BRIEF REPORTS

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Fritzsch-Xing mass matrices, V_{td}, and the CP-violating phase δ

P. S. Gill* and Manmohan Gupta
Department of Physics, Panjab University, Chandigarh-16001, India

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Natural four zeros texture mass matrices recently proposed by Fritzsch and Xing are investigated by including ''nonleading'' corrections in the context of the latest data regarding $m_t^{\rm pole}$ and $V_{\rm CKM}$ matrix elements. Apart from accommodating $m_t^{\rm pole}$ in the range 175 ± 15 GeV, $|V_{cb}|$ and $|V_{ub}/V_{cb}|=0.08\pm 0.02$, the analysis with maximal CP violation predicts $|V_{td}|=0.005-0.013$. Further, the CP-violating phase angle δ can be restricted to the ranges (i) 22°–45° and (ii) 95°–130°, concretizing the ambiguity regarding the phase of the CKM matrix. Furthermore, we find that nonleading calculations are important when the ''Cabibbo triangle'' is to be linked to the unitarity triangle. [S0556-2821(97)03017-8]

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Recently, Peccei and Wang [1] in a very interesting paper have found a possible pattern of natural mass matrices at the grand unified theory scale which are in agreement with low energy data related to the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. A concrete realization of such mass matrices at a low energy scale as shown by Wang [2] is presented by the Fritzsch and Xing (FX) ansatz [3] consisting of four zeros texture mass matrices [4]. Exploiting the idea of maximality of CP violation and relating the usual unitarity triangle with the ''Cabibbo triangle,'' they have found some very interesting results at leading order. In particular, by adding nonzero 22 elements in the ''U'' as well as ''D'' sectors to the usual Fritzsch mass matrices [5], FX have found, at leading order,

$$V_{us}=\sqrt{\frac{m_d}{m_s}}-\sqrt{\frac{m_u}{m_c}}\,e^{i\Delta\sigma}, \tag{1}$$

$$V_{cd}=\sqrt{\frac{m_u}{m_c}}-\sqrt{\frac{m_d}{m_s}}\,e^{i\Delta\sigma}, \tag{2}$$

$$\frac{V_{ub}}{V_{cb}}\approx\sqrt{\frac{m_u}{m_c}}\quad\text{and}\quad\frac{V_{td}}{V_{ts}}\approx\sqrt{\frac{m_d}{m_s}}. \tag{3}$$

Further, in the complex plane by linking the Cabibbo triangle with the usual unitarity triangle they have been able to show $\Delta\sigma\approx 90°$, implying maximal CP violation in the context of present mass matrices. The value of $\Delta\sigma$ is sensitive to variations of mass ratios m_u/m_c and m_d/m_s; however, the above conclusion about $\Delta\sigma$ is not inconsistent with the range suggested by such variations.

The purpose of the present Brief Report, on the one hand, is to find nonleading order corrections to relations (1)–(3); on the other hand, we want to examine the detailed implications of extra terms, introduced by FX in their formalism compared to earlier Fritzsch mass matrices [5], on the $V_{\rm CKM}$ phenomenology. Further, to extend the success of FX mass matrices, it becomes interesting to examine how FX mass matrices accommodate $m_t^{\rm pole}$ [6], the latest data regarding $V_{\rm CKM}$ elements V_{ub} and V_{cb}, as well as the recently found range of V_{td} due to improved QCD calculations of B^0-$\overline{B^0}$ mixing phenomenon [7]. Furthermore, it would be worthwhile to study the implications of nonleading corrections to angles of the unitarity triangle which may shed some light on the ambiguity relating to the phase of the CKM mixing matrix [8].

To this end, we have first exactly diagonalized FX matrices and calculated the corresponding $V_{\rm CKM}$. The implications of the extra terms both in ''U'' and ''D'' sectors is quite manifest in the expressions for $V_{\rm CKM}$ matrix elements derived here.

To begin with, we consider the Fritzsch-Xing mass matrices [3]: for example,

$$M_i=\begin{pmatrix} 0 & D_i & 0\\ D_i^* & C_i & B_i\\ 0 & B_i & A_i\end{pmatrix}\quad (i=u,d), \tag{4}$$

where $D_i=|D_i|e^{i\sigma_i}$ and the elements of M_i are assumed to follow the hierarchical structure, e.g., $|D_i|\ll B_i\approx C_i<A_i$. The above matrices M_i can be expressed as

$$M_i=P_i\overline{M_i}P_i^\dagger, \tag{5}$$

where the real matrices $\overline{M_i}$ may be expressed as

*Permanent address: S.G.G.S. College, Chandigarh 160026, India.

$$\overline{M_i}=\begin{pmatrix} 0 & |D_i| & 0 \\ |D_i| & C_i & B_i \\ 0 & B_i & A_i \end{pmatrix} \tag{6}$$

and

$$P_i=\mathrm{diag}(1,e^{-i\sigma_i},e^{-i\sigma_i}). \tag{7}$$

The real matrices $\overline{M_i}$ can be diagonalized exactly by orthogonal transformations: for example,

$$\overline{M_i}=O_i M_i^{\mathrm{diag}} O_i^T, \tag{8}$$

where

$$M_i^{\mathrm{diag}}=\mathrm{diag}(m_1,-m_2,m_3), \tag{9}$$

with subscripts 1, 2, 3 referring to u,c,t in the ''U'' sector and d,s,b in the ''D'' sector. The details of diagonalizing matrix O_i are given in Ref. [9].

To facilitate comparison with FX calculations as well as for a better physical understanding of the structure of V_{CKM}, we present here the approximate form of O_u. For example, by considering $m_u \ll m_c < C_u < m_t$ as well as $m_d < m_s < C_d < m_b$, the structure for O_u can be simplified and expressed as

$$O_u \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m_u}{m_c}} & \frac{m_c}{m_t}\sqrt{\frac{m_u}{m_c}\left(\frac{m_c+C_u}{m_t-C_u}\right)} \\ \sqrt{\frac{m_u}{m_c}\left(1-\frac{m_c+C_u}{m_t}\right)} & \sqrt{1-\frac{C_u}{m_t}} & \sqrt{\frac{m_c+C_u}{m_t}} \\ -\sqrt{\frac{m_u}{m_c}\left(\frac{m_c+C_u}{m_t}\right)} & -\sqrt{\frac{m_c+C_u}{m_t}} & \sqrt{1-\frac{C_u}{m_t}} \end{pmatrix}. \tag{10}$$

The matrix O_d can be obtained from O_u simply by changing $u \to d$, $c \to s$, and $t \to b$.

The mixing matrix V_{CKM} in terms of $O_{u,d}$ can be expressed as

$$V_{\mathrm{CKM}}=O_u^T P_{ud} O_d, \tag{11}$$

where

$$P_{ud}=P_u^{\dagger}P_d=\mathrm{diag}(1,e^{i\Delta\sigma},e^{i\Delta\sigma}) \quad \text{and} \quad \Delta\sigma=\sigma_u-\sigma_d. \tag{12}$$

Using Eq. (10) and retaining terms up to next-to-leading order, Eq. (11) can be simplified and written as

$$V_{\mathrm{CKM}} \cong \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}}+\sqrt{\frac{m_u}{m_c}}\,g_1 & \frac{m_s}{m_b}\sqrt{\frac{m_d}{m_s}\left(\frac{m_s+C_d}{m_b-C_d}\right)}+\sqrt{\frac{m_u}{m_c}}g_2 \\ -\sqrt{\frac{m_u}{m_c}}+\sqrt{\frac{m_d}{m_s}}\,g_1 & \sqrt{\frac{m_u m_d}{m_c m_s}}+g_1 & g_2 \\ \frac{m_c}{m_t}\sqrt{\frac{m_u}{m_c}\frac{m_c+C_u}{m_t-C_u}}-\sqrt{\frac{m_d}{m_s}}\,g_2' & -g_2 & g_1 \end{pmatrix}, \tag{13}$$

where

$$g_1=\left[\sqrt{\left(1-\frac{C_u}{m_t}\right)\left(1-\frac{C_d}{m_b}\right)}+\sqrt{\frac{m_c+C_u}{m_t}\frac{m_s+C_d}{m_t}}\right]e^{i\Delta\sigma}, \tag{14}$$

$$g_2=\left[\sqrt{\frac{m_s+C_d}{m_b}\left(1-\frac{C_u}{m_t}\right)}-\sqrt{\frac{m_c+C_u}{m_t}\left(1-\frac{C_d}{m_b}\right)}\right]e^{i\Delta\sigma}, \tag{15}$$

and

$$g_2'=\left[\sqrt{\frac{m_s+C_d}{m_b}\left(1-\frac{C_u}{m_t}\right)} -\sqrt{\frac{m_c+C_u}{m_t}\left(1-\frac{C_d+m_s}{m_b}\right)}\right]e^{i\Delta\sigma}. \tag{16}$$

The above expressions for V_{CKM} are approximate; however, for the purpose of calculations, we have employed exact expressions.

After having calculated the V_{CKM} elements, we calculate the angles of the unitarity triangle related to the decays $B_d \to \pi\pi$, $B_d \to D\pi$ and B_d^0-$\overline{B_d^0}$ mixing. The quantities usu-

TABLE I. Calculated values of R_{ub}, R_{td}, $S_{2\theta}$ $(=\sin 2\theta,\ \theta=\alpha,\beta,\gamma)$ for $M_t^{\rm pole}=175$ GeV and for different R_t values when $|V_{cb}|=0.038$ with $S_{2\alpha}$ taking negative values.

R_t	R_{ub}	R_{td}	$S_{2\alpha}$	$S_{2\beta}$	$S_{2\gamma}$
0.00	0.06	0.21	−0.18	0.52	0.66
0.02	0.07	0.20	−0.58	0.52	0.95
0.04	0.07	0.18	−0.88	0.53	1.00
0.06	0.08	0.18	−0.91	0.54	0.99
0.08	0.08	0.17	−0.96	0.54	0.96
0.10	0.08	0.16	−0.99	0.54	0.91
0.12	0.09	0.15	−1.00	0.55	0.87
0.14	0.09	0.15	−1.00	0.55	0.82
0.16	0.10	0.14	−1.00	0.56	0.75

ally discussed are $\sin 2\alpha$, $\sin 2\beta$, and $\sin 2\gamma$ whose respective relations to B decays and $V_{\rm CKM}$ elements are detailed in Ref. [10].

Before we present our results, a brief discussion about various inputs which have gone into the analysis is in order. As a first step, we have considered quark masses at 1 GeV [11], for example, $m_u=0.0051\pm0.0015$ GeV, $m_d=0.0089\pm0.0026$ GeV, $m_s=0.175\pm0.055$ GeV, $m_c=1.35\pm0.05$ GeV, and $m_b=5.3\pm0.1$ GeV. Unlike FX, we have not studied the implications of the spread in mass values; rather we have endeavored to understand the detailed implications of the variations of C_u and C_d on the CKM phenomenology. Following FX, to maximize CP violation, we have fixed $\Delta\sigma=90°$. Noting the fact that the CKM matrix elements $|V_{us}|$ and $|V_{cb}|$ are well known and have weak dependence on m_3's and C_i's, we have restricted the parameter space by first reproducing $|V_{us}|=0.22$ and $|V_{cb}|\cong0.038$ [12,13], ignoring the spread in the values of $|V_{cb}|$ as the calculated quantities hardly show any dependence on these. The above value of $|V_{cb}|$, through Eqs. (13) and (15), fixes the values of C_d for a given value of C_u.

After having fixed the values of $|V_{us}|$ and $|V_{cb}|$, we have calculated $|V_{ub}|$, $|V_{td}|$, $|V_{ts}|$, and other phenomenological quantities related to $V_{\rm CKM}$, for $m_t^{\rm pole}=175$ GeV [corresponding to $m_t(1\ {\rm GeV})\approx300$ GeV] and at different values of $R_t=C_u/m_t$. The variations with respect to m_t have not been considered as the calculated quantities do not show any significant explicit dependence on the present experimental range of $m_t^{\rm pole}$ [6].

In Tables I and II we have summarized the results of our calculations for $\sin 2\alpha$ being negative and positive, respectively. For the sake of uniformity, we have presented in the tables the ratios of $V_{\rm CKM}$ matrix elements, e.g., $R_{ub}=|V_{ub}/V_{cb}|$ and $R_{td}=|V_{td}/V_{cb}|$. A general survey of the

TABLE II. Same as in Table I with $S_{2\alpha}$ taking positive values.

R_t	R_{ub}	R_{td}	$S_{2\alpha}$	$S_{2\beta}$	$S_{2\gamma}$
0.00	0.06	0.24	0.61	0.51	−0.09
0.02	0.07	0.26	0.90	0.50	−0.47
0.04	0.08	0.27	0.98	0.50	−0.70
0.06	0.08	0.28	1.00	0.50	−0.80
0.08	0.09	0.29	1.00	0.49	−0.88
0.10	0.09	0.30	0.98	0.49	−0.93
0.12	0.10	0.30	0.96	0.49	−0.96

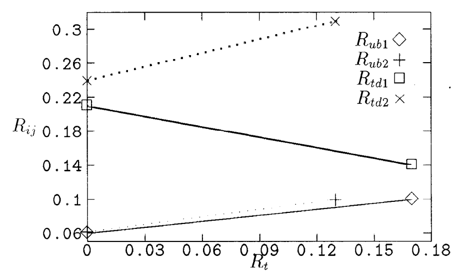

FIG. 1. Variation of calculated values of $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{cb}|$ with respect to R_t ($R_{ij}=|V_{ij}/V_{cb}|$, $ij=ub,td$), suffixes 1 and 2 corresponding to Tables I and II, respectively.

tables brings out easily that we are able to obtain R_{ub} from 0.06–0.10 [10,14] by varying R_t for $m_t^{\rm pole}=175$ GeV. This behavior of R_{ub} can be easily checked from the expressions of V_{ub} and V_{cb}. Similarly, the results for $|V_{td}|$ can encompass the presently expected range [7,10]. Coming to the angles of unitarity triangle, α, β, and γ, we find that the present values are in accordance with similar calculations by other authors [7,13,15].

For the sake of brevity we have not included in the tables our calculations regarding V_{ts}, x_s, and Jarlskog's CP-violating rephasing invariant parameter J [16]. However, a few remarks regarding these merit mention here. Interestingly, we find that $|V_{ts}|\leqslant|V_{cb}|$ for the entire range of $m_t^{\rm pole}$ and R_t. This is in accordance with expectations from the unitarity of $V_{\rm CKM}$ [10]. A departure from the above prediction would have important implications for present texture-specific mass matrices. Similarly, the parameter J and B_s^0-$\overline{B_s^0}$ mixing parameter x_s, though not shown in the tables, have been calculated to be in the acceptable ranges, e.g., $J=(1.3–2.4)\times10^{-5}$ [17] and $x_s=8-15(\sin 2\alpha>0)$, 18–45 ($\sin 2\alpha<0$) for $x_d=0.73$ [10], again in accordance with other similar calculations [7,13,15].

A careful study of the tables reveals several features which can be shown to be due to the presence of elements C_u and C_d in the FX matrices. One finds that there are two distinct ranges for V_{td}, $\sin 2\alpha$, $\sin 2\beta$, and $\sin 2\gamma$, although in the case of $\sin 2\beta$ the variations are not as much as in the case of $\sin 2\alpha$ and $\sin 2\gamma$. It is interesting to mention that $\sin 2\beta$ does not show much dependence on m_t or C_u or C_d. The narrow range of $\sin 2\beta$, despite considerable variation of parameters, provides a severe testing ground for the present texture four zeros mass matrices. The effect of the additional parameter is particularly significant in the case of $|V_{td}/V_{cb}|$ which lies in two ranges, e.g., 0.14–0.22 and 0.24–0.31, corresponding, respectively, to lower and higher value of C_d generated by Eq. (15) for a given value of C_u.

In order to have a better understanding of the significance of our results, in Fig. 1, we have shown the variation of calculated values of $|V_{ub}|$ and $|V_{td}|$ as a function of R_t. It is interesting to mention that we do not find any pronounced dependence on m_t for fixed R_t; however, the dependence on C_u and C_d is considerable. The figure also shows two distinct ranges for V_{ub} and V_{td} shown by solid and dotted lines corresponding to values of these in Tables I and II, respectively. The reason for two branches is not difficult to understand when one realizes that fixing $|V_{cb}|$ through Eqs. (13) and (15) leads to two values for C_d for a given value for C_u. A detailed investigation of the exact expressions, e.g., Eqs. (13), (15), and (16), brings out clearly the contrasting behavior of V_{ub} and V_{td}.

To understand the full significance of our results regarding the two ranges for V_{td}, $\sin 2\gamma$, etc., we would like to mention that this can be linked to the ambiguity of the phase of the CKM matrix, discussed in detail by Harris and Rosner [8]. To explore this further, we consider the exact standard parametrization of the $V_{\rm CKM}$ matrix [10]:

$$V_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{17}$$

where $c_{ij}=\cos\theta_{ij}$, $s_{ij}=\sin\theta_{ij}$, and δ corresponds to δ_{13} of the standard parametrization. In this parametrization, δ is simply equal to the angle γ of the unitarity triangle. As the value of β does not show much variation, therefore, the behavior of δ is very much related to the variations of angles α and γ. As has been mentioned in the tables, the present data allow an ambiguity in the sign of $\sin 2\gamma$ which easily translates to the ambiguity of the quadrant of δ [8]. For having a feeling of this ambiguity in the $V_{\rm CKM}$ matrix elements, one has to closely analyze the exact expression for V_{td} in Eqs. (13) and (16) along with V_{cb} in Eqs. (13) and (15).

From the tables and expression (17), one can easily find the range of δ predicted by the present set of mass matrices, as follows:

$$22° \leqslant \delta \leqslant 45° \quad \text{and} \quad 95° \leqslant \delta \leqslant 130°, \tag{18}$$

in good overlap with other similar calculations [7]. To conclude, FX mass matrices have been investigated in the context of CKM phenomenology. In particular, we have examined in detail the implications of additional elements introduced by FX in comparison with the earlier Fritzsch mass matrices. Apart from obtaining the nonleading order corrections to Eqs. (1)–(3), we have seen that the maximality of CP violation as enunciated by FX is in tune with the present data. It is very striking to note that the variation of V_{td}, due to the additional elements $C_{u,d}$, leads to a range which is very much in agreement with the results expected from the recently improved calculations of B^0-$\overline{B^0}$ mixing [7] for a fairly broad range of parameters considered here. The role played by C_u and C_d in fitting the recent data pertaining to CKM matrix elements needs to be highlighted as when either of them is zero the full range of data pertaining to V_{ub} and V_{td} cannot be fitted [18]. A precise measurement of angle β, through the decay $B \to \psi K_s^0$, and of V_{td} would certainly help in establishing the validity of present set of mass matrices. Further, in the language of Ramond, Robert, and Ross [4] it seems that present data favor texture four zeros mass matrices.

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