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Exclusive nonleptonic decays of B mesons

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The energetic exclusive two-body nonleptonic decays of B mesons are investigated in the framework of the relativistic quark model within the factorization approximation. The heavy quark expansion is used for the calculation of form factors. The obtained results are in agrement with available experimental data. [S0556-2821(97)03213-X]

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I. INTRODUCTION

The investigation of exclusive nonleptonic decays of B mesons represents an important and complicated theoretical problem. In contrast with the exclusive semileptonic decays, where the weak current matrix elements between meson states are involved, nonleptonic decays require the evaluation of hadronic matrix elements of the local four-quark operators. To simplify the analysis it is usually assumed that the matrix element of the current-current weak interaction factorizes into the product of two single current matrix elements. Thus the problem reduces to the calculation of the meson form factors, parametrizing the hadronic matrix elements of weak currents as in the case of semileptonic decays, and the meson decay constants, describing leptonic decays [1]. This makes the factorization hypothesis a very appealing assumption. However, strong interaction effects, such as final state interactions, the rescattering of the final hadrons, etc., can violate this approximation [1,2]. There are also some problems with the different renormalization point dependence of the initial and factorized amplitudes [3,4]. Thus factorization cannot be considered as a universal approach to nonleptonic decays.

There were several theoretical developments which can help to justify the factorization for certain nonleptonic decays of heavy mesons. It has been shown in Ref. [5] that factorization holds in the limit of large number of colors N_c in QCD. The leading $1/N_c$ corrections to this limit have also been considered. Moreover, intuitive arguments justifying factorization for the energetic nonleptonic decays were given by Bjorken [6] on the basis of the so-called color transparency. In these decays the final hadrons are produced in the form of pointlike color-singlet objects with a large relative momentum. And thus the hadronization of the decay products occurs after they are too far away for strongly interacting with one another, providing the possibility to avoid final state interactions. Dugan and Grinstein [3] discussed the factorization hypothesis within the heavy quark effective theory. They proved that factorization holds for the decays of a heavy B meson into a heavy D meson and a light meson, where the light quarks, which hadronize into a light meson, are highly energetic and collinear. Therefore, we have a good theoretical background to expect that factorization can be applied to the consideration of energetic nonleptonic decays of B mesons.

In this paper we calculate the branching ratios of the exclusive energetic nonleptonic decays of B mesons in the framework of the relativistic quark model on the basis of factorization. The heavy-to-heavy hadronic form factors, appearing in the factorized amplitudes, are constrained by the heavy quark effective theory (HQET) [7]. Our model explicitly satisfies all these constraints and allows the determination of the corrections in the inverse powers of the heavy quark masses up to the second order [8]. In the quoted paper we have determined the Isgur-Wise function and the subleading form factors in the whole kinematical range accessible in $B\to D$ transitions. We shall use these functions here to evolve $B\to D$ transition form factors from the point of zero recoil of the final D meson to the values of $q^2\approx m_f^2$, where m_f is a mass of the final light meson. The form factors of the heavy-to-light transitions have been calculated in our model at the point of maximum recoil of the final light meson using the expansion in inverse powers of the heavy b-quark mass from the initial B meson and in inverse powers of the large ($\sim m_b/2$) recoil momentum of the final light meson [9]. We have also determined the q^2 dependence of the form factors near this kinematical point. Thus we can calculate the heavy-to-light form factors which are necessary for the determination of energetic nonleptonic decay amplitudes. This combination of the methods of heavy quark expansion and the relativistic quark model increases the reliability of our predictions. The comparison of the results with the available experimental data will be the test of factorization.

The paper is organized as follows. In Sec. II we present the expressions for nonleptonic decay amplitudes in the factorization approximation. The relativistic quark model is described in Sec. III. In Sec. IV the heavy-to-heavy transition form factors are discussed. The heavy-to-light transition form factors are presented in Sec. V. Section VI contains our results for the branching ratios of energetic nonleptonic B

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decays and their discussion. Our conclusions are given in Sec. VII.

II. NONLEPTONIC DECAY AMPLITUDES AND FACTORIZATION

In the standard model B decays are described by the effective Hamiltonian, obtained by integrating out the heavy W boson and top quark and applying the operator product expansion. For the case of $b\rightarrow c,u$ transitions,

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cb}[c_1(\mu)O_1^{cb}+c_2(\mu)O_2^{cb}]+\frac{G_F}{\sqrt{2}}V_{ub}[c_1(\mu)O_1^{ub}+c_2(\mu)O_2^{ub}]+\cdots, \quad (1)$$

where V_{ij} are the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The Wilson coefficients $c_{1,2}(\mu)$ are evaluated perturbatively at the W scale and then they are evolved down to the renormalization scale $\mu\approx m_b$ by the renormalization-group equations. The ellipsis denotes the penguin operators, where Wilson coefficients are numerically much smaller than $c_{1,2}$ [10]. The local four-quark operators O_1 and O_2 are given by

$$O_1^{qb}=[(\tilde{d}u)_{V-A}+(\tilde{s}c)_{V-A}](\overline{q}b)_{V-A},$$

$$O_2^{qb}=(\overline{qu})_{V-A}(\tilde{d}b)_{V-A}+(\overline{qc})_{V-A}(\tilde{s}b)_{V-A}, \quad q=(u,c), \quad (2)$$

where the rotated antiquark fields are

$$\tilde{d}=V_{ud}\overline{d}+V_{us}\overline{s}, \quad \tilde{s}=V_{cd}\overline{d}+V_{cs}\overline{s}, \quad (3)$$

and for the hadronic current the following notation is used

$$(\overline{q}q')_{V-A}=\overline{q}\gamma_\mu(1-\gamma_5)q'\equiv J_\mu.$$

The factorization approach to two-body nonleptonic decays $B\rightarrow M_1(\overline{q}_1q_1')M_2(\overline{q}_2q_2')$ implies that the decay amplitude can be approximated by the product of one-particle matrix elements:

$$\langle M_1M_2|H_{\rm eff}|B\rangle=\frac{G_F}{\sqrt{2}}V_{q_1b}V_{q_2'q_2}[a_1(\mu)\langle M_1|(\overline{q}_1b)_{V-A}|B\rangle \times\langle M_2|(\overline{q}_2q_2')_{V-A}|0\rangle+a_2(\mu) \times\langle M_2|(\overline{q}_2b)_{V-A}|B\rangle \times\langle M_1|(\overline{q}_1q_1')_{V-A}|0\rangle], \quad (4)$$

where

$$a_1(\mu)=c_1(\mu)+\frac{1}{N_c}c_2(\mu), \quad a_2(\mu)=c_2(\mu)+\frac{1}{N_c}c_1(\mu), \quad (5)$$

N_c is the number of colors ($N_c=3$).

In the general case, the renormalization point (μ) dependence of the product of current operator matrix elements does not cancel the μ dependence of $a_i(\mu)$ or $c_i(\mu)$ [3,4]. Thus nonfactorizable contributions to Eq. (4) must be present in order to make the physical amplitudes independent from the renormalization scale μ. However, as is shown in [3], in the case of the production of an energetic light meson or meson resonance it is possible to justify the factorization approximation and the right-hand side of Eq. (4) is scale independent. Thus we limit our analysis of nonleptonic decays to consideration of decays with at least one energetic meson in the final state [such as $B\rightarrow D^{(*)}\pi(\rho)$ and $B\rightarrow\pi(\rho)\pi(\rho)$].

Before proceeding further, let us additionally note that in writing Eq. (4) we discarded the contribution of the color-octet currents which emerge after the Fierz transformation of color-singlet operators (2). Clearly, these currents violate factorization since they cannot provide transitions to the vacuum state. We also neglected the so-called W exchange and annihilation diagrams. In the limit $M_W\rightarrow\infty$, they are connected by the Fierz transformation and are doubly suppressed by the kinematic factor of order (m_D^2/m_B^2) and then dynamically by the decreasing form factor $F_{D\pi}(q^2=m_B^2)$ with $F_{D\pi}(0)=1$ (see Ref. [5] for details).

The coefficients (5) have been calculated at $\mu\approx m_b$ in the leading logarithmic approximation [11] as well as beyond the leading logarithmic approximation [4]. The result of Ref. [4] is

$$a_1=1.01\pm0.02 \text{ and } a_2=0.20\pm0.05, \quad (6)$$

which is close to the result of fitting experimental data [12]

$$a_1=1.03\pm0.04\pm0.06 \text{ and } a_2=0.23\pm0.01\pm0.01. \quad (7)$$

However, the a_2 prediction (6) is renormalization scheme dependent [4].

The matrix element of the current J between the vacuum and final pseudoscalar (P) or vector (V) meson states is parametrized by the decay constants $f_{P,V}$:

$$\langle P|\overline{q}\gamma^\mu\gamma_5q'|0\rangle=if_Pp_P^\mu, \quad \langle V|\overline{q}\gamma^\mu q'|0\rangle=e^\mu m_Vf_V. \quad (8)$$

The matrix elements of the weak current J between meson states have the covariant decomposition [1]

$$\langle P(p_P')|\overline{q}\gamma_\mu b|B(p_B)\rangle=\left[(p_B+p_P')_\mu-\frac{m_B^2-m_P^2}{q^2}q_\mu\right]F_1(q^2)+\frac{m_B^2-m_P^2}{q^2}q_\mu F_0(q^2), \quad (9)$$

$$\langle V(p_V')|\overline{q}\gamma_\mu b|B(p_B)\rangle=\frac{2V(q^2)}{m_B+m_V}i\epsilon_{\mu\nu\tau\sigma}e^{*\nu}p_B^\tau p_V'^{\sigma}, \quad (10)$$

$$\langle V(p_V')|\bar{q}\gamma_\mu\gamma_5 b|B(p_B)\rangle=(m_B+m_V)e^*_\mu A_1(q^2)-\frac{A_2(q^2)}{m_B+m_V}(e^*q)(p_B+p_V')_\mu-2m_V\frac{(e^*q)}{q^2}q_\mu A_3(q^2)+2m_V\frac{(e^*q)}{q^2}q_\mu A_0(q^2), \quad (11)$$

where $q=p_B-p'_{P(V)}$ and e is a polarization vector of the vector meson. The form factor $A_3(q^2)$ is the linear combination

$$A_3(q^2)=\frac{m_B+m_V}{2m_V}A_1(q^2)-\frac{m_B-m_V}{2m_V}A_2(q^2), \quad (12)$$

and in order to cancel the poles at $q^2=0$, it is necessary to require

$$F_1(0)=F_0(0), \quad A_3(0)=A_0(0). \quad (13)$$

We calculate the corresponding form factors in the framework of the relativistic quark model.

III. RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [13] of the Schrödinger-type [14],

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \quad (14)$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_a^2-m_b^2)^2}{4M^3}, \quad (15)$$

and

$$b^2(M)=\frac{[M^2-(m_a+m_b)^2][M^2-(m_a-m_b)^2]}{4M^2}. \quad (16)$$

Here $m_{a,b}$ are quark masses, M is the meson mass, and $\mathbf{p}$ is the relative momentum of quarks. While constructing the kernel of this equation $V(\mathbf{p},\mathbf{q};M)$ — the quasipotential of quark-antiquark interaction — we have assumed that the effective interaction is the sum of the one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials. We have also assumed that the vector confining potential contains the Pauli interaction. The quasipotential is defined by [15]

$$V(\mathbf{p},\mathbf{q},M)=\bar{u}_a(p)\bar{u}_b(-p)\mathcal{V}(\mathbf{p},\mathbf{q},M)u_a(q)u_b(-q)=\bar{u}_a(p)\bar{u}_b(-p)\Big\{\tfrac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_a^\mu\gamma_b^\nu+V^V_{\rm conf}(\mathbf{k})\Gamma_a^\mu\Gamma_{b;\mu}+V^S_{\rm conf}(\mathbf{k})\Big\}u_a(q)u_b(-q), \quad (17)$$

where α_S is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator, γ_μ and $u(p)$ are the Dirac matrices and spinors, and $\mathbf{k=p-q}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \quad k^0=0, \quad (18)$$

where κ is the Pauli interaction constant. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\rm conf}(r)=(1-\varepsilon)(Ar+B), \quad V^S_{\rm conf}(r)=\varepsilon(Ar+B), \quad (19)$$

reproducing $V^{\rm conf}_{\rm nonrel}(r)=V^S_{\rm conf}+V^V_{\rm conf}=Ar+B$, where ε is the mixing coefficient. The explicit expression for the quasipotential with the account of relativistic corrections of order v^2/c^2 can be found in Ref. [15]. All the parameters of our model like quark masses, parameters of linear confining potential A and B, mixing coefficient ε, and anomalous chromomagnetic quark moment κ were fixed from the analysis of meson masses [15] and radiative decays [16]. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.50$ GeV, $m_{u,d}=0.33$ GeV, and parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.30$ GeV have standard values for quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon=-1$ has been chosen from the consideration of the heavy quark expansion [8] and meson radiative decays [16], which are very sensitive to the Lorentz structure of the confining potential: the resulting leading relativistic corrections coming from vector and scalar potentials have opposite signs for the radiative $M1$ decays [16]. Finally, the universal Pauli interaction constant $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J states [15].

The meson wave functions in the rest frame have been calculated by numerical solution of the quasipotential equation (14) [17]. However, it is more convenient to use analytical expressions for meson wave functions. The examination of numerical results for the ground state wave functions of mesons containing at least one light quark has shown that they can be well approximated in the meson rest frame by the Gaussian functions

$$\Psi_M(\mathbf{p})=\left(\frac{4\pi}{\beta_M^2}\right)^{3/4}\exp\left(-\frac{\mathbf{p}^2}{2\beta_M^2}\right), \quad (20)$$

with the deviation less than 5%.

The parameters are

$$\beta_B=0.41 \text{ GeV}, \quad \beta_D=0.38 \text{ GeV}, \quad \beta_{D_s}=0.44 \text{ GeV},$$

$$\beta_{\pi(\rho)}=0.31 \text{ GeV}.$$

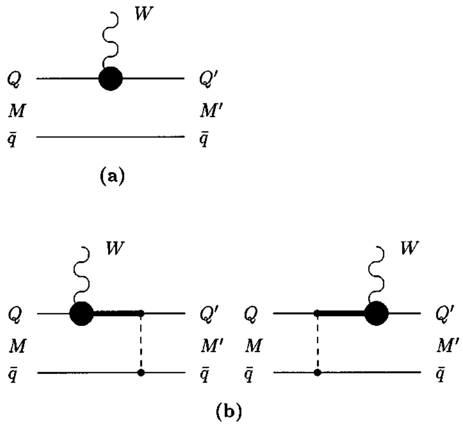

FIG. 1. (a) The lowest order vertex function $\Gamma_\mu^{(1)}$. (b) The vertex function $\Gamma_\mu^{(2)}$ with the account of the quark interaction. The dashed line corresponds to the effective potential (17). The bold line denotes the negative-energy part of the quark propagator.

The matrix element of the local current J between bound states in the quasipotential method has the form [18]

$$\langle M'|J_\mu(0)|M\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\overline{\Psi}^{b'}_{M'}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^b_M(\mathbf{q}), \tag{21}$$

where $M(M')$ is the initial (final) meson, $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function, and $\Psi^{b,b'}_{M,M'}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame.

This relation is valid for the general structure of the current $J_\mu = \overline{Q'}G_\mu Q$, where G_μ can be an arbitrary combination of Dirac matrices. The contributions to Γ come from Figs. 1(a) and 1(b). Thus the vertex functions look like

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \overline{u}_{Q'}(p_1)G_\mu u_Q(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2), \tag{22}$$

and

$$\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q}) = \overline{u}_{Q'}(p_1)\overline{u}_q(p_2) \times\left\{ G_\mu \frac{\Lambda_Q^{(-)}(k_1)}{\varepsilon_Q(k_1)+\varepsilon_Q(p_1)}\gamma_1^0\mathcal{V}(\mathbf{p}_2-\mathbf{q}_2) + \mathcal{V}(\mathbf{p}_2-\mathbf{q}_2)\frac{\Lambda_{Q'}^{(-)}(k_1')}{\varepsilon_{Q'}(k_1')+\varepsilon_{Q'}(q_1)}\gamma_1^0 G_\mu\right\} \times u_Q(q_1)u_q(q_2), \tag{23}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1(a) and 1(b), $\mathbf{k}_1=\mathbf{p}_1-\mathbf{\Delta}$, $\mathbf{k}_1'=\mathbf{q}_1+\mathbf{\Delta}$, $\mathbf{\Delta}=\mathbf{p}_M-\mathbf{p}_{M'}$, $\varepsilon(p)=(m^2+\mathbf{p}^2)^{1/2}$,

$$\Lambda^{(-)}(p)=\frac{\varepsilon(p)-[m\gamma^0+\gamma^0(\boldsymbol{\gamma}\mathbf{p})]}{2\varepsilon(p)},$$

and

$$p_{1,2}=\varepsilon_{1,2}(p)\frac{p_{M'}}{M'}\pm\sum_{i=1}^{3} n^{(i)}(p_{M'})p^i,$$

$$q_{1,2}=\varepsilon_{1,2}(q)\frac{p_M}{M}\pm\sum_{i=1}^{3} n^{(i)}(p_M)q^i,$$

here

$$n^{(i)\mu}(p)=L^\mu_{p_i}=\left\{\frac{p^i}{M},\delta_{ij}+\frac{p^ip^j}{M(E+M)}\right\},\quad E=\sqrt{p^2+M^2}.$$

Note that the contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. The form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of $q\overline{q}$ interaction.

The general structure of the current matrix element (21) is rather complicated, because it is necessary to integrate both with respect to d^3p and d^3q. The δ function in the expression for the vertex function $\Gamma^{(1)}$ permits one to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has the usual structure and can be calculated without any expansion, if the wave functions of initial and final mesons are known. The situation with the contribution of $\Gamma^{(2)}$ is different. Here, instead of the δ function, we have a complicated structure, containing the potential of the $q\overline{q}$ interaction in a meson. Thus, in general case, we cannot perform one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (21). Therefore, it is necessary to use some additional considerations. The main idea is to expand the vertex function $\Gamma^{(2)}$ in such a way that it will be possible to use the quasipotential equation (14) in order to perform one of the integrations in the current matrix element (21). The realization of such expansion differs for the cases of heavy-to-heavy ($B\to D^{(*)}$) and heavy-to-light [$B\to\pi(\rho)$] transitions.

IV. $B\to D^{(*)}$ DECAY FORM FACTORS

In the case of the heavy-to-heavy ($B\to D^{(*)}$) meson decays we have two natural expansion parameters, which are the heavy quark masses (m_b and m_c) in the initial and final meson. The most convenient point for the expansion of vertex function $\Gamma^{(2)}$ in inverse powers of the heavy quark masses is the point of zero recoil of the final D meson, where $\mathbf{\Delta}=0$ ($\mathbf{\Delta}=\mathbf{p}_B-\mathbf{p}_{D^{(*)}}$). It is easy to see that $\Gamma^{(2)}$ contributes to the current matrix element at first order of the $1/m_Q$ expansion. We limit our analysis to the consideration of the terms up to the second order. After the expansion we perform the integrations in the contribution of $\Gamma^{(2)}$ to the decay matrix element. As a result we get the expression for the current matrix element, which contains the ordinary mean values between meson wave functions and can be easily calculated numerically. The results of such calculation are given in comparison with the predictions of HQET [7] in [8]. Our model satisfies all the constraints imposed on the form factors by heavy quark symmetries and allows the determination of the Isgur-Wise and subleading form factors [8].

The q^2 dependence of form factors at leading order of the $1/m_Q$ expansion is given by

$$RF_1(q^2)=R\frac{F_0(q^2)}{1-\frac{q^2}{m_B+m_D}}=R^*V(q^2)=R^*A_0(q^2)$$
$$=R^*\frac{A_1(q^2)}{1-\frac{q^2}{m_B+m_{D^*}}}=R^*A_2(q^2)=\xi(w), \quad (24)$$

where

$$R^{(*)}=\frac{2\sqrt{m_Bm_{D^{(*)}}}}{m_B+m_{D^{(*)}}}, \quad w=\frac{m_B^2+m_{D^{(*)}}^2-q^2}{2m_Bm_{D^{(*)}}}.$$

The Isgur-Wise function in our model is

$$\xi(w)=\sqrt{\frac{2}{w+1}}\exp\left[-\left(2\rho^2-\frac{1}{2}\right)\frac{w-1}{w+1}\right], \quad (25)$$

with the slope parameter $\rho^2\simeq1.02$, which is in accordance with the recent CLEO II measurement [12] $\rho^2=1.01\pm0.15\pm0.09$.

At the first order of the $1/m_Q$ expansion four additional independent form factors arise in HQET [7]. We have determined these subleading form factors in the framework of our model [8]

$$\xi_3(w)=(\bar\Lambda-m_q)\left(1+\frac{2}{3}\frac{w-1}{w+1}\right)\xi(w),$$

$$\chi_1(w)=\bar\Lambda\frac{w-1}{w+1}\xi(w),$$

$$\chi_2(w)=-\frac{1}{32}\frac{\bar\Lambda}{w+1}\xi(w),\chi_3(w)=\frac{1}{16}\bar\Lambda\frac{w-1}{w+1}\xi(w), \quad (26)$$

where the HQET parameter $\bar\Lambda=M-m_Q$ in our model is equal to the mean value of the light quark energy in the heavy meson $\bar\Lambda=\langle\varepsilon_q\rangle\simeq0.54$ GeV.

We have also calculated the second order power corrections at the point of zero recoil of the final meson [8]. The obtained structure of the second order corrections is in accord with predictions of HQET [19]. As a result we got the values of the $B\to D^{(*)}$ transition form factors at $q^2=q^2_{\max}$ up to the second order terms. The higher order terms of the $1/m_Q$ expansion are negligibly small. However, for the consideration of the energetic nonleptonic decays of B mesons we need the form factor values at $q^2=m_f^2\approx0$ ($f=\pi,\rho,\ldots$ is a final light particle). Thus it is necessary to evolve the form factors from $q^2_{\max}$ to $q^2\approx0$. The corresponding w range is not very wide (from 1 to ~1.6). For such w values the form factors are dominated by the Isgur-Wise function [20]. Some small contributions may arise from subleading form factors. The higher order terms give very small corrections [20]. Therefore, we combine the universal Isgur-Wise q^2 dependence of form factors (24) with the subleading symmetry-breaking corrections (26). The resulting form factor w dependence is shown in Figs. 2 and 3.

The values of form factors at $q^2=0$ are

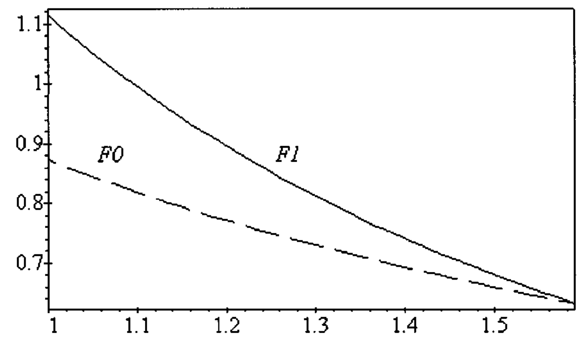

FIG. 2. The w dependence of form factors of $B\to D$ transitions.

$$F_1(0)=F_0(0)=0.63, \quad V(0)=0.79,$$
$$A_0(0)=0.63, \quad A_1(0)=0.62, \quad A_2(0)=0.61. \quad (27)$$

V. $B\to\pi(\rho)$ DECAY FORM FACTORS

In the case of heavy-to-light decays the final meson contains only light quarks (u, d). Thus, in contrast with the heavy-to-heavy transitions, we cannot expand matrix elements in inverse powers of the final quark mass. The expansion of $\Gamma^{(2)}$ only in inverse powers of the initial heavy quark mass at $\mathbf{\Delta}=0$ does not solve the problem. However, the final light meson has the large recoil momentum, in comparison with its mass, almost in the whole kinematical range. At the point of maximum recoil of the final light meson the large value of recoil momentum $|\mathbf{\Delta}_{\max}|\sim m_b/2$ allows for the expansion of decay matrix element in $1/m_b$. The contributions to this expansion come both from the inverse powers of heavy m_b from the initial B meson and from inverse powers of the recoil momentum $|\mathbf{\Delta}_{\max}|$ of the final light $\pi(\rho)$ meson. In Ref. [9] we carried out this expansion up to the second order and performed one of the integrations in the current matrix element (21), using the quasipotential equation as in the case of a heavy final meson. As a result we again get the expression for the current matrix element, which contains only the ordinary mean values between meson wave functions, but in this case at the point of maximum recoil of the final light meson.

The found values of $B\to\pi(\rho)$ form factors at the point of maximum recoil ($q^2=0$) are

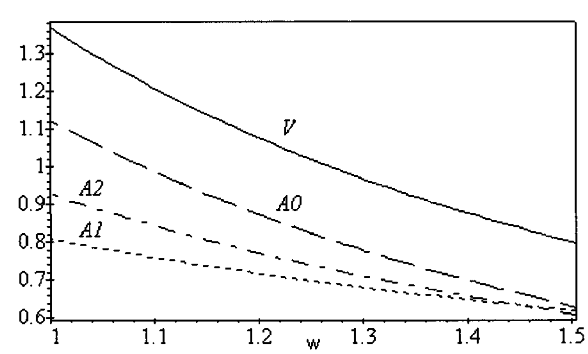

FIG. 3. The w dependence of form factors of $B\to D^*$ transitions.

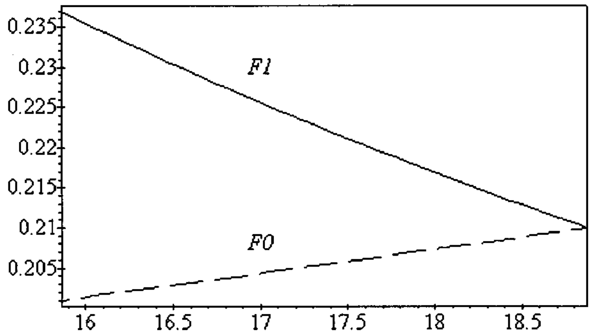

FIG. 4. The w dependence of form factors of $B\to\pi$ transitions in the kinematical region of interest for the energetic nonleptonic decays.

$$F_1(0)=F_0(0)=0.21,\quad V(0)=0.29,$$

$$A_0(0)=0.18,\quad A_1(0)=0.27,\quad A_2(0)=0.30. \tag{28}$$

The q^2 behavior of the heavy-to-light form factors near $|\mathbf{\Delta}_{\max}|$ (corresponding to $q^2=0$) is given by [9]

$$F_1(q^2)=\frac{m_B+m_\pi}{2\sqrt{m_Bm_\pi}}\tilde{\xi}(w)\mathcal{F}_1(\mathbf{\Delta}^2_{\max}), \tag{29}$$

$$F_0(q^2)=\frac{2\sqrt{m_Bm_\pi}}{m_B+m_\pi}\frac{1}{2}(1+w)\,\tilde{\xi}(w)\mathcal{F}_0(\mathbf{\Delta}^2_{\max}), \tag{30}$$

$$A_1(q^2)=\frac{2\sqrt{m_Bm_\rho}}{m_B+m_\rho}\frac{1}{2}(1+w)\,\tilde{\xi}(w)\mathcal{A}_1(\mathbf{\Delta}^2_{\max}), \tag{31}$$

$$A_{0,2}(q^2)=\frac{m_B+m_\rho}{2\sqrt{m_Bm_\rho}}\tilde{\xi}(w)\mathcal{A}_{0,2}(\mathbf{\Delta}^2_{\max}), \tag{32}$$

$$V(q^2)=\frac{m_B+m_\rho}{2\sqrt{m_Bm_\rho}}\tilde{\xi}(w)\mathcal{V}(\mathbf{\Delta}^2_{\max}), \tag{33}$$

where we have introduced the function [9]

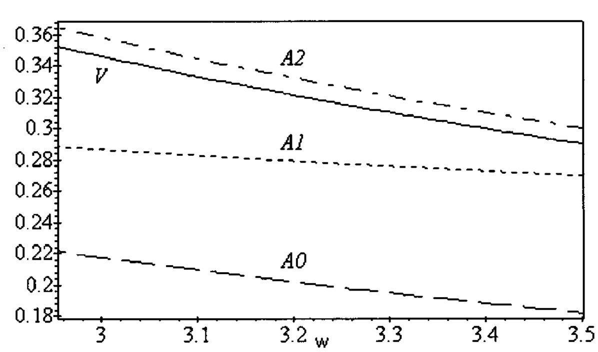

FIG. 5. The w dependence of form factors of $B\to\rho$ transitions in the kinematical region of interest for the energetic nonleptonic decays.

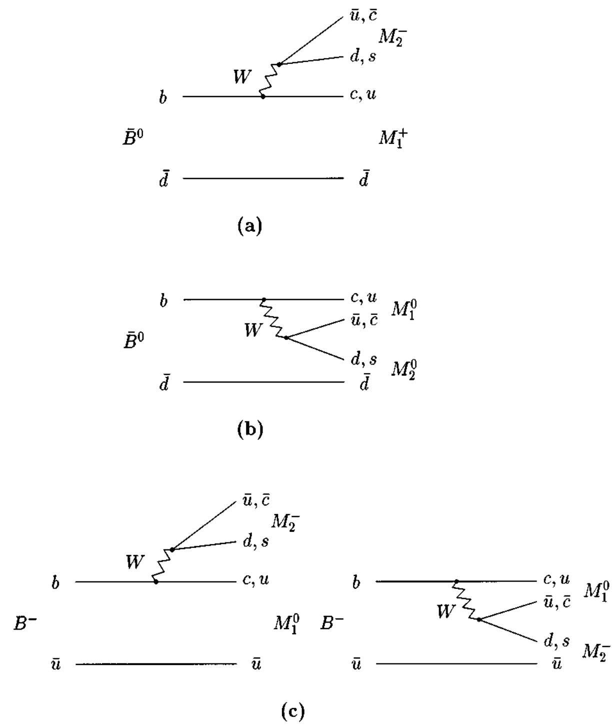

FIG. 6. Quark diagrams for two-body nonleptonic B decays: (a) "class I" decays $\overline{B^0}\to M_1^+M_2^-$; (b) "class II" decays $\overline{B^0}\to M_1^0M_2^0$; (c) "class III" decays $B^-\to M_1^0M_2^-$.

$$\tilde{\xi}(w)=\sqrt{\frac{2}{w+1}}\exp\left(-\eta\frac{\tilde{\Lambda}^2}{\beta_B^2}\frac{w-1}{w+1}\right),$$

$$\eta=\frac{2\beta_B^2}{\beta_B^2+\beta^2_{\pi(\rho)}},\quad \tilde{\Lambda}\approx 0.53\ \text{GeV}. \tag{34}$$

Equation (34) reduces to the Isgur-Wise function (25) in the limit of infinitely heavy quarks in the initial and final mesons.

It is important to note that the form factors A_1 and F_0 in Eqs. (31) and (30) have a different q^2 dependence than those of the other form factors (29), (32), and (33). This result is in contradiction with the single pole parametrization [21]. The inconsistency of combination of the pole form factors and the scaling of $B\to\pi(\rho)$ form factors with the large b-quark mass at zero recoil of a light meson has been shown in Refs. [22–24] on the basis of HQET. The form factor dependence on the heavy b-quark mass at this kinematical point is given by [22]

$$F_1(q^2_{\max})\sim m_b^{1/2},\quad F_0(q^2_{\max})\sim m_b^{-1/2},$$

$$A_1(q^2_{\max})\sim m_b^{-1/2},\quad A_2(q^2_{\max})\sim m_b^{1/2},$$

$$V(q^2_{\max})\sim m_b^{1/2}. \tag{35}$$

It is easy to see from Eqs. (29)–(34) that our model explicitly reproduces these scaling relations. The recent QCD sum rule analysis [25–27] and the light-front quark model calcu-

TABLE I. Predicted branching ratios for $B\to D^{(*)}M$ nonleptonic decays in terms of a_1 and a_2 (in percent). Our model branching ratios are quoted for values of $a_1=1.05$ and $a_2=0.25$ in comparison with experimental data (in percent). We use the values $|V_{cb}|=0.038$ and $f_D=f_{D^*}=0.220$ GeV, $f_{D_s}=f_{D_s^*}=0.260$ GeV, $f_{a_1}=0.205$ GeV [12] for our estimates.

Decay	Our result	[28]	Our result	Experiment [31]
$\overline{B^0}\to D^+\pi^-$	$0.29a_1^2$	$0.264a_1^2$	0.32	$0.31\pm0.04\pm0.02$
$\overline{B^0}\to D^+\rho^-$	$0.79a_1^2$	$0.621a_1^2$	0.87	$0.84\pm0.16\pm0.05$
$\overline{B^0}\to D^{*+}\pi^-$	$0.26a_1^2$	$0.254a_1^2$	0.28	$0.28\pm0.04\pm0.01$
$\overline{B^0}\to D^{*+}\rho^-$	$0.81a_1^2$	$0.702a_1^2$	0.88	$0.73\pm0.15\pm0.03$
$\overline{B^0}\to D^+a_1^-$	$0.78a_1^2$	$0.673a_1^2$	0.86	$0.60\pm0.22\pm0.24$
$\overline{B^0}\to D^{*+}a_1^-$	$0.92a_1^2$	$0.970a_1^2$	1.02	$1.27\pm0.30\pm0.05$
$\overline{B^0}\to D^+D_s$	$1.37a_1^2$	$1.213a_1^2$	1.51	$0.74\pm0.22\pm0.18$
$\overline{B^0}\to D^+D_s^{*-}$	$0.685a_1^2$	$0.859a_1^2$	0.75	$1.14\pm0.42\pm0.28$
$\overline{B^0}\to D^{*+}D_s^-$	$0.82a_1^2$	$0.824a_1^2$	0.90	$0.94\pm0.24\pm0.23$
$\overline{B^0}\to D^{*+}D_s^{*-}$	$2.50a_1^2$	$2.203a_1^2$	2.75	$2.00\pm0.54\pm0.49$
$\overline{B^0}\to D^0\pi^0$	$0.058a_2^2$	$0.20a_2^2$	0.0036	<0.048
$\overline{B^0}\to D^{*0}\pi^0$	$0.056a_2^2$	$0.21a_2^2$	0.0035	<0.097
$\overline{B^0}\to D^0\rho^0$	$0.053a_2^2$	$0.14a_2^2$	0.0033	<0.055
$\overline{B^0}\to D^{*0}\rho^0$	$0.156a_2^2$	$0.22a_2^2$	0.0098	<0.117
$B^-\to D^0\pi^-$	$0.29(a_1+0.64a_2)^2$	$0.265(a_1+1.230a_2)^2$	0.43	$0.50\pm0.05\pm0.02$
$B^-\to D^{*0}\pi^-$	$0.27(a_1+0.69a_2)^2$	$0.255(a_1+1.292a_2)^2$	0.40	$0.52\pm0.08\pm0.02$
$B^-\to D^0\rho^-$	$0.81(a_1+0.36a_2)^2$	$0.622(a_1+0.662a_2)^2$	1.06	$1.37\pm0.18\pm0.05$
$B^-\to D^{*0}\rho^-$	$0.83(a_1^2+0.39a_2^2 +1.15a_1a_2)$	$0.703(a_1^2+0.635a_2^2 +1.487a_1a_2)$	1.17	$1.51\pm0.30\pm0.06$
$B^-\to D^0D_s^-$	$1.40a_1^2$	$1.215a_1^2$	1.55	$1.36\pm0.28\pm0.33$
$B^-\to D^0D_s^{*-}$	$0.70a_1^2$	$0.862a_1^2$	0.77	$0.94\pm0.31\pm0.23$
$B^-\to D^{*0}D_s^-$	$0.84a_1^2$	$0.828a_1^2$	0.92	$1.18\pm0.36\pm0.29$
$B^-\to D^{*0}D_s^{*-}$	$2.56a_1^2$	$2.206a_1^2$	2.80	$2.70\pm0.81\pm0.66$

lations [24] also indicate that the q^2 dependence of the form factors A_1 and F_0 is different from those of other form factors.

The extrapolation of the q^2 dependence (29)–(33) to all values of q^2 (or w), accessible in $B\to\pi(\rho)$ transitions, introduces rather large uncertainties, because w varies in a broad kinematical range (from 1 to $\sim$19 in $B\to\pi$ and from 1 to $\sim$3.5 in $B\to\rho$). However, the w values for $B\to\pi(\rho)$ form factors which are really necessary for the consideration of energetic nonleptonic decays are limited to a rather small interval near $w_{\max}$ $(q^2=0)$. Thus, the application of formulas (29)–(33) in this region is rather reliable. We show the w dependence of $B\to\pi(\rho)$ decay form factors in Figs. 4 and 5.

VI. RESULTS AND DISCUSSION

In the factorization approximation one can distinguish three classes of B meson nonleptonic decays (see Fig. 6) [1]: the ''class I'' transitions, such as $\overline{B^0}\to M_1^+M_2^-$, where only the term with a_1 in Eq. (4) contributes (i.e., both mesons are produced by charged currents); ''class II'' transitions, such as $\overline{B^0}\to M_1^0M_2^0$, where only the term with a_2 in Eq. (4) contributes (i.e., both mesons are produced by neutral currents), and ''class III'' transitions, such as $B^-\to M_1^0M_2^-$, where both terms can contribute coherently.

The results of the calculation of the nonleptonic branching ratios for $B\to D^{(*)}$ and $B\to\pi(\rho)$ transitions are given in Tables I and II in comparison with other model predictions [28–30] and experimental data. The $B\to D^{(*)}D_s^{(*)}$ decay branching ratios are presented for completeness.

We see that our results for the ''class I'' nonleptonic $B\to D^{(*)}\pi(\rho)$ decays are close to the improved Bauer-Stech-Wirbel (BSW) model predictions [28], while our results for the ''class II'' and a_2 contributions to ''class III'' decays are smaller than those of [28]. These contributions come from $B\to\pi(\rho)$ transition form factors, which have a different q^2 behavior in our and the Bauer-Stech-Wirbel (BSW) models. The BSW model assumes universal q^2 dependence of all $B\to\pi(\rho)$ form factors. As already mentioned in our model we find the A_1 and F_0 form factors to have a different q^2 dependence than those of the other form factors [see Eqs. (30)–(33) and Figs. 4 and 5]. The form factor F_0 in our model decreases with the growing of q^2 (decreasing of w) in the kinematical range of interest for energetic nonleptonic decays (see Fig. 4). Note that our value for $B\to\pi$ form factors at $q^2=0$ is approximately 1.5 times less than that of BSW, while the values for $B\to\rho$ form factors are close in both models.

Our predictions for the branching ratios of $B\to D^{(*)}M$ nonleptonic decays presented in Table I agree with experimental data within errors. Thus we can conclude that factorization works rather well for ''class I'' and ''class III'' de-

TABLE II. Predicted branching ratios for $B\to\pi(\rho)M$ nonleptonic decays. We use the experimental values for f_π, f_ρ [12] and the value of $|V_{ub}|=0.0052$ [9] for our model estimates. All numbers are branching ratios $\times 10^5$.

Decay	Our result	Our result	[29]	[30]	Experiment [32]
$\overline{B^0}\to\pi^+\pi^-$	$0.331\vert V_{ub}\vert^2a_1^2$	0.99	1.8		<2.0
$\overline{B^0}\to\pi^+\rho^-$	$0.857\vert V_{ub}\vert^2a_1^2$	2.55	4.8		
$\overline{B^0}\to\rho^+\pi^-$	$0.234\vert V_{ub}\vert^2a_1^2$	0.70	0.4		
$\overline{B^0}\to\pi^\pm\rho^\mp$	$1.09\vert V_{ub}\vert^2a_1^2$	3.25	5.2		<8.8
$\overline{B^0}\to\rho^+\rho^-$	$0.794\vert V_{ub}\vert^2a_1^2$	2.36	1.3		<220
$\overline{B^0}\to\pi^0\pi^0$	$0.17\vert V_{ub}\vert^2a_2^2$	0.028	0.06		<0.91
$\overline{B^0}\to\pi^0\rho^0$	$0.54\vert V_{ub}\vert^2a_2^2$	0.092	0.14		<2.4
$\overline{B^0}\to\rho^0\rho^0$	$0.39\vert V_{ub}\vert^2a_2^2$	0.067	0.05		<28
$\overline{B^0}\to D_s^-\pi^+$	$0.285\vert V_{ub}\vert^2a_1^2$	0.85	8.1	1.9	<28
$\overline{B^0}\to D_s^{*-}\pi^+$	$1.06\vert V_{ub}\vert^2a_1^2$	3.1	6.1	2.7	<50
$\overline{B^0}\to D_s^-\rho^+$	$0.269\vert V_{ub}\vert^2a_1^2$	0.80	1.2	1.0	<70
$\overline{B^0}\to D_s^{*-}\rho^+$	$2.25\vert V_{ub}\vert^2a_1^2$	6.7	4.5	5.4	<80
$B^-\to\pi^0\pi^-$	$0.169\vert V_{ub}\vert^2(a_1+a_2)^2$	0.78	1.4		<1.7
$B^-\to\pi^0\rho^-$	$0.438\vert V_{ub}\vert^2(a_1+0.52a_2)^2$	1.7	2.7		<7.7
$B^-\to\rho^0\pi^-$	$0.120\vert V_{ub}\vert^2(a_1+1.95a_2)^2$	0.77	0.7		<4.3
$B^-\to\rho^0\rho^-$	$0.411\vert V_{ub}\vert^2(a_1+a_2)^2$	1.8	1.1		<100
$B^-\to D_s^-\pi^0$	$0.146\vert V_{ub}\vert^2a_1^2$	0.44	3.9	1.8	<20
$B^-\to D_s^{*-}\pi^0$	$0.545\vert V_{ub}\vert^2a_1^2$	1.6	3.0	1.3	<33
$B^-\to D_s^-\rho^0$	$0.138\vert V_{ub}\vert^2a_1^2$	0.41	0.6	0.5	<40
$B^-\to D_s^{*-}\rho^0$	$1.16\vert V_{ub}\vert^2a_1^2$	3.4	2.4	2.8	<50

cays $B\to D^{(*)}\pi(\rho)$. However, an improvement of experimental accuracy is needed to make a definite conclusion. It will be very interesting to measure the ''class II'' decay $\overline{B^0}\to D^{(*)0}\pi(\rho)^0$ branching ratios. Such measurement will be the test of factorization for ''class II'' nonleptonic decays and will help to constrain the w (or q^2) dependence of $B\to\pi(\rho)$ form factors.

We also present in Table I the predictions for $B\to D^{(*)}D_s^{(*)}$ nonleptonic decays, where only heavy mesons are present in the final state. The factorization is less justified for such decays. However, as we see from Table I our predictions, based on the factorization, are consistent with the experimental data for these decays too.

For the branching ratios of $B\to\pi(\rho)M$ nonleptonic decays presented in Table II there are only experimental upper limits at present. The measurement of these decays will allow the determination of the CKM matrix element $|V_{ub}|$, which is poorly studied. The closest experimental upper limit to the theoretical predictions is for the decay $\overline{B^0}\to\pi^+\pi^-$. It is approximately two times larger than our prediction and is very close to the result of [29]. From this upper limit on $B(\overline{B^0}\to\pi^+\pi^-)$ we get the limit on $|V_{ub}|$ in our model,

$$|V_{ub}|<7.4\times 10^{-3},$$

which is close to the value previously found from semileptonic $B\to\pi(\rho)l\nu$ decays [9]:

$$|V_{ub}|=(5.2\pm 1.3\pm 0.5)\times 10^{-3}.$$

VII. CONCLUSIONS

In this paper we have calculated the branching ratios of the energetic exclusive nonleptonic decays of B mesons on the basis of the factorization approximation. In particular, the form factors of $B\to D^{(*)}$ and $B\to\pi(\rho)$ transitions have been evaluated using the relativistic quark model and the heavy quark expansion. Such expansion has been carried out up to the second order in the heavy quark masses. Finally, the momentum dependence of leading and subleading terms of this expansion has been used for the determination of the heavy-to-heavy transition form factors at $q^2=m_f^2$, where f is a final light meson.

The overall agreement of our predictions for two-body nonleptonic decays of B mesons with the existing experimental data [31,32] shows that the factorization approximation works sufficiently well in the framework of our model. From another side, if the factorization hypothesis is taken for granted, the aforementioned agreement confirms the self-consistency of our approach, which incorporates our previously obtained results for semileptonic and leptonic decays of heavy mesons. In particular, it would be quite interesting to test the specific q^2 behavior of the heavy-to-light transition form factors F_0 and A_1 predicted by our model. Another important problem is the determination of the coefficients a_1 and a_2 via c_1 and c_2 directly from QCD. As it has been discussed already in Sec. II their values found in Ref. [4] are close to those obtained from fitting experimental data, though one should mention that c_2 is rather unstable with respect to the renormalization scheme and scale. Nevertheless, that means from our point of view that the factorization hypothesis for energetic B meson decays has more or less firm grounds within QCD, at least for the ''class I'' decays.

The situation is essentially different for D meson nonleptonic decays. In this case the best fit to the experimental data yields $a_1=1.26\pm0.10$ and $a_2=-0.51\pm0.10$ [4,12]. Meanwhile, QCD predicts [4] (the instability of the coefficient here is even stronger) $c_1=1.31\pm0.19$ and $c_2=-0.50\pm0.30$, which can only be consistent with the result of fitting data if one drops the $1/N_c$ terms in Eq. (5) and puts $a_1\simeq c_1$ and $a_2\simeq c_2$. Clearly, such an assumption would give a completely wrong result for B decays, namely, a negative sign of a_2, which is ruled out by the experimental data. This result could indicate that the factorization approach in D decays is insufficient and nonfactorizable contributions there are large. In other words the D meson is possibly not heavy enough compared to the B meson. The results of Ref. [3] seem to point in the same direction.

The successful description of nonleptonic two-body decays of B mesons makes the present approach appealing for the further consideration of $B_{s,c}$ meson decays. Work in this direction is in progress.

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