Towards determining ϕ_1 with $B\to D^{(*)}\overline{D^{(*)}}$

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We present an isospin analysis of the decay modes $B\to D\overline{D}$, $D^*\overline{D}$, $D\overline{D^*}$, and $D^*\overline{D^*}$, which allows the determination of the final-state interaction phases. As these transitions have branching ratios of the order 10^{-4} or larger, they could be useful in detecting the CP-violating phase $\phi_1\equiv\arg(-V_{cb}^*V_{cd}V_{tb}V_{td}^*)$ in the first-round experiments of a B-meson factory. The problem of penguin pollution may still be present. Once the Kobayashi-Maskawa matrix elements are known, it is possible to obtain the magnitudes and relative phases of hadronic matrix elements for $B\to D^{(*)}\overline{D^{(*)}}$. This will in turn lead to some information about the penguin pollution. [S0556-2821(97)00713-3]

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I. INTRODUCTION

Within the standard electroweak model, three angles of the Kobayashi-Maskawa (KM) unitarity triangle

$$V_{ub}^*V_{ud}+V_{cb}^*V_{cd}+V_{tb}^*V_{td}=0, \tag{1}$$

denoted by ϕ_i $(i=1,2,3)$, are determined from CP asymmetries in neutral B-meson decays to hadronic CP eigenstates [1–3]. It is expected that $\phi_1\equiv\arg(-V_{cb}^*V_{cd}V_{tb}V_{td}^*)$ can be unambiguously extracted from the decay rate difference between $B_d^0\to\psi K_S$ and $\overline{B_d^0}\to\psi K_S$. While this decay mode should be sufficient for determining ϕ_1, the initial luminosity of a B factory may require us to search for some additional decay channels which could help us establish the presence of CP violation as quickly as possible [4]. For this purpose, we shall investigate

$$B\to D\overline{D},\;\; D^*\overline{D},\;\; D\overline{D^*},\;\; D^*\overline{D^*} \tag{2}$$

in some detail.

In practical experiments the decay mode $B_d\to D^+D^-$ should have fairly large branching ratio. Under SU(3) symmetry, one can make the rough estimation

$$\begin{aligned}\mathcal{B}(B_d^0\to D^+D^-)&\sim\sin^2\theta_C\mathcal{B}(B_d^0\to D_s^+D^-)\\&\approx(3.4\pm1.9)\times10^{-4},\end{aligned} \tag{3}$$

where θ_C is the Cabibbo angle, and $\mathcal{B}(B_d^0\to D_s^+D^-)=(7\pm4)\times10^{-3}$ has been measured in experiments [5]. Also, the penguin effect in $B_d\to D^+D^-$ is expected to be smaller than that in $B_d\to\pi^+\pi^-$. Thus the CP asymmetry between $B_d^0\to D^+D^-$ and $\overline{B_d^0}\to D^+D^-$ may be dominated by the angle ϕ_1. In contrast with $B_d\to D^+D^-$, the decay modes $B_d\to D^+D^{*-}$, $D^{*+}D^-$, and $D^{*+}D^{*-}$ undergo the same weak interactions and have comparable branching ratios, although they are not the exact CP-even eigenstates [6].

Of course, the measurement of CP violation in $B_d\to D^+D^-$ cannot only cross-check the extraction of ϕ_1 from $B_d\to\psi K_S$, but also shed some light on the penguin effects and final-state interactions (FSI's) in nonleptonic B decays to double charmed mesons. For this reason, it is worth studying both $B_d\to D^+D^-$ and $B_d\to D^0\overline{D^0}$ in a model-independent approach. The similar treatment is applicable to the processes $B_d\to D\overline{D^*}$, $D^*\overline{D}$, etc.

In this work we shall carry out an isospin analysis of the processes $B\to D^{(*)}\overline{D^{(*)}}$, to relate their weak and strong phases to the relevant observables. It is found that the time-dependent measurements of $B_d\to D^+D^-$ and $B_d\to D^0\overline{D^0}$ together with the time-independent measurements of $B_u^+\to D^+\overline{D^0}$ and $B_u^-\to D^-D^0$ allow one to extract a phase parameter ϕ_1', which consists of both ϕ_1 and the penguin-induced phase information. Direct CP asymmetries in $B_d\to D^+D^-$ and $D^0\overline{D^0}$ are time-independently detectable on the $\Upsilon(4S)$ resonance. For numerical illustration, we apply the effective weak Hamiltonian and factorization approximation to $B_u^+\to D^{(*)+}\overline{D^{(*)0}}$ and $B_u^-\to D^{(*)-}D^{(*)0}$, since each of them is only involved in a single isospin amplitude. We find that their branching ratios are all above 10^{-4} and the relevant time-independent CP asymmetries may reach the 3% level. The time-dependent CP asymmetries in $B_d\to D^{(*)+}\overline{D^{(*)-}}$ and $D^{(*)0}\overline{D^{(*)0}}$ are expected to be of order one. We also emphasize that it is possible to obtain some information on the magnitudes and relative phases of hadronic matrix elements in the isospin approach, once the KM matrix elements have been known.

II. ISOSPIN ANALYSIS

The effective weak Hamiltonians, responsible for $B_u^-\to D^-D^0$, $\overline{B_d^0}\to D^+D^-$, $\overline{B_d^0}\to D^0\overline{D^0}$, and their CP-conjugate processes, have the isospin structures $|1/2,-1/2\rangle$ and $|1/2,+1/2\rangle$, respectively. The decay amplitudes of these transitions can be written in terms of the isospin amplitudes:

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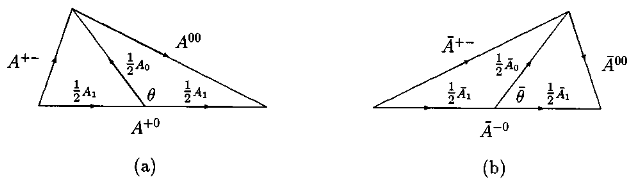

FIG. 1. The isospin triangles of $B\to D\overline{D}$ in the complex plane.

$$A^{+-}\equiv\langle D^+D^-|\mathcal{H}_{\rm eff}|B_d^0\rangle=\tfrac{1}{2}(A_1+A_0),$$

$$A^{00}\equiv\langle D^0\overline{D^0}|\mathcal{H}_{\rm eff}|B_d^0\rangle=\tfrac{1}{2}(A_1-A_0),$$

$$A^{+0}\equiv\langle D^+\overline{D^0}|\mathcal{H}_{\rm eff}|B_u^+\rangle=A_1, \tag{4}$$

and

$$\overline{A^{+-}}\equiv\langle D^+D^-|\mathcal{H}_{\rm eff}|\overline{B_d^0}\rangle=\tfrac{1}{2}(\overline{A_1}+\overline{A_0}),$$

$$\overline{A^{00}}\equiv\langle D^0\overline{D^0}|\mathcal{H}_{\rm eff}|\overline{B_d^0}\rangle=\tfrac{1}{2}(\overline{A_1}-\overline{A_0}),$$

$$\overline{A^{-0}}\equiv\langle D^-D^0|\mathcal{H}_{\rm eff}|B_u^-\rangle=\overline{A_1}. \tag{5}$$

Here A_1 $(\overline{A_1})$ and A_0 $(\overline{A_0})$ are the isospin amplitudes with $I=1$ and $I=0$, respectively. Clearly, the isospin relations (4) and (5) can be expressed as two triangles in the complex plane (see Fig. 1 for illustration):

$$A^{+-}+A^{00}=A^{+0},$$

$$\overline{A^{+-}}+\overline{A^{00}}=\overline{A^{-0}}. \tag{6}$$

One is able to determine the relative size and phase difference of isospin amplitudes A_1 $(\overline{A_1})$ and A_0 $(\overline{A_0})$ from the above triangular relations. Denoting

$$\frac{A_0}{A_1}\equiv ze^{i\theta},\quad \frac{\overline{A_0}}{\overline{A_1}}\equiv\bar{z}\,e^{i\bar{\theta}}, \tag{7}$$

then we obtain

$$z=\sqrt{\frac{2(|A^{+-}|^2+|A^{00}|^2)}{|A^{+0}|^2}-1},$$

$$\theta=\arccos\left(\frac{|A^{+-}|^2-|A^{00}|^2}{z|A^{+0}|^2}\right), \tag{8}$$

and

$$\bar{z}=\sqrt{\frac{2(|\overline{A^{+-}}|^2+|\overline{A^{00}}|^2)}{|\overline{A^{-0}}|^2}-1},$$

$$\bar{\theta}=\arccos\left(\frac{|\overline{A^{+-}}|^2-|\overline{A^{00}}|^2}{\bar{z}|\overline{A^{-0}}|^2}\right). \tag{9}$$

If $z=1$ and $\theta=0$, for example, we find that $|A^{00}|=0$, i.e., the decay mode $B_d^0\to D^0\overline{D^0}$ is forbidden.

Note that θ $(\bar{\theta})$ is in general a mixture of the weak and strong phase shifts, since both A_0 $(\overline{A_0})$ and A_1 $(\overline{A_1})$ may contain the tree-level and penguin contributions. This point can be seen more clearly if one writes the isospin amplitudes A_I and $\overline{A_I}$ $(I=1,0)$ with the help of the low-energy effective $\Delta B=\pm 1$ Hamiltonians. For example, A_I can be given as

$$A_I=\langle (D\overline{D})_I|\mathcal{H}_{\rm eff}(\Delta B=+1)|B\rangle=\frac{G_F}{\sqrt{2}}\sum_{q=u,c}[(V_{qb}^*V_{qd})S_I^q], \tag{10}$$

with

$$S_I^q=c_1\langle (D\overline{D})_I|Q_1^q|B\rangle+c_2\langle (D\overline{D})_I|Q_2^q|B\rangle$$
$$+\sum_{i=3}^{10}[c_i\langle (D\overline{D})_I|Q_i|B\rangle], \tag{11}$$

where Wilson coefficients c_i and four-quark operators Q_i at the scale $\mu=O(m_b)$ have been well defined in Ref. [7]. The expression of $\overline{A_I}$ is straightforwardly obtainable from Eq. (10) through the replacement $(V_{qb}^*V_{qd})\to(V_{qb}V_{qd}^*)$. The tree- and penguin-type hadronic matrix elements in S_I^u are expected to consist of different strong phases, and these phases should be different from those in S_I^c. This implies that the overall phases of A_1 $(\overline{A_1})$ and A_0 $(\overline{A_0})$ are nonlinear combinations of the same weak phases and the different strong phases, therefore, θ $(\bar{\theta})$ is neither purely weak nor purely strong.

Finally, it is worth mentioning that the same isospin relations hold for the decay modes $B\to D\overline{D^*}$ and $B\to D^*\overline{D}$. Of course, the isospin parameters z $(\bar{z})$ and θ $(\bar{\theta})$ in $B\to D\overline{D}$, $D\overline{D^*}$, and $D^*\overline{D}$ may be different from one another due to their different FSI's. As for $B\to D^*\overline{D^*}$, the same isospin relations hold separately for the decay amplitudes with helicity $\lambda=-1$, 0, or $+1$.

III. TIME-INDEPENDENT MEASUREMENTS

The quantities $|A^{+0}|$ and $|\overline{A^{-0}}|$ are obtainable from the time-independent measurements of decay rates of $B_u^+\to D^+\overline{D^0}$ and $B_u^-\to D^-D^0$. A determination of $|A^{+-}|$ $(|A^{00}|)$ and $|\overline{A^{+-}}|$ $(|\overline{A^{00}}|)$ is possible through the time-integrated measurements of B_d^0 vs $\overline{B_d^0}\to D^+D^-$ $(D^0\overline{D^0})$ on the $\Upsilon(4S)$ resonance, where the two produced B_d mesons are in a coherent state (with odd charge-conjugation parity) until one of them decays. In practice, one can use the semileptonic transition of one B_d meson to tag the flavor of the other meson decaying to D^+D^- or to $D^0\overline{D^0}$. The probability for observing such a joint decay event reads [3,8]

$$\mathcal{R}(l^{\pm}X^{\mp};D^+D^-)\propto|A_l|^2\left(\frac{|A^{+-}|^2+|\overline{A^{+-}}|^2}{2}\right.$$
$$\left.\mp\frac{1}{1+x_d^2}\,\frac{|A^{+-}|^2-|\overline{A^{+-}}|^2}{2}\right), \tag{12}$$

or

$$\mathcal{R}(l^{\pm}X^{\mp};D^0\overline{D^0})\propto|A_l|^2\left(\frac{|A^{00}|^2+|\overline{A^{00}}|^2}{2}\right.$$
$$\left.\mp\frac{1}{1+x_d^2}\frac{|A^{00}|^2-|\overline{A^{00}}|^2}{2}\right),\qquad(13)$$

where $|A_l|\equiv|\langle l^+X^-|\mathcal{H}_{\rm eff}|B_d^0\rangle|=|\langle l^-X^+|\mathcal{H}_{\rm eff}|\overline{B_d^0}\rangle|$ under CPT symmetry, and $x_d=\Delta m/\Gamma\approx0.73$ is a measure of B_d^0-$\overline{B_d^0}$ mixing [5]. By now the semileptonic B_d transitions such as $B_d^0\to D^{(*)-}l^+\nu_l$ and $\overline{B_d^0}\to D^{(*)+}l^-\overline{\nu}_l$ have been well reconstructed [5], i.e., $|A_l|$ has been detected independent of the above joint decay modes. Once $\mathcal{R}(l^{\pm}X^{\mp};D^+D^-)$ and $\mathcal{R}(l^{\pm}X^{\mp};D^0\overline{D^0})$ are measured, we shall be able to determine the quantities $|A^{+-}|$ ($|A^{00}|$) and $|\overline{A^{+-}}|$ ($|\overline{A^{00}}|$).

The time-independent measurements mentioned above allow one to construct the isospin triangles in Fig. 1. Consequently, the isospin parameters z ($\overline{z}$) and θ ($\overline{\theta}$) are extractable in the absence of any time-dependent measurement. If the branching ratios of $B_d^0\to D^0\overline{D^0}$ and $\overline{B_d^0}\to D^0\overline{D^0}$ are too small to be observable, then large cancellation between the isospin amplitudes A_1 ($\overline{A_1}$) and A_0 ($\overline{A_0}$) must take place. In the case that $B_d^0\to D^+D^-$ and $B_u^+\to D^+\overline{D^0}$ have been measured earlier than $B_d^0\to D^0\overline{D^0}$, a lower bound on the rate of the latter decay mode is model-independently achievable from the isospin relations obtained above. Since $\cos\theta\leqslant1$, we get from Eq. (8) that

$$\mathcal{B}(B_d^0\to D^0\overline{D^0})\geqslant\left[\sqrt{\frac{\mathcal{B}(B_d^0\to D^+D^-)}{\mathcal{B}(B_u^+\to D^+\overline{D^0})}}-1\right]^2$$
$$\times\mathcal{B}(B_u^+\to D^+\overline{D^0}),\qquad(14)$$

where tiny isospin-violating effects induced by the mass difference $m_{D^0}-m_{D^-}$ and the lifetime difference $\tau_{B_d}-\tau_{B_u}$ have been neglected. This bound should be useful to set a limit for the results of $\mathcal{B}(B_d^0\to D^0\overline{D^0})$ obtained from specific models of hadronic matrix elements. Following the same way, one can find the lower bounds for the branching ratios of $B_d^0\to D^{*0}\overline{D^0}$, $D^0\overline{D^{*0}}$, and $D^{*0}\overline{D^{*0}}$.

The nonvanishing asymmetry between the decay rates of $B_u^+\to D^+\overline{D^0}$ and $B_u^-\to D^-D^0$ signifies the existence of direct CP violation. By use of the isospin amplitudes in Eqs. (10) and (11), we obtain the CP asymmetry

$$\mathcal{A}_{\pm0}\equiv\frac{\mathcal{R}(B_u^+\to D^+\overline{D^0})-\mathcal{R}(B_u^-\to D^-D^0)}{\mathcal{R}(B_u^+\to D^+\overline{D^0})+\mathcal{R}(B_u^-\to D^-D^0)}$$
$$=2\sin\phi_3\frac{{\rm Im}(S_1^uS_1^{c*})}{N_{11}},\qquad(15)$$

where $\phi_3\equiv\arg(-V_{ub}^*V_{ud}V_{cb}V_{cd}^*)$ is an angle of the KM unitarity triangle, and N_{11} can be read from

$$N_{ij}\equiv\kappa{\rm Re}(S_i^uS_j^{u*})+\kappa^{-1}{\rm Re}(S_i^cS_j^{c*})$$
$$-\cos\phi_3{\rm Re}(S_i^uS_j^{c*}+S_j^uS_i^{c*}),\qquad(16)$$

with $\kappa\equiv|V_{ub}V_{ud}|/|V_{cb}V_{cd}|$. For the processes $B_d\to D^+D^-$ and $D^0\overline{D^0}$, pure signals of direct CP asymmetries may manifest themselves *on the* $\Upsilon(4S)$ *resonance*:

$$\mathcal{A}_{+-}\equiv\frac{\mathcal{R}(l^-X^+;D^+D^-)-\mathcal{R}(l^+X^-;D^+D^-)}{\mathcal{R}(l^-X^+;D^+D^-)+\mathcal{R}(l^+X^-;D^+D^-)}$$
$$=\frac{2\sin\phi_3}{1+x_d^2}\frac{{\rm Im}(S_1^uS_1^{c*}+S_0^uS_0^{c*}+S_1^uS_0^{c*}+S_0^uS_1^{c*})}{N_{11}+N_{00}+N_{10}+N_{01}},\qquad(17)$$

and

$$\mathcal{A}_{00}\equiv\frac{\mathcal{R}(l^-X^+;D^0\overline{D^0})-\mathcal{R}(l^+X^-;D^0\overline{D^0})}{\mathcal{R}(l^-X^+;D^0\overline{D^0})+\mathcal{R}(l^+X^-;D^0\overline{D^0})}$$
$$=\frac{2\sin\phi_3}{1+x_d^2}\frac{{\rm Im}(S_1^uS_1^{c*}+S_0^uS_0^{c*}-S_1^uS_0^{c*}-S_0^uS_1^{c*})}{N_{11}+N_{00}-N_{10}-N_{01}}.\qquad(18)$$

If the decay modes $B_d^0\to D^0\overline{D^0}$ and $\overline{B_d^0}\to D^0\overline{D^0}$ are forbidden due to the absence of final-state rescattering (i.e., $\theta\approx0$ and $z\approx1$, or $S_1^q\approx S_0^q$), then measuring the CP asymmetry $\mathcal{A}_{00}$ is practically impossible. In this case, we arrive at an interesting relation between the asymmetries $\mathcal{A}_{\pm0}$ and $\mathcal{A}_{+-}$:

$$\mathcal{A}_{\pm0}\approx(1+x_d^2)\mathcal{A}_{+-}\approx1.5\mathcal{A}_{+-}.\qquad(19)$$

The validity of this relation is testable in forthcoming experiments at a B-meson factory.

It is worthwhile at this point to give a brief comparison between the isospin language and the intuitive quark-diagram description for $B\to D^{(*)}\overline{D^{(*)}}$. Both the isospin amplitudes A_1 and A_0 are dominated by the spectator (external W emission) quark graph with the KM factor $V_{cb}^*V_{cd}$, but they also receive some small contributions from the loop-induced penguin and annihilation-type tree quark diagrams. Hence the branching ratios of $B_d^0\to D^{(*)+}D^{(*)-}$ and $B_u^+\to D^{(*)+}\overline{D^{(*)0}}$ may be of the same order. In the assumption of no final-state rescattering or channel mixing, $B_d^0\to D^{(*)0}\overline{D^{(*)0}}$ takes place only through the annihilation-type quark graphs, which are expected to have significant form-factor suppression in the factorization approximation. This argument is compatible with the isospin analysis, since the cancellation between A_1 and A_0 in A^{00} implies that the dominant spectator diagram does not contribute to $B_d^0\to D^{(*)0}\overline{D^{(*)0}}$. However, one should keep in mind that FSI effects are possible to significantly enhance the decay rate of $B_d^0\to D^{(*)0}\overline{D^{(*)0}}$ to the level comparable with that of $B_d^0\to D^{(*)+}D^{(*)-}$ or of $B_u^+\to D^{(*)+}\overline{D^{(*)0}}$, making the naive quark-diagram language a failure.

IV. TIME-DEPENDENT MEASUREMENTS

To probe the CP asymmetry induced by the interplay of direct decay and B_d^0-$\overline{B_d^0}$ mixing in $B_d \to D\overline{D}$, the time-dependent measurements are necessary on the $\Upsilon(4S)$ resonance at asymmetric B factories. In such an experimental scenario, the joint decay rates can be given as [8]

$$\mathcal{R}(l^{\pm}X^{\mp},D^{+}D^{-};t) \propto |A_l|^2 e^{-\Gamma|t|}\left[\frac{|A^{+-}|^2+|\overline{A^{+-}}|^2}{2} \mp \frac{|A^{+-}|^2-|\overline{A^{+-}}|^2}{2}\cos(x_d\Gamma t) \pm |A^{+-}|^2 \mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{+-}}}{A^{+-}}\right)\sin(x_d\Gamma t)\right] \tag{20}$$

and

$$\mathcal{R}(l^{\pm}X^{\mp},D^{0}\overline{D^{0}};t) \propto |A_l|^2 e^{-\Gamma|t|}\left[\frac{|A^{00}|^2+|\overline{A^{00}}|^2}{2} \mp \frac{|A^{00}|^2-|\overline{A^{00}}|^2}{2}\cos(x_d\Gamma t) \pm |A^{00}|^2 \mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{00}}}{A^{00}}\right)\sin(x_d\Gamma t)\right], \tag{21}$$

where t is the proper time difference between the semileptonic and nonleptonic decays,[1] and

$$\frac{q}{p} \equiv \left|\frac{q}{p}\right| \exp(-i2\phi_0) \approx \exp(-i2\phi_0) \tag{22}$$

stands for the phase information from B_d^0-$\overline{B_d^0}$ mixing [3]. For simplicity, we denote the phase difference between A_1 and $\overline{A_1}$ as

$$\varphi \equiv \frac{1}{2}\arg\left(\frac{\overline{A_1}}{A_1}\right) = \frac{1}{2}\arg\left[\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}\,\frac{S_1^c-\kappa S_1^u\exp(-i\phi_3)}{S_1^c-\kappa S_1^u\exp(+i\phi_3)}\right]. \tag{23}$$

In terms of the isospin parameters, coefficients of the $\sin(x_d\Gamma t)$ term in Eqs. (20) and (21) are given by

$$\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{+-}}}{A^{+-}}\right) = \frac{|A^{+0}\overline{A^{-0}}|}{4|A^{+-}|^2}\left[-\sin(2\phi_1')-z\sin(\theta+2\phi_1') + \bar{z}\sin(\bar{\theta}-2\phi_1')+z\bar{z}\sin(\bar{\theta}-\theta-2\phi_1')\right] \tag{24}$$

and

$$\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{00}}}{A^{00}}\right) = \frac{|A^{+0}\overline{A^{-0}}|}{4|A^{00}|^2}\left[-\sin(2\phi_1')+z\sin(\theta+2\phi_1') - \bar{z}\sin(\bar{\theta}-2\phi_1')+z\bar{z}\sin(\bar{\theta}-\theta-2\phi_1')\right], \tag{25}$$

where $\phi_1' \equiv \phi_0 - \varphi$. All the quantities on the right-hand side of Eq. (24) or (25), except ϕ_1', can be determined through the time-independent measurements of $B \to D\overline{D}$ on the $\Upsilon(4S)$ resonance. Thus measuring the CP-violating observable on the left-hand side of Eq. (24) or (25) will allow a model-independent extraction of ϕ_1'.

Let us make two remarks about the results obtained above.

(1) Within the standard model, $\phi_0 \approx \phi_1$ holds to an excellent degree of accuracy. If the tree-level quark transition $\bar{b} \to (c\bar{c})\bar{d}$ dominates the decay amplitude of $B_u^+ \to D^+\overline{D^0}$ [i.e., $|S_1^c| \gg |S_1^u|$ in Eq. (23)], then we get $\varphi \approx \arg(V_{cb}V_{cd}^*) \approx 0$ as a pure weak phase. In this case, the magnitude of ϕ_1 is extractable from the time-dependent measurement of $B_d \to D^+D^-$ or of $B_d \to D^0\overline{D^0}$ [1,2]. Note that a model-dependent estimation in the standard model gives $\varphi \sim -3°$ (see the Appendix). It is worth pointing out that the B_d^0-$\overline{B_d^0}$ mixing phase ϕ_0 can be reliably determined from the CP asymmetry in B_d^0 vs $\overline{B_d^0} \to \psi K_S$ either within or beyond the standard model.[2] Thus a comparison of ϕ_0 (extracted from $B_d \to \psi K_S$) with ϕ_1' (extracted from $B_d \to D^+D^-$ or $B_d \to D^0\overline{D^0}$) will constrain φ, which may reflect the penguin-induced phase information in $B \to D\overline{D}$.

(2) It is interesting to note that we can, in principle, obtain phases and magnitudes of the hadronic matrix elements S_I^q. On the experimental side, $|A_0|$ ($|\overline{A_0}|$), $|A_1|$ ($|\overline{A_1}|$), and θ ($\bar{\theta}$) can be determined from the time-independent measurements; and ϕ_1' can be extracted from the time-dependent measurements. If the KM phases are known, the hadronic matrix elements S_0^u, S_1^u, S_0^c, and S_1^c represent seven unknown parameters as the overall phases of them are physically irrelevant. Thus the magnitudes and relative phases of these quantities should be determinable from experimental measurements of the relevant branching ratios and CP asymmetries.

A special but interesting case is $z=\bar{z}=1$. This will lead, for arbitrary values of θ and $\bar{\theta}$, to the results

$$|A^{+-}| = |A^{+0}|\cos\frac{\theta}{2}, \quad |A^{00}| = |A^{+0}|\sin\frac{\theta}{2}, \tag{26}$$

$$|\overline{A^{+-}}| = |\overline{A^{-0}}|\cos\frac{\bar{\theta}}{2}, \quad |\overline{A^{00}}| = |\overline{A^{-0}}|\sin\frac{\bar{\theta}}{2}.$$

[1] Note that the proper time sum of the semileptonic and nonleptonic decays has been integrated out, since it will not be measured at any B-meson factory.

[2] This point relies on the condition that no significant penguin effect exists in $B_d \to \psi K_S$, and it is testable through the time-independent measurement of the decay rate difference between $B_d^0 \to \psi K_S$ and $\overline{B_d^0} \to \psi K_S$ on the $\Upsilon(4S)$ resonance.

As a straightforward consequence, one gets

$$|A^{+-}|^2+|A^{00}|^2=|A^{+0}|^2,$$

$$|\overline{A^{+-}}|^2+|\overline{A^{00}}|^2=|\overline{A^{-0}}|^2, \tag{27}$$

i.e., the two isospin triangles in Fig. 1 become right-angled triangles. If $\theta=\overline{\theta}$ is further assumed, we obtain

$$\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{+-}}}{A^{+-}}\right)=-\frac{|A^{+0}\overline{A^{-0}}|}{|A^{+-}|^2}\sin(2\phi_1')\cos^2\frac{\theta}{2},$$

$$\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{00}}}{A^{00}}\right)=-\frac{|A^{+0}\overline{A^{-0}}|}{|A^{00}|^2}\sin(2\phi_1')\sin^2\frac{\theta}{2}. \tag{28}$$

One can see that these two CP-violating quantities have the quasiseesaw dependence on the isospin phase shift θ. The magnitude of $\sin(2\phi_1')$ turns out to be

$$\sin(2\phi_1')=-\frac{1}{|A^{+0}\overline{A^{-0}}|}\left[|A^{+-}|^2\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{+-}}}{A^{+-}}\right)+|A^{00}|^2\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A^{00}}}{A^{00}}\right)\right], \tag{29}$$

apparently independent of θ.

V. SUMMARY AND CONCLUSION

We have presented an isospin analysis of the weak decays $B\to D^{(*)}\overline{D^{(*)}}$. The main results can be summarized as follows.

(a) The time-independent measurements of these transitions on the $\Upsilon(4S)$ resonance allow one to extract the isospin quantities and probe the direct CP asymmetries in them. It is possible to extract a phase parameter, which consists of the phase information from both $B_d^0-\overline{B_d^0}$ mixing and penguin diagrams, from the time-dependent measurements of $B_d\to D^+D^-$ and $D^0\overline{D^0}$. A comparison of this phase with that extracted from $B_d\to\psi K_S$ will be interesting, since their difference signifies the penguin-induced phase information (no matter whether new physics is present or not).

(b) Once the KM matrix elements have been determined, the relevant hadronic matrix elements (including their phase information) can be determined, through the isospin analysis, from some measurements of the decay rates and CP asymmetries.

In the Appendix, we have made use of the effective weak Hamiltonian and naive factorization approximation to estimate the branching ratios of $B_u^+\to D^{(*)+}\overline{D^{(*)0}}$ and $B_u^-\to D^{(*)-}D^{(*)0}$ as well as their CP asymmetries. It is remarkable that all these decay modes can well be detected in the first-round experiments of a B-meson factory. In particular, only about 10^8 $B_u^{\pm}$ events are expected to be needed for the exploration of direct CP-violating signals in them (at the 3% level).

We conclude that a careful experimental study of the decay modes $B\to D^{(*)}\overline{D^{(*)}}$ at the forthcoming B factories, before the measurements of $B\to\pi\pi$ and other charmless B decays become available, will be able to cross-check the extraction of the weak angle ϕ_1 from $B_d\to\psi K_S$, to shed some light on the penguin and FSI effects in B decays to double charmed mesons, and to probe direct CP violation in both charged and neutral B-meson systems.

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APPENDIX

Here we calculate the branching ratios of $B_u^+\to D^{(*)+}\overline{D^{(*)0}}$ and $B_u^-\to D^{(*)-}D^{(*)0}$ as well as their CP asymmetries numerically, in order to give one a feeling of ballpark numbers to be expected within the standard model. It is suitable to apply the effective weak Hamiltonian and factorization approximation to these decay modes, because each of them only has a single isospin amplitude. In contrast, $B_d\to D^{(*)+}D^{(*)-}$ or $B_d\to D^{(*)0}\overline{D^{(*)0}}$ is involved in two different isospin amplitudes; thus a direct application of the factorization approximation to such transitions may be problematic unless the FSI effects are negligibly small.

In estimating the branching ratios of $B_u^+\to D^+\overline{D^0}$, $D^{*+}\overline{D^0}$, $D^+\overline{D^{*0}}$, and $D^{*+}\overline{D^{*0}}$, it is instructive to neglect small contributions from the hadronic matrix elements $\langle D^+\overline{D^0}|Q_{1,2}^u|B_u^+\rangle$ (annihilation) and $\langle D^+\overline{D^0}|Q_{3-10}|B_u^+\rangle$ (penguin). These transitions have the weak interaction similar to that in $B_u^+\to D_s^+\overline{D^0}$, $D_s^{*+}\overline{D^0}$, $D_s^+\overline{D^{*0}}$, and $D_s^{*+}\overline{D^{*0}}$, whose decay rates have already been measured in experiments [5]. Then a comparison between the above two sets of decay modes, with the help of the factorization approximation, leads straightforwardly to the leading-order results

$$\mathcal{B}(B_u^+\to D^+\overline{D^0})\approx\frac{f_{D^+}^2}{f_{D_s^+}^2}\sin^2\theta_C\mathcal{B}(B_u^+\to D_s^+\overline{D^0}), \tag{A1}$$

$$\mathcal{B}(B_u^+\to D^+\overline{D^{*0}})\approx\frac{f_{D^+}^2}{f_{D_s^+}^2}\sin^2\theta_C\mathcal{B}(B_u^+\to D_s^+\overline{D^{*0}}),$$

$$\mathcal{B}(B_u^+\to D^{*+}\overline{D^0})\approx\frac{g_{D^{*+}}^2}{g_{D_s^{*+}}^2}\sin^2\theta_C\mathcal{B}(B_u^+\to D_s^{*+}\overline{D^0}),$$

$$\mathcal{B}(B_u^+\to D^{*+}\overline{D^{*0}})\approx\frac{g_{D^{*+}}^2}{g_{D_s^{*+}}^2}\sin^2\theta_C\mathcal{B}(B_u^+\to D_s^{*+}\overline{D^{*0}}),$$

where θ_C is the Cabibbo angle, f_X and g_{X*} ($X = D^+$ or D_s^+) are the decay constants. Since our present knowledge of f_{D^+}, $f_{D_s^+}$, $g_{D^{*+}}$, and $g_{D_s^{*+}}$ is quite poor [5], we take $f_{D^+} \approx f_{D_s^+} \approx 0.8$ and $g_{D^{*+}} \approx g_{D_s^{*+}} \approx 0.8$ for simplicity and illustration [9]. Choosing the central values of $\mathcal{B}(B_u^+ \to D_s^+ \overline{D^0})$, etc. [5], we approximately obtain $\mathcal{B}(B_u^+ \to D^+ \overline{D^0}) \approx 5.3 \times 10^{-4}$, $\mathcal{B}(B_u^+ \to D^+ \overline{D^{*0}}) \approx 3.1 \times 10^{-4}$, $\mathcal{B}(B_u^+ \to D^{*+} \overline{D^0}) \approx 3.7 \times 10^{-4}$, and $\mathcal{B}(B_u^+ \to D^{*+} \overline{D^{*0}}) \approx 7.1 \times 10^{-4}$. From this rough estimation one can see that the above decay modes are definitely detectable in the first-round experiments of a B-meson factory.

To roughly estimate the CP asymmetry between $B_u^+ \to D^+ \overline{D^0}$ and $B_u^- \to D^- D^0$, we take the timelike penguin contribution into account [10]. The annihilation and spacelike penguin effects are expected to be negligible if we insist on the significant form-factor suppression associated with them.[3] Then the overall decay amplitudes can be calculated, by use of the QCD-improved effective weak Hamiltonian and factorization approximation, in a renormalization-scheme independent way [11,12]. Instead of repeating the technical details of such a treatment, here we only write out the resultant expressions of S_1^u and S_1^c in the assumptions made above:

$$
\begin{aligned}
S_1^u \propto & \left(\frac{\overline{c_3}}{3} + \overline{c_4} + \frac{\overline{c_9}}{3} + \overline{c_{10}} \right) + \left(\frac{\overline{c_5}}{3} + \overline{c_6} + \frac{\overline{c_7}}{3} + \overline{c_8} \right) \xi_c \\
& + \frac{1+\xi_c}{9\pi} \left[\overline{c_2} \alpha_s + \left(\overline{c_1} + \frac{\overline{c_2}}{3} \right) \alpha_e \right] \left[\tfrac{10}{9} + F_u(k^2) \right], \\
S_1^c \propto & \left(\frac{\overline{c_1}}{3} + \overline{c_2} + \frac{\overline{c_3}}{3} + \overline{c_4} + \frac{\overline{c_9}}{3} + \overline{c_{10}} \right) \\
& + \left(\frac{\overline{c_5}}{3} + \overline{c_6} + \frac{\overline{c_7}}{3} + \overline{c_8} \right) \xi_c \\
& + \frac{1+\xi_c}{9\pi} \left[\overline{c_2} \alpha_s + \left(\overline{c_1} + \frac{\overline{c_2}}{3} \right) \alpha_e \right] \left[\tfrac{10}{9} + F_c(k^2) \right],
\end{aligned}
\tag{A2}
$$

where the common hadronic matrix element $\langle D^+ | (\overline{c}d)_{V-A} | 0 \rangle \langle \overline{D^0} | (\overline{b}c)_{V-A} | B_u^+ \rangle$ has been singled out from S_1^u and S_1^c. In Eq. (A2), α_s and α_e are the strong and electroweak coupling constants, respectively; $\overline{c_i}$ stands for the renormalization-scheme-independent Wilson coefficient; $\xi_c = 2m_{D^+}^2 / [m_c(m_b - m_c)]$ arises from the transformation of $(V-A)(V+A)$ currents into $(V-A)(V-A)$ ones for Q_{5-8}; and $F_q(k^2)$ denotes the penguin loop-integral function with momentum transfer k at the scale $\mu = O(m_b)$:

$$
F_q(k^2) = 4 \int_0^1 dx x(1-x) \ln \left[\frac{m_q^2 - k^2 x(1-x)}{m_b^2} \right]. \tag{A3}
$$

The absorptive part of $F_q(k^2)$ emerges for $k^2 \geqslant 4m_q^2$, leading to the possibility of direct CP violation [10].

One can calculate S_1^u and S_1^c for the decay modes $B_u^+ \to D^{*+} \overline{D^0}$, $D^+ \overline{D^{*0}}$, and $D^{*+} \overline{D^{*0}}$ using the same factorization approximation. If the polarizations of final-state vector mesons are summed over, we arrive at the same formulas as Eq. (A2) with $\xi_c = 0$ for $D^{*+} \overline{D^0}$, $\xi_c = -2m_{D^+}^2 / [m_c(m_b + m_c)]$ for $D^+ \overline{D^{*0}}$, and $\xi_c = 0$ for $D^{*+} \overline{D^{*0}}$. Of course, such results depend upon the assumptions made above and cannot be taken too seriously.

With the help of Eqs. (A2) and (A3), one is able to evaluate the CP asymmetry $\mathcal{A}_{\pm 0}$ defined in Eq. (15) and the phase parameter φ given in Eq. (23). For illustration, we typically choose $m_u = 5$ MeV, $m_c = 1.35$ GeV, $m_b = 5$ GeV, and $m_t = 174$ GeV. The Wolfenstein parameters are taken to be $\lambda = 0.22$, $A = 0.81$, $\rho = 0.05$, and $\eta = 0.36$. We adopt values of the Wilson coefficients $\overline{c_i}$ obtained in Ref. [13]. The unknown penguin momentum transfer k^2 is treated as a free parameter changing from $0.01 m_b^2$ to m_b^2. A few points can be drawn from the explicit numerical calculations.

(a) The QCD (gluonic) penguin effect plays the dominant role in the overall penguin amplitude, while the electroweak penguin effect is negligibly small. At $k^2 = 4m_c^2 \approx 0.3 m_b^2$ both $\mathcal{A}_{\pm 0}$ and $\tan(2\varphi)$ undergo a remarkable change in magnitude.

(b) The CP asymmetries $\mathcal{A}_{\pm 0}$ between $B_u^+ \to D^{(*)+} \overline{D^{(*)0}}$ and $B_u^- \to D^{(*)-} D^{(*)0}$ have the same sign and are of the order 3%. The relative change of each asymmetry due to the uncertain penguin momentum transfer k^2 is less than 15%.

(c) With the inputs listed above, the phase parameter φ is estimated to be around $-3°$. Considering the large uncertainties associated with the inputs and the approach itself, we believe that a significant deviation of φ from zero (e.g., $\varphi \sim -10°$) cannot be excluded even within the standard model.

(d) Observation of the above CP-violating signals to three standard deviations needs about 10^8 $B_u^{\pm}$ events, if the composite detection efficiency is at the 10% level. Such measurements are possible in the first-round experiments at the forthcoming B factories.

In the case that the decay channels $B_d \to D^{(*)0} \overline{D^{(*)0}}$ are significantly suppressed, we expect that the direct CP asymmetries in $B_d \to D^{(*)+} D^{(*)-}$ are comparable in magnitude with those in $B_u^+ \to D^{(*)+} \overline{D^{(*)0}}$ vs $B_u^- \to D^{(*)-} D^{(*)0}$ [see Eq. (19) for illustration]. Nevertheless, much more $B_d^0 \overline{B_d^0}$ events are needed to detect the former on the $\Upsilon(4S)$ resonance due to the cost for flavor tagging. Measurement of direct CP-violating signals in $B_d \to D^{(*)+} D^{(*)-}$ in the second-round experiments of a B-meson factory is likely.

[3]However, one should keep in mind that such an argument may not be on solid ground and has to be examined after some theoretical (experimental) progress is made in a deeper understanding of the dynamics of nonleptonic B decays.

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