High temperature dimensional reduction of the MSSM and other multiscalar models in the $\sin^2\theta_W=0$ limit

Marta Losada*

Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855

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Using dimensional reduction we construct an effective 3D theory of the minimal supersymmetric standard model in the $\sin^2\theta_W=0$ limit at finite temperature. The final effective theory is obtained after three successive stages of integration out of massive particles. We obtain the full one-loop relation between the couplings of the reduced theory and the underlying 4D couplings and masses. The procedure is also applied to a general two Higgs doublet model and the next to minimal supersymmetric standard model. [S0556-2821(97)04615-8]

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I. INTRODUCTION

A crucial problem in particle physics is the understanding of the observed baryon asymmetry of the universe. A possible scenario which has been explored involves physics at the electroweak scale. For electroweak baryogenesis to occur via a sufficiently strong first order phase transition, such that baryon number violation is suppressed after the phase transition, the sphaleron transition rate in the low temperature phase must be less than the universe expansion rate [1]. This requires that the ratio of the vacuum expectation value in the broken phase to the phase transition temperature (T_c) must be [2]

$$\frac{v(T_c)}{T_c}\gtrsim 1, \tag{1}$$

which in turn places constraints on the Higgs structure of the low temperature theory. In fact, these requirements may not be fulfilled in the standard model at least for experimentally allowed values of the Higgs boson mass [2,3].

In order to study the properties of the phase transition, standard perturbation techniques (with resummation) were initially utilized. However, there are two intrinsic problems with perturbation theory. First of all, the loop expansion parameter ($g^2T/M_W\sim\lambda/g^2$) is proportional to the zero temperature ratio of Higgs boson mass to the W boson mass. This implies that unless the Higgs boson is sufficiently light the perturbative approximations will break down when higher order corrections are included [4]. Second, high temperature gauge theories are subject to infrared divergences arising from massless gauge boson modes in the theory [5]. In the broken phase these divergences are absent because the gauge bosons of the broken symmetry have nonzero ϕ-dependent masses. Consequently, even when the aforementioned perturbative effects are under control, nonperturbative physics in the unbroken phase, for values of $\phi\sim 0$, can limit the reliability of the perturbative calculations of the phase transition. In other words, knowledge of the effective potential for small values of the scalar field is required to study the properties of the phase transition, and the assumptions underlying perturbative calculations must be verified by calculations which include nonperturbative effects.

Using the fact that in the symmetric phase at finite temperature the long distance theory is described by a three-dimensional (3D) bosonic gauge theory, Kajantie and coworkers [6–8] devised a procedure which separates the perturbative and nonperturbative aspects of the study of the electroweak phase transition. As explained in Sec. II A, due to antiperiodicity of the boundary condition at finite temperature, fermions in the theory acquire thermal masses proportional to the temperature $\sim\pi T$. The bosonic field decomposition into thermal modes contains a static mode which does not have a thermal mass, as well as modes with masses on the order of $\sim\pi T$. The construction of the effective theory for the static modes amounts to perturbatively integrating out the effects of all the massive modes. As a consequence of the interaction with the heavy modes, the masses of some of the static scalar fields are modified enough that they become heavy ($\sim gT$) and one can further construct a second effective 3D theory describing only the light fields. This reduced three-dimensional gauge theory for the light scalars can then be analyzed by numerical (lattice) calculations so that all nonperturbative effects are handled correctly.

Kajantie *et al.* reduced the standard model to a 3D gauge theory with a single light scalar, which they studied on the lattice. They concluded that many, but not all, features of the phase transition were similar to results of perturbation theory. The results showed that there is no value of the Higgs boson mass in the minimal supersymmetric standard model which is compatible with correct electroweak symmetry breaking in the low temperature theory, given the measured large top mass [7].

Consequently, it is of interest to analyze extensions of the standard model to see whether there are any cases in which there is a sufficiently strong first order phase transition. Using the numerical results of Kajantie *et al.* we can include nonperturbative effects for any 4D theory which reduces to an effective three-dimensional theory containing a single light scalar coupled to SU(2) gauge fields. This is the generic situation at the phase transition, except when the theory is fine-tuned, even when the theory has multiple scalar fields as in supersymmetry, general two-Higgs-doublet models, etc.

*On leave of absence from Centro Internacional de Física and Universidad Antonio Nariño, Sante Fe de Bogotá, Colombia.

A feature of multiscalar models is the inclusion of tree-level masses for all scalars. Additionally, some of the scalar fields may carry color charge. This implies that in the unbroken phase the static modes of the scalars can acquire masses between πT and gT. In particular, the masses of the squarks static modes receive corrections proportional to $\sim g_s T$. Consequently, unlike the standard model case, we encounter an additional intermediate effective theory in which the static modes with masses in this range are integrated out before the final reduction procedure of modes with masses $\sim gT$ is performed. For supersymmetric theories the static squark (slepton) modes are integrated out at this second stage.

To summarize, the construction of the effective theories in supersymmetric models proceeds in three stages. First, we integrate out the nonstatic modes with masses on the order of $\sim \pi T$. We are then left with a 3D theory of the bosonic sector of the 4D theory. The resulting theory contains massive static modes: see below. We next integrate out the squarks and sleptons obtaining a general two-Higgs-doublet model. Generically, at the phase transition only one of the Higgs fields is heavy, with a mass on the order of $\sim gT$, and it can be integrated out in the third stage.

For the generic case, the reduction procedure leaves us with the same effective theory as in [7], defined by the effective couplings $\overline{\lambda}_3$ and g_3^2 with a different relationship which defines these quantities in terms of the masses and couplings of the 4D theory. As explained by Kajantie *et al.* [8], the dynamics of the electroweak phase transition is governed by the quantity

$$x_c = \frac{\overline{\lambda}_3}{g_3^2} \tag{2}$$

at the critical temperature, and the constraint given by Eq. (1) translates into [7]

$$x_c < 0.04. \tag{3}$$

In this paper we present the relations between the masses and couplings of the reduced theory and the parameters of the underlying four-dimensional theory for three models: the minimal supersymmetric standard model (MSSM), a general two-Higgs-doublet model (2HDM), and the next to minimal supersymmetric standard model (NMSSM) in the $\sin^2\theta_W = 0$ limit. The analysis of the electroweak phase transition for the MSSM will be the subject of a second paper [9]. In Sec. II A we review the finite temperature formalism which is the basis for the construction of the three-dimensional theory. Sections II B, II C, and II D discuss the three effective theories for the MSSM as we integrate out in successive stages: (1) the nonstatic modes, (2) the heavy squarks and sleptons, and (3) the heavy Higgs and the temporal component of the SU(2) gauge field A_0. Section II E discusses the nongeneric case in which there are two light scalar fields in the final effective theory. We present a short discussion of other multiscalar models in Sec. III. Our conclusions are given in Sec. IV. Appendix A introduces the MSSM 4D Lagrangian, which is the model discussed in the text. Appendixes B 1 and B 2 give the explicit expressions for our results of the first and second stages. In Appendix B 3 we give the results for the case in which there are two light scalar Higgs fields in the final effective theory. The results of the application of the procedure to the two-Higgs-doublet model and the NMSSM is given in Appendixes C 1 and C 2. Finally, we present some useful high temperature formulas in Appendix D.

II. THREE-DIMENSIONAL THEORY

A. Effective theory

A given theory in four dimensions at finite temperature reduces, in the high temperature limit, to an effective 3D theory describing the static degrees of freedom. This is the statement of dimensional reduction [8,10–12].

The action at finite temperature is given by

$$S = \int_o^\beta d\tau \int d^3x \mathcal{L}, \tag{4}$$

where $\beta = 1/T$, such that the theory is characterized by bosonic (fermionic) fields obeying periodic (antiperiodic) boundary conditions in Euclidean time [13]. The finite temperature expansion of the fields in our theory is given by

$$\mathcal{S}(x,\tau) = T^{1/2}\left[\mathcal{S}_0(x) + \sum_{n\neq 0} \mathcal{S}_n(x) e^{i\omega_B \tau}\right], \tag{5}$$

$$\Psi(x,\tau) = T^{1/2}\sum_n \psi_n e^{i\omega_F \tau}, \tag{6}$$

where $\mathcal{S}$ and Ψ represent bosonic and fermionic fields, respectively. $\mathcal{S}_0$ is the static component of the bosonic field, for which $\omega_B = 0$. The resulting propagators are of the form $(k^2 + \omega_{B(F)}^2 + m^2)^{-1}$, where $\omega_B = 2\pi n T$, $\omega_F = \pi(2n+1)T$ for bosons and fermions, respectively. That is, nonstatic modes have masses $\sim \pi T$ and thus all of the fermionic modes may be integrated out at large T. Our calculations will be carried out in Landau gauge. By integrating over τ and using the orthonormality of the modes we can obtain the terms in the Lagrangian describing the static modes, the nonstatic modes, and finally the interaction terms between the heavy and light modes. Our expressions are valid only when the high temperature expansion is adequate. That is, the masses of the particles must fulfill the condition

$$m \lesssim 2\pi T. \tag{7}$$

B. First stage

We are interested in studying the MSSM with $\sin^2\theta_W = 0$. As discussed in Ref. [7], the effects arising from the U(1) subgroup for the standard model are small. For the MSSM, the relevant coupling constants for the construction of the effective theories are the strong gauge coupling and the top Yukawa coupling. Additionally, we mention that up to the moment no Monte Carlo simulations have been performed to include the nonperturbative physics of interest for a SU(2)×U(1) dimensionally reduced theory. Consequently, we work in the $\sin^2\theta_W = 0$ limit, ignoring the hypercharge U(1) gauge boson and gaugino contributions. In this section we describe in detail the dimensional reduction of the MSSM. In the appendixes we treat a general 2HDM model and supersymmetric models with an additional gauge singlet superfield. The Lagrangian for the MSSM is given in Appendix A, where we introduce our notation. Most of the discussions in the paper will be general enough to include all three models. We leave the notation and particular results corresponding to the 2HDM and NMSSM to Appendix C.

Starting with the 4D theory we generalize the procedure delineated by Kajantie *et al.* to obtain a 3D effective theory. The first stage corresponds to the integration of the massive (nonstatic) modes. As a result, all fermions are integrated out and we are left with a three-dimensional bosonic theory. We can systematically construct the dimensionally reduced theory and relate its parameters to those of the more fundamental four-dimensional theory by computing the effective interactions among the static modes generated by integrating out the $n\neq 0$ modes. This procedure is perturbative in the coupling constants of the theory and we will calculate consistently throughout to order g^4. As explained in [7,8], for our purposes it is sufficient to perform a one-loop calculation as it provides the specified accuracy and the determination of the critical temperature is precise enough as the temperature dependence in the ratio in Eq. (2) enters only through logarithmic terms.

As a result of the first stage of integration the $n=0$ bosonic modes in the theory acquire a thermal mass to leading order (excluding logarithmic corrections) of the generic form

$$m^2(T)=m^2+\nu T^2, \tag{8}$$

where m^2 is the tree-level mass squared, which for gauge bosons is identically zero. As explained above the quantity ν is determined from the one-loop integration and in particular it remains zero for the spatial components of the vector field A_i. This reflects the fact that the A_i fields are precisely the gauge fields of the 3D theory. For the temporal component of the SU(2) gauge field A_0, which is a gauge-triplet of scalars in the effective theory, $\nu\neq 0$. This implies that the temporal mode acquires a mass, the so-called Debye mass,[1] which will be on the order of $\sim gT$. Scalar particles, on the other hand, have a tree-level mass as well as a nonzero value of ν. In the high temperature approximation, their masses are of order $(m^2+g_s^2T^2)^{1/2}$ for squarks. For top squarks the contribution proportional to the top Yukawa coupling squared can also be significant. The rest of the scalars in the theory will have masses on the order of $(m^2+g^2T^2)^{1/2}$. Higher order effects are suppressed by powers of m^2/T^2.

The exact value of the tree-level mass is important as it may lead to the existence of a very light scalar particle. This may have several different consequences, but specifically, if the tree-level mass nearly cancels the term proportional to T^2, one could be close to a phase transition. We will discuss the nongeneric case in which there are two light scalar particles in the final effective theory in Sec. II E.

In order to establish our notation we write the 3D scalar potential after the first stage:

$$\begin{aligned}
V(A_0,\phi_1,\phi_2,Q_i,U_i,D_i)={}&\frac{1}{2}M_D^2A_0A_0+H(A_0A_0)\left(\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\sum_i Q_i^\dagger Q_i\right)+M_1^2\phi_1^\dagger\phi_1+M_2^2\phi_2^\dagger\phi_2\\
&+M_3^2(\phi_1^\dagger\phi_2+\phi_2^\dagger\phi_1)+\Lambda_1(\phi_1^\dagger\phi_1)^2+\Lambda_2(\phi_2^\dagger\phi_2)^2+\Lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\\
&+\Lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)+\sum_i \Lambda_3^{Q_i1}(\phi_1^\dagger\phi_1)(Q_i^\dagger Q_i)+\Lambda_3^{Q_i2}(\phi_2^\dagger\phi_2)(Q_i^\dagger Q_i)\\
&+(\Lambda_4^{Q_i1}+\overline{f_{d_i}^{L}}^2)(\phi_1^\dagger Q_i)(Q_i^\dagger\phi_1)+(\Lambda_4^{Q_i2}+\overline{f_{u_i}^{L}}^2)|\epsilon_{\alpha\beta}\phi_2^\alpha Q_i^\beta|^2+M_Q^iQ_i^\dagger Q_i\\
&+M_u^iU_i^\dagger U_i+M_d^iD_i^\dagger D_i+\overline{f_{d_i}^{R}}^2(\phi_1^\dagger\phi_1)(D_i^\dagger D_i)+\overline{f_{u_i}^{R}}^2(\phi_2^\dagger\phi_2)(U_i^\dagger U_i)+\overline{Af}_{d_i}\phi_1^\dagger Q_iD_i\\
&-\epsilon_{\alpha\beta}\overline{Af}_{u_i}\phi_2^\alpha Q_i^\beta U_i+\overline{\mu f}_{d_i}\phi_2^\dagger Q_iD_i^*-\epsilon_{\alpha\beta}\overline{\mu f}_{u_i}\phi_1^\alpha Q_i^{\beta *}U_i^*+\text{H.c.},
\end{aligned} \tag{9}$$

where α, β are SU(2) indices. Note that the fields in Eq. (9) are the static components of the scalar fields, properly renormalized, and having dimension $[\text{GeV}]^{1/2}$. Quartic couplings have dimensions of [GeV] and trilinear couplings involving $\overline{A}$ or $\overline{\mu}$ have dimension of $[\text{GeV}]^{3/2}$. We have omitted the terms corresponding to the scalar leptons and the quartic A_0 term.

The full expressions for the effective masses and couplings are given in Appendix B 1 where we also show the diagrams contributing to the calculations for each one of the parameters. We would like to point out some of the features of the calculations, though most technical details are discussed in Appendix B.

The expressions for the 3D parameters contain temperature-dependent logarithmic corrections denoted L_b and L_f. In general the coefficients of these quantities are the corresponding bosonic and fermionic contributions to the β functions for the masses and couplings in the underlying theory [6]. In the 4D theory the quartic scalar couplings (Higgs boson self-couplings and part of the interactions between Higgs bosons and squarks and sleptons) are fixed

[1]Similarly the longitudinal SU(3) gauge field acquires a mass $\sim g_sT$.

above the supersymmetry- (SUSY-) breaking scale in terms of the gauge couplings. In the zero temperature theory, at energy scales above the SUSY-breaking scale, or if there is no particle decoupling, the β function coefficients are related by the same algebraic relation as the couplings themselves [14]. This is no longer true if one considers particles decoupling in the 4D theory below the SUSY scale. Similarly, in the finite temperature theory the relation between the (scale-dependent part of the) 3D expressions of the couplings differs, upon one-loop integration of heavy modes, from the expression relating the β functions of the zero temperature theory. As a by-product our 3D expressions for the couplings and masses, yield the full one-loop β function coefficients in the zero temperature theory, including particle decoupling, for the three models we have considered.

The static modes squark (slepton) masses are nondegenerate as a result of the integration procedure even if their masses are taken to be degenerate in the 4D theory. Not only do right- and left-handed-type squarks acquire different values for their masses, so do up- and down-type right handed squarks. This occurs because right-handed squarks do not couple to SU(2) gauge fields, their quartic coupling to Higgs bosons is proportional to their corresponding Yukawa coupling, and the trilinear coupling to Higgs bosons differs for up- and down-type right-handed squarks.

As in the case of the standard model, the gauge coupling between the spatial magnetic field and scalars in the three-dimensional theory is not equal to the quartic coupling between the A_0 field and scalars. In addition, at the next stage of integration-out, this latter quartic scalar coupling will be different for each type of scalar field to which the A_0 couples to as a consequence of the soft SUSY-breaking trilinear couplings.

C. Second stage

The aim of the second and third stages is twofold. First, as explained qualitatively above, after the first stage of integration we are still left with several different mass scales, while the purpose of an effective theory is to have only one characteristic mass scale. Second, if we can construct an effective theory in which we are left with only one light scalar particle, then we have arrived at exactly the same theory which has already been analyzed on the lattice by Kajantie *et al.* [7].

What we define as second stage is necessary only when the mass of the squarks and sleptons is such that the high temperature expansion is valid. If the squarks and sleptons were extremely massive, they would have decoupled in the four-dimensional theory, or alternatively a low temperature expansion might be applicable [15]. After obtaining our reduced theory we must verify that the nonrenormalizable terms of the effective theory are indeed suppressed. That is, we must check that higher order corrections to the scalar potential at the critical temperature do not change qualitatively our results.

For the MSSM and NMSSM the second stage corresponds to the integration of heavy squarks and sleptons. We include the sleptons even though their masses do not have contributions $\sim g_s T$, since their tree-level mass at some high scale is presumably $\gtrsim gT$. The results are also applicable to the case in which we study a purely SU(2) gauge theory with multiple scalars, if we ignore all of the contributions which include g_s. For the MSSM the resultant theory after the second stage is described by a 2HDM with complicated expressions, in terms of the 4D couplings and masses, for the masses and strengths of interactions. The 3D potential for the scalar fields A_0, ϕ_1, and ϕ_2 is

$$\begin{aligned} V(A_0,\phi_1,\phi_2) = {} & \tfrac{1}{2}\overline{M_D^2}A_0A_0 + \overline{H_1}(A_0A_0)\phi_1^\dagger\phi_1 \\ & + \overline{H_2}(A_0A_0)\phi_2^\dagger\phi_2 + \overline{M_1^2}\phi_1^\dagger\phi_1 + \overline{M_2^2}\phi_2^\dagger\phi_2 \\ & + \overline{M_3^2}(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + \overline{\Lambda_1}(\phi_1^\dagger\phi_1)^2 \\ & + \overline{\Lambda_2}(\phi_2^\dagger\phi_2)^2 + \overline{\Lambda_3}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ & + \overline{\Lambda_4}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1), \end{aligned} \tag{10}$$

where we have labeled the couplings and masses after the second stage with overbars and for simplicity we maintain the same notation for the fields. The explicit expressions for the parameters are given in Appendix B 2.

D. Third stage

After the second stage the scalar fields we are left with are the two Higgs doublets and the A_0 triplet. Our objective is to be able to use the nonperturbative results for a theory with only one light scalar at the phase transition. This corresponds to the generic case; we find that only with fine-tuning can one have two light Higgs fields in addition to the spatial gauge fields. The critical temperature for a first order phase transition lies between the temperature of phase coexistence and the temperature at which the curvature of the potential is zero in some direction of field space at the origin. The two latter values are generally close but not identical for a theory with a single light scalar field at the phase transition [3]. We take the value of the critical temperature to be that at which there is a direction in field space at the origin of the Higgs potential for which the transition to the minimum of the potential in the broken phase can occur classically. This implies that at the phase transition at least one of the thermal masses of the Higgs bosons must become zero and then negative as the temperature decreases. At this temperature we can determine the mass of the other Higgs doublet; if it is heavy $\sim gT$, it can be integrated out in a third stage, together with the A_0 field. The mixing angle which determines which combination of the Higgs doublets can be integrated out depends on the temperature. We stress that as long as the variation of the mixing angle with temperature is negligible for temperatures close to the critical temperature, the strength of the phase transition, determined by the ratio λ_3/g_3^2, has a weak temperature dependence [9]. That is to say, with the estimate for the critical temperature as described above we can determine to a very good accuracy the strength of the phase transition.

To determine which is the correct scalar Higgs field which is heavy at the critical temperature one can analyze the eigenvalues of the mass matrix as a function of the temperature.

The critical temperature T_c, for which only one of the eigenvalues of the mass matrix is zero, is determined from the equation

$$\overline{M_1^2}(T_c)\overline{M_2^2}(T_c)=(\overline{M_3^2})^2(T_c). \tag{11}$$

Upon diagonalization of the scalar mass matrix the expression for the mass of the heavy Higgs field at the critical temperature is

$$\nu^2(T_c)=\overline{M_1^2}(T_c)+\overline{M_2^2}(T_c). \tag{12}$$

We denote by α_i the quartic Higgs coupling interactions in the second stage after diagonalization to the mass eigenstate basis requiring one light Higgs boson, where α_1 is the quartic self-coupling of the massless Higgs field and α_3, α_4 are quartic couplings between the light and heavy Higgs scalar fields:

$$\alpha_1=\overline{\Lambda}_1\cos^4\theta+\overline{\Lambda}_2\sin^4\theta+(\overline{\Lambda}_3+\overline{\Lambda}_4)\cos^2\theta\sin^2\theta, \tag{13}$$

$$\alpha_3=(2\overline{\Lambda}_1+2\overline{\Lambda}_2)\cos^2\theta\ \sin^2\theta+\overline{\Lambda}_3(\cos^4\theta+\sin^4\theta) -2\overline{\Lambda}_4\cos^2\theta\ \sin^2\theta, \tag{14}$$

$$\alpha_4=(2\overline{\Lambda}_1+2\overline{\Lambda}_2)\cos^2\theta\ \sin^2\theta-2\overline{\Lambda}_3\cos^2\sin^2\theta +\overline{\Lambda}_4(\cos^4\theta+\sin^4\theta), \tag{15}$$

and

$$\tan 2\theta=\frac{2\overline{M_3^2}}{(\overline{M_1^2}-\overline{M_2^2})}. \tag{16}$$

We now proceed to the third stage in which we integrate out the massive scalars which are left in the theory.[2] We are then left with an expression for the strengths of the interactions of the static magnetic fields and the light Higgs field, in terms of the quantities of the previous stage.[3] In particular, we obtain the expression for the effective 3D gauge coupling,

$$g_3^2=\overline{G^2}\left[1-\frac{\overline{G^2}}{24\pi}\left(\frac{1}{2\nu(T_c)}+\frac{1}{\overline{M_D}}\right)\right], \tag{17}$$

and the effective Higgs self-coupling,

$$\overline{\lambda}_3=\alpha_1-\left(\alpha_3^2+\frac{\alpha_4^2}{2}+\alpha_3\alpha_4\right)\frac{1}{8\pi\nu(T_c)} -\frac{3(\overline{H}_1\cos^2\theta+\overline{H}_2\sin^2\theta)^2}{8\pi\overline{M}_D}, \tag{18}$$

in terms of the original 4D parameters and the temperature. We point out that there is no wave function renormalization at the third stage as there is no trilinear coupling between the light Higgs and the heavy scalars.

If any of the squark (slepton) thermal masses were of the order of M_D rather than $\gg M_D$ they would be integrated out at this point instead of previously. If, for example, we suppose the sleptons to be light, the modifications to the second stage equations would be such that the sums in the equations of Appendix B 2 would not run over the sleptons. Furthermore, at the third stage after the rotation to the mass eigenstate basis the sleptons could be integrated out together with the heavy Higgs boson and the A_0. The results of the third stage, Eqs. (17) and (18), would contain additional terms of the same form as the contributions in the second stage. However, it is clear that the expressions for the α_i, $\overline{G^2}$, $\overline{M}_D$, $\overline{H_1^2}$, $\overline{H_2^2}$, and $\nu(T_c)$ would in general be different. We point out that the trilinear slepton–Higgs-boson coupling would vary after the rotation to the mass eigenstate basis and the light Higgs field would suffer a wave function renormalization. In addition, if the slepton(s) is nearly massless, then we cannot integrate it out; if we had, some nonrenormalizable terms would not be suppressed.

E. Nongeneric case

In this section we present a short discussion of the nongeneric case in which there are two light Higgs fields in the final 3D theory. If the parameters are fine-tuned, we could have a theory with two or more light scalar particles whose interactions are described by some potential.[4] In this case the infrared behavior must be studied with new numerical simulations.

This fine-tuned scenario can be realized in several different ways: two light Higgs bosons (two doublets, a doublet and a singlet, etc.); a Higgs boson and a slepton; a Higgs boson and a squark (top squark). In this last case the main features are a screened SU(3) A_0 field, and spatial A_i gluonic fields which are not decoupled from the squark in the 3D theory. Numerical calculations must also take this into account and the scalar octet should be integrated out.[5] For parameters of the MSSM such that two scalars remain light, at the third stage only A_0 is integrated out. The expression of the two-Higgs-doublet potential for the case with two light Higgs bosons is given in Appendix B 3.

III. OTHER MODELS

In Appendix C we give the full results of for the parameters of the effective 3D theory for the 2HDM and the

[2]We have explicitly checked that the precise order of integration out of the A_0 field, before or after diagonalization, is not relevant up to terms $\sim g^6$.

[3]There is a mass term for the lighest scalar field resulting from the final stage on integration. Its expression is not included as it is not necessary for the analysis of the phase transition. In fact, one may argue that this quantity is the appropiate one to evaluate the critical temperature. However, we have checked that the difference in T_c from Eq. (11) and by requiring the lighest scalar mass to be zero is irrelevant.

[4]The authors of Ref. [16] have suggested a scenario in which the right-handed top squark and one Higgs boson are light. For this case as they have pointed out one must be careful with color- (and charge-) breaking minima of the scalar potential, or in the slepton case lepton number violation.

[5]In the generic case for the MSSM with one light Higgs, we did not have to worry about the SU(3) fields once the squarks have been integrated out as they decouple from the rest of the particles in the theory, even though there is a Debye mass for the longitudinal gluonic field, etc.

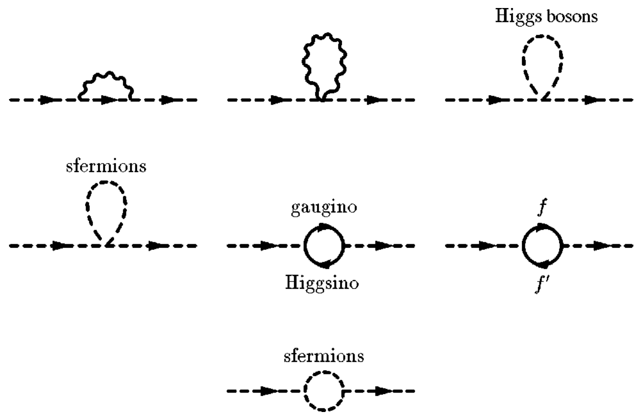

FIG. 1. Feynman diagrams contributing to the mass of the scalar Higgs bosons and to wave function renormalization.

NMSSM. Here we summarize salient characteristics of these models.

The reduction of a general 2HDM to a three-dimensional effective theory is realized in only two stages and and the main differences with the MSSM are that the Higgs couplings are not fixed in terms of the gauge couplings, there are additional scalar interaction terms, there are no superpartner contributions to the theory, the SU(3) gauge bosons completely decouple once the fermions are integrated out, and the off-diagonal Higgs boson mass term acquires a dominant contribution on the order of $\sim T^2$. All of the above can in principle change considerably the dependence of the critical temperature on the parameters in the theory.

The reduction procedure with the addition of a singlet superfield to the MSSM has the following features: It introduces additional couplings in the scalar and Higgsino sector which are not determined in terms of g_s or g; the first stage 3D parameters G, H, M_D, M_{Q_i}, M_{u_i}, and M_{d_i} do not receive additional contributions; there are additional contributions to the wave function renormalizations of ϕ_1 and ϕ_2; and for values of the parameters for which the mass of the scalar singlet is on the order of the SU(2) Debye mass, after the second stage in which squarks and sleptons are integrated out, we are left with three scalar Higgs fields.

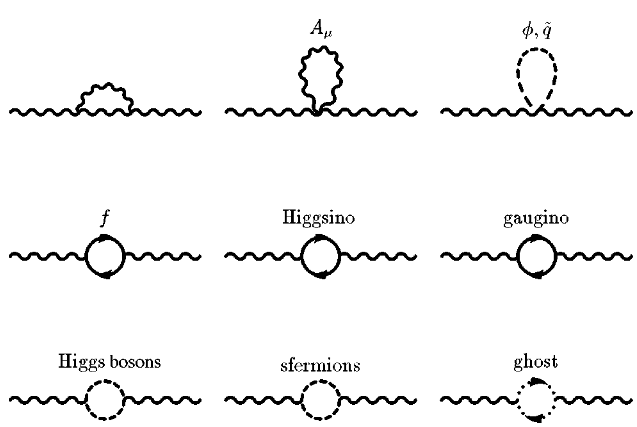

FIG. 2. Diagrams contributing to the mass of the A_0 field and wave function renormalization of the gauge fields. We use a wavy line for both spatial and temporal components of the gauge fields.

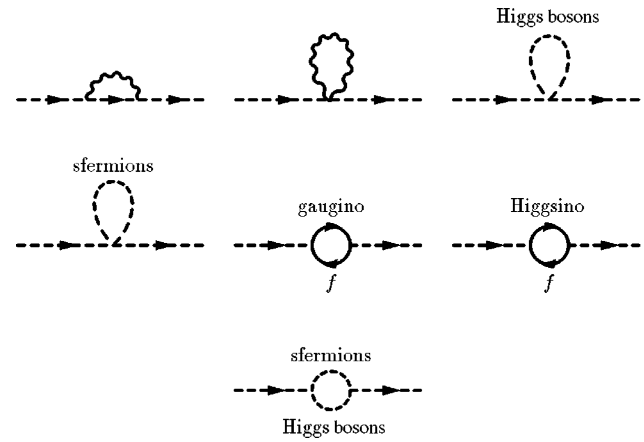

FIG. 3. Diagrams contributing to the mass and wave function renormalization of the squarks and sleptons.

IV. CONCLUSIONS

We have constructed, in the high temperature limit, effective three-dimensional theories for the MSSM, a general 2HDM, and the NMSSM which contain a single light scalar field. We obtained the full one-loop relation between the couplings of the effective theory and the underlying 4D couplings and masses. For the case that two Higgs scalars are light at the phase transition, we have also given the expression for the two-Higgs-doublet potential whose infrared behavior must be studied with numerical methods.

The original parameters of these theories can now be related to physical parameters at the electroweak scale. For the effective theories containing a single light scalar Higgs boson, this will allow us to evaluate the quantity $x_c = \lambda_3 / g_3^2$ as a function of the physical parameters. In this way, we can determine for which regions of parameter space the electroweak phase transition may be sufficiently first order. The results for the MSSM will be presented elsewhere [9].

ACKNOWLEDGMENTS

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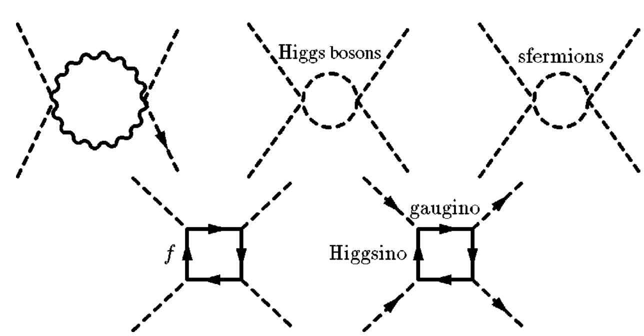

FIG. 4. Diagrams contributing to the quartic Higgs boson couplings.

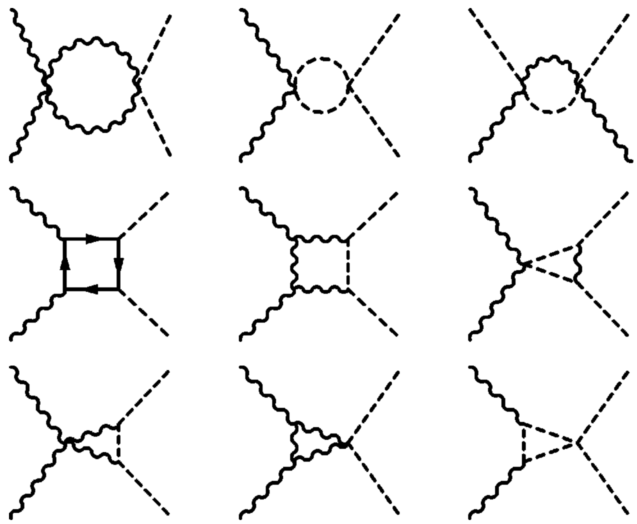

FIG. 5. Diagrams contributing to the gauge and quartic A_0-scalar couplings.

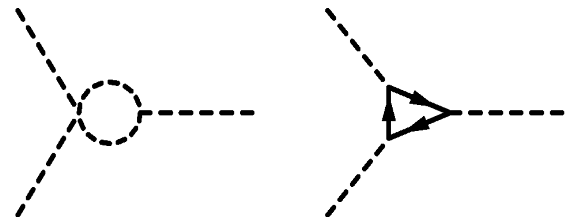

FIG. 6. Diagrams contributing to the trilinear scalar couplings.

APPENDIX A: MSSM IN FOUR DIMENSIONS

Our four-dimensional Lagrangian will be a supersymmetric SU(3)×SU(2) gauge theory with the same particle content as in the MSSM with the exclusion of U(1) vector particles and corresponding superpartner [17–19].

The MSSM chiral superfield content is

$$\hat{L}=\begin{pmatrix}\hat{\nu}_e\\ \hat{e}\end{pmatrix}\hat{E}^c, \tag{A1}$$

$$\hat{Q}=\begin{pmatrix}\hat{u}\\ \hat{d}\end{pmatrix}\hat{U}^c\hat{D}^c, \tag{A2}$$

$$\hat{\phi}_1=\begin{pmatrix}\hat{\phi}_1^0\\ -\hat{\phi}_1^-\end{pmatrix}, \quad \hat{\phi}_2=\begin{pmatrix}\hat{\phi}_2^+\\ \hat{\phi}_2^0\end{pmatrix}. \tag{A3}$$

When we refer to the scalar component of the superfield we will drop the caret.

Instead of writing out explicitly the full 4D Lagrangian we will define only the quantities we will need to refer to. In particular, the Yukawa interactions are derived from the superpotential, which for the MSSM is

$$W=\mu(\hat{\phi}_1^0\hat{\phi}_2^0+\hat{\phi}_1^+\hat{\phi}_2^-)+f_u(\hat{\phi}_2^0\hat{u}-\hat{d}\hat{\phi}_2^+)\hat{U}^c$$
$$+f_d(\hat{\phi}_1^0\hat{d}+\hat{u}\hat{\phi}_1^-)\hat{D}^c+f_e(\hat{\phi}_1^0\hat{e}+\hat{\nu}_e\hat{\phi}_1^-)\hat{E}^c. \tag{A4}$$

In order to maintain supersymmetry's virtue of stabilizing the electroweak scale via the cancellation of quadratic divergences it is standard to introduce SUSY-breaking terms which do not reintroduce this type of divergence, so-called soft SUSY-breaking terms. In particular, there are new scalar interactions proportional to terms in the superpotential as well as mass terms for scalars, gauginos, and Higgsinos. The scalar interactions are obtained replacing each chiral superfield in the superpotential by its corresponding scalar component. Without any further assumptions we would have an extraordinary amount of parameters which make it extremely difficult to do phenomenology. To simplify our parameter space we assume above the SUSY-breaking scale: (1) a unified gaugino mass $m_{1/2},m_{\tilde{g}}$; (2) common mass for squarks and sleptons, m_0^2; (3) a universal A parameter. In the formulas presented in this paper we have kept all Yukawa coupling dependence, although with the exception of the top Yukawa coupling these contributions generally can be dropped.

The scalar Higgs self-interactions generate, along with the soft terms for the scalar Higgs fields, a two-Higgs-doublet potential of the form

$$V(\phi_1,\phi_2)=m_1^2\phi_1^\dagger\phi_1+m_2^2\phi_2^\dagger\phi_2+m_3^2(\phi_1^\dagger\phi_2+\phi_2^\dagger\phi_1)$$
$$+\lambda_1(\phi_1^\dagger\phi_1)^2+\lambda_2(\phi_2^\dagger\phi_2)^2+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)$$
$$+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1), \tag{A5}$$

in which the quartic couplings are fixed in terms of the gauge coupling constants. We comment that in order to express the scalar potential in this way the ϕ_1 field has been SU(2) rotated. All λ_i are real and fixed by supersymmetry at some high scale to be

$$\lambda_1=\frac{g^2}{8}, \quad \lambda_2=\frac{g^2}{8}, \tag{A6}$$

$$\lambda_3=\frac{g^2}{4}, \quad \lambda_4=-\frac{g^2}{2}, \tag{A7}$$

in the $g'=0$ limit. As is well known the model contains five physical Higgs bosons: a charged pair, two neutral CP-even scalars, and a neutral CP-odd scalar [17–19].

APPENDIX B: EXPLICIT RELATIONSHIPS BETWEEN PARAMETERS

1. First stage parameters

The explicit relations between the 3D coupling constants and masses expressed in terms of underlying 4D couplings and the temperature, obtained as a result of one-loop integration, are given below. These results reduce to the partial results given for the MSSM in the literature [20,21], as well as the standard model results [8], by taking the appropiate limit. The formulas of Appendix D were used to obtain the final results. N, N_f, and N_s denote the SU(N) gauge group, number of fermions doublets, and number of scalar doublets, respectively. N_c is the number of colors and it is taken to be 1 for (s)leptons but we do not insert an explicit index for simplicity. $N_{\rm sq}$ is the number of squark and slepton doublets. The index i is a generation index. As the values of the A and μ parameters are not known we have kept the explicit dependence on these quantities throughout the calculation.

The thermal masses for the Higgs scalars are given by the evaluation of the diagrams in Fig. 1:[6]

$$M_1^2=m_1^2\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ f_{d_i}^2\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}\right)+\frac{3}{16}\ g^2T^2+N_c\sum_i\ f_{d_i}^2\ \frac{T^2}{12}+T^2\left(\frac{\lambda_1}{2}+\frac{\lambda_3}{6}+\frac{\lambda_4}{12}\right)+N_c\sum_i\ \frac{f_{d_i}^2}{6}\ T^2+N_{\rm sq}\left(\lambda_3+\frac{\lambda_4}{2}\right)\ \frac{T^2}{6}+\frac{g^2}{8}\ T^2-\frac{L_b}{16\pi^2}\times\left(6\lambda_1m_1^2+2\lambda_3m_2^2+\lambda_4m_2^2+2N_cm_0^2\sum_i\ f_{d_i}^2+N_c\sum_i\ (f_{d_i}^2A^2+f_{u_i}^2\mu^2)+N_{\rm sq}m_0^2(2\lambda_3+\lambda_4)\right)+3(\mu^2g^2+m_{1/2}^2g^2)\ \frac{L_f}{16\pi^2}, \tag{B1}$$

$$M_2^2=m_2^2\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ f_{u_i}^2\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}\right)+\frac{3}{16}\ g^2T^2+N_c\sum_i\ f_{u_i}^2\ \frac{T^2}{12}+T^2\left(\frac{\lambda_2}{2}+\frac{\lambda_3}{6}+\frac{\lambda_4}{12}\right)+N_c\sum_i\ \frac{f_{u_i}^2}{6}\ T^2+N_{\rm sq}\left(\lambda_3+\frac{\lambda_4}{2}\right)\ \frac{T^2}{6}+\frac{g^2}{8}\ T^2-\frac{L_b}{16\pi^2}\times\left(6\lambda_2m_2^2+2\lambda_3m_1^2+\lambda_4m_1^2+2N_cm_0^2\sum_i\ f_{u_i}^2+N_c\sum_i\ (f_{u_i}^2A^2+f_{d_i}^2\mu^2)+N_{\rm sq}m_0^2(2\lambda_3+\lambda_4)\right)+3(\mu^2g^2+m_{1/2}^2g^2)\ \frac{L_f}{16\pi^2}, \tag{B2}$$

$$M_3^2=m_3^2\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ \left(\frac{f_{d_i}^2}{2}+\frac{f_{u_i}^2}{2}\right)\times\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}\right]-\frac{L_b}{16\pi^2}\times\left((2\lambda_4+\lambda_3)m_3^2+N_cA\mu\sum_i\ (f_{d_i}^2+f_{u_i}^2)\right)+3(\mu g^2m_{1/2})\ \frac{L_f}{16\pi^2}, \tag{B3}$$

where

$$L_b=2\ \ln\frac{\overline{\mu_4}e^{\gamma}}{4\pi T}=-\ln 4\pi T^2+\gamma+\ln\mu_4^2, \tag{B4}$$

$$L_f=L_b+4\ \ln 2. \tag{B5}$$

μ_4 is the mass scale defined by the modified minimal subtraction ($\overline{\rm MS}$) scheme. For every 3D parameter, the brackets multiplying the corresponding 4D parameter contain the wave function renormalization correction. We mention specifically that the scalar-gauge boson loop contributes only to the wave function renormalization of the field, while the fermionic loops contribute to the wave function renormalization and the mass.

The Debye mass induced for the temporal component of the SU(2) gauge field has additional contributions from those of the standard model arising from Higgsino, squark, slepton, and chargino contributions as shown in Fig. 2:

$$M_D^2=\frac{g^2T^2}{6}\ (6+N_s+N_F/2+N_H/2). \tag{B6}$$

Figure 3 shows the diagrams which contribute to the mass terms for a squark (slepton) doublet. For the up and down right-handed squarks (sleptons) we must neglect the diagrams with gauge boson and gaugino loops:

$$M_Q^{i2}=m_0^2\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-(f_{d_i}^2+f_{u_i}^2)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right)+\frac{2}{3}\ g_s^2T^2+\frac{3}{16}\ g^2T^2+(f_{d_i}^2+f_{u_i}^2)\ \frac{T^2}{12}+T^2\left(\frac{\lambda_1}{2}+\frac{\lambda_3}{3}+\frac{\lambda_4}{6}\right)+(f_{d_i}^2+f_{u_i}^2)\ \frac{T^2}{6}+\frac{g^2}{8}\ T^2-\frac{L_b}{16\pi^2}\ \left(\frac{4}{3}\ g_s^2m_0^2+6\lambda_1m_0^2+m_0^2(f_{u_i}^2+f_{d_i}^2)+f_{d_i}^2m_1^2+f_{u_i}^2m_2^2+(f_{d_i}^2A^2+f_{u_i}^2\mu^2)+(f_{u_i}^2A^2+f_{d_i}^2\mu^2)+2\lambda_3(m_1^2+m_2^2)+\lambda_4(m_1^2+m_2^2)\right)+\left(3m_{1/2}^2g^2+\frac{16}{3}\ m_{\tilde g}^2g_s^2\right)\ \frac{L_f}{16\pi^2}+2\mu^2(f_{u_i}^2+f_{d_i}^2)\ \frac{L_f}{16\pi^2}+\frac{T^2}{24}\ (N_{\rm sq}-1)(4\lambda_3+2\lambda_4)-(N_{\rm sq}-1)m_0^2(2\lambda_3+\lambda_4)\ \frac{L_b}{16\pi^2}, \tag{B7}$$

[6]Figures were drawn using feynmf.mf.

$$M_u^{i^2}=m_0^2\left(1-2f_{u_i}^2\frac{L_f}{16\pi^2}+4g_s^2\frac{L_b}{16\pi^2}-\frac{8}{3}g_s^2\frac{L_f}{16\pi^2}\right)+\frac{2}{3}g_s^2T^2+f_{u_i}^2\frac{T^2}{3}+f_{u_i}^2\frac{T^2}{6}$$
$$-\frac{L_b}{16\pi^2}\left[\frac{4}{3}g_s^2m_0^2+f_{u_i}^2(2m_2^2+2\mu^2+2A^2+2m_0^2)\right]+\left(4\mu^2f_{u_i}^2+\frac{16}{3}m_{\tilde g}^2g_s^2\right)\frac{L_f}{16\pi^2}, \tag{B8}$$

$$M_d^{i^2}=m_0^2\left(1-2f_{d_i}^2\frac{L_f}{16\pi^2}+4g_s^2\frac{L_b}{16\pi^2}-\frac{8}{3}g_s^2\frac{L_f}{16\pi^2}\right)+\frac{2}{3}g_s^2T^2+f_{d_i}^2\frac{T^2}{3}+f_{d_i}^2\frac{T^2}{6}$$
$$-\frac{L_b}{16\pi^2}\left[\frac{4}{3}g_s^2m_0^2+f_{d_i}^2(2m_1^2+2\mu^2+2A^2+2m_0^2)\right]+\left(4\mu^2f_{d_i}^2+\frac{16}{3}m_{\tilde g}^2g_s^2\right)\frac{L_f}{16\pi^2}. \tag{B9}$$

The expressions for the slepton masses are omitted although they may be readily obtained by excluding the g_s corrections, noting that there is no right-handed sneutrino and dropping all $f_{u_i}^2$ contributions to M_Q and M_{d_i}. This is because the sleptons do not have a Yukawa-type coupling to the ϕ_2 field.

In Fig. 4 we show the diagrams contributing to the quartic Higgs boson couplings. The full expressions for the scalar couplings are

$$\Lambda_1=\lambda_1T\left(1+\frac{9}{2}g^2\frac{L_b}{16\pi^2}-2N_c\sum_i f_{d_i}^2\frac{L_f}{16\pi^2}-3g^2\frac{L_f}{16\pi^2}\right)+T\left[-\frac{9}{16}g^4\frac{L_b}{16\pi^2}+\frac{3}{8}\frac{g^4}{16\pi^2}+N_c\sum_i f_{d_i}^4\frac{L_f}{16\pi^2}\right.$$
$$-\left(12\lambda_1^2+\lambda_3^2+\frac{\lambda_4^2}{2}+\lambda_3\lambda_4\right)\frac{L_b}{16\pi^2}+N_c\left(-(\lambda_3+\lambda_4)\sum_i f_{d_i}^2\frac{L_b}{16\pi^2}-\sum_i f_{d_i}^4\frac{L_b}{16\pi^2}\right)$$
$$\left.+N_{\rm sq}\left(-\frac{(\lambda_3+\lambda_4)^2}{2}-\frac{\lambda_3^2}{2}\right)\frac{L_b}{16\pi^2}+\frac{5}{4}g^4\frac{L_f}{16\pi^2}\right], \tag{B10}$$

$$\Lambda_2=\lambda_2T\left(1+\frac{9}{2}g^2\frac{L_b}{16\pi^2}-2N_c\sum_i f_{u_i}^2\frac{L_f}{16\pi^2}-3g^2\frac{L_f}{16\pi^2}\right)+T\left[-\frac{9}{16}g^4\frac{L_b}{16\pi^2}+\frac{3}{8}\frac{g^4}{16\pi^2}+N_c\sum_i f_{u_i}^4\frac{L_f}{16\pi^2}\right.$$
$$-\left(12\lambda_2^2+\lambda_3^2+\frac{\lambda_4^2}{2}+\lambda_3\lambda_4\right)\frac{L_b}{16\pi^2}+N_c\left[-(\lambda_3+\lambda_4)\sum_i f_{u_i}^2\frac{L_b}{16\pi^2}-\sum_i f_{u_i}^4\frac{L_b}{16\pi^2}\right]$$
$$\left.+N_{\rm sq}\left(-\frac{(\lambda_3+\lambda_4)^2}{2}-\frac{\lambda_3^2}{2}\right)\frac{L_b}{16\pi^2}+\frac{5}{4}g^4\frac{L_f}{16\pi^2}\right], \tag{B11}$$

$$\Lambda_3=\lambda_3T\left(1+\frac{9}{2}g^2\frac{L_b}{16\pi^2}-N_c(f_{u_i}^2+f_{d_i}^2)\frac{L_f}{16\pi^2}-3g^2\frac{L_f}{16\pi^2}\right)+T\left[-\frac{9}{8}g^4\frac{L_b}{16\pi^2}+\frac{3}{4}\frac{g^4}{16\pi^2}-(6\lambda_1\lambda_3+6\lambda_2\lambda_3+2\lambda_1\lambda_4\right.$$
$$+2\lambda_2\lambda_4+2\lambda_3^2+\lambda_4^2)\frac{L_b}{16\pi^2}-N_c\sum_i((\lambda_3+\lambda_4)(f_{d_i}^2+f_{u_i}^2)+2f_{u_i}^2f_{d_i}^2)\frac{L_b}{16\pi^2}-N_{\rm sq}(2\lambda_3^2+2\lambda_3\lambda_4+\lambda_4^2)\frac{L_b}{16\pi^2}$$
$$\left.+2N_c\sum_i f_{u_i}^2f_{d_i}^2\frac{L_f}{16\pi^2}+\frac{5}{2}g^4\frac{L_f}{16\pi^2}\right], \tag{B12}$$

$$\Lambda_4=\lambda_4T\left(1+\frac{9}{2}g^2\frac{L_b}{16\pi^2}-N_c\sum_i(f_{u_i}^2+f_{d_i}^2)\frac{L_f}{16\pi^2}-3g^2\frac{L_f}{16\pi^2}\right)+T\left[-(2\lambda_1\lambda_4+2\lambda_2\lambda_4+2\lambda_4^2+4\lambda_3\lambda_4)\frac{L_b}{16\pi^2}\right.$$
$$\left.+N_c\sum_i\left(\lambda_4(f_{d_i}^2+f_{u_i}^2)+2f_{u_i}^2f_{d_i}^2\right)\frac{L_b}{16\pi^2}+N_{\rm sq}\lambda_4^2\frac{L_b}{16\pi^2}-2N_c\sum_i f_{u_i}^2f_{d_i}^2\frac{L_f}{16\pi^2}-2g^4\frac{L_f}{16\pi^2}\right]. \tag{B13}$$

There are similar diagrams to those in Fig. 4 for the the quartic couplings of the Higgs bosons to squarks and sleptons which have not been shown. They make the following contributions to these couplings:

$$\overline{f_{d_i}^{L}}^2 = f_{d_i}^2 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2} - \left(N_c \sum_i\ f_{d_i}^2 + f_{d_i}^2 + f_{u_i}^2\right)\ \frac{L_f}{16\pi^2} - 3g^2\ \frac{L_f}{16\pi^2} + 4g_s^2\ \frac{L_b}{16\pi^2} - \frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-\frac{T}{2}\ \left[\left(\lambda_4^2 - \lambda_4 f_{u_i}^2 + 4\lambda_4 f_{d_i}^2 + 2\ f_{d_i}^4 + 4\lambda_3\lambda_4 + 4\lambda_3 f_{d_i}^2 + 2\lambda_2\lambda_4 + 2\lambda_1 f_{d_i}^2 + 2\lambda_1\lambda_4 + 2\lambda_2 f_{d_i}^2 + \frac{4}{3}\ (\lambda_4 + f_{d_i}^2) g_s^2\right)\ \frac{L_b}{16\pi^2}\right.$$
$$\left. - \left((N_{\mathrm{sq}} - 1)(\lambda_4^2) + N_c \lambda_4 \sum_i\ f_{d_i}^2 - \lambda_4 f_{d_i}^2\right)\ \frac{L_b}{16\pi^2} + \left(2g^4 - \frac{16}{3}\ f_{d_i}^2 g_s^2\right)\ \frac{L_f}{16\pi^2}\right], \tag{B14}$$

$$\Lambda_3^{Q_i 1} = \lambda_3 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2} - \left(N_c \sum_i\ f_{d_i}^2 + f_{d_i}^2 + f_{u_i}^2\right)\ \frac{L_f}{16\pi^2} - 3g^2\ \frac{L_f}{16\pi^2} + 4g_s^2\ \frac{L_b}{16\pi^2} - \frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$+T\left[-\frac{9}{8}\ g^4\ \frac{L_b}{16\pi^2} + \frac{3}{4}\ \frac{g^4}{16\pi^2} - \left(\frac{4}{3}\ \lambda_3 g_s^2 + 4\lambda_3^2 + 6\lambda_1\lambda_3 + 2\lambda_4^2 + 2\lambda_4 f_{d_i}^2 + 2\ f_{d_i}^4 + 6\lambda_2\lambda_3 + 2\lambda_3\lambda_4 + \lambda_4 f_{u_i}^2 + \lambda_3 f_{u_i}^2\right.\right.$$
$$\left. + 2\lambda_1\lambda_4 + 2\lambda_2 f_{d_i}^2 + 2\lambda_2\lambda_4 + 2\lambda_1 f_{d_i}^2\right)\ \frac{L_b}{16\pi^2} - \left((N_{\mathrm{sq}} - 1)(2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2) + N_c(\lambda_3 + \lambda_4)\sum_i\ f_{d_i}^2 - (\lambda_3 + \lambda_4) f_{d_i}^2\right)$$
$$\left.\times \frac{L_b}{16\pi^2} + \left(2\ f_{d_i}^4 + \frac{5g^4}{2}\right)\ \frac{L_f}{16\pi^2}\right], \tag{B15}$$

$$\Lambda_4^{Q_i 1} = \lambda_4 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2} - \left(N_c \sum_i\ f_{d_i}^2 + f_{d_i}^2 + f_{u_i}^2\right)\ \frac{L_f}{16\pi^2} - 3g^2\ \frac{L_f}{16\pi^2} + 4g_s^2\ \frac{L_b}{16\pi^2} - \frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-\frac{T}{2}\ \left[\left(\lambda_4^2 - \lambda_4 f_{u_i}^2 + 4\lambda_4 f_{d_i}^2 + 2\ f_{d_i}^4 + 4\lambda_3\lambda_4 + 4\lambda_3 f_{d_i}^2 + 2\lambda_2\lambda_4 + 2\lambda_1 f_{d_i}^2 + 2\lambda_1\lambda_4 + 2\lambda_2 f_{d_i}^2 + \frac{4}{3}\ (\lambda_4 + f_{d_i}^2) g_s^2\right)\ \frac{L_b}{16\pi^2}\right.$$
$$\left. - \left((N_{\mathrm{sq}} - 1)(\lambda_4^2) + N_c \lambda_4 \sum_i\ f_{d_i}^2 - \lambda_4 f_{d_i}^2\right)\ \frac{L_b}{16\pi^2} + \left(2g^4 - \frac{16}{3}\ f_{d_i}^2 g_s^2\right)\frac{L_f}{16\pi^2}\right], \tag{B16}$$

$$\overline{f_{u_i}^{L}}^2 = f_{u_i}^2 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2} - \left(N_c \sum_i\ f_{u_i}^2 + f_{u_i}^2 + f_{d_i}^2\right)\ \frac{L_f}{16\pi^2} - 3g^2\ \frac{L_f}{16\pi^2} + 4g_s^2\ \frac{L_b}{16\pi^2} - \frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-\frac{T}{2}\ \left[\left(\lambda_4^2 - \lambda_4 f_{d_i}^2 + 4\lambda_4 f_{u_i}^2 + 2\ f_{u_i}^4 + 4\lambda_3\lambda_4 + 4\lambda_3 f_{u_i}^2 + 2\lambda_2\lambda_4 + 2\lambda_1 f_{u_i}^2 + 2\lambda_1\lambda_4 + 2\lambda_2 f_{u_i}^2 + \frac{4}{3}\ (\lambda_4 + f_{u_i}^2) g_s^2\right)\ \frac{L_b}{16\pi^2}\right.$$
$$\left. - \left((N_{\mathrm{sq}} - 1)(\lambda_4^2) + N_c \lambda_4 \sum_i\ f_{u_i}^2 - \lambda_4 f_{u_i}^2\right)\ \frac{L_b}{16\pi^2} + \left(2g^4 - \frac{16}{3}\ f_{u_i}^2 g_s^2\right)\ \frac{L_f}{16\pi^2}\right], \tag{B17}$$

$$\Lambda_3^{Q_i 2} = \lambda_3 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2} - \left(N_c \sum_i\ f_{u_i}^2 + f_{u_i}^2 + f_{d_i}^2\right)\ \frac{L_f}{16\pi^2} - 3g^2\ \frac{L_f}{16\pi^2} + 4g_s^2\ \frac{L_b}{16\pi^2} - \frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$+T\left[-\frac{9}{8}\ g^4\ \frac{L_b}{16\pi^2} + \frac{3}{4}\ \frac{g^4}{16\pi^2} - \left(\frac{4}{3}\ \lambda_3 g_s^2 + 4\lambda_3^2 + 6\lambda_1\lambda_3 + 2\lambda_4^2 + 2\lambda_4 f_{u_i}^2 + 2\ f_{u_i}^4 + 6\lambda_2\lambda_3 + 2\lambda_3\lambda_4 + \lambda_4 f_{d_i}^2 + \lambda_3 f_{d_i}^2\right.\right.$$
$$\left. + 2\lambda_1\lambda_4 + 2\lambda_2 f_{u_i}^2 + 2\lambda_2\lambda_4 + 2\lambda_1 f_{u_i}^2\right)\ \frac{L_b}{16\pi^2} - \left((N_{\mathrm{sq}} - 1)(2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2) N_c(\lambda_3 + \lambda_4)\sum_i\ f_{u_i}^2 - (\lambda_3 + \lambda_4) f_{u_i}^2\right)$$
$$\left.\times \frac{L_b}{16\pi^2} + \left(2\ f_{u_i}^4 + \frac{5g^4}{2}\right)\ \frac{L_f}{16\pi^2}\right], \tag{B18}$$

$$\Lambda_4^{Q_i2}=\lambda_4 T\left[1+\frac{9}{2}\ g^2\ \frac{L_b}{16\pi^2}-\left(N_c\sum_i\ f_{u_i}^2+f_{u_i}^2+f_{d_i}^2\right)\ \frac{L_f}{16\pi^2}-3g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-\frac{T}{2}\left[\left(\lambda_4^2-\lambda_4 f_{d_i}^2+4\lambda_4 f_{u_i}^2+2\ f_{u_i}^4+4\lambda_3\lambda_4+4\lambda_3 f_{u_i}^2+2\lambda_2\lambda_4+2\lambda_1 f_{u_i}^2+2\lambda_1\lambda_4+2\lambda_2 f_{u_i}^2+\frac{4}{3}\ (\lambda_4+f_{u_i}^2)g_s^2\right)\ \frac{L_b}{16\pi^2}\right.$$
$$\left.-\left((N_{\rm sq}-1)(\lambda_4^2)+N_c\lambda_4\sum_i\ f_{u_i}^2-\lambda_4 f_{u_i}^2\right)\ \frac{L_b}{16\pi^2}+\left(2g^4-\frac{16}{3}\ f_{u_i}^2 g_s^2\right)\ \frac{L_f}{16\pi^2}\right], \tag{B19}$$

$$\overline{f_{d_i}^{R}}^2=f_{d_i}^2 T\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(N_c\sum_i\ f_{d_i}^2+2\ f_{d_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$+T\left[-\frac{L_b}{16\pi^2}\ \left(\frac{4}{3}\ f_{d_i}^2 g_s^2+(2\lambda_3+\lambda_4+f_{u_i}^2)f_{d_i}^2+3f_{d_i}^4+6\lambda_1 f_{d_i}^2\right)+\frac{L_f}{16\pi^2}\ \left(\frac{16}{3}\ f_{d_i}^2 g_s^2+2\ f_{d_i}^4+3f_{d_i}^2 g^2\right)\right], \tag{B20}$$

$$\overline{f_{u_i}^{R}}^2=f_{u_i}^2 T\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(N_c\sum_i\ f_{u_i}^2+2\ f_{u_i}^2\right)\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$+T\left[-\frac{L_b}{16\pi^2}\ \left(\frac{4}{3}\ f_{u_i}^2 g_s^2+(2\lambda_3+\lambda_4+f_{d_i}^2)f_{u_i}^2+3f_{u_i}^4+6\lambda_2 f_{u_i}^2\right)+\frac{L_f}{16\pi^2}\ \left(\frac{16}{3}\ f_{u_i}^2 g_s^2+2\ f_{u_i}^4+3f_{u_i}^2 g^2\right)\right]. \tag{B21}$$

We would like to point out that the $T/2$ factor in the expressions for $\Lambda_4^{Q_i1}$, $\overline{f_{d_i}^{L}}^2$, $\Lambda_4^{Q_i2}$, and $\overline{f_{u_i}^{L}}^2$ is due to the fact that the sum of the first two is the full quartic coupling to the ϕ_1 field, and analogously for the second pair.

As explained in [8] we can obtain the gauge and quartic A_0-scalar–doublet couplings from the same set of diagrams depicted in Fig. 5:

$$G^2=g^2 T\left[1+\frac{g^2}{16\pi^2}\ \left(\frac{44-N_s}{6}\ L_b-\frac{1}{3}\ (N_f+N_H+4)L_f+\frac{2}{3}\right)\right], \tag{B22}$$

$$H=\frac{g^2 T}{4}\ \left[1+\frac{g^2}{16\pi^2}\ \left(\frac{44-N_s}{6}\ L_b-\frac{1}{3}\ (N_f+N_H+4)L_f\right)+\frac{1}{16\pi^2}\right.$$
$$\left.\times\left(\frac{35}{6}\ g^2-\frac{N_s}{3}\ g^2+\frac{g^2}{3}\ (N_F+N_H+4)+12\lambda_1+2(N_s-1)(2\lambda_3+\lambda_4)\right)\right]. \tag{B23}$$

The scalar trilinear couplings are also modified as can be seen in Fig. 6:

$$\overline{Af}_{u_i}=Af_{u_i}T^{1/2}\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(\frac{f_{d_i}^2}{2}+\frac{N_c}{2}\ \sum_i\ f_{u_i}^2+\frac{3}{2}\ f_{u_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-Af_{u_i}T^{1/2}\left(\lambda_3+2\lambda_4+3f_{u_i}^2+N_c\sum_i\ f_{u_i}^2+f_{d_i}^2-\frac{4}{3}\ g_s^2\right)\ \frac{L_b}{16\pi^2}-m_{1/2}f_{u_i}T^{1/2}(3g^2)\ \frac{L_f}{16\pi^2}-m_{\tilde g}f_{u_i}T^{1/2}\left(\frac{16}{3}\ g_s^2\right)\ \frac{L_f}{16\pi^2}, \tag{B24}$$

$$\overline{Af}_{d_i}=Af_{d_i}T^{1/2}\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(\frac{f_{u_i}^2}{2}+\frac{N_c}{2}\ \sum_i\ f_{d_i}^2+\frac{3}{2}\ f_{d_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-Af_{d_i}T^{1/2}\left(\lambda_3+2\lambda_4+3f_{d_i}^2+N_c\sum_i\ f_{d_i}^2+f_{u_i}^2-\frac{4}{3}\ g_s^2\right)\ \frac{L_b}{16\pi^2}-m_{1/2}f_{d_i}T^{1/2}(3g^2)\ \frac{L_f}{16\pi^2}-m_{\tilde g}f_{d_i}T^{1/2}\left(\frac{16}{3}\ g_s^2\right)\ \frac{L_f}{16\pi^2}, \tag{B25}$$

$$\overline{\mu f}_{u_i}=\mu f_{u_i}T^{1/2}\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(\frac{N_c}{2}\ \sum_i\ f_{d_i}^2+\frac{1}{2}\ f_{d_i}^2+\frac{3}{2}\ f_{u_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}\right]$$
$$-\mu f_{u_i}T^{1/2}\left(\lambda_3-\lambda_4+N_c\sum_i\ f_{u_i}^2-2\ f_{d_i}^2-\frac{4}{3}\ g_s^2\right)\ \frac{L_b}{16\pi^2}+\mu f_{u_i}T^{1/2}(2\ f_{d_i}^2-3g^2)\ \frac{L_f}{16\pi^2}, \tag{B26}$$

$$\overline{\mu f}_{d_i}=\mu f_{d_i}T^{1/2}\left[1+\frac{9}{4}\,g^2\,\frac{L_b}{16\pi^2}-\left(\frac{N_c}{2}\sum_i\,f_{u_i}^2+\frac{1}{2}\,f_{u_i}^2+\frac{3}{2}\,f_{d_i}^2\right)\,\frac{L_f}{16\pi^2}-\frac{3}{2}\,g^2\,\frac{L_f}{16\pi^2}+4g_s^2\,\frac{L_b}{16\pi^2}-\frac{8}{3}\,g_s^2\,\frac{L_f}{16\pi^2}\right]$$
$$-\mu f_{d_i}T^{1/2}\left(\lambda_3-\lambda_4+N_c\sum_i\,f_{d_i}^2-2\,f_{u_i}^2-\frac{4}{3}\,g_s^2\right)\,\frac{L_b}{16\pi^2}+\mu f_{d_i}T^{1/2}(2\,f_{u_i}^2-3g^2)\,\frac{L_f}{16\pi^2}. \quad \text{(B27)}$$

We comment that at zero temperature the β function coefficients for the four trilinear couplings given above, which are products of two parameters, can be obtained from the running of each parameter separately. This is true up to an arbitrary number of loops.

2. Second stage parameters

The gauge coupling is given by the expression

$$\overline{G^2}=G^2\left(1-\sum_i\,N_c\,\frac{G^2}{48\pi M_Q^i}\right). \quad \text{(B28)}$$

A clear difference appears now in the coupling of A_0 to ϕ_1 and ϕ_2 which is not protected by any symmetry. How large this difference is depends strongly on the values of the soft breaking parameters. In general, the expressions simplify considerably if we ignore the trilinear scalar interaction terms. Additional diagrams, shown in Fig. 7, which were suppressed by powers of T^{-2} for the first stage, are included. We point out that the box diagram with two external scalar Higgs legs only contributes to the four-point function which determines $\overline{G^2}$, as there is no trilinear $A_0\phi\phi$ interactions in the three-dimensional theory:

$$\overline{H_1}=H\left(1-\sum_i\,\frac{N_cf_{d_i}^2\overline{A^2}}{12\pi(M_Q^i+M_d^i)^3}-\sum_i\,\frac{N_cf_{u_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_u^i)^3}\right)$$
$$-\sum_i\,N_c\left[H(2\Lambda_3^{Q_i1}+\Lambda_4^{Q_i1})\,\frac{1}{8\pi M_Q^i}+H\overline{f_{d_i}^{L^2}}\,\frac{1}{8\pi M_Q^i}\right.$$
$$\left.-H\,\frac{1}{8\pi M_Q^i}\,\left(\frac{f_{d_i}^2\overline{A^2}}{(M_Q^i+M_d^i)^2}+\frac{f_{u_i}^2\overline{\mu^2}}{(M_Q^i+M_u^i)^2}\right)\right], \quad \text{(B29)}$$

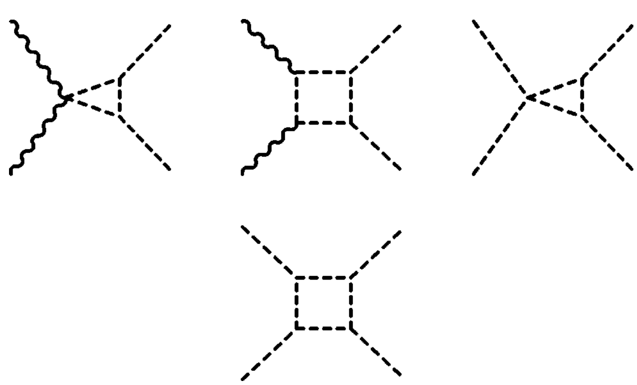

FIG. 7. Additional diagrams included in the second stage.

$$\overline{H_2}=H\left(1-\sum_i\,\frac{N_cf_{u_i}^2\overline{A^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i\,\frac{N_cf_{d_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_d^i)^3}\right)$$
$$-\sum_i\,N_c\left[H(2\Lambda_3^{Q_i2}+\Lambda_4^{Q_i2})\,\frac{1}{8\pi M_Q^i}+H\overline{f_{u_i}^{L^2}}\,\frac{1}{8\pi M_Q^i}\right.$$
$$\left.-H\,\frac{1}{8\pi M_Q^i}\,\left(\frac{f_{u_i}^2\overline{A^2}}{(M_Q^i+M_u^i)^2}+\frac{f_{d_i}^2\overline{\mu^2}}{(M_Q^i+M_d^i)^2}\right)\right]. \quad \text{(B30)}$$

The elements of the scalar Higgs doublet mass matrix are now given by

$$\overline{M_1^2}=M_1^2\left(1-\sum_i\,\frac{N_cf_{d_i}^2\overline{A^2}}{12\pi(M_Q^i+M_d^i)^3}-\sum_i\,\frac{N_cf_{u_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_u^i)^3}\right)$$
$$-\sum_i\,N_c\left((\overline{f_{d_i}^{L^2}}+2\Lambda_3^{Q_i1}+\Lambda_4^{Q_i1})\,\frac{M_Q^i}{4\pi}+\overline{f_{d_i}^{R^2}}\,\frac{M_d^i}{4\pi}+\frac{f_{d_i}^2\overline{A^2}}{4\pi(M_Q^i+M_d^i)}+\frac{f_{u_i}^2\overline{\mu^2}}{4\pi(M_Q^i+M_u^i)}\right), \quad \text{(B31)}$$

$$\overline{M_2^2}=M_2^2\left(1-\sum_i\,\frac{N_cf_{u_i}^2\overline{A^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i\,\frac{N_cf_{d_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_d^i)^3}\right)$$
$$-\sum_i\,N_c\left((\overline{f_{u_i}^{L^2}}+2\Lambda_3^{Q_i2}+\Lambda_4^{Q_i2})\,\frac{M_Q^i}{4\pi}+\overline{f_{u_i}^{R^2}}\,\frac{M_u^i}{4\pi}+\frac{f_{u_i}^2\overline{A^2}}{4\pi(M_Q^i+M_u^i)}+\frac{f_{d_i}^2\overline{\mu^2}}{4\pi(M_Q^i+M_d^i)}\right), \quad \text{(B32)}$$

$$\overline{M_3^2}=M_3^2\left(1-\sum_i\,\frac{N_cf_{d_i}^2\overline{A^2}}{24\pi(M_Q^i+M_d^i)^3}-\sum_i\,\frac{N_cf_{u_i}^2\overline{\mu^2}}{24\pi(M_Q^i+M_u^i)^3}-\sum_i\,\frac{N_cf_{u_i}^2\overline{A^2}}{24\pi(M_Q^i+M_u^i)^3}-\sum_i\,\frac{N_cf_{d_i}^2\overline{\mu^2}}{24\pi(M_Q^i+M_d^i)^3}\right)$$
$$-\sum_i\,N_c\left(\frac{f_{u_i}^2\overline{A\mu}}{4\pi(M_Q^i+M_u^i)}+\frac{f_{d_i}^2\overline{\mu A}}{4\pi(M_Q^i+M_d^i)}\right). \quad \text{(B33)}$$

The resulting Debye mass from the second stage of integration is

$$\overline{M_D^2}=M_D^2-\sum_i N_c H\,\frac{M_Q^i}{4\pi}. \tag{B34}$$

We remark that the previous quantity is always positive for values of the parameters for which both perturbation theory and the high temperature expansion are valid.

The resulting quartic Higgs boson couplings are

$$\begin{aligned}\overline{\Lambda_1}=&\Lambda_1\left(1-\sum_i \frac{N_c f_{d_i}^2\overline{A^2}}{6\pi(M_Q^i+M_d^i)^3}-\sum_i \frac{N_c f_{u_i}^2\overline{\mu^2}}{6\pi(M_Q^i+M_u^i)^3}\right)+\sum_i N_c\left(-\frac{(\Lambda_3^{Qi1}+\Lambda_4^{Qi1})^2}{2}\frac{1}{8\pi M_Q^i}-\frac{\Lambda_3^{Q1^2}}{2}\frac{1}{8\pi M_Q^i}\right.\\&-(\Lambda_3^{Qi1}+\Lambda_4^{Qi1})\overline{f_{d_i}^{L^2}}\,\frac{1}{8\pi M_Q^i}-\frac{\overline{f_{d_i}^{L^4}}}{2}\frac{1}{8\pi M_Q^i}-\frac{\overline{f_{d_i}^{R^4}}}{2}\frac{1}{8\pi M_d^i}+f_{d_i}^2\overline{A^2}(\Lambda_3^{Qi1}+\Lambda_4^{Qi1}+\overline{f_{d_i}^{L^2}})\,\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_d^i)^2}\\&+f_{u_i}^2\overline{\mu^2}\Lambda_3^{Qi1}\,\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_u^i)^2}+f_{d_i}^2\overline{A^2 f_{d_i}^{R^2}}\,\frac{1}{8\pi M_d^i}\frac{1}{(M_Q^i+M_d^i)^2}-f_{u_i}^4\overline{\mu^4}\,\frac{1}{8\pi M_u^i M_Q^i}\frac{1}{(M_Q^i+M_u^i)^3}\\&\left.-f_{d_i}^4\overline{A^4}\,\frac{1}{8\pi M_d^i M_Q^i}\frac{1}{(M_Q^i+M_d^i)^3}\right),\end{aligned} \tag{B35}$$

$$\begin{aligned}\overline{\Lambda_2}=&\Lambda_2\left(1-\sum_i \frac{N_c f_{u_i}^2\overline{A^2}}{6\pi(M_Q^i+M_u^i)^3}-\sum_i \frac{N_c f_{d_i}^2\overline{\mu^2}}{6\pi(M_Q^i+M_d^i)^3}\right)+\sum_i N_c\left(-\frac{(\Lambda_3^{Qi2}+\Lambda_4^{Qi2})^2}{2}\frac{1}{8\pi M_Q^i}-\frac{\Lambda_3^{Q2^2}}{2}\frac{1}{8\pi M_Q^i}\right.\\&-(\Lambda_3^{Qi2}+\Lambda_4^{Qi2})\overline{f_{u_i}^{L^2}}\,\frac{1}{8\pi M_Q^i}-\frac{\overline{f_{u_i}^{L^4}}}{2}\frac{1}{8\pi M_Q^i}-\frac{\overline{f_{u_i}^{R^4}}}{2}\frac{1}{8\pi M_u^i}+f_{u_i}^2\overline{A^2}(\Lambda_3^{Qi2}+\Lambda_4^{Qi2}+\overline{f_{u_i}^{L^2}})\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_d^i)^2}\\&+f_{d_i}^2\overline{\mu^2}\Lambda_3^{Qi2}\,\frac{1}{8\pi M_u^i}\frac{1}{(M_Q^i+M_u^i)^2}+f_{u_i}^2\overline{A^2}\,\overline{f_{u_i}^{R^2}}\frac{1}{8\pi M_u^i}\frac{1}{(M_Q^i+M_u^i)^2}\\&\left.-f_{d_i}^4\overline{\mu^4}\frac{1}{8\pi M_d^i M_Q^i}\frac{1}{(M_Q^i+M_d^i)^3}-f_{u_i}^4\overline{A^4}\,\frac{1}{8\pi M_u^i M_Q^i}\frac{1}{(M_Q^i+M_u^i)^3}\right),\end{aligned} \tag{B36}$$

$$\begin{aligned}\overline{\Lambda_3}=&\Lambda_3\left(1-\sum_i \frac{N_c f_{d_i}^2\overline{A^2}}{12\pi(M_Q^i+M_d^i)^3}-\sum_i \frac{N_c f_{u_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i \frac{N_c f_{u_i}^2\overline{A^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i \frac{N_c f_{d_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_d^i)^3}\right)\\&+\sum_i N_c\left(-(2\Lambda_3^{Qi1}\Lambda_3^{Qi2}+\Lambda_4^{Qi1}\Lambda_4^{Qi2}+\Lambda_3^{Qi1}\Lambda_4^{Qi2}+\Lambda_3^{Qi2}\Lambda_4^{Qi1})\,\frac{1}{8\pi M_Q^i}\right.\\&-(\overline{f_{u_i}^{L^2}}\Lambda_4^{Qi1}+\overline{f_{d_i}^{L^2}}\Lambda_4^{Qi2}+2\overline{f_{u_i}^{L^2}}\,\overline{f_{d_i}^{L^2}}+\overline{f_{u_i}^{L^2}}\Lambda_3^{Qi1}+\overline{f_{d_i}^{L^2}}\Lambda_3^{Qi2})\,\frac{1}{8\pi M_Q^i}+f_{u_i}^2(\overline{f_{d_i}^{L^2}}\,\overline{A^2}+\overline{\mu^2}\Lambda_3^{Qi2}+\overline{A^2}\Lambda_4^{Qi1}\\&+\Lambda_3^{Qi1}\overline{A^2})\,\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_u^i)^2}+f_{d_i}^2(\overline{f_{u_i}^{L^2}}\,\overline{A^2}+\overline{\mu^2}\Lambda_3^{Qi1}+\overline{A^2}\Lambda_4^{Qi2}+\Lambda_3^{Qi2}\overline{A^2})\,\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_d^i)^2}\\&+f_{u_i}^2\overline{\mu^2}\,\overline{f_{u_i}^{R^2}}\,\frac{1}{8\pi M_u^i}\frac{1}{(M_Q^i+M_u^i)^2}+f_{d_i}^2\overline{\mu^2}\,\overline{f_{d_i}^{R^2}}\,\frac{1}{8\pi M_d^i}\frac{1}{(M_Q^i+M_d^i)^2}-2[(f_{d_i}^2 f_{u_i}^2\overline{\mu^4}-2\,f_{d_i}^2 f_{u_i}^2\overline{A^2}\,\overline{\mu^2}\\&\left.+f_{d_i}^2 f_{u_i}^2\overline{A^4})f(M_Q^i,M_u^i,M_d^i)+f_{d_i}^4\overline{A^2}\,\overline{\mu^2}f(M_Q^i,M_d^i,M_d^i)+f_{u_i}^4\overline{A^2}\,\overline{\mu^2}f(M_Q^i,M_u^i,M_u^i)]\right),\end{aligned} \tag{B37}$$

$$\overline{\Lambda_4}=\Lambda_4\left(1-\sum_i \frac{N_c f_{d_i}^2\overline{A^2}}{12\pi(M_Q^i+M_d^i)^3}-\sum_i \frac{N_c f_{u_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i \frac{N_c f_{u_i}^2\overline{A^2}}{12\pi(M_Q^i+M_u^i)^3}-\sum_i \frac{N_c f_{d_i}^2\overline{\mu^2}}{12\pi(M_Q^i+M_d^i)^3}\right)$$
$$+\sum_i N_c\Bigg(\Lambda_4^{Q_i1}\Lambda_4^{Q_i2}\frac{1}{8\pi M_Q^i}+(\overline{f_{u_i}^L}^2\Lambda_4^{Q_i1}+\overline{f_{d_i}^L}^2\Lambda_4^{Q_i2}+2\overline{f_{u_i}^L}^2\overline{f_{d_i}^L}^2)\frac{1}{8\pi M_Q^i}$$
$$+[f_{u_i}^2\overline{\mu^2}(\overline{f_{u_i}^L}^2+\Lambda_4^{Q_i2})-(\overline{f_{d_i}^L}^2+\Lambda_4^{Q_i1})f_{u_i}^2\overline{A^2}]\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_u^i)^2}+[f_{d_i}^2\overline{\mu^2}(\overline{f_{d_i}^L}^2+\Lambda_4^{Q_i1})$$
$$-(\overline{f_{u_i}^L}^2+\Lambda_4^{Q_i2})f_{d_i}^2\overline{A^2}]\frac{1}{8\pi M_Q^i}\frac{1}{(M_Q^i+M_d^i)^2}-2[-(f_{d_i}^2f_{u_i}^2\overline{\mu^4}-2f_{d_i}^2f_{u_i}^2\overline{A^2}\,\overline{\mu^2}+f_{d_i}^2f_{u_i}^2\overline{A^4})f(M_Q^i,M_u^i,M_d^i)$$
$$+2f_{u_i}^4\overline{A^2}\,\overline{\mu^2}f(M_Q^i,M_u^i,M_u^i)+2f_{d_i}^4\overline{A^2}\,\overline{\mu^2}f(M_Q^i,M_u^i,M_d^i)]\Bigg),\tag{B38}$$

where

$$f(m_1,m_2,m_3)=\frac{1}{8\pi}\frac{2m_1+m_2+m_3}{m_1(m_1+m_3)^2(m_1+m_2)^2(m_2+m_3)}.\tag{B39}$$

3. Two light Higgs bosons

As mentioned in Sec. II E for the case in which both eigenvalues of the mass matrix of the Higgs doublets are such that we cannot integrate out one of the scalar Higgs fields, the third stage corresponds to the integration of only the A_0 field. Since there is no trilinear $A_0\phi\phi$ interaction term, there is no wave function renormalization at this stage. Consequently, the two-Higgs-doublet potential is

$$V(A_0,\phi_1,\phi_2)=\overline{m_1^2}\phi_1^\dagger\phi_1+\overline{m_2^2}\phi_2^\dagger\phi_2+\overline{m_3^2}(\phi_1^\dagger\phi_2+\phi_2^\dagger\phi_1)+\overline{\lambda_1}(\phi_1^\dagger\phi_1)^2+\overline{\lambda_2}(\phi_2^\dagger\phi_2)^2+\overline{\lambda_3}(\phi_1^\dagger\phi_1)\times(\phi_2^\dagger\phi_2)+\overline{\lambda_4}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1),\tag{B40}$$

where

$$\overline{m_1^2}=\overline{M_1^2}-\overline{H_1}\frac{\overline{M_D}}{4\pi},\tag{B41}$$

$$\overline{m_2^2}=\overline{M_2^2}-\overline{H_2}\frac{\overline{M_D}}{4\pi},\tag{B42}$$

$$\overline{\lambda_1}=\overline{\Lambda_1}-3\frac{\overline{H_1^2}}{8\pi\overline{M_D}},\tag{B43}$$

$$\overline{\lambda_2}=\overline{\Lambda_2}-3\frac{\overline{H_2^2}}{8\pi\overline{M_D}},\tag{B44}$$

$$\overline{\lambda_3}=\overline{\Lambda_3}-6\frac{\overline{H_1}\,\overline{H_2}}{8_D},\tag{B45}$$

and $\overline{m_3^2}=\overline{M_3^2}$ and $\overline{\lambda_4}=\overline{\Lambda_4}$.

APPENDIX C: 2HDM AND NMSSM

Our discussion of the 2HDM and NMSSN will be brief as we have already introduced all of the relevant points in presenting the effective theory for the MSSM. We will limit ourselves as much as possible to giving our results after each stage.

1. Two-Higgs-doublet model

In the case of a general two-Higgs-doublet model the scalar potential can contain additional quartic terms of the form

$$\Delta V=\lambda_5(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_2)+\lambda_6(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1)+\lambda_7(\phi_2^\dagger\phi_2)\times(\phi_2^\dagger\phi_1)+\text{H.c.}\tag{C1}$$

In this case the values of the λ_i are not expressed in terms of the weak coupling constant. We take all parameters to be real.

The reduction procedure differs from that of the MSSM because the model does not contain superpartners. This implies that we will have only two stages of reduction. The first one would once again correspond to the integration out of the heavy nonstatic modes. Consequently, the SU(3) gauge particles decouple when the fermions are eliminated. The resulting theory for the static modes is be described by a scalar potential with scalar masses

$$M_D^2=\frac{g^2T^2}{6}(4+N_s+N_F/2),\tag{C2}$$

$$\Delta M_1^2=-6m_3^2\lambda_6\frac{L_b}{16\pi^2},\tag{C3}$$

$$\Delta M_2^2=-6m_3^2\lambda_7\frac{L_b}{16\pi^2},\tag{C4}$$

$$\Delta M_3^2=\frac{T^2}{4}(\lambda_6+\lambda_7)-(12\lambda_5m_3^2+3\lambda_6m_1^2+3\lambda_7m_2^2)\frac{L_b}{16\pi^2}.\tag{C5}$$

We note that, unlike in the MSSM, the M_3^2 term receives a contribution proportional to T^2, which is in fact the dominant correction. The quartic Higgs boson couplings are modified by the terms

$$\Delta\Lambda_1=-T(2\lambda_5^2+6\lambda_6^2)\,\frac{L_b}{16\pi^2}, \tag{C6}$$

$$\Delta\Lambda_2=-T(2\lambda_5^2+6\lambda_7^2)\,\frac{L_b}{16\pi^2}, \tag{C7}$$

$$\Delta\Lambda_3=-T(4\lambda_5^2+2\lambda_6^2+8\lambda_6\lambda_7+2\lambda_7^2)\,\frac{L_b}{16\pi^2}, \tag{C8}$$

$$\Delta\Lambda_4=-T(32\lambda_5^2+5\lambda_6^2+2\lambda_6\lambda_7+5\lambda_7^2)\,\frac{L_b}{16\pi^2}, \tag{C9}$$

$$\begin{aligned}\Lambda_5=\lambda_5T&\left[1+\frac{9}{2}\,g^2\,\frac{L_b}{16\pi^2}-N_c\sum_i\,(f_{u_i}^2+f_{d_i}^2)\,\frac{L_f}{16\pi^2}\right]\\&-T(4\lambda_1\lambda_5+4\lambda_2\lambda_5+8\lambda_3\lambda_5+12\lambda_4\lambda_5\\&+5\lambda_6^2+2\lambda_6\lambda_7+5\lambda_7^2)\,\frac{L_b}{16\pi^2},\end{aligned} \tag{C10}$$

$$\begin{aligned}\Lambda_6=\lambda_6T&\left[1+\frac{9}{2}\,g^2\,\frac{L_b}{16\pi^2}-N_c\sum_i\left(\frac{f_{u_i}^2}{2}+\frac{3f_{d_i}^2}{2}\right)\frac{L_f}{16\pi^2}\right]\\&-T(12\lambda_1\lambda_6+3\lambda_3\lambda_6+4\lambda_4\lambda_6+10\lambda_5\lambda_6+3\lambda_3\lambda_7\\&+2\lambda_4\lambda_7+2\lambda_5\lambda_7)\,\frac{L_b}{16\pi^2},\end{aligned} \tag{C11}$$

$$\begin{aligned}\Lambda_7=\lambda_7T&\left[1+\frac{9}{2}\,g^2\,\frac{L_b}{16\pi^2}-N_c\sum_i\left(\frac{f_{d_i}^2}{2}+\frac{3f_{u_i}^2}{2}\right)\frac{L_f}{16\pi^2}\right]\\&-T(12\lambda_2\lambda_7+3\lambda_3\lambda_7+4\lambda_4\lambda_7+10\lambda_5\lambda_7+3\lambda_3\lambda_6\\&+2\lambda_4\lambda_6+2\lambda_5\lambda_6)\,\frac{L_b}{16\pi^2}.\end{aligned} \tag{C12}$$

We have written above only the additional contributions but we remind the reader that superpartner contributions to the formulas in Appendix B must be dropped. This is true for the G and H couplings as well, which do not receive new additional contributions from the extra interaction terms.

For the second stage there are two possibilities. First, as in the generic case of the MSSM, after the first stage one Higgs boson is much heavier than the other and it can be integrated out with the A_0, field after the mass matrix has been diagonalized. This is completely analogous to the procedure in Sec. IV D, with the parameters changed as indicated above (ignoring all overbars in the parameters of Sec. II D). The expressions for the α_i in Eqs. (13), (14), and (15) have additional contributions from the Λ_5, Λ_6, and Λ_7 terms:

$$\Delta\alpha_1=2\Lambda_5\cos^2\,\theta\sin^2\theta+2\Lambda_6\cos^4\theta\sin\theta+\Lambda_7\sin^3\theta\cos\theta, \tag{C13}$$

$$\begin{aligned}\Delta\alpha_3=&-4\Lambda_5\cos^2\theta\sin^2\theta-(2\Lambda_6-2\Lambda_7)(\cos^3\theta\sin\theta\\&-\sin^3\theta\cos\theta),\end{aligned} \tag{C14}$$

$$\begin{aligned}\Delta\alpha_4=&-4\Lambda_5\cos^2\theta\sin^2\theta-(2\Lambda_6-2\Lambda_7)\\&\times(\cos^3\theta\sin\theta-\sin^3\theta\cos\theta).\end{aligned} \tag{C15}$$

The second possibility is that both Higgs fields are light, in which case only the A_0 field is integrated out at this second stage. This is identical to the situation described in Appendix B 3. The quantities Λ_5, Λ_6, and Λ_7 are not modified by the A_0 field.

2. Next to minimal supersymmetric standard model

If we now turn to the supersymmetric case with an additional singlet superfield $\hat{N}$, we will have additional terms in the superpotential of the form [17,18]

$$\Delta W=\lambda(\hat{\phi}_1^0\hat{\phi}_2^0+\hat{\phi}_1^+\phi_2^-)\hat{N}-\frac{k}{3}\,\hat{N}^3-r\hat{N}. \tag{C16}$$

Consequently, the extra terms in the scalar potential, including additional soft SUSY-breaking terms, are

$$\begin{aligned}\Delta V=&\,m_NN^*N+m_4\phi_1^\dagger\phi_2N+\tfrac{1}{3}\,m_5N^3+m_7^2N^2+\lambda_5(\phi_1^\dagger\phi_1)\\&\times(N^*N)+\lambda_6(\phi_2^\dagger\phi_2)(N^*N)+\lambda_7(\phi_1^\dagger\phi_2)N^{*2}\\&+\lambda_8(N^*N)^2+\lambda_9(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\\&+\lambda\mu N(\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2)+\lambda f_{d_i}\phi_2^\dagger Q_iD_i^*N^*\\&-\epsilon_{\alpha\beta}\lambda f_{u_i}\phi_1^\alpha Q_i^{\beta*}U_i^*N^*+\text{H.c.}\end{aligned} \tag{C17}$$

The quartic couplings are expressed in terms of the parameters in the superpotential at the SUSY scale by

$$\lambda_5=\lambda^2,\quad \lambda_6=\lambda^2,\quad \lambda_9=\lambda^2, \tag{C18}$$

$$\lambda_7=-\lambda k,\quad \lambda_8=k^2. \tag{C19}$$

We will have three reduction stages just as in the MSSM. For the first stage, we see that the G and H couplings and the weak and strong Debye masses are not modified because the particles we have introduced are gauge singlets. The 3D squarks masses are also not modified by the introduction of the singlets. We mention that there are additional contributions to the wave function renormalization of the ϕ_1 and ϕ_2 fields from the fermionic loops involving the singlet Higgsino. For the scalar Higgs doublet masses and quartic couplings we have additional contributions

$$\Delta M_1^2=(\lambda_5+\lambda_9+\lambda^2)\frac{T^2}{12}-m_1^2\lambda^2\frac{L_f}{16\pi^2}-(m_2^2\lambda_9+m_N^2\lambda_5+m_4^2+2\lambda^2\mu^2)\frac{L_b}{16\pi^2}+2\lambda^2\mu^2\frac{L_f}{16\pi^2}, \tag{C20}$$

$$\Delta M_2^2=(\lambda_6+\lambda_9+\lambda^2)\frac{T^2}{12}-m_2^2\lambda^2\frac{L_f}{16\pi^2}-(m_1^2\lambda_9+m_N^2\lambda_6+m_4^2+2\lambda^2\mu^2)\frac{L_b}{16\pi^2}+2\lambda^2\mu^2\frac{L_f}{16\pi^2}, \tag{C21}$$

$$\Delta M_3^2=-m_3^2\lambda^2\frac{L_f}{16\pi^2}-(2m_3^2\lambda_9+2\lambda\mu m_4-2\lambda_7 m_7^2)\frac{L_b}{16\pi^2}, \tag{C22}$$

$$\Delta\Lambda_1=-T\left[2\lambda_1\lambda^2\frac{L_f}{16\pi^2}+\left(\frac{\lambda_5^2}{2}+\frac{\lambda_9^2}{2}+(\lambda_3+\lambda_4)\lambda_9\right)\frac{L_b}{16\pi^2}-\lambda^4\frac{L_f}{16\pi^2}\right], \tag{C23}$$

$$\Delta\Lambda_2=-T\left[2\lambda_2\lambda^2\frac{L_f}{16\pi^2}+\left(\frac{\lambda_6^2}{2}+\frac{\lambda_9^2}{2}+(\lambda_3+\lambda_4)\lambda_9\right)\frac{L_b}{16\pi^2}-\lambda^4\frac{L_f}{16\pi^2}\right], \tag{C24}$$

$$\Delta\Lambda_3=-T\left(2\lambda_3\lambda^2\frac{L_f}{16\pi^2}+(\lambda_5\lambda_6+2\lambda_1\lambda_9+2\lambda_2\lambda_9+2\lambda_4\lambda_9+\lambda_9^2)\frac{L_b}{16\pi^2}\right)+2\lambda^4\frac{L_f}{16\pi^2}, \tag{C25}$$

$$\begin{aligned}\Lambda_4=&(\lambda_4+\lambda_9)T\left(1+\frac{9}{2}g^2\frac{L_b}{16\pi^2}-N_c\sum_i(f_{u_i}^2+f_{d_i}^2)\frac{L_f}{16\pi^2}-3g^2\frac{L_f}{16\pi^2}-2\lambda^2\frac{L_f}{16\pi^2}\right)\\&+T\Bigg[-[2\lambda_1(\lambda_4+\lambda_9)+2\lambda_2(\lambda_4+\lambda_9)+2(\lambda_4+\lambda_9)^2+4\lambda_3(\lambda_4+\lambda_9)+2\lambda_7^2]\frac{L_b}{16\pi^2}\\&+N_c\sum_i\left(\lambda_4(f_{d_i}^2+f_{u_i}^2)+2f_{u_i}^2f_{d_i}^2\right)\frac{L_b}{16\pi^2}+N_{sq}\lambda_4^2\frac{L_b}{16\pi^2}-2N_c\sum_i f_{u_i}^2f_{d_i}^2\frac{L_f}{16\pi^2}-2g^4\frac{L_f}{16\pi^2}\Bigg].\end{aligned} \tag{C26}$$

The interaction terms between the scalar Higgs doublets and the squarks and sleptons are modified by

$$\Delta\Lambda_3^{Q_i1}=T\left[-\lambda_3\lambda^2\frac{L_f}{16\pi^2}-[\lambda^2 f_{u_i}^2+\lambda_9(\lambda_3+\lambda_4+f_{u_i}^2)]\frac{L_b}{16\pi^2}+2\lambda^2 f_{u_i}^2\frac{L_f}{16\pi^2}\right], \tag{C27}$$

$$\Delta\Lambda_3^{Q_i2}=T\left[-\lambda_3\lambda^2\frac{L_f}{16\pi^2}-[\lambda^2 f_{d_i}^2+\lambda_9(\lambda_3+\lambda_4+f_{d_i}^2)]\frac{L_b}{16\pi^2}+2\lambda^2 f_{d_i}^2\frac{L_f}{16\pi^2}\right], \tag{C28}$$

$$\Delta(\overline{f_{d_i}^L}^2+\Lambda_4^{Q_i1})=T\left[-(f_{d_i}^2+\lambda_4)\lambda^2\frac{L_f}{16\pi^2}+[\lambda^2 f_{u_i}^2+\lambda_9(f_{u_i}^2+\lambda_4)]\frac{L_b}{16\pi^2}-2\lambda^2 f_{u_i}^2\frac{L_f}{16\pi^2}\right], \tag{C29}$$

$$\Delta(\overline{f_{u_i}^L}^2+\Lambda_4^{Q_i2})=T\left[-(f_{u_i}^2+\lambda_4)\lambda^2\frac{L_f}{16\pi^2}+[\lambda^2 f_{d_i}^2+\lambda_9(f_{d_i}^2+\lambda_4)]\frac{L_b}{16\pi^2}-2\lambda^2 f_{d_i}^2\frac{L_f}{16\pi^2}\right], \tag{C30}$$

$$\Delta\overline{f_{u_i}^R}^2=T\left(-f_{u_i}^2\lambda^2\frac{L_f}{16\pi^2}-2\lambda^2 f_{u_i}^2\frac{L_b}{16\pi^2}+2\lambda^2 f_{u_i}^2\frac{L_f}{16\pi^2}\right), \tag{C31}$$

$$\Delta\overline{f_{d_i}^R}^2=T\left(-f_{d_i}^2\lambda^2\frac{L_f}{16\pi^2}-2\lambda^2 f_{d_i}^2\frac{L_b}{16\pi^2}+2\lambda^2 f_{d_i}^2\frac{L_f}{16\pi^2}\right), \tag{C32}$$

$$\Delta\overline{Af}_{u_i}=-T^{1/2}\left(Af_{u_i}\frac{\lambda^2}{2}\frac{L_f}{16\pi^2}+\lambda f_{u_i}m_4\frac{L_b}{16\pi^2}\right), \tag{C33}$$

$$\Delta\overline{Af}_{d_i}=-T^{1/2}\left(f_{d_i}\frac{\lambda^2}{2}\frac{L_f}{16\pi^2}+\lambda f_{d_i}m_4\frac{L_b}{16\pi^2}\right), \tag{C34}$$

$$\Delta\overline{\mu f}_{u_i}=-T^{1/2}\left(\mu f_{u_i}\frac{\lambda^2}{2}\frac{L_f}{16\pi^2}+\lambda^2 f_{u_i}\mu\frac{L_b}{16\pi^2}\right), \tag{C35}$$

$$\Delta\overline{\mu f}_{d_i}=-T^{1/2}\left(\mu f_{d_i}\frac{\lambda^2}{2}\frac{L_f}{16\pi^2}+\lambda^2 f_{d_i}\mu\frac{L_b}{16\pi^2}\right). \tag{C36}$$

The 3D expressions after the first stage for the mass and interaction terms of the singlet Higgs are

$$\Lambda_5=\lambda_5 T\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ f_{d_i}^2\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}-(2k^2+3\lambda^2)\ \frac{L_f}{16\pi^2}\right)$$
$$-T\left[\left(6\lambda_1\lambda_5+2\lambda_5^2+2\lambda_3\lambda_6+\lambda_4\lambda_6+4\lambda_7^2+4\lambda_5\lambda_8+N_c\sum_i\ f_{u_i}^2\lambda^2\right)\ \frac{L_b}{16\pi^2}-(3g^2\lambda^2+8k^2\lambda^2+2\lambda^4)\ \frac{L_f}{16\pi^2}\right], \quad \text{(C37)}$$

$$\Lambda_6=\lambda_6 T\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ f_{u_i}^2\frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}-(2k^2+3\lambda^2)\ \frac{L_f}{16\pi^2}\right)-T\left[\left(6\lambda_2\lambda_6+2\lambda_6^2+2\lambda_3\lambda_5+\lambda_4\lambda_5+4\lambda_7^2\right.\right.$$
$$\left.\left.+4\lambda_6\lambda_8+N_c\sum_i\ f_{d_i}^2\lambda^2\right)\ \frac{L_b}{16\pi^2}-(3g^2\lambda^2+8k^2\lambda^2+2\lambda^4)\ \frac{L_f}{16\pi^2}\right], \quad \text{(C38)}$$

$$\Lambda_7=\lambda_7 T\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ \left(\frac{f_{u_i}^2}{2}+\frac{f_{d_i}^2}{2}\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}-(2k^2+3\lambda^2)\ \frac{L_f}{16\pi^2}\right]$$
$$-T\left((2\lambda_6\lambda_7+2\lambda_7\lambda_8+2\lambda_4\lambda_7+2\lambda_5\lambda_7+\lambda_3\lambda_7)\ \frac{L_b}{16\pi^2}-2k\lambda^3\ \frac{L_f}{16\pi^2}\right), \quad \text{(C39)}$$

$$\Lambda_8=\lambda_8 T\left(1-4(k^2+\lambda^2)\ \frac{L_f}{16\pi^2}\right)-T\left((\lambda_5^2+\lambda_6^2+2\lambda_7^2+10\lambda_8^2)\ \frac{L_b}{16\pi^2}-(8k^4+2\lambda^4)\ \frac{L_f}{16\pi^2}\right), \quad \text{(C40)}$$

$$M_5=m_5 T^{1/2}\left[1-3(k^2+\lambda^2)\ \frac{L_f}{16\pi^2}\right]-3T^{1/2}\left((2\lambda_7 m_4+2\lambda_8 m_5+2\lambda\lambda_6\mu+2\lambda\lambda_5\mu)\ \frac{L_b}{16\pi^2}-4\lambda^3\mu\ \frac{L_f}{16\pi^2}\right). \quad \text{(C41)}$$

$$M_4=m_4 T^{1/2}\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-N_c\sum_i\ \left(\frac{f_{u_i}^2}{2}+\frac{f_{d_i}^2}{2}\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}-(k^2+2\lambda^2)\ \frac{L_f}{16\pi^2}\right]$$
$$-T^{1/2}\left[\left(4\lambda\lambda_7\mu+\lambda_3 m_4+2(\lambda_4+\lambda_9)m_4+\lambda_5 m_4+\lambda_6 m_4+2\lambda_7 m_5+A\lambda N_c\sum_i\ (f_{d_i}^2+f_{u_i}^2)\right)\right.$$
$$\left.\times\frac{L_b}{16\pi^2}-(3g^2\lambda m_{1/2}+4k\lambda^2\mu)\ \frac{L_f}{16\pi^2}\right], \quad \text{(C42)}$$

$$M_N=m_N^2\left(1-2(k^2+\lambda^2)\ \frac{L_f}{16\pi^2}\right)+(k^2+\lambda^2+\lambda_5+\lambda_6+2\lambda_8)\ \frac{T^2}{6}-\frac{L_b}{16\pi^2}$$
$$\times(8\lambda^2\mu^2+2\lambda_5^2 m_1^2+2\lambda_6^2 m_2^2+4\lambda_7^2 m_3^2+4\lambda_8^2 m_N^2+2m_4^2+2m_5^2-4m_7^2\lambda_8)+12\lambda^2\mu^2\ \frac{L_f}{16\pi^2}, \quad \text{(C43)}$$

$$\Lambda f_{u_i}=\Lambda f_{u_i} T\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(\frac{N_c}{2}\sum_i\ f_{d_i}^2+\frac{1}{2}\ f_{d_i}^2+\frac{3}{2}\ f_{u_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}\right.$$
$$\left.-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}-\left(\frac{3}{2}\ \lambda^2+k^2\right)\ \frac{L_f}{16\pi^2}\right]-T\left[\frac{L_b}{16\pi^2}\ \left(-2\lambda f_{u_i} f_{d_i}^2-\frac{4}{3}\ \lambda f_{u_i} g_s^2+\lambda f_{u_i}\lambda_3+\lambda f_{u_i}\lambda_5-\lambda f_{u_i}\lambda_4+N_c\lambda f_{u_i}\sum_i\ f_{u_i}^2\right)\right.$$
$$\left.-\frac{L_f}{16\pi^2}\ (-2\lambda f_{u_i} f_{d_i}^2+3\lambda f_{u_i} g^2)\right], \quad \text{(C44)}$$

$$\Lambda f_{d_i}=\Lambda f_{d_i} T\left[1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2}-\left(\frac{N_c}{2}\sum_i\ f_{u_i}^2+\frac{1}{2}\ f_{u_i}^2+\frac{3}{2}\ f_{d_i}^2\right)\ \frac{L_f}{16\pi^2}-\frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2}+4g_s^2\ \frac{L_b}{16\pi^2}\right.$$
$$\left.-\frac{8}{3}\ g_s^2\ \frac{L_f}{16\pi^2}-\left(\frac{3}{2}\ \lambda^2+k^2\right)\ \frac{L_f}{16\pi^2}\right]-T\left[\frac{L_b}{16\pi^2}\ \left(-2\lambda f_{d_i} f_{u_i}^2-\frac{4}{3}\ \lambda f_{d_i} g_s^2+\lambda f_{d_i}\lambda_3+\lambda f_{d_i}\lambda_6-\lambda f_{d_i}\lambda_4+N_c\lambda f_{d_i}\sum_i\ f_{d_i}^2\right)\right.$$
$$\left.-\frac{L_f}{16\pi^2}\ (-2\lambda f_{d_i} f_{u_i}^2+3\lambda f_{d_i} g^2)\right]. \quad \text{(C45)}$$

We denote the 3D coupling of the trilinear $\phi_1^\dagger \phi_1 N$ term by J_1 and similarly for the $\phi_2^\dagger \phi_2 N$ term by J_2:

$$J_1 = \lambda \mu T^{1/2}\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2} - N_c \sum_i\ f_{d_i}^2 \frac{L_f}{16\pi^2} - \frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2} - (k^2+2\lambda^2)\ \frac{L_f}{16\pi^2}\right)$$
$$- T^{1/2}\left[\left(6\lambda\lambda_1\mu + 2\lambda\lambda_3\mu + \lambda\lambda_4\mu + 2\lambda\lambda_5\mu + N_c\sum_i\ \lambda f_{u_i}^2\mu\right)\ \frac{L_b}{16\pi^2} - (3g^2\lambda\mu + 2\lambda^3\mu)\ \frac{L_f}{16\pi^2}\right], \quad \text{(C46)}$$

$$J_2 = \lambda \mu T^{1/2}\left(1+\frac{9}{4}\ g^2\ \frac{L_b}{16\pi^2} - N_c \sum_i\ f_{u_i}^2 \frac{L_f}{16\pi^2} - \frac{3}{2}\ g^2\ \frac{L_f}{16\pi^2} - (k^2+2\lambda^2)\ \frac{L_f}{16\pi^2}\right) - T^{1/2}\left[\left(6\lambda\lambda_2\mu + 2\lambda\lambda_3\mu + \lambda\lambda_4\mu + 2\lambda\lambda_6\mu\right.\right.$$
$$\left.\left. + N_c\sum_i\ \lambda f_{d_i}^2\mu\right)\ \frac{L_b}{16\pi^2} - (3g^2\lambda\mu + 2\lambda^3\mu)\ \frac{L_f}{16\pi^2}\right]. \quad \text{(C47)}$$

The second stage proceeds just like in the MSSM. The interaction terms between doublet and singlet Higgs fields are modified by

$$\overline{\Lambda_5} = \Lambda_5\left(1 - \sum_i\ \frac{N_c f_{d_i}^2 \overline{A^2}}{12\pi(M_Q^i + M_d^i)^3} - \sum_i\ \frac{N_c f_{u_i}^2 \overline{\mu^2}}{12\pi(M_Q^i + M_u^i)^3}\right)$$
$$- \sum_i\ \frac{\Lambda^2 f_{u_i}^2}{4\pi(M_Q^i + M_u^i)}, \quad \text{(C48)}$$

$$\overline{\Lambda_6} = \Lambda_6\left(1 - \sum_i\ \frac{N_c f_{u_i}^2 \overline{A^2}}{12\pi(M_Q^i + M_u^i)^3} - \sum_i\ \frac{N_c f_{d_i}^2 \overline{\mu^2}}{12\pi(M_Q^i + M_d^i)^3}\right)$$
$$- \sum_i\ \frac{\Lambda^2 f_{d_i}^2}{4\pi(M_Q^i + M_d^i)}, \quad \text{(C49)}$$

$$\overline{M_4} = M_4\left(1 - \sum_i\ \frac{N_c f_{u_i}^2 \overline{A^2}}{24\pi(M_Q^i + M_u^i)^3} - \sum_i\ \frac{N_c f_{d_i}^2 \overline{\mu^2}}{24\pi(M_Q^i + M_d^i)^3}\right.$$
$$\left. - \sum_i\ \frac{N_c f_{d_i}^2 \overline{A^2}}{24\pi(M_Q^i + M_d^i)^3} - \sum_i\ \frac{N_c f_{u_i}^2 \overline{\mu^2}}{24\pi(M_Q^i + M_u^i)^3}\right)$$
$$- \sum_i\ N_c\left(\frac{\Lambda \overline{A} f_{d_i}^2}{4\pi(M_Q^i + M_d^i)} + \frac{\Lambda \overline{A} f_{u_i}^2}{4\pi(M_Q^i + M_u^i)}\right), \quad \text{(C50)}$$

$$\overline{J_1} = J_1\left(1 - \sum_i\ \frac{N_c f_{d_i}^2 \overline{A^2}}{12\pi(M_Q^i + M_d^i)^3} - \sum_i\ \frac{N_c f_{u_i}^2 \overline{\mu^2}}{12\pi(M_Q^i + M_u^i)^3}\right)$$
$$- N_c\sum_i\ \frac{\Lambda \overline{\mu} \overline{f}_{u_i}^2}{4\pi(M_Q^i + M_u^i)}, \quad \text{(C51)}$$

$$\overline{J_2} = J_2\left(1 - \sum_i\ \frac{N_c f_{u_i}^2 \overline{A^2}}{12\pi(M_Q^i + M_d^i)^3}\right.$$
$$\left. + - \sum_i\ \frac{N_c f_{d_i}^2 \overline{\mu^2}}{12\pi(M_Q^i + M_u^i)^3}\right) - N_c\sum_i\ \frac{\Lambda \overline{\mu} \overline{f}_{d_i}^2}{4\pi(M_Q^i + M_d^i)}. \quad \text{(C52)}$$

There are no triangle and box diagram corrections to the above second stage quantities.

The scalars in the resulting theory are two Higgs doublets, a Higgs singlet, and the A_0 triplet. Depending on the values of the parameters which determine the 3D mass of the singlet, if it is heavy ($\sim gT$), it can be integrated out at the third stage after the diagonalization of the scalar doublets mass matrix. We remind the reader that we have determined the critical temperature by finding the direction in which the curvature of the potential vanishes at the origin. Consequently, there are no mixing terms in the mass matrix between the doublets and the singlet Higgs fields. The additional contributions to $\overline{\lambda_3}$ from the singlet, including wave function corrections, are

$$\Delta\overline{\lambda_3} = -\alpha_1\left(\frac{\alpha_5^2 + \alpha_6^2}{6\pi[\nu(T_c) + M_N]^3}\right) - \frac{\alpha_7^2}{2}\ \frac{1}{8\pi M_N}$$
$$- 2\Lambda_7^2 \cos^2\theta \sin^2\theta\ \frac{1}{8\pi M_N} + \alpha_7(\alpha_5^2 + \alpha_6^2)$$
$$\times \frac{1}{8\pi\nu(T_c)}\ \frac{1}{[\nu(T_c) + M_N]^2} - (\alpha_5^2 + \alpha_6^2)^2$$
$$\times \frac{1}{8\pi\nu(T_c) M_N}\ \frac{1}{[\nu(T_c) + M_N]^3}, \quad \text{(C53)}$$

where

$$\alpha_5 = \overline{M}_4 \cos^2\theta - \overline{J}_1 \sin\theta\cos\theta + \overline{J}_2 \sin\theta\cos\theta, \quad \text{(C54)}$$

$$\alpha_6 = -\overline{M}_4 \sin^2\theta - \overline{J}_1 \sin\theta\cos\theta + \overline{J}_2 \sin\theta\cos\theta, \quad \text{(C55)}$$

$$\alpha_7 = \overline{\Lambda}_5 \cos^2\theta + \overline{\Lambda}_6 \sin^2\theta. \quad \text{(C56)}$$

APPENDIX D: FINITE TEMPERATURE FORMULAS

In this appendix we include the basic integrals which appear in the calculation over the nonstatic modes. These results can be derived from formulas presented in the literature. We refer the reader to Refs. [8,22,23] and references within for more details regarding finite temperature formulas.

1. $m=0$

Let us consider first the massless case. We can define the quantities

$$A_s=\sum\!\!\!\!\!\!\int \frac{1}{p^{2s}}=2\mu^{2\epsilon}T\frac{\Gamma(-D/2+s)}{(4\pi)^{D/2}\Gamma(s)}\,(2\pi T)^{-2s+D}\zeta(2s-D), \tag{D1}$$

with $p^2=\vec{p}^{\,2}+\omega_B^2$, $\omega_B=2\pi nT$, and $D=3-2\epsilon$. For bosonic sums, $n=0$ is excluded, where

$$\sum\!\!\!\!\!\!\int=\mu^{2\epsilon}T\sum_{p_0}\int\frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}}. \tag{D2}$$

Similarly we have, for fermionic excitations,

$$B_j=\sum\!\!\!\!\!\!\int \frac{1}{\tilde{p}^{2s}}, \tag{D3}$$

with $\tilde{p}^2=\vec{p}^{\,2}+\omega_F^2$, and $\omega_F=2\pi(n+1/2)T$. Using the fact that

$$A_s+B_s=2^{2s-D}A_s, \tag{D4}$$

we can easily determine the fermionic contributions in terms of the bosonic integrals. Generalizing, we can write

$$B_s^{\alpha_1\cdots\alpha_k}=(2^{2s-D-k}-1)A_s^{\alpha_1\cdots\alpha_k}, \tag{D5}$$

where the superscripts $\alpha_1\cdots\alpha_k$ indicate additional powers of momenta in the integrals. In particular we have

$$A_1=\frac{T^2}{12}+O(\epsilon) \tag{D6}$$

and

$$A_2=\frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon}+L_b\right), \tag{D7}$$

where

$$\begin{aligned}L_b&=2\ln\frac{\overline{\mu_4}e^{\gamma}}{4\pi T}\\&=-\ln 4\pi T^2+\gamma+\ln\mu_4^2.\end{aligned} \tag{D8}$$

$\overline{\mu_4}$ is defined by the $\overline{\rm MS}$ scheme.

2. $m\neq 0$

If we now include the effect of masses, our formulas will be modified in the following way:

$$\begin{aligned}A_s(m)=\sum\!\!\!\!\!\!\int \frac{1}{(p^2+m^2)^s}&=\mu^{2\epsilon}T\,\frac{\Gamma(-D/2+s)}{(4\pi)^{D/2}\Gamma(s)}\,(2\pi T)^{-2s+D}\\&\times\left[\zeta\left(2s-D;\ \frac{m}{2\pi T}\right)-\left(\frac{m}{2\pi T}\right)^{(-2s+D)}\right],\end{aligned} \tag{D9}$$

where

$$\zeta(\sigma;\nu)=\sum_{n=-\infty}^{n=\infty}(n^2+\nu^2)^{-\sigma}. \tag{D10}$$

It is easy to verify that

$$A_s(m)=-(s-1)^{-1}\,\frac{\partial}{\partial m^2}\,A_{s-1}(m), \tag{D11}$$

and for high temperature, dropping the $O(\epsilon)$ terms, we can use the expansion

$$A_1(m)=\frac{T^2}{12}-\frac{m^2}{16\pi^2}\left(\frac{1}{\epsilon}+L_b\right)+O\left(\frac{m^4}{T^2}\right) \tag{D12}$$

and

$$A_2(m)=\frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon}+L_b\right)+\cdots\,. \tag{D13}$$

We can now extend our considerations to express the fermionic integrals in terms of the bosonic ones, obtaining

$$B_s^{\alpha_1\cdots\alpha_k}(m)=2^{2s-D-k}A_s^{\alpha_1\cdots\alpha_k}(2m)-A_s^{\alpha_1\cdots\alpha_k}(m). \tag{D14}$$

Let us write the explicit results for

$$A_s^i=\sum\!\!\!\!\!\!\int \frac{p_i}{(p^2+m^2)^s}=0, \tag{D15}$$

$$A_s^0=\sum\!\!\!\!\!\!\int \frac{p_0}{(p^2+m^2)^s}=0, \tag{D16}$$

$$\begin{aligned}A_s^{ij}&=\sum\!\!\!\!\!\!\int \frac{p_ip_j}{(p^2+m^2)^s}=A_1(s)\,\delta_{ij},\\A_s^{00}&=\sum\!\!\!\!\!\!\int \frac{p_0p_0}{(p^2+m^2)^s}=A_1(s)+A_2(s),\end{aligned} \tag{D17}$$

where

$$\begin{aligned}A_i(s)&=-(s-1)^{-1}\,\frac{\partial}{\partial m^2}\,A_i(s-1),\\A_1(1)&=\frac{m^4}{64\pi^2}\left(\frac{1}{\epsilon}+L_b\right)+\frac{1}{24}\,m^2T^2,\\A_2(1)&=-\frac{2\pi^2}{45}\,T^4+\frac{1}{32\pi^2}\,m^4+\frac{1}{12}\,m^2T^2+\cdots\,.\end{aligned} \tag{D18}$$

We would also like to write the explicit results for the integrals

$$C(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{1}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}=A_2, \tag{D19}$$

$$\begin{aligned}C^i(p,m_1,m_2)&=\sum\!\!\!\!\!\!\int \frac{k_i}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}\\&=p_iA_2/2,\end{aligned} \tag{D20}$$

$$C^{ij}(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{k_i k_j}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=p_i p_j C_{21}+g_{ij}C_{22}, \tag{D21}$$

with

$$C_{21}=A_2/3, \tag{D22}$$

$$C_{22}=\frac{T^2}{24}-\left(m_1^2+m_2^2+\frac{p^2}{3}\right)A_2/4, \tag{D23}$$

$$C^{00}(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{k_0^2}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=-\frac{T^2}{24}-\frac{1}{64\pi^2}\left(\frac{1}{\epsilon}+L_b+2\right)$$
$$\times\left(m_1^2+m_2^2+\frac{p^2}{3}\right), \tag{D24}$$

$$E^{ij}(p,m_1,m_2)$$
$$=\sum\!\!\!\!\!\!\int \frac{k_i k_j}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$\times\frac{1}{(k-p')^2+\omega_n^2+m_3^2}$$
$$=\delta_{ij}\,\frac{1}{4}\,\frac{1}{16\pi^2}\left(\frac{1}{\epsilon}+L_b\right), \tag{D25}$$

$$E^{00}(p,m_1,m_2)$$
$$=\sum\!\!\!\!\!\!\int \frac{k_0^2}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$\times\frac{1}{(k-p')^2+\omega_n^2+m_3^2}$$
$$=\frac{1}{16\pi^2}\left[\frac{1}{4}\left(\frac{1}{\epsilon}+L_b\right)+\frac{1}{2}\right]. \tag{D26}$$

We would like to relate the fermionic integrals of this type to the bosonic ones:

$$D^{\alpha_1\cdots\alpha_m}(p)=\sum\!\!\!\!\!\!\int \frac{\tilde{k}_{\alpha_1}\cdots\tilde{k}_{\alpha_m}}{\tilde{k}^2+\omega_n^2}\,\frac{1}{(\tilde{k}-\tilde{p})^2+\omega_n^2}, \tag{D27}$$

with, once again, $\tilde{p}^2=\vec{p}^{\,2}+\omega_F^2$, and $\omega_F=2\pi(n+1/2)T$. We obtain

$$D(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{1}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=B_2, \tag{D28}$$

$$D^i(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{k_i}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=p_i B_2/2, \tag{D29}$$

$$D^{ij}(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{k_i k_j}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=p_i p_j D_{21}+g_{ij}D_{22}, \tag{D30}$$

explicitly,

$$D_{21}=B_2/3, \tag{D31}$$

$$D_{22}=-\frac{T^2}{48}-\left(m_1^2+m_2^2+\frac{p^2}{3}\right)B_2/4, \tag{D32}$$

$$D^{oo}(p,m_1,m_2)=\sum\!\!\!\!\!\!\int \frac{k_o^2}{k^2+\omega_n^2+m_1^2}\,\frac{1}{(k-p)^2+\omega_n^2+m_2^2}$$
$$=\frac{T^2}{48}-\frac{1}{64\pi^2}\left(\frac{1}{\epsilon}+L_f+2\right)\left(m_1^2+m_2^2+\frac{p^2}{3}\right), \tag{D33}$$

where $L_f=L_b+4\ln 2$.

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