Transverse momentum and Sudakov effects in exclusive QCD processes: $\gamma^*\gamma\pi^0$ form factor

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We analyze effects due to transverse degrees of freedom in QCD calculations of the fundamental hard exclusive amplitude of a $\gamma^*\gamma\to\pi^0$ transition. A detailed discussion is given of the relation between the modified factorization approach (MFA) of Sterman *et al.* and standard factorization (SFA). Working in the Feynman gauge, we construct basic building blocks of the MFA from the one-loop coefficient function of the SFA, demonstrating that Sudakov effects are distinctly different from higher-twist corrections. We show also that the handbag-type diagram, contrary to naive expectations, does not contain an infinite chain of $(M^2/Q^2)^n$ corrections: they come only from diagrams with transverse gluons emitted from the hard propagator. A simpler picture emerges within the QCD sum rule approach: the sum over soft $\overline{q}G\cdots Gq$ Fock components is dual to $\overline{q}q$ states generated by the local axial vector current. We combine the results based on QCD sum rules with perturbative QCD radiative corrections and observe that the gap between our curves for the asymptotic and CZ distribution amplitudes is sufficiently large for an experimental discrimination between them.
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I. INTRODUCTION

The form factor $F_{\gamma^*\gamma^*\pi^0}(q_1^2,q_2^2)$ relating two (in general, virtual) photons with the lightest hadron, the pion, plays a crucial role in the studies of exclusive processes in quantum chromodynamics. With only one hadron involved, it has the simplest structure analogous to that of the form factors of deep inelastic scattering. At large photon virtualities, comparing the perturbative QCD (PQCD) predictions [1–6] with experimental data, one can get important information about the shape of the pion distribution amplitude $\varphi_\pi(x)$. Because of its relation to the axial anomaly [7], the $\gamma^*\gamma^*\pi^0$ form factor has been an object of intensive studies since the 1960's [8–13]. Experimentally, $F_{\gamma^*\gamma^*\pi^0}(q_1^2,q_2^2)$, for a small virtuality of one of the photons, $q_1^2\approx 0$, was measured only recently at e^+e^- colliders by the CELLO [14] and CLEO [15] Collaborations (in the latter case, only a preliminary announcement of the results was made). The possibility to measure $F_{\gamma^*\gamma^*\pi^0}(q_1^2\approx 0,q_2^2)$ at fixed-target machines such as the Continuous Electron Beam Accelerator Facility of Jefferson Lab was also discussed [16]. These measurements inspired the studies of the momentum dependence of this form factor within various models of the nonperturbative quark dynamics [17–27].

For a detailed comparison of PQCD predictions with experimental data, one should have reliable estimates of possible corrections to the lowest-order handbag contribution, in particular, those due to the gluon radiation and higher-twist effects. Within the standard PQCD factorization approach, the one-loop radiative corrections to the coefficient function were calculated in Refs. [4–6]. The authors of Refs. [21,28] incorporated the modified factorization approach of Sterman and collaborators [29,30] in which the factorization formula invloves an extra integration over the impact parameter $b_\perp$ and Sudakov double logarithms of $[\alpha_s\ln^2(b_\perp^2)]^n$ type are summed to all orders. In Refs. [21,28] it was claimed that such an analysis takes into account some transverse-momentum effects neglected within the standard factorization approach [1,31–33]. Incorporating the transverse-momentum-dependent wave function $\Psi(x,k_\perp)$, Jakob *et al.* [21] also proposed a model for the effects due to the intrinsic (primordial) transverse momentum.

Another attempt to take into account the transverse momentum effects was made by Cao *et al.* [26], where the light-cone formalism expression [1] for the $\gamma^*\gamma\to\pi^0$ was used. Adopting an exponential ansatz for the transverse momentum dependence of the wave function, the authors observed large ''higher-twist'' corrections, with the conclusion that it is difficult in such a situation to make a clear distinction between different shapes of the pion distribution amplitude.

In this paper, we will discuss various types of transverse momentum effects for the $\gamma^*\gamma\pi^0$ form factor. First, we briefly outline the derivation of the leading-twist PQCD formula for this process using a covariant factorization approach [34,31,35] similar to the operator product expansion (OPE) formalism. In this framework, we identify the basic types of the higher-twist corrections neglected in the leading-twist approximation. We show, in particular, that for massless quarks in a scalar theory no intrinsic transverse momentum effects are neglected in the handbag diagram: because of the simple singularity structure of the massless quark propagator, such effects can be taken into account exactly and lead to negligible pion mass corrections m_π^2/Q^2 only. In QCD, the handbag diagram contains a twist-four term interpretable as a $O(k_\perp^2)$ correction, but no terms corresponding to higher powers of $k_\perp^2$. Hence, the infinite tower of $(M^2/Q^2)^n$ corrections is generated by operators corresponding to higher $\overline{q}G\cdots Gq$ Fock components. In Sec. II, we also discuss the structure of

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the results for the one-loop radiative corrections [4–6] calculated within the standard factorization approach [31,1,33,36,37].

In Sec. III, we give a detailed one-loop derivation of the basic formulas of the modified factorization approach (MFA). We write the relevant one-loop integrals in Sudakov variables used in [29,30], introduce the impact parameter $b_\perp$, as the Fourier conjugate variable to the transverse momentum $k_\perp$, and reproduce (at one loop) the structure of the modified factorization [29]. In contrast (and complementary) to the original analysis, we use Feynman gauge which allows us to make a direct graph by graph comparison with the results [4–6] obtained within the standard factorization approach (SFA). Since the modified factorization formulas appear as an intermediate step in our calculations which eventually produce the results of the SFA, the two types of factorization give identical results at any finite order of perturbation theory. The difference between the two approaches is only in different organizations of all-order summation of higher-loop terms. Namely, in the MFA, the Sudakov-type double logarithms $[\alpha_s \ln^2(Qb_\perp)]^n$ are treated as logarithmic enhancements and are summed over all orders to produce a factor suppressing the contributions from the large-b region. In the standard approach, the $[\alpha_s \ln^2(Qb)]^n$ terms are integrated over $b_\perp$ and included order by order. We show that for the $\gamma^*\gamma\pi^0$ form factor the use of the SFA procedure is well justified since the results of the $b_\perp$ integration produce rather mild corrections ($\sim$20% at one loop). Another lesson from our detailed one-loop study of the MFA is that though the factorization formula of the MFA explicitly involves an integral over the impact parameter $b_\perp$ (or transverse momentum $k_\perp$), the results of such an integration do not produce power suppressed contributions. Thus, despite the claims made, e.g., in Refs. [38,39,21] higher-twist corrections are not included in the MFA .

In Sec. IV, we discuss two recent attempts [21,26] to model the intrinsic momentum corrections for the $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor. The approach of Jakob *et al.* [21] is based on the extrapolation of the modified factorization formula into the nonperturbative region. At large impact parameters b, the Sudakov suppression factor is supplemented by the nonperturbative wave function $\widetilde{\Psi}(x,b)$ reflecting the effects due to the primordial transverse momentum distribution. However, since terms which were inessential for the derivation of the Sudakov factor at large Q^2 may be quite important for small Q^2, it is not clear for which Q^2 region such an extrapolation is sufficiently accurate. We observe, in particular, that instead of producing the $Q^2=0$ value dictated by the axial anomaly [7,40], the extrapolation formula gives a logarithmically divergent result suggesting that the extrapolation should not go down to very low Q^2. Cao *et al.* [26] use the expression for the $\bar{q}q$ Fock state contribution to $F_{\gamma^*\gamma\pi^0}(Q^2)$ derived in the light-cone formalism by Brodsky and Lepage [1]. This expression involves no approximations and has correct limits both for small and large Q^2. In particular, we demonstrate that, in full accordance with our general analysis, it contains no higher-twist contributions. Still, one should take into account that the $\bar{q}q$ term, by definition, does not include the contribution due to higher $\bar{q}G\cdots Gq$ Fock components of the pion light-cone wave function. As shown in Ref. [2], the latter coincides in the real photon limit $Q^2=0$ with that of the $\bar{q}q$ Fock component and doubles the total result at this point. Clearly, the inclusion (or at least modeling) of this contribution is necessary for a consistent description of subasymptotic effects. Comparing the approaches of Refs. [21,26], we emphasize that they incorporate two completely different light-cone schemes. The light-cone formalism of Brodsky and Lepage [1] used in Ref. [26] is equivalent to incorporating the infinite momentum frame. On the other hand, the approach of Ref. [21] (and that of the underlying papers [29,30]) is based on the Sudakov decomposition. The basic difference between the two light-cone approaches is that the momentum of the virtual photon in the $\gamma^*\gamma\to\pi^0$ process is dominated by the transverse component in the BL light-cone scheme while it is purely longitudinal in the Sudakov approach.

In Sec. V, we use QCD sum rule ideas to get a model for the $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor which reproduces both the $Q^2=0$ constraint imposed by the axial anomaly and the lowest-order PQCD results for high Q^2. We show also that the results obtained on the basis of QCD sum rules and quark-hadron duality can be interpreted in terms of the effective valence wave function which absorbs information about soft dynamics of higher Fock components of the standard light-cone approach. Combining these results with PQCD radiative corrections, we obtain an expression depending on the choice of the low-energy distribution amplitude. The difference between our results for the asymptotic and Chernyak-Zhitnitsky (CZ) distribution amplitudes is sufficiently large for an unambiguous experimental discrimination between these two possibilities.

II. FACTORIZATION

A. Structure of factorization

We define the form factor $F_{\gamma^*\gamma^*\pi^0}(q_1^2,q_2^2)$ of the $\gamma^*\gamma^*\to\pi^0$ transition through the matrix element

$$4\pi\int \langle \pi,\vec{p}|T\{J_\mu(X)\ J_\nu(0)\}|0\rangle e^{-iq_1X}d^4X$$
$$=ie^2\sqrt{2}\ \epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F_{\gamma^*\gamma^*\pi^0}(q_1^2,q_2^2), \tag{2.1}$$

where J_μ is the electromagnetic current of the light quarks

$$J_\mu=e_u\bar{u}\gamma_\mu u+e_d\bar{d}\gamma_\mu d \tag{2.2}$$

and $|\pi,\vec{p}\rangle$ is a one-pion state with the four-momentum p. Note, that our definition (aimed at getting a simple coefficient for the spectral density for the triangle anomaly diagram, see Sec. V) differs from that in Refs. [1,21,26] by factor $\sqrt{2}/4\pi$. Experimentally, the most favorable situation is when one of the photons is real or almost real: $q_1^2\sim 0$. In this case, we will denote the form factor by $F_{\gamma^*\gamma\pi^0}(Q^2)$, where $Q^2\equiv -q_2^2$ is the virtuality of the other photon. It should be sufficiently large for PQCD to be applicable. In general, a powerlike behavior of $F_{\gamma^*\gamma\pi^0}(Q^2)$ in the large-Q^2 limit can be generated by three basic regimes (see Fig. 1).

The dominant contribution is provided by the first regime [Fig. 1(a)] which corresponds to large virtuality flow through a subgraph V containing both photon vertices. The power-

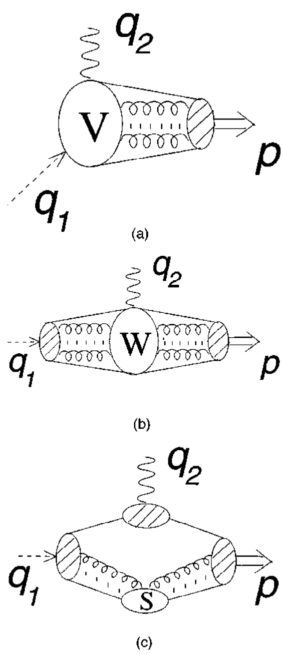

FIG. 1. Structure of factorization for the $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor at large Q^2.

counting estimate for the large-Q^2 behavior of such a configuration with arbitrary number of external lines of V is given by (see Refs. [22,24])

$$F(Q^2) \lesssim Q^{-\sum_i t_i}, \tag{2.3}$$

where t_i's are twists (dimension minus spin) of the quark and gluon external lines of V, with $t=1$ for the quarks and $t=0$ for the gluons in a covariant gauge. Hence, for the leading term, one should take the minimal number of quark lines (two in our case) while the number of the gluonic A fields is arbitrary. Generically, the leading contribution of this type can be written as

$$F_{\gamma^*\gamma^*\pi^0}(q_1,q_2) = \int C(\xi,\eta,q_1,q_2;\mu^2) \times \langle p|\mathcal{O}(\xi,\eta)|0\rangle|_{\mu^2} d^4\xi d^4\eta, \tag{2.4}$$

where the parameter μ^2 is the factorization scale, $C(\xi,\eta,q_1,q_2)$ corresponds to the short-distance amplitude with two external quark lines, and $\mathcal{O}(\xi,\eta)$ is a composite operator $\mathcal{O}(\xi,\eta) \sim \overline{q(\xi)}\gamma_5\gamma_\nu E(\xi,\eta;A)q(\eta)$. The path-ordered exponential

$$E(\xi,\eta;A) \equiv P\exp\left(ig\int_\eta^\xi A_\mu(z)dz^\mu \right)$$

of the gluonic field A results from summation over external gluon lines of V. For the quark propagator, e.g., one has

$$S^c(\xi-\eta) + \int S^c(\xi-z)\gamma^\mu g A_\mu(z) S^c(z-\eta) d^4z + \cdots = E(\xi,\eta;A) S^c(\xi-\eta)[1+O(G)], \tag{2.5}$$

where $O(G)$ depends on the gluonic fields through the gluon field strength tensor $G_{\mu\nu}$ and its covariant derivatives. Since $G_{\mu\nu}$ is asymmetric with respect to the interchange of the indices μ, ν, it should be treated as a twist-one field.

Basically, the contribution (2.4) is analogous to the quark-antiquark term of the standard operator product expansion for $J^\alpha(0)J^\beta(z)$. In this form, the operator $\mathcal{O}(\xi,\eta)$ still contains nonleading twist terms. To get the lowest-twist part, we should expand $\mathcal{O}(\xi,\eta)$ into the Taylor series

$$\overline{q(\xi)}\gamma_5\gamma_\nu E(\xi,\eta;A)q(\eta) = \sum_{n=0}^{\infty} \frac{1}{n!}\Delta^{\nu_1}\Delta^{\nu_2}\cdots\Delta^{\nu_n} \times \overline{q(\xi)}\gamma_5\gamma_\nu D_{\nu_1}D_{\nu_2}\cdots D_{\nu_n}q(\xi);$$

$$\Delta = \eta - \xi, \tag{2.6}$$

and pick out only the symmetric-traceless part $\overline{q}\gamma_5\{\gamma_\nu D_{\nu_1}D_{\nu_2}\cdots D_{\nu_n}\}q$ of each local operator from this expansion. The traces correspond to operators with contracted covariant derivatives $D^\nu D_\nu$ which, for dimensional reasons, are accompanied by powers of the interval $(\xi-\eta)^2$. Likewise, the $(\xi-\eta)^2$ factors produce extra powers of z^2 after integration over ξ and η. Finally, each power of z^2 results in an extra power of $1/Q^2$, i.e., each pair of contracted covariant derivatives $D^\nu\cdots D_\nu$ in a higher-twist operator produces $1/Q^2$ suppression at large Q^2. Hence, the twist-two part of $\mathcal{O}(\xi,\eta)$ corresponds to the lowest term of the expansion over $(\xi-\eta)^2$:

$$\mathcal{O}(\xi,\eta) = \mathcal{O}(\xi,\eta)|_{(\xi-\eta)^2=0} + O[(\xi-\eta)^2]. \tag{2.7}$$

The light-cone matrix element can be parametrized in terms of the pion distribution amplitude (DA) $\varphi_\pi(x)$:

$$\langle 0|\mathcal{O}_\nu(\xi,\eta)|\pi^0,p\rangle|_{(\xi-\eta)^2=0} = ip_\nu \int_0^1 e^{-ix(\xi p)-i\overline{x}(\eta p)}\varphi_\pi(x)dx \ , \tag{2.8}$$

which gives the probability amplitude that the fast-moving pion is a $\overline{q}q$ pair with its longitudinal momentum p shared among the quarks in fractions x and $\overline{x} \equiv (1-x)$ (throughout the paper, we use the ''bar'' convention for the momentum fractions: $\overline{x} \equiv 1-x$, $\overline{y} \equiv 1-y$, etc.). Substituting this representation into the generic expression (2.4), we obtain the hard scattering formula

$$F_{\gamma^*\gamma\pi^0}(q_1,q_2) = \frac{4\pi}{3}\int_0^1 T(q_1,q_2;xp,\overline{x}p)\varphi_\pi(x)dx, \tag{2.9}$$

where the factor $4\pi/3$ is due to our normalization of the form factor and $T(q_1,q_2;k,\overline{k})$ is the amplitude for the subprocess $\gamma(q_1)\gamma^*(q_2) \to \overline{q(\overline{k})}q(k)$. Calculating this lowest-twist amplitude in the momentum representation, we should realize that the neglect of the higher-twist operators having extra D^2 is equivalent to taking $k^2=0$, $\overline{k}^2=0$ for the external quark momenta. In general, this limit is singular for diagrams with loops, and one should regulate the resulting mass

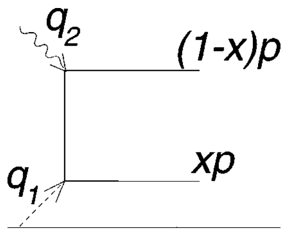

FIG. 2. Lowest-order diagram.

singularities $\ln k^2$ in some way, e.g., by dimensional regularization or by taking massive quarks and $k^2=m_q^2$. In the latter case, only the logarithmic m_q dependence should be kept in the final result: keeping the power terms m_q^2/Q^2 exceeds, for light quarks, the accuracy of the method. The subsequent procedure is to split the logarithms $\ln(Q^2/m^2)$ into the long-distance and short-distance parts $\ln(Q^2/m^2)=\ln(Q^2/\mu^2)+\ln(\mu^2/m^2)$ and absorb the long-distance ones $\ln(\mu^2/m^2)$ into the pion distribution amplitude: $\varphi_\pi(x)\to\varphi_\pi(x;\mu)$.

Thus, the lowest-twist contribution corresponds to the parton picture in which only the longitudinal (proportional to p) components of the external quark momenta appear. In the lowest order (see Fig. 2), the amplitude for transition of two photons into the quark-antiquark pair with collinear lightlike momenta xp, $\overline{x}p$ is given[1] by the quark propagator:

$$T_0(x,Q^2)=\frac{1}{-(q_1-xp)^2}=\frac{1}{xQ^2}. \tag{2.10}$$

and the PQCD result [1] for the large-Q^2 behavior of the form factor is

$$F_{\gamma^*\gamma\pi}(Q^2)=\frac{4\pi}{3}\int_0^1\frac{\varphi_\pi(x)}{xQ^2}dx\equiv\frac{4\pi f_\pi}{3Q^2}I_0. \tag{2.11}$$

Necessary nonperturbative information is accumulated in the same integral,

$$I_0=\frac{1}{f_\pi}\int_0^1\frac{\varphi_\pi(x)}{x}dx=\frac{Q^2}{f_\pi}\int_0^1 T_0(x,Q^2)\varphi_\pi(x)dx, \tag{2.12}$$

that appears in the one-gluon-exchange diagram for the pion electromagnetic form factor [32,41,42]. The value of I depends on the shape of the pion distribution amplitude (DA) $\varphi_\pi(x)$. In particular, using the asymptotic form [32,41]

$$\varphi_\pi^{\rm as}(x)=6f_\pi x(1-x) \tag{2.13}$$

gives $I_0^{\rm as}=3$. If one takes the Chernyak-Zhitnitsky ansatz [43]

$$\varphi_\pi^{\rm CZ}(x)=30f_\pi x(1-x)(1-2x)^2, \tag{2.14}$$

the integral I_0 increases by a sizable factor of 5/3: $I_0^{\rm CZ}=5$ and one can hope that this difference can be used for an experimental discrimination between the two competing models for the pion DA.

Since one of the photons has a small virtuality, one should, in principle, also take into account the regime [see Fig. 1(b)] involving a long-distance propagation in the q_1 channel, with large momentum flowing through a central subgraph W containing only the virtual photon vertex. In the lowest order, this subgraph corresponds to a hard-gluon exchange, just as in the asymptotically leading PQCD contribution to the pion electromagnetic form factor. The power counting for such a contribution into $F_{\gamma^*\gamma\pi^0}(Q^2)$ is given by

$$F(Q^2)\lesssim Q^{-t_{\mathcal{O}_1}-t_{\mathcal{O}_2}}, \tag{2.15}$$

where $t_{\mathcal{O}_i}$, $i=1,2$ are the twists of composite operators $\mathcal{O}_i$ corresponding to the q_1 and p channel, respectively. Taking into account that twist of a gauge-invariant color-singlet composite operator $\mathcal{O}_i$ cannot be less than 2, we conclude that this regime gives a nonleading $O(1/Q^4)$ contribution.

The third regime [Fig. 1(c)] corresponds to Feynman mechanism, i.e., to a situation when the passive quark is soft. Using the wave function terminology, we can say that $F_{\gamma^*\gamma\pi^0}(Q^2)$ in this regime is given by an overlap of soft wave functions describing the initial and final state. This contribution also behaves as $1/Q^4$ at large Q^2.

B. Handbag diagram and transverse momentum

For the OPE contribution, the simplest power corrections come either from the traces of the two-body operator $\mathcal{O}(x,y)$ which appears in the handbag diagram or from a direct insertion of gluon lines with physical polarizations into the propagator connecting the photon vertices. Since $D^\nu D_\nu$ can be interpreted in the momentum representation as the (generalized) virtuality k^2 of the quark field, the higher-twist operators containing $D^\nu D_\nu$ looks like a natural candidate for description of the effects due to the transverse momentum of the quarks. However, there are some practically important amplitudes which, due to their simple singularity structure, are "protected" from the towers of $(D^2)^n$-type higher-twist corrections. The most well-known example is given by the classic "handbag" diagram for deep inelastic scattering. The lowest-order diagram for the $\gamma^*\gamma\to\pi^0$ form factor (Fig. 2) has similar properties. Consider its analogue in a toy scalar model:

$$F(q_2,p)=\frac{1}{4\pi^2}\int e^{-iq_2z}\langle 0|\phi(0)\phi(z)|p\rangle\frac{d^4z}{z^2}. \tag{2.16}$$

The first term in the z^2 expansion for the matrix element,

$$\langle 0|\phi(0)\phi(z)|p\rangle=\xi_2(zp)+z^2\xi_4(zp)+(z^2)^2\xi_6(zp)+\cdots, \tag{2.17}$$

corresponds to the twist-two distribution amplitude while subsequent terms correspond to operators containing an increasing number of ∂^2's. It is straightforward to observe that, while the twist-two term produces the $1/Q^2$ contribution, the twist-four term is accompanied by an extra z^2 factor which

[1]In fact, there are two diagrams obtained from one another by the interchange of photon vertices. However, because of the symmetry of the distribution amplitude $\varphi_\pi(x)=\varphi_\pi(1-x)$, their contributions can be united.

completely kills the $1/z^2$ singularity of the quark propagator, and d^4z integration gives $\delta^4(q-xp)$, which is invisible for large Q^2. The same is evidently true for all the terms accompanied by higher powers of z^2. This means that the handbag diagram contains only one term with a powerlike behavior for large Q^2: it cannot generate higher powers of $1/Q^2$ which one could interpret as the $(\langle k^2\rangle/Q^2)^n$ expansion. Since only the $z^2=0$ projection of the bilocal operator survives, we can parametrize

$$\langle 0|\phi(0)\phi(z)|p\rangle=\int_0^1\varphi(x)e^{-i\bar{x}(zp)}dx+\cdots, \quad (2.18)$$

where the ellipsis stands for terms producing the ''invisible'' contributions, and write the lowest-order term as

$$F(q_2,p)=-\int_0^1\frac{\varphi(x)}{(q_2-\bar{x}p)^2}dx=-\int_0^1\frac{\varphi(x)}{(q_1-xp)^2}dx$$
$$=\int_0^1\frac{\varphi(x)}{xQ^2+x\bar{x}p^2}dx. \quad (2.19)$$

Hence, the handbag contribution in this case contains only the hadron-mass corrections (see [44]), but it gives no information about finite-size effects. In the momentum representation, the origin of this phenomenon can be traced to the fact that a straightforward expansion of the propagator is just in terms of traceless combinations:

$$\frac{1}{(q-k)^2}=\theta(|k|<|q|)\sum_{n=0}^{\infty}\frac{2^n}{(q^2)^{n+1}}q^{\mu_1}\cdots q^{\mu_n}\{k_{\mu_1}\cdots k_{\mu_n}\}$$
$$+\theta(|k|>|q|)\sum_{n=0}^{\infty}\frac{2^n}{(k^2)^{n+1}}$$
$$\times q^{\mu_1}\cdots q^{\mu_n}\{k_{\mu_1}\cdots k_{\mu_n}\}. \quad (2.20)$$

The handbag contribution corresponds to $|k|<|q|$, and this part of Eq. (2.20) without any approximation produces an expression equivalent to treating the k momentum as purely longitudinal $k=\bar{x}p$.

It is worth noting here that though the hadron-mass corrections have a *powerlike* behavior $(p^2/Q^2)^n$, they should not be classified as *higher-twist* corrections: they result from the kinematic hadron-mass dependence of the *lowest-twist* contribution. For deep inelastic scattering, the possibility to calculate the target-mass corrections within the lowest-twist contribution is known as the ξ-scaling phenomenon [45,46]. As emphasized by Ellis *et al.* [44], the ξ-scaling phenomenon can be also understood in terms of the primordial transverse momentum, if one takes into account that, for the lowest-twist term, the transverse momentum distribution is totally due to the nonzero hadron mass, i.e., it has a purely kinematic nature and for this reason can be calculated exactly. The quark propagator in QCD has a stronger singularity $\hat{z}/z^4$. As a result, the handbag-type contribution in QCD contains a twist-four operator with extra D^2 [47], but no operators with higher powers of D^2.

One may argue that there is another part in Eq. (2.20), when k is large (i.e., $|k|>|q|$). In this case, the k line corresponds to high virtualities. If such a large momentum goes directly into the soft hadronic wave function, the Q^2 behavior of such a contribution repeats the k^2 dependence of the soft wave function, i.e., very rapidly (say, exponentially) decreases with Q^2 (see Sec. IV C below for an explicit illustration). A more favorable possibility is when the large momentum by-passes the wave function. Such a configuration can give a leading-power contribution. In the latter case, the large virtuality flows through several lines forming a subgraph with the same (minimal possible) number of external quark lines as the lowest-order leading twist contribution. In the QCD factorization scheme, the relevant contribution produces a part of a higher-order coefficient function [see Fig. 1(a)].

C. One-loop radiative correction to the coefficient function

At one loop, the coefficient function for the $\gamma^*\gamma\to\pi^0$ form factor was calculated in Refs. [4–6]:

$$T(x,Q^2;\mu^2)=\frac{1}{xQ^2}\left\{1+C_F\frac{\alpha_s}{2\pi}\left[\left(\frac{3}{2}+\ln x\right)\ln(Q^2/\mu^2)\right.\right.$$
$$\left.\left.+\frac{1}{2}\ln^2x-\frac{x\ln x}{2(1-x)}-\frac{9}{2}\right]\right\}. \quad (2.21)$$

In full compliance with the factorization theorems [31,1] (see also [34,48,49]), the one-loop contribution contains no Sudakov double logarithms $\ln^2Q^2$ of the large momentum transfer Q. Physically, this result is due to the color neutrality of the pion. In the axial gauge, the Sudakov double logarithms appear in the box diagram (c) in Fig. 3 but they are cancelled by similar terms from the quark self-energy corrections in Figs. 3(d) and 3(e). In Feynman gauge, the double logarithms $\ln^2Q^2$ simply do not appear in any one-loop diagram. It is easy to check that the term containing the logarithm $\ln(Q^2/\mu^2)$ has the form of convolution

$$\frac{1}{xQ^2}C_F\frac{\alpha_s}{2\pi}\left(\frac{3}{2}+\ln x\right)=\int_0^1\frac{1}{\xi Q^2}V(\xi,x)\,d\xi \quad (2.22)$$

of the lowest-order (''Born'') term $T_0(\xi,Q^2)=1/\xi Q^2$ and the kernel

$$V(\xi,x)=\frac{\alpha_s}{2\pi}C_F\left[\frac{\xi}{x}\,\theta(\xi<x)\left(1+\frac{1}{x-\xi}\right)\right.$$
$$\left.+\frac{\bar{\xi}}{\bar{x}}\theta(\xi>x)\left(1+\frac{1}{\xi-x}\right)\right]_+ \quad (2.23)$$

governing the evolution of the pion distribution amplitude. The + operation is defined here, as usual [50], by

$$[F(\xi,x)]_+=F(\xi,x)-\delta(\xi-x)\int_0^1F(\zeta,x)\,d\zeta. \quad (2.24)$$

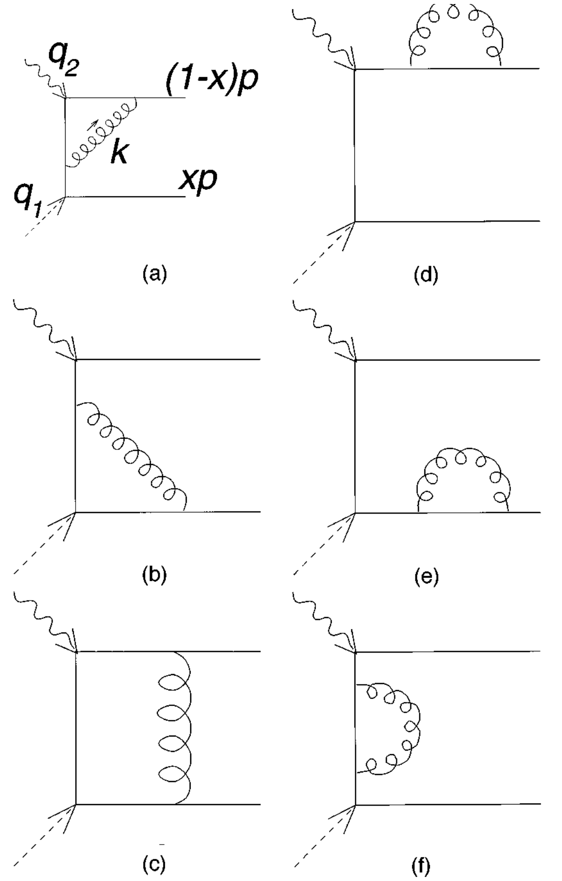

FIG. 3. One-loop diagrams.

Since the asymptotic distribution amplitude is the eigenfunction of the evolution kernel $V(\xi,x)$ corresponding to zero eigenvalue,

$$\int_0^1 V(\xi,x)\ \varphi^{\rm as}(x)\ dx=0, \tag{2.25}$$

the coefficient $\frac{3}{2}+\ln x$ of the $\ln(Q^2/\mu^2)$ term vanishes after the x integration with $\varphi^{\rm as}(x)$. Hence, the size of the one-loop correction for the asymptotic DA is μ independent and determined by the remaining terms. The I integral

$$I\equiv\frac{Q^2}{f_\pi}\int_0^1 T(x,Q^2)\ \varphi_\pi(x)\ dx \tag{2.26}$$

[cf. Eq. (2.12)] then can be written as

$$I\ |_{\varphi=\varphi^{\rm as}}=3\left\{1-\frac{5}{2}\ C_F\frac{\alpha_s}{2\pi}\right\}. \tag{2.27}$$

The negative coefficient $-5/2$ here comes from the constant term $-9/2$ [see Eq. (2.21)] partially compensated by two logarithmic terms which give together $+2$, with $+7/4$ generated by the $\frac{1}{2}\ln^2 x$ contribution and $+1/4$ by $-x\ln x/[2(1-x)]$ term. With $C_F=4/3$, the net factor is $[1-\frac{5}{3}\alpha_s/\pi]$. Hence, for $\alpha_s/\pi\approx 0.1$, the one-loop correction is less than 20% and the α_s/π expansion looks ''reasonably convergent.'' Taking the CZ form for $\varphi(x;\mu)$, we get

$$I|_{\varphi(x,\mu)=\varphi^{\rm CZ}(x)}=5\left\{1-C_F\frac{\alpha_s}{2\pi}\left(\frac{5}{6}\ln(Q^2/\mu^2)+\frac{49}{72}\right)\right\}. \tag{2.28}$$

Again, the negative coefficient $-49/72$ comes from the $-9/2$ term compensated by an increased contribution from the logarithmic terms: $\frac{1}{2}\ln^2 x$ gives $+263/72$ and $-x\ln x/[2(1-x)]$ gives 1/6. For $\mu=Q$, the one-loop modified factor is $[1-\frac{49}{108}(\alpha_s/\pi)]$, i.e., the total correction is smaller than that for the asymptotic DA. Since the result is μ dependent in this case, by an appropriate choice of μ, namely, taking $\mu=e^{49/120}Q\approx 1.5\ Q$ we can formally get a vanishing $\mathcal{O}(\alpha_s)$ correction. Then the one-loop expression for the form factor would coincide with the lowest-order formula, but with the distribution amplitude $\varphi_\pi^{\rm CZ}(x;\mu)$ evolved to the scale $\mu\approx 1.5\ Q$. However, at this scale, $\varphi_\pi(x;\mu)$ does not necessarily have the CZ form. To treat the evolution in a consistent way, we set the boundary condition that $\varphi_\pi^{\rm CZ}(x;\mu)$ has the canonical CZ form $\varphi_\pi^{\rm CZ}(x)\equiv 30\ f_\pi\ x\bar{x}(1-2x)^2$ at some specific scale $\mu=Q_0$ (the original derivation [43] assumes $Q_0=0.5$ GeV). Taking into account that $\varphi_\pi^{\rm CZ}(x)$ is a combination of two lowest eigenfunctions of the evolution kernel, we can write the solution of the evolution equation in the leading logarithm approximation:

$$\varphi_\pi^{\rm CZ}(x;\mu)=\varphi_\pi^{\rm as}(x)+\{\varphi_\pi^{\rm CZ}(x)-\varphi_\pi^{\rm as}(x)\}\left[\frac{\ln Q_0^2/\Lambda^2}{\ln\mu^2/\Lambda^2}\right]^{\gamma_2/\beta_0}, \tag{2.29}$$

where $\gamma_2=50/9$ is the relevant anomalous dimension and $\beta_0=11-\frac{2}{3}N_f$ is the lowest coefficient of the QCD β function. In what follows, we take $N_f=3$ and $\beta_0=9$. Choosing $\mu=Q$, we get, for the I integral (see also [51]),

$$I\ |_{\varphi_\pi(x,Q_0)=\varphi_\pi^{\rm CZ}(x)}=3\left\{1-\frac{5}{3}\ \frac{\alpha_s}{\pi}\right\}\left(1-\left[\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right]^{50/81}\right)+5\left\{1-\frac{49}{108}\ \frac{\alpha_s}{\pi}\right\}\left[\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right]^{50/81}. \tag{2.30}$$

Note that the $\ln^2 x$ term generates a larger positive contribution for $\varphi_\pi^{\rm CZ}(x)$ because $\varphi_\pi^{\rm CZ}(x)$ is more concentrated in the end-point region $x\sim 0$ than $\varphi_\pi^{\rm as}(x)$. Furthermore, if the distribution amplitude is extremely concentrated in the end-point region $x\sim 0$, a positive contribution from the $\frac{1}{2}\ln^2 x$ term dominates the correction and generates a large positive net effect. In such a situation, the one-loop correction vanishes only if $\mu=aQ$ with $a<1$. The broader the DA, the smaller should be the parameter a which reduces the one-loop expression to the lowest-order one. Since the effective normalization scale is smaller for a broader DA, perturbative QCD applicability is postponed to higher Q^2. One may speculate that this phenomenon simply indicates that for a broad DA the quark virtuality xQ^2 is a more natural choice for the effective factorization scale than the photon virtuality Q^2 (i.e., $a\sim\langle x\rangle$) and PQCD is applicable only if the average xQ^2 rather than Q^2 itself is large enough. One faces a similar situation studying the PQCD contribution to the pion form factor. The average virtuality $\langle xyQ^2\rangle$ of the exchanged gluon in that case is essentially smaller than Q^2 and one may question both the *self-consistency* and *reliability* of the PQCD analysis at accessible energies [52,53]. In Ref. [30], it was argued that due to the Sudakov effects in the impact parameter space, the PQCD treatment of the lowest-twist one-gluon-exchange term for the pion form factor is *self-consistent*[2] at smaller Q^2 than suggested by the estimates of the magnitude of the average gluon virtuality xyQ^2. One may expect that similar effects manifest themselves also in the $\gamma^*\gamma\pi^0$ form factor. Indeed, our numerical analysis of the one-loop correction shows that taking $a=1$ (rather than $a=\langle x\rangle$) provides a good choice for the factorization scale. It is accompanied by a small one-loop correction even for a broad DA of CZ type.

It is worth noting here that, even without incorporating the impact parameter representation, one can observe some traces of the Sudakov effects in the structure of the one-loop coefficient function in the region of small fractions x. As explained earlier, the one-loop term is obtained by calculating the $\gamma^*\gamma\to\overline{q}q$ amplitude for massive on-shell quarks with subsequent absorption of the mass logarithms in the form $\ln(\mu^2/m^2)$ into the distribution amplitude. When the virtuality xQ^2 of the quark line connecting the photon vertices becomes small, the vertex correction for the virtual photon [Fig. 3(a)] is dominated (in Feynman gauge) by the *off-shell* Sudakov double logarithm which can be written as

$$-\frac{\alpha_s}{2\pi}\,C_F\ln\frac{Q^2}{m^2}\ln\frac{Q^2}{xQ^2},$$

where xQ^2 is the virtuality of the hard quark. Of course, since this virtuality is parametrically of the order of Q^2, we get only a single logarithm with respect to Q^2, namely, $(\alpha_s/2\pi)\ C_F\ln(Q^2/m^2)\ln x$ [cf. (2.21)], just as required for factorization. However, if we write the sum of two terms

$$\frac{\alpha_s}{4\pi}\,C_F\left[\ln^2 x+2\ln\frac{Q^2}{m^2}\ln x\right]$$

which dominate the small-x region as

$$\frac{\alpha_s}{4\pi}\,C_F\left[\ln^2\frac{xQ^2}{m^2}-\ln^2\frac{Q^2}{m^2}\right],$$

we see that it converts into the standard *on-shell* Sudakov double logarithm

$$-\frac{\alpha_s}{4\pi}\,C_F\ln^2\frac{Q^2}{m^2}$$

when $xQ^2\sim m^2$. Of course, the region where xQ^2 is parametrically of the order of the IR cutoff m^2 is outside the formal applicability region of the factorization approach, and there is no surprise that double logarithms of Q^2 appear there. Note the well-known difference $\alpha_s/2\pi\to\alpha_s/4\pi$ between the *off*- and *on-shell* forms of the double logarithms. In higher orders, Sudakov logarithms are expected to exponentiate producing the Sudakov form factor[3] $\exp[-(\alpha_s/4\pi)\ C_F\ln^2(Q^2/m^2)]$, and the region of very small xQ^2 is relatively suppressed due to Sudakov effects.

This also means that taking $\mu^2\sim xQ^2$ in Eq. (2.21) is not an optimal choice, since it is accompanied by a negative rather than vanishing correction. Indeed, the original motivation to take a lower scale $\mu<Q$ was to compensate the positive contribution from the $\ln^2 x$ term. However, taking $\mu^2\sim xQ^2$ in Eq. (2.21) for a wide DA generates a negative $(-\ln^2 x)$ term which overkills the original positive $\frac{1}{2}\ln^2 x$ term and converts its sign in the net result. A negative correction, in its turn, suggests that a larger factorization scale is a better choice. This indicates that, for a broad DA, the typical distances probed in the hard subprocess are larger than those corresponding to $1/Q^2$ but smaller than those corresponding to the inverse of the average quark virtuality xQ^2.

As we will see in the next section, the modified factorization [30] is similar to the choice $\mu^2\sim xQ^2$ and for this reason it is accompanied by a negative correction. We will also explicitly show that the latter, in full accordance with the MFA analysis [29], can be explained by Sudakov effects in the impact parameter space.

III. ONE-LOOP RADIATIVE CORRECTIONS AND TRANSVERSE MOMENTUM

A. Vertex correction for virtual photon and Sudakov effects

To establish the connection between standard and modified factorization approaches, we give below a rather detailed discussion of the structure of the one-loop coefficient function using the Sudakov decomposition for the loop momenta. We use the same definition of transverse momentum $k_\perp$ as in Refs. [29,30], introduce the impact parameter $b_\perp$, and then translate our results into $b_\perp$ space. To be able to make a diagram by diagram comparison with Ref. [5], we use the Feynman gauge. This also allows us to give an independent one-loop derivation of the $b_\perp$ space Sudakov effects which complements the general approach [29] based on the analysis in the axial gauge.[4] We find it also instructive to demonstrate how the $b_\perp$-space double logarithms appear in a situation in which double logarithms of Q^2 are absent in any diagram.

We start with the diagram 3(a) which is the most natural suspect in a search for Sudakov effects in Feynman gauge. According to general rules, calculating the coefficient function one should assume that external quarks carry purely longitudinal lightlike momenta xp and $\overline{x}p$. Using p and q_1 (abbreviated in this section to q for convenience) as the basic

[2]Note, that self-consistency of the PQCD expansion (small α_s corrections) for the lowest-twist term does not necessarily mean that PQCD is reliable, since power corrections $(M^2/Q^2)^n$ can still be large (see discussion at the end of Sec. V).

[3]For the pion EM form factor, exponentiation of a similar combination $(C_F\alpha_s/4\pi)[\ln^2(xyQ^2/m^2)-\ln^2(Q^2/m^2)]$ suggested in Ref. [36] was verified by a two-loop calculation [54].

[4]In a recent paper [55], Li gave a covariant gauge derivation of the modified factorization for inclusive processes and heavy-quark decays. However, in technical implementation, his approach is quite different from ours.

Sudakov light-cone variables, we write the momentum k of the emitted gluon as

$$k=(\xi-x)p+\eta q+k_{\perp} \tag{3.1}$$

and then take the η integral by residue. After that, the contribution of Fig. 3(a) (and any other one-loop diagram) can be schematically written as

$$T_i^{(1)}(x,Q^2)=\frac{\alpha_s}{2\pi}\,C_F\int_0^1 d\xi\int M_i(x,Q^2;\xi,k_{\perp})\,\frac{d^2k_{\perp}}{2\pi}$$

$$\equiv\frac{\alpha_s}{2\pi}\,C_F\,t_i(x,Q^2). \tag{3.2}$$

The internal amplitude $M_a(x,Q^2;\xi,k_{\perp})$ for the diagram 3(a) is given by

$$M_a(x,Q^2;\xi,k_{\perp})=\frac{1}{xQ^2}\left\{-\left(\frac{\bar\xi}{\bar x}\right)\frac{Q^2+k_{\perp}^2/\bar\xi}{k_{\perp}^2[\xi Q^2+k_{\perp}^2/\bar\xi]}\,\theta(\xi>x)\right.$$

$$\left.+\frac{k_{\perp}^2\,\theta(\xi<x)}{[\xi Q^2+k_{\perp}^2/\bar\xi][\xi(x-\xi)Q^2+xk_{\perp}^2]}\right\}. \tag{3.3}$$

The $k_{\perp}$ integral diverges both in the $k_{\perp}\to\infty$ and $k_{\perp}\to 0$ limits. The ultraviolet large-$k_{\perp}$ divergences (they are actually irrelevant to our analysis) are removed by the R operation, while the low-$k_{\perp}$ collinear divergences can be regulated by taking massive quarks. In that case, $k_{\perp}^2\to k_{\perp}^2+m^2$ and the small-$k_{\perp}$ divergence (collinear singularity) is converted into the mass logarithm $\ln(Q^2/m^2)$ generating the evolution of the pion distribution amplitude. The Sudakov effects are also related to the $1/k_{\perp}^2$ singularity. It is easy to check that the coefficient in front of $1/k_{\perp}^2$ in the singular part,

$$M_a^{\mathrm{sing}}(x,Q^2;\xi,k_{\perp})=-\frac{1}{xQ^2}\,\frac{Q^2}{k_{\perp}^2[\xi Q^2+k_{\perp}^2/\bar\xi]}\left(\frac{\bar\xi}{\bar x}\right)\theta(\xi>x), \tag{3.4}$$

has the form of the product of the Born term $1/\xi Q^2$ and the relevant part

$$V_a(\xi,x)=\left(\frac{\bar\xi}{\bar x}\,\frac{\theta(\xi>x)}{\xi-x}\right)_+ \tag{3.5}$$

of the evolution kernel (2.23). Note, that calculating the evolution logarithm $\ln Q^2/m^2$ from $d^2k_{\perp}/k_{\perp}^2$, one can take $k_{\perp}=0$ (''neglect $k_{\perp}$'') in all other places, in particular, in the denominator factor $\xi Q^2+k_{\perp}^2/\bar\xi$. However, nothing prevents us from going beyond the leading logarithm approximation. Keeping the $k_{\perp}^2$ terms, we can take into account those contributions which do not have logarithmic behavior with respect to m^2 or Q^2. We will see that among them, there are ''Sudakov'' terms with a specific double-logarithmic dependence on the impact parameter $b_{\perp}$, the variable which is Fourier-conjugate to the transverse momentum $k_{\perp}$. To separate the contributions related to the evolution kernel from those corresponding to Sudakov effects, we first make the decomposition

$$-\frac{1}{[\xi Q^2+k_{\perp}^2/\bar\xi]xQ^2}=\left(\frac{1}{\xi Q^2+k_{\perp}^2/\bar\xi}-\frac{1}{xQ^2}\right)\frac{1}{(\xi-x)Q^2+k_{\perp}^2/\bar\xi} \tag{3.6}$$

and notice that the denominator factor $\xi Q^2+k_{\perp}^2/\bar\xi$ reduces to xQ^2 when $\xi=x$ and $k_{\perp}=0$. Hence, we can write

$$\frac{2\pi}{Q^2}\,t_a^{\mathrm{sing}}(x,Q^2)=-\int_x^1 d\xi\int d^2k_{\perp}\,\frac{\bar\xi/\bar x}{k_{\perp}^2[\xi Q^2+k_{\perp}^2/\bar\xi]xQ^2}=\int_x^1 d\xi\int\frac{d^2k_{\perp}}{\xi Q^2+k_{\perp}^2/\bar\xi}\left\{\frac{\bar\xi/\bar x}{k_{\perp}^2[(\xi-x)Q^2+k_{\perp}^2/\bar\xi]}\right.$$

$$\left.-\delta(\xi-x)\,\delta^2(k_{\perp})\int_x^1 d\zeta\int d^2\tilde k_{\perp}\,\frac{\bar\zeta/\bar x}{\tilde k_{\perp}^2[(\zeta-x)Q^2+\tilde k_{\perp}^2/\bar\zeta]}\right\}. \tag{3.7}$$

To disentangle the product of the δ functions in the ξ and $k_{\perp}$ variables, we rewrite Eq. (3.7) as

$$\int_0^1 d\xi\int\frac{d^2k_{\perp}}{\xi Q^2+k_{\perp}^2/\bar\xi}\left\{\frac{1}{k_{\perp}^2}\left(\frac{(\bar\xi/\bar x)\;\theta(\xi>x)}{(\xi-x)Q^2+k_{\perp}^2/\bar\xi}\right)_+\right.$$

$$\left.+\delta(\xi-x)\int_x^1\frac{\bar\zeta}{\bar x}\left(\frac{1}{k_{\perp}^2[(\zeta-x)Q^2+k_{\perp}^2/\bar\zeta]}-\delta^2(k_{\perp})\int\frac{d^2\tilde k_{\perp}}{\tilde k_{\perp}^2[(\zeta-x)Q^2+\tilde k_{\perp}^2/\bar\zeta]}\right)d\zeta\right\}, \tag{3.8}$$

where the combination

$$\left(\frac{(\bar\xi/\bar x)\;\theta(\xi>x)}{(\xi-x)Q^2+k_{\perp}^2/\bar\xi}\right)_+\equiv\frac{(\bar\xi/\bar x)\;\theta(\xi>x)}{(\xi-x)Q^2+k_{\perp}^2/\bar\xi}-\delta(\xi-x)\int_0^1\frac{(\bar\zeta/\bar x)\;\theta(\zeta>x)}{(\zeta-x)Q^2+k_{\perp}^2/\bar\zeta}d\zeta \tag{3.9}$$

is an analogue of the ''plus'' operation for the case when the transverse momentum is present. Similarly, the expression

$$\frac{1}{k_\perp^2[(\zeta-x)Q^2+k_\perp^2/\bar{\zeta}]}-\delta^2(k_\perp)\int\frac{d^2\tilde{k}_\perp}{\tilde{k}_\perp^2[(\zeta-x)Q^2+\tilde{k}_\perp^2/\bar{\zeta}]} \tag{3.10}$$

can be interpreted as a ''plus'' distribution with respect to $k_\perp$. Extracting the pure $1/k_\perp^2$ singularity from the $(\cdots)_+$ term in Eq. (3.8),

$$\frac{1}{k_\perp^2}\left(\frac{(\bar{\xi}/\bar{x})\ \theta(\xi>x)}{[(\xi-x)Q^2+k_\perp^2/\bar{\xi}]}\right)_+=\frac{1}{Q^2k_\perp^2}\left(\frac{(\bar{\xi}/\bar{x})\ \theta(\xi>x)}{(\xi-x)}\right)_+-\frac{1}{Q^2}\left(\frac{(\bar{\xi}/\bar{x})\ \theta(\xi>x)}{(\xi-x)[\bar{\xi}(\xi-x)Q^2+k_\perp^2]}\right)_+, \tag{3.11}$$

we can write Eq. (3.8) in the impact parameter representation as

$$t_a^{\rm sing}(x,Q^2)=\frac{1}{2\pi}\int_0^1 d\xi\int \quad B(\xi;bQ)[V_a(\xi,x)L(bm)+E_a(x,\xi;bQ)+\delta(\xi-x)S_a(x,bQ)]\ d^2b_\perp . \tag{3.12}$$

The function $B(\xi;bQ)$ gives the Born term in the b space:

$$B(\xi;bQ)=\frac{1}{2\pi}\int\frac{e^{-ik_\perp b_\perp}}{\xi Q^2+k_\perp^2/\bar{\xi}}d^2k_\perp=\bar{\xi}K_0(bQ\sqrt{\xi\bar{\xi}}), \tag{3.13}$$

where $b=|b_\perp|$ and $K_0(z)$ is the modified Bessel function. By $L(bm)$ we denote a regularized version of the integral resulting from the first term in Eq. (3.11):

$$L(bm)=\mathrm{Reg}_{(m)}\left\{\frac{1}{2\pi}\int d^2k_\perp\frac{e^{ik_\perp b_\perp}}{k_\perp^2}\right\}. \tag{3.14}$$

In particular, if the integral is regulated by $1/k_\perp^2\to 1/(k_\perp^2+m^2)$, then $L(bm)=K_0(bm)$. The function $L(bm)$ containing the mass logarithm $\ln(mb)$ is multiplied by the relevant part $V_a(\xi,x)$ of the evolution kernel. As discussed in the preceding section, the mass singularity $\ln(m)$ must be absorbed [in the form $\ln(m/\mu)$, where μ is the factorization scale] into the redefinition of the distribution amplitude: $\varphi_\pi(x)\to\varphi_\pi(x;\mu)$. The second term in Eq. (3.11) is given by the function $E(x,\xi;bQ)$ which also contains the evolution kernel $V_a(\xi,x)$:

$$E_a(x,\xi;bQ)=-\frac{1}{2\pi}\int e^{ik_\perp b_\perp}\left(\frac{(\bar{\xi}/\bar{x})\ \theta(\xi>x)}{(\xi-x)[\bar{\xi}(\xi-x)Q^2+k_\perp^2]}\right)_+ d^2k_\perp=-\left[\frac{\bar{\xi}}{\bar{x}}\frac{\theta(\xi>x)}{\xi-x}\ K_0(bQ\sqrt{(\xi-x)\bar{\xi}})\right]_+. \tag{3.15}$$

It is easy to notice that both the Born term $B(\xi;bQ)$ and the evolution-related terms $L(bm)$ and $E_a(x,\xi;bQ)$ exponentially decrease at large b, since the function $K_0(b\cdots)$ behaves as $\exp(-b\cdots)$ in this limit. On the other hand, the ''Sudakov'' term

$$S_a(x;bQ)=\frac{1}{2\pi}\int d^2k_\perp\frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\times\int_x^1\left(\frac{\bar{\zeta}^2}{\bar{x}}\right)\ \frac{d\zeta}{\bar{\zeta}(\zeta-x)+k_\perp^2/Q^2} \tag{3.16}$$

accompanied by $\delta(\xi-x)$ in Eq. (3.12) has a completely different behavior at large b. Indeed, changing the variable ζ in the above integral as $1-\zeta=y\bar{x}$, we rewrite Eq. (3.16) in the form

$$S_a(x;Qb)=\frac{1}{2\pi}\int d^2k_\perp\frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\int_0^1\frac{y^2dy}{y\bar{y}+k_\perp^2/\bar{x}^2Q^2}\equiv s(\bar{x}Qb). \tag{3.17}$$

According to this representation, the function $s(\bar{x}Qb)$ vanishes as $b\to 0$. In the opposite limit of large impact parameters, it has a double-logarithmic dependence on b. To see this, we integrate first over y and then over $k_\perp$ taking into account that the factor $(e^{ik_\perp b_\perp}-1)$ provides, in the limit of large b, an effective IR cutoff at $k_\perp\sim 1/b$. As a result, we obtain the large-b behavior of $s(\bar{x}Qb)$ [29]:

$$s(\bar{x}Qb)\approx\frac{1}{2\pi}\int d^2k_\perp\frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\ \ln\left(\frac{\bar{x}Q}{k_\perp}\right)\approx\int_{1/b}\frac{dk_\perp}{k_\perp}\ \ln\left(\frac{k_\perp}{\bar{x}Q}\right)\approx-\frac{1}{2}\ \ln^2(\bar{x}Qb),\quad 1/\Lambda_{\rm QCD}\gg b\gg 1/Q. \tag{3.18}$$

To be on the safe side, we included the $1/\Lambda_{\rm QCD}\gg b$ restriction to emphasize that these results are only valid in the region where one can trust PQCD expressions for quark and gluon propagators.

Integrating $s(\overline{x}Qb)$ with the Born term gives, for small x, a negative double logarithm $-\frac{1}{2}\ln^2 x$. As discussed above, such a correction is expected when one uses xQ^2 as the factorization scale. Indeed, for small x, the Born term is a function of xb^2Q^2. Hence, the choice $\mu^2=1/b^2$ is essentially equivalent to setting $\mu^2 \sim xQ^2$.

In Ref. [29], it was shown that the b-space double logarithms exponentiate in higher orders. In the double logarithmic approximation, they give the suppression factor

$$\exp\left\{-\frac{\alpha_s}{4\pi}C_F \ln^2(\overline{x}Qb)\right\} \tag{3.19}$$

for large b. The running of the coupling constant induces the next-to-leading logarithms (see [56,57]). To get them, one should put $\alpha_s(k_\perp^2)=4\pi/(\beta_0 \ln k_\perp^2/\Lambda^2)$ under the integral:

$$\alpha_s\, C_F\, s(\overline{x}Qb) \to \frac{C_F}{2\pi}\int d^2k_\perp \frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\,\alpha_s(k_\perp^2) \times \int_0^1 \frac{y^2\, dy}{y\overline{y}+k_\perp^2/\overline{x}^2 Q^2}. \tag{3.20}$$

In general, the Sudakov effects are governed by the *eikonal* [58,59,29] (or *cusp* [60–62]) anomalous dimension

$$\Gamma_{\text{cusp}}(\alpha_s)=\frac{C_F\alpha_s}{\pi}\left\{1+\frac{\alpha_s}{\pi}\left[N_c\left(\frac{67}{36}-\frac{\pi^2}{12}\right)-\frac{5}{18}N_f\right]+\cdots\right\}. \tag{3.21}$$

Clearly, only the α_s term of $\Gamma_{\text{cusp}}(\alpha_s)$ manifests itself in a one-loop calculation. To get further corrections [29], one should substitute $C_F\alpha_s/\pi$ in Eq. (3.20) by $\Gamma_{\text{cusp}}(\alpha_s)$ and also use a two-loop expression for $\alpha_s(k_\perp^2)$ and $s(\overline{x}Qb)$ [29]. Here, we restricted our analysis to the one-loop level.

B. Vertex correction for the real photon

For the real photon, the contribution of the vertex correction diagram 3(b) is given by

$$M_b(x,Q,\xi,k_\perp)=\frac{1}{xQ^2}\left\{\frac{\xi}{x}\frac{(x-\xi)Q^2+xk_\perp^2}{k_\perp^2[\xi(x-\xi)Q^2+xk_\perp^2]}\right\}\theta(\xi<x). \tag{3.22}$$

Again, we concentrate on the term singular at $k_\perp=0$. It is convenient to split it into two parts. The first part is obtained by taking xQ^2 from the $(x-\xi)Q^2$ term in the numerator and the second one by taking $(-\xi Q^2)$. We represent the first part as

$$\left(\frac{\xi}{x}\right)\frac{1}{k_\perp^2[\xi(x-\xi)Q^2+xk_\perp^2]}=\frac{1}{k_\perp^2}\left(\frac{1}{\xi Q^2+k_\perp^2/\overline{\xi}}\right)\frac{\xi^2/x}{\xi(x-\xi)+xk_\perp^2/Q^2}+\frac{\xi/x}{[\xi\overline{\xi}Q^2+k_\perp^2][\xi(x-\xi)Q^2+xk_\perp^2]}. \tag{3.23}$$

The last term here produces no divergences both for large and small $k_\perp$. The $1/k_\perp^2$ singularity is contained in the first term which we arranged to have a form of a product of the same Born term $1/(\xi Q^2+k_\perp^2/\overline{\xi})$ with a factor looking as a $k_\perp$-modified evolution kernel. Then we write this factor as a sum of a ''plus'' term and a $\delta(x-\xi)$ term:

$$\frac{(\xi^2/x)\;\theta(\xi<x)}{\xi(x-\xi)+xk_\perp^2/Q^2}=\left(\frac{(\xi^2/x)\;\theta(\xi<x)}{\xi(x-\xi)+xk_\perp^2/Q^2}\right)_+ +\delta(x-\xi)\int_0^1\frac{(\zeta^2/x)\;\theta(\zeta<x)}{\zeta(x-\zeta)+xk_\perp^2/Q^2}\,d\zeta. \tag{3.24}$$

As a result, the total contribution associated with the $k_\perp=0$ singularity can be written as

$$t_b^{sing}(x,Q^2)=\int_0^1 d\xi\int\frac{d^2k_\perp}{\xi Q^2+k_\perp^2/\overline{\xi}}\left\{\frac{1}{k_\perp^2}\left(\frac{(\xi^2/x)\;\theta(\xi<x)}{\xi(x-\xi)+xk_\perp^2/Q^2}\right)_+ +\delta(\xi-x)\int_0^x\frac{\zeta^2}{x}\left(\frac{1}{k_\perp^2[\zeta(x-\zeta)+xk_\perp^2/Q^2]} -\delta^2(k_\perp)\int\frac{d^2\tilde{k}_\perp}{\tilde{k}_\perp^2[\zeta(x-\zeta)+x\tilde{k}_\perp^2/Q^2]}\right)d\zeta\right\}, \tag{3.25}$$

where the $\delta^2(k_\perp)$ term comes from the second, ''$-\xi Q^2$'' part of the original expression (3.22).

From this decomposition, we obtain the mass singularity term

$$\left(\frac{\xi}{x}\frac{\theta(\xi<x)}{(x-\xi)}\right)_+ L(bm)\equiv V_b(\xi,x)\;L(bm), \tag{3.26}$$

the evolution-related contribution

$$E_b(x,\xi;b)=-\left[\frac{\xi}{x}\frac{\theta(\xi<x)}{(x-\xi)}K_0(bQ\sqrt{\xi(x-\xi)/x}\,)\right]_+, \tag{3.27}$$

and the Sudakov term

$$S_b(x;bQ)=\frac{1}{2\pi}\int d^2k_\perp \frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\int_0^x\left(\frac{\zeta^2}{x}\right) \frac{d\zeta}{\zeta(x-\zeta)+xk_\perp^2/Q^2}=s(\sqrt{x}Qb). \tag{3.28}$$

For large b, the latter behaves as

$$S_b(x;bQ)\approx-\frac{1}{2}\ln^2(\sqrt{x}Qb). \tag{3.29}$$

By analogy with $S_a(x;bQ)$ which is a function of $\overline{x}Qb$ we might expect that $S_b(x;bQ)$ should be a function of xQb. Our calculation above shows that $S_b(x;bQ)$ is a function of $\sqrt{x}Qb$. That this result is not unreasonable, can be justified in the following way. Note, that for small x, both the Born term $B(x;bQ)$ and our $S_b(x;bQ)$ are the functions of the same combination xb^2Q^2. Hence, integrating the product $B(x;bQ)S_b(x;bQ)$ over b just gives $1/Q^2$ multiplied by a constant factor: no $\ln x$ terms are produced. On the other hand, a $\ln^2 x$ term would appear if $S_b(x;bQ)$ would behave as $\ln^2(xQb)$ for large b. The explicit expression for diagram 3(b) given in Ref. [5] has no $\ln^2 x$ terms.

C. Box and self-energy diagrams

In the Feynman gauge, the box diagram 3(c) contribution in QCD,

$$M_c(x,Q;\xi,k_\perp)=\frac{1}{xQ^2}\left\{\frac{x(\overline{\xi^2}Q^2+k_\perp^2)}{\overline{x}k_\perp^2[\xi\overline{\xi}Q^2+k_\perp^2]}-\frac{(x-\xi)^2Q^2+xk_\perp^2}{\overline{x}k_\perp^2[\xi(x-\xi)Q^2+xk_\perp^2]}\,\theta(\xi<x)\right\}, \tag{3.30}$$

only by a numerical factor differs from that in a model with scalar or pseudoscalar gluons, in which Sudakov effects are absent. Hence, the $k_\perp=0$ singularity produces only the evolution effects:

$$M_c(x;\xi,k_\perp)=\frac{1}{k_\perp^2}\,V_c(x,\xi)\,\frac{1}{\xi Q^2}+\cdots, \tag{3.31}$$

where $V_c(x,\xi)$ is the relevant part,

$$V_c(x,\xi)=\frac{\xi}{x}\,\theta(\xi<x)+\frac{\overline{\xi}}{\overline{x}}\theta(\xi>x), \tag{3.32}$$

of the evolution kernel. Note, that $V_c(x,\xi)$ does not have a ‘‘plus’’ form by itself. The missing $\delta(x-\xi)$ terms are provided by two quark self-energy diagrams 3(d) and 3(e):

$$M_{d+e}=-\frac{1}{xQ^2}\,\delta(x-\xi)\,\frac{1}{k_\perp^2}\int_0^1\left[\frac{\overline{\zeta}}{\overline{x}}\theta(\zeta>x)+\frac{\zeta}{x}\,\theta(\zeta<x)\right]d\zeta$$

$$=-\frac{1}{2xQ^2k_\perp^2}\,\delta(x-\xi). \tag{3.33}$$

The third self-energy diagram 3(f) has only the UV divergence

$$M_f=-\frac{1}{xQ^2}\,\delta(x-\xi)\int_0^x\frac{\zeta/x}{\zeta(x-\zeta)Q^2+xk_\perp^2}\,d\zeta\,d^2k_\perp\,. \tag{3.34}$$

Combining evolution kernels from all the diagrams above, one obtains the total evolution kernel $V(\xi,x)$ (2.23).

D. Standard vs modified factorization

Summarizing the findings of the previous subsections, we write the sum of the lowest-order term and one-loop diagrams in the impact parameter representation as

$$F_{\gamma^*\gamma\pi^0}(Q^2)=\frac{4\pi}{3}\int_0^1\left\{\frac{1}{xQ^2}+\frac{\alpha_s}{2\pi}\,C_F\int_0^1 d\xi\int B(\xi;bQ)[V(\xi,x)\;L(bm)+E(\xi,x;bQ)\right.$$

$$\left.+\delta(\xi-x)S(x,bQ)+R(\xi,x;bQ)]\,\frac{d^2b_\perp}{2\pi}\right\}\varphi_\pi(x)\,dx, \tag{3.35}$$

where $B(\xi;bQ)$ is the b version of the Born term (3.13), $V(\xi,x)$ is the total evolution kernel, $E(x,\xi;bQ)$ is the sum of the evolution-related terms such as (3.15), (3.27), $S(x,bQ)$ is the total Sudakov term given by Eqs. (3.17), (3.28), and $R(\xi,x;bQ)$ accumulates all the remaining contributions coming from terms regular at $k_\perp=0$. Integrating over b and specifying the prescription for the renormalized distribution amplitude $\varphi_\pi(x;\mu)$, one would get the result (2.21) of the standard factorization scheme. In particular, the term $\frac{1}{2}\ln^2 x$, most sensitive to the width of the distribution amplitude $\varphi_\pi(x;\mu)$, comes from a negative contribution $-\frac{1}{2}\ln^2 x$ due to the Sudakov term $S(x,bQ)$ and a positive contribution $\ln^2 x$ coming from the m-independent part of the convolution

$$\int_0^1 d\xi \int B(\xi;bQ)\otimes V(\xi,x)L(bm)\,\frac{d^2b_\perp}{2\pi}$$

$$=\frac{1}{xQ^2}\left\{\left(\frac{3}{2}+\ln x\right)\ln(Q^2/m^2)+\ln^2 x+f(x)\right\}. \quad (3.36)$$

This convolution also contains terms denoted by $f(x)$ which are less singular at $x=0$. The total sum vanishes when integrated with the nonevolving asymptotic distribution amplitude $\varphi_\pi(x)$. It does not vanish, however, when integrated with DA's differing from $\varphi_\pi^{\rm as}(x)$.

The logarithmic mass singularity $\ln m$ contained in the evolution term $V(\xi,x)\ L(bm)$ is eliminated by absorbing it into the renormalized DA. The procedure used in the modified factorization approach of Refs. [29,30] is to absorb $\ln(mb)$. As a result, one obtains the pion distribution amplitude $\varphi_\pi(x;1/b)$ normalized at the scale $\mu=1/b$. Making such a choice, one should realize that b is an integration variable and, to preserve the acquired precision, one must use the evolution equation to get $\varphi_\pi(x;1/b)$ for all relevant values of b. In particular, if the distribution amplitude is assumed to have a CZ-type shape for large b, it should be evolved towards the asymptotic shape for smaller b using Eq. (2.29). Modeling $\varphi_\pi(x;1/b)$ by a function of x only amounts to neglecting the m-independent part of the convolution $B(\xi;bQ)\otimes V(\xi,x)L(bm)$ (3.36). As noted before, this contribution contains $\ln^2 x$, hence, for extremely wide distribution amplitudes it can exceed that coming from the Sudakov term which only contains $(-\frac{1}{2}\ln^2 x)$.

In the formal $b=0$ limit, the function $\varphi_\pi(x;1/b)$ evolved according to the leading logarithm approximation formula (2.29), coincides with $\varphi_\pi^{\rm as}(x)$. However, the function $E(x,\xi;bQ)$ also develops a logarithmic singularity for small b, because

$$K_0(Qb\cdots)=-\ln(Qb)+\cdots$$

for small b. Hence, two $\ln(b)$ singularities present in Eqs. (3.12),(3.35) compensate each other in the $b\to 0$ limit and the net coefficient in front of the evolution kernel is $\ln(Q/m)$: the distribution amplitude evolves in fact only to the scale $b_{\rm min}\sim 1/Q$ corresponding to the resolving power of the external probe. Absorbing $\ln(Q/m)$ into the renormalized distribution amplitude one would get $\varphi(x)\to\varphi(x;Q)$, with the large external momentum Q serving now as a factorization scale. Such a choice is usually made in the standard factorization approach, in which μ is either a fixed constant, e.g., $\mu=1$ GeV or proportional to the external momentum $\mu=aQ$, with a being a fixed number. In particular, one can optimize the choice of the parameter a by taking the value producing the shape of $\varphi(x;1/b)$ averaged over the essential region of the b integration. Another point is that the PQCD evolution of $\varphi(x;\mu)$ is reliable only in a restricted region $\mu\gtrsim\mu_0$. Since the modified factorization involves integration over all b, we formally need to know the distribution amplitude $\varphi(x;1/b)$ outside the perturbative region $b\lesssim 1/\mu_0$. One should remember, however, that the Born term $K_0(Qb\sqrt{x\bar{x}})$ for finite x exponentially suppresses the large-b region. As a result, essential impact parameters b are $\sim 1/Q$. The suppression by the Born term disappears for small x when the effective scale becomes $1/\sqrt{xQ^2}$ rather than $1/Q$. In this case, the suppression of the large-b region is provided by the exponentiation of the Sudakov terms which is the crucial element of the modified factorization approach [29,30]. As a result of the exponentiation, the series of $[\alpha_s\ln^2(Qb)]^n$ terms, each of which tends to infinity as $b\to\infty$, is substituted by the exponential of Eq. (3.19) type rapidly vanishing with growing b. Of course, for finite x, the Born term $K_0(Qb\sqrt{x\bar{x}})$ provides even stronger suppression of the large-b region and the influence of the Sudakov factor is minor. Only for small x do Sudakov effects become important. The relevant combination $\bar{x}Qb$ in the Sudakov term of the diagram 3(a) converts into Qb, and the exponentiated Sudakov factor plays a primary role in squeezing the size of essential impact parameters. A special role of the small x values in the $b_\perp$ integration is reflected by the $-\frac{1}{2}\ln^2 x$ term resulting from the convolution of the Born term with the one-loop Sudakov factor

$$\frac{1}{2\pi}\int B(x;Qb)\ S(x;Qb)\ d^2b_\perp=\frac{1}{xQ^2}\left(-\frac{1}{2}\ln^2 x-g(x)\right), \quad (3.37)$$

where $g(x)$ stands for less singular terms. After integration with the asymptotic distribution amplitude, the $[-\frac{1}{2}\ln^2 x-g(x)]$ term gives approximately $-9/4+0.05$, to be compared with the magnitude $-5/2$ of the total one-loop correction [see discussion after Eq. (2.27)]. Hence, the total one-loop correction in the case of the asymptotic DA is very close to the contribution of the Sudakov term alone (the deviation is only 12%). If the higher-loop corrections can be also approximated by the Sudakov contribution, then the exponentiated form would produce the all-order result in a rather compact form.

Discussing the numerical significance of the Sudakov terms, we should keep in mind that all the logarithmic enhancements $\ln^2(Qb)$ are perfectly integrable and that the region of small x, where the Sudakov terms are important, is small itself: after $b_\perp$ and x integrations, there are no especially large contributions in the final result. The total one-loop correction is only about 20%. Hence, the exponentiation of the Sudakov terms would alter the one-loop corrected result for the form factor by just a few percent, which is similar to the accuracy of approximating the total contribution by the Sudakov term at one loop. Note also that a few percent change may be smaller than the contribution generated by the one-loop terms $E(x,\xi,Qb)$, $R(x,\xi,Qb)$ and the effects due to the b dependence of the renormalized distribution amplitude $\varphi_\pi(x;1/b)$. Moreover, for a wide DA, the latter are comparable to or exceeding the Sudakov contributions. In principle, one can try to explicitly include these corrections within the MFA framework, but the result would not have a simple form anymore. In this situation, instead of dealing with convolutions of Bessel functions, one may prefer to use the result (2.21) of the standard factorization approach which has a simple form with easily controllable accuracy. Another bonus of using the SFA is the ability of $\varphi_\pi(x;Q)$ to fully absorb the necessary nonperturbative information: increasing Q we do not need to make any assumptions about the shape of $\varphi_\pi(x;\mu)$ at smaller values $\mu<Q$ of the factorization scale μ.

IV. INCLUSION OF PRIMORDIAL TRANSVERSE MOMENTUM

A. Brodsky-Lepage interpolation

Despite our persistent efforts, we failed so far to find any traces of contributions capable of producing a series of transverse-momentum-related power corrections to the leading PQCD result. Recall that we investigated first the higher-twist contributions due to operators with contracted covariant derivatives $D^\mu \cdots D_\mu$ which are the standard candidates to describe the $k_\perp$ effects in the OPE-like factorization approaches. We observed that, for the simplest handbag diagram, these operators do not produce the expected infinite chain of $(1/Q^2)^n$ power corrections. Then we studied one-loop radiative corrections in the Sudakov and impact-parameter representations. Our results are in full accord with the corresponding expressions of the MFA [29,30]. But they also completely agree with the one-loop results [4–6] of the SFA, i.e., they do not contain any power corrections. Nevertheless, $F_{\gamma^*\gamma\pi^0}(Q^2)\sim 1/Q^2$ cannot be a true behavior of $F_{\gamma^*\gamma\pi^0}(Q^2)$ in the low-Q^2 region, especially since the $Q^2=0$ limit of $F_{\gamma^*\gamma\pi^0}(Q^2)$ is known to be finite and normalized by the $\pi^0\to\gamma\gamma$ decay rate. The value of $F_{\gamma^*\gamma\pi^0}(0)$ in QCD [40] is fixed by the axial anomaly [7]

$$F_{\gamma^*\gamma\pi^0}(0)=\frac{1}{\pi f_\pi}. \tag{4.1}$$

If the shape of the pion DA is specified, the large-Q^2 behavior is also known. For the asymptotic DA,

$$F^{\text{as}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{4\pi f_\pi}{Q^2}. \tag{4.2}$$

Long ago, Brodsky and Lepage [3] proposed the interpolation formula

$$F^{\text{int,BL}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi f_\pi(1+Q^2/4\pi^2 f_\pi^2)}\equiv\frac{1}{\pi f_\pi(1+Q^2/s_0)}, \tag{4.3}$$

which reproduces both the $Q^2=0$ value (4.1) and the high-Q^2 behavior given by Eq. (4.2). The BL-interpolation formula (4.3) has a monopole form with the scale $s_0=4\pi^2 f_\pi^2\approx 0.67\ \text{GeV}^2$ numerically close to the ρ-meson mass squared: $m_\rho^2\approx 0.6\ \text{GeV}^2$. Thus, the BL interpolation suggests a form similar to that based on the vector meson dominance (VMD) expectation $F_{\gamma^*\gamma\pi^0}(Q^2)=1/[\pi f_\pi(1+Q^2/m_\rho^2)]$. In the VMD approach, the ρ-meson mass m_ρ serves as a parameter determining the pion charge radius, and it is only natural to expect that the tower of $(s_0/Q^2)^N$ corrections suggested by the BL-interpolation formula can be explained by intrinsic transverse momentum effects. The only problem is *how* to get Eq. (4.3) (or anything similar to it) from QCD, i.e., how to construct an expression which would provide a good model both in perturbative and nonperturbative regimes. Before proposing our variant of the solution to this problem, let us discuss briefly two recent attempts [21,26] to include intrinsic transverse momentum effects into the description of the $\gamma^*\gamma\pi^0$ form factor.

B. Extrapolation of perturbative results

As emphasized above, despite the fact that the denominator of the Born term $1/(\xi Q^2+k_\perp^2/\bar{\xi})$ is $k_\perp$ modified compared to its collinear approximation ξQ^2, convoluting $B(\xi;bQ)$ with $S(\xi;bQ)$ one would enjoy no power modifications of the canonical $1/Q^2$ behavior, i.e., the transverse-momentum effects included in the Sudakov term and other one-loop corrections do not correspond to any higher-twist contributions. The obvious reason is that, apart from the IR regulator mass m [producing a logarithmic dependence $\ln m$ which is absorbed into $\varphi(x;\mu)$], the large momentum Q is the only scale that appears in the relevant $k_\perp$ integrals.

In general, the fact that some contribution is written as an integral over the transverse momentum $k_\perp$ or the impact parameter $b_\perp$ does not necessarily mean that something beyond the leading twist is included. To illustrate this point, we note that even the lowest-order, ‘‘purely collinear’’ contribution (2.11) can be written in the impact-parameter representation. A possible form is suggested by the one-loop calculation

$$F_0(Q^2)=\frac{2}{3}\int_0^1 dx\int \bar{x}K_0(\sqrt{x\bar{x}b^2Q^2})\varphi_\pi(x)\ d^2b, \tag{4.4}$$

where $\bar{x}K_0(\sqrt{x\bar{x}b^2Q^2})$ is the impact-parameter profile of the modified propagator $1/(xQ^2+k_\perp^2/\bar{x})$ [see Eq. (3.13)]. Though the b version of the quark propagator explicitly depends on b, integrating over b in Eq. (4.4) gives a simple power result $1/Q^2$ without any subleading power corrections. This phenomenon can be traced to the absence of the b dependence in the distribution amplitude. In the momentum representation, Eq. (4.4) is equivalent to using $\varphi_\pi(x)\delta^2(k_\perp)$ for the $\bar{q}q\pi$ vertex:

$$F_0(Q^2)=\frac{4\pi}{3}\int_0^1 dx\int\frac{\varphi_\pi(x)\delta^2(k_\perp)}{xQ^2+k_\perp^2/\bar{x}}d^2k_\perp . \tag{4.5}$$

However, as we have seen in the preceding section, radiative corrections generate terms with less trivial $k_\perp$ dependence. In particular, the one-loop correction contains $\alpha_s/k_\perp^2$ terms. As a result, the $k_\perp$-dependence of the $\bar{q}q\pi$ vertex at one loop is

$$\varphi_\pi(\xi)\delta^2(k_\perp)+\frac{\alpha_s}{(2\pi)^2k_\perp^2}\int_0^1 V(\xi,x)\ \varphi_\pi(x)\ dx+\cdots\ . \tag{4.6}$$

In the impact parameter representation, the sum of $\delta^2(k_\perp)$ and $1/k_\perp^2$ terms is converted into a more suggestive combination

$$\varphi_\pi(\xi)-\frac{\alpha_s}{2\pi}\ln(bm)\int_0^1 V(\xi,x)\ \varphi_\pi(x)\ dx, \tag{4.7}$$

which can be understood as the two first terms of the α_s expansion of the expression for the leading-logarithm evolved distribution amplitude $\varphi(\xi,1/b)$ written symbolically as

$$\exp\left[-\frac{\alpha_s}{2\pi}\ln(bm)V\right]\otimes\varphi.$$

Since all the conclusions made from the studies of one-loop corrections are based on perturbative analysis, strictly speaking, they are only applicable to transverse momenta which are large enough.[5] Furthermore, there are no special reasons to expect that formulas derived for momenta $k_\perp$ generated by perturbative gluon radiation are still true in the small-$k_\perp$ region dominated by primordial (or intrinsic) transverse momentum. Still, it is tempting to extend the leading-logarithm convolution formula

$$F(Q^2)=\frac{2}{3}\int_0^1 dx\int K_0(\sqrt{x\bar{x}}Qb)\ \varphi(x;1/b)\ d^2b \tag{4.8}$$

into the nonperturbative region. To do this, we should substitute the distribution amplitude $\varphi(x;1/b)$ by a function which reflects (or models) the nonperturbative b dependence.

In the light-cone approach [1], the basic object is the wave function $\Psi(x,k_\perp)$ which depends both on the fraction variable x and transverse momentum $k_\perp$. In QCD, it is customary to split $\Psi(x,k_\perp)$ into two components. The soft component $\Psi^{\text{soft}}(x,k_\perp)$ is due to the nonperturbative part of the QCD interaction and its width is determined by the size of the relevant $\bar{q}q$ bound state. It is expected that $\Psi^{\text{soft}}(x,k_\perp)$ rapidly (e.g., exponentially) decreases for large $k_\perp^2$. In our perturbative lowest-twist treatment above, the soft wave function $\Psi^{\text{soft}}(x,k_\perp)$ was imitated by $\varphi_\pi(x)\delta^2(k_\perp)$. The PQCD interaction (gluon radiative corrections) produces the hard component $\Psi^{\text{hard}}(x,k_\perp)$ which behaves as $\alpha_s/k_\perp^2$ at large $k_\perp$. The distribution amplitude $\varphi_\pi(x)$ can be treated as the integral of the wave function $\Psi(x,k_\perp)$ over $k_\perp$ (see [1]):

$$\varphi_\pi(x)=\frac{\sqrt{6}}{(2\pi)^3}\int\ \Psi(x,k_\perp)\ d^2k_\perp\ . \tag{4.9}$$

For $\Psi^{\text{soft}}(x,k_\perp)$, this integral perfectly converges. However, the perturbative $1/k_\perp^2$ tail generates logarithmic divergences. Hence, one should supplement this definition by some regularization procedure specified by a cutoff parameter μ: $\varphi_\pi(x)\to\varphi_\pi(x,\mu)$. The ''cutoff'' should be understood in a broad sense. It may be imposed literally $k_\perp^2<\mu^2$ or one can use more gentle procedures based, say, on dimensional regularization. In other words, $\varphi_\pi(x)$ is a scheme-dependent object: $\varphi_\pi(x)\to\varphi_\pi^{(S)}(x)$. The choice of a specific scheme S is a matter of convenience. In particular, the Fourier transform

$$\tilde{\Psi}(x,b)=\frac{1}{(2\pi)^2}\int\ e^{-ik_\perp b_\perp}\Psi(x,k_\perp)\ d^2k_\perp \tag{4.10}$$

to the impact parameter representation can also be treated[6] (at least, for small b) as a regularization scheme for the integral defining the distribution amplitude:

$$\varphi_\pi^{(F)}(x;\mu=1/b)=\frac{\sqrt{6}}{2\pi}\ \tilde{\Psi}(x,b);\quad b\to 0. \tag{4.11}$$

This observation suggests the extrapolation of the convolution formula into the nonperturbative region by substituting $\varphi(x;1/b)$ in Eq. (4.8) by the b-space wave function $\tilde{\Psi}(x,b)$ (see Ref. [21]). Since the $k_\perp$ effects are only essential when xQ^2 (i.e., x) is small, one can either use the original combination $\sqrt{x\bar{x}}Qb$ in the argument of the Born term $K_0(\sqrt{x\bar{x}}Qb)$ or substitute it by $\sqrt{x}Qb$. In particular, a modified version of the convolution formula (4.8) written in the $k_\perp$ representation,

$$F_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi^2\sqrt{6}}\int_0^1 dx\ \int\ \frac{\Psi(x,k_\perp)}{xQ^2+k_\perp^2}\ d^2k_\perp\ , \tag{4.12}$$

is the starting point of the analysis by Jakob *et al.* [21]. In this expression, a simpler form $xQ^2+k_\perp^2$ is used for the modified denominator of the ''hard'' quark propagator instead of the combination $xQ^2+k_\perp^2/\bar{x}$ which appears in our Eq. (3.13). However, since the difference is proportional to $k_\perp^2$ and vanishes for $x=0$, the two forms have essentially the same footing. As a model for $\Psi(x,k_\perp)$, Jakob *et al.* [21] use the ansatz [2] with the exponential dependence on the combination $k_\perp^2/x\bar{x}$ (or Gaussian dependence on $k_\perp$). We write it in a form similar to that used in Ref. [21]:

$$\Psi^{(G)}(x,k_\perp)=\frac{4\pi^2}{\sigma\sqrt{6}}\ \frac{\varphi_\pi(x)}{x\bar{x}}\ \exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right), \tag{4.13}$$

[5] In particular, speaking about the double-logarithmic behavior ''at large b'' we imply that b may be much larger numerically than $1/Q$ but is still within the PQCD applicability range.

[6] The basic difference between $\varphi_\pi(x;1/b)$ and $\tilde{\Psi}(x,b)$ is that $\int_0^1\varphi_\pi(x;\mu)dx$ is given by the same constant f_π for any μ while $\int_0^1\tilde{\Psi}(x,b)dx$ in general depends on b.

where σ is the width parameter and $\varphi_\pi(x)$ is the desired pion distribution amplitude.[7] In the $b_\perp$ representation, the model wave function is

$$\tilde{\Psi}^{(G)}(x,b_\perp)=\frac{2\pi}{\sqrt{6}}\,\varphi_\pi(x)\exp\left(-\frac{1}{2}\,b_\perp^2\sigma x\bar{x}\right). \quad (4.14)$$

The model is restricted by two conditions taken from Ref. [2]. First, the two-body Fock component of the pion light-cone wave function $\Psi(x,k_\perp)$ is required to satisfy the constraint

$$\int_0^1 dx \int \Psi(x,k_\perp)\,\frac{d^2k_\perp}{16\pi^3}=\frac{f_\pi}{2\sqrt{6}} \quad (4.15)$$

imposed by the $\pi\to\mu\nu$ rate. This gives the usual normalization condition for the pion DA

$$\int_0^1 \varphi_\pi(x)\;dx=f_\pi. \quad (4.16)$$

The second condition specifies the value of the x integral of $\Psi(x,k_\perp)$ at zero transverse momentum

$$\int_0^1 \Psi(x,k_\perp=0)\;dx=\frac{\sqrt{6}}{f_\pi}. \quad (4.17)$$

For the model ansatz (4.13), this condition results in the following constraint for the I_0 integral:

$$I_0\equiv\frac{1}{f_\pi}\int_0^1 \varphi_\pi(x)\,\frac{dx}{x}=\frac{3\sigma}{s_0}. \quad (4.18)$$

In obtaining Eq. (4.18), we incorporated the symmetry property $\varphi_\pi(x)=\varphi_\pi(\bar{x})$ of the pion DA and used again the notation s_0 for the important combination $4\pi^2f_\pi^2$. Since $I_0^{\rm as}=3$ and $I_0^{\rm CZ}=5$, the width parameters are $\sigma^{\rm as}=s_0\approx0.67\ \mathrm{GeV}^2$ and $\sigma^{\rm CZ}=\frac{5}{3}s_0\approx1.11\ \mathrm{GeV}^2$.

In the form (4.17), the second condition was derived in Ref. [2] from the requirement that the $\pi^0\to\gamma\gamma$ decay rate [or $F_{\gamma^*\gamma\pi^0}(Q^2=0)$ which is the same] calculated within the light-cone approach coincides with that given by the axial anomaly. It is easy to see, however, that in the $Q^2\to0$ limit, the $k_\perp$ integral in Eq. (4.12) logarithmically diverges in the small-$k_\perp$ region for any function which is nonvanishing at $k_\perp=0$. Note, that $\Psi(x,k_\perp=0)$ cannot vanish if we wish to satisfy the condition (4.17). Rather ironically, the condition which presumably should secure the correct value for $F_{\gamma^*\gamma\pi^0}(Q^2)$ at $Q^2=0$ guarantees instead that the extrapolation formula diverges at that point. This gives a clear warning that one should be very careful using the simplest extrapolation: it is difficult to judge *a priori* how reliably the formula failing for $Q^2=0$ models the subasymptotic effects for moderate Q^2. The authors of Ref. [21] also include the Sudakov exponential in which they take a symmetric combination $s(\bar{x}Qb)+s(xQb)$. As noted earlier, our one-loop calculation in Sec. III B shows that for $F_{\gamma^*\gamma\pi^0}(Q^2)$ one should use $s(\sqrt{x}Qb)$ instead of $s(xQb)$. Our final observation is that expanding Eq. (4.12) in $k_\perp^2/Q^2$ one would get an infinite series of power corrections under the x integral. According to our general result, the handbag diagram should not produce a chain of higher-twist contributions. Hence, the extrapolation formula cannot be interpreted simply as a transverse-momentum-corrected expression for the handbag diagram.

C. Transverse momentum in the light-cone formalism

Another attempt to model the subasymptotic corrections was made in Ref. [26]. It is based on the Brodsky-Lepage formula [1] for the two-body (i.e., $\bar{q}q$) contribution to the $\gamma^*\gamma\pi^0$ form factor in the light-cone formalism:

$$(\epsilon_\perp\times q_\perp)F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi^2\sqrt{6}}\int_0^1 dx\int\frac{[\epsilon_\perp\times(xq_\perp+k_\perp)]}{(xq_\perp+k_\perp)^2-i\epsilon}\times\Psi(x,k_\perp)\;d^2k_\perp. \quad (4.19)$$

Here, $q_\perp$ is a two-dimensional vector in the transverse plane satisfying $q_\perp^2=Q^2$, $\epsilon_\perp$ is a vector orthogonal to $q_\perp$ and also lying in the transverse plane [1], and the cross denotes the vector product. Again, the wave function is chosen in the Gaussian form (4.13) satisfying the constraints[8] (4.15) and (4.17). Though the integrand of Eq. (4.19) looks rather singular, there are no problems with the convergence of the $k_\perp$ integral in the $q_\perp\to0$ limit. The result is finite, since

$$\left.\frac{q_\perp^\alpha+k_\perp^\alpha}{(q_\perp+k_\perp)^2-i\epsilon}\right|_{q_\perp\to0}=\pi\;\delta^2(k_\perp)\;q_\perp^\alpha \quad (4.20)$$

for any test function $\Psi(x,k_\perp)$ which depends on $k_\perp$ through $k_\perp^2$. Because of the $\delta^2(k_\perp)$ function, the $Q^2=0$ result is determined by the wave function at zero transverse momentum.

In Ref. [26], it is claimed that the $k_\perp/Q$ expansion of Eq. (4.19) produces large ''higher-twist'' corrections to the leading-twist result. In fact, when $\Psi(x,k_\perp)$ has an exponential $k_\perp^2$ dependence, it is trivial to calculate the $k_\perp$ integral explicitly:

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[7]In the original model [2] $k_\perp^2$ appears in the combination $k_\perp^2+M_q^2$, where M_q is the constituent quark mass. As a result, the distribution amplitude $\varphi_\pi(x)$ is exponentially suppressed as $\exp[-M_q^2/2\sigma x\bar{x}]$ in the end-point regions. Jakob *et al.*, however, follow Chibisov and Zhitnitsky [63] who insist that the constituent quark mass M_q should not appear in QCD-motivated models for $\Psi(x,k_\perp)$. In particular, M_q does not appear in the model wave function $\Psi^{(\rm LD)}(x,k_\perp)$ [23] based on local quark-hadron duality (see Sec. V below): only the current quark masses m_q (usually set to zero for u and d quarks) are present in QCD Feynman integrals.

[8]As emphasized recently by Kroll [51], Cao *et al.* use constituent quark masses $M_q\sim330$ MeV which produces a strong exponential suppression $\exp[-M_q^2/2\sigma x\bar{x}]$ of the end-point regions. As a result, the I-integral for the DA corresponding to their ''CZ'' model is 3.71 rather than 5, i.e., despite zero at $x=1/2$, such a model gives a rather narrow DA, which is closer in this sense to the asymptotic DA rather than to the original CZ one.

$$F^{\overline{q}q}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{4\pi}{3}\int_0^1\frac{\varphi_\pi(x)}{xQ^2}\left[1-\exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right]dx \qquad (4.21)$$

to see that the correction term in the integrand of Eq. (4.21) has an exponentially decreasing rather than a power behavior for large Q^2. This result agrees with our general statement that the handbag diagram contains no higher-twist contributions. Our analysis works in this case since the Brodsky-Lepage formula (4.19) corresponds to the handbag contribution written in the light-cone variables without any approximation. Just as in the covariant treatment, the naively expected series of power corrections $(\langle k_\perp^2\rangle/Q^2)^n$ does not appear because the expansion of

$$\frac{xq_\perp+k_\perp}{(xq_\perp+k_\perp)^2} \qquad (4.22)$$

contains only traceless combinations. Indeed, multiplying Eq. (4.22) by $q_\perp/Q^2$ and defining $(k_\perp q_\perp)=|k_\perp|Q\cos\phi$, we obtain

$$\left(\frac{1}{Q^2}\right)\frac{xQ^2+|k_\perp|Q\cos\phi}{x^2Q^2+2x|k_\perp|Q\cos\phi+k_\perp^2}=\frac{1}{xQ^2}\left\{\theta(|k_\perp|<xQ)+\sum_{n=1}^{\infty}(-1)^n\left[\left(\frac{|k_\perp|}{xQ}\right)^n\theta(|k_\perp|<xQ)-\left(\frac{xQ}{|k_\perp|}\right)^n\theta(|k_\perp|>xQ)\right]\cos(n\phi)\right\}. \qquad (4.23)$$

For a wave function $\Psi(x,k_\perp)$ depending on $k_\perp$ through $k_\perp^2$ only, all the oscillating terms proportional to $\cos(n\phi)$ [i.e., to Chebyshev polynomials $T_n(\cos\phi)$ corresponding to traceless combinations in two dimensions] vanish after the angular integration. Only the $n=0$ term written outside the sum over n gives a nonzero result. Hence, for the wave functions of $\Psi(x,k_\perp)=\psi(x,k_\perp^2)$ type, we can write

$$F^{\overline{q}q}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{2}{\pi\sqrt{6}}\int_0^1\frac{dx}{xQ^2}\int_0^{xQ}\psi(x,k_\perp^2)\;k_\perp dk_\perp\,. \qquad (4.24)$$

This means that the leading $1/xQ^2$ term in Eq. (4.21) comes from the integral over all $k_\perp$'s while the exponential correction appears because the integration region in Eq. (4.24) is restricted by $k_\perp<xQ$. Another subtlety is that the $Q^2=0$ value

$$F^{\overline{q}q}(Q^2=0)=\frac{1}{2\pi f_\pi}$$

dictated by Eqs. (4.17) and (4.20) [and manifest in Eq. (4.21)] gives only a half of what is needed to get the correct $\pi^0\to\gamma\gamma$ rate (4.1). As explained in Ref. [2], the other half comes from the term which can be interpreted as the contribution of the $\overline{q}q\gamma$ Fock component of the pion wave function. In a formal PQCD diagrammatics, this contribution is represented by graphs containing the gluons coupling to the quark line between the photon vertices. For high Q^2, such diagrams correspond to higher-twist corrections associated with the $\overline{q}G\cdots Gq$ operators. In this sense, the result of Ref. [2] is equivalent to a nonperturbative constraint on the $Q^2\to 0$ limit of such contributions. One can expect that the $\overline{q}q\gamma$ contribution decreases as $1/Q^4$ or faster for large Q^2 since it contains higher twists only. Interpretation of this contribution in terms of the $\overline{q}q\gamma$ Fock component is restricted to the case of real γ: Ref. [2] gives no expression beyond the $Q^2=0$ point. In Ref. [26] this contribution is not included. However, if the terms which double the result for $Q^2=0$ are not included, it is premature to make specific quantitative statements about the size of subasymptotic corrections in the region of moderate Q^2.

We may also wonder *why* the formulas (4.12) and (4.19) corresponding to two attempts to include the primordial transverse momentum have such a strikingly different analytic structure. In particular, the denominator of the integrand of Eq. (4.19) vanishes for $k_\perp=-xq_\perp$ while that of Eq. (4.12) is finite for all $k_\perp$ provided that $q_\perp\neq 0$. The answer is very simple: the two expressions imply two different definitions of what is longitudinal and what is transverse. Equation (4.12) is based on the Sudakov decomposition in which the momentum q_1 of the real photon has only the light-cone "plus" component while the momentum p of the pion has only the light-cone "minus" component. As a result, the momentum transfer $q_2=p-q_1$ in the Sudakov variables is purely longitudinal and has both plus and minus components, with $q_2^2=-2(q_1p)$. On the other hand, the Brodsky-Lepage formula corresponds to the infinite momentum frame in which the plus components of q_1 and p coincide. The plus component of the momentum transfer q_2 vanishes in this frame, but q_2 has a nonzero transverse component $q_\perp$, with $|q_\perp|=Q$ or $q_2^2=-q_\perp^2$. Evidently, the two frames cannot be obtained from one another by a boost. Furthermore, one should not expect a diagram by diagram correspondence between the two approaches. The main purpose of imposing the requirement $q_2^+=0$ in the light-cone approach is to avoid the Z graphs. However, in Sudakov variables (and in any approach in which q_2 has a nonzero plus component) the Z graphs should be added to reproduce the light-cone result (see [64]).

Both the approaches [21,26] discussed above fail to reproduce the $Q^2=0$ value corresponding to the axial anomaly. Our point of view is that complying with the anomaly constraint should be a minimal requirement for any model of subasymptotic effects in the $\gamma^*\gamma\pi^0$ form factor. A maximalist attitude is that such a fundamental constraint should be satisfied automatically rather than imposed as an external condition. This can be only realized in an approach which is directly related to QCD and produces anomaly as a consequence of QCD dynamics.

V. QUARK-HADRON DUALITY AND EFFECTIVE WAVE FUNCTION

A. QCD sum rule calculation of f_π and local duality

QCD sum rules provide us with the approach which deals both with perturbative and nonperturbative aspects of QCD. The basic idea of the QCD sum rule approach [65] proposed by Shifman, Vainshtein, and Zakharov (SVZ) is the quark-hadron duality, i.e., the possibility to describe one and the same object in terms of either quark, gluon, or hadronic fields. To get information about the pion, the QCD sum rule practitioners usually analyze correlators involving the axial vector current. In particular, to calculate f_π one should consider the $p_\mu p_\nu$-part of the correlator of two axial vector currents

$$\Pi^{\mu\nu}(p)=i\int e^{ipx}\langle 0|T\,(j_{5\mu}(x)\ j_{5\nu}(0)\,)|\,0\rangle\ d^4x$$
$$=p_\mu p_\nu \Pi_2(p^2)-g_{\mu\nu}\Pi_1(p^2). \tag{5.1}$$

The dispersion relation

$$\Pi_2(p^2)=\frac{1}{\pi}\int_0^\infty \frac{\rho(s)}{s-p^2}ds+\text{``subtractions''} \tag{5.2}$$

represents $\Pi_2(p^2)$ as an integral over hadronic spectrum with the spectral density $\rho^{\rm hadron}(s)$ determined by projections

$$\langle 0|j_{5\mu}(0)|\pi;P\rangle=if_\pi P_\mu, \tag{5.3}$$

etc., of the axial current onto hadronic states

$$\rho^{\rm hadron}(s)=\pi f_\pi^2\delta(s-m_\pi^2)+\pi f_{A_1}^2\delta(s-m_{A_1}^2)$$
$$+\text{``higher states''} \tag{5.4}$$

($f_\pi^{\rm exp}\approx 130.7$ MeV in our normalization). On the other hand, when the probing virtuality is negative and large, one can use the operator product expansion

$$\Pi_2(p^2)=\Pi_2^{\rm quark}(p^2)+\frac{A}{p^4}\langle\alpha_s GG\rangle+\frac{B}{p^6}\alpha_s\langle\overline{q}q\rangle^2+\cdots, \tag{5.5}$$

where $\Pi_2^{\rm quark}(p^2)$ is the perturbative version of $\Pi_2(p^2)$ given by a sum of PQCD Feynman diagrams while the condensate terms $\langle GG\rangle$, $\langle\overline{q}q\rangle$, etc. [with perturbatively calculable coefficients A,B, see Eq. (5.10) below], describe or parametrize the nontrivial structure of the QCD vacuum. For the quark amplitude $\Pi_2^{\rm quark}(p^2)$, one can also write down the dispersion relation (5.2), with $\rho(s)$ substituted by its perturbative analogue $\rho^{\rm quark}(s)$:

$$\rho^{\rm quark}(s)=\frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}+\cdots\right) \tag{5.6}$$

(we neglect light quark masses). Hence, for large $-p^2$, one can write

$$\frac{1}{\pi}\int_0^\infty \frac{\rho^{\rm hadron}(s)-\rho^{\rm quark}(s)}{s-p^2}ds\ =\frac{A}{p^4}\langle\alpha_s GG\rangle+\frac{B}{p^6}\alpha_s\langle\overline{q}q\rangle^2+\cdots. \tag{5.7}$$

This expression essentially states that the condensate terms describe the difference between the quark and hadron spectra. At this point, using the known values of the condensates, one can try to construct a model for the hadronic spectrum. In the axial vector current channel, one has an infinitely narrow pion peak $\rho_\pi=\pi f_\pi^2\delta(s-m_\pi^2)$, a rather wide peak at $s\approx 1.6\ {\rm GeV}^2$ corresponding to A_1 and then a ''continuum'' at higher energies. The simplest approximation is to treat A_1 also as a part of the continuum, i.e., to use the model

$$\rho^{\rm hadron}(s)\approx\pi f_\pi^2\delta(s-m_\pi^2)+\rho^{\rm quark}(s)\ \theta(s\geqslant s_0), \tag{5.8}$$

in which all the higher resonances including the A_1 are approximated by the quark spectral density starting at some effective threshold s_0. Neglecting the pion mass and requiring the best agreement between the two sides of the resulting sum rule

$$\frac{f_\pi^2}{p^2}=\frac{1}{\pi}\int_0^{s_0}\frac{\rho^{\rm quark}(s)}{s-p^2}ds\ +\frac{A}{p^4}\alpha_s\langle GG\rangle+\frac{B}{p^6}\alpha_s\langle\overline{q}q\rangle^2+\cdots \tag{5.9}$$

in the region of large p^2, we can fit the remaining parameters f_π and s_0 characterizing the model spectrum. In practice, the more convenient SVZ-borelized version [65] of this sum rule

$$f_\pi^2=\frac{1}{\pi}\int_0^{s_0}\rho^{\rm quark}(s)e^{-s/M^2}ds\ +\frac{\alpha_s\langle GG\rangle}{12\pi M^2}+\frac{176\pi\alpha_s\langle\overline{q}q\rangle^2}{81M^4}+\cdots \tag{5.10}$$

is used for actual fitting. Using the standard values for the condensates $\langle GG\rangle$, $\langle\overline{q}q\rangle^2$, the scale s_0 is adjusted to get an (almost) constant result for the right-hand side of Eq. (5.10) starting with the minimal possible value of the SVZ-Borel parameter M^2. The magnitude of f_π extracted in this way, is very close to its experimental value $f_\pi^{\rm exp}\approx 130$ MeV.

Of course, changing the values of the condensates, one would get the best stability for a different magnitude of the effective threshold s_0, and the resulting value of f_π would also change. There exists an evident correlation between the values of f_π and s_0 since, in the $M^2\to\infty$ limit, the sum rule reduces to the local duality relation

$$f_\pi^2=\frac{1}{\pi}\int_0^{s_0}\rho^{\rm quark}(s)\ ds. \tag{5.11}$$

Using the explicit lowest-order expression $\rho_0^{\rm quark}(s)=1/4\pi$, we get

$$s_0=4\pi^2 f_\pi^2. \tag{5.12}$$

Note that $s_0=4\pi^2 f_\pi^2$ coincides with the combination which appears in the Brodsky-Lepage interpolation formula (4.3).

B. Quark-hadron duality for the $F_{\gamma^*\gamma^*\pi^0}(Q^2)$ form factor

Information about the $\gamma^*\gamma^*\to\pi^0$ form factor can be extracted from the three-point correlation function [66]

$$\mathcal{F}_{\alpha\mu\nu}(q_1,q_2)=\frac{4\pi}{i\sqrt{2}}\int d^4x\; d^4y\;\; e^{-iq_1x-iq_2y}\times\langle 0|T\{J_\mu(x)J_\nu(y)\; j_{5\alpha}(0)\}|0\rangle \tag{5.13}$$

calculated in the region where all the virtualities $q_1^2\equiv -q^2$, $q_2^2\equiv -Q^2$ and $p^2=(q_1+q_2)^2$ are spacelike.

The form factor $F_{\gamma^*\gamma^*\pi^0}(q^2,Q^2)$ appears in the invariant amplitude $F(p^2,q^2,Q^2)$ corresponding to the tensor structure $\epsilon_{\mu\nu\rho\sigma}p_\alpha q_1^\rho q_2^\sigma$. The dispersion relation for the three-point amplitude

$$F(p^2,q^2,Q^2)=\frac{1}{\pi}\int_0^\infty \frac{\rho(s,q^2,Q^2)}{s-p^2}\,ds+\;\;\text{``subtractions''} \tag{5.14}$$

specifies the relevant spectral density $\rho(s,q^2,Q^2)$. For the hadronic spectrum we assume again the ''first resonance plus perturbative continuum'' ansatz

$$\rho^{\text{hadron}}(s,q^2,Q^2)=\pi f_\pi F_{\gamma^*\gamma^*\pi^0}(q^2,Q^2)\,\delta(s-m_\pi^2)+\theta(s>s_0)\;\rho^{\text{quark}}(s,q^2,Q^2). \tag{5.15}$$

The lowest-order perturbative spectral density $\rho^{\text{quark}}(s,q^2,Q^2)$ is given by the Feynman parameter representation

$$\rho^{\text{quark}}(s,q^2,Q^2)=2\int_0^1 \delta\!\left(s-\frac{q^2x_1x_3+Q^2x_2x_3}{x_1x_2}\right)\times\delta\!\left(1-\sum_{i=1}^{3}x_i\right)dx_1dx_2dx_3. \tag{5.16}$$

Scaling the integration variables: $x_1+x_2=y$, $x_2=xy$, $x_1=(1-x)y\equiv\bar{x}y$ and taking trivial integrals over x_3 and y, we get

$$\rho^{\text{quark}}(s,q^2,Q^2)=2\int_0^1 \frac{x\bar{x}(xQ^2+\bar{x}q^2)^2}{[sx\bar{x}+xQ^2+\bar{x}q^2]^3}\,dx. \tag{5.17}$$

The variable x here can be treated as the light-cone fraction of the pion momentum p carried by one of the quarks. In particular, the denominator of the integrand in Eq. (5.17) is related to that of the hard quark propagator: $(q_1-xp)^2=-(xQ^2+\bar{x}q^2+sx\bar{x})$.

Putting one photon on shell, $q^2=0$, we can easily calculate the x integral:

$$\rho^{\text{quark}}(s,q^2=0,Q^2)=2\int_0^1 \frac{x\bar{x}(xQ^2)^2}{[sx\bar{x}+xQ^2]^3}\,dx\;=\frac{Q^2}{(s+Q^2)^2}. \tag{5.18}$$

This result explicitly shows that if the larger virtuality Q^2 also tends to zero, the spectral density $\rho^{\text{quark}}(s,Q^2)$ becomes narrower and higher, approaching $\delta(s)$ in the $Q^2\to 0$ limit (see [67]). Thus, the perturbative triangle diagram dictates that two real photons can produce only a single massless pseudoscalar state: there are no other states in the spectrum of final hadrons (see [68]). As Q^2 increases, the spectral function broadens, i.e., higher states can also be produced.

A detailed study of the QCD sum rule for the $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor was performed in Refs. [22,24]. The results of this investigation are rather close to those based on the simple local quark-hadron duality ansatz:

$$F^{\text{LD}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi f_\pi}\int_0^{s_0}\rho^{\text{quark}}(s,Q^2)\;ds. \tag{5.19}$$

Using the explicit expression for $\rho^{\text{quark}}(s,Q^2)$, we can write

$$F^{\text{LD}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{2}{\pi f_\pi}\int_0^1 dx\int_0^{s_0}\frac{x\bar{x}(xQ^2)^2}{[sx\bar{x}+xQ^2]^3}\,ds =\frac{1}{\pi f_\pi(1+Q^2/s_0)}. \tag{5.20}$$

This result coincides with the Brodsky-Lepage interpolation formula (4.3).

C. Effective wave function

The formulas based on the local quark-hadron duality prescription can be interpreted in terms of the effective two-body light-cone wave function [23]. Consider the lowest-order perturbative spectral density for the two-point correlator. It can be written as the Cutkosky-cut quark loop integral

$$\rho^{\text{quark}}(s)=\frac{3}{2\pi^2}\int \frac{k_+}{p_+}\left(1-\frac{k_+}{p_+}\right)\theta(k_+)\;\;\delta(k^2)\times\theta(p_+-k_+)\,\delta[(p-k)^2]d^4k \tag{5.21}$$

where $s\equiv p^2$. Introducing the light-cone variables for p and k,

$$p=\{p_+\equiv P,p_-=s/P,p_\perp=0\};\qquad k=\{k_+\equiv xP,k_-,k_\perp\},$$

and integrating over k_-, we get

$$\rho^{\text{quark}}(s)=\frac{3}{2\pi^2}\int_0^1 dx\int\;\delta\!\left(s-\frac{k_\perp^2}{x\bar{x}}\right)\,d^2k_\perp\,. \tag{5.22}$$

The δ function here expresses the fact that the light-cone combination $k_\perp^2/x\bar{x}$ coincides with $s\equiv p^2$, the invariant mass of the $\bar{q}q$ pair. Substituting this expression for $\rho^{\text{quark}}(s)$ into the local duality formula (5.11), we obtain the following representation for f_π^2:

$$f_\pi^2=\frac{3}{2\pi^3}\int_0^1 dx\int\ \theta(k_\perp^2\leqslant x\bar{x}s_0)\ d^2k_\perp\,. \qquad (5.23)$$

It has a structure similar to the expression for f_π in the light-cone formalism [1] [cf. Eq. (4.15)]:

$$f_\pi=\sqrt{6}\ \int_0^1 dx\int\ \Psi(x,k_\perp)\ \frac{d^2k_\perp}{8\pi^3}. \qquad (5.24)$$

To cast the local duality result (5.23) into the form of Eq. (5.24), we introduce the ''local duality'' wave function for the pion:

$$\Psi^{\mathrm{LD}}(x,k_\perp)=\frac{2\sqrt{6}}{f_\pi}\ \theta(k_\perp^2\leqslant x\bar{x}s_0). \qquad (5.25)$$

The specific form dictated by the local duality implies that $\Psi^{\mathrm{LD}}(x,k_\perp)$ simply imposes a sharp cutoff at $k_\perp^2 x\bar{x}=s_0$. In the $b_\perp$ space, the effective wave function can be written as

$$\tilde{\Psi}^{\mathrm{LD}}(x,b_\perp)=\frac{\sqrt{6}}{\pi f_\pi b_\perp}\sqrt{x\bar{x}s_0}\ J_1(b_\perp\sqrt{x\bar{x}s_0}), \qquad (5.26)$$

where $J_1(z)$ is the Bessel function.

D. Effective wave function and $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor

Consider now the local duality expression (5.20) for $F_{\gamma^*\gamma\pi^0}(Q^2)$. Replacing s, the invariant mass of the $\bar{q}q$ pair, by its light-cone equivalent $k_\perp^2/x\bar{x}$, we get $F^{\mathrm{LD}}_{\gamma^*\gamma\pi^0}(Q^2)$ as an integral over the longitudinal momentum fraction x and the transverse momentum $k_\perp$:

$$F^{\mathrm{LD}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{2}{\pi^2 f_\pi}\int_0^1 dx\ \int\ \frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\times\theta(k_\perp^2\leqslant x\bar{x}s_0)\ d^2k_\perp\,. \qquad (5.27)$$

Now, introducing the effective wave function $\Psi^{\mathrm{LD}}(x,k_\perp)$ given by Eq. (5.25), we write $F^{\mathrm{LD}}(Q^2)$ in the ''light-cone form''

$$F^{\mathrm{LD}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi^2\sqrt{6}}\int_0^1 dx\ \int\ \frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\times\Psi^{\mathrm{LD}}(x,k_\perp)\ d^2k_\perp\,. \qquad (5.28)$$

In the impact parameter representation, this formula is

$$F^{\mathrm{LD}}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{2\pi\sqrt{6}}\int_0^1 dx\int\ xQ^2b^2\times K_2(\sqrt{x}bQ)\tilde{\Psi}^{\mathrm{LD}}(x,b_\perp)\ d^2b_\perp\,. \qquad (5.29)$$

The function $K_2(\sqrt{x}bQ)$, where $K_2(z)$ is the modified Bessel function, originates from the new version of the Born term written in the b space

$$\tilde{B}(x;bQ)\equiv\frac{1}{2\pi}\int\ e^{-ik_\perp b_\perp}\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\ d^2k_\perp=\frac{1}{4}\ xQ^2b^2K_2(\sqrt{x}bQ). \qquad (5.30)$$

Note that $\tilde{B}(x;bQ)$ is finite for $b=0$: $\tilde{B}(x;0)=1$ while the ''old'' Born term $B(x;bQ)=\bar{x}K_0(\sqrt{x\bar{x}}bQ)$ (3.13) has a logarithmic singularity at the origin of the b space. The expression (5.28) looks similar to the extrapolation formula (4.12). Furthermore, since

$$\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}=\frac{1}{xQ^2+k_\perp^2}-\frac{2k_\perp^2}{(xQ^2+k_\perp^2)^2}+\frac{k_\perp^4}{(xQ^2+k_\perp^2)^3}, \qquad (5.31)$$

the two $k_\perp$ modifications of the hard quark propagator $1/xQ^2$ differ only by $O(k_\perp^2)$ terms invisible in the analysis of effects induced by the $1/k_\perp^2$ singularity at small $k_\perp$. However, this difference is very essential when one extrapolates into the region of small Q^2. To demonstrate this, let us analyze Eq. (5.28) in some particular limits. For real photons, using the fact that

$$\frac{\mu^4}{(\mu^2+k_\perp^2)^3}\to\frac{1}{2}\ \delta(k_\perp^2) \qquad (5.32)$$

in the $\mu^2\to 0$ limit, we obtain that the $\pi^0\to\gamma\gamma$ decay rate is determined by the magnitude of the LD wave function at zero transverse momentum:

$$F^{\mathrm{LD}}_{\gamma^*\gamma\pi^0}(0)=\frac{1}{2\pi\sqrt{6}}\int_0^1\ \Psi^{\mathrm{LD}}(x,k_\perp=0)dx. \qquad (5.33)$$

This requirement is similar to that in the Brodsky-Lepage formalism. However, according to the explicit form (5.25) of $\Psi^{\mathrm{LD}}(x,k_\perp=0)$, the integral (5.33) is twice larger than the constraint (4.17) imposed on the valence $\bar{q}q$ light-cone wave function. As a result, the local duality formula exactly reproduces the $F_{\gamma^*\gamma\pi^0}(0)$ value (4.1) dictated by the axial anomaly. This outcome can be interpreted by saying that $\Psi^{\mathrm{LD}}(x,k_\perp)$ is an *effective* wave function (see [69]) describing the soft content of all $\bar{q}G\cdots Gq$ Fock components of the usual light-cone approach (see also [63]). Note, that higher-order radiative corrections to the perturbative spectral density $\rho^{\mathrm{quark}}(s,Q^2)$ are explicitly accompanied by the $\alpha_s(\mu_R^2)/\pi$ factors per each extra loop. After integration over the duality interval $0\leqslant s\leqslant s_0$, there are two physical scales: s_0 and Q^2. At low Q^2, the duality interval s_0 sets the scale at the low-momentum end of the UV-divergent integrals, hence, a natural choice for the normalization scale μ_R is $\mu_R^2\sim s_0$. At high Q^2, the short-distance dominated parts of the higher-order corrections should reproduce the PQCD results which suggest $\mu_R^2\sim Q^2$ for these terms. In any case, suppression by at least $\alpha_s(s_0)/\pi\sim 0.1$ per each extra loop is guaranteed. Since $s_0\gg\Lambda^2$, the gluonic corrections to $\rho^{\mathrm{quark}}(s,Q^2)$ are suppressed by powers of $\alpha_s(s_0)/\pi\sim 0.1$. In other words, the higher-order diagrams contributing to

$\rho^{\rm quark}(s,Q^2)$ correspond to exchange of hard gluons whose wave lengths are larger than $1/\sqrt{s_0}$.

When Q^2 is so large that the $k_\perp^2$ term can be neglected, we get the expression

$$F^{\rm LD}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi^2\sqrt{6}}\int_0^1\frac{dx}{xQ^2}\int \Psi^{\rm LD}(x,k_\perp)\, d^2k_\perp + O(1/Q^4). \quad (5.34)$$

Identifying the wave function integrated over the transverse momentum with the pion distribution amplitude

$$\varphi^{\rm LD}_\pi(x)\equiv\frac{\sqrt{6}}{(2\pi)^3}\int \Psi^{\rm LD}(x,k_\perp)\, d^2k_\perp = 6f_\pi x(1-x), \quad (5.35)$$

we obtain the lowest-order PQCD formula (2.11):

$$F_{\gamma^*\gamma\pi^0}(Q^2)|_{Q^2\to\infty}=\frac{4\pi}{3}\int_0^1\frac{\varphi_\pi(x)}{xQ^2}\,dx+O(1/Q^4) \quad (5.36)$$

for the large-Q^2 behavior of the $\gamma^*\gamma\to\pi^0$ transition form factor.

To summarize, the local duality formula (5.20) exactly reproduces the Brodsky-Lepage interpolation (4.3) between the $Q^2=0$ value $1/\pi f_\pi$ fixed by the axial anomaly and the leading large-Q^2 term $4\pi f_\pi/Q^2$ calculated for the asymptotic form of the pion distribution amplitude.

The application of the local duality ansatz in a general situation when both photons are virtual was discussed in Ref. [23]. The basic formula written in terms of the effective wave function is given by

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2)=\frac{1}{\pi f_\pi}\int_0^{s_0}\rho^{\rm quark}(s,q^2,\,Q^2)\,ds = \frac{2}{\pi f_\pi}\int_0^1 dx\int\frac{x\bar{x}(xQ^2+\bar{x}q^2)^2}{[k_\perp^2+xQ^2+\bar{x}q^2]^3} \times\Psi^{\rm LD}(x,k_\perp)\,d^2k_\perp . \quad (5.37)$$

For $q^2=Q^2=0$ it satisfies the anomaly constraint (4.1), while when both q^2 and Q^2 are large it reduces to the PQCD formula

$$F_{\gamma^*\gamma^*\pi^0}(Q^2)|_{q^2,Q^2\to\infty}=\frac{4\pi}{3}\int_0^1\frac{\varphi_\pi(x)}{xQ^2+\bar{x}q^2}\,dx+O(1/Q^4). \quad (5.38)$$

E. Extended local duality

Note, that the pion distribution amplitude (5.35) produced by the local duality prescription coincides with the asymptotic DA. To model wave functions corresponding to DA's different from $\varphi_\pi^{\rm as}(x)$, we propose to use the sharp cutoff analogue of the Gaussian model (4.13):

$$\Psi^{\rm (LD)}(x,k_\perp)=\frac{8\pi^2}{\sigma\sqrt{6}}\,\frac{\varphi_\pi(x)}{x\bar{x}}\,\theta(k_\perp^2\leqslant x\bar{x}\sigma), \quad (5.39)$$

where σ is again the width parameter and $\varphi_\pi(x)$ the desired DA, which satisfies the standard f_π-normalization constraint (4.16). To guarantee the anomaly result for the $\pi^0\to\gamma\gamma$ rate, we impose the following constraint on the x integral of $\Psi^{\rm (LD)}(x,k_\perp)$ at zero transverse momentum:

$$\int_0^1 \Psi^{\rm (LD)}(x,k_\perp=0)\,dx=\frac{2\sqrt{6}}{f_\pi}. \quad (5.40)$$

Substituting the model ansatz (5.39), we derive from this constraint the condition for the standard integral I_0

$$I_0\equiv\frac{1}{f_\pi}\int_0^1\varphi_\pi(x)\frac{dx}{x}=\frac{3\sigma}{s_0}, \quad (5.41)$$

where s_0 is the basic combination $s_0=4\pi^2f_\pi^2$. Taking $I_0^{\rm as}=3$ and $I_0^{\rm CZ}=5$, we fix the width parameters $\sigma^{\rm as}=s_0$ and $\sigma^{\rm CZ}=\frac{5}{3}s_0\approx1.11$ GeV^2. Note, that in the CZ calculation [43], the duality interval was 0.75 GeV^2 for the zeroth moment of the DA and 1.5 GeV^2 for the second one; our effective duality interval $\sigma^{\rm CZ}$ for the CZ-type DA appears to be the average of these two. Using the ansatz (5.39) in Eq. (5.28) and integrating over the transverse momentum, we obtain

$$F^{\rm LD}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{2\pi}{3}\int_0^1\frac{\varphi_\pi(x)}{x\bar{x}\sigma}\left[1-\frac{1}{(1+\bar{x}\sigma/Q^2)^2}\right]dx. \quad (5.42)$$

This formula has correct limits both for $Q^2=0$ and large Q^2. For the asymptotic distribution amplitude, Eq. (5.42) produces the expression (5.20) coinciding with the Brodsky-Lepage interpolation formula. For the Chernyak-Zhitnitsky DA we get

$$F^{\rm LD,CZ}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi f_\pi}\left\{\frac{1}{1+Q^2/\sigma}-\frac{2Q^2}{\sigma+Q^2} +12\frac{Q^4}{\sigma^2}\left[\left(1+\frac{2Q^2}{\sigma}\right)\ln\left(1+\frac{\sigma}{Q^2}\right)-2\right]\right\}. \quad (5.43)$$

Despite its apparent complexity, this expression is very close numerically to the simplest interpolation

$$F^{\rm int,CZ}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{1}{\pi f_\pi(1+Q^2/\sigma^{\rm CZ})} \quad (5.44)$$

between the anomaly value at $Q^2=0$ and the PQCD result $F^{\rm PQCD,\,CZ}_{\gamma^*\gamma\pi^0}(Q^2)=\frac{5}{3}(4\pi f_\pi/Q^2)$ calculated for the CZ distribution amplitude.

Thus, Eqs. (5.20), (5.44) model the modification of the basic I_0 integral by power corrections. On the other hand, the modification of I_0 by radiative corrections is described by Eqs. (2.27), (2.29). Though we obtained these two types of modifications in a completely independent way, it is tempting to combine them in a single expression. A self-consistent, but a rather time-consuming way to do this is to calculate the spectral density $\rho^{\rm quark}(s,Q^2)$ to two loops and apply the local duality prescription. Then both the radiative and power corrections would result from the same expression. We leave such a calculation for a future investigation.

In the absence of a completely unified approach, we can try to get an interpolating formula by combining the two independent calculations described above. A natural idea is to write all the one-loop diagrams in the b representation *a lá* modified factorization and then substitute $\varphi_\pi(x,1/b)$ by $\widetilde{\Psi}(x,b)$ and the Born factor $\overline{\xi}K_0(\sqrt{\xi\overline{\xi}}bQ)$ by the modified version $\frac{1}{4}\xi Q^2 b^2 K_2(\sqrt{\xi}bQ)$. This will give a more reliable behavior in the small-Q^2 region where the corrections are dominated by power terms. However, changing the structure of the Born factor would affect the radiative corrections and spoil the results at the high-Q^2 end, where one should exactly reproduce the PQCD results. Since the perturbative corrections are rather small, we expect that a self-consistent inclusion of radiative corrections should be rather close to a simple product of the nonperturbative $1/(1+Q^2/\sigma)$ factors and perturbative corrections from Eqs. (2.27), (2.29). Such a product gives

$$F^{\mathrm{as}}_{\gamma^*\gamma\pi^0}(Q^2)\approx\frac{1}{\pi f_\pi(1+Q^2/s_0)}\left\{1-\frac{5}{3}\frac{\alpha_s(Q^2)}{\pi}\right\} \tag{5.45}$$

for the asymptotic form of the pion DA, and

$$F^{\mathrm{CZ}}_{\gamma^*\gamma\pi^0}(Q^2)\approx\frac{1}{\pi f_\pi}\left\{\frac{1}{1+Q^2/s_0}\left\{1-\frac{5}{3}\frac{\alpha_s(Q^2)}{\pi}\right\}\left[1-\left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right)^{50/81}\right]+\frac{1}{1+\frac{3}{5}Q^2/s_0}\left\{1-\frac{49}{108}\frac{\alpha_s(Q^2)}{\pi}\right\}\left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right)^{50/81}\right\} \tag{5.46}$$

for the case when the pion DA $\varphi_\pi(x;\mu)$ coincides with $\varphi_\pi^{\mathrm{CZ}}(x)$ for $\mu=Q_0$. These expressions have necessary interpolating properties: in the absence of radiative corrections they coincide with the local duality expressions, while for large Q^2, when the power corrections can be ignored, they reproduce PQCD results. From Fig. 4, one can see that the curves for $F^{\mathrm{as}}_{\gamma^*\gamma\pi^0}(Q^2)$ and $F^{\mathrm{CZ}}_{\gamma^*\gamma\pi^0}(Q^2)$ (with $Q_0\approx0.5$ GeV [33]) in this model are sufficiently separated from each other which allows for an unambiguous experimental discrimination between them.

It is instructive to make a more detailed comparison of the relative size of perturbative $O(\alpha_s)$ and nonperturbative σ/Q^2 corrections. Taking $\Lambda=200$ MeV, we observe that the perturbative correction for the asymptotic DA changes the lowest-order result by $\lesssim30\%$ for $Q^2\gtrsim0.5$ GeV^2. This means that the PQCD expansion for the lowest-twist term in this case is self-consistent for Q^2 as low as 0.5 GeV^2. On the other hand, the power correction s_0/Q^2 exceeds 70% for all $Q^2\lesssim1$ GeV^2. This clearly indicates that PQCD results are not reliable below 1 GeV^2. To reduce the ratio s_0/Q^2 to the 20% level, one should take $Q^2\gtrsim3$ GeV^2. This is an illustration of the well-known statement (see, e.g., [65]) that reliability of simplest PQCD formulas is limited in first place by power corrections rather than by the increasing value of the QCD running coupling $\alpha_s(Q^2)$. The crucial fact here is that the scale $s_0\approx0.7$ GeV^2 determining the deviation from the PQCD $1/Q^2$ behavior is much larger than Λ^2. It is also much larger than other typical nonperturbative scales such as the square of the constituent quark mass $M_q^2\sim0.1$ GeV^2 or the average transverse momentum $\langle k_\perp^2\rangle$ [in the LD model (5.25), $\langle k_\perp^2\rangle^{\mathrm{LD}}=s_0/10\approx0.07$ GeV^2]. This observation can be easily explained by the fact that $k_\perp^2$ present in the modified Born term (5.28) is added to xQ^2 rather than to Q^2. This enhances the relative size of power corrections by a factor such as $1/\langle x\rangle$. In full accordance with the statements made in Refs. [52,53], the onset of the Q^2 region where the lowest-order PQCD result is reliable (in the sense that PQCD gives a good approximation) is determined by the size of the average virtuality xQ^2 of the ''hard'' quark. If its value is too small, PQCD is unreliable even if the effective coupling α_s is negligible and perturbation theory for the lowest-twist contribution is self-consistent.

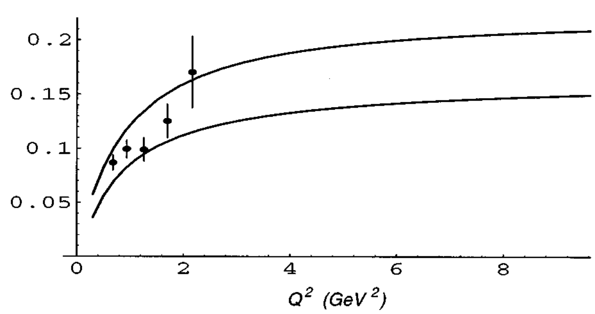

FIG. 4. Combination $\sqrt{2}Q^2F_{\gamma^*\gamma\pi^0}(Q^2)/4\pi$ (measured in GeV^2 and equivalent to $Q^2\widetilde{F}_{\gamma^*\gamma\pi^0}(Q^2)$, with the form factor $\widetilde{F}_{\gamma^*\gamma\pi^0}(Q^2)$ normalized according to definition adopted in Refs. [21,26,38] as a function of Q^2. The lower curve corresponds to our model with the asymptotic DA [Eq. (5.45)] and the upper one is based on Eq. (5.46). Data are taken from CELLO Collaboration publication [14]. Preliminary CLEO data [15] (not shown) are very close to the lower curve.

VI. CONCLUSIONS

In this paper, we discussed the status of QCD-based theoretical predictions for the $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor. As we repeatedly emphasized, in this case one deals with a rather favorable situation when QCD fixes both the $Q^2=0$ value (dictated by the axial anomaly) and the large-Q^2 behavior governed by perturbative QCD. Still, constructing a dynamically supported interpolation between the two limits, it is very important to adequately reproduce at moderate Q^2 the corrections to the asymptotic PQCD result, both perturbative and nonperturbative.

Working within the framework of the standard PQCD factorization approach (SFA), which allows one to unambiguously separate the contributions having different power-law behavior at large Q^2, we gave a detailed analysis of the one-loop coefficient function for the leading twist-two contribution. To explore the role of the transverse degrees of freedom, we wrote the relevant Feynman integrals in the Sudakov representation and showed how the SFA produces the basic building blocks of the modified factorization approach (MFA) [29], such as the Sudakov-type double logarithms $\ln^2(b)$ with respect to the impact parameter $b_\perp$ which is a Fourier conjugate to the transverse momentum $k_\perp$. The fact that we derived the Sudakov effects within the lowest-twist contribution of the SFA, explicitly demonstrates that they should not be confused with the higher-twist effects. In other words, though the Sudakov terms are given by integrals over $b_\perp$ (or $k_\perp$), they are purely perturbative and do not produce power corrections to the lowest-order PQCD result.

Furthermore, we observed that the power corrections $1/Q^2$ due to the intrinsic transverse momentum are rather elusive both within the OPE-type factorization and the light-cone approach of Brodsky and Lepage. Contrary to naive parton expectations, the simplest handbag-type diagram in both cases does not produce an infinite tower of $(1/Q^2)^n$ terms: such a series is generated by contributions corresponding to physical (transverse) gluons emitted from the hard propagator connecting the photon vertices. It goes without saying that an explicit summation of such terms is a formidable task in both of these approaches. A simpler picture emerges within the QCD sum rule approach in which the infinite sum over the soft parts of the $\bar{q}G\cdots Gq$ Fock components is dual to the $\bar{q}q$ states generated by the local axial current. An important observation establishing the connection between the QCD sum rule and light-cone approaches is that integrating the invariant mass s of the $\bar{q}q$ pair over the pion duality interval $0 \leqslant s \leqslant s_0$ is equivalent to using the effective two-body wave function $\Psi^{\mathrm{LD}}(x,k_\perp)$. The result obtained from the local quark-hadron duality (LD) ansatz applied to the lowest-order triangle diagram coincides with the Brodsky-Lepage interpolation formula [3], i.e., it reproduces both the $Q^2=0$ value specified by the axial anomaly and the high-Q^2 PQCD behavior with the normalization corresponding to the asymptotic distribution amplitude for the pion. To test the sensitivity to the shape of the pion distribution amplitude, we proposed a model for the effective wave function $\Psi^{\mathrm{LD}}(x,k_\perp)$ which reduces to the desired DA after the $k_\perp$ integration and still provides the correct limits for the form factor both at low and high Q^2.

In our analysis, the regions of small and large transverse momenta (responsible for power $1/Q^2$ and α_s corrections, respectively) were studied separately, within the frameworks of two different approaches. In spite of this, the basic results written in terms of the $k_\perp$ integrals look rather similar. A major challenge for a future study is the construction of a unified approach in which both the nonperturbative power-suppressed terms and the perturbative radiative corrections emerge from the expansion of the same expression. The quark-hadron duality approach provides a framework in which such a self-consistent unification is guaranteed. The *only* missing ingredient is the perturbative spectral density $\rho^{\mathrm{quark}}(s,Q^2)$ at the two-loop level.

There are two further improvements which should be made in the perturbative part of the problem. First, it is necessary to fix the argument of the running coupling constant α_s. In our analysis, we either left it unspecified and estimated the corrections assuming that $\alpha_s/\pi \approx 0.1$ or took $\Lambda = 200$ MeV in the one-loop expression for $\alpha_s(Q^2)$. However, for a precise comparison with experimental data, estimating the magnitude of the α_s correction one should explicitly specify the UV renormalization scheme, fix the parameter μ_R in the argument of the running coupling $\alpha_s(\mu_R)$, and use the proper value of the QCD scale Λ. A very effective scale-fixing prescription is provided by the Brodsky-Lepage-Mackenzie (BLM) approach [70]. To use the BLM prescription, one should calculate two-loop PQCD corrections to the coefficient function containing quark loop insertions into the gluon propagator. Another problem is the inclusion of the effects due to the two-loop evolution of the pion distribution amplitude [71–73]. Originally, the relevant corrections expanded in terms of a few lowest eigenfunctions of the one-loop kernel, were found to be tiny [6]. Recent progress [74] in understanding the structure of the two-loop evolution suggests that higher harmonics cannot be neglected, and the size of the two-loop evolution corrections is somewhat larger than estimated in [6]. However, our preliminary numerical estimates [75] of the effects due to the modified evolution developed in Ref. [76] do not indicate appreciable changes for the I integral.

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