Electroweak breaking and the μ problem in supergravity models with an additional U(1)

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We consider electroweak symmetry breaking in supersymmetric models with an extra nonanomalous $U(1)'$ gauge symmetry and an extra standard-model singlet scalar S. For appropriate charges the $U(1)'$ forbids an elementary μ term, but an effective μ is generated by the VEV of S, leading to a natural solution to the μ problem. There are a variety of scenarios leading to acceptably small Z-Z' mixing and other phenomenological consequences, all of which involve some but not excessive fine-tuning. One class, driven by a large trilinear soft supersymmetry-breaking term, implies small mixing, a light Z' (e.g., 200 GeV), and an electroweak phase transition that may be first order at the tree level. In another class, with $m_S^2<0$ (radiative breaking), the typical scale of dimensional parameters, including $M_{Z'}$ and the effective μ, is $\sim 1\text{TeV}$, but the electroweak scale is smaller due to cancellations. We relate the soft supersymmetry-breaking parameters at the electroweak scale to those at the string scale, choosing Yukawa couplings as determined within a class of string models. We find that one does not obtain either scenario for universal soft supersymmetry-breaking mass parameters at the string scale and no exotic multiplets contributing to the renormalization group equations. However, either scenario is possible when the assumption of universal soft breaking is relaxed. Radiative breaking can also be generated by exotics, which are expected in most string models. [S0556-2821(97)02117-6]

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I. INTRODUCTION

The simplest gauge extension of the standard model involves one or more additional U(1) symmetries and their associated extra Z bosons. Such U(1)'s often emerge in the breaking of grand unified theories (GUT's) or in string compactifications, for example.

There has been much phenomenological work on the implications of such heavy Z's for precision electroweak observables and for future hadron and e^+e^- colliders. Present [1] and future [2] limits as well as search and diagnostic capabilities depend on the Z' mass, mixing with the Z, gauge couplings, and chiral charges of the ordinary quarks and leptons, and are thus very model dependent. For many typical (especially GUT-motivated) models the limits on the Z-Z' mixing are around a few $\times 10^{-3}$. The lower limits on the Z' mass are typically around 500 GeV, usually dominated by direct searches at the Fermilab Tevatron ($p\bar{p}\to Z' \to \ell^+\ell^-$) [3], but with constraints from precision electroweak tests often competitive. Recently, a number of authors [4] have postulated that a possible excess of $Z\to b\bar{b}$ events at the CERN e^+e^- collider LEP could be accounted for by the mixing between the Z and a leptophobic (hadrophilic) Z' which mainly couples to quarks, but the most recent LEP data, especially from ALEPH, have considerably weakened the case that there is an excess [5]. In the future it should be possible to discover a heavy Z' at the CERN Large Hadron Collider (LHC) for masses up to around 10 TeV. Diagnostics of its couplings at the LHC or Next Linear Collider (NLC) (which have complementary capabilities) should be possible up to a few TeV [2].

In addition to being a useful signature of the underlying theory, an additional $U(1)'$ would have important theoretical implications. For example, an extra $U(1)'$ breaking at the electroweak scale in a supersymmetric extension of the standard model could solve the μ problem [6–9], by forbidding an elementary μ term but inducing an effective μ at the electroweak scale by the $U(1)'$ breaking. This possibility is one of the major motivations of this paper. There are also implications for baryogenesis. One popular scenario is that a lepton asymmetry [10] (or an asymmetry in some other quantum number) was created by the out of equilibrium decay of a superheavy particle (e.g., a heavy Majorana neutrino) long before the electroweak transition, and then converted to a baryon asymmetry by sphaleron effects. Such a mechanism would not be consistent with an additional $U(1)'$ at the TeV or electroweak scale unless the Majorana neutrino were neutral under the $U(1)'$. On the other hand, an extra $U(1)'$ might be useful for electroweak baryogenesis, with cosmic strings providing the needed ''out of equilibrium'' ingredient [11].

Much of the phenomenological work on extra Z's has been of the lamp-post variety, i.e., there was no strong motivation to think that an extra Z' would actually be light enough to observe. Certainly, in ordinary GUT's there is no robust prediction for the mass scale of the $U(1)'$ breaking. In supersymmetric models there are constraints on the breaking scale, which are usually of order a TeV, because the $U(1)'$ D term may induce masses of order of the breaking for all scalars which carry the $U(1)'$ charge [12]. However, that is more a phenomenological constraint than a theoretical prediction, and it can be evaded if the breaking occurs along a D-flat direction.

However, it was recently argued [8] that for a large class of string models with extra $U(1)$'s, the breaking should be at the electroweak scale and certainly not larger than a TeV. The string models considered in [8] are based on $N=1$ supersymmetric string models with the standard model (SM) gauge group $SU(2)_L\times U(1)_Y\times SU(3)_C$, three families, and at least two standard model doublets, i.e., models with at least the particle content of the minimal supersymmetric standard model (MSSM). A number of such models are based on fermionic ($Z_2 \times Z_2$) orbifold constructions [13–16] at a particular point in moduli space.

Such models suffer from a number of phenomenological problems (see Sec. II in [8] for a detailed discussion), and many such models are already excluded experimentally. Nevertheless, there is a strong motivation to search for such Z' bosons and also for the exotic [vector under SU(2)] supermultiplets with which they are usually associated. In addition, they provide a useful testing ground to address the issues of U(1)$'$ breaking within a large class of string models.

The relevant models are those in which (a) there is a nonanomalous U(1)$'$ which does not acquire a large mass from string or shadow sector dynamics, so that its mass must come from symmetry breaking in the observable sector, and (b) the soft supersymmetry breaking is such that all scalar mass-squared terms are positive and of the same order of magnitude at the string scale, which is the case for most gravity mediated hidden sector models (but not necessarily for the gauge mediated supersymmetry-breaking models that have been of recent interest).

Under these assumptions, the U(1)$'$ breaking may be radiative [8]. It can take place if there are Yukawa couplings of order 1 of a scalar which is a standard model singlet [but which carries a U(1$'$) charge] to exotic particles. This is expected in many string models, for which all nonzero Yukawa couplings are typically of the same magnitude, i.e., they are the same as the gauge coupling at the string scale up to a coefficient of order unity. These can drive the scalar mass-squared to a negative value at low energies, which is typically of the same order as the Higgs boson mass-squared, so that the electroweak and U(1)$'$-breaking scales are comparable, both being controlled by the same soft supersymmetry-breaking scale.[1]

In [8], a model was considered in which *only one* (e.g., H_2) of the two SM Higgs doublets has nonzero couplings in the superpotential and contributes to the electroweak breaking; i.e., this model roughly corresponds to the large $\tan\beta$ scenario in the MSSM. The radiative symmetry breaking can take place with $M_{Z'} \sim 1$ TeV, and sufficiently small Z-Z' mixing angle (not yet excluded by the direct and indirect heavy Z' constraints), provided the U(1)$'$ charge assignments for the H_2 and the SM singlet S [responsible for the symmetry breaking of U(1)$'$] have the same sign.

In this paper we consider the more general case with the two SM doublets $H_{1,2}$ now coupled to the SM singlet $\hat{S}$ in the superpotential with the term $h_s \hat{S} \hat{H}_1 \cdot \hat{H}_2$. In this case, the U(1)$'$ charges of $\hat{H}_{1,2}$ and $\hat{S}$ must sum to zero. This term provides an effective μ term $h_s \langle S \rangle$, once S acquires a nonzero vacuum expectation value.

Because of this additional term in the superpotential, a rich spectrum of possible symmetry-breaking scenarios emerges. In particular, we concentrate on a set of phenomenologically viable scenarios with small Z-Z' mixing ($\lesssim 10^{-3}$) and $M_{Z'}$ in the range $\lesssim 1$ TeV. We also insist on no dangerous color breaking minimum, e.g., no negative squark mass-squared parameters or large trilinear soft supersymmetry-breaking terms that involve squarks. We find various ranges of parameters that allow for such symmetry-breaking scenarios. However, all these cases involve some degree of fine-tuning of parameters, either at the electroweak scale or at the string scale.[2] A few percent of the parameter space gives a phenomenologically acceptable U(1)$'$ symmetry-breaking scenario. This fact is important since it implies that in this class of string models there is a reasonable probability that the heavy Z' is in the experimentally observable region (and not required to become massive at the string scale). In addition, these models provide an elegant solution to the μ problem, complementary to that of the Giudice-Masiero mechanism [17].[3]

In Sec. II we give explicit expressions for the scalar potential, vector boson masses, scalar masses and related sparticle masses, and introduce certain definitions and conventions that will be used throughout the work. In Sec. III, we present scenarios to obtain a small Z-Z' mixing angle based on that portion of parameter space in which the trilinear coupling is much greater than the soft mass parameters. In this case $M_{Z'}$ is typically comparable to M_Z (e.g., 200 GeV) and $\tan\beta \sim 1$. This scenario is only viable for certain (e.g., leptophobic) couplings. One version of the model has a first-order electroweak phase transition at tree level and thus has potentially interesting cosmological consequences.

In Sec. IV, we present a scenario in which the singlet acquires a large VEV so that $M_{Z'} \simeq 1$ TeV. In this case, all of the dimensional parameters in the scalar potential are of ~ 1 TeV) and the smaller electroweak scale is due to a cancellation of parameters.

In Section V, we use the renormalization group to relate the electroweak scale supersymmetry-breaking parameters to those at the string scale. We first assume the minimal particle content, consisting of the MSSM particles, the additional singlet, and the Z'. We present the results of the numerical integration of the renormalization group equations (RGE's) for the parameters of the model as a function of their boundary conditions at the string scale. With the minimal particle content, we conclude that it is necessary to invoke nonuniversal values of the soft supersymmetry-breaking parameters at the string scale to reach the desired low-energy region of parameter space. Several examples of boundary conditions at the string scale are presented which lead to the phenomenologically acceptable scenarios of Secs. III and IV. We also discuss the implications of additional exotic matter in the RGE's, and conclude that with additional SU(3) triplets, for example, the large singlet vacuum expectation value (VEV) scenario is possible with universal boundary conditions.

The RGE's are presented in Appendix A. In Appendix B, we present the details of the numerical results, and we give

[1]In some cases the breaking will be at an intermediate scale if there is a D-flat direction involving two scalars both of which have large Yukawa couplings.

[2]However, the tuning involved is no worse than that in the MSSM in the case for which the electroweak scale M_Z is small compared to μ, e.g., for $\mu \sim 1$ TeV

[3]With additional U(1)$'s$ the required terms in the Kähler potential are absent; thus the Giudice-Masiero mechanism is not applicable. Other possible solutions are surveyed in [8].

semianalytic solutions of the RGE's. Finally, in Appendix C we present examples of models with anomaly-free $\mathrm{U}(1)'$.

Our goal is to explore the general features of electroweak breaking in a class of string models, not to construct a specific model. We therefore focus on the gauge- and symmetry-breaking sectors of the theory and only specify the $\mathrm{U}(1)'$ charges when we present concrete numerical examples.

II. ELECTROWEAK SYMMETRY BREAKING

The gauge group is extended to $G=\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_{Y'}$ with the couplings g_3, g_2, g_Y, g_1', respectively.[4] The particle content is given by the left-handed chiral superfields $\hat{L}_i\sim(1,2,-1/2,Q_L)$, $\hat{E}_i^c\sim(1,1,1,Q_E)$, $\hat{Q}_i\sim(3,2,1/6,Q_Q)$, $\hat{U}_i^c\sim(\overline{3},1,-2/3,Q_U)$, $\hat{D}_i^c\sim(\overline{3},1,1/3,Q_D)$, $\hat{H}_1\sim(1,2,-1/2,Q_1)$, $\hat{H}_2\sim(1,2,1/2,Q_2)$, $\hat{S}\sim(1,1,0,Q_S)$, where the subscript i is the family index.

The superpotential for our model is[5]

$$W=h_s\hat{S}\hat{H}_1\cdot\hat{H}_2+h_Q\hat{U}_3^c\hat{Q}_3\cdot\hat{H}_2. \tag{1}$$

The form of Eq. (1) is motivated by string models [19], in which a given Higgs doublet (i.e., $\hat{H}_2$) only has Yukawa couplings to a single (third) family. This family index will not be displayed in the rest of the paper.

Gauge invariance of W under $\mathrm{U}(1)'$ requires $Q_1+Q_2+Q_S=0$. The effective μ parameter is generated by the VEV $\langle S\rangle=s/\sqrt{2}$, and will then be given by $\mu_s=h_s s/\sqrt{2}$.

Within string models there is no mechanism for supersymmetry breaking with quantitative predictive power. We thus parametrize supersymmetry breaking with the most general soft supersymmetry-breaking mass parameters. The soft supersymmetry-breaking Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{SB}=&\left(-\sum_i M_i\lambda_i\lambda_i+Ah_sSH_1\cdot H_2+A_Qh_QU^cQ\cdot H_2\right.\\&\left.+\mathrm{H.c.}\right)-m_1^2|H_1|^2-m_2^2|H_2|^2-m_S^2|S|^2-m_Q^2|Q|^2\\&-m_U^2|U|^2-m_D^2|D|^2-m_E^2|E|^2-m_L^2|L|^2,\end{aligned} \tag{2}$$

where the λ_i are gauginos, and the other fields are the scalar components of the corresponding supermultiplets. Gauge symmetry breaking is now driven by the vacuum expectation values of the doublets H_1, H_2 and the singlet S. The Higgs potential is the sum of three pieces:

$$V=V_F+V_D+V_{\mathrm{soft}}, \tag{3}$$

with

$$V_F=|h_s|^2[|H_1\cdot H_2|^2+|S|^2(|H_1|^2+|H_2|^2)], \tag{4}$$

$$\begin{aligned}V_D=&\frac{G^2}{8}(|H_2|^2-|H_1|^2)^2+\frac{g_2^2}{2}|H_1^\dagger H_2|^2\\&+\frac{g_1'^2}{2}(Q_1|H_1|^2+Q_2|H_2|^2+Q_S|S|^2)^2,\end{aligned} \tag{5}$$

$$V_{\mathrm{soft}}=m_1^2|H_1|^2+m_2^2|H_2|^2+m_S^2|S|^2-(Ah_sSH_1\cdot H_2+\mathrm{H.c.}), \tag{6}$$

where $G^2=g_Y^2+g_2^2$, and

$$H_1=\begin{pmatrix}H_1^0\\H_1^-\end{pmatrix},\quad H_2=\begin{pmatrix}H_2^+\\H_2^0\end{pmatrix}. \tag{7}$$

By an appropriate choice of the global phases of the fields, we can take Ah_s real and positive without loss of generality. By a suitable gauge rotation we can also make $\langle H_2^+\rangle=0$ and take $\langle H_2^0\rangle=v_2/\sqrt{2}$ and $\langle S\rangle=s/\sqrt{2}$ real and positive. The requirement $\langle H_1^-\rangle=0$ in the vacuum is equivalent to requiring the squared mass of the physical charged scalar to be positive and imposes some constraint on the parameter space of the model, as will be shown later. There is no room for explicit or spontaneous CP violation in the potential (3) so that $\langle H_1^0\rangle=v_1/\sqrt{2}$ is real. Furthermore, with our choice $Ah_s>0$ one has $v_1>0$ in the true minimum.

Even after the replacement of h_sS by $h_s\langle S\rangle=\mu_s\sqrt{2}$, V differs from the MSSM by additional terms quadratic in the H_i in V_F and V_D. The minimization conditions when all VEV's are nonzero give[6]

$$\begin{aligned}m_1^2=&m_3^2\tan\beta-\frac{1}{8}G^2v^2\cos2\beta-\frac{1}{2}g'^2_1Q_1(\overline{Q_H}v^2+Q_Ss^2)\\&-\frac{1}{2}h_s^2(v^2\sin^2\beta+s^2),\end{aligned} \tag{8}$$

$$\begin{aligned}m_2^2=&m_3^2\cot\beta+\frac{1}{8}G^2v^2\cos2\beta-\frac{1}{2}g'^2_1Q_2(\overline{Q_H}v^2+Q_Ss^2)\\&-\frac{1}{2}h_s^2(v^2\cos^2\beta+s^2),\end{aligned} \tag{9}$$

$$m_S^2=m_3^2\frac{v^2}{s^2}\sin\beta\cos\beta-\frac{1}{2}g'^2_1Q_S(\overline{Q_H}v^2+Q_Ss^2)-\frac{1}{2}h_s^2v^2, \tag{10}$$

where $m_3^2=(h_s/\sqrt{2})As$, $\overline{Q_H}=Q_1\cos^2\beta+Q_2\sin^2\beta$, $v^2=v_1^2+v_2^2$, and $\tan\beta=v_2/v_1$.

To ensure that the extremum at (v_1,v_2,s) is a minimum of the potential, the squared masses of Higgs scalars should be positive. In addition, $V(v_1,v_2,s)<0$ should also hold for

[4] Here $g_Y=\sqrt{3/5}g_1$, where g_1 is the grand unified theory (GUT) normalized coupling. That is, g_Y is the coupling usually called g' in the standard model.

[5] The $\mathrm{U}(1)'$ forbids not only an elementary $\mu\hat{H}_1\cdot\hat{H}_2$ term in the superpotential, but also a term $\hat{S}^3$. Such a term is needed in the NMSSM [18] to avoid the appearance of an axion after symmetry breaking. In our model, this massless pseudoscalar is eaten by the Z'. Also, unlike in the NMSSM the discrete symmetry is embedded in the gauge symmetry and thus there is no domain wall problem.

[6] For a more precise analysis of the model, beyond the scope of this paper, it would be necessary to include one-loop corrections, which can have a non-negligible effect [20].

the minimum to be acceptable. Even if all these conditions are satisfied, the minimum is not guaranteed to be the global minimum of the potential. Whether it is still acceptable will depend on the location and depth of the other possible minima and of the barrier height and width between the minima [21].

Letting Z' be the gauge boson associated with $U(1)'$, the Z-Z' mass-squared matrix is given by

$$(M^2)_{Z-Z'}=\begin{pmatrix} M_Z^2 & \Delta^2 \\ \Delta^2 & M_{Z'}^2 \end{pmatrix}, \tag{11}$$

where

$$M_Z^2=\frac{1}{4}G^2(v_1^2+v_2^2), \tag{12}$$

$$M_{Z'}^2=g'^2_1(v_1^2Q_1^2+v_2^2Q_2^2+s^2Q_S^2), \tag{13}$$

$$\Delta^2=\frac{1}{2}g'_1\,G(v_1^2Q_1-v_2^2Q_2). \tag{14}$$

The eigenvalues of this matrix are

$$M_{Z_1,Z_2}^2=\frac{1}{2}[M_Z^2+M_{Z'}^2\mp\sqrt{(M_Z^2-M_{Z'}^2)^2+4\Delta^4}]. \tag{15}$$

The Z-Z' mixing angle $\alpha_{Z\text{-}Z'}$ is given by

$$\alpha_{Z\text{-}Z'}=\frac{1}{2}\arctan\left(\frac{2\Delta^2}{M_{Z'}^2-M_Z^2}\right). \tag{16}$$

Phenomenological constraints typically require this mixing angle to be less than a few times 10^{-3} [1], although values as much as ten times larger may be possible in some models with a light Z' (e.g., $M_{Z_2}/M_{Z_1}\sim 2$) and certain (e.g., leptophobic) couplings. Then, with good precision $M_{Z_1}^2=G^2v^2/4$ so that $v=246$ GeV is fixed.

The spectrum of physical Higgs bosons after symmetry breaking consists of three neutral CP even scalars (h_i^0, $i=1,2,3$), one CP odd pseudoscalar (A^0), and a pair of charged Higgs bosons ($H^\pm$), that is, one scalar more than in the MSSM. The tree-level masses of the Higgs bosons are

$$m_{A^0}^2=\frac{\sqrt{2}Ah_s s}{\sin 2\beta}\left[1+\frac{v^2}{4s^2}\sin^2 2\beta\right], \tag{17}$$

which is never negative, and

$$m_{H^\pm}^2=M_W^2+\frac{\sqrt{2}Ah_s s}{\sin 2\beta}-\frac{1}{2}h_s^2v^2. \tag{18}$$

$m_{H^\pm}^2$ could be lighter than the W boson due to the negative third contribution. It could even be negative for some choices of the parameters.

Masses for the three neutral scalars can be obtained by diagonalizing the corresponding 3×3 mass matrix, which, in the basis $\{H_1^{0r}\equiv \mathrm{Re}(H_1^0)\sqrt{2},H_2^{0r},S^{0r}\}$, reads

$$(M^2)_{h^0}=\begin{pmatrix} \kappa_1^2v_1^2+m_3^2\tan\beta & \kappa_{12}v_1v_2-m_3^2 & \kappa_{1s}v_1s-m_3^2\dfrac{v_2}{s} \\ \kappa_{12}v_1v_2-m_3^2 & \kappa_2^2v_2^2+m_3^2\cot\beta & \kappa_{2s}v_2s-m_3^2\dfrac{v_1}{s} \\ \kappa_{1s}v_1s-m_3^2\dfrac{v_2}{s} & \kappa_{2s}v_2s-m_3^2\dfrac{v_1}{s} & \kappa_s^2s^2+m_3^2\dfrac{v_1v_2}{s^2} \end{pmatrix}, \tag{19}$$

with $\kappa_i^2=G^2/4+g'^2_1Q_i^2$, $\kappa_{12}=h_s^2+g'^2_1Q_1Q_2-G^2/4$, $\kappa_{is}=h_s^2+g'^2_1Q_iQ_S$, and $\kappa_s^2=g'^2_1Q_S^2$.

It is simple to obtain some useful information from the structure of this matrix. The tree-level mass of the lightest scalar h_1^0 satisfies the bound

$$m_{h_1^0}^2\leq M_Z^2\cos^2 2\beta+\frac{1}{2}h_s^2v^2\sin^2 2\beta+g'^2_1\overline{Q_H^2}v^2. \tag{20}$$

The first term is the usual MSSM tree-level bound. The second contribution comes from F terms and appears also in the NMSSM [18], while the third is a D-term contribution from the $U(1)'$ and thus is a particular feature of this type of models [22,23]. In contrast to the MSSM, h_1^0 can be heavier than M_Z at tree level. In addition, radiative corrections [24] will also be sizeable. This indicates that h_1^0 can easily escape detection at LEP II. For $m_{h_1^0}$ within the kinematical reach the composition of h_1^0 will determine its production cross sections (e.g., through $Z\rightarrow Z^*h_1^0$). In particular, the h_1^0ZZ coupling, and thus the cross section, are reduced if h_1^0 has a significant singlet admixture. However, when that suppression takes place h_2^0 also tends to be light [25]. Actually, in the limit of $h_1^0\rightarrow S^{0r}$ the mass of h_2^0 satisfies the limit (20). In the event that both h_1^0 and h_2^0 have a substantial singlet component, h_3^0 will also tend to be light.

In the general case, when the masses governing the scalar mass matrix ($m_{A^0},M_Z,M_{Z'}$) have comparable magnitudes, the scalar states h_i^0 will be complicated mixtures of the interaction eigenstates. When there is some hierarchy in those masses, it is possible to make definite statements about the composition of the mass eigenstates.

$(H1)$ If $M_{Z'} \gg m_A \sim M_Z$ the heavier scalar is singlet dominated ($h_3^0 \sim S^{0r}$) with mass $m_{h_3^0} \sim M_{Z'}$. The two lighter states are mixtures of H_1^{0r} and H_2^{0r} (with some mixing angle much like in the MSSM, although with masses in a different range) with masses around $m_A \sim M_Z$. More precisely, the lightest scalar h_1^0 satisfies the (approximate) mass bound

$$m_{h_1^0}^2 \lesssim M_Z^2 \cos^2 2\beta + h_s^2 v^2 \left[\frac{1}{2}\sin^2 2\beta - \frac{h_s^2}{g'^2_1 Q_S^2} - 2\frac{\overline{Q_H}}{Q_S} \right]. \tag{21}$$

$(H2)$ When $M_{Z'} \gg m_A \gg M_Z$ the two lighter mixed states of case $(H1)$ have a definite composition: $h_2^0 \sim H_1^{0r}\sin\beta - H_2^{0r}\cos\beta$ with mass $\sim m_A$ and $h_1^0 \simeq H_1^{0r}\cos\beta + H_2^{0r}\sin\beta$ with mass saturating the bound (21). In this limit h_1^0 has standard model couplings.

$(H3)$ If $m_A \gg M_{Z'}, M_Z$ then $m_{h_1^0}^2$ goes to negative values. This means that the electroweak vacuum ceases to be a minimum and turns into a saddle point; the minimum of the potential lies at some other point in field space and the symmetry breaking is not in accord with the observed values of the gauge boson masses.

More details about the Higgs spectrum in particular scenarios will be given in the next sections.

The parameter μ_s also plays an important role in the chargino-neutralino sector. Remembering that $\mu_s = h_s s/\sqrt{2}$, the masses for the two charginos $\tilde{\chi}_{1,2}^{\pm}$ are given by the MSSM formula

$$m_{\tilde{\chi}_{1,2}^{\pm}}^2 = \frac{1}{2}\{M_2^2 + \mu_s^2 + 2M_W^2 \pm \sqrt{(M_2^2+\mu_s^2+2M_W^2)^2 - 4(M_2\mu_s - M_W^2 \sin 2\beta)^2}\}, \tag{22}$$

where M_2 is the SU(2) gaugino mass. The following bounds result from (22):

$$m_{\tilde{\chi}_1^{\pm}}^2 \leq \begin{cases} \mu_s^2 + 2M_W^2\cos^2\beta, \\ M_2^2 + 2M_W^2 \sin^2\beta, \end{cases} \tag{23}$$

and the following limiting cases hold:

$$m_{\tilde{\chi}_1^{\pm}}^2 \to \begin{cases} \mu_s^2 & (M_2^2 \gg \mu_s^2, 2M_W^2\cos^2\beta), \\ M_2^2 & (\mu_s^2 \gg M_2^2, 2M_W^2 \sin^2\beta). \end{cases} \tag{24}$$

In the first (second) case, the lightest chargino is predominantly a Higgsino (gaugino).

Preliminary LEP results, including data collected at $\sqrt{s} = 172$ GeV set a 95% C.L. lower limit on the chargino mass of about $70-85$ GeV [26]. The weaker value corresponds to light enough $\tilde{e}^{\pm}, \tilde{\nu}_e$, which can interfere destructively in the $e^+e^- \to \chi^+\chi^-$ cross section. For definiteness we impose in our analysis $m_{\tilde{\chi}_1^{\pm}} > 80$ GeV. Eqs. (23), (24) imply that this lower bound puts a significant constraint on the parameter space of the model if $h_s s$ is relatively small (roughly $h_s s < M_Z$). In general, some parameter region around $M_2\mu_s = M_W^2 \sin 2\beta$ will always be excluded (for parameter values satisfying exactly that condition, $m_{\tilde{\chi}_1^{\pm}}^2 = 0$).

In the neutralino sector, there is an extra U(1)$'$ zino and the Higgsino $\tilde{S}$ as well as the four MSSM neutralinos. The 6×6 mass matrix reads (in the basis $\{\tilde{B}', \tilde{B}, \widetilde{W_3}, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S}\}$)

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1' & 0 & 0 & g_1' Q_1 v_1 & g_1' Q_2 v_2 & g_1' Q_S s \\ 0 & M_1 & 0 & -\frac{1}{2} g_Y v_1 & \frac{1}{2} g_Y v_2 & 0 \\ 0 & 0 & M_2 & \frac{1}{2} g v_1 & -\frac{1}{2} g v_2 & 0 \\ g_1' Q_1 v_1 & -\frac{1}{2} g_Y v_1 & \frac{1}{2} g v_1 & 0 & -\mu_s & -\mu_s \frac{v_2}{s} \\ g_1' Q_2 v_2 & \frac{1}{2} g_Y v_2 & -\frac{1}{2} g v_2 & -\mu_s & 0 & -\mu_s \frac{v_1}{s} \\ g_1' Q_S s & 0 & 0 & -\mu_s \frac{v_2}{s} & -\mu_s \frac{v_1}{s} & 0 \end{pmatrix}, \tag{25}$$

where M_1 and M_1' are the gaugino masses associated with U(1) and U(1)$'$, respectively.

For general values of the parameters in this matrix, the mass eigenstates will be complicated mixtures of Higgsinos and gauginos. It is useful to consider some limiting cases.

($N1$) If $M_1' = M_1 = M_2 = \mu_s = 0$ there are two massless neutralinos. One is a pure photino ($\widetilde{\chi}_1^0 = \widetilde{\gamma} = \cos\theta_W \widetilde{B} + \sin\theta_W \widetilde{W}_3$) and the other a pure Higgsino $\widetilde{\chi}_2^0 = (\widetilde{H}_1^0 \sin\beta + \widetilde{H}_2^0 \cos\beta)\cos\alpha + \widetilde{S}\sin\alpha$ with $\tan\alpha = (v/s)\sin\beta\cos\beta$. The rest of the neutralinos will have masses controlled by M_Z and $M_{Z'}$.

($N2$) If $M_i^2, \mu_s^2 \gg M_Z^2$, two of the eigenstates are just $\widetilde{B}$ and $\widetilde{W}_3$ with masses $|M_1|$ and $|M_2|$, respectively. Next, two Higgsinos, $\widetilde{H}_1^0 \sin\beta + \widetilde{H}_2^0 \cos\beta$ and $\widetilde{H}_2^0 \sin\beta - \widetilde{H}_1^0 \cos\beta$, are nearly degenerate with mass $|\mu_s|$. The remaining two neutralinos are mixtures of $\widetilde{B}'$ and $\widetilde{S}$, and we can consider two different simple situations; first, if $M_1'^2 \gg g_1'^2 Q_S^2 s^2$, then $\widetilde{B}$ has mass $|M_1'|$ while $\widetilde{S}$ is light, with mass controlled by M_Z. In the other case, with $M_1'^2 \ll g_1'^2 Q_S^2 s^2$, they have masses $m_{\widetilde{\chi}^0}^2 = g_1'^2 Q_S^2 s^2 \pm g_1' Q_S M_1' s$.

($N3$) If $\mu_s^2 \gg M_i^2, M_Z^2$ (which naturally requires $s \gg v$, hence $M_{Z'}^2 \gg M_i^2, M_Z^2$), the approximate eigenstates are $(\widetilde{B}' \pm \widetilde{S})/\sqrt{2}$ with mass $M_{Z'}$; $\widetilde{B}$, $\widetilde{W}_3$, with masses $|M_1|, |M_2|$, respectively, and $(\widetilde{H}_1^0 \pm \widetilde{H}_2^0)/\sqrt{2}$ with mass $|\mu_s|$.

In the next sections we will give numerical examples of the pattern expected for charginos and neutralinos in different scenarios.

Masses for the rest of the MSSM particles (squarks and sleptons) can be obtained directly from the MSSM formulas by setting $\mu = \mu_s = h_s s/\sqrt{2}$ and adding the pertinent D-term diagonal contributions from the U(1)$'$ [12]:

$$\delta m_i^2 = \frac{1}{2} g'^2_1 Q_i (Q_1 v_1^2 + Q_2 v_2^2 + Q_S s^2), \qquad (26)$$

where Q_i is the U(1)$'$ charge of the corresponding particle. This extra term can produce significant mass deviations with respect to the minimal model and plays an important role in the connection between parameters at the electroweak and string scales. However, in the low-energy analysis, its effect can always be absorbed in the unknown soft supersymmetry-breaking mass squared parameters.

Before proceeding with the analysis of different scenarios it is useful to compare the present model with the simplified version discussed in Ref. [8]. That version contained one Higgs doublet and one singlet, with U(1)$'$ charges Q_H and Q_S, respectively. It was shown that a sufficiently heavy Z' (with mass up to ~ 1 TeV) with small mixing to the Z could be obtained for the case $Q_H Q_S > 0$, which would allow cancellations so that M_Z and v can be small compared to $|m_H|$, $|m_S|$ and s. The more realistic case with two Higgs doublets offers several advantages. First, there can be a cancellation in the off-diagonal Z-Z' mass matrix element (14) if $Q_1 Q_2 > 0$. In addition, the presence of a trilinear coupling in the superpotential (forbidden by SU(2) in the model of [8]) qualitatively changes the Higgs potential, allowing for a richer pattern of symmetry-breaking mechanisms. In particular, the condition $Q_H Q_S > 0$ (that in our model would be generalized to $\overline{Q_H Q_S} > 0$) is no longer necessary.

We can classify the symmetry-breaking scenarios in three different categories according to the value of the singlet VEV.

(i) $s = 0$. This corresponds to the case of the breaking driven only by the two Higgs doublets (this would be the typical case if the soft mass of the singlet remains positive). The Z' boson would acquire mass of the same order as the Z, and many other particles (Higgs bosons, charginos, and neutralinos) would tend to be dangerously light ($\mu_s = 0$ now). There is in principle the possibility of a small Z-Z' mixing due to the cancellation mechanism described and one could arrange the parameters to barely satisfy experimental constraints. However, this requires considerable fine-tuning, and we do not pursue this singular scenario further.

(ii) $s \sim v$. This case would naturally give $M_{Z'} \gtrsim M_Z$ (if $g_1' Q$ is not too small) and a small μ_s (thus some sparticles will be expected to be light). One requires $Q_1 = Q_2$ to have negligible Z-Z' mixing. Such models are allowed for leptophobic couplings [4]. Particularly interesting examples of this type of scenario will be presented in the next section.

(iii) $s \gg v$. In this case $M_{Z'} \gg M_Z$ and μ_s and m_3^2 are naturally large. The Z-Z' mixing is suppressed by the large mass $M_{Z'}$ (in addition to any accidental cancellation for particular choices of charges), eventually relaxing the constraint $Q_1 Q_2 > 0$. As $M_{Z'}$ increases more fine-tuning is needed to keep M_Z light. This case will be studied in Sec. IV.

III. LARGE TRILINEAR COUPLING SCENARIO

For the sake of simplifying the analysis, the soft supersymmetry-breaking mass parameters can be written in terms of dimensionless parameters c_i and an overall mass scale M_0:

$$m_1^2 = c_1^2 M_0^2, \quad m_2^2 = c_2^2 M_0^2, \quad m_S^2 = c_S^2 M_0^2, \quad A = c_A M_0. \qquad (27)$$

Since these are the only dimensional parameters in Eq. (3), one can conveniently parametrize the VEV's as

$$v_1 = f_1 M_0, \quad v_2 = f_2 M_0, \quad s = f_s M_0. \qquad (28)$$

We first minimize the potential (3) with respect to the dimensionless parameters f_i defined through Eqs. (27),(28) and then go to physical shell by choosing

$$M_0 = \frac{v}{\sqrt{f_1^2 + f_2^2}}, \qquad (29)$$

where $v = 246$ GeV sets the scale of electroweak symmetry breaking.

In contrast to the usual MSSM potential, V in Eq. (3) has an important trilinear term involving only the Higgs fields. Therefore, one can consider a symmetry-breaking scenario driven by this large trilinear term, as opposed to the more common situation in which the value of the minimum is determined mainly by the signs and magnitudes of the soft mass-squared parameters c_1^2, c_2^2, and c_S^2. If c_A is sufficiently large compared to the soft mass-squared parameters, a c_A-induced minimum occurs with

$$f_1 \sim f_2 \sim f_s \sim \frac{c_A}{\sqrt{2}\, h_s}, \tag{30}$$

where we have also assumed that h_s is large enough so that V_F dominates the D terms. Equation (30) corresponds to $v_1 \sim v_2 \sim s \sim 174$ GeV.

In the limit of large c_A, the relative signs and the magnitudes of the soft mass-squared parameters are not important since they contribute negligibly to the location of the minimum. However, if the values of the soft mass squared parameters are nearly the same, Eq. (30) is reached for intermediate values of c_A. In the present low-energy analysis, we assume for definiteness that $|c_1^2| \sim |c_2^2| \sim |c_S^2|$. This relation is very fine-tuned in the context of the renormalization group analysis, as discussed in Sec. V.

From Eqs. (11)–(14) it is clear that $M_{Z'}$ will generally be comparable to M_Z in the large c_A case, with the exact value depending on $g_1' Q_{1,2,S}$ (which we assume are of the same order of magnitude as G). Thus, the only phenomenologically allowed possibility is to have negligible mixing (and then only for small couplings to the ordinary leptons). From Eq. (14), we see that this occurs for $Q_1 = Q_2$, in which case $\Delta^2 \to 0$ for $f_1 \sim f_2$. Both D terms in Eq. (5) vanish in this case for large c_A. Therefore, in what follows we choose $Q_1 = Q_2$.

In the large c_A solution (30), M_0 in Eq. (29) becomes

$$M_0 = \frac{h_s v}{c_A}, \tag{31}$$

and

$$\tan\beta = \frac{f_2}{f_1} \simeq 1. \tag{32}$$

The Z' mass is simply given by

$$M_{Z'}^2 = 3 Q_1^2 g'^2_1 v^2, \tag{33}$$

and

$$A = h_s v. \tag{34}$$

Using Eqs. (30), (31) in the expressions for the Higgs masses in Eqs. (17)–(19), the limiting values for the Higgs masses are

$$m_{A^0} \simeq \sqrt{\frac{3}{2}}\, h_s v,$$

$$m_{H^\pm} \simeq \frac{1}{2}\sqrt{g_2^2 + 2h_s^2}\, v,$$

$$m_{h_1^0} \simeq \frac{h_s}{\sqrt{2}}\, v, \tag{35}$$

$$m_{h_2^0} \simeq \frac{1}{2}\sqrt{G^2 + 2h_s^2}\, v,$$

$$m_{h_3^0} \simeq \sqrt{3 g_{1'}^2 Q_1^2 + \frac{h_s^2}{2}}\, v.$$

Only $m_{h_3^0}$ depends explicitly on the U(1)$'$ charges. If a particular model allows h_s to be much smaller than the gauge couplings, A^0 and h_1^0 become light and $m_{H^\pm} \simeq M_W$, $m_{h_2^0} \simeq M_Z$.

Chargino and neutralino masses depend on the gaugino masses of the SU(2)$_L$, U(1)$_Y$, and U(1)$'$ groups, and we discuss their spectrum later in this section. In the c_A-induced minimum the effective μ parameter takes the form

$$\mu_s = \frac{h_s s}{\sqrt{2}} \simeq \frac{h_s v}{2}. \tag{36}$$

This produces a small μ parameter, $\mu_s \simeq 86$ GeV for $h_s \simeq 0.7$.

To illustrate this scenario we take[7]

$$|c_1^2| = |c_2^2| = |c_S^2| = 1, \tag{37}$$

and let c_A vary from 0 to 10. Motivated by the renormalization group analysis in Sec. V, we take $h_s = 0.7$. We also take $Q_1 = Q_2 = -1$ and $g'^2_1 = \frac{5}{3} G^2 \sin^2\theta_W$, as is suggested by simple version of gauge unification, and remark occasionally on different choices.

A. Hybrid minimum

First, consider

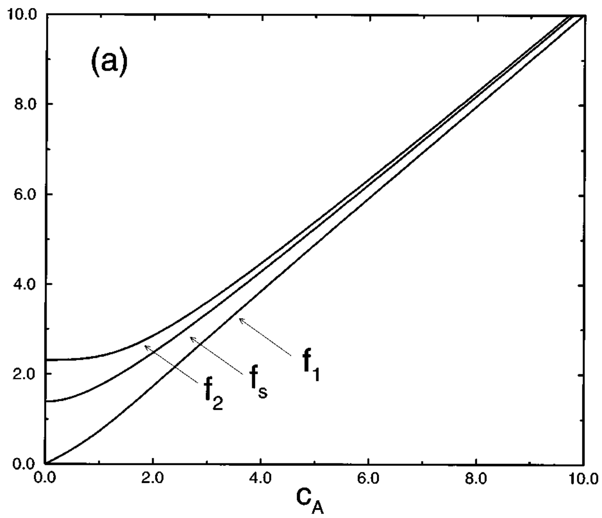
$$c_1^2 = 1\ ,\ c_2^2 = -1\ ,\ c_S^2 = -1 \tag{38}$$

with c_A varying from 0 to 10. We call this choice "hybrid," since for small c_A the minimum will be determined by these soft mass-squared parameters, and for large c_A their signs and magnitudes will be irrelevant and a minimum described by Eq. (30) will occur. Though we are ultimately interested in the large c_A minimum, we describe the properties of physical quantities in the whole c_A range.

Figure 1(a) shows the variations of the dimensionless field VEV's with c_A. For large values of c_A, the effects of the quadratic mass parameters are unimportant and Eq. (30) becomes almost exact. It is mainly because of the biasing of the soft mass-squared parameters (c_2^2 and c_S^2 are negative) that f_1, f_2, and f_s approach their large c_A character gradually.

Taking $M_Z = 91.19$ GeV the mass ratios M_{Z_1}/M_Z, M_{Z_2}/M_Z, M_0/M_Z, the Z-Z' mixing angle α, and $\tan\beta$ are shown as a function of c_A in Fig. 1(b) for the values of quadratic mass parameters in Eq. (38). We see that $M_{Z_1} \to M_Z$, $\tan\beta \to 1$, and $\alpha \to 0$ for large c_A; for example,

[7]Models which differ by a simultaneous rescaling of all of the c's are equivalent since M_0 is chosen to give the observed $v = 246$ GeV.



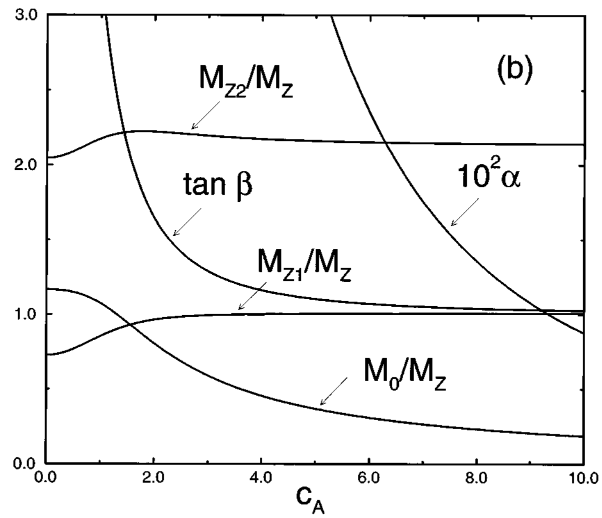


FIG. 1. (a): c_A dependence of the dimensionless field VEV's for the hybrid minimum. (b): c_A dependence of various dimensionless quantities for the hybrid minimum.

$\tan\beta = 1.03$ and $\alpha = 8.8\times 10^{-3}$ for $c_A = 10$. With our specific $\mathrm{U}(1)'$ charge assignments, $M_{Z_2}/M_Z \rightarrow 2.14$ ($M_{Z_2} \simeq 196$ GeV) for large c_A. As we observe from Fig. 1(a), the gap between f_1 and f_2 decreases rather gradually, and thus it is necessary to have larger values of c_A to obtain a smaller $Z-Z'$ mixing angle.

Figure 2 shows the variation of the scalar masses as a function of c_A for the values of soft mass-squared parameters given by Eq. (38). For large enough c_A, all masses reach their asymptotic values given by Eq. (35): $m_{H^\pm} \simeq 146$ GeV, $m_{A^0} \simeq 211$ GeV, $m^0_{h_3} \simeq 230$ GeV, $m^0_{h_2} \simeq 152$ GeV, $m^0_{h_1} \simeq 122$ GeV.

For the particular parameters in this example, the gauge symmetry is broken to $\mathrm{U}(1)_{\mathrm{EM}}$ for all values of c_A. However, for smaller $\mathrm{U}(1)'$ couplings or charges or larger values of h_s, the global minimum is $f_1 = f_s = 0$, $f_2 \neq 0$ for values of c_A smaller than some critical value, so that an additional U(1) is unbroken. This is due to the positive quartic terms in V_F [Eq. (4)], which dominate the D terms for large h_s or small charges. The symmetry is broken to the desired $\mathrm{U}(1)_{\mathrm{EM}}$ as c_A increases through this critical value, with the

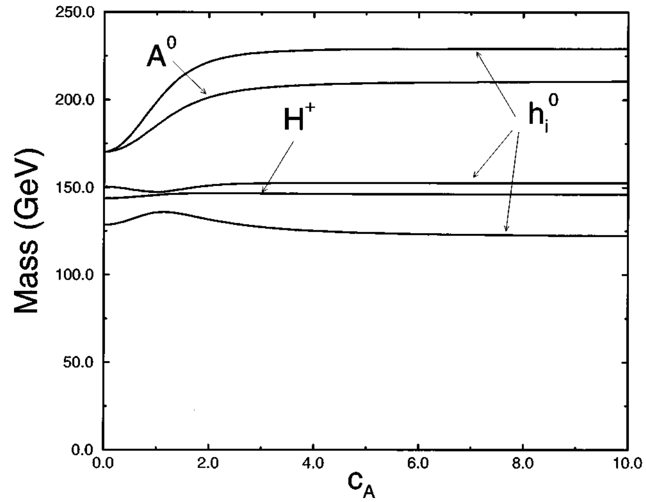


FIG. 2. c_A dependence of the Higgs boson masses for the hybrid minimum.

values of the f_i varying continuously (as in a second-order phase transition). In the large c_A limit, all quantities are controlled by Eq. (30).

B. Pure trilinear coupling minimum

For a second example, we take

$$c_1^2 = 1, \quad c_2^2 = 1, \quad c_S^2 = 1 \tag{39}$$

and vary c_A from 0 to 10. The origin is a minimum, and a deeper minimum with nonvanishing fields can only be induced by c_A.

Figure 3(a) shows the variations of the dimensionless field VEV's with c_A. For $c_A > c_A^{\mathrm{crit}} = 3$ all the fields are nonzero and identical, for our choices of the other parameters, approaching the values in Eq. (30) for large c_A.

In Fig. 3(b) we plot the dimensionless quantities M_{Z_1}/M_Z, M_{Z_2}/M_Z, M_0/M_Z, the Z-Z' mixing angle α, and $\tan\beta$ as a function of c_A for the $c_A > c_A^{\mathrm{crit}}$ portion of the total range. In this minimum $M_{Z_1} = M_Z$, $M_{Z_2} = 196$ GeV, $\alpha = 0$, $\tan\beta = 1$, and $M_0 = h_s v / c_A$. For other small positive values of the quadratic mass parameters, the minimum will again be induced by c_A, and the same values will be reached asymptotically. Fig. 4(a) shows the variation of scalar masses as a function of c_A for the soft mass-squared parameters of Eq. (39).

In Fig. 4(b) we investigate the f_1 dependence of the dimensionless potential for different values of c_A and for the mass parameters in Eq. (39). For each value of f_1, V is minimized with respect to f_2 and f_s. The straight dotted line at $V = 0$ serves as a reference to separate the two distinct minima. For all $c_A < c_A^{\mathrm{crit}}$, the global minimum is at $f_1 = f_2 = f_s = 0$. For $c_A > c_A^{\mathrm{crit}} = 3$ the minimum at $f_1 \neq 0$ becomes the true minimum and the gauge symmetry is broken. Passage of the system from one minimum to the other requires quantum tunneling through the barrier. Presumably, as the universe cooled it would have first been stuck in the local minimum, and could have eventually tunneled to the global minimum, with implications for baryogenesis [27]. As is clear from Fig. 4(b), the height of the barrier is very small

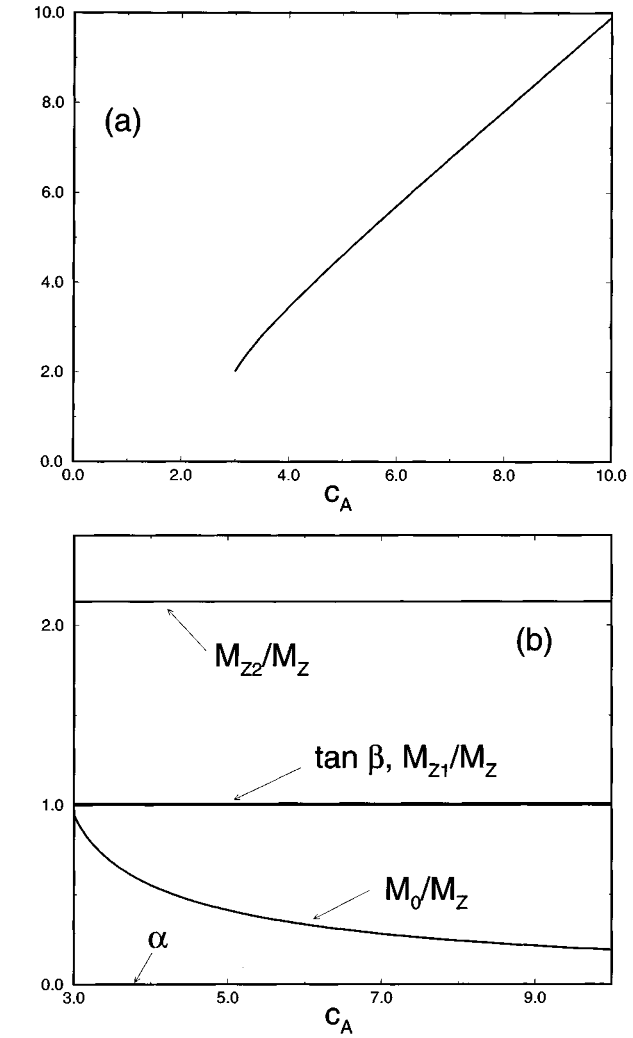


FIG. 3. (a) c_A dependence of the dimensionless VEV's and (b) various dimensionless quantities for the pure trilinear coupling minimum.

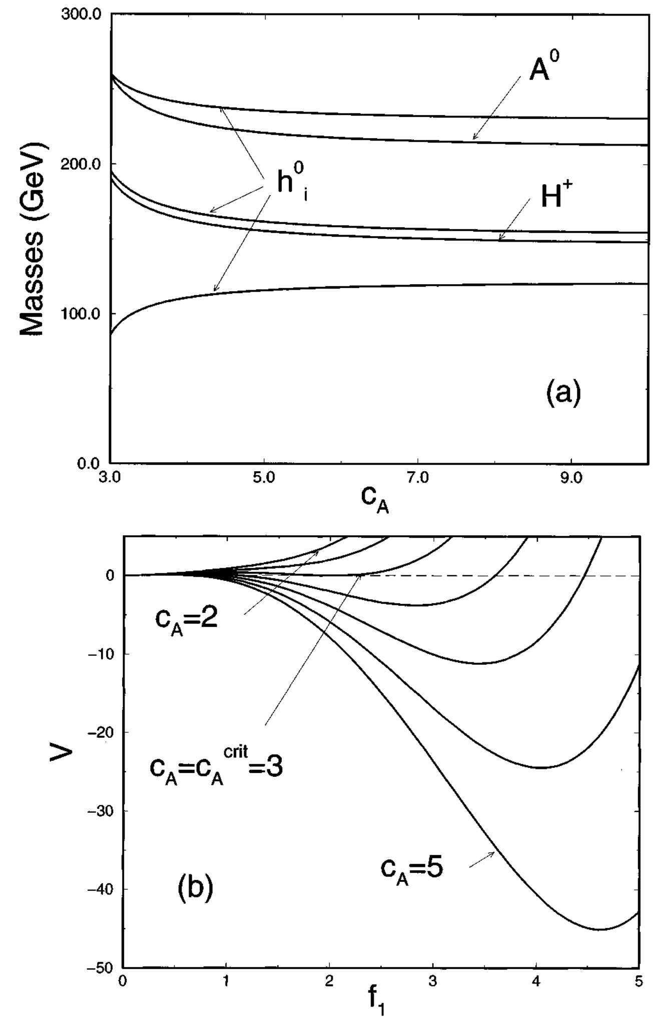


FIG. 4. (a): c_A dependence of the Higgs boson masses for the pure c_A minimum. (b): Variation of the potential with f_1 for the parameter values in Eq. (39) and different values of c_A from 2 to 5 in steps of 0.5.

compared to the depth of the minimum for the large values of c_A required to get small enough $\alpha_{Z-Z'}$. In that case there is no danger of a large supercooling and the transition can proceed without posing a cosmological problem.[8] However, a detailed discussion of the cosmological implications of this model is beyond the scope of this paper.

In summary, the negative soft mass-squared parameters in the hybrid minimum introduce a splitting among the fields for small c_A. The gap between f_1 and f_2 decreases gradually as a function of c_A, which indicates that large values of c_A are required to obtain a sufficiently small Z-Z' mixing angle. In the case of the pure trilinear coupling minimum, there is no bias from the soft mass-squared parameters and one can obtain a small mixing angle in a reasonable range of c_A values. However, in the large c_A limit the two minima have the same limiting properties solely determined by the value of the trilinear coupling.

In Fig. 5 we plot the chargino and neutralino masses as a function of the $SU(2)_L$ gaugino mass M_2 (with M_1 and M_1' as dictated by universality) in the large c_A minimum (30). Figure 5(a) shows the chargino masses together with the LEP lower bound. If $M_2 \gtrsim 1100$ GeV or $M_2 \lesssim -40$ GeV, m_{χ_1} is above the LEP bound. For $M_2 \to \infty$, m_{χ_1} approaches μ_s from below, and for $M_2 \to -\infty$, m_{χ_1} approaches μ_s from above.

In Fig. 5(b), we show the M_2 variation of the neutralino masses in the large c_A minimum. In this scenario, the neutralino mass matrix takes a simple form if $Q_1 = Q_2 \equiv Q$ and $\tan\beta = 1$. The matrix decomposes in two 3×3 matrices [in the basis $(\widetilde{B}, \widetilde{W}_3, \widetilde{H}_2^0 \sin\beta - \widetilde{H}_1^0 \cos\beta)$, $(\widetilde{B}', \widetilde{H}_1^0 \sin\beta + \widetilde{H}_2^0 \cos\beta, \widetilde{S}^0)$]. The first of them has a 2×2 submatrix identical to the chargino mass matrix. For $g_1 = 0$ the three eigenvalues are exactly equal to M_1 and $m_{\widetilde{\chi}^{\pm}_{1,2}}$. The presence of a nonzero g_1 slightly changes the picture, with the deviations largest when M_1 is close to $m_{\widetilde{\chi}^{\pm}_{1,2}}$. This behavior is shown in Fig. 5(b) where these particular three eigenvalues are singled out by solid lines. The second 3×3 matrix has one eigenvalue equal to $2\mu_s$, independent of the gaugino masses. The other two eigenvalues are

[8] We thank P.J. Steinhardt for a discussion on this point.

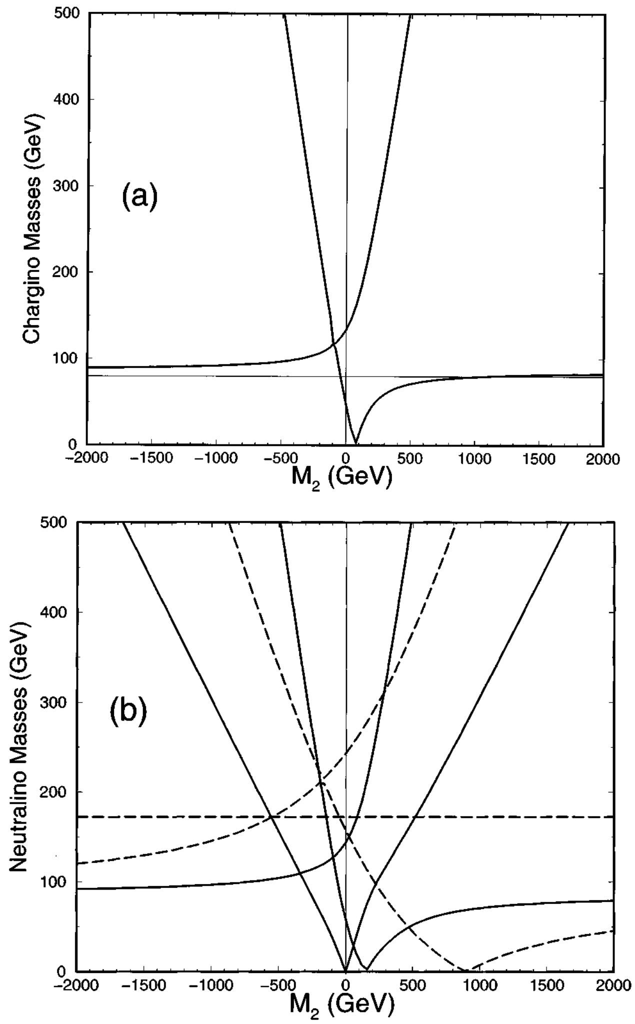


FIG. 5. Variation of the chargino masses (a) and the neutralino masses (b) with the $SU(2)_L$ gaugino mass M_2 in the large c_A minimum.

$$m_{\tilde{\chi}^0}=\frac{1}{2}[M_1'+\mu_s\pm\sqrt{(M_1'-\mu_s)^2+12g_1'^2Q^2v^2}]. \quad (40)$$

These three eigenvalues are plotted in Fig. 5(b) with dashed lines. For $M_1'\mu_s=3g_1'^2Q^2v^2$ one of the neutralino masses from Eq. (40) goes to zero. If the lightest chargino is to satisfy the LEP bound, the LSP is the χ_1^0 neutralino.

IV. LARGE S SCENARIO

Unless $g_1'Q_S$ is large, $M_{Z'}\gg M_Z$ requires $s\gg v$. In that case it is convenient to examine the $U(1)'$ breaking first, separately from $SU(2)\times$ $U(1)$ breaking, which will represent only a small correction. The breaking of the $U(1)'$ is triggered by the running of the soft mass m_S^2 towards negative values in the infrared. As a result the singlet gets a VEV [see Eq. (10)]

$$s^2\simeq\frac{-2m_S^2}{g_1'^2Q_S^2}. \quad (41)$$

That is, $M_{Z'}^2\sim-2m_S^2(\mu=S)$.

The presence of this large singlet VEV influences, already at the tree level, $SU(2)\times U(1)$ breaking, which is governed by the minimization conditions (8),(9). Let us rewrite these conditions in a form that resembles the MSSM ones:

$$-m_3^2=\frac{1}{2}\Bigg[(\widetilde{m}_1^2-\widetilde{m}_2^2)\tan2\beta+\left(M_Z^2-\frac{1}{2}h_s^2v^2\right)\sin2\beta+\frac{1}{2}g_1'^2(Q_1-Q_2)\overline{Q_H}v^2\tan2\beta\Bigg],$$

$$\mu_s^2=\frac{\widetilde{m}_2^2\sin^2\beta-\widetilde{m}_1^2\cos^2\beta}{\cos2\beta}-\frac{M_Z^2}{2}-\frac{1}{2}g_1'^2v^2\frac{Q_1^2\cos^4\beta-Q_2^2\sin^4\beta}{\cos2\beta}, \quad (42)$$

where $\widetilde{m}_i^2=m_i^2+\frac{1}{2}g_1'^2Q_iQ_Ss^2$ are the Higgs doublet soft masses corrected by the singlet VEV. The MSSM case would be recovered by setting $g_1'=h_s=0$ (but keeping μ_s fixed). The last term in Eq. (42) is negligible if there is a cancellation in the off-diagonal Z-Z' mass term (14). It is interesting to note that $\widetilde{m}_i^2+\mu_s^2$ (the effective Higgs mass terms in the potential) can be made negative by the S contribution. Then $SU(2)\times U(1)$ breaking can be triggered by the previous $U(1)'$ breaking. This is yet another alternative to the usual radiative breaking [although the breaking of the $U(1)'$ is itself radiative].

Turning back to the minimization equations (42) one would naturally expect $v^2\sim s^2$. The lightness of M_Z compared to $M_{Z'}$ results from a cancellation of different mass terms of order $M_{Z'}$. The fine-tuning involved is then roughly given by the ratio $M_{Z'}/M_Z$. It is illustrative to look at this cancellation in more detail. Consider first the case of the MSSM. By naturalness one usually assumes that soft supersymmetry-breaking mass parameters are at most of ~1 TeV. If the soft mass parameters are as heavy as that limit, then some fine-tuning is needed to get M_Z one order of magnitude lower. We will take this as the limit of admissible (low-energy) fine-tuning. As already mentioned, the μ parameter in the MSSM does not naturally satisfy that constraint. Consider next the simple model discussed in [8] with one single Higgs doublet. For large s, the cancellation to be enforced is

$$m_H^2+\frac{1}{2}g_1'^2Q_HQ_Ss^2=O(M_Z^2), \quad (43)$$

where m_H^2 is the Higgs soft mass-squared parameter. One sees that $Q_HQ_S>0$ is needed for the cancellation to occur [note that, if $m_H^2>0$, corresponding to a nonradiative breaking of $SU(2)\times U(1)$, the opposite condition $Q_HQ_S<0$ would be required]. Substituting Eq. (41) in Eq. (43) and imposing $|m_H^2|\lesssim1$ TeV^2 one arrives at the condition

$$\left(\frac{M_{Z'}}{1\ \text{TeV}}\right)^2\lesssim\frac{|Q_S|}{|Q_H|}. \quad (44)$$

From this, it follows that the only possibility of having $M_{Z'}$ significantly heavier than 1 TeV without excessive fine-tuning to keep M_Z light is to have $|Q_H| \ll |Q_S|$. The natural possibility is to have $Q_H=0$; that would correspond to a U(1)$'$ trivially decoupled from electroweak breaking.

In the case of two Higgs doublets, we can similarly require that μ_s^2, m_3^2, and $\widetilde{m}_{1,2}^2$ are at most ~ 1 TeV^2. Then we arrive at the condition

$$\left(\frac{M_{Z'}}{1\ \ \mathrm{TeV}}\right)^2 \lesssim \min\left\{\frac{|Q_S|}{|Q_1|}, \frac{|Q_S|}{|Q_2|}, \frac{g'^2_1 Q_S^2}{h_s^2}\right\}, \qquad (45)$$

and also

$$\left(\frac{A}{1\ \ \mathrm{TeV}}\right)\left(\frac{M_{Z'}}{1\ \ \mathrm{TeV}}\right) \lesssim \frac{g_1'|Q_S|}{h_s}. \qquad (46)$$

Consider first the case of h_s^2 small compared to $g'^2_1 Q_S^2$. This means μ_s is small compared to $M_{Z'}$ so that no restriction comes from the μ_s condition in Eq. (45). In this case,

$$\left(\frac{M_{Z'}}{1\ \ \mathrm{TeV}}\right)^2 \lesssim \min\left\{\frac{|Q_1+Q_2|}{|Q_1|}, \frac{|Q_1+Q_2|}{|Q_2|}\right\} \equiv m \leqslant 2. \quad (47)$$

There is a maximum value of m ($m=2$, reached for $Q_1=Q_2$) and it is not possible to decouple the Z' from electroweak breaking by a large hierarchy between the charges because of the constraint $Q_1+Q_2+Q_S=0$. If h_s^2 is larger than $g'^2_1 Q_S^2$ (that is, $\mu_s^2 \gg M_{Z'}^2$), then the minimum in Eq. (45) goes to zero, which indicates that $M_{Z'} \ll \mu_s \sim 1$ TeV to avoid a large fine-tuning. We conclude that, to have $M_{z'} \gg 1$ TeV requires excessive fine-tuning in both cases. From Eq. (46) we also find a natural upper limit to impose on the A parameter:

$$h_s A \lesssim g_1'|Q_S| O(1\ \ \mathrm{TeV}). \qquad (48)$$

In addition, the Z-Z' mixing should be small enough. For moderate values of $M_{Z'}$ (say 500 GeV), small Z-Z' mixing requires a small off-diagonal element in the Z,Z' mass matrix. In fact, this matrix element vanishes for some value of $\tan\beta$ if $Q_1 Q_2 > 0$. More precisely, $\theta_{Z-Z'} \leqslant \delta\theta$ if $\tan\beta$ is in the interval

$$\tan\beta \simeq \sqrt{Q_1/Q_2}\left[1 \pm \delta\theta \frac{G(Q_1+Q_2)}{4 g_1' Q_1 Q_2}\frac{M_{Z'}^2}{M_Z^2}\right] \quad (Q_1 Q_2 > 0) \qquad (49)$$

(with $\theta_{Z-Z'}=0$ for the central value). This quantifies the fine-tuning required in $\tan\beta$. This effect reduces the fraction of acceptable parameter space for low values of $M_{Z'}$. The reduction is less important for a Z' closer to the upper natural limit of 1 TeV, where a good cancellation in the off-diagonal Z-Z' mass term is not required and eventually the condition $Q_1 Q_2 > 0$ can be relaxed.

The pattern of the spectrum of physical Higgs bosons in the large s case is particularly simple. As discussed in Sec. II, one neutral scalar h_1^0 remains below the bound (20) and approaches the value (21). The pseudoscalar A^0 mass, $m_{A^0}^2 \simeq \sqrt{2} A h_s s/\sin 2\beta$ is naturally expected to be large (unless $A h_s$ is very small) and in that case, one of the neutral scalars and the charged Higgs boson are approximately degenerate

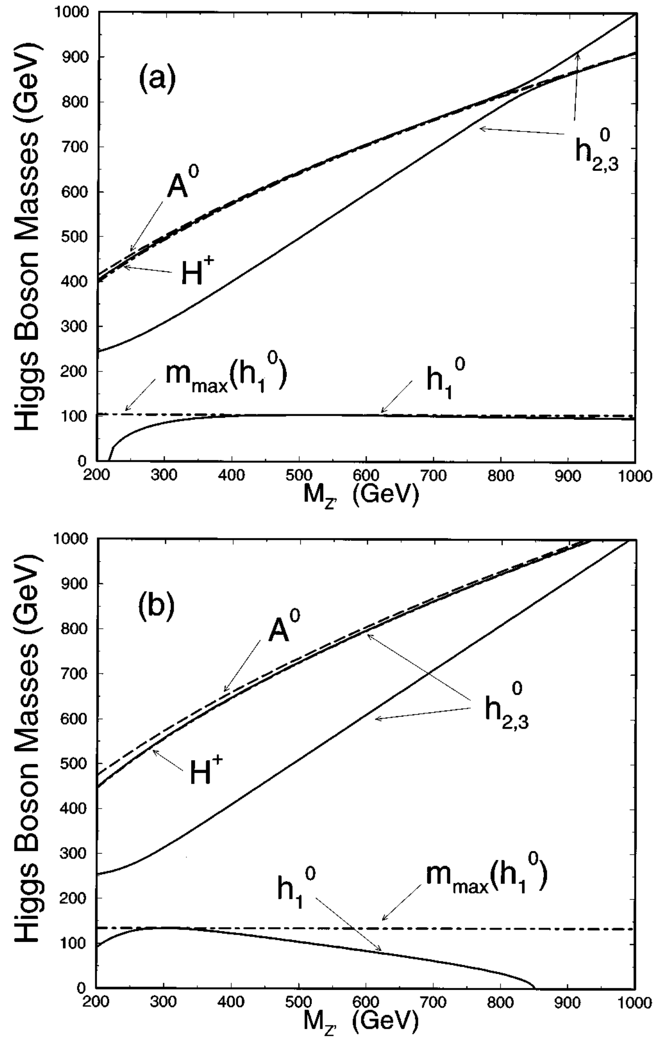


FIG. 6. Variation of the Higgs spectrum with $M_{Z'}$ for $A=500$ GeV, $Q_1=Q_2=-1/2$, and (a) $h_s=0.5$ and $\tan\beta=1.5$; (b) $h_s=0.7$ and $\tan\beta=1$. There are three scalars (solid lines) one pseudoscalar (dashed line) and one charged pair (dash-dotted). The horizontal dash-dotted line gives the bound (20).

with A^0, completing a full SU(2) doublet (H^0,A^0,$H^{\pm}$) not involved in SU(2) breaking. The lightest neutral scalar is basically the (real part of the) neutral component of the Higgs doublet which is involved in the SU(2) breaking and has then a very small singlet component. The third neutral scalar has mass controlled by $M_{Z'}$ and is basically the singlet. This mass pattern can be clearly seen in Fig. 6 for different choices of couplings and U(1)$'$ charges.

The mass of the lightest Higgs boson is of particular interest. The limiting value (21) for $m_{h_1^0}$ can be bigger or smaller than the MSSM upper bound $M_Z^2\cos^2 2\beta$ depending on couplings and charge assignments. Note that the D-term contribution $g'^2_1\overline{Q_H^2}v^2$ in Eq. (20) is exactly compensated after integrating out S and disappears in this decoupling limit. However, this exact cancellation does not take place for the F-term contributions. The behavior of $m_{h_1^0}$ as a function of $M_{Z'}$ is shown in Fig. 7(a) for two different cases. Horizontal dash-dotted lines give the upper bound [Eq. (20)], the MSSM bound $M_Z|\cos 2\beta|$ (which is zero in the figure),

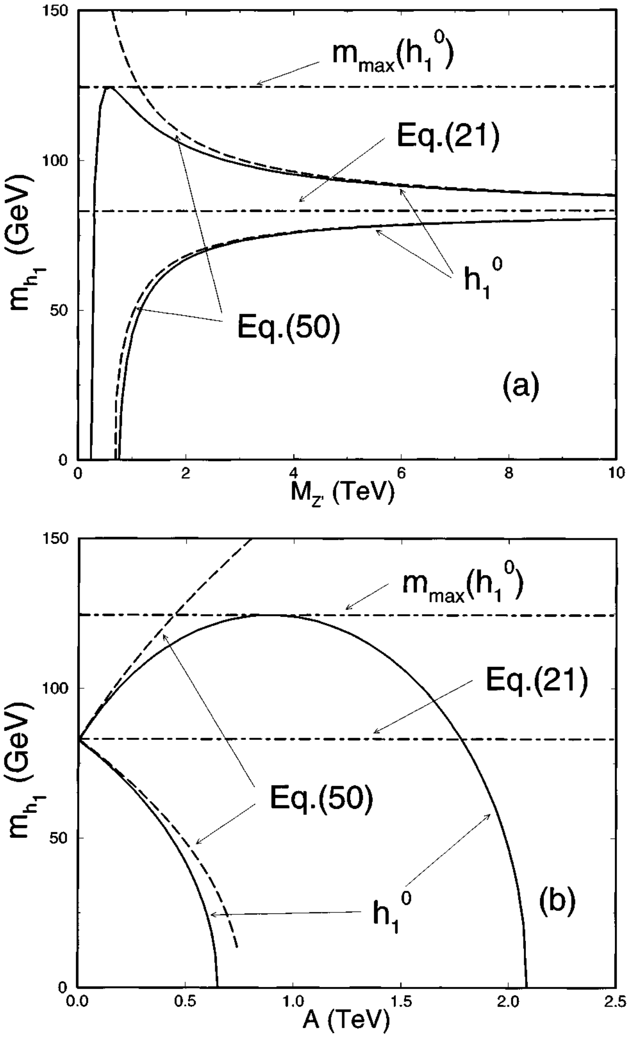


FIG. 7. Mass of the lightest Higgs scalar h_1^0 (solid line): (a) as a function of $M_{Z'}$ showing the decoupling limit $s \gg v$ for two different cases with $A=500$ GeV, $\tan\beta=1$. The upper curve has $Q_1=Q_2=-3/5$, $h_s \simeq 0.6$ and the lower $Q_1=Q_2=-1$ and $h_s \simeq 0.3$; (b) as a function of A for $M_{Z'}=1$ TeV in the two same cases. Dashed and dash-dotted lines give different mass bounds and limits as discussed in the text.

and the asymptotic value Eq. (21) [to make the figure simpler the parameters have been chosen such that Eqs. (20) and (21) are the same in both cases]. Figure 7(a) shows an example for which the asymptotic value is bigger than the MSSM upper bound. This value is approached slowly. After including subdominant terms $O(m_A^2/M_{Z'}^2)$ in Eq. (21), one obtains

$$m_{h_1^0}^2 \to M_Z^2 \cos^2 2\beta + h_s^2 v^2 \left[\frac{1}{2}\sin^2 2\beta - \frac{h_s^2}{g'^2_1 Q_S^2} - 2\frac{\overline{Q_H}}{Q_S} \right] + \sqrt{2} h_s \frac{A}{g'^2_1 Q_S^2 s} (h_s^2 + g'^2_1 Q_S \overline{Q_H}) v^2 \sin 2\beta. \tag{50}$$

This approximation is represented by dashed lines in Fig. 7(a) and gives $m_{h_1^0}$ rather precisely for large $M_{Z'}$. The sign of $K = h_s^2 + g'^2_1 Q_S \overline{Q_H}$ determines whether the asymptotic value is reached from below ($K<0$) or above ($K>0$).

In Fig. 7(b), we show the dependence of $m_{h_1^0}$ on A for fixed $M_{Z'}$ in the same two cases of Fig. 7(a). For small A, we are in case $H1$ of Sec. II and the inequality (21) holds (actually, it is saturated for the parameters chosen). The approximation (50) works well in that region. For larger A, $m_{h_1^0}$ increases or decreases depending on the sign of K. In both cases, when A grows beyond $m_A \sim M_{Z'}$, $m_{h_1^0}^2$ drops to negative values [case $H3$ in Sec. II]. The minimum of the potential does not give a correct electroweak breaking; for sufficiently large A the pattern of VEV's is similar to the one encountered in the large c_A case of the previous section, but the gauge boson masses would be much larger than the observed values. This behavior differs from the MSSM where $m_{h_1^0}$ always increases with larger m_A until the upper bound is saturated.

For some values of the parameters the large s asymptotic value for $m_{h_1^0}^2$, as computed from Eq. (21), is negative. An example of this case is shown in Fig. 6(b). In such cases there is an upper bound on $M_{Z'}$ beyond which the vacuum would be destabilized.

Next we show typical examples of the neutralino-chargino spectra. In Figs. 8(a) and 9(a) we fix $M_2=500$ GeV (assuming that M_1 and M_1' have values as dictated by universality) and show the dependence on the mass of the Z' boson of the masses in the neutralino-chargino sector. Figures 8(b) and 9(b) instead show the variation of the masses with M_2 for a fixed value of $M_{Z'}=500$ GeV. In Fig. 8(a), we clearly see how the chargino masses are controlled by M_2 (fixed) and μ_s (growing linearly with $M_{Z'}$). For low $M_{Z'}$, meaning $\mu_s < M_2$, the lighter chargino mass follows μ_s and the heavier mass is nearly constant and equal to M_2. This role is interchanged after crossing the $\mu_s \sim M_2$ region. The same behavior is manifest in Fig. 8(b), where μ_s is kept constant and M_2 varies.

In Figs. 9(a) and 9(b) we plot the spectrum of neutralinos for the same two cases. In Fig. 9(a), for large $M_{Z'}$ we have $M_i^2, \mu_s^2 \gg M_Z^2$ and the masses follow the pattern described in the discussion [case ($N2$)] after Eq. (25): the two lower (solid) curves asymptotically flattening approach $|M_1|$ and $|M_2|$ and correspond to $\tilde{B}$ and $\tilde{W}_3$, respectively. Then there are two (dashed) curves for the doublet Higgsinos tending to $|\mu_s|$ and finally two (dash-dotted) curves for two $\tilde{B}'$-$\tilde{S}$ mixed states with masses $M_{Z'} \pm M_1'/2$. Also note that two neutralino states follow closely the chargino pattern of Fig. 8(a).

Concerning the nature of the LSP, the lightest neutralino is the natural candidate in these models. In particular, we see that the LSP is mostly $\tilde{B}$. For large gaugino masses however, if $M'^2_1 \gg M_{Z'}^2$, the lightest neutralino is the singlino $\tilde{S}$ whose mass is then of the order of M_Z. This possibility is realized in the case shown in Fig. 9(b).

V. RENORMALIZATION GROUP ANALYSIS

We now turn to the renormalization group analysis of the model presented in Sec. II to determine what boundary conditions at the string scale are required to reach the desired low-energy parameter space as described in Secs. III and IV.

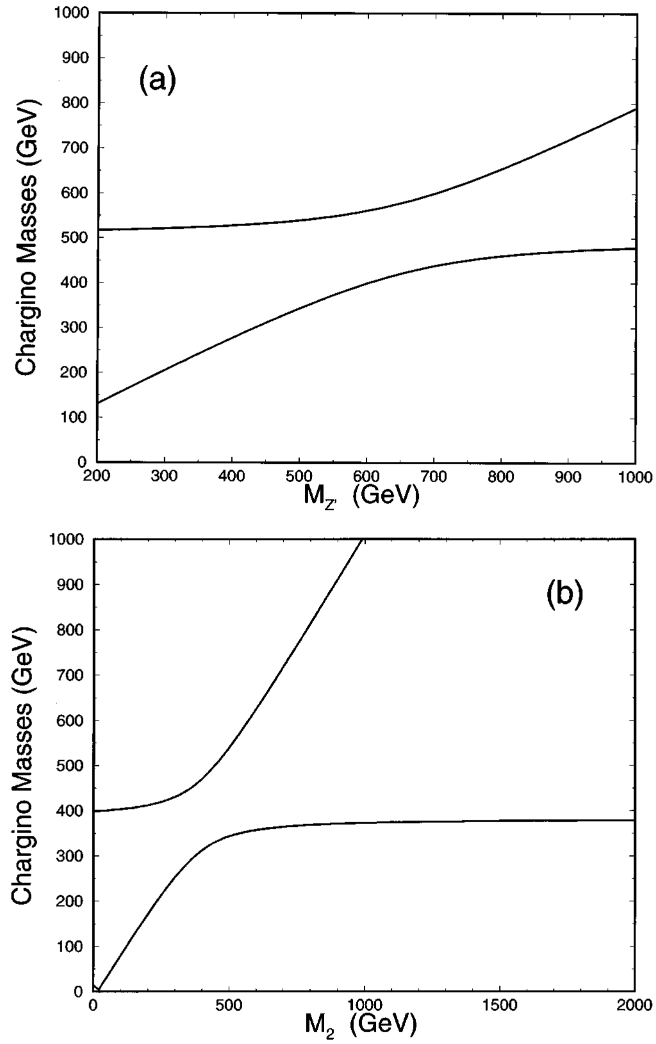


FIG. 8. Chargino masses (a) as a function of $M_{Z'}$ for $M_2 = 500$ GeV; (b) as a function of M_2 for $M_{Z'} = 500$ GeV. (We fix M_1 and M_1' by universality, $Q_1 = Q_2 = -1/2$, $h_s = 0.5$, and $\tan\beta = 1.5$.)

As our model is motivated from string theory, we normalize the gauge couplings so that at the string scale

$$g_3^0 = g_2^0 = g_1^0 = g_1'^0 = g_0. \tag{51}$$

In (weakly coupled) heterotic string theory this relation among the couplings is valid for the level one Kač-Moody models.[9] This is approximately consistent with the observed gauge coupling unification, which occurs at $M_G \simeq 3\times 10^{16}$ GeV, one order of magnitude below $M_{\rm string} \simeq 5\times 10^{17}$ GeV; this difference introduces a numerically small inconsistency in our analysis.

String models based on fermionic $(Z_2\times Z_2)$ orbifold constructions [13]–[16] at a special point in moduli space possess the feature that the couplings of the trilinear terms in the superpotential are equal for the fields whose string vertex operators do not involve additional (real) world-sheet fermion fields [with conformal dimension (1/2,1/2)].[10] In this case, the trilinear coupling is $h_i^0 = g_0\sqrt{2}$. For a majority of models all of the observable fields are of that type. However, for fields whose string vertex operators involve one such world-sheet fermion field the trilinear coupling is $h^0 = g_0$. Since in the vertex operator one can add at most *two* such world-sheet fermions [they now saturate (1,1) conformal dimension of the vertex operator], the trilinear coupling with one such field is $h^0 = g_0/\sqrt{2}$ (which is then the smallest possible nonzero value of the Yukawa coupling). In the latter case, however, such fields usually correspond to exotics.

[9]For the Kač-Moody level $k\neq 1$ the relationship among the coupling constants is altered by adding appropriate factors of $\sqrt{k}$ in the equation.

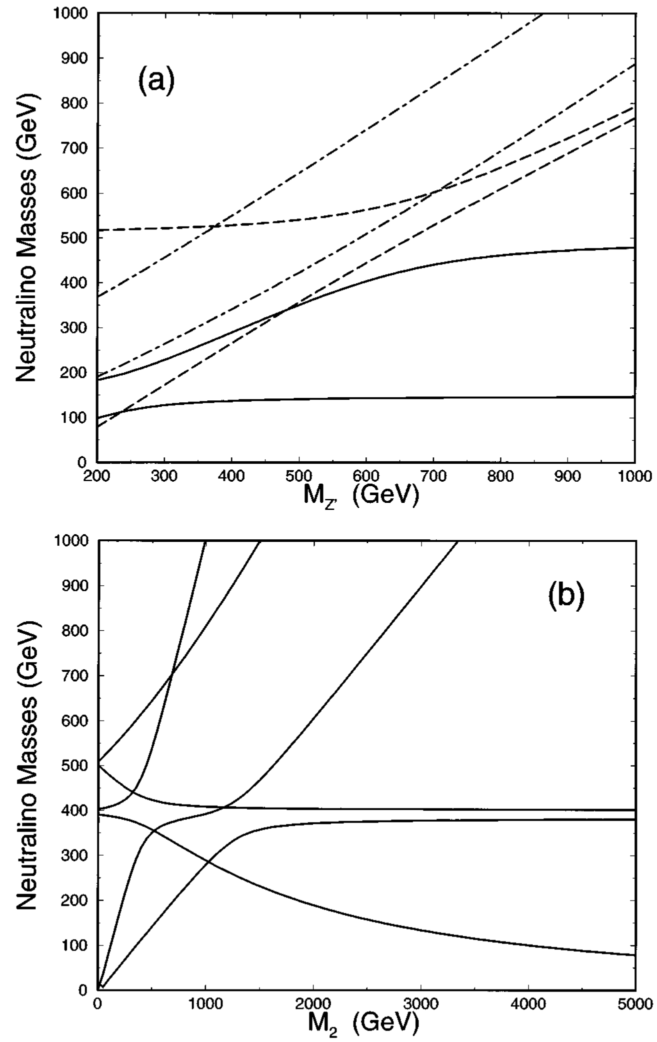


FIG. 9. Neutralino masses (a) as a function of $M_{Z'}$ for $M_2 = 500$ GeV; (b) as a function of M_2 for $M_{Z'} = 500$ GeV. (We fix M_1 and M_1' by universality, $Q_1 = Q_2 = -1/2$, $h_s = 0.5$, and $\tan\beta = 1.5$)

Thus, for the sake of simplicity we assume that the boundary conditions for the Yukawa couplings are given by

$$h_Q^0 = h_S^0 = g_0\sqrt{2}, \tag{52}$$

where g_0 is defined in Eq. (51). Using the RGE's of the MSSM (i.e., in the absence of trilinear couplings of h_2 to exotics), this value of the Yukawa coupling h_Q^0 determines

[10]We thank G. Cleaver for a discussion on this point.

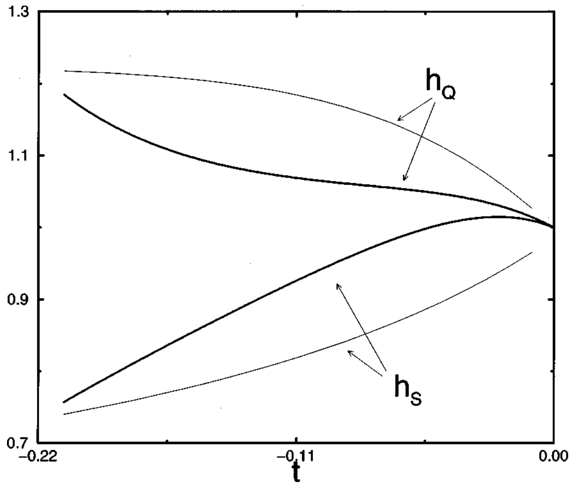


FIG. 10. Scale dependence of the Yukawa couplings (bold curves are for exact solutions). $t=(1/16\pi^2)\ln\mu/M_{\rm String}$, such that $t\sim-0.2$ at the electroweak scale.

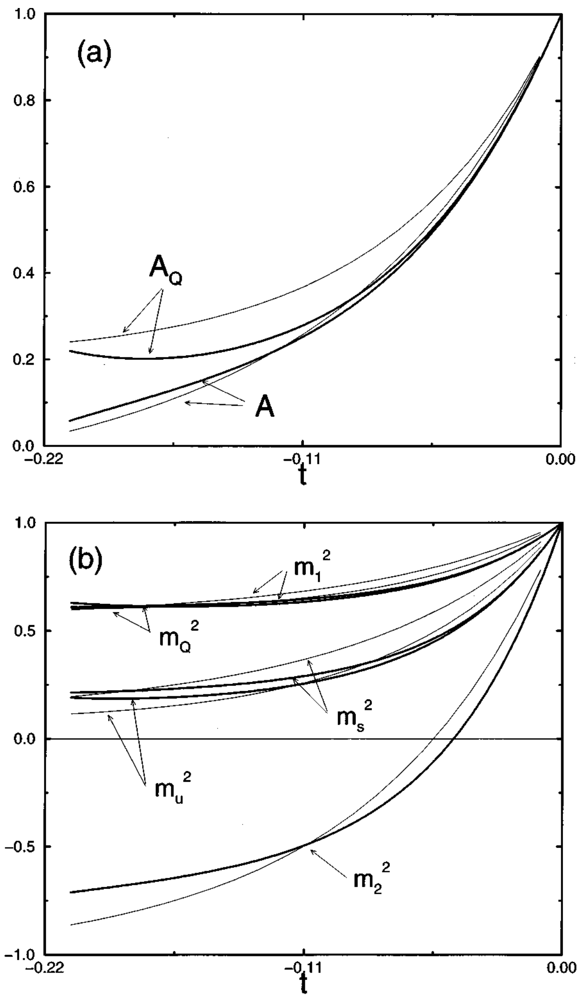


FIG. 11. Scale dependence of the dimensionless trilinear coupling parameters (a) and soft mass-squared parameters (b). Bold curves are for exact solutions.

the value of h_Q at M_Z. When combined with the VEV of H_2, which ensures the correct electroweak symmetry-breaking vacuum, this result yields a prediction for the top quark mass in the range of $\sim 170-200$ GeV [19].

We first consider universal boundary conditions for the soft supersymmetry-breaking mass parameters at the string scale.

Universal scalar soft mass-squared parameters:

$$m_1^{0\,2}=m_2^{0\,2}=m_S^{0\,2}=m_U^{0\,2}=m_Q^{0\,2}=M_0^2. \tag{53}$$

Universal gaugino masses:

$$M_3^0=M_2^0=M_1^0=M_1'^0=M_{1/2}=C_{1/2}M_0. \tag{54}$$

Universal trilinear couplings:

$$A^0=A_Q^0=C_0M_0. \tag{55}$$

As a second step, we will allow for nonuniversal initial conditions for the trilinear couplings and the soft mass-squared parameters, such that in general

$$A_i^0=c_{A_i}^0M_0, \tag{56}$$

$$m_i^{0\,2}=c_i^{0\,2}M_0^2. \tag{57}$$

The one-loop RGE's for the parameters are presented in Appendix A. We assume a minimal particle content, consistent with the superpotential (1). The renormalization group analysis of the model depends on the choice of U(1)$'$ charges of the theory, that enter the RGE's for the U(1)$'$ gauge coupling and gaugino. In general, the U(1) factors have a small effect in the RGE's of the other parameters due to the small magnitudes of the U(1) gauge couplings and gaugino masses. The U(1) factors are neglected in the running of the parameters in the semianalytic approach, which is often a good approximation. In the numerical analysis, we choose for definiteness the U(1)$'$ charges $Q_1=Q_2=-1$, $Q_L=Q_Q=-1/2$, and most of the U(1) factors are retained.[11]

We have solved the RGE's numerically, and investigated the evolution of the parameters for a wide range of boundary conditions. With a specific choice of the boundary conditions of the Yukawa couplings, we have obtained the numerical solutions for the parameters at the electroweak scale as a function of the initial values of the trilinear couplings and soft mass-squared parameters. The results are qualitatively the same with other choices of initial values of the Yukawa couplings motivated by string theory; thus for definiteness we consider only the case with initial Yukawa couplings given by Eq. (52). To further our understanding of the evolution of these parameters, we have also derived semianalytic solutions of the RGE's. The numerical and semianalytic solutions are presented and discussed in detail in Appendix B, and shown in some representative graphs. With the numerical results (B7)–(B13), we are able to investigate systematically the effect of the choice of boundary conditions on the evolution of the trilinear couplings and the soft mass-squared parameters.

[11]The factors S_1 and S_1' defined in Appendix A are not included in the numerical analysis of the RGE's, as discussed in Appendix C.

TABLE I. Large c_A minimum: $M_{Z_2}=196$ GeV, $\alpha_{Z-Z'}=0.0$, $\tan\beta=1.0$, $m_h=126$ GeV, and $Q_1=Q_2=-1$. We present the values of (M_3, M_2) at the electroweak scale. The gluino mass is $|M_3|$.

	(a)		(b)		(c)	
	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$
m_1^2 (GeV)^2	$(40.8)^2$	$(740)^2$	$(40.8)^2$	$(518)^2$	$(40.8)^2$	$(713)^2$
m_2^2 (GeV)^2	$(40.8)^2$	$(2690)^2$	$(40.8)^2$	$(892)^2$	$(40.8)^2$	$(1180)^2$
m_S^2 (GeV)^2	$(40.8)^2$	$(1090)^2$	$(40.8)^2$	$(736)^2$	$(40.8)^2$	$(1020)^2$
m_U^2 (GeV)^2	$(290)^2$	$(1840)^2$	$(150)^2$	$(549)^2$	$(180)^2$	$(581)^2$
m_Q^2 (GeV)^2	$(380)^2$	$(1110)^2$	$(250)^2$	$(409)^2$	$(300)^2$	$(320)^2$
A (GeV)	204	-4230	204	1290	204	2670
A_Q (GeV)	405	-6740	-175.0	803	-275	2680
$M_{1/2}$ (GeV)	(1160, 328)	400	$(-289, -82)$	-100	$(-578, -164)$	-200

First, we consider the case of universal boundary conditions, as stated in Eqs. (53)–(55), assuming that the only contributions to the RGE's are from the MSSM supermultiplets, $\hat{S}$, and Z' vector multiplet. An example of universal boundary conditions is presented in Figs. 10 and 11, which show the scale dependence of the Yukawa couplings, the dimensionless trilinear couplings, and the dimensionless soft mass-squared parameters, for $C_0=1.0$ and $C_{1/2}=0.1$. The dimensionless quantities are related to the physical parameters by rescaling with M_0, which is defined in Eq. (29).

These graphs illustrate the general features of universal initial boundary conditions: $h_Q(M_Z)$ is larger than $h_s(M_Z)$, $A_Q(M_Z)$ is larger than $A(M_Z)$ for $C_{1/2}\gtrsim 0.019\ C_0$, and $m_2^2(M_Z)$ is negative while the other mass-squared parameters are positive at the electroweak scale. This behavior can be seen from the solutions (B7)–(B13), and the semianalytic solutions discussed in Appendix B. These solutions also demonstrate that the initial value of the gaugino mass parameter $M_{1/2}$ directly controls the splitting of the low-energy values of the trilinear couplings and the mass-squared parameters.

These results indicate that the values of the low-energy parameters obtained with universal boundary conditions at the string scale (and assuming no exotic supermultiplets) do not lie within the phenomenologically acceptable region of parameter space. The large trilinear coupling scenario of Sec. III requires $c_A \gg c_1^2 \sim c_2^2 \sim c_S^2$ at the electroweak scale, which clearly does not follow from Fig. 11. The scenario of Sec. IV also does not result from universal initial conditions; Fig. 11(b) demonstrates that while $m_2^2(M_Z)$ is negative, $m_S^2(M_Z)$ is positive, so the singlet does not develop the large VEV necessary for this minimum.

Therefore, we must relax our assumptions of universality and/or of no exotics to reach the desired low-energy parameter space. We first consider the possibility of nonuniversal (but of the same order of magnitude) trilinear couplings and soft mass-squared parameters at the string scale. In most cases, we must choose $M_{1/2}$ small compared to other soft masses at the string scale. The value of $M_{1/2}$ must also be chosen to satisfy the phenomenological bounds on the chargino masses and the gluino masses at the electroweak scale. The boundary conditions are chosen to avoid a dangerous color-breaking minimum [28], which could result from negative squark mass-squares or large values of A_Q at the electroweak scale.[12] Negative squark mass-squares (including both the supersymmetric and soft-breaking contributions) are always unacceptable, because they imply that the standardlike minimum is an unstable saddle point. We present several illustrative examples of nonuniversal boundary conditions, the resulting low-energy parameters, and the relevant physical quantities in Tables I–VI. For economy of presentation, we display the low-energy values of the SU(3) and SU(2) gaugino masses (M_3, M_2) explicitly in the last line of each table, and do not present the values of M_1 and M_1', which follow from the assumption of universal gaugino masses [see Eqs. (B5) and (54)].

In Table I, we present a set of examples of boundary conditions that lead to the special case of the large trilinear coupling scenario in which $c_1^2(M_Z)=c_2^2(M_Z)=c_S^2(M_Z)=1.0$, and $c_A(M_Z)=5.0$. This special case, chosen for definiteness to address the effect of the large value of c_A, has the Z-Z' mixing angle identically zero, as discussed in Sec. III. In each case, the initial values of the gaugino mass and the trilinear couplings must be chosen such that A takes a large value compared to the soft mass-squared parameters at the electroweak scale. This can be obtained either by choosing A_Q^0 negative, choosing A^0 much larger than A_Q^0, or taking $M_{1/2}$ negative. The initial values of the soft mass-squared parameters also must be chosen carefully so that $m_1^2=m_2^2=m_S^2$ at the electroweak scale, which clearly is very fine-tuned. In each example, the initial values of the parameters are much larger than the low-energy values of the soft mass-squared parameters of the singlet and the Higgs bosons.

In Table II, we present examples of the more general case of the large c_A minimum in which the magnitudes of the soft supersymmetry-breaking mass-squared parameters are not exactly equal at the electroweak scale. The first example II(a) has the values $c_A(M_Z)=5.0$, $c_1^2(M_Z)=1.1$, $c_2^2(M_Z)=0.9$, and $c_S^2(M_Z)=1.0$; these small deviations in the low-energy

[12] Moderate trilinear terms involving squarks may be allowable because the charge-color breaking minimum may be only local or, if global, may be separated from the standardlike minimum by a large barrier. Whether a color- and charge-breaking global minimum is allowed depends on the cosmological history and on the tunneling rate from the standardlike minimum [21].

TABLE II. Large c_A minimum: (a) $M_{Z_2}=196$ GeV, $\alpha_{Z-Z'}=7.8\times 10^{-3}$, $\tan\beta=1.02$, $m_h=125$ GeV; (b) $M_{Z_2}=196$ GeV, $\alpha_{Z-Z'}=1.9\times 10^{-3}$, $\tan\beta=1.01$, $m_h=130$ GeV; (c) $M_{Z_2}=197$ GeV, $\alpha_{Z-Z'}=9.3\times 10^{-3}$, $\tan\beta=1.03$, $m_h=131$ GeV. In all cases $Q_1=Q_2=-1$.

	(a)		(b)		(c)	
	M_Z	M_{string}	M_Z	M_{string}	M_Z	M_{string}
m_1^2 (GeV)^2	$(42.8)^2$	$(562)^2$	$(19.9)^2$	$(684)^2$	$(23.4)^2$	$(686)^2$
m_2^2 (GeV)^2	$(38.7)^2$	$(1110)^2$	$(18.0)^2$	$(1240)^2$	$(13.5)^2$	$(1250)^2$
m_S^2 (GeV)^2	$(40.8)^2$	$(802)^2$	$(19.0)^2$	$(979)^2$	$(19.1)^2$	$(982)^2$
m_U^2 (GeV)^2	$(201)^2$	$(704)^2$	$(245)^2$	$(701)^2$	$(245)^2$	$(703)^2$
m_Q^2 (GeV)^2	$(400)^2$	$(556)^2$	$(380)^2$	$(469)^2$	$(380)^2$	$(471)^2$
A (GeV)	204	1180	190	2330	191	2330
A_Q (GeV)	-270	689	-290	2290	-290	2290
$M_{1/2}$ (GeV)	$(-434, -123)$	-150	$(-578, -164)$	-200	$(-578, -164)$	-200

values of the soft mass-squared parameters yield a mixing angle around 10^{-2}, which may be barely allowable for $M_{Z'}\sim 200$ GeV. Smaller mixing may be obtained for larger values of c_A, such as $c_A(M_Z)=10.0$, as shown in II(b), with the values of the dimensionless low-energy soft mass-squared parameters as above. Example II(c) also has $c_A(M_Z)=10.0$, but $c_1^2(M_Z)=1.5$, $c_2^2(M_Z)=0.5$, and $c_S^2(M_Z)=1.0$, and the mixing angle is again of $\sim 10^{-2}$. These examples are presented to emphasize the increase in the hierarchy between the values of the parameters at the string scale and the low-energy values with the increasing value of c_A. The comparison of examples II(b) and II(c) demonstrates the fine-tuning required at the string scale (as well as at the electroweak scale) for this scenario. The values of the soft mass-squared parameters at the string scale are very similar, yet the resulting low-energy parameters yield quite different values for the mixing angle.

Table III shows examples that yield the hybrid minimum of the large c_A scenario discussed in Sec. III, for $c_A(M_Z)=5.0$, $c_A(M_Z)=8.0$, and $c_A(M_Z)=10.0$ [cases (a), (b), and (c), respectively]. Large values of $c_A(M_Z)$ are needed to obtain a small enough mixing angle when the low-energy soft mass-squared parameters differ in magnitude or sign. This in turn causes the values of the parameters at the string scale to be much larger in magnitude than those at the electroweak scale, similar to the results presented in Table II.

In Table IV, we present examples of boundary conditions that lead to the case (large s scenario) described in Sec. IV. The initial values of the parameters are chosen to lead to the negative value of m_S^2 at the electroweak scale required for this scenario. In addition, we choose values of the squark soft mass-squared parameters such that the masses of the squarks will not be made negative when adding the large U(1)$'$ D-term contribution (26). In this case, $M_{Z_2}=1$ TeV, $\tan\beta=1$, and the Z-Z' mixing angle is zero; the last two results are due to our assumption that $c_1^2(M_Z)=c_2^2(M_Z)$, which requires fine-tuned boundary conditions. In addition, m_S^2 is negative at low energies, while the other soft mass-squared parameters are positive. This requires taking the initial values of the parameters very large relative to the low-energy values, and choosing $m_2^{0\,2}$ large compared to the initial values of the other soft mass-squared parameters. In this minimum, the chargino mass constraint is satisfied as long as $|M_{1/2}|$ is chosen large enough.

Table V presents more typical examples of boundary conditions which lead to the large s minimum with $M_{Z_2}=1$ TeV, $\tan\beta=2$, and a nonzero mixing angle. In each example, the initial values of the mass parameters are larger (by a factor 5–10) than the low-energy values. In comparison with the results of Table IV, in most cases the magnitude of $m_2^{0\,2}$ need not be taken as large relative to the other soft mass-squared parameters, because in this case m_2^2 is allowed to be negative at the electroweak scale.

In Table VI, we present examples which lead to a case of the large s minimum with a lighter Z' mass ($\sim$700 GeV), a

TABLE III. Hybrid minimum: (a) $M_{Z_2}=200$ GeV, $\alpha_{Z-Z'}=3.4\times 10^{-2}$, $\tan\beta=1.11$, $m_h=135$ GeV; (b) $M_{Z_2}=198$ GeV, $\alpha_{Z-Z'}=1.4\times 10^{-2}$, $\tan\beta=1.04$, $m_h=134$ GeV; (c) $M_{Z_2}=197$ GeV, $\alpha_{Z-Z'}=9.3\times 10^{-3}$, $\tan\beta=1.03$, $m_h=133$ GeV. In all cases $Q_1=Q_2=-1$.

	(a)		(b)		(c)	
	M_Z	M_{string}	M_Z	M_{string}	M_Z	M_{string}
m_1^2 (GeV)^2	$(36.5)^2$	$(545)^2$	$(23.1)^2$	$(560)^2$	$(18.5)^2$	$(582)^2$
m_2^2 (GeV)^2	$-(36.5)^2$	$(892)^2$	$-(23.1)^2$	$(939)^2$	$-(18.5)^2$	$(959)^2$
m_S^2 (GeV)^2	$-(36.5)^2$	$(784)^2$	$-(23.1)^2$	$(807)^2$	$-(18.5)^2$	$(837)^2$
m_U^2 (GeV)^2	$(210)^2$	$(301)^2$	$(208)^2$	$(367)^2$	$(180)^2$	$(364)^2$
m_Q^2 (GeV)^2	$(405)^2$	$(217)^2$	$(410)^2$	$(271)^2$	$(397)^2$	$(260)^2$
A (GeV)	183	1920	184	1940	185	2110
A_Q (GeV)	-310	1790	-311	1810	-301	2020
$M_{1/2}$ (GeV)	$(-578, -164)$	-200	$(-578, -164)$	-200	$(-578, -164)$	-200

TABLE IV. Large s minimum: $M_{Z_2}=1$ TeV, $\alpha_{Z-Z'}=0.0$, $\tan\beta=1.0$, $m_h=172$ GeV, and $Q_1=Q_2=-1$.

	(a)		(b)		(c)	
	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$
m_1^2 (GeV)^2	$(430)^2$	$(1660)^2$	$(430)^2$	$(969)^2$	$(430)^2$	$(713)^2$
m_2^2 (GeV)^2	$(430)^2$	$(3240)^2$	$(430)^2$	$(1880)^2$	$(430)^2$	$(2350)^2$
m_S^2 (GeV)^2	$-(701)^2$	$(2150)^2$	$-(701)^2$	$(1030)^2$	$-(701)^2$	$(478)^2$
m_U^2 (GeV)^2	$(450)^2$	$(2300)^2$	$(425)^2$	$(1190)^2$	$(425)^2$	$(1630)^2$
m_Q^2 (GeV)^2	$(511)^2$	$(1660)^2$	$(475)^2$	$(782)^2$	$(495)^2$	$(1030)^2$
A (GeV)	500	3150	500	808	500	-1460
A_Q (GeV)	190	1760	339	-1410	363	-4440
$M_{1/2}$ (GeV)	(289, 82)	100	(746, 212)	258	(995, 282)	344

nonzero mixing angle and $\tan\beta=1.4$. This case has a different choice of U(1)$'$ charges $Q_1=-1$, $Q_2=-1/2$. Once again, the initial values of the parameters are larger than the values of the parameters at the electroweak scale. As in Table V, $m_2^2(M_Z)$ is negative, so in most of the examples the magnitude of $m_2^{0\,2}$ is comparable to the initial values of the other soft mass-squared parameters.

In summary, without exotic particles it is necessary to invoke nonuniversal trilinear couplings and soft mass-squared parameters at the string scale to reach either scenario. In most cases, small initial values of the gaugino masses relative to the soft mass-squared parameters are required, such that $M_{1/2}\ll m_i^0$. It is also necessary to have $m_i^{0\,2}\gg m_i^2(M_Z)$ for the large trilinear coupling scenario, and for many of the examples that lead to the large s minimum. With these generic features of the values of the parameters at the string scale, it is possible to reach the phenomenologically viable low-energy parameter space with the minimal particle content.

Another possibility is to add to our model by considering exotic particles, as are expected in many string models. One example involves color triplets $\hat{D}_1\sim(3,1,Y_{D_1},Q_{D_1})$ and $\hat{D}_2\sim(\overline{3},1,Y_{D_2},Q_{D_2})$ which couple to the singlet through the additional term in the superpotential

$$W=h_D\hat{S}\hat{D}_1\hat{D}_2. \tag{58}$$

The presence of these exotics affects the running of the SU(3) and U(1) gauge couplings. Taken by themselves they would destroy the gauge coupling unification.[13] Thus, one must assume that $\hat{D}_{1,2}$ are associated with other exotics so that the gauge unification is restored. One example would be for $\hat{D}_i$ to be part of a complete GUT supermultiplet. Examples of anomaly-free models consistent with gauge unification are given in Appendix C. Clearly, the implications are very model dependent. A precise numerical analysis of the associated renormalization group equations of such models is beyond the scope of this paper. However, it is useful to consider the consequences of these exotics on the low-energy parameter space using a semianalytic approach. With the additional color triplets, a large singlet VEV is guaranteed with *universal boundary conditions*, as m_S^2 is negative at the electroweak scale. This was shown in [8] in the limit in which the gaugino masses and trilinear couplings can be neglected. The additional coupling of the singlet to the exotic triplets increases the overall weight driving m_S^2 negative in its RGE in analogy with m_2^2, as discussed in Appendix B. In contrast, the large trilinear coupling scenario is more difficult to obtain in this case. The presence of the new trilinear coupling A_D acts to lower the fixed point value of A further, such that at low energies $A_D(M_Z)\sim A_Q(M_Z)\gg A(M_Z)$. Universal boundary conditions would not lead to this minimum; the initial values of the trilinear couplings and the soft supersymmetry-breaking mass-squared parameters would have to be chosen to invert this hierarchy and obtain similar values of m_1^2, m_2^2, and m_S^2 at the electroweak scale.

VI. CONCLUSIONS

In this paper, we explored the features of the supersymmetric standard model with an additional nonanomalous U(1)$'$ gauge symmetry. The model is a "minimal" extension of the minimal supersymmetric standard model (MSSM), with one standard model singlet chiral superfield $\hat{S}$ added to the MSSM particle content. The U(1)$'$ charges are chosen to allow the trilinear coupling of $\hat{S}$ to the MSSM doublet chiral superfields $\hat{H}_{1,2}$ in the superpotential. This choice of U(1)$'$ charges implies that the bilinear coupling of the two doublets $\hat{H}_{1,2}$ is absent; hence, there is no elementary μ parameter in the superpotential. However, when S (scalar component of $\hat{S}$) acquires a nonzero vacuum expectation value (VEV), this trilinear term generates an effective μ term, which leads to a natural solution of the μ problem.

The gauge structure, particle content, and nature of the couplings of this type of model are key ingredients of a large class of $N=1$ supersymmetric string models based on fermionic constructions (e.g., $Z_2\times Z_2$ asymmetric orbifolds) at a particular point in moduli space. Within this approach, we identified the minimal particle content and their couplings in the supersymmetric part of the theory which are necessary to address the symmetry-breaking patterns. Thus, we ignored the difficult problems associated with the couplings of additional exotic particles in such string models. Another difficulty of this class of string models is the absence of a mechanism for supersymmetry breaking with unique quantitative

[13]Small $\sim10\%$ corrections to the RGE predictions, which could be due to exotics, may even be desirable, due to the values of the predicted unification scale and α_3.

TABLE V. Large s minimum: $M_{Z_2}=1$ TeV, $\alpha_{Z-Z'}=6.3\times10^{-3}$, $\tan\beta=2.0$, $m_h=163$ GeV, and $Q_1=Q_2=-1$.

	(a)		(b)		(c)	
	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$	M_Z	$M_{\rm string}$
m_1^2 (GeV)^2	$(427)^2$	$(937)^2$	$(427)^2$	$(696)^2$	$(427)^2$	$(665)^2$
m_2^2 (GeV)^2	$-(173)^2$	$(1810)^2$	$-(173)^2$	$(2450)^2$	$-(173)^2$	$(1190)^2$
m_S^2 (GeV)^2	$-(704)^2$	$(949)^2$	$-(704)^2$	$(383)^2$	$-(704)^2$	$(174)^2$
m_U^2 (GeV)^2	$(200)^2$	$(1310)^2$	$(310)^2$	$(1870)^2$	$(262)^2$	$(887)^2$
m_Q^2 (GeV)^2	$(380)^2$	$(974)^2$	$(400)^2$	$(1280)^2$	$(362)^2$	$(674)^2$
A (GeV)	250	2710	250	-2670	250	1460
A_Q (GeV)	-109	2460	210	-4810	178	696
$M_{1/2}$ (GeV)	$(-289, -82)$	-100	(723, 205)	250	(289, 82)	100

predictions. We chose to parametrize the supersymmetry breaking with a general set of soft supersymmetry-breaking mass parameters.

The analysis given in this paper generalizes the work of [8], which investigated the gauge symmetry-breaking pattern of the above class of string models in the limit of a large $\tan\beta$ scenario. We have addressed the nature of phenomenologically acceptable electroweak symmetry-breaking scenarios and the resulting particle spectrum in detail. In addition, we have analyzed the RGE's of the model to explore the range of parameters at the string scale which leads to the phenomenologically viable low-energy parameter space.

We summarize the main results of the analysis as follows.

Gauge symmetry-breaking scenarios. We found a rich structure of phenomenologically acceptable gauge symmetry-breaking patterns, which involved a certain but not excessive amount of fine-tuning of the parameters. The symmetry breaking necessarily takes place in the electroweak energy range.[14] For a range of the parameters which comprises a few percent of the full parameter space, the Z-Z' mixing is acceptably small and the Z' mass is sufficiently large. The symmetry-breaking patterns fall into two characteristic classes:

(i) *Large trilinear coupling scenario.* The symmetry breaking is driven by a large value of the soft supersymmetry-breaking trilinear coupling. When the trilinear coupling is larger than the scalar soft mass parameters by a factor of 5 to 10, the VEV's of $H_{1,2}$, and S are approximately equal. For equal U(1)$'$ charges for $\hat{H}_1$ and $\hat{H}_2$, the Z-Z' mixing is suppressed; it can be easily ensured to be $<10^{-3}$. The Z' is light, with mass $\sim$200 GeV. In this scenario, the electroweak phase transition may be first order with potentially interesting cosmological implications.

(ii) *Large singlet VEV scenario.* In this case, the symmetry breaking is driven by a negative mass squared term for S. Its absolute magnitude is in general larger than that of the mass squared terms for $H_{1,2}$. A certain fine-tuning of the soft mass parameters is needed to ensure acceptably small Z-Z' mixing. This scenario is viable (for different ranges of parameters) without imposing additional constraints on the U(1)$'$ charges of the Higgs fields. The Z' mass is typically in the range of 1 TeV. It is interesting to note that the range of mass parameters for this scenario is similar to that of the MSSM.

Renormalization group analysis. We have also explored the relationship between the values of the soft supersymmetry-breaking mass parameters at the electroweak scale and the values at the string scale by analyzing the RGE's of the model. We have solved the RGE's numerically as a function of the boundary conditions at the string scale. We have also derived semianalytic solutions of the RGE's to further our understanding of the evolution of the parameters. In the analysis, we chose the initial values of the Yukawa couplings (of the Higgs fields to the singlet and of the Higgs field to the third quark family) to be of the order of magnitude of the gauge coupling, as determined in a class of string models based on the fermionic construction. These couplings provide a dominant contribution to the RGE's of the soft mass parameters.

We found that with the minimal particle content, universal soft supersymmetry-breaking mass parameters at the string scale do not yield the phenomenologically acceptable range of parameters at the electroweak scale. The results which lead to the phenomenologically acceptable low-energy parameter space can be classified as follows.

(i) *Nonuniversal boundary conditions.* With the minimal particle content, nonuniversal soft supersymmetry-breaking mass parameters are required at the string scale to obtain the viable gauge symmetry-breaking scenarios previously described. In most cases, the gaugino masses at the string scale must be chosen small relative to the other soft supersymmetry-breaking mass parameters. For the large trilinear coupling scenario, the soft mass-squared parameters at the string scale are about a factor of 10 larger than their values at the electroweak scale.[15]

(ii) *Additional exotics.* Many string models predict the existence of additional exotic particles, such as additional SU(3) triplets which couple to $\hat{S}$ with Yukawa couplings of the order of the gauge couplings. The presence of such exotic

[14]The scale of U(1)$'$ symmetry breaking can be in the $10^{10}-10^{14}$ GeV range for the case of more than one SM singlet and the appropriate choices of their U(1)$'$ charges [8].

[15]In a large class of models for supersymmetry breaking, the values of these mass parameters at the string scale are closely related to the value of the gravitino mass.

TABLE VI. Large s minimum: $M_{Z_2}=700$ GeV, $\alpha_{Z-Z'}=1.4\times10^{-4}$, $\tan\beta=1.4$, $m_h=120$ GeV, and $Q_1=-1$, $Q_2=-1/2$.

	(a)		(b)		(c)	
	M_Z	M_{string}	M_Z	M_{string}	M_Z	M_{string}
m_1^2 (GeV)^2	$(207)^2$	$(480)^2$	$(207)^2$	$(407)^2$	$(207)^2$	$(546)^2$
m_2^2 (GeV)^2	$-(354)^2$	$(824)^2$	$-(354)^2$	$(622)^2$	$-(354)^2$	$(2360)^2$
m_S^2 (GeV)^2	$-(499)^2$	$(381)^2$	$-(499)^2$	$(82.8)^2$	$-(499)^2$	$(558)^2$
m_U^2 (GeV)^2	$(200)^2$	$(527)^2$	$(262)^2$	$(472)^2$	$(242)^2$	$(1780)^2$
m_Q^2 (GeV)^2	$(350)^2$	$(384)^2$	$(362)^2$	$(389)^2$	$(384)^2$	$(1190)^2$
A (GeV)	250	2340	250	1520	250	-2760
A_Q (GeV)	-190	2440	263	1030	200	-5140
$M_{1/2}$ (GeV)	$(-463, -132)$	-160	(361, 103)	125	(795, 226)	275

particles can modify the RGE analysis significantly.[16] Using the semianalytic approach, we determined that, for example, additional color triplets ensure a large value of the singlet VEV even with universal boundary conditions. This indicates that the latter scenario is obtainable for universal soft mass parameters at the string scale when such exotics are present. In the limit of small gaugino masses and trilinear couplings, this result was exhibited numerically in [8]. In contrast, the large trilinear coupling scenario is more difficult to obtain with additional exotic particles. We found that nonuniversal boundary conditions for the soft supersymmetry-breaking trilinear couplings are required to reach this scenario.

The analysis presented in this paper exhibits the viability and predictive power of supersymmetric models with an additional U(1)$'$, whose gauge structure, particle content, and nature of couplings are key ingredients of a large class of string vacua. For a range of soft supersymmetry-breaking parameters at the string scale, such models allow for interesting gauge symmetry-breaking scenarios, which can be tested at future colliders.

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APPENDIX A: RENORMALIZATION GROUP EQUATIONS

We present the renormalization group equations for the gauge couplings, gaugino masses, Yukawa couplings, trilinear couplings, and soft mass-squared parameters for the model.[17] In the following equations, S_1 and S_1' are defined to be

$$S_1=\sum_a Y_a m_a^2=\sum_{i=1}^{N_F}(m_{Ei}^2-m_{Li}^2+m_{Qi}^2+m_{Di}^2-2m_{Ui}^2)-m_1^2+m_2^2, \tag{A1}$$

$$S_1'=\sum_a Q_a m_a^2=\sum_{i=1}^{N_F}(Q_{Ei}m_{Ei}^2+2Q_{Li}m_{Li}^2+6Q_{Qi}m_{Qi}^2+3Q_{Di}m_{Di}^2+3Q_{Ui}m_{Ui}^2)+2Q_1m_1^2+2Q_2m_2^2+Q_Sm_S^2, \tag{A2}$$

N_F denotes the number of families, and the scale variable is given by

$$t=\frac{1}{16\pi^2}\ln\frac{\mu}{M_{\text{string}}}. \tag{A3}$$

The normalization of the U(1)$'$ gauge coupling is model dependent. For definiteness, we choose to normalize the gauge couplings by requiring that the gauge couplings and charges satisfy the constraint that $g_i^{0\,2}$ Tr Q^2 is constant, where the trace is evaluated over one family. With the choice of U(1)$'$ charges used in the renormalization group analysis, $g_1'(t)$ is numerically very similar to $g_1(t)$.

Gauge couplings:

$$\frac{d}{dt}g_3=(2N_F-9)g_3^3, \tag{A4}$$

$$\frac{d}{dt}g_2=(2N_F-5)g_2^3, \tag{A5}$$

$$\frac{d}{dt}g_1=\left(2N_F+\frac{3}{5}\right)g_1^3, \tag{A6}$$

$$\frac{d}{dt}g_1'=(2N_F+2r)g_1'^3, \tag{A7}$$

where

[16] Since such exotics destroy the gauge coupling unification, one has to assume that there are additional exotics (that, however, do not couple to $\hat{S}$), so that the gauge coupling unification is restored.

[17] We do not present the renormalization group equations for the soft mass-squared parameters of the staus and the bottom squarks, as they do not influence directly the symmetry-breaking pattern. These terms are included in the definitions (A1) and (A2).

$$r=\frac{2(Q_1^2+Q_2^2)+Q_S^2}{6Q_Q^2+3(Q_U^2+Q_D^2)+2Q_L^2+Q_E^2}. \tag{A8}$$

Gaugino masses:

$$\frac{d}{dt}M_3=2(2N_F-9)g_3^2M_3, \tag{A9}$$

$$\frac{d}{dt}M_2=2(2N_F-5)g_2^2M_2, \tag{A10}$$

$$\frac{d}{dt}M_1=2\left(2N_F+\frac{3}{5}\right)g_1^2M_1, \tag{A11}$$

$$\frac{d}{dt}M_1'=2(2N_F+2r)g_1'^2M_1'. \tag{A12}$$

Yukawa couplings:

$$\frac{d}{dt}h_s=h_s\{4h_s^2+3h_Q^2-[3g_2^2+g_1^2+2g_1'^2(Q_1^2+Q_2^2+Q_S^2)]\}, \tag{A13}$$

$$\frac{d}{dt}h_Q=h_Q\left\{6h_Q^2+h_s^2-\left(\frac{16}{3}g_3^2+3g_2^2+\frac{13}{9}g_1^2 +2g_1'^2(Q_U^2+Q_Q^2+Q_2^2)\right)\right\}. \tag{A14}$$

Trilinear couplings:

$$\frac{d}{dt}A=8h_s^2A+6h_Q^2A_Q-2[3M_2g_2^2+M_1g_1^2 +2M_1'g_1'^2(Q_1^2+Q_2^2+Q_S^2)], \tag{A15}$$

$$\frac{d}{dt}A_Q=12h_Q^2A_Q+2h_s^2A-2\left(\frac{16}{3}M_3g_3^2+3M_2g_2^2+\frac{13}{9}M_1g_1^2 +2M_1'g_1'^2(Q_U^2+Q_Q^2+Q_2^2)\right). \tag{A16}$$

Soft scalar mass-squared parameters:

$$\frac{d}{dt}m_S^2=4(m_S^2+m_1^2+m_2^2+A^2)h_s^2-8M_1'^2g_1'^2Q_S^2+2Q_Sg_1'^2S_1', \tag{A17}$$

$$\frac{d}{dt}m_1^2=2(m_S^2+m_1^2+m_2^2+A^2)h_s^2-8\left(\frac{3}{4}M_2^2g_2^2+\frac{1}{4}M_1^2g_1^2 +M_1'^2g_1'^2Q_1^2\right)-g_1^2S_1+2g_1'^2Q_1S_1', \tag{A18}$$

$$\frac{d}{dt}m_2^2=2(m_S^2+m_1^2+m_2^2+A^2)h_s^2+6(m_2^2+m_{Q3}^2+m_{U3}^2 +A_Q^2)h_Q^2-8\left(\frac{3}{4}M_2^2g_2^2+\frac{1}{4}M_1^2g_1^2+M_1'^2g_1'^2Q_2^2\right) +g_1^2S_1+2g_1'^2Q_2S_1', \tag{A19}$$

$$\frac{d}{dt}m_{U3}^2=4(m_2^2+m_{Q3}^2+m_{U3}^2+A_Q^2)h_Q^2-8\left(\frac{4}{3}M_3^2g_3^2+\frac{4}{9}M_1^2g_1^2 +M_1'^2g_1'^2Q_U^2\right)-\frac{4}{3}g_1^2S_1+2g_1'^2Q_US_1', \tag{A20}$$

$$\frac{d}{dt}m_{Q3}^2=2(m_2^2+m_{Q3}^2+m_{U3}^2+A_Q^2)h_Q^2-8\left(\frac{4}{3}M_3^2g_3^2+\frac{3}{4}M_2^2g_2^2 +\frac{1}{36}M_1^2g_1^2+M_1'^2g_1'^2Q_Q^2\right)+\frac{1}{3}g_1^2S_1+2g_1'^2Q_QS_1'. \tag{A21}$$

APPENDIX B: SOLUTIONS OF RGE's

1. Numerical results

The RGE's for the gauge couplings and gaugino masses with the initial conditions (51) and (54) can be solved to yield

$$g_3^2(t)=\frac{g_0^2}{1-2(2N_F-9)g_0^2t}, \tag{B1}$$

$$g_2^2(t)=\frac{g_0^2}{1-2(2N_F-5)g_0^2t}, \tag{B2}$$

$$g_1^2(t)=\frac{g_0^2}{1-2(2N_F+3/5)g_0^2t}, \tag{B3}$$

$$g_1'^2(t)=\frac{g_0^2}{1-2(2N_F+2r)g_0^2t}, \tag{B4}$$

where r is defined in Eq. (A8), and

$$M_i(t)=M_i^0\frac{g_i^2(t)}{g_0^2}. \tag{B5}$$

These solutions are inserted in the RGE's for the other parameters, which we integrated numerically. As a concrete example, we choose the U(1)$'$ charges $Q_1=Q_2=-1$, $Q_Q=Q_L=-1/2$, and initial values of the Yukawa couplings $h_Q^0=h_s^0=g_0\sqrt{2}$. The Yukawa couplings are given by

$$h_s(M_Z)=0.758\ ,\ h_Q(M_Z)=1.186. \tag{B6}$$

The trilinear couplings take the values

$$A_Q(M_Z)=-0.039\,A^0+0.07\,A_Q^0+1.84\,M_{1/2}, \tag{B7}$$

$$A(M_Z)=0.283\,A^0-0.215\,A_Q^0-0.12\,M_{1/2}. \tag{B8}$$

Finally, the soft mass-squared parameters are given by

$$m_1^2(M_Z) = -0.13\, m_2^{0\,2} + 0.8\, m_1^{0\,2} - 0.2\, m_S^{0\,2} + 0.071\, m_U^{0\,2} + 0.071\, m_Q^{0\,2} - 0.053\, A^{0\,2} + 0.0061\, A_Q^{0\,2} + 0.72\,(M_{1/2})^2 + 0.035\, A^0 A_Q^0 - 0.065\, A^0 M_{1/2} + 0.056\, A_Q^0 M_{1/2}, \tag{B9}$$

$$m_2^2(M_Z) = 0.45\, m_2^{0\,2} - 0.12\, m_1^{0\,2} - 0.12\, m_S^{0\,2} - 0.43\, m_U^{0\,2} - 0.43\, m_Q^{0\,2} - 0.027\, A^{0\,2} - 0.025\, A_Q^{0\,2} - 3.81\,(M_{1/2})^2 + 0.029\, A^0 A_Q^0 + 0.026\, A^0 M_{1/2} - 0.13\, A_Q^0 M_{1/2}, \tag{B10}$$

$$m_S^2(M_Z) = -0.26\, m_2^{0\,2} - 0.4\, m_1^{0\,2} + 0.6\, m_S^{0\,2} + 0.14\, m_U^{0\,2} + 0.14\, m_Q^{0\,2} - 0.1\, A^{0\,2} + 0.01\, A_Q^{0\,2} + 0.82\,(M_{1/2})^2 + 0.07\, A^0 A_Q^0 - 0.13\, A^0 M_{1/2} + 0.11\, A_Q^0 M_{1/2}, \tag{B11}$$

$$m_U^2(M_Z) = -0.28\, m_2^{0\,2} + 0.05\, m_1^{0\,2} + 0.05\, m_S^{0\,2} + 0.67\, m_U^{0\,2} - 0.33\, m_Q^{0\,2} + 0.017\, A^{0\,2} - 0.021\, A_Q^{0\,2} + 4.18\,(M_{1/2})^2 - 0.004\, A^0 A_Q^0 + 0.06\, A^0 M_{1/2} - 0.13\, A_Q^0 M_{1/2}, \tag{B12}$$

$$m_Q^2(M_Z) = -0.14\, m_2^{0\,2} + 0.027\, m_1^{0\,2} + 0.027\, m_S^{0\,2} - 0.17\, m_U^{0\,2} + 0.83\, m_Q^{0\,2} + 0.0086\, A^{0\,2} - 0.01\, A_Q^{0\,2} + 5.6\,(M_{1/2})^2 - 0.002\, A^0 A_Q^0 + 0.03\, A^0 M_{1/2} - 0.064\, A_Q^0 M_{1/2}. \tag{B13}$$

We have also obtained results for different choices of the initial values of the Yukawa couplings as can appear in a class of models. The low-energy results do not change significantly. For example, with $h_Q^0 = g_0\sqrt{2}$ and $h_S^0 = g_0$, the values of the coefficients do not change more than 10%.

2. Semianalytic solutions

In the following section we present approximate analytical solutions to the RGE's. To solve the RGE's, we first make the approximation that the gauge couplings (B1)–(B4) are replaced by their average values

$$g_i = \frac{1}{2}[g_i(M_Z) + g_0] \quad (i = 3,2,1,1'). \tag{B14}$$

Similarly, we replace the gaugino masses (B5) with

$$M_i = \frac{1}{2}[M_i(M_Z) + M_{1/2}]\ . \tag{B15}$$

This yields the respective values 1.00, 0.69, 0.59, 0.58 for the gauge couplings g_3, g_2, g_1, g_1', and $2.07M_{1/2}$, $0.91M_{1/2}$, $0.70M_{1/2}$, $0.69M_{1/2}$ for the gaugino masses M_3, M_2, M_1, M_1'. Under these approximations, we can solve the coupled equations for the Yukawa couplings by noticing that with the choice of initial conditions, h_Q remains relatively close to its fixed point value,[18] while h_s evolves significantly. The approximate solution is

$$h_s^2(t) = \frac{\tilde{g}_S{}^2}{1-(1-\tilde{g}_S^2/h_S^{0\,2})e^{7\tilde{g}_S^2 t}}, \tag{B16}$$

$$h_Q^2(t) = \frac{\tilde{g}_Q{}^2}{1-(1-\tilde{g}_Q^2/h_Q^{0\,2})e^{12\tilde{g}_Q^2 t}}, \tag{B17}$$

in which

$$\tilde{g}_S^2 = \frac{1}{7}\left(3g_2^2 - \frac{16}{3}g_3^2 + \frac{5}{9}g_1^2 + 2(2Q_S^2 + 2Q_1^2 + Q_2^2 - Q_U^2 - Q_Q^2)\ g_1'^2\right), \tag{B18}$$

$$\tilde{g}_Q{}^2 = \frac{1}{6}\left(\frac{16}{3}g_3^2 + 3g_2^2 + g_1^2 + 2(Q_U^2 + Q_Q^2 + Q_2^2)\ g_1'^2 - \overline{h_S^2}\right), \tag{B19}$$

$$\overline{h}_S = \frac{1}{2}[h_S^0 + h_s(M_Z)], \tag{B20}$$

$$\overline{h}_Q = \frac{1}{2}[h_Q^0 + h_Q(M_Z)]. \tag{B21}$$

As a first approximation to solve for the trilinear couplings, we use the averaged Yukawa couplings, averaged gaugino masses, and averaged gauge couplings, and the U(1) factors are neglected for simplicity. The equations are then solved to yield

$$A_Q(t) = \alpha_{1Q}e^{\lambda_1 t} + \alpha_{2Q}e^{\lambda_2 t} - \beta_{1Q}e^{\lambda_1 t} + \beta_{2Q}e^{\lambda_2 t} - A_{QP}, \tag{B22}$$

$$A(t) = \alpha_{1S}e^{\lambda_1 t} + \alpha_{2S}e^{\lambda_2 t} - \beta_{1S}e^{\lambda_1 t} + \beta_{2S}e^{\lambda_2 t} - A_{SP}, \tag{B23}$$

where the initial condition-dependent α and β coefficients are

$$\alpha_{iQ} = \alpha_i(A_Q^0, A_S^0), \tag{B24}$$

$$\beta_{iQ} = \alpha_i(A_{QP}, A_{SP}), \tag{B25}$$

$$\alpha_{iS} = \alpha_{iQ}\frac{(\lambda_i - 12\overline{h_Q^2})}{2\overline{h_S^2}}, \tag{B26}$$

$$\beta_{iS} = \beta_{iQ}\frac{(\lambda_i - 12\overline{h_Q^2})}{2\overline{h_S^2}}. \tag{B27}$$

We have introduced some shorthand notation:

[18]The gauge couplings run, so this is not a fixed point in the exact sense. However, this approach is valid in the limit that Eq. (B14) holds.

$$\alpha_1(A,B)=\frac{A(12\overline{h_Q^2}-\lambda_2)+2\overline{h_S^2}B}{\lambda_1-\lambda_2}, \tag{B28}$$

$$\alpha_2(A,B)=\frac{A(\lambda_1-12\overline{h_Q^2})-2\overline{h_S^2}B}{\lambda_1-\lambda_2}, \tag{B29}$$

$$\lambda_{1,2}=6\overline{h_Q^2}+4\overline{h_S^2}\pm\sqrt{(6\overline{h_Q^2}-4\overline{h_S^2})^2+12\overline{h_Q^2}\,\overline{h_S^2}}, \tag{B30}$$

$$A_{QP}=\frac{(128/3)g_3^2M_3+18g_2^2M_2}{42\overline{h_Q^2}}, \tag{B31}$$

$$A_{SP}=\frac{-(32/3)g_3^2M_3+6g_2^2M_2}{14\overline{h_S^2}}. \tag{B32}$$

With the approximations (B14), (B15), (B19), and (B20), the fixed point values are $A_{QP}=2.3M_{1/2}$ and $A_{SP}=-1.8M_{1/2}$. This analysis slightly overestimates the splitting of the fixed point values, but shows the tendency for $A_Q(M_Z)$ to be larger than $A(M_Z)$ for $M_{1/2}$ positive.

The equations for the trilinear terms can also be solved when the running of the SU(3) gauge coupling and gaugino are included. The others are neglected for simplicity, as the SU(3) gauge coupling is dominant. In this case the solutions are

$$A_Q(t)=\alpha_{1Q}e^{\lambda_1 t}+\alpha_{2Q}e^{\lambda_2 t}+M_{1/2}f_Q(I_1(t),I_2(t)), \tag{B33}$$

$$A(t)=\alpha_{1S}e^{\lambda_1 t}+\alpha_{2S}e^{\lambda_2 t}+M_{1/2}f_S(I_1(t),I_2(t)) \ , \tag{B34}$$

in which

$$f_Q=-\frac{16}{9}\,\frac{e^{\lambda_1 t}(12\overline{h_Q^2}-\lambda_2)I_1(t)+e^{\lambda_2 t}(\lambda_1-12\overline{h_Q^2})I_2(t)}{\lambda_1-\lambda_2}, \tag{B35}$$

$$f_S=-\frac{16}{9}\,\frac{6\overline{h_Q^2}(e^{\lambda_1 t}I_1(t)-e^{\lambda_2 t}I_2(t))}{\lambda_1-\lambda_2}, \tag{B36}$$

and the functions $I_i(t)$ are defined by

$$I_i(t)=\int_{x=0}^{6g_0^2 t}e^{-\lambda_i x/6g_0^2}\frac{dx}{(1+x)^2}. \tag{B37}$$

To solve the RGE's for the soft mass-squared parameters, only the SU(3) gauge coupling and gaugino are included in the analysis. To obtain relatively compact approximate analytical solutions, the trilinear couplings are also replaced by their average values:

$$\overline{A_Q}=\frac{1}{2}[A_Q^0+A_Q(M_Z)], \tag{B38}$$

$$\overline{A}=\frac{1}{2}[A^0+A(M_Z)]. \tag{B39}$$

With these further approximations, it is useful to consider the solutions for the sums defined by

$$\Sigma_1=m_Q^2+m_U^2+m_2^2, \tag{B40}$$

$$\Sigma_2=m_S^2+m_1^2+m_2^2. \tag{B41}$$

The solutions are given by

$$\Sigma_1(t)=(\gamma_1+\rho_1)e^{\lambda_1 t}+(\gamma_2+\rho_2)e^{\lambda_2 t}-\Delta_1, \tag{B42}$$

$$\Sigma_2(t)=(\delta_1+\eta_1)e^{\lambda_1 t}+(\delta_2+\eta_2)e^{\lambda_2 t}-\Delta_2, \tag{B43}$$

in which

$$\gamma_i=\alpha_i(m_Q^{0\,2}+m_U^{0\,2}+m_2^{0\,2},m_S^{0\,2}+m_1^{0\,2}+m_2^{0\,2}), \tag{B44}$$

$$\rho_i=\alpha_i(\Delta_1,\Delta_2), \tag{B45}$$

$$\delta_i=\gamma_i\frac{(\lambda_i-12\overline{h_Q^2})}{2\overline{h_S^2}}, \tag{B46}$$

$$\eta_i=\rho_i\frac{(\lambda_i-12\overline{h_Q^2})}{2\overline{h_S^2}}, \tag{B47}$$

$$\Delta_1=\overline{A_Q^2}-\frac{128}{63}\frac{g_3^2M_3^2}{\overline{h_Q^2}}, \tag{B48}$$

$$\Delta_2=\overline{A^2}+\frac{32}{21}\frac{g_3^2M_3^2}{\overline{h_S^2}}. \tag{B49}$$

The renormalization group equations for the individual mass-squared parameters may then be integrated explicitly to yield

$$\begin{aligned}m_1^2(t)=&\frac{5}{7}m_1^{0\,2}-\frac{1}{7}m_2^{0\,2}-\frac{2}{7}m_S^{0\,2}+\frac{1}{7}m_U^{0\,2}+\frac{1}{7}m_Q^{0\,2}+\frac{1}{7}\Delta_1\\&-\frac{2}{7}\Delta_2-3.05g_3^2M_3^2t\\&+2\overline{h_S^2}\left\{\frac{\eta_1+\delta_1}{\lambda_1}e^{\lambda_1 t}+\frac{\eta_2+\delta_2}{\lambda_2}e^{\lambda_2 t}\right\},\end{aligned} \tag{B50}$$

$$\begin{aligned}m_2^2(t)=&-\frac{1}{7}m_1^{0\,2}+\frac{3}{7}m_2^{0\,2}-\frac{1}{7}m_S^{0\,2}-\frac{3}{7}m_U^{0\,2}-\frac{3}{7}m_Q^{0\,2}-\frac{3}{7}\Delta_1\\&-\frac{1}{7}\Delta_2+9.14g_3^2M_3^2t\\&+\frac{2\overline{h_S^2}(\eta_1+\delta_1)+6\overline{h_Q^2}(\gamma_1+\rho_1)}{\lambda_1}\,e^{\lambda_1 t}\\&+\frac{2\overline{h_S^2}(\eta_2+\delta_2)+6\overline{h_Q^2}(\gamma_2+\rho_2)}{\lambda_2}\,e^{\lambda_2 t},\end{aligned} \tag{B51}$$

$$m_S^2(t) = -\frac{4}{7}m_1^{0\,2} - \frac{2}{7}m_2^{0\,2} + \frac{3}{7}m_S^{0\,2} + \frac{2}{7}m_U^{0\,2} + \frac{2}{7}m_Q^{0\,2} + \frac{2}{7}\Delta_1 - \frac{4}{7}\Delta_2 - 6.1 g_3^2 M_3^2 t + 4\overline{h_S^2} \times \left\{ \frac{\eta_1+\delta_1}{\lambda_1} e^{\lambda_1 t} + \frac{\eta_2+\delta_2}{\lambda_2} e^{\lambda_2 t} \right\}, \tag{B52}$$

$$m_U^2(t) = \frac{2}{21}m_1^{0\,2} - \frac{6}{21}m_2^{0\,2} + \frac{2}{21}m_S^{0\,2} + \frac{13}{21}m_U^{0\,2} - \frac{8}{21}m_Q^{0\,2} + \frac{8}{21}\Delta_1 + \frac{2}{21}\Delta_2 - 2.54 g_3^2 M_3^2 t + 4\overline{h_Q^2}\left\{ \frac{\rho_1+\gamma_1}{\lambda_1} e^{\lambda_1 t} + \frac{\rho_2+\gamma_2}{\lambda_2} e^{\lambda_2 t} \right\}, \tag{B53}$$

$$m_Q^2(t) = \frac{1}{21}m_1^{0\,2} - \frac{2}{21}m_2^{0\,2} + \frac{1}{21}m_S^{0\,2} - \frac{4}{21}m_U^{0\,2} + \frac{17}{21}m_Q^{0\,2} - \frac{4}{21}\Delta_1 + \frac{1}{21}\Delta_2 - 6.6 g_3^2 M_3^2 t + 2\overline{h_Q^2}\left\{ \frac{\rho_1+\gamma_1}{\lambda_1} e^{\lambda_1 t} + \frac{\rho_2+\gamma_2}{\lambda_2} e^{\lambda_2 t} \right\}. \tag{B54}$$

These solutions are valid in the limit of small initial gaugino masses, such that their contribution to the evolution of the trilinear couplings and the mass squares is small. When this condition is not satisfied, the SU(3) gaugino masses and gauge couplings control the evolution of all the parameters, and the approximation of neglecting the running of the gaugino masses and gauge couplings breaks down. As stated above, it is possible to incorporate the running of the SU(3) gauge coupling and gaugino in solving the equations for the trilinear couplings and obtain solutions to these equations that are in better agreement with the exact solutions. This is also possible for the soft mass-squared parameters, but the solutions are cumbersome and thus do not yield much physical insight, so they are not presented here.

In the limit in which the gaugino masses and trilinear couplings are neglected (Δ_i, ρ_i, and η_i are zero), it is possible to use the semianalytic expressions to show that with universal initial conditions, the only soft mass-squared parameter that will run negative is m_2^2. In this limit, Eqs. (B42) and (B43) approach zero asymptotically. Therefore, in the asymptotic limit the appropriate sums of the individual mass-squared parameters must also approach zero. Since H_2 couples both to the quarks and the singlet in the superpotential, it has a greater weight driving it negative in its RGE (A19), and it will be negative at low energies. The other soft mass-squared parameters have smaller group theoretical prefactors, and in the asymptotic limit they must be positive to compensate for the negative value of m_2^2. This indicates that the other soft mass-squared parameters are necessarily positive at the electroweak scale, as the asymptotes dominate the low-energy behavior. Although the solution of the RGE's requires a choice of average values of the Yukawa couplings, the asymptotes of the mass-squared parameters do not depend on the Yukawa couplings; it is only the group theoretical factors present in the RGE's that lead to this result.

This also indicates why it becomes so simple to have m_S^2 negative when we add exotics that couple to the singlet in the superpotential. This increases the effective group theoretical factor in the RGE for m_S^2, so it is naturally negative at the electroweak scale for universal boundary conditions.

APPENDIX C: NONANOMALOUS U(1)′

In this work, we consider the phenomenological consequences of an additional nonanomalous U(1)′ symmetry. The requirement that the U(1)′ symmetry be anomaly-free severely constrains the U(1)′ charge assignments of the theory; the charges must be chosen so that the U(1)′ triangle anomaly and the mixed anomalies cancel. Furthermore, we require that the charges forbid an elementary μ term ($Q_1+Q_2 \neq 0$) but allow our induced μ term ($Q_1+Q_2+Q_S=0$). Finally, we require (for models involving light exotic supermultiplets) that the approximate gauge unification under the standard model group be respected. In this appendix, we display two models which satisfy these constraints and provide "existence" proofs. One involves *ad hoc* charge assignments for the minimal particle content, and the other is GUT motivated and involves exotics. The construction of realistic string-derived models is beyond the scope of this paper.

In the model we consider with the MSSM particle content and one additional singlet, for which approximate gauge unification is respected, the anomaly constraints are

$$0 = \sum_i (2Q_{Qi} + Q_{Ui} + Q_{Di}), \tag{C1}$$

$$0 = \sum_i (3Q_{Qi} + Q_{Li}) + Q_1 + Q_2, \tag{C2}$$

$$0 = \sum_i \left(\frac{1}{6}Q_{Qi} + \frac{1}{3}Q_{Di} + \frac{4}{3}Q_{Ui} + \frac{1}{2}Q_{Li} + Q_{Ei} \right) + \frac{1}{2}(Q_1+Q_2), \tag{C3}$$

$$0 = \sum_i (Q_{Qi}^2 + Q_{Di}^2 - 2Q_{Ui}^2 - Q_{Li}^2 + Q_{Ei}^2) - Q_1^2 + Q_2^2, \tag{C4}$$

$$0 = \sum_i (6Q_{Qi}^3 + 3Q_{Di}^3 + 3Q_{Ui}^3 + 2Q_{Li}^3 + Q_{Ei}^3) + 2Q_1^3 + 2Q_2^3 + Q_S^3. \tag{C5}$$

The first four constraints correspond to the mixed anomalies with SU(3), SU(2), $[\mathrm{U}(1)_Y]^2$, and $\mathrm{U}(1)_Y$, respectively. The final equation is the U(1)′ triangle anomaly condition.

There are also constraints from the requirements of gauge invariance:

$$Q_{U3} + Q_{Q3} + Q_2 = 0, \tag{C6}$$

$$Q_1 + Q_2 + Q_S = 0, \tag{C7}$$

where Eqs. (C6) and (C7) follow from the existence of a Yukawa interaction for the t quark mass and a term to generate an effective μ parameter, respectively. We do not require the existence of Yukawa interactions for leptons ($Q_E+Q_L+Q_1=0$) or down-type quarks ($Q_D+Q_Q+Q_1=0$). This is consistent with our superpotential (1), which does not include Yukawa couplings for these superfields. This implies in general that these fields must have masses generated by other mechanisms (e.g., higher-dimensional terms in the superpotential and/or extra fields in the model). In one of the examples below we obtain that the condition $Q_E+Q_L+Q_1=0$ is automatically satisifed for the third generation, so that the mass of the tau lepton can be generated by higher-dimensional terms. However, $Q_D+Q_Q+Q_1\neq 0$ in that model, so that the bottom quark mass (and the masses of the first two generations) generated by higher-dimensional terms would be suppressed by powers of the U(1)$'$ breaking scale, and are thus too small.

We have been able to find examples of charge assignments for our model which satisfy the anomaly [Eqs. (C1)–(C5)] and gauge invariance [Eqs. (C6)–(C7)] constraints. One simple possibility is

$$\begin{aligned} &Q_{E3}=Q_2-Q_1, && Q_{L3}=-Q_2,\\ &Q_{Q3}=-\tfrac{1}{3}Q_1, && Q_S=-(Q_1+Q_2),\\ &Q_{D3}=\tfrac{1}{3}(Q_1+3Q_2), && Q_{U3}=\tfrac{1}{3}(Q_1-3Q_2), \end{aligned} \tag{C8}$$

for arbitrary Q_1 and Q_2, and the first and second families have zero U(1)$'$ charges (other examples with nonzero charges for all three families can easily be constructed). This choice is consistent with string models where U(1)$'$ charges for quarks and leptons of different families are *not equal* in general.

We now consider the effects of neglecting the U(1) factors (A1) and (A2) in the analysis of the RGE's for the soft mass parameters. It is straightforward to derive the evolution equations for S_1 and S_1'; if the charge assignments are such that the conditions for anomaly cancellation and gauge invariance of the superpotential [Eqs. (C1)–(C7)] are satisfied, one obtains a homogeneous coupled system involving only S_1, S_1', the U(1) gauge couplings, and the U(1)$'$ charges. For universal soft mass-squared parameters at the string scale, S_1 and S_1' are manifestly zero when the anomaly conditions are satisfied, and they remain zero from $M_{\rm string}$ to M_Z. When there are nonuniversal soft mass-squared parameters, S_1 and S_1' have nonzero initial values. In the semianalytic approach in which the gauge couplings are replaced by their average values, it is possible to solve this coupled system for our example of U(1)$'$ charge assignments, and show that the system exponentially decays. Therefore, these factors become less important, and neglecting them in the RGE analysis is well justified.

As an example of a GUT-motivated U(1)$'$, we consider the ψ [2], which occurs in the breaking of E$_6$ to SO(10)$\times$U(1)$_\psi$. It is not our intention to consider GUT's per se, but rather to use this as an existence proof of acceptable U(1)$'$ quantum numbers. The theory will be anomaly-free if the matter supermultiplets transform according to

$$3\times 27_L+n(27_L+27_L^*), \tag{C9}$$

where 27_L and $27_L^*\sim(27_R)^\dagger$ refer to 27-plets of E_6. Since the 27_L and 27_L^* pairs are vector, any submultiplets can have a string (or GUT) scale mass and decouple without breaking the U(1)$_\psi$ or introducing anomalies, and indeed in most string models one expects only parts of the $27_L+27_L^*$ to be present in the observable sector.

It is convenient to display the decomposition of the 27_L under the SU(5)$\times$U(1)$_\psi$ subgroup,

$$\begin{aligned} 27_L\to{}&(10,1)_L+(5^*,1)_L+(1,1)_L+(5,-2)_L+(5^*,-2)_L\\ &+(1,4)_L, \end{aligned} \tag{C10}$$

where the first and second quantities are the SU(5) multiplet and $\sqrt{24}Q_\psi$, respectively. In Eq. (C10), the $(10,1)_L+(5^*,1)_L$ constitutes an ordinary family, $(1,1)_L$ and $(1,4)_L$ are standard model singlets, and $(5,-2)_L+(5^*,-2)_L$ are exotic multiplets which form a vector pair under the standard model gauge group but are chiral under U(1)$_\psi$. In particular, $(5,-2)_L$ consists of D_L and h_2, where D is a color-triplet charge $-1/3$ quark and h_2 has the standard model quantum numbers of the H_2. Similarly, $(5^*,-2)_L$ consists of $\overline{D_L}$ and h_1, where h_1 has the quantum number of either the H_1 or a lepton doublet.

Any of the three h_1's and three h_2's have the appropriate quantum numbers to be the MSSM Higgs doublets. Furthermore, the $(1,4)_L$ could be the singlet S, with the two Yukawa couplings in Eq. (1) allowed by U(1)$_\psi$. An exotic $h_D\hat{S}\hat{D}\hat{\overline{D}}$ coupling, as in Eq. (58), is also allowed. Hence, a model consisting of three 27-plets has most of the ingredients needed to display the considerations of this paper, albeit with additional singlets and $(5,-2)_L+(5^*,-2)_L$ pairs.

The model as such is not consistent with the observed approximate gauge unification. The two extra $(5,-2)_L+(5^*,-2)_L$ pairs and the singlets do not affect the standard model gauge unification at one-loop. However, the D and $\overline{D}$ associated with the two Higgs doublets destroy the unification, and they cannot be made superheavy without breaking the U(1)$_\psi$ and also introducing anomalies in the effective low-energy theory.

Gauge unification can be restored without introducing anomalies by adding a single $27_L+27_L^*$ pair, and assuming, for example, that only the Higgs-like doublets h_2 and h_3 associated with the $(5,-2)_L$ (from 27_L) and $(5^*,+2)_L$ (from 27_L^*) remain in the observable sector. The h_2 is equivalent to the h_2's from the other 27-plets, while the h_3 is similar to the h_1 multiplets, except that it has the opposite Q_ψ. The h_3 is not a candidate for the H_1, because its Q_ψ would not allow the Yukawa interactions in Eq. (1) needed to generate an effective μ [an elementary μ is allowed by U(1)$_\psi$ is this case] or the effective Yukawa interactions (e.g., generated by higher-dimension terms in the superpotential) for the down-type quarks and electrons. Thus, in this model the Higgs multiplets (or at least H_1) are not associated with the extra $27_L+27_L^*$, although the latter are needed for gauge unification. This is not an *ad hoc* assumption, but a consequence of the allowed couplings; the model actually has eight Higgs-like doublets, four h_2's, three h_1's, and one h_3. Assuming positive soft mass squares at the Planck scale, the only fields to actually acquire VEV's will be those which have the necessary Yukawa interactions in Eq. (1) and possibly Eq. (58), i.e., an h_1 and h_2 pair.

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