Gauge technique for heavy quarks

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It is possible to determine an off-shell propagator for heavy quarks to order $1/m$ in mass and in any covariant gauge ξ, which applies universally to all the quarks, by using the gauge technique. Given a simple ansatz for the gluon propagator, the result for the leading behavior of the quark propagator is $S(v\cdot k)=\Gamma(1+2\alpha_\xi)[(1+\gamma\cdot v)/2v\cdot k](-v\cdot k/\Lambda)^{2\alpha_\xi}{}_0F_2[1+\alpha_\xi,3/2+\alpha_\xi;\,\alpha\xi(v\cdot k)^2/3\pi\Lambda^2]$, where v is the velocity of the particle containing the heavy quark, Λ is a QCD mass scale, and $\alpha_\xi\equiv-\alpha(2+\xi)/3\pi$. The above result is a totally reliable deduction from the assumed gluon propagator in the infrared limit ($\sim 1/q^2$) and accounts for soft-gluon corrections to the fermion in internal loops.
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I. INTRODUCTION

The QCD quark Lagrangian (N flavors) is endowed with a higher symmetry in the limit of equal quark velocity which applies even when the quark masses are different [1]. Thus it generalizes the old U(N)×U(N) supersymmetry [2] for the equal mass case. Provided that the momentum transfer to the gluons is not much greater than the QCD scale one can thereby deduce a number of relations between transition amplitudes, which seem to be borne out by experiment. It has become customary [3] to attribute a velocity v to the hadron containing a constituent heavy quark so that the momentum of the quark in a bound state is written $p=mv+k$, where k denotes the residual quark momentum, itself associated with the light material that makes up the hadron. As a result one can show that the ''free'' quark propagator in the $m\to\infty$ limit is simply given by $S=(1+\gamma\cdot v)/2v\cdot k$ and one can use this in subsequent leading order calculations of various matrix elements.

In this paper we would like to show that one can improve on S by taking account of soft-gluon corrections. The result of the dressing is to provide a propagator which contains the characteristic QCD scale Λ and which coincides with the free one in the limit of vanishing gluon coupling $\alpha=g^2/4\pi$. Thus this propagator applies just as well to all the quarks in the heavy mass limit [4] and does not jeopardize the prevailing higher symmetry. In order to derive it we use the ''gauge technique'' [5] for QCD, which is known to be a reliable method in the infrared and ultraviolet limit [6]. The technique itself was originally devised by Salam to provide a closed equation for the source propagator in QED, in a manner which respected the Ward-Takahashi identity, and it was later extended to non-Abelian gauge theories. It has gained wide currency today as a useful way of uncovering the nonperturbative behavior of Green's functions. In one version of the gauge technique, it produces a self-consistent equation for the quark spectral function in any gauge, from which the propagator follows [7]. In the next section we set out the velocity projector decomposition of the propagator. Next we derive the effective vertex for soft gluons and finally we solve the equation in question, obtaining the result quoted in the abstract; there we also compare the result with QED where the scale Λ is missing.

II. VELOCITY PROJECTIONS

When one substitutes $p=mv+k$ in the free quark propagator, the resulting expression,

$$S(p)=\frac{1}{m(\gamma\cdot v-1)+\gamma\cdot k}, \tag{1}$$

has to be taken in the limit $m\to\infty$ in order to discern the resulting (leading) dependence on four-velocity v (with $v^2=1$). Now any 4×4 quark matrix M for a particular flavor, like the propagator, can be decomposed into projections using $P_\pm=(1\pm\gamma\cdot v)/2$ according to

$$M=P_+M_{++}P_++P_+M_{+-}P_-+P_-M_{-+}P_+ \\ +P_-M_{--}P_- .$$

Equivalently, the M_{ij} sector is defined through $M_{ij}=P_iMP_j$. This has the effect of resolving the 4×4 matrix into four separate 2×2 matrices:

$$M\Rightarrow\begin{pmatrix} M_{++} & M_{+-}\\ M_{-+} & M_{--}\end{pmatrix}.$$

In this basis,

$$\gamma\cdot v\Rightarrow\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\qquad \gamma_\mu\Rightarrow\begin{pmatrix} v_\mu & \gamma_\mu\\ \gamma_\mu & -v_\mu\end{pmatrix}.$$

In particular the inverse free propagator resolves to

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$$S^{-1}(p)=m(\gamma\cdot v-1)+\gamma\cdot k\Rightarrow\begin{pmatrix} k\cdot v & \gamma\cdot k\\ \gamma\cdot k & -k\cdot v-2m\end{pmatrix}, \tag{2}$$

and correspondingly,

$$S(p)\Rightarrow\frac{1}{k^2+(k\cdot v)^2+2mk\cdot v}\begin{pmatrix} k\cdot v+2m & \gamma\cdot k\\ \gamma\cdot k & -k\cdot v\end{pmatrix}. \tag{3}$$

Thus, up to order $1/m^2$, the free S decomposes into

$$S(p)\Rightarrow\frac{1}{k\cdot v}\begin{pmatrix} 1-\dfrac{k^2}{2mk\cdot v} & \dfrac{\gamma\cdot k}{2m}\\ \dfrac{\gamma\cdot k}{2m} & -\dfrac{k\cdot v}{2m}\end{pmatrix}+O\left(\frac{1}{m^2}\right), \tag{4}$$

from which one infers that the leading large component is $S_{++}\sim 1/k\cdot v$.

Let us now consider the full propagator in a covariant gauge, which is best written in the Lehmann-Kallen-like spectral form for our purposes:

$$S(p)=\int\frac{\rho(W)dW}{\gamma\cdot p-W+i\eta\epsilon(W)},\qquad \int dW\equiv\left(\int_{-\infty}^{-m}+\int_{m}^{\infty}\right)dW, \tag{5}$$

for an ordinary sort of particle. In the heavy quark limit we anticipate that the negative energy cut is ''far away'' and that the main contribution from soft gluons will arise in the vicinity of $W=m$, to within a region of order Λ. The free propagator is of course obtained just by setting $\rho(W)=\delta(W-m)$ above. Put $W=m+\omega$, $p=mv+k$ and take velocity projections as in Eq. (4), to obtain

$$S(p)\Rightarrow\int\frac{\rho(\omega)d\omega}{2m(v\cdot k-\omega)+k^2-\omega^2}\times\begin{pmatrix} 2m+k\cdot v+\omega & \gamma\cdot k\\ \gamma\cdot k & -k\cdot v+\omega\end{pmatrix}. \tag{6}$$

We see that the dressed propagator is still dominated by its S_{++} component, which assumes a very simple form, $\int d\omega\rho(\omega)/(v\cdot k-\omega)$ despite the inclusion of QCD interactions. More generally, to order $1/m$, we get

$$S\Rightarrow\int\frac{\rho(\omega)d\omega}{v\cdot k-\omega}\begin{pmatrix} 1+\dfrac{(v\cdot k)^2-k^2}{2m(v\cdot k-\omega)} & \dfrac{\gamma\cdot k}{2m}\\ \dfrac{\gamma\cdot k}{2m} & \dfrac{\omega-v\cdot k}{2m}\end{pmatrix}+O\left(\frac{1}{m^2}\right).$$

III. APPLICATION OF THE GAUGE TECHNIQUE

The next stage involves the solution of the Dyson-Schwinger (DS) equation for the propagator, while taking cognizance of the (longitudinal) Ward-Takahashi identity,

$$(p-p')^{\mu}S(p')\Gamma_{\mu}(p',p)S(p)=S(p')-S(p), \tag{7}$$

because it can lead to a self-consistent equation for S, if we ignore certain transverse terms in the vertex Γ_{μ}. If one were to incorporate the transverse Takahashi identity [8] as well, one would in effect be solving the full field theory; but the transverse identity—in anything but two dimensions [9]—brings in other vertices leading to a system of equations which is actually not closed, unless one makes further drastic truncations [10]. Alternatively if one knew the full solution of the DS equation in any particular gauge, one would be able to determine it in any other gauge via the Landau-Khalatnikov-Zumino gauge covariance [11] relations.

We have none of these luxuries. The gauge technique does its best to solve Eq. (7) in the form stated, while making sure that the singularities in the nontruncated Green's function are properly included. It does not solve the inverse form of Eq. (7) because that would give a linear relation between Γ and S^{-1} and produce a difficult *nonlinear* equation for the inverse propagator; besides which, it is not obvious how to handle the heavy quark limit for the inverse propagator—which is dominated by its S^{-1}_{--} projection, conversely to S_{++}. The gauge technique [5] starts off by defining the obvious longitudinal solution to the vertex,

$$S(p')\Gamma^{\parallel}_{\mu}(p',p)S(p)\equiv\int dW\rho(W)\frac{1}{\gamma\cdot p'-W}\gamma_{\mu}\frac{1}{\gamma\cdot p-W}, \tag{8}$$

as a weighted mass integral. One can readily check that Eq. (8) obeys Eq. (7) automatically, but of course the full vertex must include transverse additions $\Gamma^{\perp}$; these are unknown unless one has some knowledge about them through perturbation theory [12] or examines equations for higher-order Green's functions [13] or makes use of the transverse identity [8], which is essentially equivalent. It is worth pointing out that such transverse terms are soft, vanishing with the vector meson momentum. For that reason the gauge technique is a clearly reliable tool in the infrared limit, though it is also gauge covariant in the ultraviolet regime [6] as it happens; it is only at intermediate energies that transverse corrections to Eq. (8) play an important role.

Returning to heavy quarks, let us expand solution (8) in powers of $1/m$ by writing it as

$$S(p')\Gamma^{\parallel}_{\mu}(p',p)S(p)=\int d\omega\rho(\omega)\frac{1}{m(\gamma\cdot v-1)+\gamma\cdot k'-\omega}\gamma_{\mu}\frac{1}{m(\gamma\cdot v-1)+\gamma\cdot k-\omega} \tag{9}$$

and taking velocity projections

$$\Rightarrow\int\frac{\rho(\omega)d\omega}{(v\cdot k'-\omega)(v\cdot k-\omega)}\begin{pmatrix} v_\mu\left[1+\frac{(v\cdot k')^2-k'^2}{2m(v\cdot k'-\omega)}+\frac{(v\cdot k)^2-k^2}{2m(v\cdot k-\omega)}\right]+\frac{\gamma\cdot k'\gamma_\mu+\gamma_\mu\gamma\cdot k}{2m} & \frac{v_\mu\gamma\cdot k+\gamma_\mu(\omega-k\cdot v)}{2m} \\ \frac{\gamma\cdot k'\cdot v_\mu+\gamma_\mu(\omega-k'\cdot v)}{2m} & 0 \end{pmatrix} \tag{10}$$

up to order $1/m^2$. Note that representations (8) and (10) are *exact* for small $p-p'$ in the complete Green's function (7) for the reasons we have already given.

The next step is to use approximation (8) in the DS equation, which we write in the renormalized form

$$Z^{-1}=(\gamma\cdot p-m+\delta m)S(p)+i\frac{g^2}{(2\pi)^4}\frac{\lambda^i}{2}\int d^4qS(p)\Gamma_\mu(p,p-q)S(p)\gamma_\nu D^{\mu\nu}(q)\frac{\lambda^i}{2}. \tag{11}$$

Recalling the connection, $Z^{-1}=\int\rho(W)dW$, the spectral form of the equation is

$$\int\frac{\rho(W)dW}{\gamma\cdot p-W}[W-m+\delta m+\Sigma(p,W)]=0, \tag{12}$$

where

$$\Sigma(p,W)=i\frac{g^2}{(2\pi)^4}\frac{\lambda^i}{2}\int d^4q\gamma_\mu\frac{1}{\gamma.(p-q)-W}\gamma_\nu D^{\mu\nu}(q)\frac{\lambda^i}{2} \tag{13}$$

is the self-energy for a quark of mass W due to gluons in first-order perturbation theory.

At this point we carry out the heavy quark expansion and take velocity projections to arrive at

$$\int\frac{\rho(\omega)d\omega}{v\cdot k-\omega}\left[(\omega+\delta m)\begin{pmatrix}1+\frac{(v\cdot k)^2-k^2}{2m(v\cdot k-\omega)} & \frac{\gamma\cdot k}{2m}\\ \frac{\gamma\cdot k}{2m} & \frac{\omega-v\cdot k}{2m}\end{pmatrix}+\begin{pmatrix}\Sigma_{++}(v\cdot k,\omega) & \Sigma_{+-}(v\cdot k,\omega)\\ \Sigma_{-+}(v\cdot k,\omega) & \Sigma_{--}(v\cdot k,\omega)\end{pmatrix}\right]=0, \tag{14}$$

where, after summing over colors (hence the factor of 4/3),

$$\Sigma(v\cdot k,\omega)\Rightarrow i\frac{4g^2/3}{(2\pi)^4}\int\frac{d^4qD^{\mu\nu}(q)}{v\cdot(k-q)-\omega}\begin{pmatrix} v_\mu v_\nu[1+O(1/m)] & v_\mu\gamma_\nu+O(1/m)\\ \frac{\gamma\cdot kv_\mu v_\nu}{2m}+\frac{(\omega-k\cdot v)\gamma_\mu v_\nu}{2m} & \frac{\gamma\cdot kv_\mu\gamma_\nu}{2m}-\frac{(\omega-k\cdot v)\gamma_\mu\gamma_\nu}{2m}\end{pmatrix} \tag{15}$$

and up to order $(1/m)$.

IV. THE SPECTRAL EQUATION

To make any further progress and determine the spectral function ρ and thence the propagator, we need to make some further approximations and/or assumptions about the behavior of the gluon. It is generally accepted that the gluons are massless so that the propagator $D(q)$ is at least as singular as $1/q^2$; it is also known that in the ultraviolet regime this is subject to well-defined logarthmic damping; the behavior for small q^2, where the strong force enslaves color, is more mysterious and there have been suggestions that $D(q)$ could be as singular as $1/q^4$, that it plateaus, or even that one should not be using QCD at all but an effective field theory incorporating chiral symmetry with real mesons. What is certain is the occurrence of a mass scale Λ demarcating the ultraviolet from the infrared regime of D. As we are only interested in soft-gluon effects on the heavy quark lines, we will adopt a gluon propagator which implies masslessness, which cuts off in the ultraviolet and which introduces the fundamental QCD mass scale Λ. This scale, above which hard-gluon processes become important, is not to be regarded purely as an ultraviolet cutoff (that usually lies way above all normal masses), but as an intrinsic scale associated with the running of the non-Abelian coupling constant. For our purposes it is quite enough to use an effective

$$D_{\mu\nu}(q)=\left(-\eta_{\mu\nu}+\xi\frac{q_\mu q_\nu}{q^2}\right)\frac{\Lambda^2}{q^2(\Lambda^2-q^2)} \tag{16}$$

knowing its limitations full well. It incorporates the main things we want and also includes a covariant gauge parameter ξ. If other readers wish to modify D with a more sophisticated and perhaps more realistic expression, they can repeat our calculations below; while that is sure to alter the precise form of our answers, we believe it will not affect the main features of our results in a very significant way.

Returning to the largest component of Eq. (15), we have to consider the spectral equation

$$0=\int \frac{\rho(\omega)d\omega}{v\cdot k-\omega}[\omega+\delta m+\Sigma_{++}(v\cdot k,\omega)], \qquad (17)$$

$$\Sigma_{++}(v\cdot k,\omega)=i\frac{4g^2/3}{(2\pi)^4}\int d^4q\frac{\xi(v\cdot q)^2/q^2-1}{v\cdot(k-q)-\omega}\times\left(\frac{1}{q^2}-\frac{1}{q^2-\Lambda^2}\right). \qquad (18)$$

A straightforward but messy calculation gives

$$\Sigma_{++}(\omega,\omega')=\frac{4\alpha}{3\pi}\Bigg[(\omega-\omega')\left(1+\frac{\xi}{2}-\xi\frac{(\omega-\omega')^2}{\Lambda^2}\right)\times\ln\frac{2(\omega-\omega')}{\Lambda}+\frac{1}{4}\xi(\omega-\omega')+\frac{1}{2}\left(1-\xi\frac{(\omega-\omega')^2}{\Lambda^2}\right)\sqrt{(\omega-\omega')^2-\Lambda^2}\ln\times\frac{\omega-\omega'-\sqrt{(\omega-\omega')^2-\Lambda^2}}{\omega-\omega'+\sqrt{(\omega-\omega')^2-\Lambda^2}}\Bigg] \qquad (19)$$

to leading order and in a general gauge. First we solve the spectral equation (17)

$$0=\int d\omega'\rho(\omega')\Bigg[\frac{\omega'+\delta m}{\omega-\omega'}+\frac{4\alpha}{3\pi}\Bigg(\ln\frac{2(\omega-\omega')}{\Lambda}+\frac{1}{2}\sqrt{(\omega-\omega')^2-\Lambda^2}\ln\frac{\omega-\omega'-\sqrt{(\omega-\omega')^2-\Lambda^2}}{\omega-\omega'+\sqrt{(\omega-\omega')^2-\Lambda^2}}\Bigg)\Bigg] \qquad (20)$$

in the Fermi-Feynman gauge $\xi=0$ to discover what is going on. By taking the imaginary part of Eq. (20), we obtain

$$0=(\omega+\delta m-2\alpha\Lambda/3)\rho(\omega)-(4\alpha/3\pi)\int_\omega^\infty\rho(\omega')d\omega'. \qquad (21)$$

This has the solution

$$\rho(\omega)\propto(\omega+\delta m-2\alpha\Lambda/3)^{-1+4\alpha/3\pi}$$

and, since the self-mass is given by

$$\int (\omega+\delta m)\rho(\omega)d\omega=0,$$

this fixes $\delta m=2\alpha\Lambda/3$. It makes good sense, being governed by the QCD mass scale and gluon coupling. One last matter is the proportionality factor: we must ensure that $\rho(\omega)$ reduces to $\delta(\omega)$ when $\alpha\to 0$. Hence we choose the overall constant so that the result for the spectral function for $\xi=0$ is neat and compact; namely,

$$\rho_{\xi=0}(\omega)=\frac{1}{\omega\Gamma(4\alpha/3\pi)}\left(\frac{\Lambda}{\omega}\right)^{4\alpha/3\pi}. \qquad (22)$$

A bonus of this choice is that the heavy quark propagator simplifies to the elegant nonperturbative expression:

$$S_{\xi=0}(v\cdot k)=\Gamma\left(1-\frac{4\alpha}{3\pi}\right)\frac{1+\gamma\cdot v}{2v\cdot k}\left(-\frac{\Lambda}{v\cdot k}\right)^{4\alpha/3\pi}. \qquad (23)$$

In the limit $\alpha\to 0$ one recovers the free result $1/(v\cdot k)$ for S.

Now we turn to the general gauge ξ. Noting that the ξ-dependent part of $\Sigma_{++}(v\cdot k,\omega)$ vanishes at the threshold $v\cdot k=\omega$, the gauge dependence of ρ arises purely from the imaginary part of Σ_{++}. In this way Eq. (21) gets modified to

$$0=(\omega+\delta m-2\alpha/3\pi)\rho(\omega)-\frac{4\alpha}{3\pi}\int_\omega^\infty d\omega'\rho(\omega')\times\left[1+\frac{\xi}{2}-\xi\frac{(\omega-\omega')^2}{\Lambda^2}\right]. \qquad (24)$$

Once again the self-mass condition requires $\delta m=2\alpha\Lambda/3$, which is satisfyingly *gauge-independent*. The resulting integral equation for the spectral function is a little bit harder to solve now; nevertheless one may establish that it reduces to a generalized hypergeometric function:

$$\rho(\omega)=\frac{1}{\omega\Gamma(-\alpha_\xi)}\left(\frac{\Lambda}{\omega}\right)^{-2\alpha_\xi}{}_0F_2\left(1+\alpha_\xi,3/2+\alpha_\xi;\frac{\alpha\xi\omega^2}{3\pi\Lambda^2}\right), \qquad (25)$$

where $\alpha_\xi\equiv-\alpha(2+\xi)/3\pi$. Thereupon [14] the heavy quark propagator becomes

$$S(v\cdot k)=\Gamma(1+2\alpha_\xi)\frac{1+\gamma\cdot v}{2v\cdot k}\left(-\frac{\Lambda}{v\cdot k}\right)^{-2\alpha_\xi}\times{}_0F_2\left(1+\alpha_\xi,3/2+\alpha_\xi;\frac{\alpha\xi(v\cdot k)^2}{3\pi\Lambda^2}\right)+O(1/m). \qquad (26)$$

This is universal to all the quarks and could be used to estimate the soft-gluon corrections in loops which result from dressing fermion lines and their vertices. Notice that when $\xi=-2$, namely in the Fried-Yennie gauge, the result simplifies to

$$S\to\frac{1+\gamma\cdot v}{2v\cdot k}{}_0F_2\left(1,\frac{3}{2},\frac{2\alpha(v\cdot k)^2}{3\pi\Lambda^2}\right).$$

In any case Eq. (26) agrees with the gauge covariance identities [11] in the infrared regime connected with our particular gluon propagator. However, a word of caution: the result (26) does not take account of gluon self-interactions; those will somehow need to be included separately in heavy quark calculations.

There is one further test of our work. One needs to verify that to order $1/m$ the other, ''small component'' sectors in the propagator velocity projections are correctly determined by Eq. (25) because they are fixed in terms of the leading ρ. We have indeed checked this out: the S_{+-} sector produces precisely the same equation as Eq. (24), while the S_{--} sector is nothing but the self-mass condition, $\int(\omega+\delta m)\rho(\omega)d\omega=0$, which we have already settled [15].

Finally it is worth comparing the answers with the gauge technique solutions for scalar QED say. Those solutions do not have the benefit of an intrinsic cutoff; rather the source mass itself acts as the cutoff and the results read

$$m^2\rho(W)=\frac{(W^2/m^2-1)^{-1-2a_\xi}}{\Gamma(-2a_\xi)}{}_2F_1\times\left(-a_\xi,1-a_\xi;-2a_\xi;1-\frac{W^2}{m^2}\right),$$

$$a_\xi=(\xi+2)\alpha/4\pi,$$

$$m^2S(p)=\Gamma(1+a_\xi)\Gamma(2+a_\xi){}_2F_1(1+a_\xi,2+a_\xi;2;p^2/m^2).$$

We notice a strong similarity with Eqs. (25) and (26), which becomes greater when one substitutes $p=mv+k$ and expands to order $1/m$. The only change is that for QED m takes the place of Λ as the argument of the hypergeometric function. In our case of course, Λ has physical meaning ($\sim$300 MeV) and should not be taken to infinity. However if one were to consider $\Lambda \gg v\cdot k$, one would obtain the behavior $S(v\cdot k)\sim(v\cdot k)^{-1-2\alpha(2+\xi)/3\pi}$. In some ways the Bloch-Nordsieck approximation to QCD—whereby the γ_μ-matrix vertex is simply replaced by v_μ—resembles our approach to this problem since it does away with the matrix algebra too. The Bloch-Nordsieck method has a long history [16] and it has been studied in various contexts [17], especially in QED, where it reproduces the behavior stated above apart from a color factor 4/3 in the exponent. It also is subject to the same criticisms as our analysis, as far as the hard gluons are concerned.

It only remains to obtain the nonleading behavior of S in the various sectors. This entails solving the spectral equation up to order $1/m$ and is where our use of approximations (9) and (16) start to look a bit suspect because they are connected with gluons which carry off appreciable momentum. Taking account of those effects and determining mass dependence of the heavy quark Lagrangian is a nice subject for future research.

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