Phenomenological analysis of heavy hadron lifetimes

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A phenomenological analysis of lifetimes of bottom and charmed hadrons within the framework of the heavy quark expansion is performed. The baryon matrix element is evaluated using the bag model and the nonrelativistic quark model. We find that bottom-baryon lifetimes follow the pattern $\tau(\Omega_b)\simeq\tau(\Xi_b^-)>\tau(\Lambda_b)\simeq\tau(\Xi_b^0)$. However, neither the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ nor the absolute decay rates of the Λ_b baryon and B mesons can be explained. One way of solving both difficulties is to allow the presence of linear $1/m_Q$ corrections by scaling the inclusive nonleptonic width with the fifth power of the hadron mass m_{H_Q} rather than the heavy quark mass m_Q. The hierarchy of bottom baryon lifetimes is dramatically modified to $\tau(\Lambda_b)>\tau(\Xi_b^-)>\tau(\Xi_b^0)>\tau(\Omega_b)$: The longest-lived Ω_b among bottom baryons in the OPE prescription now becomes the shortest lived. The replacement of m_Q by m_{H_Q} in nonleptonic widths is natural and justified in the PQCD-based factorization approach formulated in terms of hadron-level kinematics. For inclusive charmed baryon decays, we argue that since the heavy quark expansion does not converge, local duality cannot be tested in this case. We show that while the ansatz of substituting the heavy quark mass by the hadron mass provides a much better description of the charmed-baryon lifetime *ratios*, it appears unnatural and unpredictive for describing the *absolute* inclusive decay rates of charmed baryons, contrary to the bottom case.
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I. INTRODUCTION

The lifetime differences among the charmed mesons D^+, D^0 and charmed baryons have been studied extensively both experimentally and theoretically since the late 1970s. It was realized very early that the naive parton model gives the same lifetimes for all heavy particles containing a heavy quark Q and that the underlying mechanism for the decay width differences and the lifetime hierarchy of heavy hadrons comes mainly from the nonspectator effects such as W exchange and Pauli interference due to the identical quarks produced in heavy quark decay and in the wave function (for a review, see [1,2]). The nonspectator effects were expressed in the 1980s in terms of local four-quark operators by relating the total widths to the imaginary part of certain forward scattering amplitudes [3–5]. (The nonspectator effects for charmed baryons were first studied in [6].) With the advent of heavy quark effective theory (HQET), it was recognized in the early 1990s that nonperturbative corrections to the parton picture can be systematically expanded in powers of $1/m_Q$ [7,8]. Subsequently, it was demonstrated that this $1/m_Q$ expansion is applicable not only to global quantities such as lifetimes, but also to local quantities, e.g., the lepton spectrum in the semileptonic decays of heavy hadrons [9]. Therefore, the above-mentioned phenomenological work in the 1980s acquired a firm theoretical footing in the 1990s, namely, the heavy quark expansion (HQE), which is a generalization of the operator product expansion (OPE) in $1/m_Q$. Within this QCD-based framework, some phenomenological assumptions can be turned into some coherent and quantitative statements and nonperturbative effects can be systematically studied. As an example, consider the baryon matrix element of the two-quark operator $\langle\Lambda_b|\overline{b}b|\Lambda_b\rangle$. The conventional quark-model evaluation of this matrix element is model dependent:

$$\frac{\langle\Lambda_b|\overline{b}b|\Lambda_b\rangle}{2m_{\Lambda_b}}=\begin{cases}1 & \text{NQM},\\ \int d^3r[u_b^2(r)-v_b^2(r)] & \text{bag model},\end{cases} \tag{1.1}$$

where $u(r)$ and $v(r)$ are the large and small components, respectively, of the quark wave function. However, the matrix element (1.1), which is equal to unity in the nonrelativistic quark model (NQM), becomes smaller in the bag model due to the contribution from the lower component of the quark wave function. In the HQE approach, it is given by [see Eq. (2.8) below]

$$\frac{\langle\Lambda_b|\overline{b}b|\Lambda_b\rangle}{2m_{\Lambda_b}}=1+\frac{1}{2m_b^2}\left(\frac{\langle\Lambda_b|\overline{b}(iD_\perp)^2b|\Lambda_b\rangle}{2m_{\Lambda_b}}\right)+\frac{1}{4m_b^2}\left(\frac{\langle\Lambda_b|\overline{b}\sigma\cdot Gb|\Lambda_b\rangle}{2m_{\Lambda_b}}\right)+O(1/m_b^3), \tag{1.2}$$

with $D_\perp^\mu=\partial^\mu-v^\mu v\cdot D$. This expression is not only model independent but also contains nonperturbative kinetic and chromomagnetic effects which are either absent or overlooked in the earlier quark-model calculations.

Based on the OPE approach for the analysis of inclusive weak decays, predictions for the ratios of bottom hadron lifetimes have been made by several groups. The first correction to bottom hadron lifetimes is of order $1/m_b^2$ and it is model independent [10]:

$$\frac{\tau(B^-)}{\tau(B_d)}=1+O(1/m_b^3),\quad \frac{\tau(B_s)}{\tau(B_d)}=(1.00\pm0.01)+O(1/m_b^3),$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=0.98+O(1/m_b^3). \tag{1.3}$$

The $1/m_b^2$ corrections are small and essentially canceled out in the lifetime ratios. Nonspectator effects in inclusive decays due to the Pauli interference and W-exchange contributions account for $1/m_b^3$ corrections and they have two eminent features: First, the estimate of nonspectator effects is model dependent; the hadronic four-quark matrix elements are usually evaluated by assuming the factorization approximation for mesons and the quark model for baryons. Second, $1/m_b^3$ corrections can be quite significant due to a phase-space enhancement by a factor of $16\pi^2$. Predictions made in [11] for lifetime ratios of bottom hadrons are

$$\frac{\tau(B^-)}{\tau(B_d)}=1.0+0.05\left(\frac{f_B}{200\ \ \mathrm{MeV}}\right)^2,\quad \frac{\tau(\Lambda_b)}{\tau(B_d)}\gtrsim 0.9. \tag{1.4}$$

Experimentally [12], while the B^- and B_d lifetimes are very close, it appears that the Λ_b lifetime is significantly shorter than the B meson one:

$$\frac{\tau(B^-)}{\tau(B_d)}=1.06\pm 0.04,$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=0.79\pm 0.06\quad (\text{world average}). \tag{1.5}$$

It should be mentioned that while the world average value for $\tau(\Lambda_b)/\tau(B_d)$ is dominated by CERN e^+e^- collider LEP experiments [12], the Collider Detector at Fermilab (CDF) experiment alone yields [13]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=0.87\pm 0.11\quad (\text{CDF}). \tag{1.6}$$

It is thus important to confirm the lifetime ratio experimentally in the near future. Evidently, the conflict between experiment Eqs. (1.5) and theoretical expectations from Eqs. (1.3) or (1.4) is striking and intriguing. This has motivated several subsequent studies trying to understand the enhancement of the Λ_b decay rate [10,14–17]. For example, a model-independent analysis in [10] gives

$$\frac{\tau(B^-)}{\tau(B_d)}\simeq 1+0.03B_1+0.004B_2-0.70\varepsilon_1+0.20\varepsilon_2,$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}\simeq 0.98-0.17\varepsilon_1+0.20\varepsilon_2-(0.012+0.021\widetilde{B})r, \tag{1.7}$$

where ε_i, B_i, $\widetilde{B}$, and r are the hadronic parameters to be introduced below in Sec. II. Note that while the ratio $\tau(B^-)/\tau(B_d)$ is predicted to be greater than unity in [11] [see Eqs. (1.4)], it was argued in [10] that the unknown nonfactorizable contributions in Eqs. (1.7) characterized by ε_i make it impossible to have reliable predictions on the magnitude of the lifetime ratio and even the sign of corrections. Since the measured ratio of $\tau(B^-)/\tau(B_d)$ is very close to unity, it follows from Eqs. (1.7) that $\varepsilon_1 \approx 0.3\varepsilon_2$ [10]. Then it is clear that the data for the ratio $\tau(\Lambda_b)/\tau(B_d)$ cannot be accommodated by the theoretical prediction (1.7) without invoking a too large value of r or $\widetilde{B}$, which is expected to be order unity. It is reasonable to conclude that the $1/m_b^3$ corrections in the heavy quark expansion do not suffice to describe the observed lifetime differences between Λ_b and B_d.

In order to employ the OPE approach to compute inclusive weak decays of heavy hadrons, some sort of quark-hadron duality has to be assumed (for an extensive discussion of quark-hadron duality and its violation, see [18,19]). Consider the inclusive semileptonic decay. The OPE cannot be carried out on the physical cut in the complex $v\cdot q$ plane since $T^{\mu\nu}$, the time-ordered product of two currents, along the physical cut is dominated by physical intermediate hadron states which are nonperturbative in nature. To compute $T^{\mu\nu}$ or the Wilson coefficients by perturbative QCD, the OPE has to be performed in the unphysical region far away from the physical cut. The question is then how to relate the operator product expansion for $T^{\mu\nu}$ in the unphysical region to the physical quantities in the physical Minkowski space. Since the physically observable quantity is related to the imaginary part of $T^{\mu\nu}$, it can be reliably computed by deforming the contour of integration into the unphysical region [20,18], provided that the physical quantity involves certain integrals of $T^{\mu\nu}$ in the physical region. This procedure is called ''global duality'' [18]. Global quark-hadron duality also means that the hadronic cross section is dual or matching to the OPE-based quark cross section. However, unlike the total cross section in e^+e^- annihilation, there is a small portion of the contour near the physical cut where global duality can no longer be applied. As stressed in [18], one must resort to local duality to justify the use of the OPE in this small region. Fortunately, the contribution is of order $\Lambda_{\mathrm{QCD}}/m_Q$ and can be neglected for quantities smeared over an energy scale of order Λ_{QCD}.

Global quark-hadron duality for inclusive semileptonic decays, namely, the matching between the hadronic and OPE-based expressions for decay widths or smeared spectra in semileptonic B and Λ_b decays, has been explicitly proved to the first two terms in $1/m_b$ expansion and the first order in α_s in the Shifman-Voloshin (SV) limit [21]. The hadronic decay rate is calculated by summing over all allowed exclusive decay channels. In the SV limit for B meson decays via $b\to c$ transitions, the dominant hadronic final states are the D and D^*. (At zero recoil, the quark-mixing-favored semileptonic decays of a B meson in the heavy quark limit can only produce a D or D^* meson [21,22].) The exclusive decay rates or distributions for $B\to(D+D^*)\ell\bar{\nu}$ depend on hadron masses, whereas the inclusive decay rates evaluated by the OPE depend on quark masses. Global duality is then proved by showing explicitly the equality of inclusive and exclusive decay rates. Note that this proof of global duality in QCD is valid only in the SV limit. Beyond this limit, it becomes difficult to sum over all allowed exclusive semileptonic decay channels and evaluate all of them. It was shown recently in [23] that a proof of quark-hadron global duality in the general kinematic region to order $(\Lambda_{\mathrm{QCD}}/m_B)^2$ can be achieved in the PQCD-based factorization approach, which is formulated in terms of meson-level kinematics rather than the quark-level one. It was demonstrated explicitly in [23] that the integrated quark-level spectrum equals the hadron-level spectrum and that linear $1/m_b$ corrections to the total decay rate are nontrivially canceled out, in agreement with the OPE expectation [7,8].[1]

Unlike the semileptonic inclusive decays in which the use of the OPE is validated by deforming the contour away from the physical cut, it is pointed out in [18] that there is no external momentum q in inclusive nonleptonic decays which allows analytic continuation into the complex plane. Therefore, the OPE is *a priori* not justified in this case and local duality has to be invoked in order to apply the OPE directly in the physical region. It is obvious that local quark-hadron duality is less firm and secure than global duality, although its validity has been proved to the first two terms in $1/m_Q$ expansion and first order in α_s in the SV limit under the factorization hypothesis [24]. It should be stressed that quark-hadron duality is *exact* in the heavy quark limit, but its systematical $1/m_Q$ expansion is still lacking. It is very likely that $1/m_Q$ corrections to quark-hadron duality behave differently for inclusive semileptonic and nonleptonic decays. Motivated by the conflict between theory and experiment for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$, it was suggested in [16] that the assumption of local duality is not correct for nonleptonic inclusive widths and that the presence of linear $1/m_b$ corrections is strongly indicated by the data. Moreover, the $1/m_b$ corrections are well described by the simple ansatz that the heavy quark mass m_Q is replaced by the decaying hadron mass in the m_Q^5 factor in front of all nonleptonic widths. It is easily seen that the factor $(m_B/m_{\Lambda_b})^5=0.73$ is very close to the observed value of $\tau(\Lambda_b)/\tau(B_d)$. Under this ansatz, a much better description of the lifetimes of bottom and charmed hadrons was shown in [16]. Irrespective of the lifetime ratio problem, there is another important reason why this ansatz is welcome. The absolute decay rate of the B meson predicted in the OPE approach is at least 20% smaller than the experimental value (see Sec. III below). We shall show in Sec. III that the discrepancy between theory and experiment is greatly improved when the nonleptonic width scales with m_B^5.

In the aforementioned factorization approach of [23], the nonleptonic width $\Gamma_{\rm NL}^{\rm had}$ of bottom hadrons scales with $m_{H_b}^5$. Local duality means that a replacement of meson-level kinematics by quark kinematics, for example, $m_{H_b}=m_b(1+\overline{\Lambda}_{H_b}/m_b+\cdots)$, . . . , etc., will turn $\Gamma_{\rm NL}^{\rm had}$ into $\Gamma_{\rm NL}^{\rm OPE}$, the OPE-based decay rate. Consequently, the relation between the violation of local duality and the above-mentioned ansatz will become natural in the factorization approach.

In the present paper we will study nonspectator effects in inclusive nonleptonic and semileptonic decays and analyze the lifetime pattern of heavy hadrons. In particular, we focus on the lifetimes of heavy baryons and study the implications of broken local duality. We will demonstrate that the lifetime hierarchy of bottom baryons is dramatically modified when the quark mass is replaced by the hadron mass in nonleptonic widths. The layout of this paper is organized as follows. In Sec. II we give general heavy quark expansion expressions for inclusive nonleptonic and semileptonic widths and pay attention to the evaluation of baryon four-quark matrix elements and the nonperturbative parameter λ_2 for baryons. We then study bottom-hadron lifetimes in Sec. III and apply the ansatz mentioned above. In Sec. IV we examine the applicability of the same prescription to charmed baryon decays. Discussions and conclusions are given in Sec. V.

II. FRAMEWORK

In this section we write down the general expressions for the inclusive decay widths of heavy hadrons and evaluate the relevant hadronic matrix elements. It is known that the inclusive decay rate is governed by the imaginary part of an effective nonlocal forward transition operator T. When the energy released in the decay is large enough, the nonlocal effective action can be recast as an infinite series of local operators with coefficients containing inverse powers of the heavy quark mass m_Q. Under this heavy quark expansion, the inclusive nonleptonic decay rate of a heavy hadron H_Q containing a heavy quark Q is given by [7,8]

$$\begin{aligned}\Gamma_{\rm NL}(H_Q)=&\frac{G_F^2m_Q^5}{192\pi^3}N_c\xi\frac{1}{2m_{H_Q}}\Bigg[\left(c_1^2+c_2^2+\frac{2c_1c_2}{N_c}\right)\Bigg([I_0(x,0,0)+I_0(x,x,0)]\langle H_Q|\overline{Q}Q|H_Q\rangle\\&-\frac{1}{m_Q^2}[I_1(x,0,0)+I_1(x,x,0)]\langle H_Q|\overline{Q}\sigma\cdot GQ|H_Q\rangle\Bigg)\\&-\frac{4}{m_Q^2}\frac{2c_1c_2}{N_c}(I_2(x,0,0)+I_2(x,x,0))\langle H_Q|\overline{Q}\sigma\cdot GQ|H_Q\rangle\Bigg]+\frac{1}{2m_{H_Q}}\langle H_Q|\mathcal{L}_{\rm nspec}|H_Q\rangle+O(1/m_Q^4),\end{aligned}\tag{2.1}$$

where $\sigma\cdot G=\sigma_{\mu\nu}G^{\mu\nu}$,$c_1,c_2$ are Wilson coefficient functions, $N_c=3$ is the number of color, the factor ξ takes care of the relevant Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, for example, $\xi=|V_{cb}V_{ud}|^2$ for quark-mixing-favored bottom decay, and I_0, I_1, and I_2 are phase-space factors,

[1]The absence of linear $1/m_b$ corrections to decay widths is trivial in the SV limit since the inclusive decay rates depend on $\Delta M=m_B-m_D$ rather than m_B, and $\Delta M=\delta m+O(1/m_b^2)$ with $\delta m=m_b-m_c$ [21].

$$I_0(x,0,0)=(1-x^2)(1-8x+x^2)-12x^2\ln x,$$
$$I_1(x,0,0)=\frac{1}{2}\left(2-x\frac{d}{dx}\right)I_0(x,0,0)=(1-x)^4,$$
$$I_2(x,0,0)=(1-x)^3, \tag{2.2}$$

for the $b\to c\bar{u}d$ ($x=m_c^2/m_b^2$) or $c\to su\bar{d}$ ($x=m_s^2/m_c^2$) transition, and

$$I_0(x,x,0)=v(1-14x-2x^2-12x^3)+24x^2(1-x^2)\ln\frac{1+v}{1-v},$$
$$I_1(x,x,0)=\frac{1}{2}\left(2-x\frac{d}{dx}\right)I_0(x,x,0),$$
$$I_2(x,x,0)=v\left(1+\frac{x}{2}+3x^2\right)-3x(1-2x^2)\ln\frac{1+v}{1-v}, \tag{2.3}$$

for the $b\to cc\bar{s}$ transition with $v\equiv\sqrt{1-4x}$.

The dimension-6 four-quark operators $\mathcal{L}_{\rm nspec}$ in Eq. (2.1) describe nonspectator effects in inclusive decays of heavy hadrons and are given by [3–5]

$$\begin{aligned}\mathcal{L}_{\rm nspec}=&\frac{G_F^2m_Q^2}{2\pi}\xi(1-x)^2\{(c_1^2+c_2^2)(\bar{Q}Q)(\bar{q}_1q_1)+2c_1c_2(\bar{Q}q_1)(\bar{q}_1Q)\}-\frac{G_F^2m_Q^2}{6\pi}\xi\Bigg\{c_1^2(1-x)^2\Bigg[\left(1+\frac{x}{2}\right)(\bar{Q}Q)(\bar{q}_2q_2)\\&-(1+2x)\bar{Q}^\alpha(1-\gamma_5)q_2^\beta\bar{q}_2^\beta(1+\gamma_5)Q^\alpha\Bigg]+(2c_1c_2+N_cc_2^2)(1-x)^2\Bigg[\left(1+\frac{x}{2}\right)(\bar{Q}q_2)(\bar{q}_2Q)-(1+2x)\bar{Q}\\&\times(1-\gamma_5)q_2\bar{q}_2(1+\gamma_5)Q\Bigg]\Bigg\}-\frac{G_F^2m_Q^2}{6\pi}\xi\{c_1^2\sqrt{1-4x}[(1-x)(\bar{Q}Q)(\bar{q}_3q_3)-(1+2x)\bar{Q}^\alpha(1-\gamma_5)q_3^\beta\bar{q}_3^\beta(1+\gamma_5)Q^\alpha]\\&+(2c_1c_2+N_cc_2^2)\sqrt{1-4x}[(1-x)(\bar{Q}q_3)(\bar{q}_3Q)-(1+2x)\bar{Q}(1-\gamma_5)q_3\bar{q}_3(1+\gamma_5)Q]\}\\&-\frac{G_F^2m_Q^2}{6\pi}\xi\Bigg\{c_2^2(1-x)^2\Bigg[\left(1+\frac{x}{2}\right)(\bar{Q}Q)(\bar{q}_3q_3)-(1+2x)\bar{Q}^\alpha(1-\gamma_5)q_3^\beta\bar{q}_3^\beta(1+\gamma_5)Q^\alpha\Bigg]+(2c_1c_2+N_cc_1^2)(1-x)^2\\&\times\Bigg[\left(1+\frac{x}{2}\right)(\bar{Q}q_3)(\bar{q}_3Q)-(1+2x)\bar{Q}(1-\gamma_5)q_3\bar{q}_3(1+\gamma_5)Q\Bigg]\Bigg\},\end{aligned} \tag{2.4}$$

where $(\bar{q}'q)\equiv\bar{q}'\gamma_\mu(1-\gamma_5)q$, and α,β are color indices. Note that for charm decay, $Q=c$, $q_1=u$, $q_2=d$, and $q_3=s$, and for bottom decay, $Q=b$, $q_1=u$, $q_2=d$, and $q_3=s$. The last term in Eq. (2.4) is due to the constructive interference of the s quark and hence it occurs only in charmed baryon decays. The third term in Eq. (2.4) exists only in bottom decays with $c\bar{c}$ intermediate states. For inclusive nonleptonic decays of heavy mesons, the first term in Eq. (2.4) corresponds to a Pauli interference and the second and third terms to W-exchange contributions. For heavy baryon decays, the first term is a W-exchange contribution and the rest are interference terms. The phase-space suppression factors, e.g., $(1-x)^2$, $\sqrt{1-4x}$, . . . , etc., in Eq. (2.4) are derived in [25,10].

Several remarks are in order. (i) There is no linear $1/m_Q$ corrections to the inclusive decay rate due to the lack of gauge-invariant dimension-4 operators [20,7], a consequence known as Luke's theorem [26]. Nonperturbative corrections start at order $1/m_Q^2$. (ii) It is clear from Eqs. (2.1) and (2.4) that there is a two-body phase-space enhancement factor of $16\pi^2$ for nonspectator effects relative to the three-body phase space for heavy quark decay. This implies that nonspectator effects, being of order $1/m_Q^3$, are comparable to and even exceed the $1/m_Q^2$ terms. (iii) For charmed meson decay, the $1/N_c$ correction to $\Gamma_{\rm NL}$ characterized by the term $(2c_1c_2/N_c)\langle H_c|\bar{c}c|H_c\rangle$ is found to be compensated by the nonperturbative gluonic effect [i.e., the term proportional to $I_2(x,0,0)$]. This cancellation is small for B meson decay due to the smallness of $1/m_b^2$. This indicates that the rule of discarding $1/N_c$ terms [27] is operative in charm decays but not so for the B meson case. (iv) Thus far the Wilson coefficients and four-quark operators in Eq. (2.4) are renormalized at the heavy quark mass scale. Sometimes the so-called hybrid renormalization [5,28] is performed to evolve the four-quark operators (not the Wilson coefficients) from m_Q down to a low energy scale, say, a typical hadronic scale $\mu_{\rm had}$. The underlying reason is that the factorizable approximation for meson matrix elements and the quark model for baryon matrix elements are believed to be more reliable at the scale $\mu_{\rm had}$. The evolution from m_Q down to $\mu_{\rm had}$ will in general introduce new structures such as penguin operators. However, in the present paper we will follow [10] to employ Eqs. (2.1) and (2.4) as our starting point for describing inclusive weak decays since it is equivalent to first evaluating the four-quark matrix elements renormalized at the m_Q scale and then relating them to the hadronic matrix elements renormalized at $\mu_{\rm had}$ through the renormalization group equation, provided that the effect of penguin operators is neglected.

For inclusive semileptonic decays, apart from the heavy quark decay contribution there is an additional nonspectator effect in charmed baryon semileptonic decay originating from the Pauli interference of the s quark [29]. We are now ready to deduce the inclusive semileptonic widths from Eq. (2.1) and the last term in Eq. (2.4) by putting $c_1=1$, $c_2=0$, and $N_c=1$:

$$\Gamma_{\rm SL}(H_Q)=\frac{G_F^2m_Q^5}{192\pi^3}|V_{\rm CKM}|^2\,\frac{\eta(x,x_\ell,0)}{2m_{H_Q}}\times\left[I_0(x,0,0)\langle H_Q|\overline{Q}Q|H_Q\rangle-\frac{1}{m_Q^2}I_1(x,0,0)\times\langle H_Q|\overline{Q}\sigma\cdot GQ|H_Q\rangle\right]-\frac{G_F^2m_c^2}{6\pi}|V_{cs}|^2\times\frac{1}{2m_{H_c}}(1-x)^2\left[\left(1+\frac{x}{2}\right)(\overline{c}s)(\overline{s}c)-(1+2x)\,\overline{c}(1-\gamma_5)s\,\overline{s}(1+\gamma_5)c\right],\qquad(2.5)$$

where $\eta(x,x_\ell,0)$ with $x_\ell=(m_\ell/m_Q)^2$ is the QCD radiative correction to the semileptonic decay rate. Its general analytic expression is given in [30]. The special case $\eta(x,0,0)$ is given in [31] and it can be approximated numerically by [32]

$$\eta(x,0,0)\cong1-\frac{2\alpha_s}{3\pi}\left[\left(\pi^2-\frac{31}{4}\right)(1-\sqrt{x})^2+\frac{3}{2}\right].\qquad(2.6)$$

With $x=0$ and the replacement $\alpha_s\to\frac{3}{4}\alpha$, Eq. (2.6) is reduced to the well-known QED correction to the muon decay. The second term in Eq. (2.5) occurs only in the semileptonic decay of Ξ_c and Ω_c baryons.

We next turn to the two-body matrix elements $\langle H_Q|\overline{Q}Q|H_Q\rangle$. The use of the equation of motion

$$\overline{Q}Q=\overline{Q}\not{v}Q+\frac{1}{2m_Q^2}\overline{Q}(iD_\perp)^2Q+\frac{1}{4m_Q^2}\overline{Q}\sigma\cdot GQ+O(1/m_Q^3),\qquad(2.7)$$

with $D_\perp^\mu=\partial^\mu-v^\mu v\cdot D$, leads to

$$\frac{\langle H_Q|\overline{Q}Q|H_Q\rangle}{2m_{H_Q}}=1-\frac{K_H}{2m_Q^2}+\frac{G_H}{2m_Q^2},\qquad(2.8)$$

with

$$K_H\equiv-\frac{1}{2m_{H_Q}}\langle H_Q|\overline{Q(iD_\perp)}^2Q|H_Q\rangle=-\lambda_1,$$

$$G_H\equiv\frac{1}{2m_{H_Q}}\langle H_Q|\overline{Q}\tfrac{1}{2}\sigma\cdot GQ|H_Q\rangle=d_H\lambda_2.\qquad(2.9)$$

The mass of the heavy hadron H_Q is then of the form

$$m_{H_Q}=m_Q+\overline{\Lambda}_{H_Q}-\frac{\lambda_1}{2m_Q}-\frac{d_H\lambda_2}{2m_Q},\qquad(2.10)$$

where the three nonperturbative HQET parameters $\overline{\Lambda}_{H_Q}$, λ_1, and λ_2 are independent of the heavy quark mass and in general $\overline{\Lambda}_{H_Q}$ is different for different heavy hadrons. Since $\sigma\cdot G\sim\vec{S}_Q\cdot\vec{B}$ and since the chromomagnetic field is produced by the light cloud inside the heavy hadron, it is clear that $\sigma\cdot G$ is proportional to $\vec{S}_Q\cdot\vec{S}_\ell$, where $\vec{S}_Q$ ($\vec{S}_\ell$) is the spin operator of the heavy quark (light cloud). More precisely,

$$d_H=-\langle H_Q|4\vec{S}_Q\cdot\vec{S}_\ell|H_Q\rangle=-2[S_{\rm tot}(S_{\rm tot}+1)-S_Q(S_Q+1)-S_\ell(S_\ell+1)].\qquad(2.11)$$

Therefore, $d_H=3$ for B,D mesons, $d_H=-1$ for B^*,D^* mesons, $d_H=0$ for the antitriplet baryon T_Q, $d_H=4$ for the spin-$\frac{1}{2}$ sextet baryon S_Q, and $d_H=-2$ for the spin-$\frac{3}{2}$ sextet baryon S_Q^*. It follows from Eq. (2.10) that

$$\lambda_2^{\rm meson}=\frac{1}{4}(m_{P*}^2-m_P^2)=\begin{cases}0.12\ \ {\rm GeV}^2 & \text{for the } B \text{ meson},\\ 0.14\ \ {\rm GeV}^2 & \text{for the } D \text{ meson},\end{cases}$$

$$\lambda_2^{\rm baryon}=\frac{1}{6}(m_{S_Q^*}^2-m_{S_Q}^2).\qquad(2.12)$$

The values of $\lambda_2^{\rm baryon}$ will be fixed later. As for the kinetic energy parameter λ_1 we use [33]

$$\lambda_1^{\rm meson}\sim\lambda_1^{\rm baryon}=-(0.4\pm0.2)\ \ {\rm GeV}^2.\qquad(2.13)$$

This leads to

$$m_b-m_c=(\langle m_B\rangle-\langle m_D\rangle)\left(1-\frac{\lambda_1}{2\langle m_B\rangle\langle m_D\rangle}\right)=(3.40\pm0.03)\ \ {\rm GeV},\qquad(2.14)$$

where $\langle m_P\rangle=\frac{1}{4}(m_P+3m_{P*})$ denotes the spin-averaged meson mass.

We will follow [10] to parametrize the hadronic matrix elements in a model-independent way. For meson matrix elements of four-quark operators, we follow [10] to define the parameters B_i and ε_i:

$$\langle \overline{B_q}|(\overline{b}q)(\overline{q}b)|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 B_1,$$

$$\langle \overline{B_q}|\overline{b}(1-\gamma_5)q\overline{q}(1+\gamma_5)b|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 B_2,$$

$$\langle \overline{B_q}|(\overline{b}t^a q)(\overline{q}t^a b)|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 \varepsilon_1,$$

$$\langle \overline{B_q}|\overline{b}t^a(1-\gamma_5)q\overline{q}t^a(1+\gamma_5)b|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 \varepsilon_2, \qquad (2.15)$$

where $(\overline{q'}t^a q) \equiv \overline{q'}t^a\gamma_\mu(1-\gamma_5)q$ and $t^a = \lambda^a/2$. Under the factorization approximation, B_i and ε_i are given by $B_i = 1$ and $\varepsilon_i = 0$, but they will be treated as free parameters here. As a consequence of Eqs. (2.15), we obtain

$$\langle \overline{B_q}|(\overline{b}b)(\overline{q}q)|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 (\tfrac{1}{3}B_1 + 2\varepsilon_1),$$

$$\langle \overline{B_q}|\overline{b}^\alpha(1-\gamma_5)q^\beta\overline{q}^\beta(1+\gamma_5)b^\alpha|\overline{B_q}\rangle = f_{B_q}^2 m_{B_q}^2 (\tfrac{1}{3}B_2 + 2\varepsilon_2). \qquad (2.16)$$

As for the baryon matrix elements of four-quark operators we have to rely on the quark model. We first consider the MIT bag model [34] and define three four-quark overlap integrals:

$$a_q = \int d^3r[u_q^2(r)u_Q^2(r) + v_q^2(r)v_Q^2(r)],$$

$$b_q = \int d^3r[u_q^2(r)v_Q^2(r) + v_q^2(r)u_Q^2(r)],$$

$$c_q = \int d^3r u_q(r)v_q(r)u_Q(r)v_Q(r), \qquad (2.17)$$

which are expressed in terms of the large and small components $u(r)$ and $v(r)$, respectively, of the quark wave function. For the antitriplet heavy baryon T_Q or the sextet heavy baryon Ω_Q (recall that only the Ω_c^0 and Ω_b^- of the sextet baryons decay weakly), the four baryon matrix elements

$$\langle T_Q|(\overline{Q}q)(\overline{q}Q)|T_Q\rangle, \quad \langle T_Q|(\overline{Q}Q)(\overline{q}q)|T_Q\rangle,$$

$$\langle T_Q|\overline{Q}(1-\gamma_5)q\overline{q}(1+\gamma_5)Q|T_Q\rangle,$$

$$\langle T_Q|\overline{Q}^\alpha(1-\gamma_5)q^\beta\overline{q}^\beta(1+\gamma_5)Q^\alpha|T_Q\rangle$$

are not all independent. First of all, we have

$$\langle T_Q|(\overline{Q}q)(\overline{q}Q)|T_Q\rangle = -(a_q+b_q)(2m_{T_Q}),$$

$$\langle \Omega_Q|(\overline{Q}s)(\overline{s}Q)|\Omega_Q\rangle = -\tfrac{1}{3}(18a_s + 2b_s + 32c_s)(2m_{\Omega_Q}) \qquad (2.18)$$

(see e.g., Ref. [35] for the technical detail of the bag model evaluation), where we have taken into account the fact that there are two valence s quarks in the wave function of the Ω_Q. Second, since the color wave function for a baryon is totally antisymmetric, the matrix element of $(\overline{Q}Q)(\overline{q}q)$ is the same as that of $(\overline{Q}q)(\overline{q}Q)$ except for a sign difference. Thus we follow [10] to define a parameter $\widetilde{B}$:

$$\langle T_Q|(\overline{Q}Q)(\overline{q}q)|T_Q\rangle = -\widetilde{B}\langle T_Q|(\overline{Q}q)(\overline{q}Q)|T_Q\rangle,$$

$$\langle \Omega_Q|(\overline{Q}Q)(\overline{s}s)|\Omega_Q\rangle = -\widetilde{B}\langle \Omega_Q|(\overline{Q}s)(\overline{s}Q)|\Omega_Q\rangle, \qquad (2.19)$$

so that $\widetilde{B} = 1$ in the valence-quark approximation. Third, it is straightforward to show that

$$\langle T_Q|\overline{Q}^\alpha\gamma_\mu\gamma_5 Q^\beta\overline{q}^\beta\gamma^\mu(1-\gamma_5)q^\alpha|T_Q\rangle = 0,$$

$$\langle \Omega_Q|\overline{Q}^\alpha\gamma_\mu\gamma_5 Q^\beta\overline{q}^\beta\gamma^\mu(1-\gamma_5)q^\alpha|\Omega_Q\rangle = 4\left(a - \frac{b}{3}\right)(2m_{\Omega_Q}). \qquad (2.20)$$

The first relation in Eqs. (2.20) is actually a model-independent consequence of heavy quark spin symmetry [10]. Since

$$\overline{Q}^\alpha\gamma_\mu\gamma_5 Q^\beta\overline{q}^\beta\gamma^\mu(1-\gamma_5)q^\alpha = -\overline{Q}(1-\gamma_5)q\overline{q}(1+\gamma_5)Q - \tfrac{1}{2}(\overline{Q}q)(\overline{q}Q), \qquad (2.21)$$

it follows from Eqs. (2.20) that

$$\begin{aligned}\langle T_Q|\overline{Q}^\alpha(1-\gamma_5)q^\beta\overline{q}^\beta(1+\gamma_5)Q^\alpha|T_Q\rangle &= -\widetilde{B}\langle T_Q|\overline{Q}(1-\gamma_5)q\overline{q}(1+\gamma_5)Q|T_Q\rangle \\ &= -\tfrac{1}{2}\widetilde{B}(a_q+b_q)(2m_{T_Q}),\end{aligned}$$

$$\begin{aligned}\langle \Omega_Q|\overline{Q}^\alpha(1-\gamma_5)s^\beta\overline{s}^\beta(1+\gamma_5)Q^\alpha|\Omega_Q\rangle &= -\widetilde{B}\langle \Omega_Q|\overline{Q}(1-\gamma_5)s\overline{s}(1+\gamma_5)Q|\Omega_Q\rangle \\ &= \widetilde{B}(a_s - \tfrac{5}{3}b_s - \tfrac{16}{3}c_s)(2m_{\Omega_Q}).\end{aligned} \qquad (2.22)$$

In the nonrelativistic quark model (NQM), baryon matrix elements of four-quark operators are the same as that of Eqs. (2.18) and (2.22) except for the replacement

$$a_q \to |\psi_{Qq}(0)|^2 = \int d^3r u_q^2(r)u_Q^2(r), \quad b_q \to 0, \quad c_q \to 0. \qquad (2.23)$$

In general, the strength of destructive Pauli interference and W exchange is governed by $a_q + b_q$ in the bag model and $|\psi(0)|^2$ in the NQM. However, it is well known in hyperon decay that the bag model calculation of $a_q + b_q$ gives a much smaller value than the nonrelativistic estimate of $|\psi(0)|^2$: $a_u + b_u \sim 3\times 10^{-3}$ GeV^3, while $|\psi(0)|^2 \sim 10^{-2}$ GeV^3. We shall see later that this also occurs in bottom baryon decay. As pointed out in [36], naively one may be tempted to conclude that the relativistic models are presumably more reliable. For example, the lower component of the wave function is needed to reduce the NQM prediction $g_A = \frac{5}{3}$ to the experimental value of 1.25. However, the difference between $a_u + b_u$ and $|\psi(0)|^2$ is not simply attributed to relativistic corrections; it arises essentially from the distinction in the spatial scale of the wave function especially at the origin. As a consequence, both models give a quite different quantitative description for processes sensitive to $|\psi(0)|^2$. It has been long advocated in [37] that a small value of $|\psi(0)|^2$ should be discarded since a realistic potential that fits to the orbital-excitation spectrum yields $\langle \delta(\vec{r}_1-\vec{r}_2)\rangle \sim 10^{-2}$ GeV^3. Empirically, it also appears that the NQM works better for charmed baryon decays [4,36].

In the following we will consider the NQM estimate of baryon matrix elements. Consider $|\psi^{\Lambda_b}_{bq}(0)|^2$ as an example. A straightforward calculation of hyperfine splitting between Σ_b and Λ_b yields [38]

$$m_{\Sigma_b}-m_{\Lambda_b}=\frac{16\pi}{9}\alpha_s(m_b)\frac{m_b-m_q}{m_b m_q^2}|\psi^{\Lambda_b}_{bq}(0)|^2, \quad (2.24)$$

where the equality $|\psi^{\Sigma_b}_{bq}(0)|^2=|\psi^{\Lambda_b}_{bq}(0)|^2$ has been assumed. The uncertainties in Eq. (2.24) associated with $\alpha_s(m_b)$ and the constituent quark mass m_q can be reduced by introducing the B meson wave function at the origin squared, $|\psi^{B}_{b\bar{q}}(0)|^2=\frac{1}{12}f_B^2 m_B$, which is related to the B^* and B mass difference by $m_{B^*}-m_B=\frac{32}{9}\pi\alpha_s(m_b)|\psi^B_{b\bar{q}}(0)|^2/(m_b m_q)$. Hence,

$$|\psi^{\Lambda_b}_{bq}(0)|^2=\frac{2m_q}{m_b-m_q}\frac{m_{\Sigma_b}-m_{\Lambda_b}}{m_{B^*}-m_B}|\psi^B_{b\bar{q}}(0)|^2. \quad (2.25)$$

Another method is proposed by Rosner [15] to consider the hyperfine splittings of Σ_b and B separately so that

$$|\psi^{\Lambda_b}_{bq}(0)|^2=|\psi^{\Sigma_b}_{bq}(0)|^2=\frac{4}{3}\frac{m_{\Sigma_b^*}-m_{\Sigma_b}}{m_{B^*}-m_B}|\psi^B_{b\bar{q}}(0)|^2. \quad (2.26)$$

This method is supposed to be most reliable as $|\psi_{bq}(0)|^2$ thus determined does not depend on α_s and m_q directly. Numerically, we find that Eqs. (2.25) and (2.26) both give very similar results. Defining the wave function ratio

$$r=\left|\frac{\psi^{\Lambda_b}_{bq}(0)}{\psi^B_{b\bar{q}}(0)}\right|^2, \quad (2.27)$$

the baryon matrix elements in Eqs. (2.18) and (2.22) can be recast to

$$\langle T_b|(\bar{b}b)(\bar{q}q)|T_b\rangle=-\tilde{B}\langle T_b|(\bar{b}q)(\bar{q}b)|T_b\rangle$$
$$=\tfrac{1}{12}f_{B_q}^2 m_{B_q} r\tilde{B}(2m_{T_b}),$$

$$\langle T_b|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|T_b\rangle=\tfrac{1}{24}f_{B_q}^2 m_{B_q} r(2m_{T_b}),$$

$$\langle T_b|\bar{b}^{\alpha}(1-\gamma_5)q^{\beta}\bar{q}^{\beta}(1+\gamma_5)b^{\alpha}|T_b\rangle$$
$$=-\tfrac{1}{24}f_{B_q}^2 m_{B_q} r\tilde{B}(2m_{T_b}),$$

$$\langle \Omega_b|(\bar{b}b)(\bar{s}s)|\Omega_b\rangle=-\tilde{B}\langle \Omega_b|(\bar{b}s)(\bar{s}b)|\Omega_b\rangle$$
$$=\tfrac{1}{2}f_{B_q}^2 m_{B_q} r\tilde{B}(2m_{\Omega_b}),$$

$$\langle \Omega_b|\bar{b}(1-\gamma_5)s\bar{s}(1+\gamma_5)b|\Omega_b\rangle=-\tfrac{1}{12}f_{B_q}^2 m_{B_q} r(2m_{\Omega_b}),$$

$$\langle \Omega_b|\bar{b}^{\alpha}(1-\gamma_5)s^{\beta}\bar{s}^{\beta}(1+\gamma_5)b^{\alpha}|\Omega_b\rangle=\tfrac{1}{12}f_{B_q}^2 m_{B_q} r\tilde{B}(2m_{\Omega_b}), \quad (2.28)$$

where f_{B_q} is the decay constant of the meson $\overline{B_q}$.

To estimate $|\psi_{bq}(0)|^2$ and the parameter r in the NQM, we find, from Eq. (2.26),[2]

$$r_{\Lambda_b}=\frac{4}{3}\frac{m_{\Sigma_b^*}-m_{\Sigma_b}}{m_{B^*}-m_B}, \quad r_{\Xi_b}=\frac{4}{3}\frac{m_{\Xi_b^*}-m_{\Xi'_b}}{m_{B^*}-m_B},$$

$$r_{\Omega_b}=\frac{4}{3}\frac{m_{\Omega_b^*}-m_{\Omega_b}}{m_{B^*}-m_B}, \quad (2.29)$$

and likewise for r_{Λ_c}, r_{Ξ_c}, and r_{Ω_c}, where $\Xi'_{b,c}$ denote spin-$\frac{1}{2}$ sextets. Heavy baryon masses have been studied in [39] in $1/m_Q$ and $1/N_c$ expansions within the HQET framework. The chromomagnetic mass splittings for charmed baryons are given by [39]

$$m_{\Sigma_c^*}-m_{\Sigma_c}=65.7\pm 2.3 \text{ MeV},$$
$$m_{\Xi_c^*}-m_{\Xi'_c}=63.2\pm 2.6 \text{ MeV},$$
$$m_{\Omega_c^*}-m_{\Omega_c}=60.6\pm 5.7 \text{ MeV}, \quad (2.30)$$

where precise measurements of Σ_c^* and Ξ_c^* have been reported by CLEO [40]. It is evident that the heavy-quark spin-violating mass relation [39]

$$(m_{\Sigma_c^*}-m_{\Sigma_c})+(m_{\Omega_c^*}-m_{\Omega_c})=2(m_{\Xi_c^*}-m_{\Xi'_c}) \quad (2.31)$$

is very accurate. It follows that

$$m_{\Sigma_b^*}-m_{\Sigma_b}=\left(\frac{m_c}{m_b}\right)(m_{\Sigma_c^*}-m_{\Sigma_c})=21.0 \text{ MeV} \quad (2.32)$$

for $m_b=5$ GeV, $m_c=1.6$ GeV (see below). This mass splitting is substantially smaller than the preliminary result $m_{\Sigma_b^*}-m_{\Sigma_b}=(56\pm 16)$ MeV reported by the DELPHI Collaboration [41]. Since the measured mass difference of Σ_c^* and Σ_c is around 66 MeV [cf. Eqs. (2.30)], a large hyperfine splitting of order 55 MeV for the Σ_b baryon is very unlikely. Likewise,

$$m_{\Xi_b^*}-m_{\Xi'_b}=20.2 \text{ MeV}, \quad m_{\Omega_b^*}-m_{\Omega_b}=19.4 \text{ MeV}. \quad (2.33)$$

Because $\Delta m_B=m_{B^*}-m_B=45.7\pm 0.4$ MeV and $\Delta m_D=m_{D^*}-m_D\cong 143$ MeV [42] [note that Δm_B and Δm_D obey the same scaling relation as Eq. (2.32)], we find

$$r_{\Lambda_c}\cong r_{\Lambda_b}=0.61, \quad r_{\Xi_c}\cong r_{\Xi_b}=0.59, \quad r_{\Omega_c}\cong r_{\Omega_b}=0.53, \quad (2.34)$$

[2]Our result for r_{Λ_b} is the same as [15] but different from [10] in which r_{Λ_b} is given by $\frac{4}{3}(m^2_{\Sigma_b^*}-m^2_{\Sigma_b})/(m^2_{B^*}-m^2_B)$.

and

$$|\psi_{bq}^{\Lambda_b}(0)|^2=0.87\times10^{-2}\ \ \mathrm{GeV}^3,$$

$$|\psi_{bq}^{\Xi_b}(0)|^2=0.84\times10^{-2}\ \ \mathrm{GeV}^3,$$

$$|\psi_{bq}^{\Omega_b}(0)|^2=0.81\times10^{-2}\ \ \mathrm{GeV}^3, \tag{2.35}$$

for $f_{B_q}=180$ MeV [43]. An estimate in the QCD sum rule analysis yields $r\simeq0.1-0.3$ [17]. Therefore, the NQM estimate of $|\psi_{bq}(0)|^2$ is indeed larger than the analogous bag model quantity: $a_q+b_q\sim3\times10^{-3}$ GeV^3. However, for the charmed baryon we obtain $|\psi_{cq}^{\Lambda_c}(0)|^2=3.8\times10^{-3}$ GeV^3 for $f_D=200$ MeV [43], which is smaller than those in bottom or hyperon decay. It seems that the smallness of $|\psi_{cq}^{\Lambda_c}(0)|^2$ is ascribed to the assumption that the D meson wave function at the origin squared, $|\psi_{c\bar{q}}^{D}(0)|^2$, is given by $\frac{1}{12}f_D^2m_D$. We will come back to this point in Sec. IV. By comparing Eqs. (2.28) with Eq. (2.18) we see that r is of order 0.20 in the bag model.

Finally we are ready to estimate the HQET parameter $\lambda_2^{\mathrm{baryon}}$ [see Eq. (2.12)]. Using the baryon masses [39]

$$m_{\Sigma_c}=2452.9\ \ \mathrm{MeV},\quad m_{\Xi_c'}=2580.8\ \ \mathrm{MeV},$$

$$m_{\Omega_c}=2699.9\ \ \mathrm{MeV},$$

$$m_{\Sigma_b}=5824.2\ \ \mathrm{MeV},\quad m_{\Xi_b'}=5950.9\ \ \mathrm{MeV},$$

$$m_{\Omega_b}=6068.7\ \ \mathrm{MeV}, \tag{2.36}$$

and Eqs. (2.30)–(2.33) we find

$$\lambda_2^{\mathrm{baryon}}=\begin{cases}0.055\ \ \mathrm{GeV}^2 & \text{for charmed baryons,}\\ 0.041\ \ \mathrm{GeV}^2 & \text{for } \Sigma_b,\\ 0.040\ \ \mathrm{GeV}^2 & \text{for } \Xi_b',\\ 0.039\ \ \mathrm{GeV}^2 & \text{for } \Omega_b.\end{cases} \tag{2.37}$$

It is interesting to note that the large-N_c relation [39]

$$\lambda_2^{\mathrm{meson}}\sim N_c\lambda_2^{\mathrm{baryon}} \tag{2.38}$$

is fairly satisfied especially for bottom hadrons.

III. LIFETIMES OF BOTTOM HADRONS

Using the formulism described in the last section, semileptonic and nonleptonic widths are calculated in this section. We shall first try to fix the heavy quark pole mass from the measured inclusive semileptonic decay rate. The semileptonic width of the B meson given by Eq. (2.5),

$$\Gamma_{\mathrm{SL}}(B\to Xe\bar{\nu})=\frac{G_F^2m_b^5}{192\pi^3}|V_{cb}|^2\eta(x,0,0)\times\left\{I_0(x,0,0)\frac{\langle\bar{B}|\bar{b}b|\bar{B}\rangle}{2m_B}-\frac{1}{m_b^2}I_1(x,0,0)\frac{\langle\bar{B}|\bar{b}\sigma\cdot Gb|\bar{B}\rangle}{2m_B}\right\}, \tag{3.1}$$

has the salient feature that empirically $\Gamma_{\mathrm{SL}}(B)$ is very insensitive to the choice of m_b as long as m_b-m_c, which is free of renormalon ambiguity, is fixed according to Eq. (2.14). Hence, we may use the measured $\Gamma_{\mathrm{SL}}(D)$ to fix m_c to be 1.6 GeV (see Sec. IV below) which in turn implies $m_b=5$ GeV, in excellent agreement with the pole mass determined from lattice QCD: $m_b=5.0\pm0.2$ GeV [44]. Since $x=(m_c/m_b)^2=0.1024$, the phase-space factors I_i in Eqs. (2.2) and (2.3) read

$$I_0(x,0,0)=0.476,\quad I_0(x,x,0)=0.147,\quad I_1(x,0,0)=0.649,$$

$$I_1(x,x,0)=0.328,\quad I_2(x,0,0)=0.723,\quad I_2(x,x,0)=0.220. \tag{3.2}$$

From Eq. (3.1) we obtain

$$\Gamma(B\to Xe\bar{\nu})=4.44\times10^{-14}\ \ \mathrm{GeV},$$

$$\Gamma_{\mathrm{SL}}(B)=2.24\Gamma(B\to Xe\bar{\nu})=9.95\times10^{-14}\ \ \mathrm{GeV}, \tag{3.3}$$

for $|V_{cb}|=0.039$, where use of Eqs. (2.8), (2.9), (2.12), and (2.13) has been made, for example,

$$\frac{\langle\bar{B}|\bar{b}\sigma\cdot Gb|\bar{B}\rangle}{2m_B}=6\lambda_2^{\mathrm{meson}}=0.72\ \ \mathrm{GeV}^2. \tag{3.4}$$

Since the phase space for the τ semileptonic decay mode relative to that of the e mode is 0.24:1, this accounts for the factor 2.24 in Eqs. (3.3). The result (3.3) agrees very well with experiment [42]:

$$\Gamma(B^-/B^0\ \ \text{admixture}\to Xe\bar{\nu})=(4.31\pm0.17)\times10^{-14}\ \ \mathrm{GeV}. \tag{3.5}$$

Likewise, we find, for bottom baryon semileptonic decays,

$$\Gamma(\Lambda_b\to Xe\bar{\nu})=\Gamma(\Xi_b\to Xe\bar{\nu})=4.59\times10^{-14}\ \ \mathrm{GeV},$$

$$\Gamma(\Omega_b\to Xe\bar{\nu})=4.53\times10^{-14}\ \ \mathrm{GeV}, \tag{3.6}$$

and hence

$$\Gamma_{\mathrm{SL}}(\Lambda_b)=\Gamma_{\mathrm{SL}}(\Xi_b)=2.24\Gamma(\Lambda_b\to Xe\bar{\nu})=1.027\times10^{-13}\ \ \mathrm{GeV},$$

$$\Gamma_{\mathrm{SL}}(\Omega_b)=2.24\Gamma(\Omega_b\to Xe\bar{\nu})=1.014\times10^{-13}\ \ \mathrm{GeV}. \tag{3.7}$$

Note that the tiny difference between $\Gamma_{\mathrm{SL}}(\Lambda_b)$ and $\Gamma_{\mathrm{SL}}(\Omega_b)$ arises from the fact that the chromomagnetic operator contributes to the matrix element of Ω_b but not to Λ_b (or Ξ_b) as the light degrees of freedom in the latter are spinless; that is,

$$\frac{\langle\Lambda_b|\overline{b}\sigma\cdot Gb|\Lambda_b\rangle}{2m_{\Lambda_b}}=0,$$

$$\frac{\langle\Omega_b|\overline{b}\sigma\cdot Gb|\Omega_b\rangle}{2m_{\Omega_b}}=8\lambda_2^{\rm baryon}=0.31\ \ {\rm GeV}^2. \qquad (3.8)$$

To compute the nonleptonic decay rate we apply the Wilson coefficient functions

$$c_1(\mu)=1.14,\quad c_2(\mu)=-0.31, \qquad (3.9)$$

which are evaluated at $\mu=4.4$ GeV to the leading logarithmic approximation (see Table XIII of [45]). From Eq. (2.1) the nonleptonic widths of bottom baryons arising from b quark decay are found to be

$$\Gamma^{\rm dec}(B)=2.216\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma^{\rm dec}(\Omega_b)=2.217\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma^{\rm dec}(\Lambda_b)=\Gamma^{\rm dec}(\Xi_b)=2.220\times10^{-13}\ \ {\rm GeV}. \qquad (3.10)$$

We see that the b quark decay contribution $\Gamma^{\rm dec}$ is very similar for bottom hadrons even though the chromomagnetic mass splitting is different among them. Therefore, to $O(1/m_b^3)$ we obtain

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}\cong\frac{\tau(\Xi_b)}{\tau(B_d)}\cong\frac{\tau(\Omega_b)}{\tau(B_d)}=0.99+O(1/m_b^3). \qquad (3.11)$$

We next turn to the nonspectator effects of order $1/m_b^3$. The Pauli interference in inclusive nonleptonic B^- decay and the W-exchange contribution to B_d can be evaluated from the first and second terms in Eq. (2.4):

$$\Gamma^{\rm ann}(B_d)=-\Gamma_0\,\eta_{\rm nspec}\{(1-x)^2(1+\tfrac{1}{2}x)[(\tfrac{1}{3}c_1^2+2c_1c_2+N_cc_2^2)B_1+2c_1^2\varepsilon_1]-(1-x)^2(1+2x)\times[(\tfrac{1}{3}c_1^2+2c_1c_2+N_cc_2^2)B_2+2c_1^2\varepsilon_2]\},$$

$$\Gamma_-^{\rm int}(B^-)=\Gamma_0\,\eta_{\rm nspec}(1-x)^2[(c_1^2+c_2^2)(B_1+6\varepsilon_1)+6c_1c_2B_1], \qquad (3.12)$$

with [10]

$$\Gamma_0=\frac{G_F^2m_b^5}{192\pi^3}|V_{cb}V_{ud}|^2,\qquad \eta_{\rm nspec}=16\pi^2\frac{f_B^2m_B}{m_b^3}, \qquad (3.13)$$

where we have applied Eqs. (2.15) and (2.16) and neglected the Cabibbo-suppressed W-exchange contribution to B_d. As stressed in [10], the coefficients of B_i in Eqs. (3.12) are one to two orders of magnitude smaller than that of ε_i. Therefore, the contributions of B_i can be safely neglected at least in $\Gamma^{\rm ann}(B_d)$. Numerically,

$$\Gamma^{\rm ann}(B_d)=(-0.491\varepsilon_1+0.563\varepsilon_2)\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma_-^{\rm int}(B^-)=(-0.130B_1+1.505\varepsilon_1)\times10^{-13}\ \ {\rm GeV}. \qquad (3.14)$$

Beyond the factorization approximation, ε_i may receive nonfactorizable contributions. A QCD sum rule estimate gives $\varepsilon_1\approx-0.15$ and $\varepsilon_2\approx0$ [25]. This implies a constructive W exchange to B_d and a destructive Pauli interference to B^-.

As for the nonspectator effects in nonleptonic decays of bottom baryons we obtain from Eq. (2.4) that

$$\Gamma^{\rm ann}(\Lambda_b)=\Gamma_0\,\eta_{\rm nspec}r(1-x)^2[\widetilde{B}(c_1^2+c_2^2)-2c_1c_2],$$

$$\Gamma_-^{\rm int}(\Lambda_b)=-\frac{1}{4}\Gamma_0\,\eta_{\rm nspec}r\left[(1-x)^2(1+x)+\left|\frac{V_{cd}}{V_{ud}}\right|^2\sqrt{1-4x}\right]\times(\widetilde{B}c_1^2-2c_1c_2-N_cc_2^2),$$

$$\Gamma^{\rm ann}(\Xi_b^0)=\Gamma^{\rm ann}(\Lambda_b),\qquad \Gamma_-^{\rm int}(\Xi_b^0)=\Gamma_-^{\rm int}(\Lambda_b),$$

$$\Gamma_-^{\rm int}(\Xi_b^-)=-\frac{1}{4}\Gamma_0\,\eta_{\rm nspec}r[(1-x)^2(1+x)+\sqrt{1-4x}]\times(\widetilde{B}c_1^2-2c_1c_2-N_cc_2^2),$$

$$\Gamma_-^{\rm int}(\Omega_b^-)=-\frac{1}{6}\Gamma_0\,\eta_{\rm nspec}r\sqrt{1-4x}(5-8x)\times(\widetilde{B}c_1^2-2c_1c_2-N_cc_2^2), \qquad (3.15)$$

where use has been made of Eq. (2.28). Note that there is no W-exchange contribution to the Ξ_b^- and Ω_b and that there are two Cabibbo-allowed Pauli interference terms in Ξ_b^- decay, and one Cabibbo-allowed as well as one Cabibbo-suppressed interferences in Λ_b decay. It is easily seen that under the valence-quark approximation, i.e., $\widetilde{B}=1$, the W-exchange contribution $\Gamma^{\rm ann}$ is proportional to $c_-=(c_1-c_2)/2$ as the four-quark operator $O_+=(\overline{q}_1q_2)(\overline{q}_3q_4)+(\overline{q}_1q_4)(\overline{q}_3q_2)$ is symmetric in color indices whereas the color wave function for a baryon is totally antisymmetric. Writing

$$\Gamma_{\rm NL}=\Gamma^{\rm dec}+\Gamma^{\rm ann}+\Gamma_-^{\rm int}, \qquad (3.16)$$

the numerical results for nonleptonic inclusive decay rates are

$$\Gamma_{\rm NL}(\Lambda_b)=[2.220+(0.042+0.058\widetilde{B})r]\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma_{\rm NL}(\Xi_b^0)=[2.220+(0.043+0.066\widetilde{B})r]\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma_{\rm NL}(\Xi_b^-)=[2.220-(0.037+0.114\widetilde{B})r]\times10^{-13}\ \ {\rm GeV},$$

$$\Gamma_{\rm NL}(\Omega_b)=[2.217-(0.043+0.133\widetilde{B})r]\times10^{-13}\ \ {\rm GeV}, \qquad (3.17)$$

where for later convenience we have normalized the parameter r in Eqs. (3.17) to r_{Λ_b} [see Eqs. (2.34)]; that is, we have taken into account the SU(3)-breaking effect for r. Note that ε_i and B_i in Eqs. (3.14) and $\widetilde{B}$ and r in Eqs. (3.17) are all renormalized at $\mu = 4.4$ GeV.

Before proceeding, it is worth emphasizing the difference between the W-exchange contributions in the inclusive nonleptonic decays of the B meson and the bottom baryon. It is conventionally argued that W exchange in heavy meson decay is suppressed by helicity and color mismatch. For example, W-exchange in B decay is helicity suppressed by a factor of $16\pi^2(f_B/m_B)^2$ relative to the heavy quark decay amplitude.[3] By contrast, W exchange in baryon decay is neither helicity nor color suppressed. The diquark Qq system in the heavy baryon can have a spin-0 configuration and the decay of a spin-0 (not spin-1) state into two quarks is not subject to helicity suppression.

Since $\widetilde{B}$ is of order unity and $r \sim 0.60$, it is evident from Eqs. (3.17) and (3.10) that the bottom baryon lifetimes follow the pattern (see also Table I)

$$\tau(\Omega_b^-) \simeq \tau(\Xi_b^-) > \tau(\Lambda_b^0) \simeq \tau(\Xi_b^0). \qquad (3.18)$$

This pattern originates from the fact that while Λ_b, Ξ_b^0, Ξ_b^-, and Ω_b all receive contributions from destructive Pauli interference, only Λ_b and Ξ_b^0 have W exchange and that $\Gamma_-^{\rm int}$ is largest in Ω_b due to the presence of two valence s quarks in its quark content. We shall see shortly that this lifetime pattern is dramatically modified when the b quark mass is replaced by the bottom baryon mass in nonleptonic widths.

It follows from Eqs. (3.3), (3.7), (3.10), (3.14), and (3.17) that

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.99 - 0.15\varepsilon_1 + 0.17\varepsilon_2 - (0.013 + 0.018\widetilde{B})r, \qquad (3.19)$$

which is a model-independent result. This is consistent with the result (1.7) obtained in [10] with ε_i, $\widetilde{B}$, and r renormalized at $\mu = 4.85$ GeV and with $f_B = 200$ MeV. As stated in the Introduction, ε_1 and ε_2 obey the constraint $\varepsilon_1 \approx 0.3\varepsilon_2$; then, it is quite difficult, if not impossible, to accommodate the experimental value (1.5) for $\tau(\Lambda_b)/\tau(B_d)$ without invoking too large a value of r and/or $\widetilde{B}$. We will argue below that the contribution of $-0.15\varepsilon_1 + \cdots - 0.018\widetilde{B}r$ in Eq. (3.19) is at most of order 6%. We hasten to remark that the current CDF result (1.6) for the lifetime ratio is consistent with the theoretical prediction [see, however, a comment after Eq. (3.22)].

Irrespective of the above-mentioned lifetime ratio problem, there exists another serious difficulty; namely, the predicted absolute decay width of the B or Λ_b hadron based on the heavy quark expansion [see Eqs. (3.3), (3.7), (3.10), and (3.17)] is too small compared to the experimental values [12]:

$$\Gamma(B_d) = (4.246^{+0.094}_{-0.125}) \times 10^{-13}\ \text{GeV},$$

$$\tau(B_d) = (1.55 \pm 0.04)\ \text{ps},$$

$$\Gamma(B^-) = (3.965^{+0.098}_{-0.093}) \times 10^{-13}\ \text{GeV},$$

$$\tau(B^-) = (1.66 \pm 0.04)\ \text{ps},$$

$$\Gamma(\Lambda_b) = (5.351^{+0.422}_{-0.365}) \times 10^{-13}\ \text{GeV},$$

$$\tau(\Lambda_b) = (1.23 \pm 0.09)\ \text{ps}. \qquad (3.20)$$

Obviously, even if the destructive contribution $\Gamma_-^{\rm int}(B^-)$ is not taken into account, the result $\Gamma^{\rm dec}(B) + \Gamma_{\rm SL}(B) = 3.211 \times 10^{-13}$ GeV is too small by about 20% to account for the observed decay rate of B^-.[4] To compute the decay widths of bottom baryons, we have to specify the values of $\widetilde{B}$ and r. Since $\widetilde{B} = 1$ in the valence-quark approximation and since the wave function squared ratio r is evaluated using the quark model, it is reasonable to assume that the NQM and the valence-quark approximation are most reliable when the

[3] It had been claimed that soft gluon emission from the initial quark line or soft gluon content in the initial wave function can vitiate both helicity and color suppression [46]. The net effect is that the factor f_B/m_B is effectively replaced by f_B/m_q, where m_q is the constituent quark mass of the antiquark in the $\bar{B}$ meson [47]. As a consequence, contributions of W exchange will exhibit powerlike $(m_B/m_q)^2$ enhancement and this renders the treatment of the heavy quark expansion for W exchange invalid. This issue was resolved by Bigi and Uraltsev [47] who showed that such powerlike enhancement does not arise for fully inclusive transitions and the soft gluon effect merely amounts to renormalizing the coefficients of four-quark operators.

[4] The problem with the absolute total decay width $\Gamma(B)$ of the B meson is intimately related to the problem with the B meson semileptonic branching ratio $B_{\rm SL}$. The theoretical prediction for $B_{\rm SL}$ is in general above 12.5% [48], while experimentally $B_{\rm SL} = (10.23 \pm 0.39)\%$ [49]. In our case we obtain $B_{\rm SL} \gtrsim 13.8\%$. Several scenarios have been put forward in the past to resolve the discrepancy between theory and experiment for $B_{\rm SL}$ or $\Gamma(B)$. Here we mention two of the possibilities. (i) Since the theoretical results depend on the scale μ to renormalize $\alpha_s(\mu)$ and the Wilson coefficients $c_{1,2}(\mu)$, one may choose a low renormalization scale, $\mu/m_b \sim 0.3 - 0.5$, to accommodate the data [10]. Local duality holds in this scenario. (ii) Next-to-leading order QCD radiative corrections to nonleptonic decay will increase the rate for $b \to c\bar{c}s$ substantially and decrease $B_{\rm SL}$ [50,51]. Using the result of [50], we find that the QCD effect will bring $B_{\rm SL}$ down by 1% and hence $B_{\rm SL} \gtrsim 12.7\%$. It was suggested in [52,18] that a failure of local duality in the $b \to c\bar{c}s$ channel, which has a smaller energy release than that in $b \to c\bar{u}d$, will further enhance $\Gamma(B)$ and suppress $B_{\rm SL}$. However, this explanation encounters a problem: The charm counting n_c will increase and become as large as 1.30 [50], which is too large compared to the experimental value $n_c = 1.12 \pm 0.05$ [49]. One way out of this difficulty for n_c is proposed in [53] that a sizable fraction of $b \to c\bar{c}s$ transitions can be seen as charmless $b \to s$ processes. In the present paper we will not pursue any of the aforementioned possibilities as none of them can explain the lifetime difference between Λ_b and B_d. The recipe we are going to discuss below [see Eq. (3.23)] will solve all the problems with $B_{\rm SL}$, $\Gamma(B)$, n_c, and $\tau(\Lambda_b)/\tau(B_d)$.

TABLE I. Various contributions to the decay rates (in units of 10^{-13} GeV) of bottom baryons.

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma^{\rm int}_{-}$	$\Gamma_{\rm SL}$	$\Gamma^{\rm tot}$	$\tau(10^{-12}$ s)	$\tau_{\rm expt}(10^{-12}$ s)
Λ_b^0	2.220	0.145	-0.064	1.027	3.327	1.98	1.23 ± 0.09
Ξ_b^0	2.220	0.138	-0.051	1.027	3.334	1.97	
Ξ_b^-	2.220		-0.110	1.027	3.137	2.10	
Ω_b^-	2.217		-0.127	1.014	3.104	2.12	

baryon matrix elements are evaluated at a typical hadronic scale $\mu_{\rm had}$. As shown in [10], the parameters $\widetilde{B}$ and r renormalized at two different scales are related via the renormalization group equation to be

$$\widetilde{B}(\mu)r(\mu)=\widetilde{B}(\mu_{\rm had})r(\mu_{\rm had}),$$

$$\widetilde{B}(\mu)=\frac{\widetilde{B}(\mu_{\rm had})}{\kappa+(1/N_c)(\kappa-1)\widetilde{B}(\mu_{\rm had})}, \tag{3.21}$$

with

$$\kappa=\left(\frac{\alpha_s(\mu_{\rm had})}{\alpha_s(\mu)}\right)^{3N_c/2\beta_0}=\sqrt{\frac{\alpha_s(\mu_{\rm had})}{\alpha_s(\mu)}} \tag{3.22}$$

and $\beta_0=\frac{11}{3}N_c-\frac{2}{3}n_f$. Choosing $\alpha_s(\mu_{\rm had})=0.5$ and $\mu=4.4$ GeV, we obtain $\widetilde{B}(\mu)=0.59\widetilde{B}(\mu_{\rm had})\simeq0.59$ and $r(\mu)\simeq1.7r(\mu_{\rm had})$. Using $r(\mu_{\rm had})=0.61$ [see Eq. (2.34)], the calculated decay rates of bottom baryons are summarized in Table I. It is evident that the predicted Λ_b lifetime is too large by eight standard deviations. Note that while the CDF measurement (1.6) for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ can be easily accommodated in theory, it is still difficult to explain the absolute lifetime $\tau(\Lambda_b)=(1.32\pm0.15\pm0.07)$ ps measured by CDF [13].

It has been advocated in [16] that, unlike the semileptonic inclusive case, since the OPE cannot be rigorously justified for nonleptonic inclusive decays, the failure of explaining the observed lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ implies that the assumption of local duality is not correct for nonleptonic inclusive widths. It is further suggested in [16] that corrections of order $1/m_Q$ should be present and this amounts to replacing the heavy quark mass by the mass of the decaying hadron in the m_Q^5 factor in front of all nonleptonic widths. In the following we shall see that the ansatz

$$\Gamma_{\rm NL}\to\Gamma_{\rm NL}\left(\frac{m_{H_b}}{m_b}\right)^5 \tag{3.23}$$

will not only solve the short Λ_b lifetime problem but also provide the correct absolute decay rates for bottom hadrons.

Employing the hadron masses

$$m_{B_d}=5279.2\pm1.8 \text{ MeV [42]},$$

$$m_{B^-}=5278.9\pm1.8 \text{ MeV [42]},$$

$$m_{\Lambda_b}=5621\pm5 \text{ MeV [54]}, \tag{3.24}$$

we obtain

$$\Gamma_{\rm tot}(\Lambda_b)=[3.986+(0.075+0.105\widetilde{B})r]\times10^{-13} \text{ GeV} +\Gamma_{\rm SL}(\Lambda_b),$$

$$\Gamma_{\rm tot}(B_d)=[2.908+(-0.644\varepsilon_1+0.739\varepsilon_2)]\times10^{-13} \text{ GeV} +\Gamma_{\rm SL}(B),$$

$$\Gamma_{\rm tot}(B^-)=[2.907+(-0.171B_1+1.974\varepsilon_1)]\times10^{-13} \text{ GeV} +\Gamma_{\rm SL}(B), \tag{3.25}$$

with $\Gamma_{\rm SL}(\Lambda_b)$ and $\Gamma_{\rm SL}(B)$ being given by Eqs. (3.3) and (3.7), respectively. Consequently,

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=0.78-0.13\varepsilon_1+0.15\varepsilon_2-(0.015+0.021\widetilde{B})r. \tag{3.26}$$

Comparing this with Eq. (3.19) we see that the main effect of including linear $1/m_b$ corrections is to shift the central value of the lifetime ratio from 0.99 to 0.78. Moreover, the experimental value $\tau(\Lambda_b)/\tau(B_d)=0.79\pm0.06$ [12] indicates that the remaining contribution $-0.13\varepsilon_1+\cdots$ in Eq. (3.26) is at most $\pm6\%$. It is also evident from Eqs. (3.25) that the discrepancy between theory and experiment for the absolute decay width of B mesons is greatly improved.

The most dramatic effect due to the ansatz (3.23) occurs in the lifetime pattern of bottom baryons. Employing the bottom baryon masses (2.36), (3.24), and $m_{\Xi_b}=5803.7\pm7.1$ MeV,[5] some large enhancement to various nonleptonic contributions to the decay widths of bottom baryons is shown in Table II. We see that the improved Λ_b lifetime is in agreement with experiment and the new hierarchy of bottom baryon lifetimes emerges as

$$\tau(\Lambda_b^0)>\tau(\Xi_b^-)>\tau(\Xi_b^0)>\tau(\Omega_b^-), \tag{3.27}$$

which is drastically different from the previous one: The longest-lived Ω_b among bottom baryons in the conventional OPE now becomes shortest lived. Needless to say, it is of great importance to measure the hierarchy of bottom baryon lifetimes in order to test the ansatz (3.23). The branching ratios of semileptonic inclusive decays are calculated from Table II to be

[5]We have used the CDF mass of the Λ_b [see Eqs. (3.24)] to update the Ξ_b mass prediction given in [39].

TABLE II. Various contributions to the decay rates (in units of 10^{-13} GeV) of bottom baryons. The ansatz (3.23) has been applied to enhance the nonleptonic b quark decay and nonspectator effects.

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma_-^{\rm int}$	$\Gamma_{\rm SL}$	$\Gamma^{\rm tot}$	τ (10^{-12} s)	$\tau_{\rm expt}$ (10^{-12} s)
Λ_b^0	3.986	0.260	−0.116	1.027	5.157	1.28	1.23±0.09
Ξ_b^0	4.678	0.290	−0.107	1.027	5.888	1.12	
Ξ_b^-	4.678		−0.231	1.027	5.474	1.20	
Ω_b^-	5.840		−0.335	1.014	6.519	1.01	

$$\mathcal{B}(\Lambda_b\to Xe\overline{\nu})=8.9\%,\quad \mathcal{B}(\Xi_b^0\to Xe\overline{\nu})=7.8\%,$$
$$\mathcal{B}(\Xi_b^-\to Xe\overline{\nu})=8.4\%,\quad \mathcal{B}(\Omega_b\to Xe\overline{\nu})=6.9\%. \tag{3.28}$$

Since serious and precise measurements of the hierarchy of lifetimes of bottom baryons may not be available in the very near future,[6] it is thus important to carry out more precise measurement of the B_s lifetime. An application of the prescription (3.23) will modify the prediction [10]

$$\frac{\tau(B_s)}{\tau(B_d)}=1\pm O(1\%) \tag{3.29}$$

to [16]

$$\frac{\tau(B_s)}{\tau(B_d)}=0.938 \tag{3.30}$$

for the average B_s lifetime. The current world average is $\tau(B_s)/\tau(B_d)=0.98\pm0.05$ [12].

IV. LIFETIMES OF CHARMED BARYONS

In Sec. III we see that a replacement of the heavy quark mass with the decaying hadron mass in the m_Q^5 factor in front of nonleptonic widths provides a much better description of the lifetimes of the Λ_b baryon and B mesons. It is claimed in [16] that a much better fit to the charmed hadron lifetimes is also achieved if $\Gamma_{\rm NL}$ for charm decay approximately scales with the fifth power of charmed hadron masses, apart from corrections of order $1/m_c^2$. We will carefully examine the applicability of this recipe in this section. For a theoretical overview of charmed baryon lifetimes, the reader is referred to the review of Blok and Shifman [2].

We begin with the semileptonic inclusive decay of the D meson:

$$\Gamma(D\to Xe\overline{\nu})=\frac{G_F^2m_c^5}{192\pi^3}|V_{cs}|^2\eta(x,0,0)\left\{I_0(x,0,0)\frac{\langle D|\overline{c}c|D\rangle}{2m_D}-\frac{1}{m_c^2}I_1(x,0,0)\frac{\langle D|\overline{c}\sigma\cdot Gc|D\rangle}{2m_D}\right\}. \tag{4.1}$$

[6]The current LEP results for the lifetime of Ξ_b are $(1.35^{+0.37+0.15}_{-0.28-0.17})$ ps by ALEPH [55] and $(1.5^{+0.7}_{-0.4}\pm0.3)$ ps by DELPHI [56]. The average is $\tau(\Xi_b)=(1.39^{+0.34}_{-0.28})$ ps. Evidently, the uncertainty is still too large to have a meaningful test on the prediction (3.27).

We find that the experimental values for D^+ and D^0 semileptonic widths [42] can be fitted by the quark pole mass $m_c=1.6$ GeV. Taking $m_s=170$ MeV, we then have $x=(m_s/m_c)^2=0.0113$ and

$$I_0(x,0,0)=0.9166,\quad I_1(x,0,0)=0.9556,$$
$$I_2(x,0,0)=0.9665 \tag{4.2}$$

for charm decay. Repeating the same exercise for charmed baryons, we obtain the charmed baryon semileptonic decay rates

$$\Gamma(\Lambda_c\to Xe\overline{\nu})=\Gamma(\Xi_c\to Xe\overline{\nu})=1.533\times10^{-13}\ \text{GeV},$$
$$\Gamma(\Omega_c\to Xe\overline{\nu})=1.308\times10^{-13}\ \text{GeV}, \tag{4.3}$$

which are larger than that of the D meson:

$$\Gamma(D\to Xe\overline{\nu})=1.090\times10^{-13}\ \text{GeV}. \tag{4.4}$$

The prediction (4.3) for the Λ_c baryon is in good agreement with experiment:

$$\Gamma(\Lambda_c\to Xe\overline{\nu})_{\rm expt}=(1.438\pm0.543)\times10^{-13}\ \text{GeV}. \tag{4.5}$$

For charmed baryons Ξ_c and Ω_c, there is an additional contribution to the semileptonic width coming from the Pauli interference of the s quark [29]. From Eq. (2.5) we obtain

$$\Gamma^{\rm int}(\Xi_c\to Xe\overline{\nu})=\tfrac{1}{4}\Gamma_0'\eta_{\rm nspec}r_{\Xi_c}(1-x^2)(1+x),$$
$$\Gamma^{\rm int}(\Omega_c\to Xe\overline{\nu})=\tfrac{1}{6}\Gamma_0'\eta_{\rm nspec}r_{\Omega_c}(1-x^2)(5+x), \tag{4.6}$$

where we have applied Eqs. (2.28) for charmed baryon matrix elements, $\Gamma_0'=\Gamma_0/|V_{ud}|^2$ and

$$\Gamma_0=\frac{G_F^2m_c^5}{192\pi^3}|V_{cs}V_{ud}|^2,\quad \eta_{\rm nspec}=16\pi^2\frac{f_D^2m_D}{m_c^3}. \tag{4.7}$$

We shall see later that, depending on the parameter r, the nonspectator effect in the semileptonic decay of Ξ_c and Ω_c can be very significant, in particular for the latter.

We now turn to the nonleptonic inclusive decays of charmed hadrons. It is well known that the longer lifetime of D^+ relative to D^0 comes mainly from the destructive Pauli interference in D^+ decay [57,3]. However, it is also known that, depending on the parameters B_1 and especially ε_1, the Pauli interference $\Gamma_-^{\rm int}(D^+)$ in analogue to $\Gamma_-^{\rm int}(B^-)$ given by Eqs. (3.12) can be easily overestimated and may even over-

TABLE III. Various contributions to the decay rates (in units of 10^{-12} GeV) of charmed baryons. When nonspectator effects in semileptonic decay are included, the predictions are shown in parentheses. Experimental values are taken from [42].

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma^{\rm int}_-$	$\Gamma^{\rm int}_+$	$\Gamma_{\rm SL}$	$\Gamma^{\rm tot}$	τ (10^{-13} s)	$\tau_{\rm expt}$ (10^{-13} s)
Λ_c^+	0.903	0.858	−0.238		0.306	1.829	3.60	2.06 ± 0.12
Ξ_c^+	0.903	0.042	−0.226	0.423	0.306(0.498)	1.447(1.639)	4.55(4.02)	$3.5^{+0.7}_{-0.4}$
Ξ_c^0	0.903	0.817		0.423	0.306(0.498)	2.448(2.640)	2.69(2.49)	$0.98^{+0.23}_{-0.15}$
Ω_c^0	0.968	0.224		1.256	0.262(0.772)	2.710(3.220)	2.43(2.04)	0.64 ± 0.20

come the c quark decay rate $\Gamma^{\rm dec}$ so that the resulting nonleptonic width becomes negative. This certainly does not make sense. It has been discussed in great length by Chernyak [25] as how to circumvent the difficulty with the lifetime of D^+. We shall not address this issue in the present work and instead focus on the lifetimes of charmed baryons. Our purpose is to apply the ansatz similar to Eq. (3.23) and see if a better description of charmed baryon lifetimes can be achieved.

In addition to the destructive Pauli interference $\Gamma^{\rm int}_-$, there exists another Pauli interference term $\Gamma^{\rm int}_+$ in charmed baryon decay which arises from the constructive interference between the s quark produced in the c quark decay and the spectator s quark in the charmed baryon. Since the expressions of $\Gamma^{\rm ann}$ and $\Gamma^{\rm int}_-$ for charmed baryons are similar to Eqs. (3.15) for bottom baryon decays, here we will only write down the expressions for $\Gamma^{\rm int}_+$ described by the last term in Eq. (2.4):

$$\Gamma^{\rm int}_+(\Xi_c)=-\tfrac{1}{4}\Gamma_0\,\eta_{\rm nspec}r_{\Xi_c}(1-x^2)(1+x)\times(\widetilde{B}c_2^2-2c_1c_2-N_cc_1^2),$$

$$\Gamma^{\rm int}_+(\Omega_c)=-\tfrac{1}{6}\Gamma_0\,\eta_{\rm nspec}r_{\Omega_c}(1-x^2)(5+x)\times(\widetilde{B}c_2^2-2c_1c_2-N_cc_1^2). \tag{4.8}$$

It is easily seen that Eqs. (4.8) are reduced to Eqs. (4.6) when $c_1=1$, $c_2=0$, $N_c=1$, and $V_{ud}=1$. The Ξ_c^+ and Ω_c baryons also receive contributions from Cabibbo-suppressed W exchange:

$$\Gamma^{\rm ann}(\Xi_c^+)=|V_{us}/V_{ud}|^2\Gamma_0\,\eta_{\rm nspec}r_{\Xi_c}(1-x^2)\times[\widetilde{B}(c_1^2+c_2^2)-2c_1c_2],$$

$$\Gamma^{\rm ann}(\Omega_c)=6|V_{us}/V_{ud}|^2\Gamma_0\,\eta_{\rm nspec}r_{\Omega_c}(1-x^2)\times[\widetilde{B}(c_1^2+c_2^2)-2c_1c_2]. \tag{4.9}$$

The Ω_c matrix element [see Eq. (2.18)]

$$\langle\Omega_c|(\overline{cs})(\overline{sc})|\Omega_c\rangle=-6|\psi^{\Omega_c}_{cs}(0)|^2(2m_{\Omega_c}) \tag{4.10}$$

accounts for the factor of 6 in Eq. (4.9).

To proceed we employ the Wilson coefficients

$$c_1(\mu)=1.35,\quad c_2(\mu)=-0.64, \tag{4.11}$$

evaluated at the scale $\mu=1.25$ GeV. From Eqs. (3.21) and (3.22) we obtain $\widetilde{B}(\mu)\simeq0.74\widetilde{B}(\mu_{\rm had})\simeq0.74$ and $r(\mu)\simeq1.36r(\mu_{\rm had})$. Repeating the same exercise as the bottom baryon case, the results of calculations are exhibited in Table III. We see that the lifetime pattern

$$\tau(\Xi_c^+)>\tau(\Lambda_c^+)>\tau(\Xi_c^0)>\tau(\Omega_c^0) \tag{4.12}$$

is in accordance with experiment. It is evident that when nonspectator effects in semileptonic decay are included, as shown in parentheses in Table III, the discrepancy between theory and experiment is improved. This lifetime hierarchy (4.12) is qualitatively understandable. The Ξ_c^+ baryon is longest lived among charmed baryons because of the smallness of W exchange and partial cancellation between constructive and destructive Pauli interferences, while Ω_c is shortest lived due to the presence of two s quarks in the Ω_c that renders the contribution of $\Gamma^{\rm int}_+$ largely enhanced. It is also clear from Table III that, although the qualitative feature of the lifetime pattern is comprehensive, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor.

In order to have a better quantitative description of nonleptonic inclusive decays of charmed baryons, we shall follow [16] to assume that $\Gamma_{\rm NL}$ scales with $m^5_{H_c}$ instead of m_c^5:

$$\Gamma_{\rm NL}(\Lambda_c):\Gamma_{\rm NL}(\Xi_c^+):\Gamma_{\rm NL}(\Xi_c^0):\Gamma_{\rm NL}(\Omega_c)=\Gamma^{(0)}_{\rm NL}(\Lambda_c)\left(\frac{m_{\Lambda_c}}{m_c}\right)^5:\Gamma^{(0)}_{\rm NL}(\Xi_c^+)\left(\frac{m_{\Xi_c^+}}{m_c}\right)^5:\Gamma^{(0)}_{\rm NL}(\Xi_c^0)\times\left(\frac{m_{\Xi_c^0}}{m_c}\right)^5:\Gamma^{(0)}_{\rm NL}(\Omega_c)\left(\frac{m_{\Omega_c}}{m_c}\right)^5, \tag{4.13}$$

where $\Gamma^{(0)}_{\rm NL}$ is the nonleptonic decay rate calculated in the framework of the heavy quark expansion and it has the form

$$\Gamma^{(0)}_{\rm NL}=\frac{G_F^2m_c^5}{192\pi^3}[a+b/m_c^2+c/m_c^3+O(1/m_c^4)]. \tag{4.14}$$

To compute the absolute decay width, we introduce a parameter λ so that

$$\Gamma^{(0)}_{\rm NL}\rightarrow\Gamma_{\rm NL}=\lambda\Gamma^{(0)}_{\rm NL}\left(\frac{m_{H_c}}{m_c}\right)^5. \tag{4.15}$$

Unlike the ansatz (3.23) for bottom hadrons, it will become clear shortly that λ is much less than unity for charmed

TABLE IV. Same as Table III except that the ansatz (4.15) has been applied to enhance the nonleptonic c quark decay and nonspectator effects.

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma_-^{\rm int}$	$\Gamma_+^{\rm int}$	$\Gamma_{\rm SL}$	$\Gamma^{\rm tot}$	τ (10^{-13} s)	$\tau_{\rm expt}$ (10^{-13}s)
Λ_c^+	0.960	2.753	-0.884		0.306	3.136	2.10	2.06 ± 0.12
Ξ_c^+	1.404	0.195	-1.231	1.227	0.306(0.837)	1.902(2.432)	3.46(2.70)	$3.5^{+0.7}_{-0.4}$
Ξ_c^0	1.415	3.868		1.238	0.306(0.837)	6.828(7.358)	0.96(0.89)	$0.98^{+0.23}_{-0.15}$
Ω_c^0	2.389	1.668		5.775	0.262(1.675)	10.09(11.51)	0.65(0.57)	0.64 ± 0.20

hadrons. Applying the prescription (4.15), treating λ, r, and $\tilde{B}$ as free parameters, and fitting them to the data of charmed baryon lifetimes [42], we find

$$\lambda=0.18, \quad r=1.72, \quad \tilde{B}=1.46, \tag{4.16}$$

where r and $\tilde{B}$ are renormalized at $\mu=1.25$ GeV. The numerical results are summarized in Table IV. Contrary to the previous case, a prefect agreement with experiment will be achieved if nonspectator effects in semileptonic decay are not included.

Let us examine the fitted parameters (4.16) in more detail. The value $r=1.72$ is fairly reasonable as it implies $|\psi_{cq}^{\Lambda_c}(0)|^2=1.1\times10^{-2}$ GeV^3, which is consistent with those of hyperons and bottom baryons. Then, does it mean that our previous estimate of r for charmed baryons [see Eqs. (2.34)] is too small? In our opinion, the enhancement of $|\psi_{cq}^{\Lambda_c}(0)|^2$ is likely due to the fact that $|\psi_{c\bar{q}}^{D}(0)|^2$ is not simply equal to $y/12f_D^2m_D$ with $y=1$ and $f_D\approx200$ MeV. We conjecture that a more realistic value of y is probably close to 3 for charmed baryons and to unity for bottom baryons.

As for the parameter $\tilde{B}(\mu)$, it is expected to be less than unity if the valence-quark approximation is believed to be valid at a lower hadronic scale. Therefore, it is not clear to us why $\tilde{B}(\mu)$ is larger than unity and what is its implication. The smallness of λ is attributed to the fact that the inclusive nonleptonic decays of charmed baryons are not dominated by the c quark decay. Nonspectator effects of W exchange and Pauli interference terms are expected to be of order

$$16\pi^2(\Lambda_{\rm QCD}/m_c)^3\sim0.5-0.7, \tag{4.17}$$

where the factor of $16\pi^2$ is a two-body phase-space enhancement relative to the three-body phase space of heavy quark decay. Realistic calculations (see Tables III and IV) indicate that nonspectator contributions are comparable to and even dominate over the c quark decay mechanism. This implies that the charmed quark is not heavy enough (i.e., the energy release is not sufficiently large) to make a sensible and meaningful heavy quark expansion. For bottom hadrons, we see in Sec. III that at least for the Λ_b baryon and B mesons, the nonleptonic decay rate is approximated by

$$\Gamma_{\rm NL}(H_b)\approx\Gamma^{\rm dec}\left(\frac{m_{H_b}}{m_b}\right)^5, \tag{4.18}$$

where $\Gamma^{\rm dec}$ is the heavy quark decay rate. However, we find for charmed baryons that $\Gamma^{\rm dec}(m_{H_c}/m_c)^5$ are 5.36, 7.84, 7.84, and 13.24 (in units of 10^{-12} GeV), respectively, for Λ_c, Ξ_c^+, Ξ_c^0, and Ω_c, where $\Gamma^{\rm dec}$ is taken from Table III. Therefore, even in the absence of $1/m_c^2$ and $1/m_c^3$ corrections or even when the heavy quark expansion converges, the scaled nonleptonic c quark decay rate $\Gamma^{\rm dec}(m_{H_c}/m_c)^5$ already exceeds the experimental decay widths: 3.20, 1.88, 7.72, and 10.28 (in units of 10^{-12} GeV) [42], that is,

$$\Gamma^{\rm dec}\left(\frac{m_{H_c}}{m_c}\right)^5>\Gamma_{\rm tot}(H_c), \tag{4.19}$$

except for the Ξ_c^0. The presence of large nonspectator contributions (see Tables III and IV) will make the discrepancy between theory and experiment for decay widths even much worse. Hence, we have to introduce a parameter $\lambda\ll1$ to suppress the absolute rates. However, since λ is an entirely unknown parameter in theory, the recipe of scaling $\Gamma_{\rm NL}$ with the fifth power of charmed hadron mass is *ad hoc* and does not have the predictive power for the absolute decay widths. We conclude that, although the ansatz (4.13) provides a much better description of lifetime *ratios* for charmed baryons (apart from the annoying parameter $\tilde{B}$), the prescription (4.15) appears unnatural and unpredictive for describing the *absolute* inclusive decay rates of charmed baryons due to the presence of the unknown parameter λ. Since the heavy quark expansion converges very badly, local duality is thus not testable in inclusive nonleptonic charm decay.

V. DISCUSSIONS AND CONCLUSIONS

We have analyzed the lifetimes of bottom and charmed hadrons within the framework of the heavy quark expansion. Special attention is paid to the nonperturbative parameter $\lambda_2^{\rm baryon}$ and four-quark matrix elements for baryons. We found that the large-N_c relation $\lambda_2^{\rm meson}\sim N_c\lambda_2^{\rm baryon}$ is satisfactorily obeyed by bottom hadrons. We have followed [10] to parametrize the four-quark matrix elements in a model-independent way. Baryon matrix elements are evaluated using the NQM and the bag model. The bag-model estimate for bottom baryon matrix elements is smaller than that of the NQM by a factor of ~3. The hadronic parameter r defined in Eq. (2.27) is estimated in the NQM to be in the range 0.53–0.61 for both bottom and charmed baryons. Nonspectator effects in inclusive nonleptonic decays are then studied in detail. The main results of our analysis are as follows.

(1) Using the charmed quark pole mass fixed from the measured semileptonic decay widths of D^+ and D^0, we have calculated $1/m_Q^2$ nonperturbative corrections to the semileptonic inclusive widths for other heavy hadrons. We found that while $\Gamma_{\rm SL}(B)$ is very close to $\Gamma_{\rm SL}(\Lambda_b)$, $\Gamma_{\rm SL}(D)$ is

smaller than $\Gamma_{\rm SL}(\Lambda_c)$. The predicted semileptonic decay rates for the B meson and the Λ_c baryon are in good agreement with experiment. This implies that global duality is valid for inclusive semileptonic decay. For charmed baryons Ξ_c and Ω_c, there is an additional contribution to the semileptonic width coming from the constructive Pauli interference of the s quark. This interference effect is sizable for the Ξ_c and becomes overwhelming for the Ω_c.

(2) The lifetime pattern of the bottom baryons is predicted to be $\tau(\Omega_b)\simeq\tau(\Xi_b^-)>\tau(\Lambda_b)\simeq\tau(\Xi_b^0)$. Nonspectator effects due to W exchange and destructive Pauli interference account for their lifetime differences. The model-independent expression in the OPE for $\tau(\Lambda_b)/\tau(B_d)$ is given by Eq. (3.19), which is difficult to accommodate the data without invoking unnaturally too large values of hadronic parameters. Irrespective of the short Λ_b lifetime problem, the calculated absolute decay width of the charged B^- meson is at least 20% too small compared to experiment. Since the predicted $\Gamma_{\rm SL}(B)$ agrees with data, the deficit of the B meson decay rate is blamed on the nonleptonic width.

(3) Unlike the semileptonic decays, the heavy quark expansion in inclusive nonleptonic decay cannot be justified by analytic continuation into the complex plane and local duality has to be assumed in order to apply the OPE directly in the physical region. The shorter lifetime of the Λ_b relative to that of the B_d meson suggests a significant violation of quark-hadron local duality. The simple ansatz that $\Gamma_{\rm NL}\to\Gamma_{\rm NL}(m_{H_b}/m_b)^5$ not only solves the lifetime ratio problem but also provides the correct absolute decay widths for the Λ_b baryon and the B meson. The hierarchy of bottom baryon lifetimes is modified to $\tau(\Lambda_b)>\tau(\Xi_b^-)>\tau(\Xi_b^0)>\tau(\Omega_b)$: The longest-lived Ω_b among bottom baryons in the OPE approach now becomes shortest lived. This ansatz can be tested by measuring the Ξ_b lifetime in the near future. More precise measurement of the B_s lifetime provides another quick and direct test of local duality.

(4) The lifetime hierarchy $\tau(\Xi_c^+)>\tau(\Lambda_c)>\tau(\Xi_c^0)>\tau(\Omega_c)$ is qualitatively understandable in the OPE approach but not quantitatively. Apart from an annoying feature with the parameter $\widetilde{B}$, a better description of inclusive decays of charmed baryons is achieved by scaling $\Gamma_{\rm NL}$ with $m_{H_c}^5$ instead of m_c^5. Contrary to the bottom case, a small parameter $\lambda\ll 1$ has to be introduced, namely, $\Gamma_{\rm NL}\to\lambda\Gamma_{\rm NL}(m_{H_c}/m_c)^5$; otherwise, absolute decay widths of charmed baryons will be largely overestimated. Since λ is an entirely unknown parameter in theory, it renders the above prescription unnatural and less predictive. As the heavy quark expansion in charm decay converges very badly, it is meaningless to test local duality in nonleptonic inclusive decay of charmed hadrons.

We conclude that the recipe of allowing the presence of linear $1/m_Q$ corrections by scaling the nonleptonic decay widths with the fifth power of the hadron mass is operative in the bottom family but becomes unnatural in charm decay. Can this prescription be justified in a more fundamental way? It is interesting to note that a PQCD-based factorization formulism has been developed for inclusive semileptonic B meson decay [23]. This approach is formulated directly in terms of meson-level kinematics. Quark-hadron duality can be tested by comparing results obtained from quark-level kinematics and those from meson kinematics. The validity of global duality has been demonstrated in the general kinematic region up to $O(1/m_Q^2)$; $1/m_Q$ corrections to inclusive semileptonic widths are indeed nontrivially canceled out. When this factorization approach is generalized to nonleptonic decays and to heavy baryons, it is natural to expect that $\Gamma_{\rm NL}(B)/\Gamma_{\rm NL}(\Lambda_b)\approx(m_B/m_{\Lambda_b})^5$ if local duality is violated. Since the application of PQCD and hence the factorization scheme of [23] to charm decay is very marginal due to the fact that the charmed hadron scale is not sufficiently large, the scaling behavior of $\Gamma_{\rm NL}$ with $m_{H_Q}^5$ occurring in the bottom decay is no longer anticipated in inclusive nonleptonic decays of charmed hadrons.

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