

# Nonperturbative QCD contributions to the semileptonic decay width of the $B$ meson

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(Received 19 February 1997)

Nonperturbative QCD contributions to the inclusive semileptonic decay of the  $B$  meson consist of the dynamic and kinematic components. We calculate the decay width in an approach based on light-cone expansion and heavy quark effective theory, which is able to include both components of nonperturbative QCD contributions. The kinematic component results in the phase-space extension and is shown to be quantitatively crucial, which could increase the decay width significantly. We find that the semileptonic decay width is enhanced by long-distance strong interactions by  $+(9 \pm 6)\%$ . This analysis is used to determine the CKM (Cabibbo-Kobayashi-Maskawa) matrix element  $|V_{cb}|$  with a controlled theoretical error. Implications of the phase-space effects for the nonleptonic decay widths of  $b$  hadrons are briefly discussed. The experimental evidence for the phase-space effects is pointed out. [S0556-2821(97)05217-X]

PACS number(s): 13.20.He

## I. INTRODUCTION

A direct goal of studying the inclusive semileptonic  $B$  meson decay  $\bar{B} \rightarrow e \bar{\nu}_e X$  is to determine the standard model parameter  $|V_{cb}|$  accurately. The semileptonic decay width can be expressed as

$$\Gamma_{\text{SL}} = \gamma_c |V_{cb}|^2 + \gamma_u |V_{ub}|^2. \quad (1)$$

The first term in Eq. (1) results from the  $b \rightarrow c$  transition. The second term in Eq. (1) is due to the  $b \rightarrow u$  transition and is negligible in comparison with the first term since  $\gamma_u \sim \gamma_c$  and  $|V_{ub}| \ll |V_{cb}|$ . The semileptonic decay width is determined by two measured quantities: the inclusive semileptonic  $B$  decay branching ratio  $B_{\text{SL}}$  and the  $B$  meson lifetime  $\tau_B$ :

$$\Gamma_{\text{SL}} = \frac{B_{\text{SL}}}{\tau_B}. \quad (2)$$

Therefore, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $|V_{cb}|$  can be determined through

$$|V_{cb}|^2 = \frac{\Gamma_{\text{SL}}}{\gamma_c} = \frac{B_{\text{SL}}}{\gamma_c \tau_B}, \quad (3)$$

with the theoretical input  $\gamma_c$ .

Theory is needed to calculate  $\gamma_c$  and to understand quantitatively uncertainties in this calculation. The main obstacle to this end is the difficulty of taking into account nonperturbative QCD effects on the underlying weak decay process.

In recent years a heavy quark expansion approach to inclusive  $B$  decays has been developed [1–7] to account for nonperturbative QCD effects. This approach is based on the operator product expansion and the heavy quark effective theory (HQET). An operator product expansion on the time ordered product of two currents is performed. The momentum of the incoming  $b$  quark is written as  $p_b = m_b v + k$  ( $m_b$  stands for the  $b$  quark mass and  $v$  the  $B$  hadron veloc-

ity) and the residual momentum,  $k$ , is expanded in. For keeping track of the  $m_b$  dependence of matrix elements, the  $b$  quark operators in full QCD are matched onto those in heavy quark effective theory (HQET). The leading term of the expansion coincides with the free quark decay model. The next terms are computed in powers of  $1/m_b$ , where no  $1/m_b$  term appears.

The calculations [8,9] in the heavy quark expansion approach claimed that nonperturbative QCD contributions *decrease* the semileptonic decay width by a few percent with respect to the free quark decay width. There are, however, theoretical limitations in this approach [3]. The operator product expansion breaks down for low-mass final hadronic states. In particular, the endpoint singularities of the lepton spectra indicate a failure of the operator product expansion. Moreover, the truncation of the expansion enforces the use of quark kinematics rather than physical hadron kinematics. Describing the lepton spectra demands a resummation of the heavy quark expansion [10]. There remains a need to clarify the consequence of the theoretical limitations for the calculation of the semileptonic decay width, as it is desirable to improve the accuracy in the independent determination<sup>1</sup> of  $|V_{cb}|$  from the inclusive semileptonic  $B$  decay with theoretical refinements.

The resummation of the heavy quark expansion introduces [10] a distribution function (“shape function”) of the  $b$  quark in the  $B$  meson, which incorporates nonperturbative QCD effects. A similar distribution function arises [11,12] from the light-cone dominance in the inclusive semileptonic  $B$  meson decays. The introduction of the distribution function eliminates the theoretical difficulties mentioned above, namely the endpoint singularities are absent and the use of physical hadron kinematics is allowed (but does not arise “for free”).

In this paper we will use the light-cone approach [11,12] to calculate the nonperturbative QCD contributions. This ap-

<sup>1</sup>Using different experimental and theoretical methods,  $|V_{cb}|$  can be also determined independently from exclusive semileptonic  $B$  decays.

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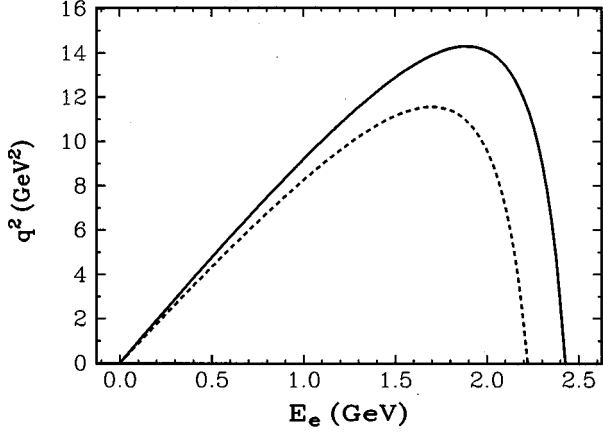


FIG. 1. Phase space for the  $b \rightarrow c$  inclusive semileptonic decay. The interior of the solid curve is the hadron level phase space (the changeableness of the mass of the final hadronic state is not shown explicitly). The interior of the dashed curve is the quark level phase space.

proach describes the decay by using the light-cone expansion and the HQET, which provides a theoretical justification for the DIS-like parton model [13]. The predicted electron energy spectrum agrees well with the experimental measurement [12]. The use of physical hadron kinematics is built into this approach explicitly, so that both dynamic and kinematic effects of nonperturbative QCD are properly taken into account. The latter is shown to be quantitatively crucial. We find an about 9% enhancement of the semileptonic decay width with respect to the free quark decay width by nonperturbative QCD contributions, in contrast to the results obtained in the heavy quark expansion approach.

The reason of the enhancement is the following. There are two components — dynamics and kinematics — of nonperturbative QCD effects on inclusive semileptonic  $B$  decays. First, the decay dynamics deviates from the free quark decay dynamics as quarks are confined in hadrons and can never be free. However, the dynamic deviation changes the decay width only slightly since the  $b$  quark inside the  $B$  meson is almost on shell. Second, the decay kinematics gets changed. The phase space extends from the quark level to the hadron level [the detailed formulas will be given below in Eqs. (13)–(15) for hadron kinematics and in Eqs. (16) and (17) for quark kinematics], shown in Fig. 1 for the  $b \rightarrow c$  decay. The phase-space extension arises from the difference in the  $B$  meson and  $b$  quark masses and the fact that the mass of the decay product quark is fixed in the free quark decay picture, while the mass of the final hadronic state is actually changeable. The phase-space effect is a dominating factor, as indicated by the replacement of the  $b$ -quark mass with the  $B$ -meson mass in the decay rate. It is thus important to include this type of contributions to the decay width. Consequently, it is conceivable that the net effect of nonperturbative QCD *enhances* the semileptonic decay width. The negative contribution found in previous studies in the heavy quark expansion approach is just a reflection of incompleteness of the calculation, which fails to take into account, in particular, a large part of nonperturbative QCD contributions due to the phase-space effect.

We will describe the approach in Sec. II and analyze non-

perturbative QCD contributions in comparison with the heavy quark expansion and extract then  $|V_{cb}|$  from the inclusive semileptonic  $B$  meson decay in Sec. III and finally conclude and discuss in Sec. IV.

## II. APPROACH

The semileptonic decay width can be split into two parts: one, denoted by  $\Gamma_{\text{nonpert}}$ , includes nonperturbative QCD contributions, the other results from perturbative QCD corrections to the decay width, denoted by  $\Gamma_{\text{pert}}$ . Namely,

$$\Gamma_{\text{SL}} = \Gamma_{\text{nonpert}} + \Gamma_{\text{pert}}. \quad (4)$$

Nonperturbative QCD effects are contained in the hadronic tensor  $W_{\mu\nu}$ . It can be written in terms of a current commutator taken between  $B$  states:

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4y e^{iq \cdot y} \langle B | [j_\mu(y), j_\nu^\dagger(0)] | B \rangle, \quad (5)$$

where  $q$  is the momentum transfer to the final lepton pair.  $|B\rangle$  refers to the  $B$ -meson state with energy  $E_B$  and is normalized according to  $\langle B | B \rangle = 2E_B (2\pi)^3 \delta^3(\mathbf{0})$ .

It is well known that integrals like the one in Eq. (5) are dominated by distances where

$$0 \leq y^2 \leq \frac{1}{q^2}. \quad (6)$$

For inclusive semileptonic  $B$ -meson decays, the momentum transfer squared  $q^2$  is timelike and varies in the physical range

$$0 \leq q^2 \leq (M_B - M_{X_{\min}})^2, \quad (7)$$

where  $M_B$  and  $M_{X_{\min}}$  represent the  $B$ -meson mass and the minimum value of the invariant mass of the hadronic final state, respectively. Because of the large  $B$ -meson mass, extended regions of phase space involve large values of  $q^2$  (see also Fig. 1). Therefore, the decay is dominated by light-cone distances between the two currents in Eq. (5). This allows to replace the commutator of the two currents with its singularity on the light cone times an operator bilocal in the  $b$  quark fields. Furthermore, the light-cone dominance enables us to expand the matrix element of the bilocal operator between  $B$ -meson states in powers of  $\Lambda_{\text{QCD}}^2/q^2$ . The leading nonperturbative effect is described by a distribution function [11,12]:

$$f(\xi) = \frac{1}{4\pi M_B^2} \int d(y \cdot P_B) e^{i\xi y \cdot P_B} \langle B | \bar{b}(0) \not{P}_B (1 - \gamma_5) \times b(y) | B \rangle |_{y^2=0}, \quad (8)$$

where  $P_B$  denotes the four-momentum of the  $B$  meson.  $f(\xi)$  is the probability of finding a  $b$ -quark with momentum  $\xi P_B$  inside the  $B$  meson. The hadronic tensor can be expressed in terms of the distribution function

$$W_{\mu\nu} = 4(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta}) \int d\xi f(\xi) \varepsilon(\xi P_{B_0} - q_0) \times \delta[(\xi P_B - q)^2 - m_c^2] (\xi P_B - q)^\alpha P_B^\beta, \quad (9)$$

where  $m_c$  is the charm quark mass.

$\Gamma_{\text{nonpert}}$  is calculated in this approach by integrating the differential decay rate in the  $B$  rest frame,

$$\Gamma_{\text{nonpert}} = \int dE_e \int dq^2 \int dM_X^2 \frac{d^3\Gamma}{dE_e dq^2 dM_X^2}, \quad (10)$$

with the differential decay rate

$$\frac{d^3\Gamma}{dE_e dq^2 dM_X^2} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3 M_B} \frac{q_0 - E_e}{\sqrt{q^2 + m_c^2}} \times \left\{ f(\xi_+) \left( 2E_e \xi_+ - \frac{q^2}{M_B} \right) - (\xi_+ \rightarrow \xi_-) \right\}, \quad (11)$$

where  $E_e$  is the electron energy and we have neglected the electron mass.  $M_X$  denotes the invariant mass of the final hadronic state. The dimensionless variables  $\xi_\pm$  reads

$$\xi_\pm = \frac{q_0 \pm \sqrt{q^2 + m_c^2}}{M_B}. \quad (12)$$

Note that there appears in the differential decay rate (11) the  $B$  meson mass rather than the  $b$  quark mass. The integration limits are specified by hadron kinematics:

$$0 \leq E_e \leq \frac{M_B}{2} \left( 1 - \frac{M_{X_{\min}}^2}{M_B^2} \right), \quad (13)$$

$$0 \leq q^2 \leq 2E_e \left( M_B - \frac{M_{X_{\min}}^2}{M_B - 2E_e} \right), \quad (14)$$

$$M_{X_{\min}}^2 \leq M_X^2 \leq (M_B - 2E_e) \left( M_B - \frac{q^2}{2E_e} \right), \quad (15)$$

which define the hadron level phase space shown in Fig. 1. For comparison, we also write down here the kinematic boundaries for the free quark decay:

$$0 \leq E_e \leq \frac{m_b}{2} \left( 1 - \frac{m_c^2}{m_b^2} \right), \quad (16)$$

$$0 \leq q^2 \leq 2E_e \left( m_b - \frac{m_c^2}{m_b - 2E_e} \right), \quad (17)$$

which are also shown in Fig. 1. It is an important feature of this approach that the calculation can be performed in the physical phase space as a large contribution of nonperturbative QCD arises from the extension of phase space from the quark level to the hadron level. It should be pointed out, however, that in theoretical calculations we take  $M_{X_{\min}} = m_c$  since we assume quark-hadron duality in our approach.

Important properties of the distribution function are derived from field theory. Due to current conservation, it is exactly normalized to unity with a support  $0 \leq \xi \leq 1$ . It obeys positivity. When the distribution function becomes the delta function,  $\delta(\xi - m_b/M_B)$ , the free quark decay is reproduced. Furthermore, the next two moments of the distribution function can be estimated in the HQET, as we shall discuss below. These two moments determine the mean value  $\mu$  and the variance  $\sigma^2$  of the distribution function, which characterize the position of the maximum and the width of it, respectively:

$$\mu \equiv M_1(0) = \bar{\xi} + M_1(\bar{\xi}), \quad (18)$$

$$\sigma^2 \equiv M_2(\mu) = M_2(\bar{\xi}) - M_1^2(\bar{\xi}), \quad (19)$$

where  $M_n(\bar{\xi})$  is the  $n$ th moment about a point  $\bar{\xi}$  of the distribution function defined by

$$M_n(\bar{\xi}) = \int_0^1 d\xi (\xi - \bar{\xi})^n f(\xi). \quad (20)$$

By definition,  $M_0(\bar{\xi}) = 1$ .

The accuracy of the theory is remarkably improved by estimating the next two moments of the distribution function in the framework of the HQET. However, it cannot yet be completely determined in QCD. For practical calculations, therefore, we shall use an ansatz for the distribution function, which respects all known properties, with two parameters  $a$  and  $b$  as follows

$$f(\xi) = N \frac{\xi(1-\xi)}{(\xi-a)^2 + b^2} \theta(\xi) \theta(1-\xi), \quad (21)$$

where  $N$  is the normalization constant. For  $a = m_b/M_B$  and  $b = 0$ , Eq. (21) becomes a delta function,  $\delta(\xi - m_b/M_B)$ , and the free quark decay is reproduced. Another form of the distribution function has been proposed in [14].

The perturbative QCD corrections to  $O(\alpha_s)$  has been calculated [15,16]. It has the form

$$\Gamma_{\text{pert}} = -\frac{2\alpha_s}{3\pi} H\left(\frac{m_c}{m_b}\right) \Gamma_b, \quad (22)$$

where  $\Gamma_b$  is the free quark decay width:

$$\Gamma_b = \Gamma_0 \Phi\left(\frac{m_c}{m_b}\right), \quad (23)$$

with

$$\Gamma_0 = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3}, \quad (24)$$

$$\Phi(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x. \quad (25)$$

An analytic expression for  $H(m_c/m_b)$  is given in [16].

The semileptonic decay width can be calculated by substituting Eqs. (10) and (22) into Eq. (4). The parameters involved and hence the sources of the theoretical error are (1) the parameters in the distribution function [for the ansatz

(21) that will be used, they are  $a$  and  $b$ ], (2) the beauty and charm quark pole masses  $m_b$  and  $m_c$ , and (3) the strong coupling constant  $\alpha_s$ .

There are several theoretical constraints on these parameters stemming from the HQET, which reduce the theoretical uncertainties considerably. We discuss them in turn.

(I) Performing a light-cone OPE and following the method of [3] to expand the matrix elements of the local operators in the HQET, the next two moments and hence the mean value  $\mu$  and the variance  $\sigma^2$  of the distribution function can be related [11] to two accessible parameters  $K_b$  and  $G_b$  up to the order of  $(\Lambda_{\text{QCD}}/m_b)^2$ :

$$\mu = \frac{m_b}{M_B} (1 + E_b), \quad (26)$$

$$\sigma^2 = \left( \frac{m_b}{M_B} \right)^2 \left( \frac{2K_b}{3} - E_b^2 \right), \quad (27)$$

where  $E_b = K_b + G_b$ . Both  $K_b$  and  $G_b$  are of order  $(\Lambda_{\text{QCD}}/m_b)^2$ . This leads to a model-independent conclusion: the distribution function is sharply peaked around  $\xi = \mu \approx m_b/M_B$  and its width is of order  $\Lambda_{\text{QCD}}/M_B$ .

For numerical analyses we need to know  $K_b$  and  $G_b$  quantitatively. The parameter  $G_b$  is related [3] to the observables:

$$m_b G_b = -\frac{3}{4} (M_{B^*} - M_B), \quad (28)$$

where the mass difference of the vector  $B^*$  and the pseudo-scalar  $B$  mesons is measured to be  $M_{B^*} - M_B = 0.046$  GeV.  $K_b$  can be reexpressed in terms of another often used parameter  $\lambda_1$  instead,

$$K_b = -\frac{\lambda_1}{2m_b^2}. \quad (29)$$

It is harder to determine  $\lambda_1$  (or  $K_b$ ). The accurate value of it is not known. Consequently, the mean value  $\mu$  and the variance  $\sigma^2$  of the distribution function are determined by the two parameters  $m_b$  and  $\lambda_1$ :  $\mu$  depends on  $m_b$  strongly and  $\lambda_1$  very weakly, while  $\sigma^2$  is sensitive essentially only to  $\lambda_1$ . Hence, the parameters  $a$  and  $b$  in the ansatz (21) for the distribution function are also determined by  $m_b$  and  $\lambda_1$ .

(II) The quark mass difference is related to  $\lambda_1$  in the HQET [9]:

$$m_b - m_c = (\bar{M}_B - \bar{M}_D) \left\{ 1 - \frac{\lambda_1}{2\bar{M}_B \bar{M}_D} + O(1/m_c^3) \right\}, \quad (30)$$

where the spin-averaged meson masses

$$\bar{M}_B = \frac{1}{4} (M_B + 3M_{B^*}) = 5.31 \text{ GeV}, \quad (31)$$

$$\bar{M}_D = \frac{1}{4} (M_D + 3M_{D^*}) = 1.97 \text{ GeV}. \quad (32)$$

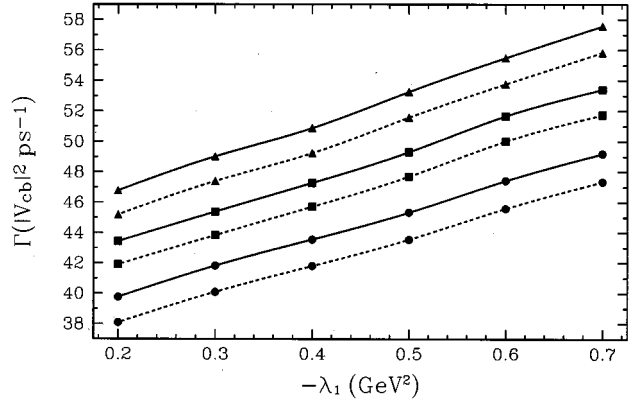


FIG. 2. Dependence of the semileptonic decay width  $\Gamma$  on the theoretical input parameters  $m_b$ ,  $\lambda_1$ , and  $\mu_r$ . The solid (dashed) curves are for the renormalization scale  $\mu_r = m_b$  ( $\mu_r = m_b/2$ ). The curves with solid dots, boxes, triangles correspond to  $m_b = 4.7, 4.9, 5.1$  GeV, respectively.

Finally, the remaining theoretical input parameters for our analysis are  $m_b$ ,  $\lambda_1$ , and  $\alpha_s$ .

### III. ANALYSIS

We evaluate  $\Gamma_{\text{nonpert}}$  and  $\Gamma_{\text{pert}}$  in the approach described above using the three input parameters. For  $m_b$  we use

$$m_b = 4.9 \pm 0.2 \text{ GeV}. \quad (33)$$

According to a QCD sum rule calculation [17], we take

$$\lambda_1 = -(0.5 \pm 0.2) \text{ GeV}^2. \quad (34)$$

As a result, the mean value and the variance of the distribution function are

$$\mu = 0.93 \pm 0.04, \quad (35)$$

$$\sigma^2 = 0.006 \pm 0.002. \quad (36)$$

A truncating of perturbative series causes the dependence of perturbative calculations on the renormalization scale  $\mu_r$ . For inclusive semileptonic  $B$  decays perturbative QCD corrections are known only to the leading order. The result given in Eq. (22) exhibits an implicit scale dependence of the strong coupling  $\alpha_s$ . We vary the scale over the range of  $m_b/2 \leq \mu_r \leq m_b$  to estimate the theoretical error due to the choice of the scale used in the argument of  $\alpha_s$ .

The dependence of the decay width  $\Gamma$  on the parameters is shown in Fig. 2. The variation of  $\Gamma$  with  $m_b$  or  $\lambda_1$  is stronger than  $\mu_r$ . The variation of  $m_b$  leads to an uncertainty of 8% in the decay width if other parameters are kept fixed. The same uncertainty in the decay width results from the variation of  $\lambda_1$ . An uncertainty of 2% in the decay width is introduced when the renormalization scale  $\mu_r$  is varied between  $m_b/2$  and  $m_b$ . In addition, the impact of the shape of the distribution function on the value of the decay width is studied. The value of the decay width is more sensitive to the variation of the mean value than the variation of the variance

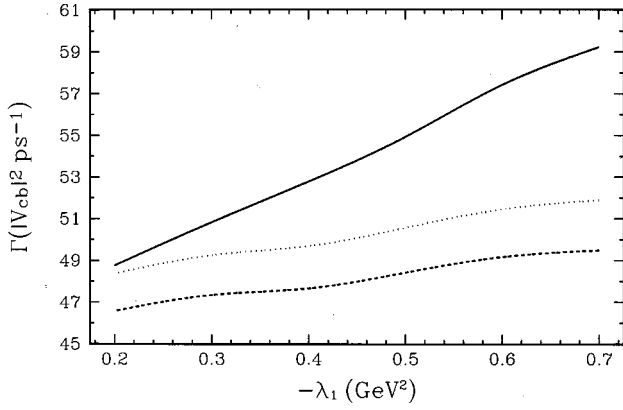


FIG. 3. Semileptonic decay width  $\Gamma$  as a function of  $\lambda_1$  calculated in our approach (solid curve), the free quark decay model (dotted curve), and the heavy quark expansion approach (dashed curve). We take  $m_b = 4.9$  GeV and  $\alpha_s = 0$ .

of the distribution function. Furthermore, we modify Eq. (21) with two more parameters  $\alpha$  and  $\beta$  to be

$$f(\xi) = N \frac{\xi(1-\xi)^\alpha}{[(\xi-a)^2 + b^2]^\beta} \theta(\xi) \theta(1-\xi). \quad (37)$$

Using Eq. (37) we find that the value of the decay width is insensitive to the change of the shape of the distribution function if the mean value and the variance of it are kept fixed. This insensitivity diminishes the model dependence. This analysis yields

$$\gamma_c = 49 \pm 9 \text{ ps}^{-1}, \quad (38)$$

with a theoretical error of 18%.

In Fig. 3, we compare the decay widths calculated in our approach, the free quark decay model, and the heavy quark expansion approach. The result in our approach shows that nonperturbative QCD contributions enhance the decay width by  $+(9 \pm 6)\%$  with respect to the free quark decay width, in contrast to the result of the heavy quark expansion approach where a reduction of the free quark decay width by  $-(4.3 \pm 0.5)\%$  is found. The change of the sign indicates that the nonperturbative effects receive a large phase-space enhancement. We also observe that the decay width calculated in our approach goes to a free quark decay limit as  $-\lambda_1$  decreases. This behavior is expected since the distribution function approaches a delta function, which reproduces the free quark decay, as  $-\lambda_1$  and hence  $\sigma^2$  decrease. Thus this behavior provides a check of calculations.

This theoretical analysis can be used to determine  $|V_{cb}|$ . Experimentally the inclusive semileptonic branching ratio  $B_{\text{SL}}$  has been measured at the  $\Upsilon(4S)$  and  $Z^0$  resonances, respectively. The lifetime  $\tau_B$  has been measured by experiments at  $Z^0$  and in  $p\bar{p}$  collisions. The average of these measurements leads to [18]

$$\Gamma_{\text{SL}} = 67.3 \pm 2.7 \text{ ns}^{-1}. \quad (39)$$

Putting it together with the theoretical value of  $\gamma_c$  given in Eq. (38), we obtain from Eq. (3)

$$|V_{cb}| = 0.0371 \pm 0.0007 \pm 0.0034, \quad (40)$$

where the first error is experimental and the second theoretical.

#### IV. CONCLUSION AND DISCUSSION

We have calculated the semileptonic decay width of the  $B$  meson using an approach based on the light-cone expansion and the HQET. Nonperturbative QCD effects are described by a single distribution function. Several important properties of the distribution function are known from QCD and the HQET of it. However, one still has to model the distribution function. Fortunately the result of the calculation of the decay width in this approach is nearly model-independent, since it is essentially only sensitive to the mean value and the variance of the distribution function, whose theoretical estimates exist. Moreover, this approach is able to take into account both dynamic and kinematic components of nonperturbative QCD effects. We have shown that including the latter is indeed quantitatively crucial, which could increase the decay width significantly. We find an enhancement of the free quark decay width by  $+(9 \pm 6)\%$  due to nonperturbative QCD contributions, contrary to the claims from the heavy quark expansion approach. As a result, a value of  $|V_{cb}|$  is extracted from the inclusive semileptonic  $B$  meson decay with a controlled theoretical error.

The main theoretical uncertainty arises from the values of the  $b$  quark mass and the HQET parameter  $\lambda_1$ . It seems possible to reduce theoretical uncertainties by a detailed fit<sup>2</sup> to the measured charged-lepton energy spectrum to determine the parameters and a calculation of the next-to-leading order perturbative QCD correction.<sup>3</sup> Future measurements of the distribution function and more theoretical efforts on calculations of hadronic matrix elements should enable to reduce further the uncertainties.

Careful inclusion of the kinematic effect of nonperturbative strong interactions is also necessary for reliable predictions for the nonleptonic decay widths of hadrons containing a  $b$  quark. The nonleptonic decay widths of  $b$  hadrons may be calculated in a similar way. The decay widths are expressed in terms of the  $b$  hadron masses rather than the  $b$  quark mass provided the phase-space effects are included, whereas according to the heavy quark expansion, the relevant mass in the decay widths should be the universal  $b$ -quark mass and no corrections of order  $1/m_b$  should be present [21]. We would anticipate an enhancement of the nonleptonic decay width by nonperturbative QCD if both dynamic and kinematic effects of it are properly taken into account. Since the phase-space effects cancel out to a large extent in the ratio of the decay widths, they are unlikely to change significantly the semileptonic branching ratio of the

<sup>2</sup>Such a fit has been done in the heavy quark expansion approach [19].

<sup>3</sup>Partial calculations of higher-order corrections exist [20].

$B$  meson or the average number of charmed hadrons produced per  $B$  decay. On the other hand, the prediction on the ratio of the  $\Lambda_b$  and  $B$  lifetimes from the heavy quark expansion approach seems to be in conflict with the data [22]. In this case phase-space effects are enlarged due to the significant difference between the  $\Lambda_b$  and  $B$  masses. The replacement of the  $b$ -quark mass with the nonuniversal  $b$ -hadron

masses results in a perfect agreement [23] between the theory and the experimental data, giving evidence for the phase-space effects.

#### ACKNOWLEDGMENTS

I would like to thank E. A. Paschos for discussions.

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