

Nonleptonic weak decays of bottom baryons

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Cabibbo-allowed two-body hadronic weak decays of bottom baryons are analyzed. Contrary to the charmed baryon sector, many channels of bottom baryon decays proceed only through the external or internal W -emission diagrams. Moreover, W exchange is likely to be suppressed in the bottom baryon sector. Consequently, the factorization approach suffices to describe most of the Cabibbo-allowed bottom baryon decays. We use the nonrelativistic quark model to evaluate heavy-to-heavy and heavy-to-light baryon form factors at zero recoil. When applied to the heavy quark limit, the quark model results do satisfy all the constraints imposed by heavy quark symmetry. The decay rates and up-down asymmetries for bottom baryons decaying into $\frac{1}{2}^+ + P(V)$ and $\frac{3}{2}^+ + P(V)$ are calculated. It is found that the up-down asymmetry is negative except for $\Omega_b \rightarrow \frac{1}{2}^+ + P(V)$ decay and for decay modes with ψ' in the final state. The prediction $\mathcal{B}(\Lambda_b \rightarrow J/\psi\Lambda) = 1.6 \times 10^{-4}$ for $|V_{cb}| = 0.038$ is consistent with the recent Collider Detector at Fermilab measurement. We also present estimates for $\Omega_c \rightarrow \frac{3}{2}^+ + P(V)$ decays and compare with various model calculations. [S0556-2821(97)03617-5]

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I. INTRODUCTION

While much new data of charmed baryon nonleptonic weak decays has become available in recent years, the experimental study of hadronic weak decays of bottom baryons is just beginning to get in gear. This is best illustrated by the decay mode $\Lambda_b \rightarrow J/\psi\Lambda$ which is interesting both experimentally and theoretically. Its branching ratio was originally measured by the UA1 Collaboration to be $(1.8 \pm 1.1) \times 10^{-2}$ [1]. However, both the Collider Detector at Fermilab (CDF) [2] and CERN e^+e^- collider LEP [3] Collaborations did not see any evidence for this decay. The theoretical situation is equally ambiguous: The predicted branching ratio ranges from 10^{-3} to 10^{-5} . Two early estimates [4,5] based on several different approaches for treating the $\Lambda_b \rightarrow \Lambda$ form factors yield a branching ratio of order 10^{-3} . It was reconsidered in [6] within the nonrelativistic quark model by taking into account the $1/m_Q$ corrections to baryonic form factors at zero recoil and the result $\mathcal{B}(\Lambda_b \rightarrow J/\psi\Lambda) = 1.1 \times 10^{-4}$ was obtained (see the erratum in [6]). Recently, it was found that $\mathcal{B}(\Lambda_b \rightarrow J/\psi\Lambda)$ is of order 10^{-5} in [7] by extracting form factors at zero recoil from experiment and in [8] by generalizing the Stech's approach for form factors to the baryon case. This issue has finally settled down experimentally: The decay $\Lambda_b \rightarrow J/\psi\Lambda$ is observed by CDF [9] and the ratio of cross section times branching fraction, $\sigma_{\Lambda_b} \mathcal{B}(\Lambda_b \rightarrow J/\psi\Lambda) / [\sigma_{B^0} \mathcal{B}(B^0 \rightarrow J/\psi K_S)]$ is measured. The branching ratio of $\Lambda_b \rightarrow J/\psi\Lambda$ turns out to be $(3.7 \pm 1.7 \pm 0.4) \times 10^{-4}$, assuming $\sigma_{\Lambda_b} / \sigma_B = 0.1/0.375$ and $\mathcal{B}(B^0 \rightarrow J/\psi K_S) = 3.7 \times 10^{-4}$. It is interesting to note that this is also the first successful measurement of exclusive hadronic decay rate of bottom baryons, even though the branching ratio of $\Lambda_b \rightarrow \Lambda\pi$ is expected to exceed that of $\Lambda_b \rightarrow J/\psi\Lambda$ by an order of magnitude. Needless to say, more and more data of bottom baryon decay data will be accumulated in the near future.

Encouraged by the consistency between experiment and

our nonrelativistic quark model calculations for $\Lambda_b \rightarrow J/\psi\Lambda$, we would like to present in this work a systematic study of exclusive nonleptonic decays of bottom baryons (for earlier studies, see [10,11]). Just as in the meson case, all hadronic weak decays of baryons can be expressed in terms of the following quark-diagram amplitudes [12]: \mathcal{A} , the external W -emission diagram; \mathcal{B} , the internal W -emission diagram; \mathcal{C} , the W -exchange diagram; and \mathcal{E} , the horizontal W -loop diagram. The external and internal W -emission diagrams are sometimes referred to as color-allowed and color-suppressed factorizable contributions. However, baryons being made out of three quarks, in contrast to two quarks for mesons, bring along several essential complications. First of all, the factorization approximation that the hadronic matrix element is factorized into the product of two matrix elements of single currents and that the nonfactorizable term such as the W -exchange contribution is negligible relative to the factorizable one is known empirically to be working reasonably well for describing the nonleptonic weak decays of heavy mesons [13]. However, this approximation is *a priori* not directly applicable to the charmed baryon case as W exchange there, manifested as pole diagrams, is no longer subject to helicity and color suppression.¹ That is, the pole contribution can be as important as the factorizable one. The

¹This is different from the naive color suppression of internal W emission. It is known in the heavy meson case that nonfactorizable contributions will render the color suppression of internal W emission ineffective. However, the W -exchange in baryon decays is not subject to color suppression even in the absence of nonfactorizable terms. A simple way to see this is to consider the large- N_c limit. Although the W exchange diagram is down by a factor of $1/N_c$ relative to the external W -emission one, it is compensated by the fact that the baryon contains N_c quarks in the limit of large N_c , thus allowing N_c different possibilities for W exchange between heavy and light quarks [14].

experimental measurement of the decay modes $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$, $\Sigma^+ \pi^0$, and $\Lambda_c^+ \rightarrow \Xi^0 K^+$, which do not receive any factorizable contributions, indicates that W exchange indeed plays an essential role in charmed baryon decays. Second, there are more possibilities in drawing the \mathcal{B} and \mathcal{C} types of amplitudes [12]; in general there exist two distinct internal W emissions and several different W -exchange diagrams and only one of the internal W -emission amplitudes is factorizable.

The nonfactorizable pole contributions to hadronic weak decays of charmed baryons have been studied in the literature [15–17]. In general, nonfactorizable s - and p -wave amplitudes for $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + P(V)$ decays (P : pseudoscalar meson, V : vector meson), for example, are dominated by $\frac{1}{2}^-$ low-lying baryon resonances and $\frac{1}{2}^+$ ground-state baryon poles, respectively. However, the estimation of pole amplitudes is a difficult and nontrivial task since it involves weak baryon matrix elements and strong-coupling constants of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ baryon states. This is the case in particular for s -wave terms as we know very little about the $\frac{1}{2}^-$ states. As a consequence, the evaluation of pole diagrams is far more uncertain than the factorizable terms. Nevertheless, the bottom baryon system has some advantages over the charmed baryon one. First, W exchange is expected to be less important in the nonleptonic decays of the former. The argument goes as follows. The W -exchange contribution to the total decay width of the heavy baryon relative to the spectator diagram is of order $R = 32\pi^2 |\psi_{Qq}(0)|^2 / m_Q^3$ [5], where the square of the wave function $|\psi_{Qq}(0)|^2$ determines the probability of finding a light quark q at the location of the heavy quark Q . Since $|\psi_{cq}(0)|^2 \sim |\psi_{bq}(0)|^2 \sim (1-2) \times 10^{-2} \text{ GeV}^2$ [5], it is clear that R is of order unity in the charmed baryon case, while it is largely suppressed in bottom baryon decays. Therefore, although W exchange plays a dramatic role in charmed baryon case (it even dominates over the spectator contribution in hadronic decays of Λ_c^+ and Ξ_c^0 [5]), it becomes negligible in inclusive hadronic decays of bottom baryons. It is thus reasonable to assume that the same suppression is also inherited in the two-body nonleptonic weak decays of bottom baryons. Second, for charmed baryon decays, there are only a few decay modes which proceed through external or internal W -emission diagram, namely, Cabibbo-allowed $\Omega_c^0 \rightarrow \Omega^- \pi^+ (\rho^+)$, $\Xi^{*0} \bar{K}^0 (\bar{K}^{*0})$ and Cabibbo-suppressed $\Lambda_c^+ \rightarrow p \phi$, $\Omega_c^0 \rightarrow \Xi^- \pi^+ (\rho^+)$. However, even at the Cabibbo-allowed level, there already exist a significant number of bottom baryon decays which receive contributions only from factorizable diagrams (see Tables II and III) and $\Lambda_b \rightarrow J/\psi \Lambda$ is one of the most noticeable examples. For these decay modes we can make a reliable estimate based on the factorization approach as they do not involve troublesome nonfactorizable pole terms. Moreover, with the aforementioned suppression of W exchange, many decay channels are dominated by external or internal W emission. Consequently, contrary to the charmed baryon case, it suffices to apply the factorization hypothesis to describe most of Cabibbo-allowed two-body nonleptonic decays of bottom baryons, and this makes the study of bottom baryon decays considerably simpler than that in charmed baryon decays.

Under the factorization approximation, the baryon decay amplitude is governed by a decay constant and form factors.

In order to study heavy-to-heavy and heavy-to-light baryon form factors, we will follow [6] to employ the nonrelativistic quark model to evaluate the form factors at zero recoil. Of course, the quark model results should be in agreement with the predictions of the heavy quark effective theory (HQET) for antitriplet-to-antitriplet heavy baryon form factors to the first order in $1/m_Q$ and for sextet-to-sextet ones to the zeroth order in $1/m_Q$. The quark model, however, has the merit that it is applicable to heavy-to-light baryonic transitions as well and accounts for $1/m_Q$ effects for sextet-to-sextet heavy baryon transition. In this paper, we will generalize the work of [6] to $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + P(V)$ transitions in order to study the decays $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + P(V)$. As the conventional practice, we then make the pole dominance assumption for the q^2 dependence to extrapolate the form factor from zero recoil to the desired q^2 point.

The layout of the present paper is organized as follows. In Sec. II we first discuss the quark-diagram amplitudes for Cabibbo-allowed bottom baryon decays. Then with the form factors calculated using the nonrelativistic quark model, the external and internal W -emission amplitudes are computed under the factorization approximation. Results of model calculations and their physical implications are discussed in Sec. III. A detail of the quark model evaluation of form factors is presented in Appendix A and the kinematics for nonleptonic decays of baryons is summarized in Appendix B.

II. NONLEPTONIC WEAK DECAYS OF BOTTOM BARYONS

A. Quark diagram classification

The light quarks of the bottom baryons belong to either a $\bar{\mathbf{3}}$ or a $\mathbf{6}$ representation of the flavor SU(3). The Λ_b^+ , Ξ_b^{0A} , and Ξ_b^{-A} form a $\bar{\mathbf{3}}$ representation and they all decay weakly. The Ω_b^+ , Ξ_b^{0S} , Ξ_b^{-S} , $\Sigma_b^{+,0,-}$ form a $\mathbf{6}$ representation; among them, however, only Ω_b^+ decays weakly.

Denoting the bottom baryon, charmed baryon, octet baryon, decuplet baryon and octet meson by B_b , B_c , $B(\mathbf{8})$, $B(\mathbf{10})$ and $M(\mathbf{8})$, respectively, the two-body nonleptonic decays of bottom baryon can be classified into

$$\begin{aligned}
 \text{(a)} \quad & B_b(\bar{\mathbf{3}}) \rightarrow B_c(\bar{\mathbf{3}}) + M(\mathbf{8}), \\
 \text{(b)} \quad & B_b(\bar{\mathbf{3}}) \rightarrow B_c(\mathbf{6}) + M(\mathbf{8}), \\
 \text{(c)} \quad & B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{8}) + M(\mathbf{8}), \\
 \text{(d)} \quad & B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10}) + M(\mathbf{8}),
 \end{aligned} \tag{2.1}$$

and

$$\begin{aligned}
 \text{(e)} \quad & B_b(\mathbf{6}) \rightarrow B_c(\mathbf{6}) + M(\mathbf{8}), \\
 \text{(f)} \quad & B_b(\mathbf{6}) \rightarrow B_c^*(\mathbf{6}) + M(\mathbf{8}), \\
 \text{(g)} \quad & B_b(\mathbf{6}) \rightarrow B_c(\bar{\mathbf{3}}) + M(\mathbf{8}),
 \end{aligned} \tag{2.2}$$

- (h) $B_b(\mathbf{6}) \rightarrow B(\mathbf{8}) + M(\mathbf{8})$,
- (i) $B_b(\mathbf{6}) \rightarrow B(\mathbf{10}) + M(\mathbf{8})$,

where B_c^* designates a spin- $\frac{3}{2}$ sextet charmed baryon. In [12] we have given a general formulation of the quark-diagram scheme for the nonleptonic weak decays of charmed baryons, which can be generalized directly to the bottom baryon case. The general quark diagrams for decays in Eqs. (2.1) and (2.2) are the external W -emission \mathcal{A} , internal W -emission diagrams \mathcal{B} and \mathcal{B}' , W -exchange diagrams \mathcal{C}_1 , \mathcal{C}_2 , and \mathcal{C}' , and the horizontal W -loop diagrams \mathcal{E} and \mathcal{E}' (see Fig. 2 of [12] for notation and details).² The quark coming from the bottom quark decay in diagram \mathcal{B}' contributes to the final meson formation, whereas it contributes to the final baryon formation in diagram \mathcal{B} . Consequently, diagram \mathcal{B}' contains factorizable contributions but \mathcal{B} does not. Note that, contrary to the charmed baryon case, the horizontal W -loop diagrams (or the so-called penguin diagrams under one-gluon-exchange approximation) can contribute to some of Cabibbo-allowed decays of bottom baryons. Since the two spectator light quarks in the heavy baryon are antisymmetrized in $B_Q(\bar{\mathbf{3}})$ and symmetrized in $B_Q(\mathbf{6})$ and since the wave function of $B(\mathbf{10})$ is totally symmetric, it is clear that factorizable amplitudes \mathcal{A} and \mathcal{B}' cannot contribute to the decays of types (b), (d), and (g). For example, decays of type (d) receive contributions only from the W -exchange and W -loop diagrams, namely \mathcal{C}_{2S} , \mathcal{C}'_S , and \mathcal{E}_S (see Fig. 1 of [12]). There are only a few Cabibbo-allowed $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10}) + M(\mathbf{8})$ decays:

$$\Lambda_b^0 \rightarrow D^0 \Delta^0, \quad D^{*0} \Delta^0; \quad \Xi_b^{0,-} \rightarrow D^0 \Sigma^{*0,-}, \quad D^{*0} \Sigma^{*0,-}. \quad (2.3)$$

They all only receive contributions from the W -exchange diagram \mathcal{C}'_S . We have shown in Tables II and III the quark diagram amplitudes for those Cabibbo-allowed bottom baryon decays that do receive contributions from the external W -emission \mathcal{A} or internal W -emission \mathcal{B}' .

B. Factorizable contributions

At the quark level, the hadronic decays of bottom baryons proceed through the above-mentioned various quark diagrams. At the hadronic level, the decay amplitudes are conventionally evaluated using factorization approximation for quark diagrams \mathcal{A} and \mathcal{B}' and pole approximation for the remaining diagrams \mathcal{B} , \mathcal{C}_1 , \mathcal{C}_2 , \dots [14–17]. Among all possible pole contributions, including resonances and continuum states, one usually focuses on the most important poles such as the low-lying $\frac{1}{2}^+$, $\frac{1}{2}^-$ states. However, it is difficult to make a reliable estimate of pole contributions since they involve baryon matrix elements and strong coupling constants of the pole states. Fortunately, among the 32 decay modes of

Cabibbo-allowed decays $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + P(V)$ listed in Table II and 8 channels of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + P(V)$ in Table III, 20 of them receive contributions only from factorizable terms. Furthermore, as discussed in the Introduction, the W -exchange contribution to the inclusive decay rate of bottom baryons relative to the spectator decay is of order $32\pi^2 |\psi_{bq}(0)|^2 / m_b^3 \sim (3-5)\%$. It is thus reasonable to assume that the same suppression persists at the exclusive two-body decay level. The penguin contributions \mathcal{E} and \mathcal{E}' to the Cabibbo-allowed decay modes, e.g., $\Lambda_b \rightarrow D_s^{(*)} \Lambda_c$, $\Xi_b \rightarrow D_s^{(*)} \Xi_c$, $\Omega_b \rightarrow D_s^{(*)} \Omega_c$ (see Table II) can be safely neglected since the Wilson coefficient $c_6(m_b)$ of the penguin operator O_6 is of order 0.04 [18] and there is no chiral enhancement in the hadronic matrix element of O_6 due to the absence of a light meson in the final state. Therefore, by neglecting the W -exchange contribution as a first-order approximation, we can make sensible predictions for most of decay modes exhibited in Tables II and III. As for the non-factorizable internal W -emission \mathcal{B} , there is no reason to argue that it is negligible.

To proceed we first consider the Cabibbo-allowed decays $B_b(\frac{1}{2}^+) \rightarrow B_f(\frac{1}{2}^+) + P(V)$. The general amplitudes are

$$\mathcal{M}[B_i(1/2^+) \rightarrow B_f(1/2^+) + P] = i \bar{u}_f(p_f) (A + B \gamma_5) u_i(p_i), \quad (2.4)$$

$$\begin{aligned} \mathcal{M}[B_i(1/2^+) \rightarrow B_f(1/2^+) + V] \\ = \bar{u}_f(p_f) \varepsilon^{*\mu} [A_1 \gamma_\mu \gamma_5 + A_2 (p_f)_\mu \gamma_5 \\ + B_1 \gamma_\mu + B_2 (p_f)_\mu] u_i(p_i), \end{aligned}$$

where ε_μ is the polarization vector of the vector meson. The QCD-corrected weak Hamiltonian responsible for Cabibbo-allowed hadronic decays of bottom baryons read

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (c_1 O_1 + c_2 O_2) + (u \rightarrow c, \quad d \rightarrow s), \quad (2.5)$$

with $O_1 = (\bar{u}s)(\bar{b}c)$ and $O_2 = (\bar{u}c)(\bar{b}s)$, where $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. Under factorization approximations, the external or internal W emission contributions to the decay amplitudes are given by

$$\begin{aligned} A &= \lambda a_{1,2} f_P (m_i - m_f) f_1(m_P^2), \\ B &= \lambda a_{1,2} f_P (m_i + m_f) g_1(m_P^2), \end{aligned} \quad (2.6)$$

and

$$\begin{aligned} A_1 &= -\lambda a_{1,2} f_V m_V [g_1(m_V^2) + g_2(m_V^2)(m_i - m_f)], \\ A_2 &= -2\lambda a_{1,2} f_V m_V g_2(m_V^2), \\ B_1 &= \lambda a_{1,2} f_V m_V [f_1(m_V^2) - f_2(m_V^2)(m_i + m_f)], \\ B_2 &= 2\lambda a_{1,2} f_V m_V f_2(m_V^2), \end{aligned} \quad (2.7)$$

²The quark diagram amplitudes \mathcal{A} , \mathcal{B} , \mathcal{B}' , \dots , in each type of hadronic decays are in general not the same. For octet baryons in the final state, each of the W -exchange and W -loop amplitudes has two more independent types: the symmetric and the antisymmetric, for example, \mathcal{C}_{1A} , \mathcal{C}_{1S} , \mathcal{E}_A , \mathcal{E}_S , \dots , etc. [12].

where $\lambda = G_F V_{cb} V_{ud}^* / \sqrt{2}$ or $G_F V_{cb} V_{cs}^* / \sqrt{2}$, depending on the final meson state under consideration, f_i and g_i are the form factors defined by ($q = p_i - p_f$)

$$\begin{aligned} & \langle B_f(p_f) | V_\mu - A_\mu | B_i(p_i) \rangle \\ &= \bar{u}_f(p_f) \{ f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu \\ & \quad - [g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q_\mu] \gamma_5 \} u_i(p_i), \end{aligned} \quad (2.8)$$

$m_i(m_f)$ is the mass of the initial (final) baryon, f_P and f_V are the decay constants of pseudoscalar and vector mesons, respectively, defined by

$$\begin{aligned} \langle 0 | A_\mu | P \rangle &= -\langle P | A_\mu | 0 \rangle = i f_P q_\mu, \\ \langle 0 | V_\mu | V \rangle &= \langle V | V_\mu | 0 \rangle = f_V m_V \varepsilon_\mu^*, \end{aligned} \quad (2.9)$$

with the normalization $f_\pi = 132$ MeV.

Since in this paper we rely heavily on the factorization approximation to describe bottom baryon decay, we digress for a moment to discuss its content. In the naive factorization approach, the coefficients a_1 for the external W emission amplitude and a_2 for internal W -emission are given by $(c_1 + c_2/3)$ and $(c_2 + c_1/3)$, respectively. However, we have learned from charm decay that the naive factorization approach never works for the decay rate of color-suppressed decay modes, though it usually operates for color-allowed decays. For example, the predicted rate of $\Lambda_c^+ \rightarrow p \phi$ in the naive approach is too small when compared with experiment [15]. This implies that the inclusion of nonfactorizable contributions is inevitable and necessary. If nonfactorizable effects amount to a redefinition of the effective parameters a_1 , a_2 and are universal (i.e., channel independent) in charm or bottom decays, then we still have a new factorization scheme with the universal parameters a_1 , a_2 to be determined from experiment. Throughout this paper, we will thus treat a_1 and a_2 as free effective parameters. The factorization hypothesis implies universal and channel independent a_1^{eff} and a_2^{eff} in charm or bottom decay.³

Since we shall consider heavy-to-heavy and heavy-to-light baryonic transitions, it is clear that HQET is not adequate for our purposes: the predictive power of HQET for baryon form factors at order $1/m_Q$ is limited only to antitriplet-to-antitriplet heavy baryonic transition. Hence, we will follow [6] to apply the nonrelativistic quark model to

evaluate the weak current-induced baryon form factors at zero recoil in the rest frame of the heavy parent baryon, where the quark model is most trustworthy. This quark model approach has the merit that it is applicable to heavy-to-heavy and heavy-to-light baryonic transitions at maximum q^2 and that it becomes meaningful to consider $1/m_q$ corrections so long as the recoil momentum is smaller than the m_q scale.

The complete quark model results for form factors f_i and g_i at zero recoil read [6]

$$\begin{aligned} f_1(q_m^2)/N_{fi} &= 1 - \frac{\Delta m}{2m_i} + \frac{\Delta m}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) (m_i + m_f - \eta \Delta m) \\ & \quad - \frac{\Delta m}{8m_i m_f m_Q} \bar{\Lambda} (m_i + m_f + \eta \Delta m), \\ f_2(q_m^2)/N_{fi} &= \frac{1}{2m_i} + \frac{1}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) [\Delta m - (m_i + m_f) \eta] \\ & \quad - \frac{\bar{\Lambda}}{8m_i m_f m_Q} [\Delta m + (m_i + m_f) \eta], \\ f_3(q_m^2)/N_{fi} &= \frac{1}{2m_i} - \frac{1}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) (m_i + m_f - \eta \Delta m) \\ & \quad + \frac{\bar{\Lambda}}{8m_i m_f m_Q} (m_i + m_f + \eta \Delta m), \quad (2.10) \\ g_1(q_m^2)/N_{fi} &= \eta + \frac{\Delta m \bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q} \right) \eta, \\ g_2(q_m^2)/N_{fi} &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q} \right) \eta, \\ g_3(q_m^2)/N_{fi} &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} + \frac{1}{m_f m_Q} \right) \eta, \end{aligned}$$

where $\bar{\Lambda} = m_f - m_q$, $\Delta m = m_i - m_f$, $q_m^2 = \Delta m^2$, $\eta = 1$ for the $\bar{\mathbf{3}}$ baryon B_i , and $\eta = -\frac{1}{3}$ for the $\mathbf{6}$ baryon B_i , and N_{fi} is a flavor factor:

$$N_{fi} = \text{flavor-spin} \langle B_f | b_q^\dagger b_Q | B_i \rangle_{\text{flavor-spin}} \quad (2.11)$$

for the heavy quark Q in the parent baryon B_i transiting into the quark q (being a heavy quark Q' or a light quark) in the daughter baryon B_f . It has been shown in [6] that the quark model predictions agree with HQET for antitriplet-to-antitriplet (e.g., $\Lambda_b \rightarrow \Lambda_c$, $\Xi_b \rightarrow \Xi_c$) form factors to order $1/m_Q$. For sextet $\Sigma_b \rightarrow \Sigma_c$ and $\Omega_b \rightarrow \Omega_c$ transitions, the HQET predicts that, to the zeroth order in $1/m_Q$ (see e.g., [22]),

³For $D(B) \rightarrow PP$ or VP decays (P denotes a pseudoscalar meson, V a vector meson), nonfactorizable effects can always be lumped into the effective parameters a_1 and a_2 . For $D(B) \rightarrow VV$ and heavy baryon decays, universal nonfactorizable terms are assumed under the factorization approximation. The first systematical study of heavy meson decays within the framework of improved factorization was carried out by Bauer, Stech, and Wirbel [19]. Theoretically, nonfactorizable terms come mainly from color-octet currents. Phenomenological analyses of D and B decay data [20,21] indicate that while the factorization hypothesis in general works reasonably well, the effective parameters $a_{1,2}$ do show some variations from channel to channel.

$$\begin{aligned}
& \langle B_f(v', s') | V_\mu | B_i(v, s) \rangle \\
&= -\frac{1}{3} \bar{u}_f(v', s') \{ [\omega \gamma_\mu - 2(v+v')_\mu] \xi_1(\omega) \\
&\quad + [(1-\omega^2) \gamma_\mu - 2(1-\omega)(v+v')_\mu] \xi_2(\omega) \} u_i(v, s), \\
& \langle B_f(v', s') | A_\mu | B_i(v, s) \rangle \\
&= \frac{1}{3} \bar{u}_f(v', s') \{ [\omega \gamma_\mu + 2(v-v')_\mu] \xi_1(\omega) \\
&\quad + [(1-\omega^2) \gamma_\mu - 2(1+\omega)(v-v')_\mu] \xi_2(\omega) \} u_i(v, s),
\end{aligned} \tag{2.12}$$

where $\omega \equiv v \cdot v'$, ξ_1 and ξ_2 are two universal baryon Isgur-Wise functions with the normalization of ξ_1 known to be $\xi_1(1)=1$. From Eq. (2.12) we obtain

$$\begin{aligned}
f_1 &= F_1 + \frac{1}{2} (m_i + m_f) \left(\frac{F_2}{m_i} + \frac{F_3}{m_f} \right), \\
f_2 &= \frac{1}{2} \left(\frac{F_2}{m_i} + \frac{F_3}{m_f} \right), \\
f_3 &= \frac{1}{2} \left(\frac{F_2}{m_i} - \frac{F_3}{m_f} \right), \\
g_1 &= G_1 - \frac{1}{2} (m_i - m_f) \left(\frac{G_2}{m_i} + \frac{G_3}{m_f} \right), \\
g_2 &= \frac{1}{2} \left(\frac{G_2}{m_i} + \frac{G_3}{m_f} \right), \\
g_3 &= \frac{1}{2} \left(\frac{G_2}{m_i} - \frac{G_3}{m_f} \right),
\end{aligned} \tag{2.13}$$

with

$$\begin{aligned}
F_1 &= -G_1 = -\frac{1}{3} [\omega \xi_1 + (1-\omega^2) \xi_2], \\
F_2 &= F_3 = \frac{2}{3} [\xi_1 + (1-\omega) \xi_2], \\
G_2 &= -G_3 = \frac{2}{3} [\xi_1 - (1+\omega) \xi_2].
\end{aligned} \tag{2.14}$$

Since $N_{fi}=1$ and $\eta=1$ for sextet-to-sextet transition, it follows from Eq. (2.10) that

$$\begin{aligned}
f_1(q_m^2) &= -\frac{1}{3} \left[1 - (m_i + m_f) \left(\frac{1}{m_i} + \frac{1}{m_f} \right) \right], \\
f_2(q_m^2) &= \frac{1}{3} \left(\frac{1}{m_i} + \frac{1}{m_f} \right), \quad f_3(q_m^2) = \frac{1}{3} \left(\frac{1}{m_i} - \frac{1}{m_f} \right), \\
g_1(q_m^2) &= -\frac{1}{3}, \quad g_2(q_m^2) = g_3(q_m^2) = 0.
\end{aligned} \tag{2.15}$$

It is easily seen that at zero recoil $\omega=1$, the quark model results (2.15) are in accord with the HQET predictions (2.13) provided that

$$\xi_2(1) = \frac{1}{2} \xi_1(1) = \frac{1}{2}. \tag{2.16}$$

This is precisely the prediction of large- N_c QCD [23].

Three remarks are in order. First, there are two different quark model calculations of baryon form factors [24,25] prior to the work [6]. An obvious criterion for testing the reliability of quark model calculations is that model results must satisfy all the constraints imposed by heavy quark symmetry. In the heavy quark limit, normalizations of heavy-to-heavy form factors and hence some relations between form factors at zero recoil are fixed by heavy quark symmetry. These constraints are not respected in [24]. While this discrepancy is improved in the work of [25], its prediction for $\Lambda_b \rightarrow \Lambda_c$ (or $\Xi_b \rightarrow \Xi_c$) form factors at order $1/m_Q$ is still too large by a factor of 2 when compared with HQET [6]. Second, the flavor factor N_{fi} (2.11) for heavy-to-light transition is usually smaller than unity (see Table I) due to the fact that SU(N) flavor symmetry is badly broken. As stressed in [25–27], it is important to take into account this flavor-suppression factor when evaluating the heavy-to-light baryon form factors. Third, in deriving the baryon matrix elements at zero recoil in the rest frame of the parent baryon, we have neglected the kinetic energy (KE) of the quark participating weak transition relative to its constituent mass M_q . This is justified in the nonrelativistic constituent quark model even when the final baryon is a hyperon or a nucleon. The kinetic energy of the QCD current quark inside the nucleon at rest is of order a few hundred MeV. In the nonrelativistic quark model this kinetic energy is essentially absorbed in the constituent mass of the constituent quark. As a result, it is a good approximation to neglect (KE/M_q) for the constituent quarks inside the nucleon (or hyperon) at rest. Of course, this approximation works best for $Q \rightarrow Q'$ transition, and fairly good for $Q \rightarrow s$ or $Q \rightarrow u(d)$ transition.

We next turn to the Cabibbo-allowed decays $B_b(\frac{1}{2}^+) \rightarrow B_f^*(\frac{3}{2}^+) + P(V)$ with the general amplitudes:

$$\begin{aligned}
& \mathcal{M}[B_i(1/2^+) \rightarrow B_f^*(3/2^+) + P] \\
&= i q_\mu \bar{u}_f^\mu(p_f) (C + D \gamma_5) u_i(p_i), \\
& \mathcal{M}[B_i(1/2^+) \rightarrow B_f^*(3/2^+) + V] \\
&= \bar{u}_f^\nu(p_f) \varepsilon^{*\mu} [g_{\nu\mu} (C_1 + D_1 \gamma_5) + p_{1\nu} \gamma_\mu (C_2 + D_2 \gamma_5) \\
&\quad + p_{1\nu} p_{2\mu} (C_3 + D_3 \gamma_5)] u_i(p_i),
\end{aligned} \tag{2.17}$$

with u^μ being the Rarita-Schwinger vector spinor for a spin- $\frac{3}{2}$ particle. The external and internal W -emission contributions under factorization approximation become

TABLE I. Nonrelativistic quark model predictions for baryonic form factors evaluated at $q^2=0$ using dipole q^2 dependence (m_i being the mass of the parent heavy baryon). Also shown are the spin and flavor factors for various baryonic transitions. Our flavor factors N_{fi} for $\Omega_c^0 \rightarrow \Omega^-$ and $\Omega_c^0 \rightarrow \Xi^{*0}$ are two times smaller than that in [32].

Transition	η	N_{fi}	$f_1(0)$	$f_2(0)m_i$	$f_3(0)m_i$	$g_1(0)$	$g_2(0)m_i$	$g_3(0)m_i$
$\Lambda_b^0 \rightarrow \Lambda_c^+$	1	1	0.530	-0.100	-0.012	0.577	-0.013	-0.109
$\Lambda_b^0 \rightarrow \Lambda^0$	1	$\frac{1}{\sqrt{3}}$	0.062	-0.025	-0.008	0.108	-0.014	-0.043
$\Lambda_b^0 \rightarrow n$	1	$\frac{1}{\sqrt{2}}$	0.045	-0.024	-0.011	0.095	-0.022	-0.051
$\Xi_b^{0,-} \rightarrow \Xi_c^{+,0}$	1	1	0.533	-0.124	-0.018	0.580	-0.019	-0.135
$\Xi_b^{0,-} \rightarrow \Xi^{0,-}$	1	$\frac{1}{\sqrt{2}}$	0.083	-0.041	-0.016	0.143	-0.027	-0.070
$\Xi_b^{0,-} \rightarrow \Sigma^{0,-}$	1	$\frac{1}{2}$	0.042	-0.028	-0.014	0.083	-0.028	-0.054
$\Xi_b^0 \rightarrow \Lambda^0$	1	$\frac{1}{2\sqrt{3}}$	0.019	-0.012	-0.006	0.041	-0.013	-0.025
$\Omega_b^- \rightarrow \Omega_c^0$	$-\frac{1}{3}$	1	0.710	0.666	-0.339	-0.195	0.009	0.056
$\Omega_b^- \rightarrow \Xi^-$	$-\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	0.102	0.103	-0.097	-0.028	0.011	0.019
$\Omega_b^- \rightarrow \Omega_c^{*0}$		1	0.902	0.451	-0.451	-0.606	-0.237	0.490
$\Omega_b^- \rightarrow \Omega^-$		1	0.320	0.160	-0.160	-0.228	-0.260	0.257
$\Omega_b^- \rightarrow \Xi^{*-}$		$\frac{1}{\sqrt{3}}$	0.158	0.079	-0.079	-0.094	-0.177	0.141
$\Omega_c^0 \rightarrow \Omega^-$		1	1.167	0.837	-0.837	-0.804	-0.916	1.006
$\Omega_c^0 \rightarrow \Xi^{*0}$		$\frac{1}{\sqrt{3}}$	0.942	0.471	-0.471	-0.390	-0.731	0.634

$$C = -\lambda a_{1,2} f_P [\bar{g}_1(m_P^2) + (m_i - m_f) \bar{g}_2(m_P^2) + (m_i E_f - m_f^2) \bar{g}_3(m_P^2)],$$

$$D = \lambda a_{1,2} f_P [\bar{f}_1(m_P^2) - (m_i + m_f) \bar{f}_2(m_P^2) + (m_i E_f - m_f^2) \bar{f}_3(m_P^2)], \quad (2.18)$$

and

$$C_i = -\lambda a_{1,2} f_V m_V \bar{g}_i(m_V^2), \quad D_i = \lambda a_{1,2} f_V m_V \bar{f}_i(m_V^2), \quad (2.19)$$

where $i=1,2,3$, and the form factors \bar{f}_i as well as \bar{g}_i are defined by

$$\begin{aligned} \langle B_f^*(p_f) | V_\mu - A_\mu | B_i(p_i) \rangle = & \bar{u}_f^\nu \{ [\bar{f}_1(q^2) g_{\nu\mu} + \bar{f}_2(q^2) p_{1\nu} \gamma_\mu \\ & + \bar{f}_3(q^2) p_{1\nu} p_{2\mu}] \gamma_5 \\ & - [\bar{g}_1(q^2) g_{\nu\mu} + \bar{g}_2(q^2) p_{1\nu} \gamma_\mu \\ & + \bar{g}_3(q^2) p_{1\nu} p_{2\mu}] \} u_i. \quad (2.20) \end{aligned}$$

In deriving Eq. (2.18) we have applied the constraint $p_\nu u^\nu(p) = 0$. As before, form factors are evaluated at zero recoil using the nonrelativistic quark model and the results are (see Appendix A for detail)

$$\bar{f}_1(q_m^2)/N_{fi} = \frac{2}{\sqrt{3}} \left(1 + \frac{\bar{\Lambda}}{2m_q} + \frac{\bar{\Lambda}}{2m_Q} \right),$$

$$\bar{f}_2(q_m^2)/N_{fi} = \frac{1}{\sqrt{3}m_i} \left(1 + \frac{\bar{\Lambda}}{2m_q} + \frac{\bar{\Lambda}}{2m_Q} \right),$$

$$\bar{f}_3(q_m^2)/N_{fi} = -\frac{1}{\sqrt{3}m_i m_f} \left(1 + \frac{\bar{\Lambda}}{2m_q} + \frac{\bar{\Lambda}}{2m_Q} \right),$$

$$\bar{g}_1(q_m^2)/N_{fi} = -\frac{2}{\sqrt{3}}, \quad (2.21)$$

$$\bar{g}_2(q_m^2)/N_{fi} = -\frac{1}{\sqrt{3}} \frac{\bar{\Lambda}}{m_q m_i},$$

$$\bar{g}_3(q_m^2)/N_{fi} = -\bar{f}_3(q_m^2)/N_{fi}.$$

TABLE II. Factorizable contributions to the decay rates (in units of 10^{10} s^{-1}) and up-down asymmetries of Cabibbo-allowed nonleptonic weak decays of bottom baryons $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + P(V)$. Also shown are the quark-diagram amplitudes for various reactions.

Decay	Diagram	Γ	α	Decay	Diagram	Γ	α
$\Lambda_b^0 \rightarrow \pi^- \Lambda_c^+$	$\mathcal{A}, \mathcal{B}, \mathcal{C}_1, \mathcal{C}_2$	$0.31a_1^2$	-0.99	$\Xi_b^{0,-} \rightarrow \pi^- \Xi_c^{+,0}$	$\mathcal{A}, \mathcal{C}_2$	$0.33a_1^2$	-1.00
$\Lambda_b^0 \rightarrow \rho^- \Lambda_c^+$	$\mathcal{A}, \mathcal{B}, \mathcal{C}_1, \mathcal{C}_2$	$0.44a_1^2$	-0.88	$\Xi_b^{0,-} \rightarrow \rho^- \Xi_c^{+,0}$	$\mathcal{A}, \mathcal{C}_2$	$0.47a_1^2$	-0.88
$\Lambda_b^0 \rightarrow D_s^- \Lambda_c^+$	$\mathcal{A}, \mathcal{E}'$	$0.93a_1^2$	-0.99	$\Xi_b^{0,-} \rightarrow D_s^- \Xi_c^{+,0}$	$\mathcal{A}, \mathcal{C}', \mathcal{E}', \mathcal{E}$	$0.99a_1^2$	-0.99
$\Lambda_b^0 \rightarrow D_s^{*-} \Lambda_c^+$	$\mathcal{A}, \mathcal{E}'$	$0.74a_1^2$	-0.36	$\Xi_b^{0,-} \rightarrow D_s^{*-} \Xi_c^{+,0}$	$\mathcal{A}, \mathcal{C}', \mathcal{E}', \mathcal{E}$	$0.78a_1^2$	-0.36
$\Lambda_b^0 \rightarrow J/\psi \Lambda^0$	\mathcal{B}'	$0.17a_2^2$	-0.10	$\Xi_b^{0,-} \rightarrow J/\psi \Xi^{0,-}$	\mathcal{B}'	$0.32a_2^2$	-0.10
$\Lambda_b^0 \rightarrow \psi' \Lambda^0$	\mathcal{B}'	$0.14a_2^2$	0.05	$\Xi_b^{0,-} \rightarrow \psi' \Xi^{0,-}$	\mathcal{B}'	$0.27a_2^2$	0.05
$\Lambda_b^0 \rightarrow D^0 n$	$\mathcal{B}', \mathcal{C}'$	$0.024a_2^2$	-0.81	$\Xi_b^{0,-} \rightarrow D^0 \Sigma^{0,-}$	$\mathcal{B}'(\mathcal{C}')^a$	$0.020a_2^2$	-0.85
$\Lambda_b^0 \rightarrow D^{*0} n$	$\mathcal{B}', \mathcal{C}'$	$0.017a_2^2$	-0.42	$\Xi_b^{0,-} \rightarrow D^{*0} \Sigma^{0,-}$	$\mathcal{B}'(\mathcal{C}')^a$	$0.014a_2^2$	-0.45
$\Xi_b^0 \rightarrow D^0 \Lambda^0$	\mathcal{B}'	$0.005a_2^2$	-0.81	$\Xi_b^0 \rightarrow D^{*0} \Lambda^0$	\mathcal{B}'	$0.003a_2^2$	-0.44
$\Omega_b^- \rightarrow \pi^- \Omega_c^0$	\mathcal{A}	$0.30a_1^2$	0.51	$\Omega_b^- \rightarrow D_s^{*-} \Omega_c^0$	$\mathcal{A}, \mathcal{B}, \mathcal{E}', \mathcal{E}$	$0.35a_1^2$	0.64
$\Omega_b^- \rightarrow \rho^- \Omega_c^0$	\mathcal{A}	$0.39a_1^2$	0.53	$\Omega_b^- \rightarrow D^0 \Xi^-$	\mathcal{B}'	$0.033a_2^2$	0.47
$\Omega_b^- \rightarrow D_s^- \Omega_c^0$	$\mathcal{A}, \mathcal{B}, \mathcal{E}', \mathcal{E}$	$1.09a_1^2$	0.42	$\Omega_b^- \rightarrow D^{*0} \Xi^-$	\mathcal{B}'	$0.014a_2^2$	0.54

^aThe decay modes $\Xi_b^0 \rightarrow D^0 \Sigma^0$, $D^{*0} \Sigma^0$ also receive W -exchange contribution \mathcal{C}' .

The above form factors are applicable to heavy-to-heavy (i.e., $\mathbf{6} \rightarrow \mathbf{6}^*$) and heavy-to-light (i.e., $\mathbf{6} \rightarrow \mathbf{10}$) baryon transitions.

In HQET the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ matrix elements are given by (see, e.g., [22])

$$\begin{aligned} \langle B_f^*(v') | V_\mu | B_i(v) \rangle &= \frac{1}{\sqrt{3}} \bar{u}_f^\nu(v') \{ (2g_{\mu\nu} + \gamma_\mu v_\nu) \xi_1 \\ &+ v_\nu [(1 - v \cdot v') \gamma_\mu - 2v'_\mu] \xi_2 \} \gamma_5 u_i(v), \end{aligned} \quad (2.22)$$

$$\begin{aligned} \langle B_f^*(v') | A_\mu | B_i(v) \rangle &= -\frac{1}{\sqrt{3}} \bar{u}_f^\nu(v') \{ (2g_{\mu\nu} - \gamma_\mu v_\nu) \xi_1 \\ &+ v_\nu [(1 + v \cdot v') \gamma_\mu - 2v'_\mu] \xi_2 \} u_i(v), \end{aligned}$$

where ξ_1 and ξ_2 are the baryon Isgur-Wise functions introduced in Eq. (2.12). We find that, at zero recoil,

$$\begin{aligned} \bar{f}_1(q_m^2) &= \frac{2}{\sqrt{3}}, & \bar{f}_2(q_m^2) &= \frac{1}{\sqrt{3}m_i}, \\ \bar{f}_3(q_m^2) &= -\frac{2}{\sqrt{3}} \frac{\xi_2(1)}{m_i m_f}, \\ \bar{g}_1(q_m^2) &= -\frac{2}{\sqrt{3}}, & \bar{g}_2(q_m^2) &= \frac{1}{\sqrt{3}m_i} [1 - 2\xi_2(1)], \\ \bar{g}_3(q_m^2) &= \frac{2}{\sqrt{3}} \frac{\xi_2(1)}{m_i m_f}. \end{aligned} \quad (2.23)$$

Since $N_{fi} = 1$ for the heavy-to-heavy transition, it is clear that the quark model results for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ form factors (2.21) in

the heavy quark limit are in agreement with the HQET predictions (2.23) with $\xi_2(1) = \frac{1}{2}$ [see Eq. (2.16)].

Since the calculation for the q^2 dependence of form factors is beyond the scope of the nonrelativistic quark model, we will follow the conventional practice to assume a pole dominance for the form-factor q^2 behavior

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_V^2)^n}, \quad g(q^2) = \frac{g(0)}{(1 - q^2/m_A^2)^n}, \quad (2.24)$$

where m_V (m_A) is the pole mass of the vector (axial-vector) meson with the same quantum number as the current under consideration. The function

$$G(q^2) = \left(\frac{1 - q_m^2/m_{\text{pole}}^2}{1 - q^2/m_{\text{pole}}^2} \right)^2$$

plays the role of the baryon Isgur-Wise function $\zeta(\omega)$ for $\Lambda_Q \rightarrow \Lambda_{Q'}$ transition, namely, $G = 1$ at $q^2 = q_m^2$. The function $\zeta(\omega)$ has been calculated in the literature in various different models [28–31]. Using the pole masses $m_V = 6.34$ GeV, $m_A = 6.73$ GeV for $\Lambda_b \rightarrow \Lambda_c$ transition, it is found in [6] that $G(q^2)$ is consistent with $\zeta(\omega)$ only if $n = 2$. Nevertheless, one should bear in mind that the q^2 behavior of form factors is probably more complicated and it is likely that a simple pole dominance only applies to a certain q^2 region.

Assuming a dipole q^2 behavior for form factors, we have tabulated in Table I the numerical values of $B_b(\frac{1}{2}^+) \rightarrow \frac{1}{2}^+$, $B_b(\frac{1}{2}^+) \rightarrow \frac{3}{2}^+$ and $B_c(\frac{1}{2}^+) \rightarrow \frac{3}{2}^+$ form factors at $q^2 = 0$ calculated using (2.10) and (2.21). Uses have been made of $|V_{cb}| = 0.038$ [33]: the constituent quark masses [light quark masses being taken from p. 619 of the Particle Data Group (PDG) book [34]]

$$m_b = 5 \text{ GeV}, \quad m_c = 1.6 \text{ GeV}, \quad m_s = 510 \text{ MeV},$$

$$m_d=322 \text{ MeV}, \quad m_u=338 \text{ MeV}, \quad (2.25)$$

the pole masses

$$\begin{aligned} b \rightarrow c: \quad m_V &= 6.34 \text{ GeV}, \quad m_A = 6.73 \text{ GeV}, \\ b \rightarrow s: \quad m_V &= 5.42 \text{ GeV}, \quad m_A = 5.86 \text{ GeV}, \\ b \rightarrow d: \quad m_V &= 5.32 \text{ GeV}, \quad m_A = 5.71 \text{ GeV}, \\ c \rightarrow s: \quad m_V &= 2.11 \text{ GeV}, \quad m_A = 2.54 \text{ GeV}, \\ c \rightarrow u: \quad m_V &= 2.01 \text{ GeV}, \quad m_A = 2.42 \text{ GeV}, \end{aligned} \quad (2.26)$$

and the bottom baryon masses

$$m_{\Lambda_b} = 5.621 \text{ GeV}, \quad m_{\Xi_b} = 5.80 \text{ GeV}, \quad m_{\Omega_b} = 6.04 \text{ GeV}. \quad (2.27)$$

Note that the CDF measurement [9] $m_{\Lambda_b} = 5621 \pm 4 \pm 3 \text{ MeV}$ has better accuracy than the PDG value $5641 \pm 50 \text{ MeV}$ [34]; the combined value is $m_{\Lambda_b} = 5621 \pm 5 \text{ MeV}$.

III. RESULTS AND DISCUSSION

With the baryon form factors tabulated in Table I we are in a position to compute the factorizable contributions to the decay rate and up-down asymmetry for Cabibbo-allowed weak decays of bottom baryons $B_b(\frac{1}{2}^+) \rightarrow \frac{1}{2}^+(\frac{3}{2}^+) + P(V)$. The factorizable external and internal W -emission amplitudes are given by (2.6), (2.7), (2.18) and (2.19). The calculated results are summarized in Tables II and III. (The formulas for decay rates and up-down asymmetries are given in Appendix B.) For decay constants we use

$$f_\pi = 132 \text{ MeV}, \quad f_D = 200 \text{ MeV} [36],$$

$$f_{D_s} = 241 \text{ MeV} [36],$$

$$f_\rho = 216 \text{ MeV}, \quad f_{J/\psi} = 395 \text{ MeV}, \quad f_{\psi'} = 293 \text{ MeV}, \quad (3.1)$$

where we have taken into account the momentum dependence of the fine-structure constant to determine $f_{J/\psi}$ and $f_{\psi'}$ from experiment. In the absence of reliable theoretical estimates for f_{D^*} and $f_{D_s^*}$, we have taken $f_{D^*} = f_D$ and $f_{D_s^*} = f_{D_s}$ for numerical calculations.

From Tables II and III we see that, except for those decay modes with ψ' in the final state and for $\Omega_b \rightarrow \frac{1}{2}^+ + P(V)$ decays, the up-down asymmetry parameter α is found to be negative.⁴ As noted in [11], the parameter α in $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + P(V)$ decay becomes -1 in the soft pseudoscalar meson or vector meson limit, i.e., $m_p \rightarrow 0$ or $m_V \rightarrow 0$. In practice, α is sensitive to m_V but not so to m_p . For example, $\alpha \approx -1$ for $\Lambda_b \rightarrow D_s \Lambda_c$ and $\Xi_b \rightarrow D_s \Xi_c$ even though the D_s meson is heavy, but it changes from $\alpha = -0.88$ for $\Lambda_b \rightarrow \rho \Lambda_c$ to -0.10 for $\Lambda_b \rightarrow J/\psi \Lambda$. As stressed in Sec. II,

⁴The parameter α of $\Lambda_b \rightarrow J/\psi \Lambda$ is estimated to be 0.25 in [7], whereas it is -0.10 in our case.

by treating a_1 and a_2 as free parameters, our predictions should be most reliable for those decay modes which proceed only through the external W -emission diagram \mathcal{A} or the internal W -emission \mathcal{B}' . Moreover, we have argued that the penguin contributions \mathcal{E}' and \mathcal{E} to Cabibbo-allowed decays are safely negligible and that the W -exchange amplitudes $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}'$ are very likely to be suppressed in bottom baryon decays. It is thus very interesting to test the suppression of W -exchange in decay modes of $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10}) + P(V)$ that proceed only through W exchange [see Eq. (2.3)] and in decays $B_b(\bar{\mathbf{3}}) \rightarrow B_c(\bar{\mathbf{3}}) + P(V)$, e.g., $\Xi_b \rightarrow \pi(\rho)\Xi_c$, $\Xi_b \rightarrow D_s^{(*)}\Xi_c$, that receive contributions from factorizable terms and W exchange. Since the nonfactorizable internal W -emission amplitude \mathcal{B} is *a priori* not negligible, our results for $\Lambda_b \rightarrow \pi(\rho)\Lambda_c$, $\Omega_b \rightarrow D_s^{(*)}\Omega_c^{(*)}$ (see Tables II and III) are subject to the uncertainties due to possible contributions from the quark diagram \mathcal{B} .

In order to have an idea about the magnitude of branching ratios, let us take $a_1 \sim 1$ as that inferred from $B \rightarrow D^{(*)}\pi(\rho)$ decays [37] and $a_2 \sim 0.28$ as that in $B \rightarrow J/\psi K^{(*)}$ decays.⁵ Using the current world average $\tau(\Lambda_b) = (1.23 \pm 0.06) \times 10^{-12} \text{ s}$ [33], we find from Table II that

$$\begin{aligned} \mathcal{B}(\Lambda_b^0 \rightarrow D_s^- \Lambda_c^+) &\cong 1.1 \times 10^{-2}, \\ \mathcal{B}(\Lambda_b^0 \rightarrow D_s^{*-} \Lambda_c^+) &\cong 9.1 \times 10^{-3}, \\ \mathcal{B}(\Lambda_b^0 \rightarrow \pi^- \Lambda_c^+) &\sim 3.8 \times 10^{-3}, \\ \mathcal{B}(\Lambda_b^0 \rightarrow \rho^- \Lambda_c^+) &\sim 5.4 \times 10^{-3}, \\ \mathcal{B}(\Lambda_b^0 \rightarrow J/\psi \Lambda) &= 1.6 \times 10^{-4}, \\ \mathcal{B}(\Lambda_b^0 \rightarrow \psi' \Lambda) &= 1.4 \times 10^{-4}. \end{aligned} \quad (3.2)$$

Our estimate for the branching ratio of $\Lambda_b \rightarrow J/\psi \Lambda$ is consistent with the CDF result [9]

$$\mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) = (3.7 \pm 1.7 \pm 0.4) \times 10^{-4}. \quad (3.3)$$

Recall that the predictions (3.2) are obtained for $|V_{cb}| = 0.038$.

Since the decay mode $\Omega_c^0 \rightarrow \pi^+ \Omega^-$ has been seen experimentally, we also show the estimate of Γ and α in Table IV for $\Omega_c^0 \rightarrow \frac{3}{2}^+ + P(V)$ decays with the relevant form factors being given in Table I. For comparison, we have displayed in Table IV the model results of Xu and Kamal [39],⁶ Körner and Krämer [14]. In the model of Xu and Kamal, the D -wave amplitude in Eq. (2.17) and hence the parameter α vanishes in the decay $\Omega_c \rightarrow \frac{3}{2}^+ + P$ due to the fact that the vector current is conserved at all q^2 in their scheme 1 and at $q^2 = 0$ in scheme 2. By contrast, the D -wave amplitude in

⁵A fit to recent measurements of $B \rightarrow J/\psi K(K^*)$ decays by CDF and CLEO yields [38] $a_2(B \rightarrow J/\psi K) = 0.30$ and $a_2(B \rightarrow J/\psi K^*) = 0.26$.

⁶The B and D amplitudes in Eq. (4) of [39], where the formulas for Γ and α in $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + P$ decay are given, should be interchanged.

TABLE III. Predicted decay rates (in units of 10^{10} s^{-1}) and up-down asymmetries for Cabibbo-allowed nonleptonic weak decays of the bottom baryon $\Omega_b^- \rightarrow \frac{3}{2}^+ + P(V)$. Also shown are the quark-diagram amplitudes for various reactions.

Decay	Diagram	Γ	α	Decay	Diagram	Γ	α
$\Omega_b^- \rightarrow \pi^- \Omega_c^{*0}$	\mathcal{A}	$0.67a_1^2$	-0.38	$\Omega_b^- \rightarrow J/\psi \Omega^-$	\mathcal{B}'	$3.15a_2^2$	-0.18
$\Omega_b^- \rightarrow \rho^- \Omega_c^{*0}$	\mathcal{A}	$0.95a_1^2$	-0.75	$\Omega_b^- \rightarrow \psi' \Omega^-$	\mathcal{B}'	$1.94a_2^2$	0.004
$\Omega_b^- \rightarrow D_s^- \Omega_c^{*0}$	$\mathcal{A}, \mathcal{B}, \mathcal{E}', \mathcal{E}$	$0.88a_1^2$	-0.22	$\Omega_b^- \rightarrow D^0 \Xi^{*-}$	\mathcal{B}'	$0.23a_2^2$	-0.80
$\Omega_b^- \rightarrow D_s^{*-} \Omega_c^{*0}$	$\mathcal{A}, \mathcal{B}, \mathcal{E}', \mathcal{E}$	$0.98a_1^2$	-0.31	$\Omega_b^- \rightarrow D^{*0} \Xi^{*-}$	\mathcal{B}'	$0.27a_2^2$	-0.38

our case does not vanish. Assuming that the form factors $\bar{f}_1, \bar{f}_2, \bar{f}_3$ have the same q^2 dependence, we see from Eqs. (2.18) and (2.21) that the amplitude D is proportional to $(E_f - m_f)/m_f$, which vanishes at $q^2 = q_{\text{max}}^2$ but not at $q^2 = m_P^2$. Contrary to the decay $\Omega_b^- \rightarrow \frac{3}{2}^+ + P(V)$, the up-down asymmetry is found to be positive in $\Omega_c^0 \rightarrow \frac{3}{2}^+ + P(V)$ decays. Note that the sign of α for $\Omega_c \rightarrow \frac{3}{2}^+ + V$ is opposite to that of [39].⁷ Therefore, it is desirable to measure the parameter α in decays $\Omega_c \rightarrow \frac{3}{2}^+ + P(V)$ to discern different models. To have an estimate of the branching ratio, we take the large- N_c values $a_1(m_c) = 1.10$, $a_2(m_c) = -0.50$ as an illustration and obtain

$$\begin{aligned}
 \mathcal{B}(\Omega_c^0 \rightarrow \pi^+ \Omega^-) &\simeq 1.0 \times 10^{-2}, \\
 \mathcal{B}(\Omega_c^0 \rightarrow \rho^+ \Omega^-) &\simeq 3.6 \times 10^{-2}, \\
 \mathcal{B}(\Omega_c^0 \rightarrow \bar{K}^0 \Xi^{*0}) &\simeq 2.5 \times 10^{-3}, \\
 \mathcal{B}(\Omega_c^0 \rightarrow \bar{K}^{*0} \Xi^{*0}) &\simeq 3.7 \times 10^{-3},
 \end{aligned} \tag{3.4}$$

where use of $\tau(\Omega_c) = 6.4 \times 10^{-14} \text{ s}$ [34] has been made.

Three important ingredients on which the calculations are built in this work are factorization, nonrelativistic quark model, and dipole q^2 behavior of form factors. The factorization hypothesis can be tested by extracting the effective parameters a_1, a_2 from data and seeing if they are channel independent. Thus far we have neglected the effects of final-state interactions which are supposed to be less important in bottom baryon decay since decay particles in the two-body final state are energetic and moving fast, allowing less time for significant final-state interactions. We have argued that, in the nonrelativistic quark model, the ratio of (KE/M_q) is small even for the constituent quark inside the nucleon (or hyperon) at rest. As for the q^2 dependence of baryon form factors, we have applied dipole dominance motivated by the consistency with the q^2 behavior of the baryon Isgur-Wise function. Nevertheless, in order to check the sensitivity of the form factor q^2 dependence, we have repeated calculations using the monopole form. Since for a given q^2 , the absolute values of the form factors in the monopole behavior are larger than that in the dipole one, it is expected that the

branching ratios obtained under the monopole ansatz will get enhanced, especially when the final-state baryons are hyperons. Numerically, we find that, while decay asymmetries remain essentially unchanged, the decay rates of $B_b(\frac{1}{2}^+) \rightarrow B_c(\frac{1}{2}^+) + P(V)$ and $B_b(\frac{1}{2}^+) \rightarrow \text{hyperon} + P(V)$ are in general enhanced by factors of ~ 1.8 and ~ 3.5 , respectively. In reality, the utilization of a simple q^2 dependence, monopole or dipole, is probably too simplified. It thus appears that major calculational uncertainties arise mainly from the *ad hoc* ansatz on the form factor q^2 behavior.

In conclusion, if the W -exchange contribution to the hadronic decays of bottom baryons is negligible, as we have argued, then the theoretical description of bottom baryons decaying into $\frac{1}{2}^+ + P(V)$ and $\frac{3}{2}^+ + P(V)$ is relatively clean since these decays either receive contributions only from external/internal W emission or are dominated by factorizable terms. The absence or the suppression of the so-called pole terms makes the study of Cabibbo-allowed decays of bottom baryons considerably simpler than that in charmed baryon decay. We have evaluated the heavy-to-heavy and heavy-to-light baryon form factors at zero recoil using the nonrelativistic quark model and reproduced the HQET results for heavy-to-heavy baryon transition. It is stressed that for heavy-to-light baryon form factors, there is a flavor-suppression factor which must be taken into account in calculations. Predictions of the decay rates and up-down asymmetries for $B_b \rightarrow \frac{1}{2}^+ + P(V)$ and $\Omega_c \rightarrow \frac{3}{2}^+ + P(V)$ are given. The parameter α is found to be negative except for $\Omega_b \rightarrow \frac{1}{2}^+ + P(V)$ decays and for those decay modes with ψ' in the final state. We also present estimates of Γ and α for $\Omega_c \rightarrow \frac{3}{2}^+ + P(V)$ decays. It is very desirable to measure the asymmetry parameter to discern different models.

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APPENDIX A: BARYON FORM FACTORS IN THE QUARK MODEL

Since the $\frac{1}{2}^+$ to $\frac{1}{2}^+$ baryon form factors have been evaluated at zero recoil in the nonrelativistic quark model [6], we will focus in this appendix on the baryon form factors in $\frac{1}{2}^+$ to $\frac{3}{2}^+$ transition. Let u^α be the Rarita-Schwinger vector-spinor for a spin- $\frac{3}{2}$ particle. The general four plane-wave solutions for u^α are (see, for example, [40])

⁷It seems to us that the sign of A_i and B_i (or C_i and D_i in our notation) in Eq. (58) of [39] should be flipped. A consequence of this sign change will render α positive in $\Omega_c \rightarrow \frac{3}{2}^+ + V$ decay.

TABLE IV. Predicted decay rates (in units of 10^{11} s^{-1}) and up-down asymmetries (in parentheses) for Cabibbo-allowed nonleptonic weak decays of the charmed baryon $\Omega_c^0 \rightarrow \frac{3}{2}^+ + P(V)$ in various models. The model calculations of Xu and Kamal are done in two different schemes [39].

Decay	This work	Xu and Kamal [39]	Körner and Krämer [14]
$\Omega_c^0 \rightarrow \pi^+ \Omega^-$	$1.33a_1^2(0.17)$	$2.13a_1^2(0)$	$2.09a_1^2(0)$
$\Omega_c^0 \rightarrow \rho^+ \Omega^-$	$4.68a_1^2(0.43)$	$11.6a_1^2(-0.08)$	$11.3a_1^2(-0.21)$
$\Omega_c^0 \rightarrow \bar{K}^0 \Xi^{*0}$	$1.53a_2^2(0.35)$	$1.00a_2^2(0)$	$0.89a_2^2(0)$
$\Omega_c^0 \rightarrow \bar{K}^{*0} \Xi^{*0}$	$2.32a_2^2(0.28)$	$4.56a_2^2(-0.09)$	$4.54a_2^2(-0.27)$

$$\begin{aligned}
u_1^\alpha &= (u_1^0, \vec{u}_1) = (0, \vec{\epsilon}_1 u_\uparrow), & \langle B_f^*(+1/2) | \vec{V} | B_i(-1/2) \rangle \\
u_2^\alpha &= (u_2^0, \vec{u}_2) = \left(\sqrt{\frac{2}{3}} \frac{|p|}{m} u_\uparrow, \frac{1}{\sqrt{3}} \vec{\epsilon}_1 u_\downarrow - \sqrt{\frac{2}{3}} \frac{E}{m} \vec{\epsilon}_3 u_\uparrow \right), & = -\bar{f}_1 \left(\frac{1}{\sqrt{3}} \vec{\epsilon}_1 \bar{u}_\downarrow - \sqrt{\frac{2}{3}} \frac{E_f}{m_f} \vec{\epsilon}_3 \bar{u}_\uparrow \right) \gamma_5 u_\downarrow + \sqrt{\frac{2}{3}} \frac{p m_i}{m_f} \\
& & \quad \times \bar{u}_\uparrow (\bar{f}_2 \vec{\gamma} \gamma_5 + \bar{f}_3 \vec{p} \gamma_5) u_\downarrow, & (A7) \\
u_3^\alpha &= (u_3^0, \vec{u}_3) = \left(\sqrt{\frac{2}{3}} \frac{|p|}{m} u_\downarrow, \frac{1}{\sqrt{3}} \vec{\epsilon}_2 u_\uparrow - \sqrt{\frac{2}{3}} \frac{E}{m} \vec{\epsilon}_3 u_\downarrow \right), & \langle B_f^*(+1/2) | \vec{A} | B_i(-1/2) \rangle \\
& & = -\bar{g}_1 \left(\frac{1}{\sqrt{3}} \vec{\epsilon}_1 \bar{u}_\downarrow - \sqrt{\frac{2}{3}} \frac{E_f}{m_f} \vec{\epsilon}_3 \bar{u}_\uparrow \right) u_\downarrow + \sqrt{\frac{2}{3}} \frac{p m_i}{m_f} \\
u_4^\alpha &= (u_4^0, \vec{u}_4) = (0, \vec{\epsilon}_2 u_\downarrow), & \quad \times \bar{u}_\uparrow (\bar{g}_2 \vec{\gamma} + \bar{g}_3 \vec{p}) u_\downarrow, & (A8)
\end{aligned}$$

in the frame where the baryon momentum \vec{p} is along the z axis, and

$$\epsilon_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad \epsilon_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}, \quad \epsilon_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (A2)$$

Note that the spin z component of the four solutions (A1) corresponds to $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, respectively. Substituting Eq. (A1) into Eq. (2.20) yields

$$\begin{aligned}
& \langle B_f^*(+1/2) | V_0 | B_i(+1/2) \rangle \\
& = \sqrt{\frac{2}{3}} \frac{p}{m_f} \bar{u}_\uparrow (\bar{f}_1 \gamma_5 + \bar{f}_2 m_i \gamma_0 \gamma_5 + \bar{f}_3 m_i E_f \gamma_5) u_\uparrow, & (A3)
\end{aligned}$$

$$\begin{aligned}
& \langle B_f^*(+1/2) | A_0 | B_i(+1/2) \rangle \\
& = \sqrt{\frac{2}{3}} \frac{p}{m_f} \bar{u}_\uparrow (\bar{g}_1 + \bar{g}_2 m_i \gamma_0 + \bar{g}_3 m_i E_f) u_\uparrow, & (A4)
\end{aligned}$$

$$\langle B_f^*(+3/2) | \vec{V} | B_i(+1/2) \rangle = -\bar{f}_1 \vec{\epsilon}_1 \bar{u}_\uparrow \gamma_5 u_\uparrow, \quad (A5)$$

$$\langle B_f^*(+3/2) | \vec{A} | B_i(+1/2) \rangle = -\bar{g}_1 \vec{\epsilon}_1 \bar{u}_\uparrow u_\uparrow, \quad (A6)$$

where \vec{p} is the momentum of the daughter baryon along the z axis in the rest frame of the parent baryon. The baryon matrix elements in Eqs. (A3)–(A8) can be evaluated in the non-relativistic quark model. Following the same procedure outlined in [6], we obtain

$$\begin{aligned}
& \langle B_f^* | V_0 | B_i \rangle / N_f = \langle 1 \rangle, \\
& \langle B_f^* | \vec{V} | B_i \rangle / N_f = -\frac{1}{2m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) \langle \vec{q} + i \vec{\sigma} \times \vec{q} \rangle \\
& \quad + \frac{\bar{\Lambda}}{4m_Q m_f} \langle \vec{q} - i \vec{\sigma} \times \vec{q} \rangle, \\
& \langle B_f^* | A_0 | B_i \rangle / N_f = \left[-\frac{1}{2m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) + \frac{\bar{\Lambda}}{4m_Q m_f} \right] \langle \vec{\sigma} \cdot \vec{q} \rangle, & (A9)
\end{aligned}$$

$$\langle B_f^* | \vec{A} | B_i \rangle / N_f = \langle \vec{\sigma} \rangle - \frac{\bar{\Lambda}}{4m_Q m_f^2} \left\langle (\vec{\sigma} \cdot \vec{q}) \vec{q} - \frac{1}{2} \vec{\sigma} q^2 \right\rangle,$$

where $\vec{q} = \vec{p}_i - \vec{p}_f$, $N_f = \sqrt{(E_f + m_f)/2m_f}$, m_q is the mass of the quark q in B_f^* coming from the decay of the heavy quark Q in B_i , and $\langle X \rangle$ stands for $\text{flavor-spin} \langle B_f^* | X | B_i \rangle_{\text{flavor-spin}}$. Form factors \bar{f}_i and \bar{g}_i are then determined from Eq. (A3) to Eq. (A9). For example, \bar{f}_1 can be determined from the x (or y) component of Eq. (A5) which is

$$\begin{aligned} \langle B_f^*(+3/2)|V_x|B_i(+1/2)\rangle &= -\frac{\bar{f}_1}{\sqrt{2}} \frac{N_f}{E_f+m_f} \chi^\dagger \vec{\sigma} \cdot \vec{q} \chi_\uparrow \\ &= \frac{\bar{f}_1}{\sqrt{2}} \frac{p N_f}{E_f+m_f}, \end{aligned} \quad (\text{A10})$$

where χ is a two-component Pauli spinor. From Eq. (A9) we find

$$\begin{aligned} \langle B_f^*(+3/2)|V_x|B_i(+1/2)\rangle &= \frac{p N_f}{4m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} + \frac{\bar{\Lambda} m_q}{2m_Q m_f}\right) \\ &\times \langle (\sigma_+ - \sigma_-) b_q^\dagger b_Q \rangle. \end{aligned} \quad (\text{A11})$$

Since $[N_{fi}]$ being defined by Eq. (2.11)

$$\begin{aligned} \text{flavor-spin} \langle B_f^*(+3/2) | (\sigma_+ - \sigma_-) b_q^\dagger b_Q | B_i(+1/2) \rangle_{\text{flavor-spin}} \\ = \frac{4}{\sqrt{6}} N_{fi}, \end{aligned} \quad (\text{A12})$$

for sextet B_i and vanishes for antitriplet B_i , it is evident that only the decay of Ω_b into $\frac{3}{2}^+ + P(V)$ can receive factorizable contributions. Indeed the decays $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10}) + P(V)$ proceed only through W exchange or W loop, as discussed in Sec. II. It follows from Eqs. (A10)–(A12) that at zero recoil

$$\bar{f}_1(q_m^2)/N_{fi} = \frac{2}{\sqrt{3}} \left(1 + \frac{\bar{\Lambda}}{2m_q} + \frac{\bar{\Lambda}}{2m_Q}\right), \quad (\text{A13})$$

which is the result shown in Eq. (2.21). The form factor \bar{f}_2 is then fixed by the x (or y) component of Eq. (A7). Substituting \bar{f}_1 and \bar{f}_2 into Eq. (A3) determines \bar{f}_3 . The remaining form factors \bar{g}_i are determined in a similar way.

APPENDIX B: KINEMATICS

In this appendix we summarize the kinematics relevant to the two-body hadronic decays of $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ (\frac{3}{2}^+) + P(V)$. With the amplitudes (2.4) for $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + P$ decay and Eq. (2.17) for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + P$, the decay rates and up-down asymmetries read

$$\begin{aligned} \Gamma(1/2^+ \rightarrow 1/2^+ + P) &= \frac{p_c}{8\pi} \left\{ \frac{(m_i+m_f)^2 - m_P^2}{m_i^2} |A|^2 \right. \\ &\quad \left. + \frac{(m_i-m_f)^2 - m_P^2}{m_i^2} |B|^2 \right\}, \\ \alpha(1/2^+ \rightarrow 1/2^+ + P) &= -\frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}, \end{aligned} \quad (\text{B1})$$

and

$$\begin{aligned} \Gamma(1/2^+ \rightarrow 3/2^+ + P) &= \frac{p_c^3}{8\pi} \left\{ \frac{(m_i-m_f)^2 - m_P^2}{m_i^2} |C|^2 \right. \\ &\quad \left. + \frac{(m_i+m_f)^2 - m_P^2}{m_i^2} |D|^2 \right\}, \\ \alpha(1/2^+ \rightarrow 3/2^+ + P) &= -\frac{2\kappa \text{Re}(C^*D)}{\kappa^2 |C|^2 + |D|^2}, \end{aligned} \quad (\text{B2})$$

where p_c is the c.m. momentum and $\kappa = p_c/(E_f+m_f) = \sqrt{(E_f-m_f)/(E_f+m_f)}$. For $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + V$ decay we have [35]⁸

$$\begin{aligned} \Gamma(1/2^+ \rightarrow 1/2^+ + V) &= \frac{p_c}{8\pi} \frac{E_f+m_f}{m_i} \left[2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2} (|S+D|^2 + |P_1|^2) \right], \\ \alpha(1/2^+ \rightarrow 1/2^+ + V) &= \frac{4m_V^2 \text{Re}(S^*P_2) + 2E_V^2 \text{Re}(S+D)^*P_1}{2m_V^2 (|S|^2 + |P_2|^2) + E_V^2 (|S+D|^2 + |P_1|^2)}, \end{aligned} \quad (\text{B3})$$

with the S , P , and D waves given by

$$\begin{aligned} S &= -A_1, \\ P_1 &= -\frac{p_c}{E_V} \left(\frac{m_i+m_f}{E_f+m_f} B_1 + m_i B_2 \right), \\ P_2 &= \frac{p_c}{E_f+m_f} B_1, \\ D &= -\frac{p_c^2}{E_V(E_f+m_f)} (A_1 - m_i A_2), \end{aligned} \quad (\text{B4})$$

where the amplitudes A_1 , A_2 , B_1 , and B_2 are defined in Eq. (2.4). However, as emphasized in [14], it is also convenient to express Γ and α in terms of the helicity amplitudes

$$h_{\lambda_f, \lambda_V; \lambda_i} = \langle B_f(\lambda_f) V(\lambda_V) | H_W | B_i(\lambda_i) \rangle \quad (\text{B5})$$

with $\lambda_i = \lambda_f - \lambda_V$. Then [14]

$$\Gamma = \frac{p_c}{32\pi m_i^2} \sum_{\lambda_f, \lambda_V} (|h_{\lambda_f, \lambda_V; 1/2}|^2 - |h_{-\lambda_f, -\lambda_V; -1/2}|^2),$$

⁸The formulas for the decay rate of $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + V$ decay given in [15,6] contain some errors which are corrected in errata.

$$\alpha = \sum_{\lambda_f, \lambda_V} \frac{(|h_{\lambda_f, \lambda_V; 1/2}|^2 - |h_{-\lambda_f, -\lambda_V; -1/2}|^2)}{(|h_{\lambda_f, \lambda_V; 1/2}|^2 + |h_{-\lambda_f, -\lambda_V; -1/2}|^2)}. \quad (\text{B6})$$

The helicity amplitudes for $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + V$ decay are given by [14]

$$H_{\lambda_1, \lambda_2; 1/2}^{\text{PV (PC)}} = H_{\lambda_1, \lambda_2; 1/2} \mp H_{-\lambda_1, -\lambda_2; -1/2},$$

$$H_{-1/2, -1; 1/2}^{\text{PV (PC)}} = 2 \begin{Bmatrix} \sqrt{Q_+} A_1 \\ -\sqrt{Q_-} B_1 \end{Bmatrix}, \quad (\text{B7})$$

$$H_{1/2, 0; 1/2}^{\text{PV (PC)}} = \sqrt{\frac{2}{m_V}} \begin{Bmatrix} \sqrt{Q_+} (m_i - m_f) A_1 - \sqrt{Q_-} m_i p_c A_2 \\ \sqrt{Q_-} (m_i + m_f) B_1 + \sqrt{Q_+} m_i p_c B_2 \end{Bmatrix},$$

where the upper (lower) entry is for parity-violating (-conserving) helicity amplitude, and

$$Q_{\pm} = (m_i \pm m_f)^2 - m_V^2 = 2m_i(E_f \pm m_f). \quad (\text{B8})$$

Note that the helicity amplitudes for $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + V$ decay shown in Eq. (20) of [14] are too large by a factor of $\sqrt{2}$. One can check explicitly that the decay rates and up-down asymmetries evaluated using the partial-wave method and the helicity-amplitude method are equivalent. For completeness, we also list the helicity amplitudes for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ + V$ decay [14]:

$$H_{\lambda_1, \lambda_2; 1/2}^{\text{PV (PC)}} = H_{\lambda_1, \lambda_2; 1/2} \pm H_{-\lambda_1, -\lambda_2; -1/2},$$

$$H_{3/2, 1; 1/2}^{\text{PV (PC)}} = 2 \begin{Bmatrix} -\sqrt{Q_+} C_1 \\ \sqrt{Q_-} D_1 \end{Bmatrix}, \quad (\text{B9})$$

$$H_{-1/2, -1; 1/2}^{\text{PV (PC)}} = \frac{2}{\sqrt{3}} \begin{Bmatrix} -\sqrt{Q_+} [C_1 - 2(Q_-/m_f) C_2] \\ \sqrt{Q_-} [D_1 - 2(Q_+/m_f) D_2] \end{Bmatrix},$$

$$H_{1/2, 0; 1/2}^{\text{PV (PC)}} = \frac{2\sqrt{2}}{\sqrt{3}m_f m_V} \begin{Bmatrix} -\sqrt{Q_+} [\frac{1}{2}(m_i^2 - m_f^2 - m_V^2) C_1 + Q_- (m_i + m_f) C_2 + m_i^2 p_c^2 C_3] \\ \sqrt{Q_-} [\frac{1}{2}(m_i^2 - m_f^2 - m_V^2) D_1 - Q_+ (m_i - m_f) D_2 + m_i^2 p_c^2 D_3] \end{Bmatrix}.$$

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