

## Magnetic field spectrum in a plasma in thermal equilibrium in the epoch of primordial nucleosynthesis

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The low-frequency magnetic field spectrum in the primordial plasma is of particular interest as a possible origin of magnetic fields in the universe (e.g., Tajima and co-workers and Cable and Tajima). We derive the magnetic field spectrum in the primordial plasma, in particular, at the epoch of primordial nucleosynthesis. The pioneering study of Cable and Tajima of the electromagnetic fluctuations, based on the *fluctuation-dissipation theorem*, is extended. Our model describes both the thermal and collisional effects in a plasma. It is based on a kinetic description with the Bhatnagar, Gross, and Krook (BGK) collision term. It is shown that the zero-frequency peak found by Cable and Tajima decreases. At high frequencies, the blackbody spectrum is obtained naturally without the necessity of the link procedure used by them. At low frequencies ( $\omega \leq 4\omega_{pe}$ , where  $\omega_{pe}$  is the electron plasma frequency), it is shown that the magnetic field spectrum has more energy than the blackbody spectrum in vacuum. [S0556-2821(97)02318-7]

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### I. INTRODUCTION

Although plasma is the main constituent of the primordial universe, very few previous studies deal directly with plasma phenomena. The effect of a plasma in cosmology normally has been studied with respect to the origin of the magnetic field. For example, the study of Harrison [1] elaborates a model of the origin of the magnetic field due to turbulence in the primordial plasma. There have been studies [2] that analyze the effect of a magnetic field on primordial nucleosynthesis. Other studies, such as that of Halcomb and Tajima [3], deduced the dispersion relation of waves, taking into account the expansion of the universe. There still lacks a general study of plasma phenomena related to cosmology.

A plasma in thermal equilibrium sustains fluctuations of the magnetic field (even for a nonmagnetized plasma). This study is concerned with the study of the magnetic field spectrum in the primordial plasma.

The electromagnetic fluctuations in a plasma have been studied in numerous works, including those of Dawson [4], Rostoker, Aamodt, and Eldridge [5], Sitenko and Gurin [6], and Akhiezer, Akhiezer, and Sitenko [7]. Most of the results are compiled in the books of Sitenko and Akhiezer *et al.* [8,9]. Little attention has been given to the question of how the magnetic field spectrum looks in a plasma. A naive answer to this question might be that it is a blackbody spectrum with a cutoff at the plasma frequency, knowing that photons only propagate in a plasma for  $\omega > \omega_p$ , where  $\omega_p$  is the plasma frequency. This is not true, however, due to the magnetic fluctuations of the plasma.

Cable and Tajima and Tajima and co-workers [10–12] performed a broad study of the magnetic field fluctuations in a plasma. They based their analyses on the *fluctuation-dissipation theorem*. They were concerned, in particular,

with the low-frequency spectrum of fluctuations, because, as they pointed out, no expression exists.

The fluctuation-dissipation theorem [8,9] predicts the intensity of electromagnetic fluctuations. The intensity of such fluctuations is highly dependent on how the plasma is described, in particular, on the dissipation mechanisms used.

Cable and Tajima and Tajima and co-workers [10–12] studied the magnetic field fluctuations for several cases. Two of their descriptions concern the primordial plasma which we are interested in, which is an isotropic, nonmagnetized, and nondegenerate plasma: (a) a cold, gaseous plasma and (b) a warm, gaseous plasma described by kinetic theory.

In their study, Cable and Tajima (CT) in case (a) used the *cold plasma* description with a constant collision frequency. In case (b) they analyzed the spectrum of fluctuations only for low frequencies, with the *warm plasma* description for phase velocity  $\omega/k$  less or equal to the thermal velocity of the electrons  $v_e$  and the ions  $v_i$  in a collisionless description.

For the cold plasma description the spectrum that they obtain has a large zero-frequency peak. As the frequency is increased, the spectrum first drops below the blackbody spectrum in vacuum, then becomes the blackbody spectrum at high frequencies. In case (b), for the warm plasma description, the analysis was made only for the low-frequency regime and they argued that the zero-frequency peak is present as well. They argue that the energy contained in the peak is approximately equal to the energy *lost* by the plasma cutoff effect.

In order to obtain a correct magnetic field spectrum, it is necessary to describe the plasma in the most complete way possible, taking into account thermal and collisional effects in a unified description.

In this study we extend the pioneering work of CT, presenting a model that includes, in a unified description, collisional and thermal effects. Our model is based on kinetic theory incorporating thermal effects for all frequencies and wave numbers (not only for  $\omega/k \leq v_e, v_i$ ). In order to describe the collisions that exist in the plasma, we used a model collision term. This collision term describes binary

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collisions, as used in the work by CT. In this way, we extend the previous model describing thermal and collisional effects for all frequencies and wave numbers. Their description, the cold plasma and the warm plasma description in the collisionless case, are special cases of this model. [A pure collisionless treatment is unreal, such as case (b) extended to all frequencies, since if there were no collisions, then only Cherenkov emission could produce fluctuations, and there are no particles traveling fast enough to emit light waves.]

However, for a fully ionized plasma as is our case, a treatment that takes into account collisions in a more complete way is necessary. Our model, an extension of the CT model, describes the basic features of a kinetic description.

We present in Sec. II the general expressions of the magnetic field fluctuations based on the fluctuation-dissipation theorem. We review the cold plasma description in Sec. II A and the warm plasma description in the collisionless case in Sec. II B. In Sec. II C we present a general discussion and criticism of the assumptions made by CT. In Sec. III we present our model. Finally, in Sec. IV, we discuss the results and present our conclusions.

## II. MAGNETIC FIELD FLUCTUATIONS

The spectrum of fluctuations of the electric field in a plasma, given by the fluctuation-dissipation theorem (for the deduction of the fluctuation-dissipation theorem from the general relation of fluctuations in a plasma see [8,9]) is

$$\frac{1}{8\pi} \langle E_i E_j \rangle_{\mathbf{k}\omega} = \frac{i}{2} \frac{\hbar}{e^{\hbar\omega/T} - 1} (\Lambda_{ji}^{-1} - \Lambda_{ij}^{-1*}), \quad (2.1)$$

where

$$\Lambda_{ij}(\omega, \mathbf{k}) = \frac{k^2 c^2}{\omega^2} \left( \frac{k_i k_j}{k^2} - \delta_{ij} \right) + \varepsilon_{ij}(\omega, \mathbf{k}), \quad (2.2)$$

where  $\varepsilon_{ij}(\omega, \mathbf{k})$  is the dielectric tensor of the plasma. For an isotropic plasma,

$$\Lambda_{ij} = \frac{k_i k_j}{k^2} \varepsilon_L + \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \left( \varepsilon_T - \frac{k^2 c^2}{\omega^2} \right), \quad (2.3)$$

where  $\varepsilon_L$  and  $\varepsilon_T$  are, respectively, the longitudinal and transverse dielectric permittivities of the plasma. In this case [8],

$$\langle E_i E_j \rangle_{\mathbf{k}\omega} = 8\pi \frac{\hbar}{e^{\hbar\omega/T} - 1} \left\{ \frac{k_i k_j}{k^2} \frac{\text{Im}\varepsilon_L}{|\varepsilon_L|^2} + \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{\text{Im}\varepsilon_T}{\left| \varepsilon_T - \left( \frac{kc}{\omega} \right)^2 \right|^2} \right\}. \quad (2.4)$$

Using the fact that  $\mathbf{B}_{\mathbf{k}\omega} = (c/\omega) \mathbf{k} \times \mathbf{E}_{\mathbf{k}\omega}$ , we have the expression for the magnetic field fluctuations:

$$\langle B_i B_j \rangle_{\mathbf{k}\omega} = 8\pi \frac{\hbar}{e^{\hbar\omega/T} - 1} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \left( \frac{kc}{\omega} \right)^2 \frac{\text{Im}\varepsilon_T}{\left| \varepsilon_T - \left( \frac{kc}{\omega} \right)^2 \right|^2}. \quad (2.5)$$

So,

$$\frac{\langle B^2 \rangle_{\mathbf{k}\omega}}{8\pi} = 2 \frac{\hbar}{e^{\hbar\omega/T} - 1} \left( \frac{kc}{\omega} \right)^2 \frac{\text{Im}\varepsilon_T}{\left| \varepsilon_T - \left( \frac{kc}{\omega} \right)^2 \right|^2}. \quad (2.6)$$

An intuitive way to understand the above expression, is that the fluctuation-dissipation theorem takes into account the emission and absorption processes in a plasma, and knowing that in equilibrium they are equal, the fluctuation level is evaluated. As Tajima *et al.* point out [12], ‘‘an individual mode decays by a certain dissipation, giving up energy to particles or other modes, while particles (or other modes) excite new modes and repeat the process and the amount of fluctuations is related to the dissipation.’’

In short, to determine the fluctuations of the magnetic field in a plasma in equilibrium or quasiequilibrium, it is sufficient to know the transverse dielectric permittivity of the plasma, in particular, the dissipation mechanisms present for each frequency and wave number ( $\text{Im}\varepsilon_T$ ). This depends on the treatment used to describe the plasma.

Another important feature of the magnetic field spectral distribution can be seen from Eq. (2.6). The equation

$$\varepsilon_T(\omega, \mathbf{k}) - \left( \frac{kc}{\omega} \right)^2 = 0 \quad (2.7)$$

determines the transverse eigenfrequencies of the plasma. Therefore, the magnetic field spectral distribution has steep maxima at the frequencies that correspond to the transverse plasma eigenfrequencies. In the transparency region ( $\text{Im}\varepsilon_T \ll \text{Re}\varepsilon_T$ ), the magnetic spectrum has  $\delta$ -function-like maxima near the eigenfrequencies (i.e., the frequency spectrum of the fluctuations contains only the transverse eigenfrequencies in the plasma [9]). Knowing that the photons have the dispersion relation in the plasma  $\omega^2 = \omega_p^2 + k^2 c^2$ , we note that for frequencies  $\omega \gg \omega_p$  the eigenfrequencies manifest themselves and the magnetic field spectrum behaves like a blackbody spectrum.

### A. Cold plasma description

The cold plasma description does not take into account the thermal movement of the electrons, describing them by fluid equations. In such a description, collisionless damping is not included. To include collisions in this case is straightforward, being only necessary to add a new term  $\propto \eta v$  in the fluid equations, where  $\eta$  is the collision frequency and  $v$  the velocity of the particles. We follow here the CT study.

CT used a multifluid model of the plasma (neglecting the  $\mathbf{v} \times \mathbf{B}$  forces due to the smallness of the velocities and the electromagnetic fields),

$$m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = e_\alpha \mathbf{E} - \eta_\alpha m_\alpha \mathbf{v}_\alpha, \quad (2.8)$$

where  $\alpha$  is a particle species label and  $\eta_\alpha$  is the collision frequency of species  $\alpha$ . From the above equation (performing a Fourier transformation and rearranging the terms), the dielectric tensor can be obtained:

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega(\omega + i\eta_{\alpha})} \delta_{ij}. \quad (2.9)$$

CT studied the case of an electron-positron plasma. This is the plasma that dominated the universe in the beginning of primordial nucleosynthesis, at  $T \sim 1 \text{ MeV}$ . In the case of an electron-positron plasma,  $\omega_{pe^+}^2 = \omega_{pe^-}^2$  and  $\eta_{e^+} = \eta_{e^-} = \eta$ . Equation (2.9) then becomes

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} - \frac{\omega^2}{\omega(\omega + i\eta)} \delta_{ij}, \quad (2.10)$$

where  $\omega_p^2 = \omega_{pe^+}^2 + \omega_{pe^-}^2$ .  $\eta_e$  was taken as the Coulomb collision frequency,  $\eta_e = 2.91 \times 10^{-6} n_e \ln \Lambda T^{-3/2} (\text{eV}) s^{-1}$ , where  $n_e$  is the electron (positron) density. [For the case of an electron-proton plasma, also treated by CT,  $\eta_p = 4.78 \times 10^{-18} n_e \ln \Lambda T^{-3/2} (\text{eV}) s^{-1}$ ]. That is, this collision frequency describes the binary collisions in a plasma.

For an isotropic plasma, the transverse dielectric permittivity is

$$\varepsilon_T(\omega, \mathbf{k}) = 1 - \frac{\omega_p^2}{\omega(\omega + i\eta)}. \quad (2.11)$$

Substituting Eq. (2.11) in Eq. (2.6), CT obtained the magnetic field spectrum in the cold plasma description. If relativistic temperature effects are included, the substitution  $\omega_p \rightarrow \omega_p / \sqrt{\gamma}$  is made.

The magnetic field spectrum  $\langle B^2 \rangle_{\omega}$  is found by integrating  $\langle B^2 \rangle_{k\omega}$  over wave numbers  $k$  [and dividing by  $(2\pi)^3$ ]. The  $\langle B^2 \rangle_{\omega}$  diverges for high wave numbers. CT deal with this problem, breaking the integration on wave number into two intervals. One interval runs from  $|\mathbf{k}|=0$  to  $|\mathbf{k}|=k_{\text{cut}}$ . The other interval runs from  $|\mathbf{k}|=k_{\text{cut}}$  to  $|\mathbf{k}|=\infty$ . In the first interval, they keep the collision frequency  $\eta$  finite and in the second interval they let  $\eta \rightarrow 0$  and drop the low-frequency part of the spectrum. The final expression obtained was

$$\begin{aligned} \frac{\langle B^2 \rangle_{\omega}}{8\pi} &= \frac{1}{\pi^2} \frac{\hbar \omega'}{e^{(\hbar \omega_{pe}/T)\omega' - 1}} 2 \eta' \left( \frac{\omega_{pe}}{c} \right)^3 \\ &\times \int_0^{x_{\text{cut}}} dx \frac{x^4}{(\omega'^2 + \eta'^2)x^4 + \dots} \\ &+ \frac{\hbar (\omega'^2 - \omega_p'^2)^{3/2}}{2\pi (e^{(\hbar \omega_{pe}/T)\omega' - 1})} \left( \frac{\omega_{pe}}{c} \right)^3 \Theta[\omega - \omega_{\text{hev}}], \end{aligned} \quad (2.12)$$

where  $\Theta$  is the Heaviside step function,  $\omega' = \omega/\omega_{pe}$ ,  $\omega_p' = \omega_p/\omega_{pe}$ ,  $\eta' = \eta/\omega_{pe}$  and  $x = kc/\omega_{pe}$ . The first term extends up to the frequency  $\omega_{\text{hev}} = \sqrt{k_{\text{cut}}^2 c^2 + \omega_p^2}$ . The second term is the high-frequency and high wave-number expression (i.e., the spectrum for frequencies  $\omega \geq \omega_{\text{hev}}$ ).

The justification given to this breakup procedure was that  $\eta$  should vanish smoothly as  $k \rightarrow \infty$  and as long as the results “do not critically depend on the manner in which  $\eta$  approaches zero” the abrupt cutoff should be acceptable as a crude model. No strong justification was given to the choice of  $k_{\text{cut}}$ . They chose  $x_{\text{cut}} \equiv k_{\text{cut}} c / \omega_{pe} \approx 1$ , mainly because this is the value that makes the frequency spectrum smooth at the joining of the low-frequency and the blackbody spectrum [10,12].

## B. Warm plasma description

The warm plasma description describes the plasma based on the kinetic theory that takes into account the thermal distribution of the particles. In this description collisionless damping, such as Landau damping, appears. The kinetic theory is based on the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy equations that describe a system of many particles. These equations are solved by expanding the distribution function of the many particles in terms of the plasma parameter  $g = 1/n\lambda_D^3$ , where  $\lambda_D$  is the Debye length and  $n$  is the particle density. The description that is used usually is the collisionless description, based on the Vlasov equation that does not take into account collisions (and neglecting  $\mathbf{v} \times \mathbf{B}$  forces):

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q}{m} \mathbf{E} \cdot \nabla_{\mathbf{v}} \right) f(\mathbf{x}, \mathbf{v}, t) = 0, \quad (2.13)$$

where  $\nabla \equiv \partial/\partial \mathbf{x}$  and  $\nabla_{\mathbf{v}} \equiv \partial/\partial \mathbf{v}$ . The Vlasov equation in first order (in  $g$ ) takes into account collisions, where the term on the right-hand side of the equation is now the *collision term*:

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q}{m} \mathbf{E} \cdot \nabla_{\mathbf{v}} \right) f(\mathbf{x}, \mathbf{v}, t) = \left( \frac{\partial f}{\partial t} \right)_C. \quad (2.14)$$

For the collisionless description (assuming an isotropic plasma), the transverse dielectric permittivity is obtained from Eq. (2.13). Assuming that the particles have a Maxwellian velocity distribution, the transverse dielectric permittivity is [13–16]

$$\begin{aligned} \varepsilon_T(\omega, \mathbf{k}) &= 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left\{ \phi \left( \frac{\omega}{\sqrt{2} k v_{\alpha}} \right) - i \left( \frac{\pi}{2} \right)^{1/2} \frac{\omega}{k v_{\alpha}} \right. \\ &\quad \left. \times \exp \left( - \frac{\omega^2}{2 k^2 v_{\alpha}^2} \right) \right\}, \end{aligned} \quad (2.15)$$

where  $\alpha$  is the label for each species of the plasma and  $v_{\alpha}$  is the thermal velocity for each species,  $v_{\alpha} = \sqrt{T/m_{\alpha}}$ .  $z = \omega/\sqrt{2} k v_{\alpha}$  and  $\phi(z) = 2z e^{-z^2} \int_0^z e^{x^2} dx$ .

In order to investigate thermal effects, CT used the collisionless description (i.e., the Vlasov equation in the regime where  $\omega/k$  is less than the thermal speed of the plasma constituents). They studied a hydrogen plasma; thus the region investigated was  $\omega/k \leq v_e, v_i$  where  $v_e = \sqrt{T/m}$  and  $v_i = \sqrt{T/M}$ . [The extension to an electron-positron plasma, as in case (a), is straightforward:  $M \rightarrow m$  so  $v_i \rightarrow v_{e^+}$ ]. They used, therefore, Eq. (2.15) in the limit  $z \ll 1$  and expanded the plasma dispersion function  $\phi(z)$ . Substituting the approxi-

mate expression of  $\varepsilon_T(\omega, \mathbf{k})$  (expanded in the limit of  $z \ll 1$ ) in Eq. (2.6), they obtained  $\langle B^2 \rangle_{k\omega}/8\pi$ .

CT examined only the low-frequency behavior of  $\langle B^2 \rangle_\omega$ , noting that it diverges for high wave numbers and a cutoff procedure is necessary. They assumed the same upper limit as in case (a),  $x_{\text{upper}} = x_{\text{cut}} \cong 1$ .

### C. Discussion and criticisms

For the cold plasma description, used in case (a), CT obtained a large zero-frequency peak in the magnetic field fluctuation spectrum (arguing that the warm plasma description exhibits it as well).

The total spectrum is obtained with the link of the low-frequency term to the high-frequency term [Eq. (2.12)]. The behavior of the spectrum can be seen in Fig. 1(a). We study an electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30}$  cm<sup>-3</sup>. This is the plasma in the beginning of primordial nucleosynthesis. The magnetic field spectrum  $S(\omega) \equiv \langle B^2 \rangle_\omega / 8\pi$  is divided by a normalization  $S_0 = \omega_{pe}^2 k_B T / c^3$ . The dashed curve is the first term of Eq. (2.12), the low-frequency spectrum. The dash-dot-dash curve is the second term of Eq. (2.12), the high-frequency spectrum (obtained with  $\eta \rightarrow 0$ ). We note the link point used between the two curves. The blackbody spectrum in vacuum is also plotted (solid curve). It can be seen that the general behavior is that after the peak, with increasing frequency, the spectrum drops below the blackbody spectrum. At high frequencies, it merges into the blackbody spectrum. In Fig. 1(b) we plotted only the cold plasma spectrum for low and high frequencies. In Fig. 1(c) is an electron-proton plasma at  $T = 10^9$  K and  $n_e = 5.4 \times 10^{26}$  cm<sup>-3</sup>. In the epoch of primordial nucleosynthesis, at lower temperatures, the electrons and positrons annihilate and the plasma is reduced to a plasma of protons and electrons. In Fig. 1(d) we used the same plasma as in Fig. 1(c), but plotted only the cold plasma spectrum for low and high frequencies.

CT argued that the results do not critically depend on the upper limit. We show below that this is not true. As CT noted, the divergence occurs due to the subtle interaction between matter and radiation in small scales. CT used a classical fluid equation with a constant collision frequency (Coulomb collision frequency). This collision frequency describes the binary interactions in the plasma. They chose the cutoff  $x_{\text{cut}} \cong 1$ , basically because this is the value that makes the frequency spectrum smooth at the joining of the low-frequency and the blackbody spectrum.

In Sec. VII of CT, they gave a quantum-mechanical justification of not extending  $k$  beyond  $k_{\text{cut}}$ . They argue that for  $(\hbar k)^2 / 2m \gg k_B T$  the plasma has a negligible effect on the electromagnetic spectrum. Let us call this  $k$ ,  $k_{\text{lim}} [(\hbar k_{\text{lim}})^2 / 2m = k_B T]$ .

When treating Coulomb collisions, a cutoff has to be taken, since at small distances the energy of the Coulomb interactions of the particles exceeds their kinetic energy which violates the applicability of the condition of the perturbation expansion (in the plasma parameter  $g \ll 1$ ). This occurs approximately for distances  $r_{\text{min}} \sim e^2 / T$  or, more exactly, for the distance of closest approach between a test particle and an electron in a plasma,  $k_{\text{max}} = r_{\text{min}} \cong M m v^2 / (m + M) |e q|$ , where  $M$ ,  $v$ , and  $q$  are, respectively, the mass,

velocity, and charge of the test particle [15].

Comparing the value of  $k_{\text{cut}}$ ,  $k_{\text{lim}}$ , and  $k_{\text{max}}$  we see that  $k_{\text{cut}}$  ( $x_{\text{cut}} = k_{\text{cut}} c / \omega_{pe} \cong 1$ ) is much smaller than the others. For example, for the cases that CT used, (1)  $T = 10^{10}$  K,  $n_e = 4.8 \times 10^{30}$  cm<sup>-3</sup>, (2)  $T = 10^6$  K,  $n_e = 6.5 \times 10^9$  cm<sup>-3</sup>, and (3)  $T = 10^4$  K,  $n_e = 6.5 \times 10^3$  cm<sup>-3</sup>, we found (1)  $x_{\text{lim}} \cong k_{\text{lim}} c / \omega_{pe} = 11.5$  and  $x_{\text{max}} \cong k_{\text{max}} c / \omega_{pe} = 2444.4$ , (2)  $x_{\text{lim}} = 3.1 \times 10^9$  and  $x_{\text{max}} = 6.6 \times 10^9$ , and (3)  $x_{\text{lim}} = 3.1 \times 10^{10}$  and  $x_{\text{max}} = 6.6 \times 10^{11}$ .

As  $x_{\text{cut}}$  is quite arbitrary, we decided to vary the upper limit in the first term in Eq. (2.12), until  $x_{\text{upper}} = x_{\text{max}}$  to observe the behavior of the curves. This is shown in Fig. 2 where again we plot  $S(\omega)/S_0$  vs  $\omega/\omega_{pe}$ .

We plotted the low-frequency spectrum term not only until  $\omega_{hev} = \sqrt{k_{\text{cut}}^2 c^2 + \omega_p^2}$ , as CT did, but extended it to higher frequencies. The dash-dot-dash curve has the upper limit  $x_{\text{upper}} = x_{\text{cut}}$  as CT used. The dotted curve has the upper limit  $x_{\text{upper}} = 2x_{\text{cut}}$ ; the dashed curve has the upper limit  $x_{\text{upper}} = 5x_{\text{cut}}$ ; and the dash-two dot-dash curve has the upper limit  $x_{\text{upper}} = x_{\text{max}}$ . We also plotted the blackbody spectrum in vacuum (solid curve). It can be seen that by extending the upper limit to higher and higher values we obtain more and more of the blackbody spectrum, before the spectrum drops. This is easy to understand since, as we noted before, the magnetic field spectrum, obtained from the fluctuation-dissipation theorem, contains the transverse eigenfrequencies of the plasma, the photons. When we extend the value of the upper limit, we permit higher eigenfrequencies to manifest themselves. This occurs up to the frequency when, due to the upper limit chosen, no more photons can manifest themselves.

Another feature that appears is that by increasing the upper limit the valley that appears, due to the plasma cutoff effect, becomes smaller and smaller. Eventually, for  $x_{\text{upper}} = x_{\text{max}}$ , we obtain a large peak for  $\omega \sim 0$  but without any valley. In fact, for frequencies  $\omega \leq 2\omega_{pe}$  the curve is above the blackbody spectrum in vacuum. This can be seen in Fig. 3(a), where the dashed curve is the low-frequency term [in Eq. (2.12)] extended to high frequencies (not only for  $\omega \leq \omega_{hev}$ ) with  $x_{\text{upper}} = x_{\text{max}}$  compared to the blackbody spectrum in vacuum (solid curve). In Fig. 3(b) we plotted only the cold plasma curve, extended to high frequencies, showing that with  $x_{\text{upper}} = x_{\text{max}}$  the full blackbody spectrum is reproduced.

CT argued, based on the behavior of the cold plasma spectrum with  $x_{\text{upper}} = x_{\text{cut}}$ , that the energy under the  $\omega \sim 0$  peak is approximately equal to the energy stolen from the blackbody spectrum due to the plasma cutoff. In their words, this happens because the ‘‘plasma squeezes the fluctuation energy of modes with frequency less than  $\omega_p$  into modes with frequencies very close to zero.’’

First of all, clearly, this does not happen if the upper limit is high enough in order to reproduce the full blackbody spectrum at high frequencies (with  $x_{\text{upper}} = x_{\text{max}}$ ) because the entire curve is above the blackbody spectrum. Second, we noted previously that a blackbody (photon) spectrum with a cutoff  $\omega < \omega_p$  is expected taking into account that the photons only propagate for  $\omega > \omega_p$  in a plasma, where  $\omega_p$  is the plasma frequency. However, when we speak about blackbody, we speak about modes which propagate, in our case,

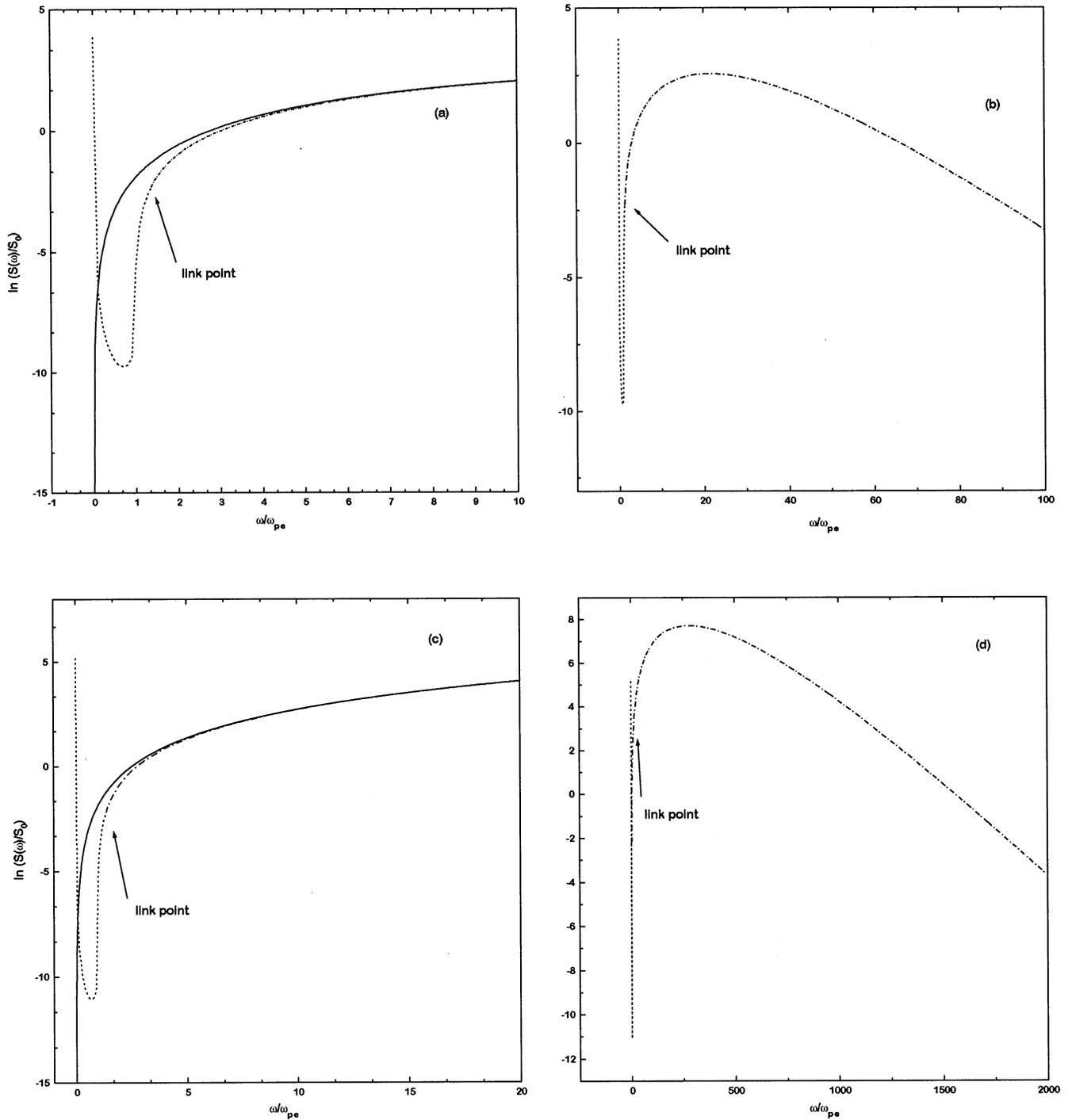


FIG. 1. The magnetic field spectrum  $\ln[S(\omega)/S_0]$  vs  $\omega/\omega_{pe}$  [where  $S(\omega) = \langle B^2 \rangle \omega / 8\pi$  and  $S_0 = \omega_{pe}^2 k_B T / c^3$  is the normalization] for the cold plasma description with  $x_{\text{upper}} = x_{\text{cut}} \cong 1$  for (a) the electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30} \text{ cm}^{-3}$  [the dashed curve is the cold plasma spectrum for low frequencies, the dash-dot-dash curve is the cold plasma spectrum for high frequencies (the link point between the two curves is indicated), and the solid curve is the blackbody spectrum in vacuum]; (b) the same as case (a), where the cold plasma spectrum is plotted extended to high frequencies; (c) the same as case (a) but for an electron-proton plasma at  $T = 10^9$  K and  $n_e = 5.4 \times 10^{26} \text{ cm}^{-3}$ ; and (d) the same as case (b) but for an electron-proton plasma at  $T = 10^9$  K and  $n_e = 5.4 \times 10^{26} \text{ cm}^{-3}$ .

photons. In vacuum, the photons have a dispersion relation  $\omega = kc$  and they are present in the entire frequency spectrum. In a plasma, when the dispersion relation is  $\omega^2 = \omega_p^2 + k^2 c^2$ , they only appear for  $\omega > \omega_p$ .

Dawson [4] deduced the radiation spectrum in a plasma by a gedanken experiment. A slab of plasma at a temperature

$T$  is put between two blackbodies at temperature  $T$ . Between the plasma and the blackbodies are vacuum regions. Radiation is emitted by the blackbodies and enters the plasma. In equilibrium, the plasma radiates the same amount of radiation that it absorbs. Dawson deduced the density of radiation in a plasma as

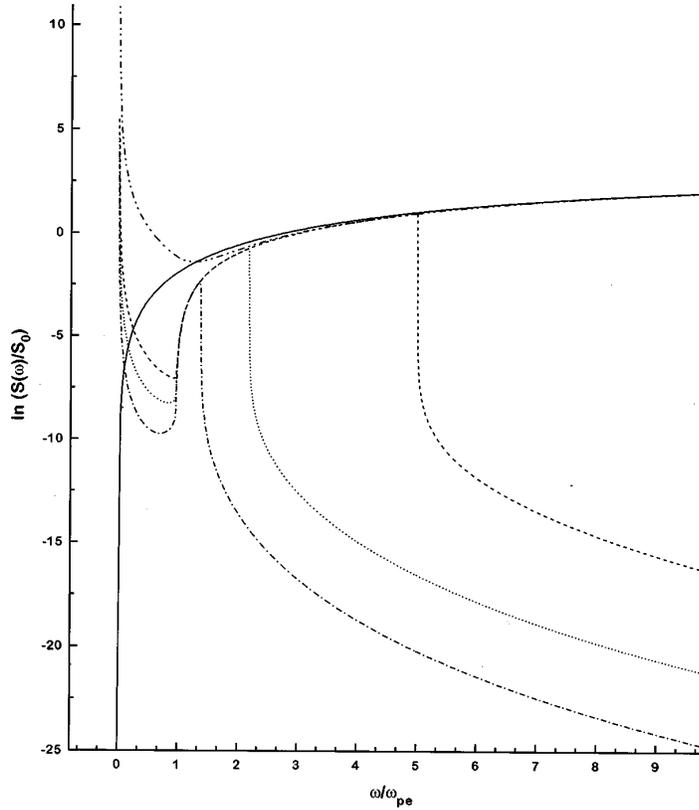


FIG. 2. The magnetic field spectrum  $\ln[S(\omega)/S_0]$  vs  $\omega/\omega_{pe}$  for the cold plasma description for various upper limits [where  $S(\omega) = \langle B^2 \rangle_\omega / 8\pi$  and  $S_0 = \omega_{pe}^2 k_B T / c^3$  is the normalization]. The plasma is an electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30}$   $\text{cm}^{-3}$ . The dash-dot-dash curve is the spectrum with  $x_{\text{upper}} = x_{\text{cut}}$ ; the dotted curve is the spectrum with  $x_{\text{upper}} = 2x_{\text{cut}}$ ; the dashed curve is the spectrum with  $x_{\text{upper}} = 5x_{\text{cut}}$ ; and the dash-double dot-dash curve is the spectrum with  $x_{\text{upper}} = x_{\text{max}}$ . The solid curve is the blackbody spectrum in vacuum.

$$U_p = \left( 1 - \frac{\omega_p^2}{\omega^2} \right) U_v, \quad (2.16)$$

where  $U_p$  is the radiation density in the plasma and  $U_v$  the blackbody radiation density in the vacuum. It can be seen that a new factor appears, due to the presence of the plasma. This kind of gedanken experiment tells us, however, only about modes that propagate. Nothing is told about the modes that do not propagate which appear due to correlations in the plasma. They are present at low frequencies,  $\omega < \omega_p$ , but they also are present at  $\omega > \omega_p$ , contributing, in addition to the photons, to the magnetic field spectrum. Our results show that this happens only for very high frequencies ( $\omega \gg \omega_p$ ) when the photons dominate the magnetic field spectrum. The magnetic field fluctuation spectrum has little to do with the photon blackbody spectrum in vacuum for  $\omega < \omega_p$ . In principle, the magnetic field spectrum can be greater, or less, than the photon blackbody spectrum in vacuum for  $\omega < \omega_p$ . The only manner to obtain the magnetic field spectrum is analyzing the magnetic field fluctuations from, for example, the fluctuation-dissipation theorem. Using the cold plasma description, for example, with the cutoff  $x_{\text{upper}} \sim x_{\text{max}}$ , we find that the magnetic spectrum has more energy than the blackbody photon spectrum in vacuum.

### III. OUR MODEL

In order to extend the work of CT and to have a more complete description of the plasma, we desire a model that includes thermal effects as well as collisional effects. For this, we need a kinetic description that takes into account

collisions. We used the Vlasov equation in first order. This equation gives collisional corrections to the Vlasov equation in zero order (collisionless case). It retains terms of order  $g$  and neglects terms of higher order. As the plasma parameter  $g$  is much less than unity (for example, as pointed out by [12], in the epoch of  $t = 10^{-2} - 10^0$  s in the primordial universe,  $g \cong 10^{-3}$ ), this is a good approximation as terms of higher order are much smaller. As noted before, the term on the right-hand side of Eq. (2.14),  $(\partial f / \partial t)_C$ , is the *collision term*. Obtaining  $(\partial f / \partial t)_C$  is a matter of great difficulty and different forms are required for various types of collisions, such as electron-electron, electron-neutral molecule, etc. Most of the expressions for  $(\partial f / \partial t)_C$  involve integral functionals of the distribution function  $f$ . The basic difficulty in taking into account the effect of collisions lies in the complexity of the solution of the kinetic equation when the correct collision integral is used. The problem can be simplified if a *model* collision term is used, that is, an approximate expression.

The Boltzmann collision term takes into account all the possible binary collisions which the particle under observation might suffer. It is applied to weakly ionized plasma, when the scattering of charged particles by neutrals is predominant. In a fully ionized plasma (as in our case), the collisions are not predominantly binary and  $(\partial f / \partial t)_C$  for a *test* particle does not derive mainly from the possibility that other particles approach very closely and abruptly deflect it. The cumulative effect of more distant particles is more important. The charged particles simultaneously interact with all the other particles in the Debye sphere, which is a large number. The Fokker-Planck collision term is appropriate for

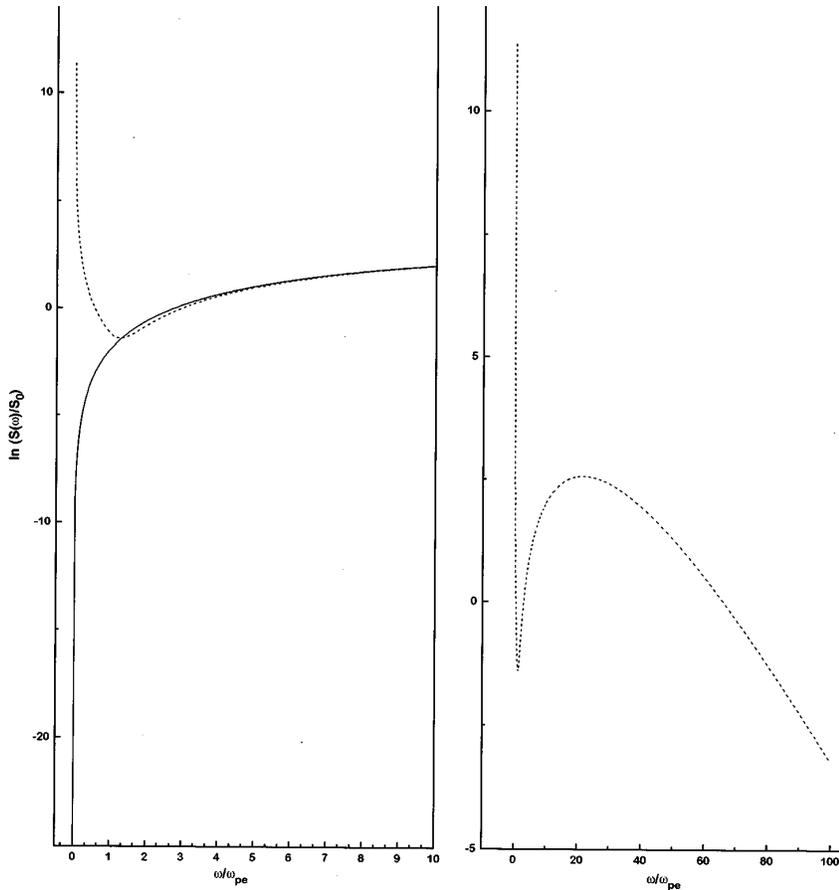


FIG. 3. The magnetic field spectrum  $\ln[S(\omega)/S_0]$  vs  $\omega/\omega_{pe}$  [where  $S(\omega) = \langle B^2 \rangle_\omega / 8\pi$  and  $S_0 = \omega_{pe}^2 k_B T / c^3$  is the normalization] for the cold plasma description with  $x_{\text{upper}} = x_{\text{max}}$  for (a) the electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30} \text{ cm}^{-3}$  (the dashed curve is the cold plasma spectrum and the solid curve is the blackbody spectrum in vacuum); and (b) the same as case (a), extended to high frequencies showing the blackbody behavior.

a fully ionized plasma. It is based in the notion that a large-angle deflection of a particle by collisions is produced more rapidly by a succession of small-angle scattering with distant particles. It takes into account the effect of microscopic fields produced by all the other particles in the plasma. A test particle then is subject to simultaneously “grazing” collisions and its progress in velocity space becomes a random walk.

We used the Bhatnagar, Gross, and Krook (BGK) collision term as a rough guide to the inclusion of collisions in the plasma. BGK is a model equation of the Boltzmann collision term. (For a derivation see Clemmow and Dougherty [16] and Alexandrov, Bogdankevich, and Rukhadze [17].) As CT, we are then only treating binary collisions. (Concerning this collision term, see, for example, the work of Sitenko and Gurin [6], who studied the effect of an effective binary collision frequency on the fluctuations in a plasma.) A more complete treatment using the Fokker-Planck collision term is necessary. However, due to its complexity, as it involves integral functionals of the distribution function  $f$ , the kinetic equations are extremely difficult to solve. We used as an effective collision frequency, the Coulomb collision frequency as did CT.

The BGK collision term conserves the number of particles. The universe at high temperatures, at the beginning of primordial nucleosynthesis ( $T \sim 1 \text{ MeV}$ ), is an electron-positron plasma. As it cools down ( $T \leq 0.5 \text{ MeV}$ ), the electrons and positrons start to annihilate and finally, at low temperatures, the plasma is reduced to a plasma of protons and electrons. Let us see if this is a good approximation in the

early universe. The rate of annihilation is given by  $\Gamma \sim nv\sigma$ , where  $n$  is the density,  $v$  the average velocity, and  $\sigma$  the average cross section. The average cross section is given by  $\sigma = (2\pi\alpha^2/s\beta)[(3-\beta^4)A/2\beta - 2 + \beta^2]$ , where  $\beta^2 = 1 - 4m^2/s$ ,  $A = \ln[(1+\beta)/(1-\beta)]$ ,  $\alpha = \frac{1}{137}$ , and  $s = (2E)^2$  with  $m$  the electron mass and  $E$  the electron energy [18]. Comparing  $\Gamma$  with the collision frequency  $\eta$  (the Coulomb collision frequency as used by CT), we see that  $\Gamma < \eta$ . For example, for  $T \sim 0.8 \text{ MeV}$ ,  $\Gamma = 4 \times 10^{16} \text{ s}^{-1}$ , and  $\eta = 4 \times 10^{17} \text{ s}^{-1}$ . Therefore, the characteristic time of the kinetic processes is shorter than for annihilation and the approximation of the conservation of the number of particles at high temperatures is valid. (As in our case, when the collision frequency is much less than the plasma frequency, the collision time is the dominant time for the kinetic processes.) At low temperatures, an electron-proton plasma has to be considered.

We note that because the electron-positron plasma does not strictly conserve the particle number due to annihilation and creation, there may not be a great advantage of using the BGK collision term, although the above paragraph indicates that the annihilation frequency is very much smaller than the collision frequency.

The plasma, in the epoch of primordial nucleosynthesis, is almost collisionless. (For example, for  $T = 10^{10} \text{ K}$ ,  $\eta/\omega_{pe} \approx 10^{-3}$ .) Therefore, a possible procedure that one might think of is to expand the dielectric permittivity in terms of  $\eta/\omega$ . However, we are interested in the spectrum for all frequencies, even  $\omega \sim 0$ , and no matter how small  $\eta$  is, we require frequencies with  $\omega < \eta$ . Thus an expansion in  $\eta/\omega$  cannot be made.

Therefore, we perform an analysis of the Vlasov equation in first order with the BGK collision term, without making any approximation,

$$\left(\frac{\partial f}{\partial t}\right)_C = -\eta(f - f_{\max}), \quad (3.1)$$

where  $\eta$  is the Coulomb collision frequency (considered constant).  $f_{\max}$  is given by  $f_{\max}(\mathbf{x}, \mathbf{v}, t) = N(\mathbf{x}, t)f_0(\mathbf{v})/N_0$  where the number density is  $N = N_0 + N_1$  and  $f = f_0 + f_1$ ,  $f_0$  being the unperturbed Maxwellian distribution. Substituting this collision term in the equation of Vlasov in first order, Eq. (2.14), performing a Fourier transformation, and rearranging the terms, we arrive at

$$f_1(\mathbf{v}) = \frac{1}{i(\omega - i\eta - \mathbf{k} \cdot \mathbf{v})} \left[ -\frac{e}{m} E_i \frac{\partial f_0}{\partial v_i} + \eta \frac{N_1}{N_0} f_0 \right]. \quad (3.2)$$

To eliminate  $N_1$  we integrate over velocity space [16]:

$$N_1 = \int f_1(\mathbf{v}) d\mathbf{v}. \quad (3.3)$$

We have the current  $\mathbf{j} = e \int v f_1(\mathbf{v}) d\mathbf{v}$ . Knowing that  $j_i = \sigma_{ij} E_j$ , then for an isotropic plasma the transverse permittivity is easily obtained (generalizing for several species):

$$\varepsilon_T(\omega, \mathbf{k}) = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left( \frac{\omega}{\sqrt{2}k v_{\alpha}} \right) Z \left( \frac{\omega + i\eta_{\alpha}}{\sqrt{2}k v_{\alpha}} \right), \quad (3.4)$$

where  $\alpha$  is the label for each species of the plasma,  $v_{\alpha}$  is the thermal velocity for each species, and  $Z(z)$  is the Fried-Conte function [19],

$$Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dt e^{t^2}}{t - z}. \quad (3.5)$$

If relativistic temperature effects are included, the substitution  $\omega_p \rightarrow \omega_p / \sqrt{\gamma}$  is made.

It can be seen that no approximation needs to be made on  $\omega / \sqrt{2}k v_{\alpha}$  and  $\eta / \sqrt{2}k v_{\alpha}$ . (Rewriting  $Z(z)$  in terms of the error function,  $Z(z) = i\sqrt{\pi} \exp(-\xi^2) [1 + \operatorname{erf}(i\xi)]$  [19], it can be easily solved numerically without having to take any asymptotic limit.)

Another thing that we need to be careful of is related to the dielectric permittivities in the region of  $\omega/k \geq c$ , where  $c$  is the velocity of light. For  $\omega/k > c$ , the term connected with Cherenkov emission in the imaginary part of the dielectric permittivities has to vanish. This is because there are no particles with velocities greater than the speed of light to produce Cherenkov emission. [The condition for Cherenkov emission of a wave  $\omega(k)$  requires particles with velocities satisfying the condition  $v = \omega/k \equiv v_p$ , where  $v_p$  is the phase velocity of the wave.] For our case, a plasma in thermal equilibrium, we assume as CT did in Sec. V of their paper [11], that the distribution function is a Maxwellian one. The

calculation of the permittivity tensor (and the dielectric permittivities) involves integral functionals of the distribution function. As pointed out by Melrose [14], the nonvanishing of the imaginary parts, in a collisionless treatment (that is, Cherenkov emission), is due entirely to the effects of unphysical particles with  $v > c$  in the Maxwellian distribution. We deal with this problem by requiring that the Cherenkov emission in the imaginary part is zero in the regime  $\omega/k > c$ . The term of the imaginary part, connected with the collisional damping, is not set to zero.

As photons have phase velocities greater than the speed of light ( $\omega/k > c$ ), the Cherenkov emission cannot produce the photons. In a *pure collisionless* treatment, the imaginary part has to be set to zero in the regime of  $\omega/k > c$ . In this case, no photons are produced and the treatment is unrealistic. In our model, which includes collisions and thermal effects, only the term connected with the Cherenkov emission has to be set to zero. In this treatment we ensure that the photons are produced only by collisions, that is, by bremsstrahlung (free-free) emission.

Therefore, by forcing the Cherenkov emission in the imaginary part to be zero in the regime of  $\omega/k > c$ , we ensure that no spurious effects contaminate the result. We do this not only in the regime of photons ( $\omega > \omega_{pe}$ ), but for all frequencies. That is, Cherenkov emission of magnetic fluctuations occurs only in physical regimes, i.e., for  $\omega/k \leq c$ . A fully relativistic treatment (needed for the primordial universe at high temperatures) is left for a future study. There, the imaginary parts of the dielectric permittivities are zero for  $v_{\phi} > c$ , where  $v_{\phi} = \omega/k$ , when  $\gamma = (1 - v_{\phi}^2/c^2)^{-1/2}$  and  $p_{\phi} = \gamma_{\phi} m v_{\phi}$  are imaginary.

In principle, the fact that the term connected with the Cherenkov emission for  $\omega/k > c$  is forced to be zero could be troublesome. This procedure, however, did not change appreciably the results. For example, the change of the intensity of the magnetic field fluctuations  $\Delta \langle B^2 \rangle_{\omega}$  (after integrating on wave number) for  $T = 7 \times 10^9$  K,  $n_e = 4.6 \times 10^{30}$  cm<sup>-3</sup> is  $10^{-7}$ ,  $10^{-3}$ , and  $10^{-5}$  for  $\omega/\omega_p = 0.1, 10.0$ , and  $100.0$ , respectively. The change is the difference between  $(\operatorname{Im} \varepsilon_T)_C \neq 0$  and  $(\operatorname{Im} \varepsilon_T)_C = 0$ , where  $(\operatorname{Im} \varepsilon_T)_C$  is the term in the imaginary part of the transverse dielectric permittivity connected with Cherenkov emission.

Before we go on, it is interesting to comment and emphasize that both the cold plasma description and the warm plasma collisionless description are particular solutions of this model. Let us consider, as CT did, an electron-positron plasma. For  $|z| \gg 1$ , where  $z = (\omega + i\eta) / \sqrt{2}k v_e$ , we expect to obtain the cold plasma approximation. In the limit of  $|z| \gg 1$ ,  $Z(z) = -1/z - 1/2z^3 + \dots - i\sqrt{\pi} z e^{-z^2}$ . The last term is due to Cherenkov emission, but as we noted before, in this limit this is a spurious solution and we need to set it to zero. Taking only the first term, and substituting in Eq. (3.4), we obtain the cold plasma dielectric permittivity, as expected.

The warm plasma collisionless description is obtained by setting  $\eta \rightarrow 0$ . In this limit, if we write  $z = x + iy$ ,  $iy = i0$ ,  $\bar{\phi}(x + i0) = \phi(x) - i\sqrt{\pi} x e^{-x^2}$  [ $Z(z) = -1/z \bar{\phi}$ ], Eq. (3.4) is equal to the warm plasma collisionless dielectric permittivity, as expected.

We substitute the dielectric permittivity, Eq. (3.4), in Eq. (2.6) obtaining the magnetic field spectrum  $\langle B^2 \rangle_{k\omega}$ . Here, a divergence at high wave numbers also occurs. Let us discuss this in detail. Our model uses a kinetic theory description with a model collision term that describes the binary collisions in the plasma. In our case, a cutoff has to be taken, since for very small distances the energy of the Coulomb interactions of the particles exceeds their kinetic energy which violates the applicability of the condition of the perturbation expansion (in the plasma parameter  $g \ll 1$ ). This occurs approximately for distances  $r_{\min} \sim e^2/T$ , or more exactly, the distance of closest approach between a test particle and an electron in a plasma,  $k_{\max} = 1/r_{\min} \cong Mmv^2/(m+M)|eq|$ , where  $M$ ,  $v$ , and  $q$  are, respectively, the mass, velocity, and charge of the test particle [15].

The cutoff procedure can only be removed, treating properly the effects of distant encounters. This can be done with the Fokker-Planck collision term. Thompson and Hubbard [20] and Hubbard [21,22], analyzed the Fokker-Planck equation and its coefficients. The diffusion and friction coefficients that appear in the Fokker-Planck equation take into account correlation effects between distant particles in the plasma. When higher order terms in the Fokker-Planck equation are calculated and summed, a term resembling the Boltzmann collision term is obtained. In their treatment, they showed that the cutoff procedure is unnecessary. However, due to the complexity of the solution of the kinetic equation with the Fokker-Planck collision term, we used the BGK collision term. With this model collision term, a cutoff is necessary and we chose  $x_{\max}$  consistent with this model collision term. A more exact treatment, however, is needed.

In Fig. 4(a) we plot the magnetic field spectrum  $S(\omega) = \langle B^2 \rangle_{\omega} / 8\pi$  (divided by the normalization  $S_0 = \omega_{pe}^2 k_B T / c^3$ ) vs  $\omega / \omega_{pe}$  for an electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30}$  cm $^{-3}$ . The dotted curve is our model and we compare it with the blackbody spectrum in vacuum (the solid curve). In Fig. 4(b), we extend the curves to high frequencies, showing the behavior of the blackbody at high frequencies. Figs. 4(c) and 4(d) are for an electron-proton plasma, with  $T = 10^9$  K and  $n_e = 5.4 \times 10^{26}$  cm $^{-3}$ .

As we commented before (Sec. II C), the results are very dependent on the cutoff chosen. Using  $x_{\max}$  as the cut-off, we obtain results that differ from the CT results: First, the peak intensity found by them for frequencies  $\omega \sim 0$  decreases. This is due to the kinetic plasma effects that smear out the peak. However, it is interesting to see that qualitative agreement between the work of CT and ours exists with respect to the zero-frequency peak. Second, we obtain the blackbody naturally for high frequencies. Third, the magnetic field spectrum has more energy than the blackbody spectrum for frequencies  $\omega \leq 4\omega_{pe}$ .

#### IV. CONCLUSIONS AND DISCUSSION

The magnetic field spectrum can be deduced from the fluctuations of the magnetic field described by the fluctuation-dissipation theorem and it is highly dependent on the way the plasma is described. We discussed the cold plasma description and the warm plasma description in the collisionless case studied by CT. We showed that  $x_{\text{cut}}$  is

much smaller than  $x_{\max}$  and  $x_{\text{lim}}$ , where  $x_{\text{lim}} = k_{\text{lim}}c / \omega_{pe}$  used in Sec. VII of CT (arguing that for  $k > k_{\text{lim}}$  the plasma has a negligible effect).  $x_{\max}$  is the cutoff used in treating binary collisions,  $(x_{\max})^{-1}$  being the distance of closest approach between a test particle and an electron in a plasma (divided by  $c / \omega_{pe}$ ).

We also showed that the fluctuation-dissipation theorem contains the eigenfrequencies of the plasma (in the transverse case), the photons. Using the cold plasma description with the upper limit  $x_{\text{upper}} = x_{\max}$ , we obtain the blackbody spectrum at high frequencies naturally, without the necessity of a *link* procedure used by CT. For this case, the valley disappears and the curve is above the blackbody spectrum in vacuum.

The calculations were made for two types of primordial plasmas: The electron-positron plasma at the beginning of the big-bang nucleosynthesis; and the electron-proton plasma at lower temperatures.

The manner to obtain the entire magnetic field spectrum is analyzing the magnetic field fluctuations. This is the only way to obtain information, not only about modes that propagate (i.e., photons), but also modes that do not propagate. The modes that do not propagate appear not only at low frequencies but also at high frequencies due to the correlations in the plasma. Only at very high frequencies does the photon contribution dominate the magnetic field spectrum. The argument used by CT, that the energy under the peak is almost equal to the energy “stolen” by the plasma cutoff effect of the blackbody in vacuum, is incorrect. There is no reason why we have to have the same energy as the blackbody spectrum in vacuum for photons for  $\omega < \omega_{pe}$ , since the photons have a different dispersion relation than the fluctuations in the plasma. In fact, using a upper limit  $x_{\text{upper}} = x_{\max}$  in the cold plasma description, for example, the spectrum obtained is above the blackbody spectrum in vacuum.

The reason why the collective modes of the plasma can have more energy for  $\omega \leq \omega_p$  than the photons in vacuum can be understood as follows. Photons are massless bosons with the dispersion relation  $\omega^2 = k^2c^2$ . For the energy interval  $0 \leq \omega \leq \omega_p$ , the wave number interval is  $k = 0$  to  $k$  equal to  $\omega_p/c$ . A relatively small amount of phase space is involved. For the collective motions of the plasma, in general, we have a larger amount of phase space. For example, for plasmons with energy  $\omega \sim \omega_p$ , the amount of phase space extends to a maximum  $k$  of  $k_D \cong \omega_p/v_t$ , which is greater than  $\omega_p/c$  for the photons. In general, for a given frequency for  $\omega < \omega_p$ , the greater phase available to the collective modes of the plasma (than that of the photons) implies more energy, or a higher spectrum.

We presented a model that incorporates, in the same description, thermal and collisional effects. We used the Vlasov equation with the BGK collision term. This collision term describes the binary collisions in the plasma. A model that takes into account collisions in a more complete way is necessary. For a fully ionized plasma it is necessary to use the Fokker-Planck collision term that takes into account the effect of the microscopic fields. Due to the complexity of the solution of the kinetic equation with such a collision term, we used the BGK collision term. This model, an extension of the CT model, describes the basic features of a kinetic description.

As we noted before, the results are very dependent on the

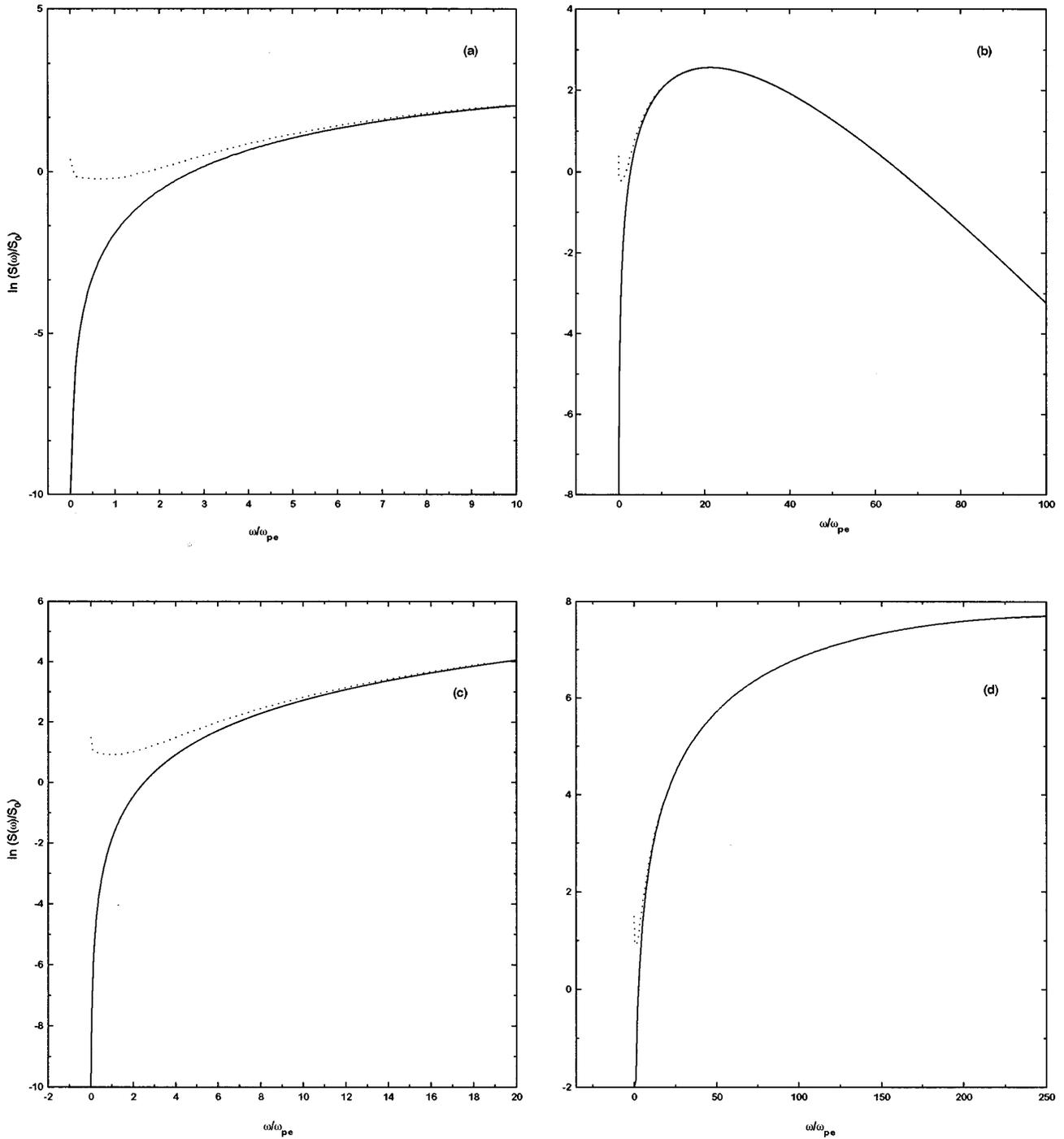


FIG. 4. The magnetic field spectrum  $\ln[S(\omega)/S_0]$  vs  $\omega/\omega_{pe}$  [where  $S(\omega) = \langle B^2 \rangle_\omega / 8\pi$  and  $S_0 = \omega_{pe}^2 k_B T / c^3$  is the normalization] for our model for (a) the electron-positron plasma at  $T = 7 \times 10^9$  K and  $n_e = 4.6 \times 10^{30}$   $\text{cm}^{-3}$  (the dotted curve is the spectrum of our model and the solid curve is the blackbody spectrum in vacuum); (b) the same as case (a), extended to high frequencies; (c) the same as case (a) but for an electron-proton plasma at  $T = 10^9$  K and  $n_e = 5.4 \times 10^{26}$   $\text{cm}^{-3}$ ; and (d) the same as case (c), extended to high frequencies.

cutoff chosen. Using  $x_{\max}$  as the cutoff, consistent with the collision term used, we obtain results that differ from the CT results. The final magnetic spectrum of a nonmagnetized plasma in thermal equilibrium has the following characteristics: (a) The peak intensity found by CT for frequencies  $\omega \sim 0$  decreases; (b) the blackbody is obtained naturally for high frequencies; and (c) the magnetic spectrum has more energy than the blackbody photon spectrum in vacuum, in

particular, for frequencies,  $\omega \leq 4\omega_{pe}$ , where  $\omega_{pe}$  is the electron plasma frequency.

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