

Analyticity, shapes of semileptonic form factors, and $\bar{B} \rightarrow \pi l \bar{\nu}$

C. Glenn Boyd*

Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213

Martin J. Savage†

Department of Physics, University of Washington, Seattle, Washington 98195

(Received 18 February 1997)

We give a pedagogical discussion of the physics underlying dispersion relation-derived parametrizations of form factors describing $B \rightarrow \pi l \bar{\nu}$ and $B \rightarrow D l \bar{\nu}$. Moments of the dispersion relations are shown to provide substantially tighter constraints on the $f_+(t)$ form factor describing $\bar{B} \rightarrow \pi l \bar{\nu}$ than the unweighted dispersion relation alone. Heavy quark spin symmetry relations between the $B \rightarrow \pi l \bar{\nu}$ and $B^* \rightarrow \pi l \bar{\nu}$ form factors enable such constraints to be tightened even further. [S0556-2821(97)01513-0]

PACS number(s): 13.20.He, 12.39.Hg

INTRODUCTION

Exclusive semileptonic decays of heavy mesons play an important role in the determination and overconstraining of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The CKM element V_{cb} has been extracted [1] from $\bar{B} \rightarrow D^* l \bar{\nu}$ and $\bar{B} \rightarrow D l \bar{\nu}$ using heavy quark symmetry [2], while the element V_{ub} has been estimated from $\bar{B} \rightarrow \pi l \bar{\nu}$ and $\bar{B} \rightarrow \rho l \bar{\nu}$ rates [3] using various models. In both cases, the normalization and shape of the relevant hadronic form factors influence the extracted value of the CKM angle. For V_{cb} , the normalization of the $B \rightarrow D^{(*)}$ matrix element at zero recoil is provided by heavy quark symmetry. However, typical extrapolations to this point use *ad hoc* parametrizations of form factors that introduce theoretical uncertainties comparable to the statistical uncertainties [4,5]. This is especially unfortunate since the uncertainty in V_{cb} feeds into unitarity-triangle constraints from CP violation observed in the kaon system as the fourth power [6]. For V_{ub} neither the normalization nor the shape is well known. The normalization near zero recoil may be obtained from lattice simulations or by combining heavy quark and chiral symmetries with measurements of related semileptonic decays in the charmed and bottom sector [7], but a parametrization away from zero recoil is necessary to compare to experimental data.

Some progress in describing the shape of such form factors has recently been made in the form of model-independent parametrizations [4,8] based on QCD dispersion relations and analyticity [9,10]. These dispersion relations lead to an infinite tower of upper and lower bounds that can be derived by using the normalizations of the form factor $F(t_i)$ at a fixed number of kinematic points t_i as input [10–12]. When the normalization is known at several points (say, five or more for $\bar{B} \rightarrow \pi l \bar{\nu}$), the upper and lower bounds are typically so tight they look like a single line. A natural ques-

tion then arises: What is the most general form consistent with the constraints from QCD? The answer to this question is the parametrization of [4]. For a generic form factor $F(t)$ describing the exclusive semileptonic decay of a \bar{B} meson to a final state meson H as a function of momentum-transfer squared $t = (p_B - p_H)^2$, the parametrization takes the form

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{k=0}^{\infty} a_k z(t; t_0)^k, \quad (1)$$

where $\phi(t)$ is a computable function arising from perturbative QCD. The function $P(t)$ depends only on the masses of mesons below the $\bar{B}\bar{H}$ pair production threshold that contribute to $\bar{B}\bar{H}$ pair-production as virtual intermediate states. The variable $z(t; t_0)$ is a kinematic function of t defined by

$$\frac{1 + z(t; t_0)}{1 - z(t; t_0)} = \sqrt{\frac{t_+ - t}{t_+ - t_0}}, \quad (2)$$

where $t_+ = (M_B + M_H)^2$ is the pair-production threshold and t_0 is a free parameter that is often [4,9–12] taken to be $t_- = (M_B - M_H)^2$, the maximum momentum-transfer squared allowed in the semileptonic decay $\bar{B} \rightarrow H l \bar{\nu}$. The coefficients a_k are unknown constants constrained to obey

$$\sum_{k=0}^{\infty} (a_k)^2 \leq 1. \quad (3)$$

The kinematic function $z(t; t_0)$ takes its minimal physical value z_{\min} at $t = t_-$, vanishes at $t = t_0$, and reaches its maximum z_{\max} at $t = 0$. Thus the sum $\sum a_k z^k$ is a series expansion about the kinematic point $t = t_0$. For $\bar{B} \rightarrow D^* l \bar{\nu}$ with $t_0 = t_-$, the maximum value of z is $z_{\max} = 0.06$, and the series in Eq. (1) can be truncated while introducing only a small error [4]. The value z_{\max} can be made even smaller by choosing an optimized value $0 \leq t_0 \leq t_-$ [8]. In that case, most form factors describing $\bar{B} \rightarrow D l \bar{\nu}$ and $\bar{B} \rightarrow D^* l \bar{\nu}$ can be parametrized with only one unknown constant to an accuracy of a few percent (assuming the normalization at zero recoil

*Electronic address: boyd@fermi.phys.cmu.edu

†Electronic address: savage@phys.washington.edu

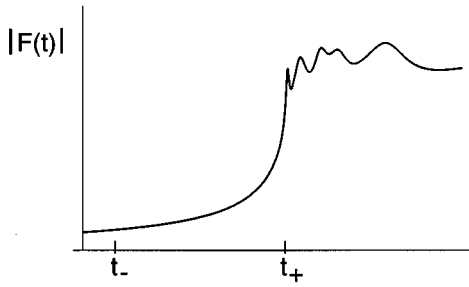


FIG. 1. The magnitude of a generic form factor $F(t)$ as a function of t . The pair-production threshold t_+ and the semileptonic end point t_- are shown schematically.

given by heavy quark symmetry). Thus the continuous function $F(t)$ has been reduced to a single constant, for example, the value of the form factor $F(t=0)$ at maximum recoil. For $\bar{B} \rightarrow \pi l \bar{\nu}$, the maximum value of z is $z_{\max} = 0.52$, but even in this case Eqs. (1) and (3) severely constrain the relevant form factor [11,12].

This remarkable constraining power can be traced to the existence of a naturally small parameter z_{\max} that arises algebraically from a conformal map. In this paper we attempt to trace the physical origin of z_{\max} in the hope of developing some intuition about the physics underlying the analyticity constraints of Eqs. (1) and (3). Further, we will incorporate two generalizations that lead to a significantly stronger constraint on the observable $\bar{B} \rightarrow \pi l \bar{\nu}$ form factor.

PHYSICAL BASIS FOR A SMALL PARAMETER

To understand heuristically why there is a small parameter associated with semileptonic heavy meson decays, consider for the moment a form factor $F(t)$ in the decay $\bar{B} \rightarrow D l \bar{\nu}$, and take $t_0 = t_-$. Crossing symmetry tells us the analytic continuation of the form factor $F(t)$ that describes semileptonic decay for $0 \leq t \leq t_-$ also describes $\bar{B}\bar{D}$ pair production for $t \geq t_+$. Figure 1 shows the general features one expects for $F(t)$ in the region $0 \leq t \leq \infty$. The form factor has a cut due to pair production beginning at $t = t_+$, as well as a series of poles from bound B_c -type states in the vicinity of t_+ . It varies rapidly near these poles, then falls smoothly from its peak values near $t \sim t_+$ to its minimum values near $t \sim 0$. It is not essential to our argument that the form factor decreases monotonically as t approaches zero, only that the variation in $F(t)$ over the semileptonic region $0 \leq t \leq t_-$ is determined by the distance to the branch cut $t = t_+$ and the magnitude of the form factor near the branch point, $F(t_+)$. For fixed $F(t_+)$, $F(t)$ varies more slowly over the semileptonic region as t_-/t_+ decreases, while for fixed t_-/t_+ , $F(t)$ also varies more slowly in the semileptonic region as $F(t_+)$ decreases. Both observation and QCD perturbation theory imply that the rate of $\bar{B}\bar{D}$ pair production cannot be arbitrarily large for $t > t_+$, and combined with the fact that $t_+ \gg t_-$ for $\bar{B} \rightarrow D l \bar{\nu}$, we expect the variation of $F(t)$ over the semileptonic region to be small. We wish to associate the small parameter z_{\max} with this variation.

Suppose the form factor can be roughly described in the physical semileptonic region by

$$F(t) \sim \frac{F_0}{(t_+ - t)^p}, \quad (4)$$

where F_0 is a constant. A reasonable measure of the variation of $F(t)$ over the physical region for the semileptonic decay is

$$\delta_F = \frac{F(t_-) - F(0)}{F(t_-) + F(0)} = \frac{t_+^p - (t_+ - t_-)^p}{t_+^p + (t_+ - t_-)^p}. \quad (5)$$

This measure depends only on the kinematic thresholds t_+, t_- and the power p . For $p=1$ the form factor is pole dominated and δ_F is similar to the Shifman-Voloshin parameter [13], $(M_B - M_D)^2 / (M_B + M_D)^2 \sim 1/4$. However, comparison with Eq. (2) reveals that δ_F can be identified with z_{\max} only if $p=1/2$ giving

$$\delta_F = \left(\frac{\sqrt{M_B} - \sqrt{M_H}}{\sqrt{M_B} + \sqrt{M_H}} \right)^2 = z_{\max}. \quad (6)$$

This value of p leads to the small value of z_{\max} for $B \rightarrow D$. Other decays such as $\bar{B} \rightarrow \rho l \bar{\nu}$, $D \rightarrow K^* l \bar{\nu}$, etc., have larger values of z_{\max} , with the largest occurring for $\bar{B} \rightarrow \pi l \bar{\nu}$. Even for this extreme case, $z_{\max} \approx 1/2$ is small enough to provide a useful expansion parameter.

On the face of it, this value of p seems rather surprising. After all, we know bound states exist and will contribute to form factors like poles. On the other hand, the dispersion relation relies on quark-hadron duality and perturbative QCD. In perturbative QCD the fundamental degrees of freedom are quarks and gluons, there are no bound states at any finite order in perturbation theory to couple to the pair-produced fermions, and the form factor has no poles. Indeed, at leading order in the parton model, the $\bar{B} \rightarrow D^* l \bar{\nu}$ form factors have the form [14] of Eq. (4),

$$F(t) = \frac{F_0}{\sqrt{t_+ - t}}, \quad (7)$$

with $p=1/2$. Given that there are bound states in nature, how can the perturbative QCD results be trustworthy? Certainly perturbative QCD cannot be used directly in the semileptonic region. However, the perturbative calculation of pair production should be reliable as long as a large region of momentum transfer is smeared over [15], or integrated over with smooth weighting functions. By constraining the magnitude of the form factor in the pair-production region, the perturbative analysis indirectly constrains the shape of the form factor in the semileptonic region.

For $\bar{B} \rightarrow \pi l \bar{\nu}$, the kinematically allowed region t_- is much larger, and the heuristic discussion above applies less clearly. An explicit derivation is required to see that, even in this case, pair-production constraints allow $F(t)$ in the semileptonic region to be expanded in powers of $z \leq z_{\max}$.

MOMENTS OF THE DISPERSION RELATION

For a general semileptonic decay $\bar{B} \rightarrow H l \bar{\nu}$, the heuristic discussion of the previous section can be made concrete by

considering the two-point function for the vector or axial vector currents $J = \bar{q} \gamma^\mu b, \bar{q} \gamma^\mu \gamma_5 b$ that arise in the charged current decay of b hadrons:

$$\begin{aligned} \Pi_J^{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T J^\mu(x) J^{\dagger\nu}(0) | 0 \rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_J^T(q^2) + g^{\mu\nu} \Pi_J^L(q^2). \end{aligned} \quad (8)$$

The polarization functions $\Pi_J^{L,T}(q^2)$ do not fall off fast enough at large q^2 for an unsubtracted dispersion relation to be finite. However, derivatives of the polarization functions do fall fast enough at high q^2 for finite dispersion relations to exist. As we wish to constrain hadronic form factors by a perturbative calculation, it is useful to define the derivatives of the polarization functions at $q^2=0$ where the partonic amplitude is well behaved, far from the physical region for $\bar{B}\bar{H}$ pair production. At $q^2=0$ the n th derivative of the polarization tensor for the current J^μ is

$$\begin{aligned} \chi_J^{(n)} &\equiv \frac{1}{3\Gamma(n+3)} \frac{\partial^{n+2} \Pi_J^{ii}(0)}{\partial (q^2)^{n+2}} = \frac{1}{3\Gamma(n+2)} \frac{\partial^{n+1} \Pi_J^T(0)}{\partial (q^2)^{n+1}} \\ &\quad - \frac{1}{3\Gamma(n+3)} \frac{\partial^{n+2} \Pi_J^L(0)}{\partial (q^2)^{n+2}} \\ &= \frac{1}{\pi} \int_0^\infty dt \frac{(1/3) \text{Im} \Pi_J^{ii}(t)}{t^{n+3}}, \end{aligned} \quad (9)$$

where $J=V,A$ for vector and axial vector currents, respectively. This dispersion relation relates the computation of $\chi_J^{(n)}$ at the unphysical value $q^2=0$ to the weighted integral over the pair-production region of the imaginary part of $\Pi_J^{ii}(q^2)$. The higher the moment n the more the integral is weighted near the pair-production threshold, so we expect the calculation to be most reliable for low moments where the smearing is largest.

It is straightforward to determine the $\chi_J^{(n)}$ in perturbative QCD. For a ratio of quark masses $u = m_q/m_b$, a one-loop (leading order) calculation of the vector current correlator gives

$$\begin{aligned} \chi_V^{(n)}(u) &= \frac{3[\Gamma(n+3)]^2}{2\pi^2 m_b^{2n+2} \Gamma(2n+6)} \left[\frac{1}{n+1} F(n+1, n+3; 2n+6; 1-u^2) \right. \\ &\quad \left. + \frac{1-u}{4(n+2)} (u F(n+2, n+4; 2n+7; 1-u^2) - F(n+2, n+3; 2n+7; 1-u^2)) \right], \end{aligned} \quad (10)$$

where $F(a, b; c; \xi)$ is a hypergeometric function. The same expression results for the axial current after the substitution $u \rightarrow -u$, i.e., $\chi_A^{(n)}(u) = \chi_V^{(n)}(-u)$. For a massless quark $m_q=0$, the expressions simplify to

$$\chi_{V,A}^{(n)}(0) = \frac{3}{4\pi^2 m_b^{2n+2}} \frac{1}{(n+1)(n+2)(n+4)}. \quad (11)$$

The correlator, Eq. (8), has also been computed at two loops [16], i.e., $O(\alpha_s)$. The higher order corrections result in a 25% increase [12] in $\chi^{(0)}(0)$.

Since production of $\bar{B}\bar{H}$ hadrons is a subset of total hadronic production, the perturbative calculation $\chi^{(n)}$ serves to constrain the analytically continued form factors for $\bar{B} \rightarrow H$ decay. More precisely, the partonic computation provides an upper bound to the smeared contributions of poles and cuts above the pair production threshold. The contribution of poles below threshold will also influence the variation of a given form factor $F(t)$ in the semileptonic region, and must be considered separately. While subthreshold contributions are not a fundamental aspect of the dispersion relation approach (for example, form factors in $D \rightarrow \pi l \bar{\nu}$ are analytic below the $D - \pi$ threshold), they are not accounted for by the perturbative calculation and must be properly handled when present [17].

We now turn to relating $\chi_J^{(n)}$ to $F(t)$. This is accomplished by inserting a sum over intermediate states into $\text{Im} \Pi_J^{ii}(q^2)$,

$$\begin{aligned} \text{Im} \Pi_J^{ii}(q^2) &= \frac{1}{2} \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^2 4E_1 E_2} \delta^{(4)}(q - p_1 - p_2) \\ &\quad \times \sum_{pol} \langle 0 | J^{\dagger i} | \bar{B}(p_1) \bar{H}(p_2) \rangle \\ &\quad \times \langle \bar{B}(p_1) \bar{H}(p_2) | J_i | 0 \rangle + \dots, \end{aligned} \quad (12)$$

where the sum is over polarizations of H and the ellipsis denotes strictly positive contributions from the B^* , higher resonances and multiparticle states. In terms of a calculable kinematic function $k(t)$ arising from two-body phase space and the Lorentz structure associated with the form factor $F(t)$, we may substitute the inequality

$$\frac{1}{3} \text{Im} \Pi_J^{ii}(t) \geq k(t) |F(t)|^2 \quad (13)$$

into Eq. (9) to get the contribution to the hadronic moment $\chi_J^{(n)}$ (hadronic) from the form factor $F(t)$ of interest,

$$\chi_J^{(n)}(\text{hadronic}) \geq \frac{n!}{\pi} \int_{t_+}^\infty \frac{k(t) |F(t)|^2}{t^{n+3}}, \quad (14)$$

where n_I is the isospin degeneracy of the $\bar{B}\bar{H}$ pair. We rely on perturbative QCD at the unphysical point $q^2=0$ (or equivalently, on global duality for suitably smeared production rates) to assert that hadronic and partonic expressions for χ are equal. Then for each n ,

$$\chi_J^{(n)}(\text{hadronic}) = \chi_J^{(n)}(u), \quad (15)$$

where $\chi_J^{(n)}(u)$ is the n th moment as computed in perturbative QCD. Therefore we have that

$$\frac{n_I}{\pi \chi_J^{(n)}(u)} \int_{t_+}^{\infty} \frac{k(t) |F(t)|^2}{t^{n+3}} \leq 1, \quad (16)$$

and hence

$$\frac{1}{\pi} \int_{t_+}^{\infty} dt |h^{(n)}(t) F(t)|^2 \leq 1, \quad (17)$$

where $h^{(n)}(t) = \sqrt{n_I k(t) / \chi_J^{(n)}(u)} t^{-(3+n)/2}$. The argument of the square root is positive since the integrand came from a production rate.

The inequality of Eq. (17) makes clear how the perturbative calculation constrains the magnitude of the form factor in the pair-production region. To constrain the form factor in the semileptonic region $0 \leq t \leq t_-$, we would like to find functions $\varphi_k(t)$ that are orthonormal with respect to the integral Eq. (17),

$$\frac{1}{\pi} \int_{t_+}^{\infty} dt \text{Re}[\varphi_k(t) \varphi_j^*(t)] = \delta_{kj}, \quad (18)$$

and that vanish somewhere in the semileptonic region, say at $0 \leq t_0 \leq t_-$. We could then expand $h^{(n)}(t)F(t)$ in terms of these basis functions and use Eq. (17) to bound the expansion coefficients.¹ If $h^{(n)}(t)F(t)$ turned out to be analytic in t outside the pair-production region, its expansion would be equally valid in the semileptonic region, and we would have a parametrization of $F(t)$ in terms of unknown, but bounded, expansion coefficients. Unfortunately, neither $h^{(n)}(t)$ nor $F(t)$ are in general analytic away from the pair-production cut. The kinematic factor $h^{(n)}(t)$ has explicit poles at $t=0$ and the form factor $F(t)$ may also have poles arising from the contribution of bound states that can interpolate between the current J and the $\bar{B}\bar{H}$ pair. For example, the experimentally accessible form factor $f_+(t)$ in $\bar{B} \rightarrow \pi l \bar{\nu}$ has a pole at $t = M_{B^*}^2$ coming from the contribution of the B^* resonance.

Fortunately, a simple pole at $t=t_p$ can be eliminated by multiplying by $z(t;t_p)$. Rewriting z [as defined in Eq. (2)] as

$$z(t;t_p) = \frac{t_p - t}{(\sqrt{t_+ - t} + \sqrt{t_+ - t_p})^2}, \quad (19)$$

¹We can choose the expansion coefficients a_k to be real so that only $\text{Re}[\varphi_k(t) \varphi_j^*(t)]$ need vanish for $k \neq j$ since it is this expression that arises in $|\sum a_k \varphi_k|^2$. In a more general case where the a_k are complex, our results go through unchanged if the inner product in Eq. (18) is redefined as $\langle f, g \rangle \equiv \lim_{\epsilon \rightarrow 0} (1/2\pi) \int_{t_+}^{\infty} dt [f(t+i\epsilon)g^*(t-i\epsilon) + f(t-i\epsilon)g^*(t+i\epsilon)]$.

makes it clear that $z(t;t_p)$ vanishes at $t=t_p$ and has magnitude one in the pair-production region, $|z(t;t_p)|=1$ for $t \geq t_+$. We can therefore construct a quantity with no poles outside the pair-production region by multiplying $h^{(n)}(t)$ and $F(t)$ by factors of $z(t;t_p)$ for each pole at t_p . To make an analytic function outside the pair-production region, we generally also need to eliminate square-root branch cuts in $h^{(n)}(t)$ that arise from factors of the H -meson three-momentum by dividing by $\sqrt{z(t;t_-)}$. The elimination of poles and cuts from $h^{(n)}(t)$ by $h^{(n)}(t) \rightarrow \tilde{P}(t)h^{(n)}(t)$, where $\tilde{P}(t)$ is a product of $z(t;0)$'s and $\sqrt{z(t;t_-)}$'s, can be automatically accomplished by replacing

$$\begin{aligned} \frac{1}{t} \rightarrow \frac{-z(t;0)}{t} &= \frac{1}{(\sqrt{t_+ - t} + \sqrt{t_+})^2}, \\ \sqrt{t_- - t} \rightarrow \sqrt{\frac{t_- - t}{z(t;t_-)}} &= \sqrt{t_+ - t} + \sqrt{t_+ - t_-}. \end{aligned} \quad (20)$$

The elimination of poles from $F(t)$ by $F(t) \rightarrow P(t)F(t)$ is accomplished by multiplying by a product $P(t) = \prod_j z(t;t_j)$ for each contributing subthreshold resonance of invariant mass squared t_j . Since $\tilde{P}(t)$ and $P(t)$ have a unit modulus along the pair-production cut [the integration region in Eq. (17)], the well-behaved quantity $\tilde{P}(t)h^{(n)}(t)P(t)F(t)$ obeys the same relation:

$$\frac{1}{\pi} \int_{t_+}^{\infty} dt |\tilde{P}(t)h^{(n)}(t)P(t)F(t)|^2 \leq 1. \quad (21)$$

Whereas $\tilde{P}(t)$ may be viewed as a technical device to smooth out the kinematic function $h^{(n)}(t)$, $P(t)$ contains essential information about the resonance structure of $F(t)$ in the unphysical region $t_- < t < t_+$. In Eq. (17), the poles in $F(t)$ above threshold are constrained by the perturbative calculation; in Eq. (21), the poles below threshold are accommodated by $P(t)$. Both sets of poles influence the shape of $F(t)$ in the semileptonic region. Since $P(t)$ depends only on the position of the poles below threshold and not on the residues, it applies for arbitrarily strong or weak residues. Therefore, we should not be surprised that the eventual effect of a nontrivial function $P(t)$ is to weaken the constraint on $F(t)$.

In terms of orthonormal functions $\varphi_k(t)$ satisfying Eq. (18), the expansion

$$\tilde{P}(t)h^{(n)}(t)P(t)F(t) = \sum_{k=0}^{\infty} a_k^{(n)} \varphi_k(t) \quad (22)$$

combines with Eq. (21) to yield

$$\sum_{k=0}^{\infty} (a_k^{(n)})^2 \leq 1, \quad (23)$$

valid for moderate values of $n \geq 0$. As the expression given in Eq. (22) is valid everywhere outside the cut in the complex t plane the form factor $F(t)$ in the region of semileptonic decay $0 \leq t \leq t_-$ is

$$F(t) = \frac{1}{\bar{P}(t)h^{(n)}(t)P(t)} \sum_{k=0}^{\infty} a_k^{(n)} \varphi_k(t). \quad (24)$$

The sum is over $k \geq 0$ because by construction $\bar{P}(t)h^{(n)}(t)P(t)F(t)$ has no poles for $t < t_+$. All that remains is to find the orthogonal polynomials $\varphi_k(t)$. This is a math problem that can be accomplished by a change of variables. In the complex t plane, the integration contour may be viewed as a segment from $+\infty$ to $t=t_+$ just below the cut and a segment from $t=t_+$ to $+\infty$ just above the cut. Defining $y = \sqrt{t-t_+}$ maps the line segments just above and below this cut onto the real y axis. The y axis in turn can be mapped onto the unit circle by the bilinear transformation $z(t; t_0) = (y - \sqrt{t_0 - t_+}) / (y + \sqrt{t_0 - t_+})$. This is precisely the change of variables in Eq. (2). Since $z^n = e^{in\theta}$ are orthonormal functions on the unit circle, we can work backwards to find

$$\varphi_k(t) \equiv \frac{1}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \left(\frac{t_+ - t_0}{t_+ - t} \right)^{1/4} \left(\frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \right)^k. \quad (25)$$

The expansion in orthonormal basis functions is simply a Taylor series in $z(t; t_0)^k$. However, the variable z does little to aid the development of physical intuition, so we continue to work with the momentum transfer t . Contact with previous literature [4,8] can be made by identifying

$$\phi^{(n)}[z(t; t_0)] = (\sqrt{t_+ - t} + \sqrt{t_+ - t_0}) \left(\frac{t_+ - t}{t_+ - t_0} \right)^{1/4} \bar{P}(t)h^{(n)}(t), \quad (26)$$

choosing $n=0$, setting $t_0 = t_-$ [or, in the case and language of [8], $t_0 = (1-N)t_+ + Nt_-$], and expressing the ‘‘Blaschke factors’’ [18–20] $z(t; t_p)$ composing $P(t)$ in terms of $z(t; t_-)$ and $z(t_p; t_-)$. With these identifications Eq. (24) becomes Eq. (1) with $\phi(z) = \phi^{(0)}(z)$ and $a_k = a_k^{(0)}$.

PARAMETRIZATIONS FOR SEMILEPTONIC FORM FACTORS

We are primarily interested in constraining form factors that describe the decay of B mesons, although the formalism

applies equally well to Λ_b baryons, or even D and K mesons if $\Pi(q^2)$ is evaluated at an appropriate spacelike q^2 . Specialization to a particular decay and form factor requires an explicit computation of the ϕ functions. For a pseudoscalar final meson H or a vector meson H^* with polarization ϵ , the various form factors in semileptonic B decay may be defined by

$$\langle H^*(p', \epsilon) | V^\mu | \bar{B}(p) \rangle = ig \epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha^* p'_\beta p_\gamma,$$

$$\langle H^*(p', \epsilon) | A^\mu | \bar{B}(p) \rangle$$

$$= f \epsilon^{*\mu} + (\epsilon^* \cdot p) [a_+(p+p')^\mu + a_-(p-p')^\mu],$$

$$\langle H(p') | V^\mu | \bar{B}(p) \rangle = f_+(p+p')^\mu + f_-(p-p')^\mu, \quad (27)$$

where it is useful to also define

$$F_1 = \frac{1}{M_H} \left[\frac{1}{2} (t_+ - t)(t_- - t) a_+ - \frac{1}{2} (t - M_B^2 + M_H^2) f \right], \quad (28)$$

with $t = (p-p')^2$. It is straightforward to determine the $k(t)$ function associated with each of the form factors:

$$k_i(t) = \frac{1}{3\pi 2^s} \left(\frac{1}{t} \right)^p [(t-t_+)(t-t_-)]^{w/2}. \quad (29)$$

For the form factors whose contribution to the rate is unsuppressed by the lepton mass, the indices s , p , and w are given by

$$k_g(t): s=5, \quad p=1, \quad w=3,$$

$$k_{F_1}(t): s=4, \quad p=2, \quad w=1,$$

$$k_f(t): s=3, \quad p=1, \quad w=1,$$

$$k_{f_+}(t): s=4, \quad p=2, \quad w=3. \quad (30)$$

The $\phi^{(n)}(t)$ functions defined in Eq. (26) for each form factor are

$$\phi_i^{(n)}(t; t_0) = \sqrt{\frac{n_I}{2^s 3 \pi \chi_J^{(n)}}} \left(\frac{t_+ - t}{t_+ - t_0} \right)^{1/4} (\sqrt{t_+ - t} + \sqrt{t_+})^{-(3+n+p)} (\sqrt{t_+ - t} + \sqrt{t_+ - t_0}) (\sqrt{t_+ - t} + \sqrt{t_+ - t_-})^{w/2} (t_+ - t)^{w/4}. \quad (31)$$

Our previous discussions allow us to see that each particular form factor F_i has the functional form

$$F_i(t) = \frac{1}{P_i(t) \phi_i^{(n)}(t; t_0)} \sum_{k=0}^{\infty} a_k^{(n)} z(t; t_0)^k, \quad (32)$$

for each moment n and expansion point t_0 , where z may be expressed as

$$z(t; t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \quad (33)$$

and $\Sigma(a_k^{(n)})^2 \leq 1$.

The Blaschke factors $P_i(t)$ depend on the masses of subthreshold resonances. For $\bar{B} \rightarrow D l \bar{\nu}$ and $\bar{B} \rightarrow D^* l \bar{\nu}$ from factors, the masses of the relevant B_c -type resonances can be rather accurately estimated from potential models [21,22]. Using the results of [21] in Eq. (19), the Blaschke factors for the form factors f and F_1 are

$$P_f(t) = P_{F_1}(t) = z(t; (6.730 \text{ GeV})^2) z(t; (6.736 \text{ GeV})^2) z(t; (7.135 \text{ GeV})^2) z(t; (7.142 \text{ GeV})^2), \quad (34)$$

while for the form factors g and f_+ they are

$$P_g(t) = z(t; (6.337 \text{ GeV})^2) z(t; (6.899 \text{ GeV})^2) z(t; (7.012 \text{ GeV})^2) z(t; (7.280 \text{ GeV})^2) \quad (35)$$

and

$$P_{f_+}(t) = z(t; (6.337 \text{ GeV})^2) z(t; (6.899 \text{ GeV})^2) z(t; (7.012 \text{ GeV})^2), \quad (36)$$

respectively. For $\bar{B} \rightarrow \pi l \bar{\nu}$, there is only one resonance and the form factor $f_+(t)$ has the Blaschke factor $P(t) = z(t; (5.325 \text{ GeV})^2)$. For $D \rightarrow \pi l \bar{\nu}$, there are no resonances, and $P(t) = 1$.

For the lowest moment ($n=0$) and $t_0 = t_-$, Eqs. (31)–(33) reproduce the results of [4], while for arbitrary t_0 they reproduce the results for mesons given in [8]. For higher moments ($n > 0$) and a given form factor F_i , they imply

$$\sum_{k=0}^{\infty} a_k^{(n)} z(t; t_0)^k = \frac{\phi_i^{(n)}(t)}{\phi_i^{(0)}(t)} \sum_{k=0}^{\infty} a_k^{(0)} z(t; t_0)^k = \sqrt{\frac{\chi_J^{(0)}}{\chi_J^{(n)}}} \left[\frac{1}{\sqrt{t_+ - t} + \sqrt{t_+}} \right]^n \sum_{k=0}^{\infty} a_k^{(0)} z(t; t_0)^k \equiv \left(\sum_j c_j^{(n)} z(t; t_0)^j \right) \sum_{k=0}^{\infty} a_k^{(0)} z(t; t_0)^k. \quad (37)$$

It is a simple matter to compute the coefficients $c_j^{(n)}$ and match powers of z . The condition $\Sigma(a_k^{(n)})^2 < 1$ then implies

$$(c_0^{(n)} a_0^{(0)})^2 + (c_0^{(n)} a_1^{(0)} + c_1^{(n)} a_0^{(0)})^2 + (c_0^{(n)} a_2^{(0)} + c_1^{(n)} a_1^{(0)} + c_2^{(n)} a_0^{(0)})^2 + \dots < 1, \quad (38)$$

for each n . For a given set of $c_j^{(n)}$ one may find that the higher moments provide tighter constraints on the $a_j^{(0)}$ than the lowest moment alone. This is the case for $B \rightarrow \pi$, for the lowest few moments. However, for $B \rightarrow D$ the higher moments do not improve the bounds imposed by the lowest moment.

CONSTRAINTS FOR $\bar{B} \rightarrow \pi l \bar{\nu}$

To make use of Eq. (32) in extracting V_{ub} from experimental measurements of $\bar{B} \rightarrow \pi l \bar{\nu}$ we should keep only a finite number of parameters $a_k^{(0)}$ and compute the maximal truncation error from the omission of higher order terms in the series. This truncation error can be minimized by optimizing t_0 and thereby decreasing z_{\max} [8]. Applying Eq. (38) further decreases the truncation error by restricting the range of higher order parameters (of course, it also restricts the range of the parameters we keep). Both effects decrease the number of parameters required to determine $f_+(t)$ at a given level of accuracy.

Experimental data on $\bar{B} \rightarrow \pi l \bar{\nu}$ are not yet available to precisely describe the shape of $f_+(t)$ and so for now we will simply illustrate the utility of the higher moments. Imagine $f_+(t)$ is known at some fixed number of kinematic points. For concreteness we choose the lattice-inspired values [12,23,24] $f_+(21 \text{ GeV}^2) = 1.7$ and $f_+(0 \text{ GeV}^2) = 0.5$ to fix $a_0^{(0)}$ and $a_1^{(0)}$ in terms of $a_2^{(0)}$. Varying $a_2^{(0)}$ subject to the zeroth moment constraint $\Sigma(a_k^{(0)})^2 < 1$ maps out the envelope of parametrizations² consistent with the $n=0$ dispersion relation. Varying $a_2^{(0)}$ subject to the constraint of the n th moment results in a smaller allowed range for $a_2^{(0)}$, for the first few n . Since the allowed range of $f_+(t)$ is proportional to the allowed range of $a_2^{(0)}$, a relative reduction in the range of $a_2^{(0)}$ leads to the same relative reduction in the width of the envelope.

The one-loop results for the allowed range of $a_2^{(0)}$ are shown in the second column of Table I. We have used a pole quark mass $m_b = M_B - \bar{\Lambda}$ corresponding to $\bar{\Lambda} = 0.4 \text{ GeV}$ [25] (varying $\bar{\Lambda}$ by $\pm 0.1 \text{ GeV}$ results in no more than a 6% change in the bounds), and fixed $t_0 = t_-$. Note that the $n=3$ bounds are tighter than the $n=0$ result by a factor of 3. In fact, the $n=6$ bound (not listed) is better by a factor of 4. However, $\chi^{(n)}$ receives known [16] corrections from two-loop perturbative graphs [$O(\alpha_s)$] and nonperturbative matrix elements. We use values of the condensates $\langle \bar{u}u \rangle|_{1 \text{ GeV}} = (-0.24 \text{ GeV})^3$ and $\alpha_s / \pi \langle G_{\mu\nu} G^{\mu\nu} \rangle = 0.02 \pm 0.02 \text{ GeV}^4$ from [27]. The third and fourth columns of Table I show the perturbative and condensate correction factors $\rho_{\text{pert}}^{(n)}$ and $\rho_{\text{cond}}^{(n)}$ defined by

$$\chi^{(n)}(\text{two loop}) = (1 + \rho_{\text{pert}}^{(n)} + \rho_{\text{cond}}^{(n)}) \chi^{(n)}(\text{one loop}). \quad (39)$$

²This procedure is equivalent to the method of computing upper and lower bounds by forming determinants of inner products often used in the literature [9–12,26]. However, Eq. (32) also dictates the shape of curves allowed inside the envelope.

TABLE I. Values of the zeroth-moment parameter $a_2^{(0)}$ consistent with the n th moment constraint. From left to right is the constraint on $a_2^{(0)}$ at one loop, the relative two-loop perturbative and condensate contributions $\rho_{\text{pert}}^{(n)}$ and $\rho_{\text{cond}}^{(n)}$, the constraint on $a_2^{(0)}$ at two loops, and the two-loop constraint on $a_2^{(0)}$ including the $B^*\pi$ spin symmetry contribution as described in the text.

n	a_2^0 (one loop)	$\rho_{\text{pert}}^{(n)}$	$\rho_{\text{cond}}^{(n)}$	a_2^0 (two loop)	$a_2^{(0)}$ (HQS)
0	$-0.77 < a_2 < 0.79$	0.23	0.01	$-0.78 < a_2 < 0.79$	$-0.54 < a_2 < 0.58$
1	$-0.45 < a_2 < 0.47$	0.32	0.06	$-0.52 < a_2 < 0.53$	$-0.40 < a_2 < 0.41$
2	$-0.33 < a_2 < 0.34$	0.38	0.14	$-0.44 < a_2 < 0.45$	$-0.36 < a_2 < 0.36$
3	$-0.26 < a_2 < 0.28$	0.42	0.28	$-0.40 < a_2 < 0.41$	$-0.35 < a_2 < 0.35$

Beyond $n=3$ the sum of these corrections approaches 100% of the leading result and the reliability of the calculation becomes questionable. Even at $n=3$, the corrections are large enough that one might worry about the convergence of the operator product expansion. We will use $n=2$ in our examples. The allowed range of $a_2^{(0)}$ using the two-loop result, including condensates, is shown in the fifth column of Table I. Although weaker than the corresponding one-loop result, the two-loop bounds for $n>0$ remain significantly more constraining than the $n=0$ bounds.

The ellipsis in Eq. (12) includes the contribution of intermediate $\bar{B}^*\pi$ states to the hadronic moments. In the heavy b -quark limit, the form factor g_* , defined by

$$\langle \pi(p') | V^\mu | \bar{B}^*(p, \epsilon) \rangle = i g_* \epsilon^{\mu\alpha\beta\gamma} \epsilon_{\alpha p'_\beta p_\gamma}, \quad (40)$$

is related to f_+ by spin symmetry when the π meson is soft (i.e., near $t=t_-$):

$$g_*(v \cdot p') = \frac{2}{M_B} f_+(v \cdot p') \left[1 + \mathcal{O}\left(\frac{1}{M}\right) \right], \quad (41)$$

where $v=p/M_B$. Including the contribution of the $\bar{B}^*\pi$ state (similar contributions were used in [28] for the $B \rightarrow B$ elastic form factor) modifies the constraint of Eq. (21) to

$$\frac{1}{\pi} \int_{t_+}^{\infty} dt [|P(t)X(t)\phi_{f_+}^{(n)} f_+(t)|^2 + |P(t)X(t)\phi_{g_*}^{(n)} g_*(t)|^2] \leq 1, \quad (42)$$

where $X(t) = [(t_+ - t_0)/(t_+ - t)]^{1/4} / (\sqrt{t_+ - t} + \sqrt{t_+ - t_0})$. One finds that $\phi_{g_*} = \phi_g$, so expanding

$$g_*(t) = \frac{1}{P(t)\phi_g^{(n)}(t)} \sum_{k=0}^{\infty} b_k^{(n)} z(t; t_0)^k, \quad (43)$$

leads to the constraint

$$\sum_{k=0}^{\infty} (a_k^{(n)})^2 + (b_k^{(n)})^2 \leq 1. \quad (44)$$

We cannot relate all of the $b_k^{(n)}$ to the $a_k^{(n)}$ because the b -quark spin symmetry is only valid near $t=t_-$. One expects that the normalization and first derivative of g_* and f_+ at t_- obey the heavy quark relation to $\sim 10\%$ for the physical B mass, so

$$b_0^{(n)} = \frac{2}{M_B} \frac{\phi_g^{(n)}(t_-)}{\phi_{f_+}^{(n)}(t_-)} a_0^{(n)} = \sqrt{2} \left(1 + \sqrt{\frac{M_\pi}{M_B}} \right)^2 a_0^{(n)},$$

$$b_1^{(n)} = 4\sqrt{2} \sqrt{\frac{M_\pi}{M_B}} a_0^{(n)} + \sqrt{2} \left(1 + \sqrt{\frac{M_\pi}{M_B}} \right)^2 a_1^{(n)}, \quad (45)$$

to $\pm 10\%$. Taking 90% of the absolute values of the right-hand sides of Eq. (45) gives lower bounds on $|b_0^{(n)}|$ and $|b_1^{(n)}|$, that can be inserted into Eq. (44) (this constraint applies as well to heavy-heavy systems, where it could be further improved by including the \bar{B}^*D^* intermediate state). The improvement on the range of $a_2^{(0)}$ from the inclusion of this additional hadronic final state is shown in the last column of Table I (using two-loop amplitudes). The effect is $\sim 30\%$ for $n=0$, decreasing to $\sim 15\%$ for $n=3$.

The $n=0$ bounds on $f_+(t)$ at two loops, without the application of spin symmetry (column five of Table I), are shown as the outer, dashed, pair of curves in Fig. 2, while the bounds from the $n=2$ improvement, including the contribution from $\bar{B}^*\pi$, are given by the inner, solid, pair of curves. The constraints arising from the $n \geq 0$ moments are a dramatic improvement over the lowest order $n=0$ constraint alone. Note that even the $n=1$ bounds in the last column of Table I represent an improvement over that of $n=0$ alone by nearly a factor of 2.

The quickly weakening constraint near $t=t_-$ would be absent had we included a normalization at zero recoil. Figure 3 shows the spin-improved, $n=2$ constraint assuming an ad-

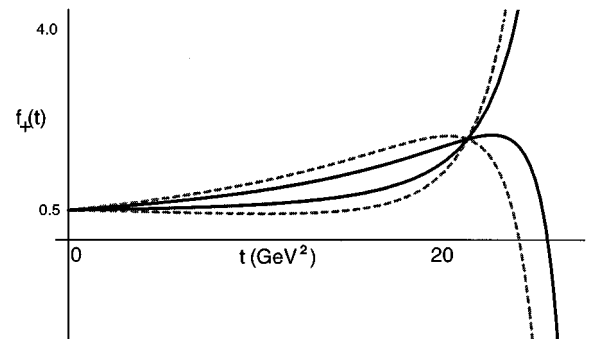


FIG. 2. Upper and lower bounds on the $\bar{B} \rightarrow \pi l \bar{\nu}$ form factor $f_+(t)$. The dashed lines are the bounds arising from the $n=0$ moment while the solid lines are the bounds arising from the $n=2$ moment. The form factor has been fixed at two points by the lattice inspired values $f_+(21 \text{ GeV}^2) = 1.7$ and $f_+(0) = 0.5$. The allowed regions are the interiors of the dashed or solid pairs of curves.

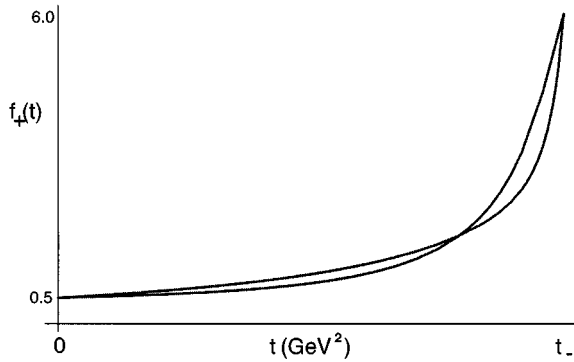


FIG. 3. Bounds on the $\bar{B} \rightarrow \pi l \bar{\nu}$ form factor $f_+(t)$ arising from three normalization points, $f_+(t_-) = 6.0$, $f_+(21 \text{ GeV}^2) = 1.7$ and $f_+(0) = 0.5$. The constraints are from moments of $n \leq 2$ as discussed in the text, including the $B^* \pi$ hadronic intermediate state using bottom-quark spin symmetry at $t = t_-$.

ditional normalization, $f_+(t_-) = 6$. It gives a feel for how tightly $f_+(t)$ can be constrained given measurements of $f_+(t)$ at only three points. In this case, $f_+(t)$ is determined to better than $\pm 15\%$ over the entire kinematic region. These plots are presented only to provide a feel for what is needed to describe $f_+(t)$ to a given accuracy. In practice, one should fit the parametrization Eq. (32) to experimental data and theoretical (lattice or heavy quark symmetry) predictions to extract V_{ub} . The work presented here should improve the precision of such extractions significantly.

CONCLUSIONS

We have discussed constraints on semileptonic form factors arising from analyticity and dispersion relations, and identified the physical origin of the naturally small parameter z_{\max} . The shape and magnitude of an analytically continued form factor in the pair production region has significant impact upon the size and shape of the form factor in the semileptonic region. Perturbative constraints on pair production therefore constrain semileptonic decay amplitudes. Such constraints take the form of parametrizations of form factors in terms of a small number of unknown, but bounded, constants a_k . Only a small number of a_k are needed to describe a given form factor because the parametrizations arise from truncated expansions in a kinematic variable $z(t; t_0)$ that is surprisingly small, $|z(t; t_0)| \leq z_{\max} = 0.065$ for $\bar{B} \rightarrow D^* l \bar{\nu}$ and $|z(t; t_0)| \leq z_{\max} = 0.52$ for $\bar{B} \rightarrow \pi l \bar{\nu}$.

We have traced the smallness of z_{\max} to the dual nature of the hadronic and partonic descriptions of QCD. Whereas in the hadronic description the form factor is a sum of poles and less singular terms, in the partonic description the form factor is always less singular than a pole. Duality then implies that the variation of the form factor $F(t)$ over the physical region $0 < t < t_-$ for the semileptonic decay $\bar{B} \rightarrow H l \bar{\nu}$ is characterized by

$$\frac{F(t_-) - F(0)}{F(t_-) + F(0)} \sim \left(\frac{\sqrt{M_B} - \sqrt{M_H}}{\sqrt{M_B} + \sqrt{M_H}} \right)^2 = z_{\max}. \quad (46)$$

The small values of z_{\max} are due to the square-root dependence on meson masses, which is in turn a consequence of the absence of poles in the partonic description of QCD.

We applied these ideas in a discussion of analyticity constraints in terms of the momentum-transfer variable t . We improved the constraints on the $f_+(t)$ form factor by $\sim 15\text{--}30\%$ by including the contribution of the $\bar{B}^* \rightarrow \pi l \bar{\nu}$ form factor using b -quark spin symmetry to relate it to the $\bar{B} \rightarrow \pi l \bar{\nu}$ form factor at zero recoil. We also improved the analyticity constraints by considering higher moments of the dispersion relation. As an illustrative example, we fixed the normalization of the $\bar{B} \rightarrow \pi l \bar{\nu}$ form factor $f_+(t)$ at two kinematic points to lattice-inspired values, and plotted the envelope of allowed parametrizations (Fig. 2). The higher moment constraints result in upper and lower bounds that are roughly twice as tight as the traditional, zeroth-moment result. Given a third normalization at zero recoil, the form factor is determined over the entire kinematic range to $\pm 15\%$, as shown in Fig. 3. This suggests that a parametrization using one normalization (possibly given by lattice or heavy quark symmetry predictions) and two free parameters could eventually be used for a model-independent extraction of V_{ub} with roughly 30% theoretical errors arising from the t dependence.

ACKNOWLEDGMENTS

We would like to thank B. Grinstein, M. Lu, and M. Wise for useful discussions. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-91-ER 40682. C.G.B. thanks the Nuclear Theory group at the University of Washington for their hospitality during part of this work.

- [1] ALEPH Collaboration, D. Buskulic *et al.*, Phys. Lett. B **359**, 236 (1995); CERN Report No. CERN-PPE-96-150, 1996 (unpublished); A. Anastassov *et al.*, Report No. CLEO CONF 96-8, ICHEP96 PA05-079 1996 (unpublished); T. Bergfeld *et al.*, Report No. CLEO CONF 96-3, ICHEP96 PA05-078, 1996 (unpublished); DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **71**, 539 (1996); OPAL Collaboration, K. Ackerstaff *et al.*, Phys. Lett. B **395**, 128 (1997).
 [2] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**,

- 527 (1990); M. B. Voloshin and M. A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)].
 [3] J. P. Alexander *et al.*, Report No. CLEO CONF 96-16, ICHEP96 PA05-81, 1996 (unpublished).
 [4] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Lett. B **353**, 306 (1995); Nucl. Phys. **B461**, 493 (1996).
 [5] S. Stone, presented at NATO Advanced Study Institute on Techniques and Concepts of High-Energy Physics, St. Croix, U.S. Virgin Islands, 1996 (unpublished).

- [6] T. Inami and C. Lim, *Prog. Theor. Phys.* **65**, 297 (1981); **65**, 1772 (1981); A. Buras *et al.*, *Nucl. Phys.* **B238**, 529 (1984); F. Gilman and M. B. Wise, *Phys. Rev. D* **27**, 1128 (1983).
- [7] N. Isgur and M. B. Wise, *Phys. Rev. D* **42**, 2388 (1990); Z. Ligeti and M. B. Wise, *ibid.* **53**, 4937 (1996).
- [8] C. G. Boyd and R. F. Lebed, *Nucl. Phys.* **B485**, 275 (1997).
- [9] N. N. Meiman, *Sov. Phys. JETP* **17**, 830 (1963); S. Okubo, *Phys. Rev. D* **3**, 2807 (1971); S. Okubo and I. Fushih, *ibid.* **4**, 2020 (1971); L-F. Li and H. Pagels, *ibid.* **3**, 2191 (1971); V. Singh and A. K. Raina, *Fortschr. Phys.* **27**, 561 (1979).
- [10] C. Bourrely, B. Machet, and E. de Rafael, *Nucl. Phys.* **B189**, 157 (1981).
- [11] C. G. Boyd, B. Grinstein, and R. F. Lebed, *Phys. Rev. Lett.* **74**, 4603 (1995).
- [12] L. Lellouch, *Nucl. Phys.* **B479**, 353 (1996).
- [13] M. B. Voloshin and M. A. Shifman, *Yad. Fiz.* **47**, 801 (1988) [*Sov. J. Nucl. Phys.* **47**, 511 (1988)].
- [14] M. Suzuki, *Nucl. Phys.* **B258**, 553 (1985).
- [15] E. C. Poggio, H. R. Quinn, and S. Weinberg, *Phys. Rev. D* **13**, 1958 (1976).
- [16] S. C. Generalis, *J. Phys. G* **16**, 367 (1990); **16**, 785 (1990).
- [17] C. E. Carlson, J. Milana, N. Isgur, T. Mannel, and W. Roberts, *Phys. Lett. B* **299**, 133 (1993); A. Falk, M. Luke, and M. B. Wise, *ibid.* **299**, 123 (1993); B. Grinstein and P. Mende, *ibid.* **299**, 127 (1993); C. Dominguez, J. Körner, and D. Pirjol, *ibid.* **301**, 373 (1993).
- [18] I. Caprini, *Z. Phys. C* **61**, 651 (1994); *Phys. Lett. B* **339**, 187 (1994).
- [19] E. de Rafael and J. Taron, *Phys. Lett. B* **282**, 215 (1992); *Phys. Rev. D* **50**, 373 (1994).
- [20] P. Duren, *Theory of H^p Spaces* (Academic, New York, 1970).
- [21] E. J. Eichten and C. Quigg, *Phys. Rev. D* **49**, 5845 (1994).
- [22] S. S. Gershtein, V. V. Kisilev, A. K. Likhoded, and A. V. Tkabladze, *Phys. Rev. D* **51**, 3613 (1995); Y.-Q. Chen and Y.-P. Kuang, *ibid.* **46**, 1165 (1992).
- [23] UKQCD Collaboration, D. Burford *et al.*, *Nucl. Phys.* **B447**, 425 (1995).
- [24] APE Collaboration, C. R. Alton *et al.*, *Phys. Lett. B* **315**, 513 (1995).
- [25] A. Kapustin, Z. Ligeti, M. B. Wise, and B. Grinstein, *Phys. Lett. B* **375**, 327 (1996); A. Falk, M. Luke, and M. Savage, *Phys. Rev. D* **53**, 6316 (1996).
- [26] D. Becirevic, Report No. LPTHE-ORSAY-96-14, hep-ph/9603298 (unpublished); *Phys. Rev. D* **54**, 6842 (1996).
- [27] P. Ball, *Phys. Rev. D* **48**, 3190 (1993); E. Braaten, S. Narison, and A. Pich, *Nucl. Phys.* **B373**, 581 (1992).
- [28] I. Caprini, *Phys. Rev. D* **53**, 4086 (1996); I. Caprini and C. Macesanu, *ibid.* **54**, 5686 (1996).