Charm and bottom semileptonic decays

Patrick J. O'Donnell and Gürsevil Turan

Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7

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We review the present status of theoretical attempts to calculate the semileptonic charm and bottom decays and then present a calculation of these decays in the light-front frame at the kinematic point $q^2 = 0$. This allows us to evaluate the form factors at the same value of q^2, even though the allowed kinematic ranges for charm and bottom decays are very different. Also, at this kinematic point the decay is given in terms of only one form factor $A_0(0)$. For the ratio of the decay rates given by the E653 collaboration we show that the determination of the ratio of the Cabibbo-Kobayashi-Maskawa matrix elements is consistent with that obtained from the unitarity constraint, though a new measurement by the E687 Collaboration is about two standard deviations too high. At present, though, the unitarity method still has greater accuracy. Since comparisons of the semileptonic decays into ρ and either electrons or muons will be available soon from the E791 Fermilab experiment, we also look at the massive muon case. We show that for a range of q^2 the $SU(3)_F$ symmetry breaking is small even though the contributions of the various helicity amplitudes becomes more complicated. For B decays, the decay $B \to K^* l \bar{l}$ at $q^2 = 0$ involves an extra form factor coming from the photon contribution and so is not amenable to the same kind of analysis, leaving only the decay $B \to K^* \nu \bar{\nu}$ as a possibility. As the mass of the decaying particle increases we note that the SU(3) symmetry becomes badly broken at $q^2 = 0$. [S0556-2821(97)01013-8]

I. INTRODUCTION

Semileptonic B- and D- meson decays constitute a subject of great interest in the physics of electroweak interactions, as they may help determine the various Cabibbo-Kobayashi-Maskawa (CKM) mixing angles. In particular, the decays involving $b \to c l \nu_l$ are eminently suitable for the heavy-quark effective theory (HQET) to determine [1] the CKM matrix V_{cb}. For exclusive decays to a final state with a u or s quark, and for the D-meson decays as a whole, it is less likely that the heavy-quark symmetries apply. Since the dynamical content of the corresponding amplitudes is contained in Lorentz-invariant form factors, to know and understand form factors of hadronic currents is very important for analyzing these decays.

However, few of these form factors have been measured experimentally, and those that have been are not known very precisely yet because of the smallness of the branching ratios associated with them. On the other hand, the theoretical calculations are hard to estimate because of the nonperturbative character of strong interactions. Here, one may resort to a model, but that introduces uncertainties that are inherent to the model itself. To overcome this difficulty, at least to some degree, many authors have studied, instead of the branching ratios of the semileptonic decays of the particular heavy mesons, their ratios at some particular kinematical points, usually at zero recoil.

For example, Sanda and Yamada [2] propose a strategy to get $|V_{ub}|$ by relating the differential decay width of the $\bar{B} \to \rho l \bar{\nu}$ to that of the process $\bar{B} \to K^* l \bar{l}$ at their respective q^2_{max} limits $[(m_{\bar{B}} - m_\rho)^2$ and $(m_{\bar{B}} - m_{K*})^2]$ using the SU(3)-flavor symmetry and heavy-quark approximation. They find

$$\frac{|V_{ub}|^2}{|V_{tb} V^*_{ts}|^2} = \frac{q^{2\bar{B} \to K^*}_{max}}{q^{2\bar{B} \to \rho}_{max}} \left(\frac{p_{K*}}{p_\rho}\right)_{lim} \left(\frac{\alpha_{QED}}{4\pi}\right)^2 2(C^2_V + C^2_A)$$

$$\times \frac{[d\Gamma(\bar{B} \to \rho l \bar{\nu})/dq^2]_{q^2 \to q^{2\bar{B} \to \rho}_{max}}}{[d\Gamma(\bar{B} \to K^* l \bar{l})/dq^2]_{q^2 \to q^{2\bar{B} \to K^*}_{max}}}, \quad (1)$$

where $(p_{K*}/p_\rho)_{lim} = \sqrt{m_\rho/m_{K*}}$ and C_V and C_A are the QCD corrected Wilson coefficients. The matrix element $|V_{ub}|$ may be determined if the right-hand side can be obtained by experiment and $|V_{ts}|$ from the unitarity condition. The problem is that in the zero recoil limit $p_{\rho,K*} \to 0$, i.e., $q^2 = q^2_{max}$, the q^2 distributions vanish due to the phase space suppression. This means that experimentally there should be no events at that point and very few in the neighborhood, making it a very difficult measurement.

Dib and Vera [3] relate $B \to \rho l \bar{\nu}$ to $D \to \rho l \bar{\nu}$ also at the point of zero recoil, using the heavy-quark symmetry to get a model-independent result to leading order in inverse powers of large masses. At the kinematical point of zero recoil $y = 1$, where $y = (m^2_I + m^2_\rho - q^2)/2m_I m_\rho$ and $I = B$ or D they find

$$\frac{d\Gamma(B \to \rho e \nu)/dy|}{d\Gamma(D \to \rho e \nu)/dy|_{y \to 1}} = \frac{|V_{ub}|^2}{|V_{cd}|^2} \left(\frac{f^{[B]}_{A_1}(1)}{f^{[D]}_{A_1}(1)}\right)^2 \left(\frac{m_B - m_\rho}{m_D - m_\rho}\right)^2. \quad (2)$$

They determine the ratio of form factors $f^{[B]}_{A_1}(1)/f^{[D]}_{A_1}(1)$ in the constituent quark model at the tree level in the HQET and also with the inclusion of short-distance QCD corrections. Their numerical result for the ratio of form factors is between 1.09 and 1.18. The parameters that cause the largest uncertainty in the ratio are m_c and μ. Again, the rates vanish at $y = 1$ due to phase space. To determine the left-hand side

of Eq. (2) experimentally one should access the region nearby and extrapolate to the point of $y = 1$.

Ligeti and Wise give a model-independent method [4] which is based on the study [5] of the double ratio of form factors $(f^{(B \to \rho)}/f^{(B \to K^*)})/(f^{(D \to \rho)}/f^{(D \to K^*)})$. They claim that this double ratio is equal to unity in the SU(3) limit, and in the limit of heavy-quark symmetry so that a determination of $|V_{ub}|$ is possible using information obtainable from the decay modes $B \to \rho l \bar{\nu}_l$, $B \to K^* \nu \bar{\nu}$, $D \to \rho \bar{l} \nu_l$, and $D \to K^* \bar{l} \nu_l$. They use a pole model to get away from the zero-recoil point. Since the maximum values for y are different for B and D decays, $y_{max} = 3.5$ and $y_{max} = 1.3$, respectively, they limit y for the B decays to lie in the range $1 < y < 1.5$. Provided $f^{(D \to \rho)}(y)/f^{(D \to K^*)}(y)$ is almost independent of y then a precise value for $|V_{ub}|$ can be extracted from the rates for $B \to K^* \nu \bar{\nu}$ and $B \to \rho l \bar{\nu}_l$ integrated over this region in y and $f^{(D \to \rho)}(1)/f^{(D \to K^*)}(1)$. However, at the present time the rare decays $B \to K^* \bar{l} l$ and $B \to K^* \nu \bar{\nu}$, have not been observed, and there is no information on the individual form factors for $D \to \rho \bar{l} \nu_l$.

In this paper, we concentrate on opposite end of the heavy meson decay kinematic spectrum, namely, vanishing four-momentum transfer $q^2 = 0$ for the ratio of $D \to \rho \bar{l} \nu_l$ to $D \to K^* \bar{l} \nu_l$ and for the ratio of $B \to \rho l \bar{\nu}_l$ to $B \to K^* \nu \bar{\nu}$. This kinematic point is the maximum recoil of the ρ or K^*; $q^2 = 0$ corresponds to different values of y in these cases and is not therefore a good point from the y, or heavy-quark approach. The motivation for choosing this kinematic point is that, first of all, there is a well-developed way in the light-front formalism [6] to deal with the point at $q^2 = 0$. Secondly, the other calculations obtain results at the zero-recoil point where it is known that the experiments should find no events so that extrapolations and pole models have to be used. Thirdly, the decay widths at $q^2 = 0$ are given in terms of only one form factor $A_0(0)$ (defined below). Finally, there is now a first report of the lattice calculation of the form factor $A_0(0)$ for the decay $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ [7], which is important phenomenologically for the determination of $|V_{ub}|$. They have determined a range of values for $A_0^{B^0 \to \rho^+}(0)$ and found that $A_0^{B^+ \to \rho^0}(0)/\sqrt{2} = (0.16 - 0.35)^{+9}_{-6}$, where the range is due to systematic uncertainty and the quoted error is statistical.

II. SEMILEPTONIC $D \to V l \nu_l$ DECAYS

We define the form factors in the semileptonic decay of a D meson $D(c \bar{q})$ into a vector meson $V(Q \bar{q})$ with the polarization vector ε^μ, by

$$\langle V(p_V, \varepsilon) | \bar{Q} \gamma_\mu (1 - \gamma_5) c | D(p_D) \rangle$$

$$= \frac{2V(q^2)}{m_D + m_V} i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_D^\alpha p_V^\beta - (m_D + m_V) \varepsilon^{*\mu} A_1(q^2)$$

$$+ \frac{(\varepsilon^* p_D)}{m_D + m_V} (p_D + p_V)^\mu A_2(q^2)$$

$$- 2m_V \frac{(\varepsilon^* p_D)}{q^2} q^\mu A(q^2). \tag{3}$$

The form factor A can be written as

$$A(q^2) = A_0(q^2) - A_3(q^2), \tag{4}$$

where

$$A_3(q^2) = \frac{m_D + m_V}{2m_V} A_1(q^2) - \frac{m_D - m_V}{2m_V} A_2(q^2) \tag{5}$$

and with $A_0(0) = A_3(0)$.

In the limit of vanishing lepton masses, the term proportional to A in Eq. (3) does not contribute to the total amplitude and hence to the decay rate. In this limit, the differential q^2 distribution of the semileptonic $D \to V l \nu_l$ decay can be written

$$\frac{d\Gamma}{dq^2} \Big|_{m_l = 0} = \frac{G^2}{(2\pi)^3} |V_{cQ}|^2 \frac{p_V q^2}{12 m_D^2} (|H_+|^2 + |H_-|^2 + |H_0|^2), \tag{6}$$

where H_+, H_-, and H_0 are the partial helicity amplitudes:

$$H_{\pm} = 2m_V A_0(q^2) + \frac{(m_D^2 - m_V^2 - q^2) A_2(q^2) \mp 2m_D p_V V(q^2)}{m_D + m_V},$$

$$H_0 = \frac{1}{\sqrt{q^2}} \left[(m_D^2 - m_V^2 - q^2) A_0(q^2) + \frac{2m_V q^2}{m_D + m_V} A_2(q^2) \right]. \tag{7}$$

We now compare the lepton spectra in the decays $D \to \rho \bar{l} \nu_l$ and $D \to K^* \bar{l} \nu_l$ at $q^2 = 0$. In the limit of vanishing lepton mass, the differential decay rate for $D \to V l \nu_l$ decay is determined by only one form factor A_0 at $q^2 = 0$:

$$\frac{d\Gamma(D \to V l \nu_l)}{dq^2} \Bigg|_{q^2 \to 0}$$

$$= \frac{G_F^2}{192\pi^3 m_D^3} |V_{cQ}|^2 (m_D^2 - m_V^2)^3 |A_0^{D \to V}(0)|^2. \tag{8}$$

Hence, the ratio of the two distributions at $q^2 = 0$ is

$$\frac{[d\Gamma(D \to \rho \bar{l} \nu_l)/dq^2]_{q^2 \to 0}}{[d\Gamma(D \to K^* \bar{l} \nu_l)/dq^2]_{q^2 \to 0}}$$

$$= \frac{|V_{cd}|^2}{|V_{cs}|^2} \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{K^*}^2} \right)^3 \frac{|A_0^{D \to \rho}(0)|^2}{|A_0^{D \to K^*}(0)|^2}. \tag{9}$$

From the experimental ratio $[d\Gamma(D \to \rho \bar{l} \nu_l)/dq^2]/[d\Gamma(D \to K^* \bar{l} \nu_l)/dq^2]_{q^2 \to 0}$ one needs knowledge of $|A_0^{D \to \rho}(0)|/|A_0^{D \to K^*}(0)|$ to extract the ratio $|V_{cd}/V_{cs}|$. This ratio is often taken to have the value unity by SU(3)-flavor symmetry.

There have been many model-dependent studies of this ratio which we show in Table I. From Table I, we see that theoretical predictions of the ratio of form factors fall in a range near 1. The procedure adopted to get $A_0(0)$ often uses Eq. (5) with the calculated values of A_1 and A_2 rather than A_0 directly. However, this indirect way to get A_0 may have

TABLE I. The form factor $A_0(0)$ of $D \to \rho$, $D \to K^*$, $B \to \rho$, and $B \to K^*$ transitions.

Reference	$D \to \rho$	$D \to K^*$	$D \to \rho/D \to K^*$	$B \to \rho$	$B \to K^*$
[14] [a,g]	0.67	0.73	0.92	0.28	
[28] [a]	0.85	0.80	1.06	0.14	
[29] [a]	0.60	0.73	0.82	0.21	
[30] [a]		0.91(0.84)		-0.37	
[31] [a]		0.74 ± 0.12		0.14 ± 0.20	
[32] [b]	0.57 ± 0.40	0.45 ± 0.30	1.27 ± 1.23	0.79 ± 0.80	
[33] [b,g]	0.52 ± 0.05	0.58 ± 0.05	0.90 ± 0.12	0.24 ± 0.02	0.30 ± 0.03
[34] [b]				0.15 ± 0.97	
[35] [b]				0.28 ± 1.1	
[36] [c]	0.74	0.59	1.25	0.24	
[37] [c,g]	0.73 ± 0.17	0.65 ± 0.14	1.12 ± 0.36	1.10 ± 0.30	
[38] [c]		0.39 ± 0.13			
[39] [d]	0.76 ± 0.25	0.72 ± 0.17	1.06 ± 0.43		
[40] [d,g]	0.64 ± 0.17	0.71 ± 0.16	0.90 ± 0.31		
[41] [d]		0.77 ± 0.29		-0.57 ± 0.65	
[7] [d,g]				$(0.22 - 0.49)^{+13}_{-8}$	
[42] [e]	$-$	0.48 ± 0.12			
[9] [f]					0.31
[10] [f]	0.69	0.78	0.88	0.32	
[11] [f,g]				0.30	0.40
This Work [g]	0.66	0.75	0.88		

[a]Quark model.
[b]QCD sum rules.
[c]HQET + chiral perturbation theory.
[d]Lattice calculation.
[e]Heavy-quark-symmetry.
[f]Light-front quark model.
[g]$A_0(0)$ is directly calculated.

some difficulties coming from the q^2 dependences of the form factors and also from possible correlations in treating the errors. This has already been commented on in Ref. [33] and we will have further remarks to make when we discuss the nonmassless lepton case.

We use the light-front quark model, which is suitable at the kinematic limit where $q^2 = 0$, to determine the same ratio. This model was developed [6] a long time ago and there have been many applications [8–13] where the details can be found.

In the light-front quark model, the quark coordinates are given by

$$p_{Q+} = xP_+, \quad \mathbf{p}_{Q\perp} = x\mathbf{P}_\perp + \mathbf{k}_\perp, \quad p_{\bar{q}+} = (1-x)P_+,$$

$$\mathbf{p}_{\bar{q}\perp} = (1-x)\mathbf{P}_\perp + \mathbf{k}_\perp, \quad 0 \leq x \leq 1, \quad \mathbf{P} = \mathbf{p}_Q + \mathbf{p}_{\bar{q}}, \quad (10)$$

where $\mathbf{k} = (k_z, \mathbf{k}_\perp)$ is the internal momentum. For $\mathbf{P}$ (and similarly for other vectors), $\mathbf{P} = (P_+, \mathbf{P}_\perp)$ with $P_+ = P_0 + P_z$ and $\mathbf{P}_\perp = (P_x, P_y)$.

To calculate the form factors, one reasonable and often used assumption for the meson wave function $\phi(x, \mathbf{k}_\perp)$ is a Gaussian-type function

$$\phi(x, \mathbf{k}_\perp) = \eta(\mathbf{k}) \sqrt{\frac{d\mathbf{k_z}}{dx}}, \quad \eta(\mathbf{k}) = (\pi\omega^2)^{-3/4} \exp(-\mathbf{k}^2/2\omega^2), \quad (11)$$

where ω is a scale parameter and x is defined through

$$x = \frac{e_Q + k_z}{e_Q + e_{\bar{q}}}, \quad e_i = \sqrt{m_i^2 + \mathbf{k}^2} (i = Q, \bar{q}). \quad (12)$$

The wave function (11) has been used in Refs. [6,9] and also in [11,12] for various applications of the light-front quark model.

A similar wave function is

$$\phi(x, \mathbf{k}_\perp) = \eta(\mathbf{k}) \sqrt{\frac{d\mathbf{k_z}}{dx}}, \quad \eta(\mathbf{k}) = N\exp(-M_0^2/8\omega^2). \quad (13)$$

Here N is the normalization constant and M_0 is the invariant mass of the quarks, which is now given by

$$M_0 = e_Q + e_{\bar{q}}. \quad (14)$$

This wave function has been also applied for heavy mesons in [10] and [13].

Another possibility is the wave function adopted in [14]:

$$\phi(x,\mathbf{k}_\perp)=N\sqrt{\frac{x(1-x)}{\pi\omega^2}}\exp\left(-\frac{M^2}{2\omega^2}\left[x-\frac{1}{2}-\frac{m_Q^2-m_{\bar{q}}^2}{2M^2}\right]^2\right)$$

$$\times\exp\left(-\frac{\mathbf{k}_\perp^2}{2\omega^2}\right), \tag{15}$$

where M is the mass of the meson. As shown in [11], the wave functions (11) and (15) satisfy the scaling law [15]

$$f_H\propto\frac{1}{\sqrt{m_h}}, \quad m_h\to\infty, \tag{16}$$

where f_H is the heavy-H-meson decay constant and m_h is the corresponding heavy quark. However, the wave function (13) does not satisfy Eq. (16) unless the parameter ω scales as the square root of the heavy-quark mass [16].

We use the wave function (11) in our calculations. The parameters for ρ and K^* are taken from Ref. [17]. In Ref. [17] the pion decay constant used has the value $f_\pi=92.4\pm0.2$ MeV and the ρ decay constant $f_\rho/m_\rho=152.9\pm3.6$ MeV, both taken from experiment. The same value of ω (called β there) is assumed for both the π and the ρ mesons. This is in line with the usual ideas of hyperfine splitting [18]. The parameters that are fitted are the quark masses, found to be $m_u=m_d=250\pm5$ MeV, where it is assumed that $\omega_{u\bar{u}}=\omega_{d\bar{d}}=\omega_{u\bar{d}}=0.3194$. A similar calculation for the kaon based on the decay constant $f_K=113.4\pm1.1$ MeV and the decay rate for $K^{*+}\to K^+\gamma$, leads to the s-quark mass $m_s=0.37\pm0.02$ MeV and the wave-function parameter $\omega_{u\bar{s}}=0.3949$ GeV. The values for the masses of u and s quarks and the wave-function parameter $\omega_{u\bar{s}}$ obtained in this way are used to evaluate the K^* decay constant $f_{K^*}=186.73$ MeV.

Using these values of the quark masses and wave-function parameter we get $A_0^{D\to\rho}(0)/A_0^{D\to K^*}(0)=0.88$, i.e., an SU(3)$_F$-breaking effect at the level of about 10%. The kinematical factor in Eq. (9) readjusts this value and the ratio of the decay rates in terms of the CKM factor becomes 0.96, which is very close to the SU(3)$_F$ symmetry limit. This result for the values of the form factors is not strongly dependent on the choice of wave function. We have calculated the same form factors also with the wave function (13) above and got a similar result. The fact that form factors do not depend on the choice of wave function can also be seen by comparing our result with the one given in the first row of Table I, which came from using the wave function (15).

The E653 Collaboration determined the following ratio of decay rates [19]:

$$\frac{\Gamma(D^+\to\rho^0\mu^+\nu)}{\Gamma(D^+\to\bar{K}^{*0}\mu^+\nu)}=0.044^{+0.031}_{-0.025}\pm0.014. \tag{17}$$

Using the values quoted by the Particle Data Group (PDG) [20], coming from the unitarity constraint to get the ratio of the CKM matrix elements $|V_{cd}/V_{cs}|$,

$$|V_{cd}|/|V_{cs}|=0.226\pm0.003, \tag{18}$$

and the values given in the last row of the table for the form factors of $D\to\rho$ and $D\to K^*$, we find

$$\frac{\Gamma(D^+\to\rho^0\mu^+\nu)}{\Gamma(D^+\to\bar{K}^{*0}\mu^+\nu)}=0.025, \tag{19}$$

which is consistent with the result of E653, though the errors are large.

A new measurement has recently been published by the E687 group [21] which gives gives the value $0.079\pm0.019\pm0.013$. In this case the predicted value of the ratio is much lower at about the 2 standard deviation level.

Lepton mass effects. Most of the theoretical and experimental analyses of the exclusive semileptonic decays assume that taking the lepton mass to be zero is a good approximation. For the electron and τ-lepton cases, the situation is relatively clear. For the electron, the zero mass approximation is good since the threshold is very close to the massless limit. On the other hand, it is obvious that one has to include lepton mass effects when analyzing semileptonic decays involving τ leptons. Since comparisons of the semileptonic decays into ρ and either electrons or muons will be available soon [22], we now discuss the massive muon case.

Two different aspects have to be considered when lepton mass effects are included in an analysis of semileptonic decays [23]. The kinematics of the decay processes change. There is also a change of a dynamical nature: when the lepton acquires a mass there can also be spin-flip contributions.

When the lepton mass is taken to be nonzero, Eq.(6) becomes [23]

$$\left.\frac{d\Gamma}{dq^2}\right|_{m_l\neq0}=\frac{G^2}{(2\pi)^3}|V_{cQ}|^2\frac{p_V(q^2-m_l^2)^2}{12m_D^2q^2}$$

$$\times\left[(|H_+|^2+|H_-|^2+|H_0|^2)\right.$$

$$\left.+\left(\frac{m_l^2}{2q^2}\right)(|H_+|^2+|H_-|^2+|H_0|^2+3|H_t|^2)\right]. \tag{20}$$

In addition to spin-1 contributions, there are off-shell spin-0 contributions proportional to $|H_t|^2$, where

$$H_t=\frac{2m_Dp_V}{\sqrt{q^2}}A_0(q^2). \tag{21}$$

In Eq. (20), spin-flip contributions bring in the characteristic flip factor $m_l^2/2q^2$ which vanishes in the zero lepton mass limit. The bounds on q^2 are given by $m_l^2\leq q^2\leq(m_D-m_V)^2$ and it is seen that because of the factor $(q^2-m_l^2)^2$ multiplying all of the helicity amplitudes, all the form factor contributions vanish at threshold $q^2=m_l^2$. This is in contrast to the case for $m_l=0$, Eq. (6), where the longitudinal helicity amplitude H_0 appears with a $1/q^2$ factor which survives at $q^2=0$ with a contribution proportional to a single form factor A_0. We have written the helicity amplitudes in terms of the form factors A_0, A_2, and V using the definition given in Ref. [8] rather than the more conventional choice
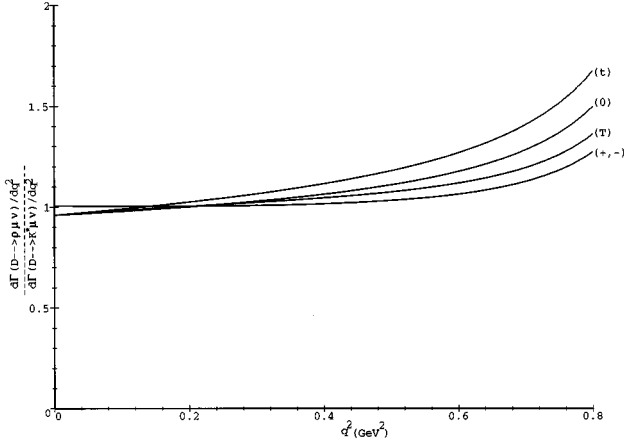
FIG. 1. q^2 spectra for $[d\Gamma(D\rightarrow\rho\mu\nu)/dq^2]/$ $[d\Gamma(D\rightarrow K^*\mu\nu)/dq^2]_{m_\mu\neq0}$ for different helicity contributions, longitudinal (0), transverse $(+,-)$, flip (t), and total (T) in units of $|V_{cd}/V_{cs}|^2$.

FIG. 2. q^2 spectra of semileptonic decay rates $D\rightarrow\rho\mu\nu_\mu$ with $m_\mu=0$ (dotted) and $m_\mu\neq0$ (full).

[20,23] of A_1, A_2, and V. This latter choice does not connect smoothly to the massless $q^2=0$ limit and gives incorrect results for SU(3)$_F$ breaking.

To investigate the SU(3)$_F$ limit when the lepton mass effect is included we need to consider q^2 dependences of the form factors. For this we use the approximation

$$F(q^2)\simeq\frac{F(0)}{1-q^2/\Lambda_1^2+q^4/\Lambda_2^4}, \qquad (22)$$

where we take the values of the parameters Λ_1 and Λ_2 from [24].

In Fig. 1 we show the q^2 spectra for the ratio $[d\Gamma(D\rightarrow\rho\mu\nu_\mu)/dq^2]/[d\Gamma(D\rightarrow K^*\mu\nu_\mu)/dq^2]_{m_\mu\neq0}$ for different helicity contributions defined in Eq. (20). As an explicit example, if we take the mass of the muon to be zero then our differential decay rate for $D\rightarrow K^*\mu\nu_\mu$ at $q^2=0$ gives a value of 5.82 in units of $|V_{cs}|^2 10^{10}$ sec^{-1} GeV^{-2}. The threshold for nonzero mass is $q^2=0.011$ GeV^2, where the decay rate vanishes. It is at $q^2=0.087$ GeV^2 that the value of 5.82 is first obtained. However, this 5.82 is now composed of three parts (see Table II), that coming from our $A_0(q^2)$ with a value of 4.15 and two other parts, one called a flip contribution (giving 1.05) with the remaining contribution of 0.62 coming from the transverse helicity part. In our model the ratio $[d\Gamma(D\rightarrow\rho\mu\nu_\mu)/dq^2]/[d\Gamma(D\rightarrow K^*\mu\nu_\mu)/dq^2]_{m_\mu\neq0}$ at this point (0.087) becomes 0.98 compared to 0.96 at $q^2=0$ in the massless limit. So, the SU(3)$_F$ limit remains steady when the simple, single form factor is re-
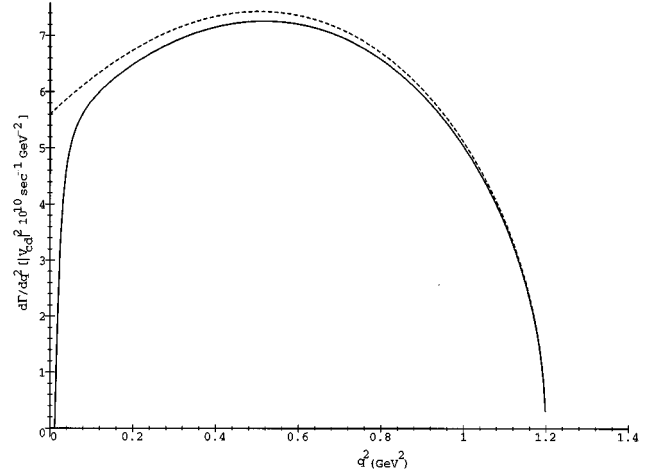
placed by a more complicated collection of form factors. This behavior breaks down only at higher q^2 when the effects of the differing masses of ρ and K^* become obvious. Figures 2 and 3 show the effect of a massive muon in $D\rightarrow\rho\mu\nu$.

III. RARE $B\rightarrow K^*\nu\bar{\nu}$ DECAY

The main reason for studying the decay $B\rightarrow K^*\nu\bar{\nu}$ is that in contrast to the decay $B\rightarrow K^*l\bar{l}$, where l is a charged lepton, its differential decay rate does not have any singularity at $q^2=0$. In the standard model, $B\rightarrow K^*\nu\bar{\nu}$ decay is governed by Z^0 penguin diagrams and box diagrams. The decay $B\rightarrow K^*l\bar{l}$ has an additional structure $\sigma_{\mu\nu}q_\nu/q^2$, which dominates the decay rate [25]. This does not occur, of course, in those calculations that stay away from the $q^2=0$ region. Moreover, the decay $B\rightarrow K^*\nu\bar{\nu}$ is a good process theoretically, since both the perturbative α_s and nonperturbative $1/m_b^2$ corrections are known to be small [26].

Contributions from the Z^0 penguin diagrams and box diagrams are sensitive functions of the top quark mass m_t. Thus, they contain an uncertainty due to the dependence of m_t on the choice of the renormalization scale μ. As stressed in Ref. [27], in order to reduce this uncertainty, it is necessary to calculate $O(\alpha_s)$ corrections to these diagrams involving internal top quark exchanges. The resulting effective Hamiltonian for $B\rightarrow K^*\nu\bar{\nu}$ decay is given [27] as

TABLE II. The first two columns show the partial helicity rates $d\Gamma_{(i)}/dq^2$, $i=(0),(+,-),(t),(T)$ for longitudinal, transverse, flip, and total contributions at $q^2=0.087$ GeV^2 in units of $|V_{CKM}|^2 10^{10}$ sec^{-1} GeV^{-2}. The last column gives the ratio.

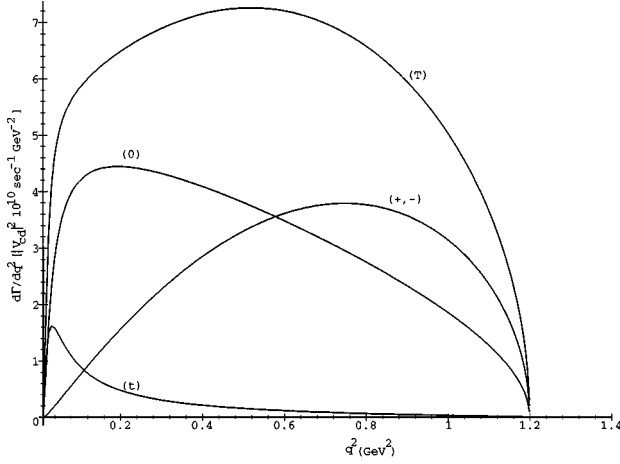| $d\Gamma_{(i)}/dq^2$	$D\rightarrow\rho$	$D\rightarrow K^*$	$D\rightarrow\rho/D\rightarrow K^*$
$d\Gamma_{(0)}/dq^2$	4.05	4.15	0.97
$d\Gamma_{(+,-)}/dq^2$	0.62	0.62	1.00
$d\Gamma_{(t)}/dq^2$	1.04	1.05	0.99
$d\Gamma_{(T)}/dq^2$	5.71	5.82	0.98

FIG. 3. Partial helicity rates $d\Gamma/dq^2$ for longitudinal (0), transverse $(+,-)$, flip (t) and total (T) contributions as a function of q^2 for $D\to\rho\mu\nu_\mu$ ($m_\mu\neq0$). The flip contribution is small but not as tiny as indicated in Ref. [22].

$$H_{\text{eff}}^{\nu\bar\nu}=\frac{4G_F}{\sqrt2}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)V_{ts}^*V_{tb}X(x_t)(\bar s\,\gamma_\mu Lb)(\bar\nu\gamma_\mu L\nu),$$

$$(23)$$

where $x_t=m_t^2/M_W^2$ and

$$X(x)=X_0(x)+\frac{\alpha_s}{4\pi}X_1(x).\qquad(24)$$

Here, X_0 represents pure electroweak one-loop contributions and X_1 results from $O(g_2^4\alpha_s)$ two-loop diagrams. We do not display here the explicit forms of $X_0(x_t)$ and $X_1(x_t)$, which can be found in Ref. [27]. At $\mu=M_W$ and $m_t=175$ GeV, we find that $X(x_t)=1.47$.

The differential decay rate for $B\to K^*\nu\bar\nu$ at zero momentum transfer is

$$\left.\frac{d\Gamma(B\to K^*\nu\bar\nu)}{dq^2}\right|_{q^2=0}$$

$$=\frac{G_F^2}{192\pi^3m_B^3}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)^2|V_{ts}^*V_{tb}|^2(m_B^2-m_{K^*}^2)^3$$

$$\times|A_0^{B\to K^*}(0)|^2|X(x_t)|^2.\qquad(25)$$

Taking $\tau_B=1.5\times10^{-12}$ sec^{-1}, $\sin^2\theta_W=0.23$, $|V_{tb}|=1$ and $A_0^{B\to K^*}(0)=0.4$ [11] and varying V_{ts} in the range $0.030\leq|V_{ts}|\leq0.048$ we find

$$4.81\times10^{-8}\text{ GeV}^{-2}\leq\frac{1}{\Gamma_{\text{tot}}}\frac{d\Gamma(B\to K^*\nu\bar\nu)}{dq^2}$$

$$\leq1.23\times10^{-7}\text{ GeV}^{-2}.\qquad(26)$$

The differential decay rate for $B\to\rho l\bar\nu_l$ is determined by the form factor A_0 at $q^2=0$, just as in the case of the D decays.

The ratio of $B\to\rho l\bar\nu_l$ to $B\to K^*\nu\bar\nu$ gives

$$\frac{[d\Gamma(B\to\rho l\bar\nu_l)/dq^2]_{q^2\to0}}{[d\Gamma(B\to K^*\nu\bar\nu)/dq^2]_{q^2\to0}}$$

$$=\frac{|V_{ub}|^2}{|V_{ts}^*V_{tb}|^2}\left(\frac{m_B^2-m_\rho^2}{m_B^2-m_{K^*}^2}\right)^3\left(\frac{2\pi\sin^2\theta_W}{\alpha}\right)^2$$

$$\times\frac{|A_0^{B\to\rho}(0)|^2}{|A_0^{B\to K^*}(0)|^2}\frac{1}{|X(x_t)|^2}$$

$$=(1.86\times10^4)\frac{|V_{ub}|^2}{|V_{ts}^*|^2}\frac{|A_0^{B\to\rho}(0)|^2}{|A_0^{B\to K^*}(0)|^2}.\qquad(27)$$

The form factors $A_0^{B\to\rho}(0)$ and $A_0^{B\to K^*}(0)$ have already been calculated [11] in the light-front quark model as shown in the table. Thus, we see $A_0^{B\to\rho}(0)/A_0^{B\to K^*}(0)=0.75$. By comparison, the corresponding result for D form factors we find $A_0^{D\to\rho}(0)/A_0^{D\to K^*}(0)=0.88$. That is, the SU(3)$_F$ breaking of the form factors becomes larger as the mass of the decaying meson increases.

IV. CONCLUSION

In this paper we have reviewed the present status of theoretical attempts to calculate the semileptonic charm and bottom decays. We then presented a calculation of these decays in the light-front frame at the kinematic point $q^2=0$. This allowed us to evaluate the form factors at the same value of q^2, even though the allowed kinematic ranges for charm and bottom decays are very different. Also, at this kinematic point the decay is given in terms of only one form factor $A_0(0)$. For the ratio of the decay rates given by the E653 Collaboration we show that the determination of the ratio of the CKM matrix elements is consistent with that obtained from the unitarity constraint. At present, though, the unitarity method still has greater accuracy. For B decays, the decay $B\to K^*l\bar l$ at $q^2=0$ involves an extra form factor coming from the photon contribution and so is not amenable to the same kind of analysis, leaving only the decay $B\to K^*\nu\bar\nu$ as a possibility. This is not an easy mode to determine experimentally.

The results obtained in our model for the form factor $A_0(0)$, for D decays, as well as other models are collected in Table I. We see that theoretical predictions of the ratio of form factors fall in a range near 1. If $A_0(0)$ is obtained from Eq. (5) with the calculated values of A_1 and A_2 then there may be difficulties coming from the q^2 dependences of the form factors and also from possible correlations in treating the errors. The comparison with QCD sum rules predictions [32,34,35] shows a similar problem (the exception is Ref. [33], where A_0 is directly calculated): the uncertainities in the results obtained using A_1 and A_2 are so large that they obscure the real value of A_0. For the nonzero lepton mass case, the use of A_1, A_2, and V does not connect smoothly to the zero lepton mass results. When A_2 is used in place of A_1 the SU(3) symmetry breaking remains small for a range of q^2, even though there is a more complicated collection of form factors.

It is interesting to note the predictions of [37] obtained in a framework based on HQET and chiral symmetries. Although their values for the form factors for $D \to \rho$ and $D \to K^*$ agree with the predictions of other models given in the table, the result of $B \to \rho$ is larger than most of the others.

It is claimed sometimes that the light-front quark model is ruled out since it typically gives a value about 15% less than one for the ratio $R = f_+^{D \to \pi}(0)/f_+^{D \to K}(0)$. The experimental value of R is obtained from the measurements of the decays $D^0 \to K^- e^+ \nu_e$ and $D^+ \to \pi^0 e^+ \nu_e$ by Mark III [43] and CLEO II [44]:

$$\frac{Br(D^+ \to \pi^0 e^+ \nu_e)}{Br(D^0 \to K^- e^+ \nu_e)} = \begin{cases} (8.5 \pm 2.7 \pm 1.4)\% & \text{Mark III,} \\ (10.5 \pm 3.9 \pm 1.3)\% & \text{CLEO II.} \end{cases}$$

To translate these results into the values of ratio R, pole dominance is assumed for the q^2 dependence of the form factors for the $\pi e \nu_e (K e \nu_e)$ decay with the mass of the vector resonance given by the mass of the $D^*(D_s^*)$ meson:

$$R = \begin{cases} (1.29 \pm 0.21 \pm 0.11) & \text{Mark III,} \\ (1.01 \pm 0.20 \pm 0.07) & \text{CLEO II.} \end{cases}$$

Given the size of these errors, it is premature to claim that a value less than unity is ruled out.

In an analysis of two-body hadronic decays $D^+ \to \pi^+ \pi^0$ and $D^0 \to K^+ \pi^-$, Chau and Cheung [45] calculated the ratio R and found that relative magnitude of the form factors should be such that $f_+^{D \to \pi}(0) > f_+^{D \to K}(0)$ in order to be consistent with the pattern of SU(3) breaking. However, this calculation relies on the large-N_c factorization approach in addition to the pole dominance assumption for the q^2 dependences of the form factors. They also neglect the final state interaction effects. These assumptions have recently been questioned by Kamal *et al.* [46]. Also, the value of the branching ratio for $D^0 \to K \pi$ may have been overestimated [47].

Finally, we note that as the mass of the decaying particle increases the SU(3) symmetry breaking becomes greater at $q^2 = 0$.

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