field. Because of the scalar field, the BD theory and general relativity must have distinctions in some domains, although they can be in agreement under the post-Newtonian approximation. In recent years, much attention has been drawn in the BD theory, in particular, in the strong field domains. A strong field appeared in the early universe. La and Steinhardt [2] have shown that the BD theory seems to be better than the Einstein theory of gravity for solving the ''graceful exit'' problem in the inflation model. This is because the scalar field in the BD theory provided a natural termination of the inflationary era via bubble nucleation without the need for finely tuned cosmological parameters.

The other example comes from the black holes in the BD theory. More recently, many authors have investigated the gravitational collapse and black hole formation in the BD theory [3–7]. It turned out that the dynamic scalar field in the BD theory plays an important role in the process of collapse and critical phenomenon. Hawking [8] proved first that in the four-dimensional vacuum BD theory, the black hole solution is just the Schwarzschild solution with a trivial constant scalar field (hereafter the black holes in this paper mean the static, asymptotically flat, and spherically symmetric solutions with horizon). Further the stability of black holes in the BD theory has been investigated in Ref. [9]. On the other hand, the vacuum BD theory can be transformed into the Einstein-massless scalar theory by using a conformal transformation. In Ref. [10], the solution to Einstein-massless scalar equations was given. Although this solution has an asymptotically flat region and the scalar field approaches zero at spacelike infinity, it exhibits a naked singularity. When the scalar field is constant, the solution reduces to the Schwarzschild case. It is also noted that the black hole solution in the vacuum BD theory corresponds to the Schwarzschild solution with a constant scalar field. This can also be confirmed from the no scalar-hair theorem by Bekenstein [11] and Saa [12].

On the other hand, it is well known that the black hole solution to Einstein-Maxwell equations is the Reissner-Nordström solution. In higher dimensions, its solution can be regarded as a simple dimensional generalization of Reissner-Nordström solution [13]. In order to investigate the distinctions between the BD and Einstein theories, it is important to see whether the black hole solution in the Brans-Dicke-Maxwell theory belongs to the Reissner-Nordström solution or its trivially dimensional extension. We find that the $D=4$ black hole solution in the BD-Maxwell theory belongs to the Reissner-Nordström solution with a constant scalar field. In higher dimensions $(D>4)$, however, one obtains the black hole solutions with the nontrivial scalar. This is because the stress-energy tensor of Maxwell field is not traceless in the higher dimensions and the action of Maxwell field is not invariant under conformal transformations. Accordingly, the Maxwell field can be regarded as the source of the scalar field in the BD theory. The main purpose of this paper is to report this result.

In the $D(\geqslant 4)$ dimensions, the action of the Brans-Dicke-Maxwell theory is given by

$$I=\frac{1}{16\pi}\int d^Dx\sqrt{-g}\left(\phi R-\frac{\omega}{\phi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi-F_{\mu\nu}F^{\mu\nu}\right), \tag{1}$$

where $R$ is the scalar curvature, $F_{\mu\nu}$ is the Maxwell field, $\omega$ is the coupling constant, and the $\phi$ denotes the BD scalar with the dimensions $G^{-1}$. Here $G$ is a $D$-dimensional Newtonian constant. In this paper, we choose units such that $c=G=1$. In this BD frame, test particles have constant rest mass and move along the geodesics. That is, matter fields are coupled to gravity only via the metric, and do not interact with the scalar $\phi$. So we introduce the Maxwell kinetic term as in Eq. (1). Varying (1) yields equations of motion:

 

$$\phi G_{\mu\nu}\equiv\phi\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)$$
$$=\frac{\omega}{\phi}\left[\nabla_\mu\phi\nabla_\nu\phi-\frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right]$$
$$+2\left(F^{\lambda}_{\mu}F_{\nu\lambda}-\frac{1}{4}g_{\mu\nu}F^2\right)+\nabla_\mu\nabla_\nu\phi-g_{\mu\nu}\nabla^2\phi, \quad (2)$$

$$0=\nabla_\mu(F^{\mu\nu}), \quad (3)$$

$$\nabla^2\phi=-\frac{d-1}{2[(d+1)\omega+(d+2)]}F^2, \quad (4)$$

where $G_{\mu\nu}$ is the Einstein tensor, $\nabla$ represents the covariant differentiation in the spacetime metric $g_{\mu\nu}$, and $d=D-3$. Solving Eqs. (2)–(4) directly is a nontrivial task because the right hand side of Eq. (2) includes the second derivatives of the scalar. We can remove this difficulty by a conformal transformation.

Considering a conformal transformation

$$g_{\mu\nu}=\Omega^2\overline{g_{\mu\nu}}, \quad (5)$$

with

$$\Omega^{-(d+1)}=\phi \quad (6)$$

and

$$\overline{\phi}=\sqrt{2a}\int^{\phi}\frac{d\phi}{\phi}=\sqrt{2a}\ln\phi, \quad a=\frac{d+2}{d+1}+\omega, \quad (7)$$

the BD-Maxwell theory (1) can be transformed into the Einstein-Maxwell theory with a minimally coupled scalar field ($\overline{\phi}$)

$$\overline{I}=\frac{1}{16\pi}\int d^Dx\sqrt{-\overline{g}}\left[\overline{R}-\frac{1}{2}(\overline{\nabla\phi})^2-e^{-b\overline{\phi}}\overline{F^2}\right], \quad (8)$$

where

$$b=\frac{d-1}{d+1}\frac{1}{\sqrt{2a}}, \quad (9)$$

$\overline{R}$ and $\overline{\nabla}$ are the scalar curvature and covariant differentiation in the new metric $\overline{g_{\mu\nu}}$, respectively. Here a few points should be stressed. First of all, Eq. (7) implies $a>0$ $[\omega>-(d+2)/(d+1)]$, and one has $\overline{\phi}=0$ at spacelike infinity. Second, the action remains unchanged under the conformal transformation ($\overline{I}$ and $I$ give us a difference of surface term associated with the scalar field). This point plays an important role in dealing with physical quantities between the two frames. Third, we stress that the BD theory (1) is only mathematically equivalent to the theory (8). In the Einstein frame (8), the test particle will take variable rest mass with spacetime and move no longer along the geodesics. Note that there exists a coupling between Maxwell field and scalar field. Finally, it is worth noting that $b=0$ when $D=4$. In four dimensions the Maxwell field is decoupled from the scalar field in the Einstein frame and further in the BD frame, it cannot be considered as the source of the scalar field $\phi$ [see Eq. (4)]. Hence it turns out that $D=4$ is a special case in the BD-Maxwell theory.

Varying the action (8), we can obtain equations of motion

$$\overline{G_{\mu\nu}}\equiv\overline{R_{\mu\nu}}-\frac{1}{2}\overline{g_{\mu\nu}}\overline{R}$$
$$=\frac{1}{2}\overline{\nabla_\mu}\overline{\phi}\overline{\nabla_\nu}\overline{\phi}-\frac{1}{4}\overline{g_{\mu\nu}}(\overline{\nabla\phi})^2$$
$$+2e^{-b\overline{\phi}}\left(\overline{F^{\lambda}_{\mu}}\overline{F_{\nu\lambda}}-\frac{1}{4}\overline{g_{\mu\nu}}\overline{F^2}\right), \quad (10)$$

$$\overline{\nabla^2}\overline{\phi}=-be^{-b\overline{\phi}}\overline{F^2}, \quad (11)$$

$$0=\overline{\nabla_\mu}(e^{-b\overline{\phi}}\overline{F^{\mu\nu}}). \quad (12)$$

Comparing Eqs. (2)–(4) with Eqs. (10)–(12), one finds that if $(\overline{g_{\mu\nu}}, \overline{\phi}, \overline{F_{\mu\nu}})$ is the solution to Eqs. (10)–(12), then

$$(g_{\mu\nu},\phi,F_{\mu\nu})=\left(\exp-\left(\frac{2}{(d+1)\sqrt{2a}}\overline{\phi}\right)\overline{g_{\mu\nu}},\right.$$
$$\left.\times\exp\left(\frac{1}{\sqrt{2a}}\overline{\phi}\right),\overline{F_{\mu\nu}}\right) \quad (13)$$

is the solution of Eqs. (2)–(4). In order to demonstrate clearly, let us consider first the absence of the Maxwell field. In four dimensions, Brans [14] constructed the static solutions in the Brans-Dicke frame. Here we will provide the solutions of higher dimensional BD theory by the conformal transformation. In the absence of Maxwell field, Eqs. (10)–(12) have the following solution with isotropic coordinates [10]:

$$d\overline{s^2}=-e^{f}dt^2+e^{-h}(dr^2+r^2d\Omega^2_{d+1}), \quad (14)$$

$$\overline{\phi}=\left[\frac{2(d+1)}{d}(1-\gamma^2)\right]^{1/2}\ln\frac{r^d-r_0^d}{r^d+r_0^d}, \quad (15)$$

where

$$e^f=\left[\frac{r^d-r_0^d}{r^d+r_0^d}\right]^{2\gamma}, \quad (16)$$

$$e^{-h}=\left[1-\frac{r_0^{2d}}{r^{2d}}\right]^{2/d}\left[\frac{r^d-r_0^d}{r^d+r_0^d}\right]^{-2\gamma/d}. \quad (17)$$

Here $\gamma$ and $r_0$ are two integration constants. Obviously, when $r\to\infty$, one has $f\to0$, $h\to0$, and $\overline{\phi}\to0$. Therefore the spacetime (14) has asymptotically flat region and the scalar field $\overline{\phi}$ vanishes at spacelike infinity. From Eq. (15) it follows $0\leqslant\gamma^2\leqslant1$. When $\gamma\in[-1,0)$, however, the solution has a ''negative'' Arnowitt-Deser-Misner (ADM) mass [10]. In particular, for $\gamma=-1$ the solution is the $D$-dimensional Schwarzschild solution with a negative mass, which describes a naked singularity spacetime. In order for the solution (14) to be a physical solution, we confine $\gamma$ to $0\leqslant\gamma\leqslant 1$. The special example is $\gamma=1$. In this case, the scalar field vanishes. Equation (14) describes a $D$-dimensional Schwarzschild black hole geometry with mass $M=2r_0^d$. In other cases, Eq. (14) describes spacetime with a naked singularity, whose singular point is $r=r_0$. To observe it explicitly, let us calculate the scalar curvature of Eq. (14). This leads to

$$\overline{R}=\frac{4d(d+1)r_0^{2d}(1-\gamma^2)r^{2(d+1)}}{(r^d+r_0^d)^{2(d+1+\gamma)/d}(r^d-r_0^d)^{2(d+1-\gamma)/d}}. \qquad (18)$$

From Eq. (18) it is shown that in the cases of $\gamma^2\neq 1$, spacetime (14) has a naked scalar curvature singularity at $r=r_0$, which cannot be removed by coordinate transformations. Therefore, in the higher dimensional ($D\geqslant 4$) Einstein-minimally coupling massless scalar field system, the only black hole solution is a $D$-dimensional Schwarzschild solution with a trivial scalar field.

In the $D$-dimensional vacuum BD theory, using Eq. (13), we can obtain its solution:

$$ds^2=\Omega^2 d\overline{s}^2=\left(\frac{r^d+r_0^d}{r^d-r_0^d}\right)^{2/(d+1}[(d+1)(1-\gamma^2)/ad]^{1/2}d\overline{s}^2, \qquad (19)$$

$$\phi=\left(\frac{r^d-r_0^d}{r^d+r_0^d}\right)^{[(d+1)(1-\gamma^2)/ad]^{1/2}}, \qquad (20)$$

where $d\overline{s}^2$ is given by Eq. (14). It is easy to show that the solution (19) has asymptotically flat space and the point $r=r_0$ corresponds to a naked singularity still. This can be found from calculating the scalar curvature of the solution (19) through the relation

$$R=\Omega^{-2}\overline{R}-2(d+2)\Omega^{-3}\overline{g^{\mu\nu}}\overline{\nabla}_\mu\overline{\nabla}_\nu\Omega$$
$$-(d+2)(d-1)\Omega^{-4}\overline{g^{\mu\nu}}\overline{\nabla}_\mu\Omega\overline{\nabla}_\nu\Omega. \qquad (21)$$

Again, when $\gamma=1$, the solution (19) is reduced to the $D$-dimensional Schwarzschild solution with the constant scalar field ($\phi=1$). In that case, the BD theory degenerates into the Einstein theory of gravitation. From Eq. (15) we find $\overline{\phi}\leqslant 0$ (because of $r_0\geqslant 0$). So the scalar $\phi$ in the BD theory belongs to the region $\phi\in(0,1]$. When the Maxwell field is absent, however, the action (8) and equations of motion (10)–(12) remain unchanged under the transformation $\overline{\phi}\to-\overline{\phi}$. Thus, we can obtain another solution of the vacuum BD theory:

$$ds^2=\left(\frac{r_d-r_0^d}{r^d+r_0^d}\right)^{2/d+1[(d+1)(1-\gamma^2)/ad]^{1/2}}d\overline{s}^2, \qquad (22)$$

$$\phi=\left(\frac{r_d+r_0^d}{r^d-r_0^d}\right)^{[(d+1)(1-\gamma^2)/ad]^{1/2}}, \qquad (23)$$

where $d\overline{s}^2$ is still given by Eq. (14). In this case, the scalar field $\phi$ takes values in the region $[1,\infty)$. But the spacetime (22) has still asymptotically flat region and the point $r=r_0$ is a curvature singularity unless $\gamma=1$. When $\gamma=1$, the scalar field is a constant and the solution (22) is the $D$-dimensional Schwarzschild solution. Thus, we emphasized again that the black hole solution of the vacuum BD theory is the Schwarzschild solution with a constant scalar field in higher dimensions.

We now turn to the charged case. Introducing the Maxwell field, the situation is changed significantly. Consulting with the conformal transformation (5)–(7), instead of Eqs. (2)–(4), Eqs. (10)–(12) can be used for looking for the solutions. The black hole solutions for the actions similar to Eq. (8) have been found in Refs. [15,16]. Considering the dual form of the black holes given in Ref. [16], we obtain the black hole solutions of Eqs. (10)–(12),

$$d\overline{s}^2=-A^2dt^2+B^2dr^2+C^2d\Omega_{d+1}^2, \qquad (24)$$

$$e^{b\overline{\phi}}=\left[1-\left(\frac{r_-}{r}\right)^d\right]^{\alpha d}, \qquad (25)$$

$$\overline{F_{tr}}=\frac{Q}{r^{d+1}}, \qquad (26)$$

where

$$A^2(r)=\left[1-\left(\frac{r_+}{r}\right)^d\right]\left[1-\left(\frac{r_-}{r}\right)^d\right]^{1-\alpha d}, \qquad (27)$$

$$B^2(r)=\left[1-\left(\frac{r_+}{r}\right)^d\right]^{-1}\left[1-\left(\frac{r_-}{r}\right)^d\right]^{\alpha-1}, \qquad (28)$$

$$C^2(r)=r^2\left[1-\left(\frac{r_-}{r}\right)^d\right]^{\alpha}, \qquad (29)$$

$$\alpha=\frac{2b^2(d+1)}{d[2d+b^2(d+1)]}. \qquad (30)$$

Here $Q$, $r_+$, and $r_-$ are integration constants. According to the Gauss theorem, the electric charge is

$$q=\frac{1}{4\pi}\int_{r\to\infty}\overline{F_{tr}}\sqrt{-\overline{g}}d^{d+1}x=\frac{A_{d+1}}{4\pi}Q, \qquad (31)$$

where $A_{d+1}$ is the volume of the $(d+1)$-dimensional unit sphere. The constant $Q$ is related to the constants $r_+$ and $r_-$:

$$Q^2=\frac{\alpha d^3(r_+r_-)^d}{2b^2}. \qquad (32)$$

From Eq. (25) we see that the scalar field is bounded everywhere, except at $r=0$. When $\alpha=0$, the solution (24)–(26) reduces to a $D$-dimensional Reissner-Nordström solution with the vanishing scalar field. Thus, the constants $r_+$ and $r_-$ can be interpreted in terms of the outer horizon and inner horizon (assuming $r_+ \geqslant r_-$) but, for a general $\alpha$, the point $r_-$ is a scalar curvature singularity. This can be seen from the scalar curvature of the solution (24):

$$\overline{R} = \frac{\alpha^2 d^4 r_-^{2d}}{2b^2 r^{2d+2}} \left[1 - \left(\frac{r_+}{r}\right)^d\right] \left[1 - \left(\frac{r_-}{r}\right)^d\right]^{-(\alpha+1)} - \frac{2(d-1)Q^2}{(d+1)r^{2d+2}} \left[1 - \left(\frac{r_-}{r}\right)^d\right]^{-\alpha}, \tag{33}$$

which diverges at $r = r_-$ unless $\alpha = 0$. This also confirms that the inner horizon of $D$-dimensional Reissner-Nordström black holes is instable. In the our case, due to the appearance of the scalar field, the inner horizon is converted into a scalar curvature singularity.

With the Euclidean action method [17,18], we obtain the ADM mass ($\overline{M}$), Hawking temperature ($\overline{T}$), and the entropy ($\overline{S}$) of the black hole solution:

$$\overline{M} = \frac{A_{d+1}}{16\pi}(d+1)[r_+^d + (1 - \alpha - \alpha d) r_-^d], \tag{34}$$

$$\overline{T} = \frac{d}{4\pi r_+} \left[1 - \left(\frac{r_-}{r_+}\right)^d\right]^{1 - \alpha(d+1)/2}, \tag{35}$$

$$\overline{S} = \frac{1}{4}\overline{\Sigma} = \frac{A_{d+1} r_+^{d+1}}{4} \left[1 - \left(\frac{r_-}{r_+}\right)^d\right]^{\alpha(d+1)/2}, \tag{36}$$

where $\overline{\Sigma}$ is the horizon area of the black hole (24). With Eq. (13), one finds the charged black hole solution in the Brans-Dicke-Maxwell theory (frame):

$$ds^2 = \Omega^2 d\overline{s^2} = \left[1 - \left(\frac{r_-}{r}\right)^d\right]^{-2\alpha d/(d-1)} d\overline{s^2}, \tag{37}$$

$$\phi = \exp\left(\frac{1}{\sqrt{2a}}\right)\overline{\phi} = \left[1 - \left(\frac{r_-}{r}\right)^d\right]^{\alpha d(d+1)/(d-1)}, \tag{38}$$

$$F_{tr} = \overline{F_{tr}} = \frac{Q}{r^{d+1}}, \tag{39}$$

where $d\overline{s^2}$ is given by Eq. (24). The charged black hole solution (37) has an asymptotically flat region. The scalar field is bounded at the horizon, vanishes at singular point $r = r_-$, and tends to $\phi = 1$ at spacelike infinity. The action (8) and its equations of motion cannot remain unchanged under the transformation $\overline{\phi} \to -\overline{\phi}$ if the Maxwell field is present. Therefore, unlike the absence of Maxwell field, we obtain solution (37)–(39). In addition, it is easy to show that the ADM mass, Hawking temperature, and the entropy of black hole (37) are still given by Eqs. (34)–(36), respectively. This is so because the Euclidean action is invariant under the conformal transformation (up to a surface term associated with the scalar field). But it seems that the entropy of solution (24) satisfies the area formula, but the entropy in solution (37) does not. This is due to the fact that the black hole entropy comes from the surface term in the Euclidean action formalism. The surface term in the Einstein frame (8) is given by

$$\overline{I}_{\rm surface} = -\frac{1}{8\pi}\int_{\partial \overline{V}} d^{d+2}x \sqrt{\overline{h}}[\overline{K} - \overline{K_0}], \tag{40}$$

where $\overline{K}$ represents the extrinsic curvature in the metric $\overline{h}$ of a constant $r > r_+$ timelike supersurface $\partial\overline{V}$. And $\overline{K_0}$ is the extrinsic curvature of vacuum background (here it is the $D$-dimensional Minkowski spacetime). One can show that in Einstein frame the entropy satisfies the 1/4 area formula (Ref. [19])

$$\overline{S} = -\frac{1}{8\pi}\int_{r_+} d^{d+2}x \sqrt{\overline{h}}[\overline{K} - \overline{K_0}] = \frac{1}{4}\overline{\Sigma}. \tag{41}$$

Instead in the BD frame (1), the surface term leads to

$$I_{\rm surface} = -\frac{1}{8\pi}\int_{\partial V} d^{d+2}x \sqrt{h}\,\phi[K - K_0]. \tag{42}$$

The black hole entropy in the Brans-Dicke frame is found to be

$$S = -\frac{1}{8\pi}\int_{r_+} d^{d+2}x \sqrt{h}\,\phi[K - K_0] = \frac{1}{4}\phi(r_+)\Sigma, \tag{43}$$

where $\Sigma$ is the area of horizon in the Brans-Dicke frame (37). It appears that due to the scalar field, the area formula is no longer valid in the BD theory [4]. But making use of Eq. (37), it is found that Eq. (41) is equal to Eq. (43) and the entropy remains unchanged under the conformal transformations.

For the Hawking temperature (35), in the Einstein frame (24), it can be calculated as

$$\overline{T} = \left.\frac{(A^2)'}{4\pi\sqrt{A^2 B^2}}\right|_{r_+}, \tag{44}$$

where a prime denotes derivative with respect to $r$. In the Brans-Dicke frame (37), it is

$$T = \left.\frac{(\Omega^2 A^2)'}{4\pi\Omega^2\sqrt{A^2 B^2}}\right|_{r_+}. \tag{45}$$

Because the conformal parameter $\Omega^2$ is regular at the horizon, one can find that $\overline{T}$ is equal to $T$. Therefore, the Hawking temperature is an invariant quantity under conformal transformations only if the transformations are regular at event horizon.

The invariance of the ADM mass of black holes can be deduced from the first law of thermodynamics:

$$dM = TdS + \cdots, \tag{46}$$

where the ellipsis means the work terms. Because the Hawking temperature and entropy are invariant quantities, the ADM mass must be invariant under the regular conformal transformations.

The thermodynamics of the black hole solution (37) is quite interesting. When $d=1$ ($\alpha=0$), it reduces to that of Reissner-Nordström black holes (see the discussion below). For $d>1$ and $r_+=r_-$, it follows from Eqs. (35) and (36) that the Hawking temperature diverges because of $\alpha(d+1)/2>1, \overline{S}=0$. This property is similar to that of the $a>1$ dilaton black holes [20].

As for the solution (37) in the Brans-Dicke-Maxwell theory, we have three points to be stressed. First, when $D=4$, the solution (37)–(39) reduces to the Reissner-Nordström solution with the constant scalar field ($\phi=1$). That is, in four dimensions, the black hole solution is just the Reissner-Nordström solution. Because the equation of motion for the scalar becomes the source-free equation [see Eqs. (4) or (11)], there is nothing to support the nontrivial scalar field. Accordingly, we have only the trivial scalar. This reason also holds for the vacuum BD theory. Second, in higher dimensions ($b\neq 0$), we have the black hole solution with the nontrivial scalar field in the BD-Maxwell theory. As is well known, the Maxwell field is allowed by the no-hair theorem of black holes. From Eqs. (4) or (11), we see that the Maxwell field can be considered as the source of the scalar field. In this way the Maxwell field supports the nontrivial scalar field. Here we remind the reader that the stress-energy tensor of the Maxwell field is traceless only in four dimensions. Also in the Einstein frame, the action of the Maxwell field is invariant under four-dimensional conformal transformations. This can be seen clearly from the action (8). Finally, we make a comment on the no-hair theorem. At present, the interpretations of the no-hair theorem seem to have two aspects. One group argues that an asymptotically flat black hole cannot carry the nontrivial scalar fields bounded on the regular horizon (see the recent proofs of the no-scalar hair theorem [11,12,21,22]). On the other hand, the no-hair theorem means that the black hole can be characterized by only a few parameters: mass, angular momentum, and electric (magnetic) charge. In the latter sense, the existence of nontrivial scalar field in the BD-Maxwell theory does not violate the no-hair theorem because the solution (37) depends on only two parameters: the ADM mass and electric charge. In fact, such scalar hairs exist largely in the black holes of string theories.

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